CHAPTER I

INTRODUCTION

Let S be a semigroup and e be an idempotent element in S. Then $\mathcal{H}$-class H_e is a subgroup of S [1]. Furthermore, if S is a completely regular semigroup, then every $\mathcal{H}$-class is a group [1]. The $\mathcal{H}$-classes and completely regular elements are important in the study of semigroups which its notations were introduced by J. A. Green [2] and J. M. Howie [1], respectively. The notions of a Γ-semigroup was introduced by M. K. Sen in 1981 [3]. In 1986 [4], M. K. Sen and N. K. Saha changed the definition, which is more general definition. Many authors tried to transfer results of semigroups or ordered semigroups to Γ-semigroups or ordered Γ-semigroups, which many classical notions of semigroups (ordered semigroups) have been extended to Γ-semigroups (ordered Γ-semigroups) (see [5], [6], [7], [8], [9], [10], [11], [12]). In [13], N. K. Saha introduced the definition of Green's relations for a Γ-semigroup, and the notions of inverse Γ-semigroups has been studied by M. K. Sen and N. K. Saha in 1990 [14]. In this thesis, many useful definitions and basic results for reference are obtained from [8]. In [15], Xhillari and Braja introduced the definition of a completely regular Γ-semigroup. The author adapts the definition which is more general definition. The notions of one-sided inverse for semigroups was studied by M. Petrich [16]. It is the main idea for this research, which one can obtain the useful statement as follows: the existence of any kind of these inverses for an element of semigroups satisfying the structure of Green's relations for semigroups and completely regular semigroups.

This research is organized into five chapters as follows. Chapter I is an introduction to the research problems. Chapter II is concerned with some well-known definitions and some useful results that will be used in our main results of this research.

In Chapter III, we organized into three sections as follows. For Section 1, we introduce the definition of a completely regular Γ-semigroup, which is more general definition of [15]. This definition will be used in the main body of the thesis. We consider and study the relationship between $\mathcal{H}$-class and a completely regular element in Γ-semigroup. In Section 2, we introduce the concepts of subgroups of a Γ-semigroup, and we also extend properties of completely regular elements which are related to the structure of Green's relations on semigroups to Γ-semigroups. Section 3, we introduce the principal novelty here is the following set of concepts on structure of Γ-semigroups: For a Γ-semigroup S and $a, b \in S$ and $\alpha, \beta \in \Gamma$, we say that b is $(\mathcal{L}, \alpha, \beta)$-inverse ($(\mathcal{R}, \alpha, \beta)$-inverse) of a if $b \in V_a^\beta(a)$ and $a \mathcal{L} b$ ($a \mathcal{R} b$); if both, we say that b is $(\mathcal{H}, \alpha, \beta)$-inverse of a. We study the existence of any kinds of these inverses of an element a of a Γ-semigroup S which is equivalent to a being a completely regular element. Moreover, we introduce the sets of all such inverses. The conclusions of the research are in Chapter IV.