CHAPTER IV

CONCLUSIONS

The following results are all main theorems of this thesis :

1. Let S be a Γ-semigroup and $e \in E_\gamma(S)$. Then H_e is a subgroup of S_γ.

2. Let S be a Γ-semigroup. Then the following statements are equivalent:

(1) S is completely regular.

(2) Each element of S lies in a subgroup of S_γ for some $\gamma \in \Gamma$.

(3) Every $\mathcal{H}$-class is a subgroup of S_δ for some $\delta \in \Gamma$.

3. Let S be a Γ-semigroup and $a \in S$, $\alpha, \beta \in \Gamma$. Then the following statements hold:

(1) If a is an (α, β)-completely regular element of S, then H_a is a subgroup of S_α and S_β with the same identity and the same inverse of a.

(2) If H_a is a subgroup of S_α and S_β with the same identity, then a is an (α, β)-completely regular element of S.

(3) a is an (α, β)-completely regular element of S if and only if H_a is a subgroup of S_α and S_β with the same identity.

4. Let S be a Γ-semigroup and $a \in S$. Then the following statements are equivalent:

(1) $a \in G_\Gamma(S)$.

(2) a is an (α, β)-completely regular element of S for some $\alpha, \beta \in \Gamma$.

(3) a has a (γ, δ)-inverse x with $a\gamma x = x\delta a$ for some $x \in S$, $\gamma, \delta \in \Gamma$.

(4) $a \in \bigcup_{e \in E_\Gamma(S)} H_e$.

5. Let S be a Γ-semigroup. Then the following two statements hold:

(1) $G_\Gamma(S)$ is the set of all (α, β)-completely regular elements of S for some $\alpha, \beta \in \Gamma$.

(2) $G_\Gamma(S) = \bigcup_{e \in E_\Gamma(S)} H_e$.

6. Let S be a Γ-semigroup, $a,b \in S$ and $\alpha,\beta \in \Gamma$. Then the following statements hold:

(1) $V_L^{(\alpha,\beta)}(a) \cap V_L^{(\alpha,\beta)}(b) = \emptyset$ or $V_L^{(\alpha,\beta)}(a) = V_L^{(\alpha,\beta)}(b)$.

(2) $V_R^{(\alpha,\beta)}(a) \cap V_R^{(\alpha,\beta)}(b) = \emptyset$ or $V_R^{(\alpha,\beta)}(a) = V_R^{(\alpha,\beta)}(b)$.

(3) $V_H^{(\alpha,\beta)}(a) \cap V_H^{(\alpha,\beta)}(b) = \emptyset$ or $V_H^{(\alpha,\beta)}(a) = V_H^{(\alpha,\beta)}(b)$.

7. Let S be a Γ-semigroup and $a \in S$. Then the following statements are equivalent:

(1) a is an (α,β)-completely regular element of S for some $\alpha,\beta \in \Gamma$.

(2) a has an $(\mathcal{H},\gamma,\delta)$-inverse for some $\gamma,\delta \in \Gamma$.

(3) a has an $(\mathcal{L},\zeta,\eta)$-inverse for some $\zeta,\eta \in \Gamma$.

(4) a has an $(\mathcal{R},\theta,\vartheta)$-inverse for some $\theta,\vartheta \in \Gamma$.

8. Let S be a Γ-semigroup, $a \in S$ and $\alpha,\beta \in \Gamma$. If a is an (α,β)-completely regular element of S, then the following statements hold:

(1) $V_L^{(\alpha,\beta)}(a) = \{f\gamma x \mid f \in E_\alpha(L_a), \gamma \in \{\alpha,\beta\}\}$

$= \{q \in S \mid a = a\alpha(q\delta a) = (a\delta a)\alpha q,\ q = (q\delta a)\alpha q = (q\alpha q)\delta a,$ where $\delta \in \{\alpha,\beta\}\},$

(2) $V_R^{(\alpha,\beta)}(a) = \{x\gamma f \mid f \in E_\beta(R_a), \gamma \in \{\alpha,\beta\}\}$

$= \{r \in S \mid a = (a\zeta r)\beta a = r\beta(a\zeta a),\ r = r\beta(a\zeta r) = a\zeta(r\beta r),$ where $\zeta \in \{\alpha,\beta\}\},$

(3) $V_H^{(\alpha,\beta)}(a) = \{x\}$

$= \{s \in S \mid a = a\eta s\eta a,\ s = s\eta a\eta s,\ a\eta s = s\eta a,$ where $\eta \in \{\alpha,\beta\}\},$

where x is both an inverse of a in H_a of S_α and S_β.