Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

Maejo International Journal of Science and Technology

ISSN 1905-7873
Available online at www.mijst.mju.ac.th

Full Paper

On a fuzzy approach for the evaluation of golf players

Micael S. Couceiro[1,2,*], **Fernando M. L. Martins**[3,4], **Filipe M. Clemente**[3,5], **Gonçalo Dias**[5] and **Rui Mendes**[3,6]

[1] Artificial Perception for Intelligent Systems and Robotics (AP4ISR), Institute of Systems and Robotics (ISR), University of Coimbra, Polo II, Coimbra, Portugal
[2] Ingeniarius, Lda., Mealhada, Portugal
[3] Polytechnic Institute of Coimbra, Coimbra College of Education, Department of Education, Portugal
[4] Institute of Telecommunications (Instituto de Telecomunicações) (IT), Covilhã, Portugal
[5] Faculty of Sport Sciences and Physical Education (FCDEF/CIDAF), University of Coimbra, Portugal
[6] Interdisciplinary Centre for the Study of Human Performance, Lisbon Technical University, Portugal

* Corresponding author, e-mail: micael@ingeniarius.pt

Received: 9 July 2013 / Accepted: 22 April 2014 / Published: 22 April 2014

Abstract: Athletes' performance may be complex to assess since multiple different metrics may be used to determine the overall performance. This paper proposes a novel fuzzy logic approach to evaluate players' accuracy in the golf putting context. This evaluation methodology merges three input parameters, namely the relative product metrics which are defined by the binary error, the radial error and the argument error. The proposed model herein is suitable to evaluate game context situations involving subjectivity, vagueness and imprecise information. Experimental results show that the evaluation of players' performance might be different than if we only had the aim of the golf game in mind, which involves placing the ball in the hole with as few shots as possible.

Keywords: golf, fuzzy logic approach, performance of golf players, golf putting

---

INTRODUCTION

Sports performance evaluation has been studied along diverse lines and within different disciplinary research frameworks, focusing on the analysis of players' behaviour (e.g. individual analysis) or teams who are part of a given competitive context (e.g. collective analysis) [1]. This has afforded a broad scientific knowledge towards the development of new performance analysis methods [2].

However, the literature shows that sports evaluation is usually carried out in an analytical or standardised manner (e.g. batteries of physical tests or physiological and psychological analysis), or in a quantitative manner, viz. through technical-tactical indicators (e.g. match statistics). Despite their usefulness, these techniques are not enough to describe players' performance as a whole, bearing in mind that they only consider the 'cause-effect' linear actions resulting from athletes' actions or match situations [3]. For instance, one should better understand the meaning of having a golf ball finish before, after or in the vicinity of the hole, and to what extent this is truly meaningful and important for the putting performance. Moreover, it should be noted that the state of the art is scarce around this topic and does not clarify in any way the difference between a golf ball that stays in the ±90° lines or ±180° lines towards the hole. In other words, the literature does not provide any theoretical support for this research question, thus reinforcing the proposal of an alternative evaluation methodology.

It is important to further investigate these aspects that can be decisive in the golf-game outcome. Furthermore, this will demystify the role of the well-known *radial error* in the final evaluation of golf putting, as in many other sports movements, by merging it with other performance evaluation metrics, herein denoted as binary error and argument error. Since a unique, highly reliable and effective performance evaluation metric is crucial for assessing the athlete's potential and overall performance, a novel approach based on fuzzy logic has been introduced [4]. This metric merges all performance metrics relevant to achieve skill mastery and improve the execution of the task [5, 6].

Fuzzy logic was structured in 1965 by Zadeh [7] at the University of California, Berkeley to deal with and represent uncertainties. Fuzzy logic becomes important as our world is not made up of completely true or false facts and contemplates intermediate logical values between 'False' (0) and 'True' (1). This means that a diffuse logical value may be found in values between 0 and 1 [8]. Fuzzy logic has been used in several applications as a multiple-criteria analysis tool. The successful development of a fuzzy model is a complex multi-step process, in which the designer is faced with a large number of alternative implementation strategies and attributes [9]. Fuzzy logic addresses such applications perfectly as it resembles human decision-making, which can generate precise solutions from certain or approximate information.

This paper covers a large set of practical applications within the golf putting and proposes a fuzzified metric that can provide both quantitative and qualitative information. This metric shows that one can devise a 'memory' that objectively provides a trend of players' performance during the execution of a given task. In this case, the player is able to monitor the motor skill progress and correct product errors resulting from the putting performance. Furthermore, the proposed approach is extremely useful for measuring the performance fluctuations and irregularities of players, as well as assessing their individual motor skill characteristics.

Notation

N Number of trials
n_i Binary value of putting *Success*
η_B Binary error metric
ε_i^x Lateral error

ε_i^y	Longitudinal error
$\dot{\varepsilon}_i$	Polar error
ε_i	Radial error module
θ_i	Argument error
μ_R	Radial error arithmetic mean
ε_{max}	Maximum radial error
η_R	Radial error metric
ρ_s	Spearman's rank correlation coefficient
η_θ	Argument error metric
μ_n	Binary error membership function
μ_ε	Radial error membership function
μ_θ	Argument error module membership function
μ_p	Consequent function of the putting performance

PERFORMANCE METRICS

This section presents three metrics to evaluate the golf putting accuracy, namely the binary error, the radial error and the argument error. The literature considers solely and explicitly the radial error as a strict measure of performance [10, 11]. The other metrics, namely the binary error and the argument error, are complementary to the radial error. The formalisation of these metrics aims at providing a comparison of the golf putting performance of various players.

