

ห้องสมุดงานวิจัย สำนักงานคณะกรรมการการวิจัยแห่งชาติ



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**THE ITERATIVE APPROXIMATION METHODS OF
GENERALIZED NONEXPANSIVE MAPPINGS FOR
FIXED POINT AND GENERALIZED
EQUILIBRIUM PROBLEMS**

UTHAI KAMRAKSA

**A Thesis Submitted to the Graduate School of Naresuan University
in Partial Fulfillment of the Requirements
for the Doctor of Philosophy Degree in Mathematics
May 2012**

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This thesis entitled "The iterative approximation methods of generalized nonexpansive mappings for fixed point and generalized equilibrium problems" submitted by Uthai Kamraksa in partial fulfillment of the requirements for the Doctor of Philosophy Degree in Mathematics is hereby approved.

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ABSTRACT

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In this research, we introduce new iterative schemes for finding a common solution of fixed point problems, variational inequality problems and equilibrium problems of some nonlinear mappings and we prove existence theorems of generalized mixed equilibrium problems. We establish the following results. Firstly, we study approximation methods for equilibrium problems, and fixed point problems in Hilbert spaces. Secondly, we study a hybrid method for variational inequality problems and equilibrium problems. Thirdly, we study strong convergence theorems for strict pseudo-contractions in Banach spaces. Finally, we study existence theorems and iterative approximation methods for generalized mixed equilibrium problems and fixed point problems in Banach space.

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