ภาคผนวก ก

การหาค่าสัมประสิทธิ์ f, g, h สำหรับสมการประสิทธิภาพของเครื่องสูบ ในบทที่ 3 มีวิธีการหา ดังนี้

$$fQ_{\max}^3 + gQ_{\max}^2 + hQ_{\max} = 0 \tag{1}$$

$$fQ_P^3 + gQ_P^2 + hQ_P = \eta_{\max} \tag{2}$$

$$3fQ_P^2 + 2gQ_P + h = 0 \tag{3}$$

$(1)\times Q_p^3 \rightarrow$ $$fQ_{\max}^3 Q_p^3 + gQ_{\max}^2 Q_p^3 + hQ_{\max} Q_p^3 = 0 \tag{4}$$

$(2)\times Q_{\max}^3 \rightarrow$ $$fQ_{\max}^3 Q_P^3 + gQ_{\max}^3 Q_P^2 + hQ_{\max}^3 Q_P = \eta_{\max} Q_{\max}^3 \tag{5}$$

$(5)-(4) \rightarrow$ $$\left(Q_{\max}^3 Q_p^2 - Q_{\max}^2 Q_p^3\right)g + \left(Q_{\max}^3 Q_p - Q_{\max} Q_p^3\right)h = \eta_{\max} Q_{\max}^3 \tag{6}$$

$(2)\times 3 \rightarrow$ $$3fQ_P^3 + 3gQ_P^2 + 3hQ_P = 3\eta_{\max} \tag{7}$$

$(3)\times Q_p \rightarrow$ $$3fQ_P^3 + 2gQ_p^2 + hQ_p = 0 \tag{8}$$

$(7)-(8) \rightarrow$ $$gQ_P^2 + 2hQ_P = 3\eta_{\max} \tag{9}$$

$(9)\times\left(Q_{\max}^3 - Q_{\max}^2 Q_p\right) \rightarrow$

$$\left(Q_{\max}^3 Q_p^2 - Q_{\max}^2 Q_p^3\right)g + 2\left(Q_{\max}^3 Q_p - Q_{\max}^2 Q_p^2\right)h = 3\eta_{\max}\left(Q_{\max}^3 - Q_{\max}^2 Q_p\right) \tag{10}$$

$(10)-(6) \rightarrow$ $$\left(Q_{\max}^3 Q_p - 2Q_{\max}^2 Q_p^2 + Q_{\max} Q_p^3\right)h = \eta_{\max}\left(2Q_{\max}^3 - 3Q_{\max}^2 Q_p\right) \tag{11}$$

$$h = \frac{\eta_{\max}\left(2Q_{\max}^3 - 3Q_{\max}^2 Q_p\right)}{Q_{\max}^3 Q_p - 2Q_{\max}^2 Q_p^2 + Q_{\max} Q_p^3} \tag{12}$$

$$= \frac{\eta_{\max} Q_{\max}^2\left(2Q_{\max} - 3Q_p\right)}{Q_{\max} Q_p\left(Q_{\max}^2 - 2Q_{\max} Q_p + Q_p^2\right)} \tag{13}$$

$$h = \frac{\eta_{\max} Q_{\max}\left(2Q_{\max} - 3Q_p\right)}{Q_p\left(Q_{\max} - Q_p\right)^2} \tag{14}$$

แทนค่า h ลงใน (9) จะได้

$$gQ_p^2 + \frac{2\eta_{\max} Q_{\max}\left(2Q_{\max} - 3Q_p\right)}{\left(Q_{\max} - Q_p\right)^2} = 3\eta_{\max} \tag{15}$$

$$gQ_p^2 = 3\eta_{\max} - \frac{2\eta_{\max} Q_{\max}\left(2Q_{\max} - 3Q_p\right)}{\left(Q_{\max} - Q_p\right)^2} \tag{16}$$

$$g = \frac{3\eta_{\max}}{Q_p^2} - \frac{2\eta_{\max} Q_{\max}\left(2Q_{\max} - 3Q_p\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{17}$$