Binary Error Evaluation

One way to evaluate the golf putting accuracy consists in computing the *binary error*. Despite not being denoted as such in the literature, this metric may be formalised based on theoretical assumptions around the disciplines of engineering, notably in the concepts inherent to binary logic [12]. Applying this logic to the golf putting performance, it is possible to evaluate the players' performance by quantifying the number of times that the ball entered the hole over the total number of trials of motor practice. For example, according to the binary logic (e.g. values 0 and 1), if the player succeeded in placing the ball into the hole, 1 would be regarded as *Success*; otherwise, the score value would be 0, i.e. *Failure*. In other words, this metric does not contemplate intermediate values [12].

In this work, we formalise this metric as:

$$\eta_B = \frac{1}{N}\sum_{i=1}^{N} n_i \tag{1}$$

where N is the total number of trials and n_i is the binary value that represents putting *Success*. That is, if the ball enters the hole, n_i = 1; otherwise, n_i = 0. However, considering the metric η_B, if the player scores as many times as he/she fails, the evaluation will be 0.5, thus completely disregarding how much he/she failed. Therefore, this result does not take into account the consistency of the player's performance, as he/she might still obtain a high score even if some of the trials were highly inaccurate.

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

Radial Error Evaluation

In recent studies focusing on the analysis of golf putting [6, 13, 14], the *radial error* has been used to examine the product measures derived from the motor performance of players. Through the analysis of both longitudinal and lateral errors, the studies obtained quantitative measures to evaluate the distance between the final position of the ball and the centre of the hole. When the player is able to place the ball into the hole, the error is considered 0 in both longitudinal and lateral components and consequently, the radial error is also 0.

In this context, a performance metric can be defined as the arithmetic mean of the radial error obtained in each trial:

$$\mu_R = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_i \tag{2}$$

where ε_i is the radial error of trial i [15], which can be obtained from the application of the Pythagorean Theorem (Figure 1) as:

$$\varepsilon_i = \sqrt{(\varepsilon_i^x)^2 + (\varepsilon_i^y)^2} \tag{3}$$

Through the analysis of metric μ_R, we can conclude that the higher its value, the worse the putting accuracy performance becomes. From Figure 1 we can observe that the legs of the triangle are defined by the lateral error ε_i^x and the longitudinal error ε_i^y while the hypotenuse corresponds to the radial error ε_i.

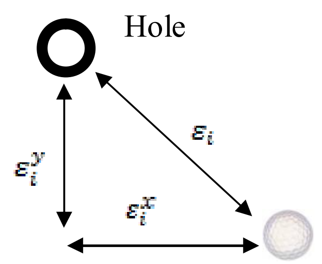

Figure 1. Representation of the three measured errors

However, by representing this metric as an absolute value, the inter-subject comparison is far from straightforward (cf. Results section). One way to overcome this constraint was presented in a previous study [15], which applied a normalised measure based on a maximum radial error ε_{max}, which depends on the evaluative and normative criteria (e.g. handicap, green limits and the player's distance to the hole), being always superior or equal to the radial error ε_i for any trial i of any player, i.e. $\varepsilon_i \leq \varepsilon_{max} \; \forall \, i$. Thus, the relative metric η_R is obtained through the expression:

$$\eta_R = \frac{1}{N}\sum_{i=1}^{N}\left(1 - \frac{\varepsilon_i}{\varepsilon_{max}}\right) \tag{4}$$

Contrary to the previous metric from equation (1), η_R provides an 'analogical' evaluation of the putting accuracy, i.e. being not solely represented by the *Success* or *Failure* of this movement. It should be noted that a trial that promotes the placement of the ball inside the hole tends to be considered only slightly better than another which results in a ball close to it. This evaluation metric does not take into account all the 'dynamics' of the putting performance since the radial error may lead to an erroneous evaluation, thus concealing the obtained results. Unlike other gestures (e.g. javelin throw), in golf putting the lateral error ε_i^x may not carry the same 'weight' as the longitudinal error ε_i^y in terms of putting performance.

Argument Error Evaluation

The concept adopted in this section dates back to the pioneering studies conducted by Isaac Newton, which paved the way to research around polar coordinates as we know them today [16]. From an operational point of view, a semi-straight line starting at the origin and any other point in the Cartesian plane (x, y) may be represented in the polar plane as a module (distance) and an argument (angle) [16]. In order to evaluate the putting accuracy performance, we herein propose a new evaluation metric that considers the radial error as the module (absolute value) of an error (denoted as polar error) represented in the polar coordinate system as:

$$\dot{\varepsilon}_i = \varepsilon_i \angle \theta_i \qquad (5)$$

where the argument θ_i is obtained through the conversion of the Cartesian coordinate system $(\varepsilon_i^x, \varepsilon_i^y)$ into the polar coordinate system $(\varepsilon_i, \theta_i)$, θ_i being obtained with the arc tangent variation *atan2* function.

