Estimating Daily Real Yields and Expected Inflations For Thailand's Financial Market*

การกำหนดอัตราดอกเบี้ยที่แท้จริงและอัตราเงินเฟ้อที่คาดเป็นรายวันสำหรับตลาดการเงินไทย

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ABSTRACT

The study proposes an approach to estimate the term structures of daily real yields and expected inflations. An affine multifactor interest model for daily real and nominal yields and daily inflation rate is considered and then aggregated for the month so that it can be estimated using aggregate daily nominal-yield and information-variable data together with monthly inflation data. Because the parameters are primarily daily, the real yields and expected inflations can be identified by the parameter estimates and the information variables on the day. The approach is applied to identify the daily term structures of real yields and expected inflations for Thailand. Using the data from March 1, 2001 to August 30, 2013, the study finds that the term structure of average real yields has a normal shape, while that of the expected inflations is flat. Inflation premiums are significantly different from zero, with positive values for shorter maturities and negative values for longer maturities from 2 years onward.

Keywords: Daily Real Yields, Affine Multifactor Interest Rate Model, Daily Real-Yield Estimation

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บทคัดย่อ

การศึกษาเสนอวิธีระบุโครงสร้างอัตราดอกเบี้ยที่แท้จริงและอัตราเงินเฟ้อที่คาดเป็นรายวัน วิธีที่เสนอพิจารณาตัวแบบจำลองเพื่อกำหนดอัตราดอกเบี้ยเป็นรายวันโดยอ้างอิงเชิงเส้นตรงกับกลุ่มปัจจัยแฝง ก่อนที่จะบวกรวมตัวแบบรายวันที่มีในเดือนให้เป็นตัวแบบสำหรับเดือน การศึกษากำหนดตัวแบบสำหรับเดือนโดยใช้ข้อมูลอัตราดอกเบี้ยและข้อมูลข่าวสารสำหรับวันที่รวมเป็นรายเดือนแล้ว ร่วมกับการใช้ข้อมูลอัตราเงินเฟ้อรายเดือน เนื่องจากค่าพารามิเตอร์ที่กำหนดได้เป็นค่าพารามิเตอร์สำหรับวัน อัตราดอกเบี้ยที่แท้จริงและอัตราเงินเฟ้อที่คาดรายวันจึงระบุได้จากค่าพารามิเตอร์และข้อมูลข่าวสารสำหรับวันที่กำลังพิจารณานั้น การศึกษาได้ประยุกต์ใช้วิธีที่เสนอสำหรับตลาดการเงินไทยโดยใช้ข้อมูลตั้งแต่วันที่ 1 มกราคม 2544 ถึงวันที่ 30 สิงหาคม 2556 และพบว่า โครงสร้างของอัตราดอกเบี้ยที่แท้จริงมีรูปทรงปกติ ในขณะที่โครงสร้างของอัตราเงินเฟ้อที่คาดมีรูปทรงแบนราบ ค่าชดเชยความเสี่ยงจากเงินเฟ้อมีระดับที่ต่างจากศูนย์อย่างมีนัยสำคัญ โดยที่ค่าชดเชยระยะสั้นเป็นบวก แล้วค่อย ๆ ลดลงจนเป็นลบสำหรับระยะตั้งแต่ 2 ปี เป็นต้นไป

คำสำคัญ : อัตราดอกเบี้ยที่แท้จริงรายวัน ตัวแบบจำลองเพื่อกำหนดอัตราดอกเบี้ยโดยอ้างอิงเชิงเส้นตรงกับกลุ่มปัจจัยแฝง การกำหนดอัตราดอกเบี้ยที่แท้จริงเป็นรายวัน

1. INTRODUCTION

Real yields and expected inflations are important information for securities trading and economic monitoring. But these two variables are not observed. In the literature, alternative estimation techniques have been proposed-among which multifactor affine interest rate models are popular due to their flexibility to explain time-varying risk premiums (Eraker (2008)). For the U.S. market, for example, Ang et al. (2008) estimated these term structures using a regime-switching factor model with inflation and nominal yield data. Chen et al. (2010) proposed a multifactor, modified quadratic term structure model and estimated the term structures using nominal-and TIPS-yield data. Recently, Ho et al. (2014) estimated the term structures, using an affine multifactor model and nominal-yield and inflation-derivatives-price data. For the U.K. market, Evans (2003) employed a regime-switching affine model and nominal- and TIPS-yield data in the estimation. Joyce et al. (2010) developed an essentially affine term structure model to estimate the structures using nominal- and real-yield data together with inflation and analyst-forecast inflation data. And, in Spain Gimero and Marques (2012) applied an affine model to estimate the structures using the data on nominal yields, inflation and Diebold-Li beta shape factors.