$$= \frac{3\eta_{\max}}{Q_p^2} - \frac{\eta_{\max}\left(4Q_{\max}^2 - 6Q_{\max} Q_p\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{18}$$

$$= \frac{3\eta_{\max}\left(Q_{\max} - Q_p\right)^2 - \eta_{\max}\left(4Q_{\max}^2 - 6Q_{\max} Q_p\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{19}$$

$$= \frac{3\eta_{\max}\left(Q_{\max}^2 - 2Q_{\max} Q_p + Q_p^2\right) - \eta_{\max}\left(4Q_{\max}^2 - 6Q_{\max} Q_p\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{20}$$

$$= \frac{\eta_{\max}\left(3Q_{\max}^2 - 6Q_{\max}Q_p + 3Q_p^2 - 4Q_{\max}^2 + 6Q_{\max}Q_p\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{21}$$

$$g = \frac{\eta_{\max}\left(3Q_p^2 - Q_{\max}^2\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{22}$$

จาก (1) จะได้

$$fQ_{\max}^3 = -gQ_{\max}^2 - hQ_{\max} \tag{23}$$

$$f = -\frac{\left(gQ_{\max} + h\right)}{Q_{\max}^2} \tag{24}$$

$$= -\frac{1}{Q_{\max}^2}\left[\frac{\eta_{\max}Q_{\max}\left(3Q_p^2 - Q_{\max}^2\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} + \frac{\eta_{\max}Q_{\max}\left(2Q_{\max} - 3Q_p\right)}{Q_p\left(Q_{\max} - Q_p\right)^2}\right] \tag{25}$$

$$= \frac{-\eta_{\max}Q_{\max}\left(3Q_p^2 - Q_{\max}^2 + 2Q_{\max}Q_p - 3Q_p^2\right)}{Q_{\max}^2Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{26}$$

$$= \frac{-\eta_{\max}Q_{\max}\left(2Q_{\max}Q_p - Q_{\max}^2\right)}{Q_{\max}^2Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{27}$$

$$= \frac{-\eta_{\max}Q_{\max}^2\left(2Q_p - Q_{\max}\right)}{Q_{\max}^2Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{28}$$

$$f = \frac{\eta_{\max}\left(Q_{\max} - 2Q_p\right)}{Q_p^2\left(Q_{\max} - Q_p\right)^2} \tag{29}$$

การแก้ระบบสมการเชิงเส้นโดยกฎของคราเมอร์ (Carmer's Rule)

$$fQ_{\max}^{3} + gQ_{\max}^{2} + hQ_{\max} = 0 \tag{1}$$

$$fQ_{P}^{3} + gQ_{P}^{2} + hQ_{P} = \eta_{\max} \tag{2}$$

$$3fQ_{P}^{2} + 2gQ_{P} + h = 0 \tag{3}$$

$$\begin{bmatrix} Q_{\max}^{3} & Q_{\max}^{2} & Q_{\max} \\ Q_{p}^{3} & Q_{p}^{2} & Q_{p} \\ 3Q_{p}^{2} & 2Q_{p} & 1 \end{bmatrix} \bullet \begin{bmatrix} f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ \eta_{\max} \\ 0 \end{bmatrix} \tag{4}$$

$$f = \frac{\begin{bmatrix} 0 & Q_{\max}^{2} & Q_{\max} \\ \eta_{\max} & Q_{p}^{2} & Q_{p} \\ 0 & 2Q_{p} & 1 \end{bmatrix}}{\begin{bmatrix} Q_{\max}^{3} & Q_{\max}^{2} & Q_{\max} \\ Q_{p}^{3} & Q_{p}^{2} & Q_{p} \\ 3Q_{p}^{2} & 2Q_{p} & 1 \end{bmatrix}} \tag{5}$$