Through the conversion from Cartesian coordinates to polar coordinates, one can obtain the quadrant where the ball is located at the end of each trial of motor practice, as well as the circular positions around the origin (hole), as shown in Figure 2. It should be noted that the y-axis is aligned with the line defined by the exit point of the ball (i.e. where the moment of the putter's impact with the ball takes place) and the centre of the hole. On the other hand, the x-axis is perpendicular to the y-axis and aligned with the centre of the hole.

Considering the example from Figure 2, one can observe that balls 1 and 2 are of the same distance from the hole, i.e. $\varepsilon_1 = \varepsilon_2$. However, ball 2 is located in the first quadrant, closer to the 0º line with Θ_2 = 10°, while ball 1 is located in the second quadrant, closer to the 90º line with Θ_1 = 100°. The question is which of the two situations represents a better performance in terms of putting accuracy.

The metrics defined in equations (2) and (4) represent the same accuracy for the example depicted in Figure 2: in neither of the two cases did the balls enter the hole and both are at the same distance from it. Nevertheless, although not explicitly discussed in the literature [9, 10, 17], a player's performance tends to be considered 'worse' when the ball finishes closer to the line that separates the first quadrant from the fourth and the second quadrant from the third, i.e. the x-axis line. Put it differently, it is preferable to obtain angles closer to 90º rather than angles closer to ±180º.

As previously stated, the lateral error ε_i^x proves to be more 'critical' than the positive longitudinal error ε_i^y. This means that combining the analysis of both the radial error ε_i and the argument of radial error θ_i, it is possible to determine whether the accuracy of a given trial is better than the other. Given the same example from Figure 2, we can conclude that the putting accuracy represented by ball 1 is higher than that represented by ball 2. It may also be observed that the putting accuracy represented by ball 3 is lowest, even taking into account that $\varepsilon_3 < \varepsilon_1$, as this ball is placed in the fourth quadrant with Θ_3 = 280°.

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

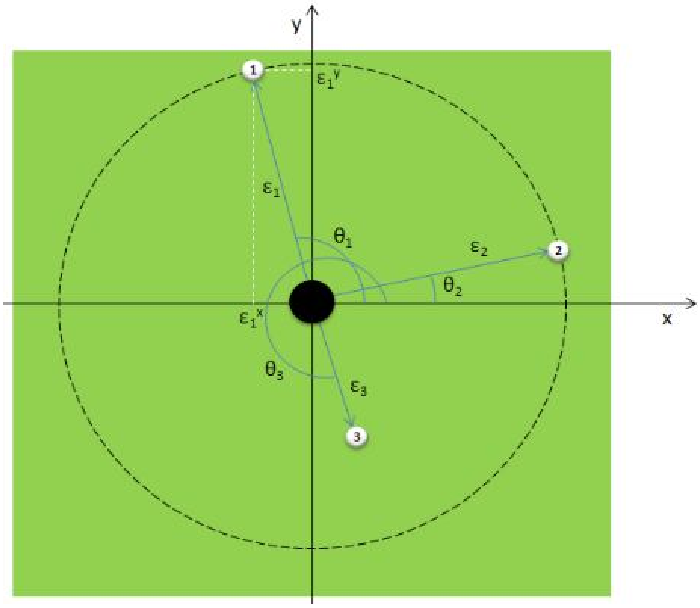

Figure 2. Graphic representation of Cartesian and polar coordinate systems around the centre of the hole

In order to compare the accuracy performance using the argument error, as before, a novel relative metric is proposed:

$$\eta_\theta = \frac{1}{N}\sum_{i=1}^{N}\left(1 - \frac{||\theta_i| - 90|}{90}\right) \tag{6}$$

To further improve the feasibility of this proposed approach, a Spearman's rank correlation test [18] was carried out between 10 golf players' handicap and their argument error, η_θ. This study consisted of 30 trials performed by each player at 4 metres from the hole without any constraints. A Spearman's rank correlation coefficient of ρ_s = -0.4893 was obtained, thus allowing the observation of a decreasing monotonic trend between players' handicap and their accuracy performance through the angular position of the balls. In other words, one can consider that as handicap decreases, putters finish closer to the 90º line.

Nevertheless, the fact that there are multiple evaluation metrics to determine a player's performance increases the complexity of the selection process. Due to the 'dynamics' of the putting performance, it may not be sufficient to consider each evaluation metric independently. It is therefore extremely important to find a way to evaluate a player's performance and simultaneously ponder the binary metric (if the ball enters the hole or not), the radial error (if it does not enter the hole, how far is it), and the argument error (if the ball does not enter the hole, what is its angular location?). Consequently, it is based on a fuzzy approach, introduced in the next section, that we will evaluate the overall performance accuracy associated with the golf putting.

