

## Allocation of reactive power service cost to distributed generators

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### **Abstract:**

*A problem of excess voltage will arise frequently as distributed generation, particularly photovoltaic and wind power generation, increases in society. One of the effective measures to cope with this problem is to supply an appropriate amount of reactive power from generators. In doing so, an appropriate cost sharing rule needs to be established. To this end, it is important to determine what amount of negative effect a unit of real power output by a distributed generator has on the voltage stability. The purpose of the study is to propose a method for measuring this effect. The proposed method is an application of the Aumann-Shapley rule, which is one of cost sharing rules among multiple goods. The method computes an amount of reactive power required for a unit of real power output by a distributed generator, which enables a system operator not only to cover the total amount of reactive power needed but also to fairly estimate the negative effect of real power output. The method may be useful for designing a different kind of feed-in tariff system that embodies the negative effect of distributed generation on an electric power system.*

**Keywords:** Distributed generation; Photovoltaic system; Reactive power; Feed-in tariff; Aumann-Shapley rule

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### **1. Introduction**

As distributed generation increases, a problem of excess voltage in an electric power system will arise more frequently. If a large amount of real power is fed from distributed generators into the grid, probable reverse power flow may cause excess voltage in parts of the grid. This problem is more serious if more photovoltaic (PV) and wind power generators are grid-connected since such generation is significantly affected by the weather so that a large amount of power is unexpectedly supplied into the grid at a time. The excess voltage must be avoided from an electrical engineering point of view.

One of the effective measures to address the problem is to supply reactive power adequately. To arrange to secure a proper amount of reactive power from generators, a system operator incurs some operational costs, whereas generators' costs of supplying reactive power can be ignored as long as their real power output is not reduced in doing so. On the other hand, it makes sense to consider that owing to the reactive power support, part of the real power can be fed into the grid. Then the resultant costs should be passed on to the real power suppliers depending on the amount of reactive power to which they owe their real power output.

The purpose of the study is to propose a method to measure the contribution of reactive power to the real power a distributed generator supplies through PV and wind power generation. In other words, it computes what amount of reactive power a unit of real power supplied by the generator requires when the total amount of real power is maximized in the system.

### **2. Material and methods**

A maximized amount of real power is successfully fed into the grid by virtue of the supply of some reactive power. However, it is not straightforward to link a unit of real power a generator supplies with an amount of reactive power required; the reactive power supplied affects not the voltage at a single bus but the voltage configuration in the power system in a complex way. To this end, a method is proposed that is an application of the Aumann-Shapley (A-S) rule (Aumann and Shapley, 1974).

Numerous studies have addressed the above problem (Choi et al., 1998; Lin et al., 2006a; Ueki et al., 2008; Koide et al., 2012). These generally computed reactive power costs to a distributed real power supplier, positing a cost function of real power production, by means of an optimization technique with constraints such as real and reactive power flow equations, power output limits, and voltage limits. These studies were based on the “final” real and reactive power output and did not take account of the marginal effect of real power output on the reactive power required.

As an alternative, the A-S rule, one of cost sharing rules among multiple goods, may be applicable. A classic application of this method is by Billera et al. (1978) to internal telephone billing rates. A monthly collection of phone calls made in a university was broken down into types that formed the basis of the monthly billings such as the time of day, destination, and type of day (weekday or weekend). Then a price per minute of call is to be set for each type, but this is not a straightforward task since the marginal cost of call generally changes as total minutes of calls are accumulating in the month.

Then the A-S rule was applied to the task in this situation. Denote by  $f(x_1, \dots, x_n)$  the minimal cost of the monthly calls  $X = (x_1, \dots, x_n)$ , where  $x_i$  represents the total monthly minutes of calls of type  $i$  made by different departments of the university. Then the price charged for calls of type  $i$  is established as  $\int_0^{x_i} f_i(tX) dt$  by the A-S rule so that the billings to type  $i$  are  $x_i \int_0^1 f_i(tX) dt$  as a whole. This pricing rule embodies not only efficiency in that total billings cover the total costs but also fairness in that all calls of a type are billed at the same rate.

Lin et al. (2005) and Lin et al. (2006b) both applied the A-S rule considering each distributed generator as a type to which an individual price is attached since the effect on voltage stability varies among the generators. Those models posited a cost function of real power production together with marginal cost bidding for real power supply (Lin et al., 2005) or a certain profit rate to the production costs (Lin et al., 2006b), defined reactive power costs as the opportunity costs, that is, the profit a generator would have obtained if it had not supplied the reactive power, and allocated the costs according to the A-S rule.