$$= \frac{2\eta_{\max}Q_{\max}Q_{p} - \eta_{\max}Q_{\max}^{2}}{Q_{\max}^{3}Q_{p}^{2} + 3Q_{\max}^{2}Q_{p}^{3} + 2Q_{\max}Q_{p}^{4} - 3Q_{\max}Q_{p}^{4} - 2Q_{\max}^{3}Q_{p}^{2} - Q_{\max}^{2}Q_{p}^{3}} \tag{6}$$

$$= \frac{\eta_{\max}Q_{\max}\left(2Q_{p} - Q_{\max}\right)}{-Q_{\max}^{3}Q_{p}^{2} + 2Q_{\max}^{2}Q_{p}^{3} - Q_{\max}Q_{p}^{4}} \tag{7}$$

$$= \frac{\eta_{\max}Q_{\max}\left(2Q_{p} - Q_{\max}\right)}{-Q_{\max}Q_{p}^{2}\left(Q_{\max}^{2} - 2Q_{\max}Q_{p} + Q_{p}^{2}\right)} \tag{8}$$

$$= \frac{\eta_{\max}Q_{\max}\left(2Q_{p} - Q_{\max}\right)}{-Q_{\max}Q_{p}^{2}\left(Q_{\max} - Q_{p}\right)^{2}} \tag{9}$$

$$f = \frac{\eta_{\max}\left(Q_{\max} - 2Q_{p}\right)}{Q_{p}^{2}\left(Q_{\max} - Q_{p}\right)^{2}} \tag{10}$$

$$g = \frac{\begin{bmatrix} Q_{\max}^3 & 0 & Q_{\max} \\ Q_p^3 & \eta_{\max} & Q_p \\ 3Q_p^2 & 0 & 1 \end{bmatrix}}{\begin{bmatrix} Q_{\max}^3 & Q_{\max}^2 & Q_{\max} \\ Q_p^3 & Q_p^2 & Q_p \\ 3Q_p^2 & 2Q_p & 1 \end{bmatrix}} \tag{11}$$

$$= \frac{\eta_{\max} Q_{\max}^3 - 3\eta_{\max} Q_{\max} Q_p^2}{-Q_{\max} Q_p^2 \left(Q_{\max} - Q_p\right)^2} \tag{12}$$

$$= \frac{-\eta_{\max} Q_{\max} \left(3Q_p^2 - Q_{\max}^2\right)}{-Q_{\max} Q_p^2 \left(Q_{\max} - Q_p\right)^2} \tag{13}$$

$$g = \frac{\eta_{\max} \left(3Q_p^2 - Q_{\max}^2\right)}{Q_p^2 \left(Q_{\max} - Q_p\right)^2} \tag{14}$$

$$h = \frac{\begin{bmatrix} Q_{\max}^3 & Q_{\max}^2 & 0 \\ Q_p^3 & Q_p^2 & \eta_{\max} \\ 3Q_p^2 & 2Q_p & 0 \end{bmatrix}}{\begin{bmatrix} Q_{\max}^3 & Q_{\max}^2 & Q_{\max} \\ Q_p^3 & Q_p^2 & Q_p \\ 3Q_p^2 & 2Q_p & 1 \end{bmatrix}} \tag{15}$$

$$= \frac{3\eta_{\max} Q_{\max}^2 Q_p^2 - 2\eta_{\max} Q_{\max}^3 Q_p}{-Q_{\max} Q_p^2 \left(Q_{\max} - Q_p\right)^2} \tag{16}$$

$$= \frac{\eta_{\max} Q_{\max}^2 Q_p \left(3Q_p - 2Q_{\max}\right)}{-Q_{\max} Q_p^2 \left(Q_{\max} - Q_p\right)^2} \tag{17}$$

$$= \frac{\eta_{\max} Q_{\max} \left(3Q_p - 2Q_{\max}\right)}{-Q_p \left(Q_{\max} - Q_p\right)^2} \tag{18}$$

$$h = \frac{\eta_{\max} Q_{\max} \left(2Q_{\max} - 3Q_p\right)}{Q_p \left(Q_{\max} - Q_p\right)^2} \tag{19}$$