FUZZY APPROACH

In the specific case of this work, it is possible through fuzzy logic to transform quantitative variables into qualitative ones by describing not only the 'total' error obtained by the player, but also the extent of his/her failure in the same trial. In order to do so, this research considers three inputs for the diffuse system, which are defined by the three measures previously introduced, namely binary error, radial error and argument error.

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

The membership function of the binary error is represented by a unitary crisp membership function as shown in equation (7) and Figure 3. In other words, the value 0 is ascribed for an unsuccessful shot (*Failure*) and for a successful one (*Success*) the value 1 is ascribed. It should be noted that this method has been used in the analysis of dynamic systems, namely artificial intelligence [9].

$$\mu_n(n_i) = \begin{cases} 0 & , n_i = 0 \\ 1 & , n_i = 1 \end{cases} \tag{7}$$

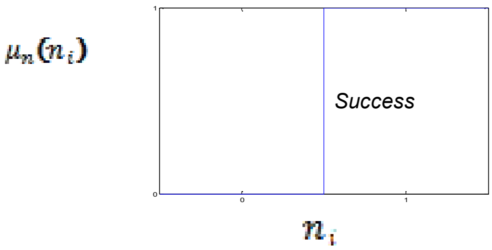

Figure 3. Binary error membership function

The membership function of the radial error may be represented as a special case of a triangular membership function (Figure 4). The smaller the radial error is, the closer the ball will be to the hole. This function may be represented as shown in equation (8):

$$\mu_\varepsilon(\varepsilon_i) = \begin{cases} \dfrac{\beta - \varepsilon_i}{\beta} & , \varepsilon_i \le \beta \\ 0 & , \varepsilon_i > \beta \end{cases} \tag{8}$$

where parameter β is equal to ε_{max} (cf. equation (4)), which, as previously stated, may be related with the green's size or the highest radial error recorded from all trials.

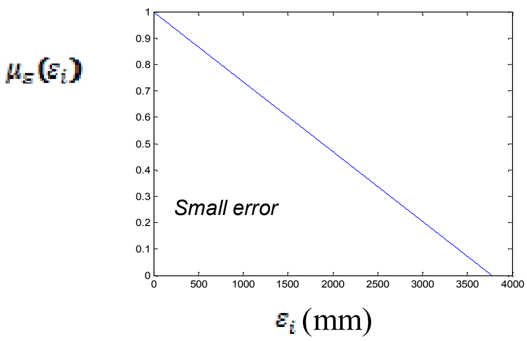

Figure 4. Radial error membership function

The membership function of the argument error is defined by a generalised bell-shaped function as shown in Figure 5. This function has one more parameter than the Gaussian function typically used:

$$\mu_\theta(|\theta_i|) = \frac{1}{\left[1 + \frac{(|\theta_i| - c)}{a}\right]^{2b}} \tag{9}$$

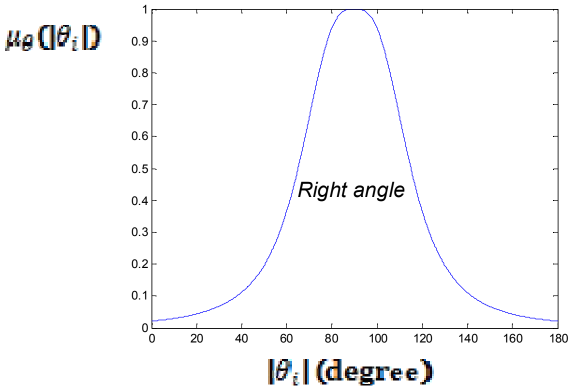

Figure 5. Argument error module membership function

We decided to use the absolute value of the argument error, $|\theta_i|$, as input, considering negative angles (third and fourth quadrants) as having the same evaluation as positive angles (first and second quadrants). Parameters a , b and c are defined considering that a ball situated on the |90|º line (right angle) has the maximum performance value as to the argument error, whereas a ball closer to the 0º or 180º line (i.e. between the first and fourth quadrants and between the second and third quadrants respectively) represents a shot with a lower performance as to the argument error. Using Matlab's Fuzzy Logic Toolbox [1], equation (9) is parameterised with a = 25, b = 1.5, c = 90.

For defuzzification, we considered Mamdani's implication [7] with the lowest (first) of the maxima. Basically, two diffuse IF-THEN rules allow the classification of a given putting trial in terms of overall performance p_i as:

IF n_i is *Success* **THEN** p_i is *Accurate with weight 1*

ELSE-IF ε_i is *Small* $\dot{AND}$ $|\theta_i|$ is *Right* **THEN** p_i is *Accurate with weight 0.8*

The first rule is the most relevant to classifying the putting success as it represents the performance metric of the golf player, contemplating whether or not the ball enters the hole. If it does, n_i= 1; then all other variables are irrelevant and the shot will be classified as *Accurate*, i.e. p_i= 1. On the other hand, if the ball does not enter the hole, n_i= 0, and the radial error needs to be pondered. It was decided that the weight of this second rule should be 0.8: the moment the player does not hit the hole, the putting is considered to have, at best, an accuracy of 80%, i.e. p_i= 0.8.

