A (homogeneous) **linear recurrence** over the integers has the form

$$u_n = a_1 u_{n-1} + a_2 u_{n-2} + \cdots + a_k u_{n-k}, \tag{1}$$

for $n \geq k \in \mathbb{N}$, where $a_1, a_2, \cdots, a_k \in \mathbb{Z}$ are integer constants. The linear recurrence equation (1) defines a unique integer sequence $(u_n)_{n=0}^{\infty}$ after the first k initial terms $u_0, u_1, \cdots, u_{k-1}$ are given. A sequence $(u_n)_{n=0}^{\infty}$ is said to be *recurrent* if it is defined by a linear recurrence. The integer k in (1) is called the *order* of the recurrence and also of the defined recurrent sequent.

We shall consider the following problem of recurrent sequences.
Positivity Problem: Let a linear recurrence (1) be given together with the initial term u_i for $i = 0, 1, \cdots, k$. Is the recurrent sequence $(u_n)_{n=0}^{\infty}$ nonnegative, i.e., does it hold that $u_n \geq 0$ for all n?

In 2006, Vesa Halava, Tero Harju and Mika Hirvensalo proved that the second order Positivity Problem is decidable.

In this thesis, we consider that the Positivity Problem for a sequence satisfying a third order linear recurrence with integer coefficients, i.e., the problem whether each element of this sequence is nonnegative, is decidable.

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Student's signature	Thesis Advisor's signature	