In contrast, the present study computes, rather than reactive power costs, an amount of reactive power requirements associated with the real power supplied by a distributed generator. There are two reasons for this modeling: First, marginal cost of PV and wind power generation is considered negligible. Second, it might be unrealistic, at least as a simple scheme, that a generator be forced to increase reactive power supply with reducing the real power output in order to make another generator's real power output increase.

The A-S rule, as is well-known, can be seen as an extension of average cost sharing and is continuous generalization of the Shapley value, which is, loosely speaking, an average of an individual's marginal contribution to coalitional payoffs when participating in a coalition in order. Hence, the proposed method could “properly” estimate the negative effect of a distributed generator's real power output on the voltage stability.

### 3. Results and discussion

Suppose that at a point in time, distributed generators have some real power to be fed into the grid but may not have all done so successfully because of the voltage constraint of the system. Then a system operator tries to maximize the sum of real power output that is actually fed into the grid by arranging a least amount of reactive power to be supplied from appropriate generators. As a result, a certain amount of real power  $P = (p_1, \dots, p_n)$ , where  $p_i$  represents a distributed generator  $i$ 's output

of real power, can be supplied together with the least amount of reactive power support in the system as a whole.

Accordingly, each distributed generator can feed the prescribed amount of real power into the grid with the help of the reactive power supply. To compute what amount of reactive power contributes to a unit of real power supplied by a distributed generator, the A-S rule is applied in three steps following Billera et al. (1978). First, identify  $t_0 \in [0, 1]$  such that for any  $t \leq t_0$ , supplying  $tP$  is free from the voltage constraint and needs no reactive power support. Second, increase  $t$  from  $t_0$  to 1 in fixed increments  $\Delta t$ , identify for each  $t$  least-amount reactive power supply necessary to have the real power  $tP$  supplied, and compute the numerical partial derivative with respect to  $i$ 's amount of real power. Third, add up these values to obtain the amount of reactive power for  $i$  to supply a unit of real power. The resultant value is multiplied by  $i$ 's amount of real power output to yield the reactive power involved in  $i$ 's real power output.

Alternatively, if the demand for real power can be considered unchanged for some sets of real power output, a data envelopment technique may be applied to the computation of reactive power requirements in the A-S rule (Hougaard et al., 2009): Assuming that real and reactive power output satisfies three conditions of convexity, decreasing returns, and free disposability, this technique can construct  $f(tP)$  from observations of real power output and associated least-amount reactive power requirements by using parametric linear programming. This technique, if available, will be a more tractable option than the first method in applying the A-S rule.

If a unit of reactive power is priced at  $c$ , each distributed generator must pay the reactive power costs equal to  $c$  multiplied by the amount of reactive power requirements computed just above. The money collected may be used for the system operator to maintain the system arranging optimal allocation of reactive power. Alternatively, it will be possible that a generator is paid for the reactive power it supplied. In this case, a distributed generator may make profits by selling both real and reactive power, improving profitability of PV and wind power generation as well as increased amounts of power sales.

There are many individuals and firms which engage in small-scale distributed generation such as residential grid-connected PV generation. It will be possible to arrange that these small-scale generators do not participate in the proposed scheme; They can either reduce real power output when the voltage constraint becomes binding at their buses or supply reactive power on their own to make it possible to feed real power into the grid. In other words, relatively large-scale distributed generators generally may participate in the proposed scheme.

#### 4. Conclusion

The study proposed a method, by applying the A-S rule, to measure the negative effect of a distributed generator's real power output on the voltage stability in that what amount of reactive power is required to supply a unit of real power under the voltage constraint. If an appropriate price is attached to a unit of reactive power, a distributed real power supplier must pay the costs depending on the reactive power requirements. In this regard, the method might help design a new kind of feed-in tariff (FIT) system.

The prevailing FIT systems are predominantly aimed at diffusing green power generation from renewable energy sources; for example, more adoption of PV systems for PV generation. Accordingly, it might no longer be necessary after a certain goal of diffusion is achieved with much reduction in the cost of PV system adoption. However, taking account of the problem of excess

voltage which the study is concerned with, a different kind of FIT system should succeed the current one after a planned level of diffusion.

The proposed method sheds light on a negative attribute of distributed generation. The value computed by the method may be incorporated into a new kind of FIT system in addition to values of positive attributes such as greenhouse gas emission reduction and energy security.

Technological aspects are significantly critical for the method to be practical in a real power system. In particular, a system operator must compute optimal supply of reactive power and procure that supply probably by making use of information technology. Discussing these features is beyond the scope of this study but collaborative studies covering relevant fields are of great importance.

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