As to the radial error, we know that the higher it is, the worse the player's performance becomes. However, this relation can only be considered linear if the argument error remains constant. If the argument error module comes close to the limits of ±180º, the putting will be considered to have a lower performance and consequently a lower accuracy. The question here is: *How accurate will the putting be when the radial error argument varies*?

This relation cannot be considered linear because of the features inherent to the putting execution. This means that the ***AND*** connective cannot be considered as usual, viz. only by considering the minimum or the product between $\mu_\varepsilon(\varepsilon_i)$ and $\mu_\theta(|\theta_i|)$. A new $\dot{AND}$ connective is then proposed to relate both membership functions while maintaining the relation previously addressed:

$$\dot{AND} = \mu_\varepsilon(\varepsilon_i) \times \mu_\theta(|\theta_i|)^{1-\mu_\varepsilon(\varepsilon_i)} \tag{10}$$

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

As a result, the lower the radial error is, i.e. $\mu_\varepsilon(\varepsilon_i) \approx 1$, the lower the influence of the argument error module results, and vice-versa.

Finally, the consequent function is defined as follows:

$$\mu_p(p_i) = \begin{cases} p_i, & \varepsilon_i \geq 0 \\ 0, & \varepsilon_i < 0 \end{cases} \quad (11)$$

where $\mu_p(p_i)$ is the overall accuracy performance evaluation of trial *i* (Figure 6).

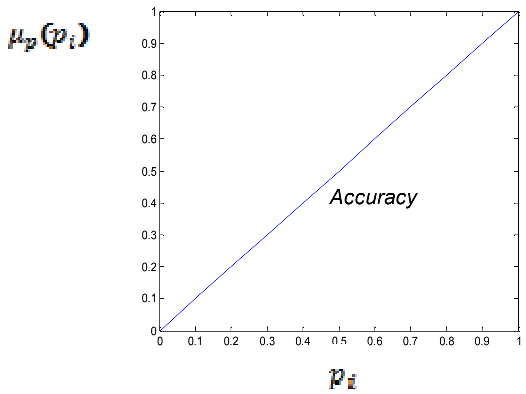

Figure 6. Consequent function of the putting performance

In order to understand the metric of the evaluation proposed, we shall consider the following example. Two golf players executed two trials each. Player 1 was able to hit the ball in the first trial: $\varepsilon_1^1 = 0$, but obtained the highest radial error of all the trials in the second shot, with an argument error of 0º: $\dot{\varepsilon}_2^1 = \varepsilon_{max}\angle 0°$. Player 2 failed both trials with a radial error 10 times lower than ε_{max}, having obtained an argument error of 90º in the first trial and 0º in the second trial: $\dot{\varepsilon}_1^2 = \frac{\varepsilon_{max}}{10}\angle 90°$ and $\dot{\varepsilon}_2^2 = \frac{\varepsilon_{max}}{10}\angle 0°$.

In this situation the scenario would be the following. As regards the binary error, player 1 would have a higher performance than that of player 2, as he/she was able to place the first ball into the hole, viz. $\eta_B^1 = 0.5 > \eta_B^2 = 0$. As regards the radial error, player 2 would be the best, with $\eta_R^2 = 0.9 > \eta_R^1 = 0.5$. Both players would present the same performance as to the argument error, with $\eta_\theta^1 = \eta_\theta^2 = 0.5$.

Given the current context, by benefiting from the fuzzy evaluation proposed in this section, we obtain $\mu_p^1 = 0.5 > \mu_p^2 = 0.36$. This means that player 1 had a 14% higher performance than player 2.

The following section compares the performance of several expert players by benefiting from the diverse metrics presented in this work.

EXPERIMENTAL RESULTS

The performance metrics previously presented were evaluated considering 10 male golfers who were adults (aged 33.8 ± 11.9), volunteers, right-handed and experts (10.8 ± 5.4 handicap). An artificial plain green carpet 10 m long, 2 m wide and 4 mm thick [15] used by Minigolf professionals, rectangular with no flaws, quite similar to the green's natural surface texture was used.

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

A real golf hole was placed at 3.5 m from the carpet ending and 1 m from each lateral extremity. Three black dots marked the putting and were placed at 2 m (*D1*), 3 m (*D2*) and 4 m (*D3*). The dots were in the same direction as the hole at 1 m from each lateral extremity of the carpet. A ramp 1 m long was placed under the carpet, levelling up the carpet surface to a height of 10 cm. A straight platform 4 m long was placed immediately after the ramp to keep that same height.

The ball's trajectory was tracked using a digital camera placed on a tripod 1.55 m high with an inclination of 22º pointing down. The camera was shot at 30 frames/sec. with a resolution of 1280×720 pixels and a focal length of 26 mm. In order to assist in the data analysis and convert the virtual into real coordinates, 13 reference points were marked on the carpet.

Three studies were designed. In the first study (*E1*), 30 trials were performed at 1, 2, 3 and 4 m away from the hole without any constraint (without ramp). In the second study (*E2*), 30 trials were performed at 2, 3 and 4 m away from the hole, with a constraint imposed by the ramp. In the third and last study (*E3*), 30 trials were performed at 2 m away from the hole with a constraint imposed by the ramp and an angle of 25º to the left and right of the hole.