The studies of real yields and expected inflations for emerging markets including Thailand are few. Mills and Wang (2006) studied real yields in Korea, Malaysia, Singapore and Thailand. The study did not rely on any interest rate model in the analysis. Because the authors define real yields as being nominal yields minus inflation, their real yields are measured with errors due to omission of inflation premiums. For Thailand, Khanthavit (2010) estimated the real curve using a two-latent-factor affine model with the nominal yield and inflation data, while Apaitan and Rungcharoenkitkul (2011) estimated the real curve using a four-macro-factor affine model with nominal yield, inflation and observed macro-variable data.

In this study, I propose to estimate the daily term structures of real yields and expected inflations for Thailand. My contributions are two folds. Firstly, Thailand is one of the most important emerging markets in the world and one of the largest economies in South East Asia. The sizes of its equity and bond markets in 2013 were approximately 350 and 275 billion USDs, respectively[1]. Moreover, the Thai government recently issued two inflation-linked bonds in 2011 and 2013 and planned to use the bonds for its fund raising in the future.

In the past, Khanthavit (2010) and Apaitan and Rungcharoenkitkul (2011) studied the term structure of real yields but did not study expected inflations. Their real curves were estimated monthly and with lag due to lagged reporting of inflation and economic variables. The Bank of Thailand (2003) also estimated real yields and expected inflation. Its real yields equaled nominal yields minus expected inflations. The expected inflations were estimated empirically using a macroeconomic model under rational expectations. The Bank's estimates were monthly and lagged due to lagged, monthly report of inflation and macro variables. Its resulting real yields might be biased because the Bank ignored inflation premiums. Finally, although inflation-linked bonds have been traded on the Thai market since 2011, the market is extremely illiquid and the reported yields are mostly indicative rather than executed yields.

[1] Based on an exchange rate of 33 baht per 1 USD.

Real yields and expected inflations are needed to support asset trading and economic monitoring. Unfortunately, the real yields and expected inflations data for Thailand are incomplete, lagged or erroneous. This study improves these data for the country.

Secondly, the study proposes a technique to estimate real yields and expected inflations on a daily basis. The daily rates are useful and important because they support more active trading of the securities--especially inflation-linked bonds, and closer monitoring of the economy. The study is aware of certain techniques and data sets in the literature that can be used for daily estimation. Chen et al. (2010) and Ho et al. (2014) are good examples because TIPS yields and inflation-derivatives prices in their studies are available daily. Yet it is difficult to apply them in Thailand or in other emerging markets. In these countries TIPS and inflation derivatives markets are young, illiquid or inexistent. The available data generally are daily nominal yields and monthly inflation.

The technique proposed in this study is practical and new. It considers an affine latent-factor interest model for daily real and nominal yields and daily inflation. Then it aggregates the model for the month so that the model can be estimated using monthly aggregate nominal yield and monthly inflation data. Because the parameter estimates are primarily daily, real yields and expected inflations can be inferred from the estimates and projected latent variables on the day.

Using Thailand's data from March 1, 2001 to August 30, 2013, the study finds that the term structure of real yields has a normal shape, while that of expected inflations is flat. Inflation premiums are significantly different from zero, with positive values for shorter maturities and negative values for longer maturities of 2 years onward.

2. THE MODEL

Affine models have been used widely to estimate real yields, expected inflations and inflation premiums. In this study, I adopt the model of Joyce et al. (2010) to describe nominal and real yields in Thailand. It is an essentially affine term structure model which relates the nominal and real yields with a set of latent factors linearly under a no-arbitrage condition in the real world. The model is flexible. It allows time-varying risk premiums and real short rate. The number of latent factors can be raised to capture complex behavior of the yields. Moreover, a latent factor model is found in previous studies to fit yields better than a macro factor model.

2.1 The Pricing of Real and Nominal Bonds

In a no-arbitrage environment, the time-t price $P_t^{n,R}$ of a zero-coupon real bond with an n-period maturity must be given by (Cochrane (2005))

$$P_t^{n,R} = E_t\{M_{t+1}M_{t+2} \dots M_{t+n}\}, \qquad (1)$$

where M_{t+j} is the real pricing kernel in j periods hence and $E_t\{.\}$ is the conditional expectation operator in the real world. The price $P_t^{n,N}$ of a zero-coupon nominal bond is given in a similar way but with the nominal pricing kernel $M^{*}_{t+j}=M_{t+j}\frac{I_{t+j-1}}{I_{t+j}}$ being substituted for M_{t+j}. I_{t+j} is the consumer price index at time $t+j$.