Binary Evaluation

Figure 7 shows the players' performance throughout the 3 experimental studies by only considering the binary error metric. Player 1 shows the best performance. He obtained a success rate of 83%, succeeding in placing 83% of the balls into the hole. Player 1 is closely followed by players 4 and 9, while player 2 presents the worst performance.

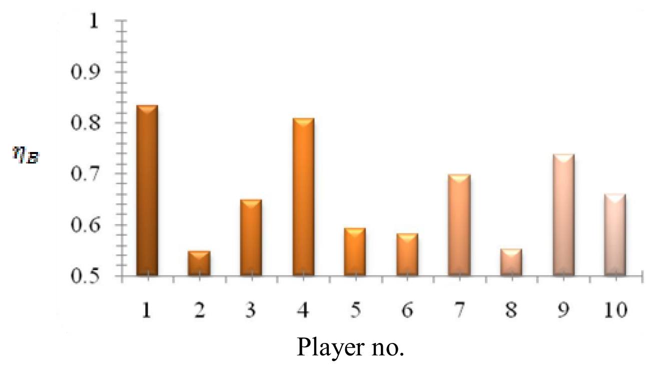

Figure 7. Binary evaluation of the players in 3 experimental studies

Radial Error Evaluation

Taking into account the radial error, Figure 8 shows the players' performance throughout the 3 experimental studies. In this study a maximum threshold value is found for player 8: $\beta = \varepsilon_{max}$ = 3778 *mm*. The lowest radial error is found for player 4, who comes closer to 1. Players 9 and 1 closely follow player 4. Finally, player 8 clearly shows the worst performance.

Argument Error Evaluation

Figure 9 shows the players' performance throughout the 3 experimental studies by only considering the argument error. One may observe that player 1 shows the best performance, being closer to 1. Players 7 and 8 follow, with player 2 having the worst performance.

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

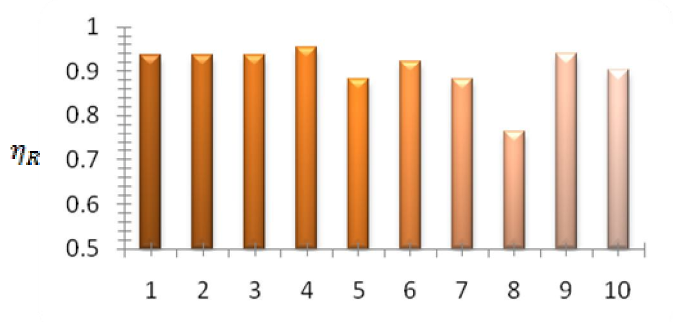

Figure 8. Radial error evaluation of the players in 3 experimental studies

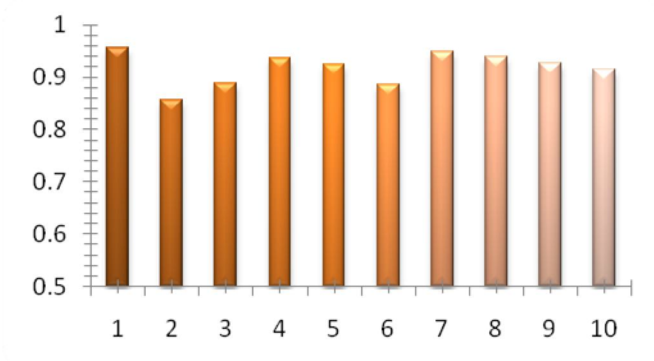

Figure 9. Argument error evaluation of the players in 3 experimental studies

Fuzzy Evaluation

To evaluate players using fuzzy logic, we benefited from the Fuzzy Inference System Editor of the Fuzzy Logic Toolbox [1]. Complementing the information previously presented, Figure 10 and Table 1 depict the players' performance throughout the 3 experimental studies relying on diffuse information**.**

In the first study player 1 achieves the best performance, followed by players 4 and 9. Player 8 shows the worst performance. In the second study player 3 achieves the best performance, followed by players 10 and 4. The data also show that faced with a constraint (ramp/slope), the accuracy performance of player 1 tends to decrease considerably when compared with the performance of player 4. Player 8 shows the worst performance. In the last study player 5 shows the best performance, placing about 75% of the balls in the hole. Players 6 and 7 follow player 5. Again, player 8 shows the worst performance. Player 4 obtains the best performance in the 3 studies (Figure 10), followed by players 1 and 9. Finally, player 8 shows the worst performance (TOTAL row from Table 1).

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

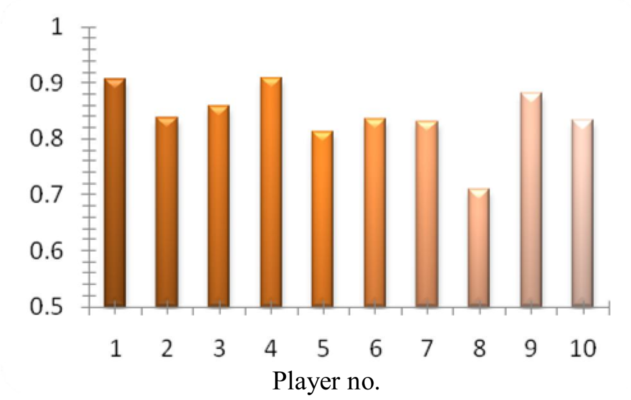

Figure 10. Fuzzy evaluation of players in 3 experimental studies

Table 1. Evaluation of fuzzy performance obtained by players throughout 3 experimental studies

Study	*S01*	*S02*	*S03*	*S04*	*S05*	*S06*	*S07*	*S08*	*S09*	*S10*
E1_1m	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9787	1.0000
E1_2m	0.9847	0.8947	0.9517	0.9757	0.8590	0.8290	0.8807	0.7917	0.9640	0.8923
E1_3m	1.0000	0.7850	0.7700	0.9860	0.7260	0.7703	0.8510	0.7657	0.9530	0.8633
E1_4m	1.0000	0.7407	0.8050	0.9367	0.8047	0.7340	0.8503	0.4550	10.000	0.6267
E1	0.9962	0.8551	0.8817	0.9746	0.8474	0.8333	0.8955	0.7531	0.9739	0.8456
E2_2m	0.9413	0.9320	0.9610	0.9647	0.8400	0.9507	0.9277	0.8820	0.9493	0.9477
E2_3m	0.9273	0.8820	0.9847	0.9777	0.8323	0.8757	0.8270	0.6777	0.8587	0.9713
E2_4m	0.8657	0.8380	0.9033	0.8953	0.7133	0.8320	0.6153	0.5763	0.8307	0.9327
E2	0.9114	0.8840	0.9497	0.9459	0.7952	0.8861	0.7900	0.7120	0.8796	0.9506
E3_ang1	0.8237	0.7523	0.7843	0.6923	0.7763	0.7407	0.7710	0.6747	0.7447	0.6823
E3_ang2	0.6157	0.7027	0.5533	0.7553	0.7590	0.7827	0.7490	0.5550	0.6437	0.5753
E3	0.7197	0.7275	0.6688	0.7238	0.7677	0.7617	0.7600	0.6148	0.6942	0.6288
TOTAL	**0.9065**	**0.8364**	**0.8570**	**0.9093**	**0.8123**	**0.8350**	**0.8302**	**0.7087**	**0.8803**	**0.8324**

Note: *E1_1m* = study 1: 1 metre; *E1_2m* = study 1: 2 metres; *E1_3m* = study 1: 3 metres; *E1_4m* = study 1: 4 metres; *E2_2m* = study 2: 2 metres; *E2_3m* = study 2: 3 metres; *E2_4m* = study 2: 4 metres; *E3_ang1* = study 3 (angle 1 – left); *E3_ang2* = study 3 (angle 2 - right); *S01* = player 1; *S02* = player 2, etc.

DISCUSSION AND CONCLUSIONS

Considering the previously presented results, it is important to retrieve as much information as possible about the putting execution so as to understand how this methods can be applied to the sports training context. If we only consider the aim of the golf game in mind, which involves placing the ball in the hole with as few shots as possible, player 1 would be considered as the best performer (cf. binary error) followed by players 4 and 9. However, if one considers the performance evaluation as regards a fuzzy logic accuracy, viz. the simultaneous analysis of the binary error, the radial error and the argument error, player 4 is the one presenting the best performance, followed by players 1 and 9.

The use of the proposed metric is suitable in evaluating situations that involve subjectivity, vagueness and imprecise information. However, the success of a fuzzy-based engine relies on the experience of selecting an adequate membership function. Therefore, an expert knowledge about the task is required in order to validate the proposed rules and membership functions. In other words, fuzzy systems need expert experience to strengthen the decision rules and to handle imprecise value in its reasoning.

This approach brings implications to the area of sports training since it aims at providing a deeper understanding of players' flaws [5]. The approach is truly important mainly in a coaching perspective to avoid overusing standard metrics that lack relevant information about a given gesture [19]. For instance, although player 1 was the best performing player most of the time, his overall performance significantly dropped when both ramp and slope constraints were added. In that sense, this multidisciplinary approach provides for the understanding of the golf putting and the acquisition of the necessary information during training and competition. Meanwhile, fuzzy logic has practical applications in other individual and team sports (e.g. tennis, football and basketball) that can benefit from this type of fuzzified metric with both quantitative and qualitative information, being mainly useful in following the performance trend of athletes' motor behaviour. Such techniques are equally effective in assessing how the athlete can stabilise his/her performance by exploring different levels of variability and complexity. Moreover, this approach is extremely useful for measuring the performance fluctuations and irregularities of both novices and experts, as well as for assessing their individual motor skill characteristics and profiles [17].

Operationally, this study introduces new evaluation metrics that are relevant in sports so as to measure the performance of athletes in laboratory and real situations for both teaching and learning. Specifically in the golf putting context, these metrics show that it is possible to devise a 'memory' that objectively provides a trend of players' performance during the execution of the task. In this case the player is able to monitor his/her motor progress and correct errors resulting from the putting performance. Moreover, the metrics also allow quantifying the result of the action and the direction of the error in the context of training and competition [17].

However, in order to consolidate the conclusions obtained in this work, it might be necessary to extend this type of metrics to other sports. In this case, and given the complexity of such extension, an interdisciplinary approach covering several areas of knowledge such as sport sciences, mathematics and engineering is proposed. The scientific contributions emerging from this interdisciplinary work can help to further understand the 'mechanics' connecting fuzzy logic to sports.

ACKNOWLEDGEMENTS

This work was supported by a PhD scholarship (SFRH/BD /73382/2010) granted to the first author by the Portuguese Foundation for Science and Technology (FCT) and the Institute of Systems and Robotics (ISR). The authors also thank the Coimbra College of Education (ESEC) and the Instituto de Telecomunicações (IT) for providing the facilities and equipment for this study. This work was also supported by the Instituto de Telecomunicações, granted by the Portuguese Foundation for Science and Technology (FCT) with the ref. PEst-OE/EEI/LA0008/2013.

REFERENCES

1. T. McGarry, D. I. Anderson, S. A. Wallace, M. D. Hughes and I. M. Franks, "Sport competition as a dynamical self-organizing system", J. *Sports Sci.*, **2002**, *20*, 771-781.
2. P. O'Donoghue, "Time-motion analysis", in "The Essentials of Performance Analysis: An Introduction" (Ed. M. Hughes and I. Franks), Routledge, London, **2008**, pp.180-205.

Maejo Int. J. Sci. Technol. **2014**, *8*(01), 86-99

3. M. Pidwirny, “Definitions of systems and models: fundamentals of physical geography”, **2011**, http://www.physicalgeography.net/fundamentals/4b.html (Accessed: July 2011).
4. M. Othman, K. R. Ku-Mahamud and A. A. Bakar, “Fuzzy evaluation method using fuzzy rule approach in multicriteria analysis”, *Yugosl. J. Oper. Res.*, **2008**, *18*, 95-107.
5. B. Kolev, P. Chountas, I. Petrounias and V. Kodogiannis, “An application of intuitionistic fuzzy relational databases in football match result predictions”, *Adv. Soft Comput.*, **2005**, *2*, 281-289.
6. E. R. Komi, J. R. Roberts and S. J. Rothberg, “Measurement and analysis of grip force during a golf shot”, *J. Sports Eng. Technol.*, **2008**, *222*, 23-35.
7. L.A. Zadeh, “Fuzzy sets”, *Inform. Control*, **1965**, *8*, 338-353.
8. E. Cox, “The Fuzzy Systems Handbook: A Practitioner's Guide to Building, Using and Maintaining Fuzzy Systems”, Academic Press, Boston, **1994**, pp.35-38.
9. W. Pedrycz and F. Gomide, “Fuzzy Systems Engineering: Toward Human-Centric Computing”, Wiley-IEEE Press, **2007**, pp.67-99.
10. D. Pelz, “Putting Bible: The Complete Guide to Mastering the Green”, Doubleday, New York, **2000**.
11. S. J. Mackenzie and D. B. Evans, “Validity and reliability of a new method for measuring putting stroke kinematics using the TOMI1 system”, *J. Sports Sci.*, **2010**, *8*, 1-9.
12. J. M. Garibaldi and E. C. Ifeachor, “Application of simulated annealing fuzzy model tuning to umbilical cord acid-base interpretation”, *IEEE Trans. Fuzzy Syst.*, **1999**, *7*, 72-84.
13. N. C. A. da Costa, L. J. Henschen, J. J. Lu and V. S. Subrahmanian, “Automatic theorem proving in paraconsistent logics: Theory and implementation”, Proceedings of 10th International Conference on Automated Deduction, New York, USA, **1990**, pp.72-86.
14. R. Mendes, R. Martins and G. Dias, “Effects of a contextual interference continuum on golf putting task”, in “Book of Abstracts do XIII Annual Congress of the European College of Sport Science” (Ed. J. Cabri, F. Alves, D. Araújo, J. Barreiros, J. Diniz and A. Veloso), Faculty of Human Kinetics (University of Lisbon), Lisbon, **2008**, pp.325-326.
15. R. Mendes, G. Dias and S. Chiviacowsky, “Contextual interference continuum on golf putting task”, *Braz. J. Motor Behav.*, **2010**, *5*, 21-22.
16. J. Stewart, “Calculus: Early Transcendental”, McGraw Hill, New York, **2007**, pp.635-688.
17. M. S. Couceiro, G. Dias, F. M. L. Martins and J. M. A. Luz, “Towards a correction metric to the putting lip-out”, Proceedings Symposium on Fractional Signals and Systems, **2011**, Coimbra, Portugal, Vol. 1, pp.138-146.
18. J. Pallant, “SPSS Survival Manual: A Step by Step Guide to Data Analysis Using the SPSS Program”, Allen and Unwin, Crows Nest (Australia), **2011**, pp.102-120.
19. G. Dias and R. Mendes, “Effects of a contextual interference continuum on golf putting task”, *Rev. Bras. Educ. Fís. Esporte*, **2010**, *24*, 545-553.