$$P_t^{n,N} = E_t\{M^*_{t+1}M^*_{t+2}\dots M^*_{t+n}\}. \tag{2}$$

2.2 Real Yields, Nominal Yields and Their Compositions

From eqs. (1) and (2), because the real yield $y_t^{n,R}$ and nominal yield $y_t^{n,N}$ are $-\frac{1}{n}Ln\{P_t^{n,R}\}$ and $-\frac{1}{n}Ln\{P_t^{n,N}\}$, up to a second order approximation the yields must equal

$$y_t^{n,R} = -\frac{1}{n}\left\{E_t\left(\textstyle\sum_{j=1}^{n} m_{t+j}\right) + \frac{1}{2}V_t\left(\textstyle\sum_{j=1}^{n} m_{t+j}\right)\right\} \tag{3.1}$$

$$y_t^{n,N} = -\frac{1}{n}\left\{E_t\left(\textstyle\sum_{j=1}^{n}(m_{t+j}-\pi_{t+j})\right) + \frac{1}{2}V_t\left(\textstyle\sum_{j=1}^{n}(m_{t+j}-\pi_{t+j})\right)\right\}, \tag{3.2}$$

where $m_{t+j} = Ln\{M_{t+j}\}$. $\pi_{t+j} = Ln\left\{\frac{I_{t+j-1}}{I_{t+j}}\right\}$ is logged inflation. $V_t(.)$ is the variance operator conditioned on the information at time t.

From eq. (3.1), the 1-period real yield $y_t^{1,R}$ is $-E_t(m_{t+1}) - \frac{1}{2}V_t(m_{t+1})$. Using this relationship, the real yield $y_t^{n,R}$ can be decomposed into

$$y_t^{n,R} = \frac{1}{n}\left\{E_t\left(\textstyle\sum_{j=1}^{n} y_{t+j-1}^{1,R}\right) - \textstyle\sum_{j=2}^{n} Cov_t\left(\textstyle\sum_{s=1}^{j-1} m_{t+s}, m_{t+j}\right)\right\}. \tag{4}$$

$Cov_t(.)$ is the conditional covariance operator. The term $\frac{1}{n}E_t\left(\sum_{j=1}^{n} y_{t+j-1}^{1,R}\right)$ is the average expected 1-period real yield. In the risk neutral world, $y_t^{n,R} = \frac{1}{n}E_t\left(\sum_{j=1}^{n} y_{t+j-1}^{1,R}\right)$. So, the term $-\frac{1}{n}\sum_{j=2}^{n} Cov_t\left(\sum_{s=1}^{j-1} m_{t+s}, m_{t+j}\right) = y_t^{n,R} - \frac{1}{n}E_t\left(\sum_{j=1}^{n} y_{t+j-1}^{1,R}\right)$ can be interpreted as being real term premium.

By definition, the break-even inflation rate is $y_t^{n,N}$ - $y_t^{n,R}$. Its structure is given by

$$y_t^{n,N} - y_t^{n,R} = \frac{1}{n}\left\{E_t\left(\textstyle\sum_{j=1}^{n}\pi_{t+j}\right) - \frac{1}{2}V_t\left(\textstyle\sum_{j=1}^{n}\pi_{t+j}\right) + Cov_t\left(\textstyle\sum_{j=1}^{n} m_{t+j}, \textstyle\sum_{j=1}^{n}\pi_{t+j}\right)\right\}. \tag{5}$$

The term $\frac{1}{n}E_t\left(\sum_{j=1}^{n}\pi_{t+j}\right)$ is the expected inflation for the next n periods. The terms $-\frac{1}{n}\frac{1}{2}V_t\left(\sum_{j=1}^{n} m_{t+j}\right)$ and $\frac{1}{n}Cov_t\left(\sum_{j=1}^{n} m_{t+j}, \sum_{j=1}^{n}\pi_{t+j}\right)$ are the Jensen's effect (or inflation convexity) and the covariance effect (Ho et al. (2014)). Their sum is the inflation premium.[2] Under the Fisher hypothesis, $y_t^{n,N} = y_t^{n,R} + \frac{1}{n}E_t\left(\sum_{j=1}^{n}\pi_{t+j}\right)$ and the inflation premium is zero.

2.3 Stochastic Behavior of Pricing Kernels

The logged, real pricing kernel m_{t+1} takes on the form as in eq. (6).

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[2] Joyce et al. (2010) called the term $\frac{1}{n}Cov_t\left(\sum_{j=1}^{n} m_{t+j}, \sum_{j=1}^{n}\pi_{t+j}\right)$ inflation premium. In Ang et al. (2008), the inflation convexity is ignored. In this study, I estimate the inflation premium from the difference between the break-even inflation and expected inflation.

$$m_{t+1} = -(\bar{r} + \boldsymbol{\gamma}'\mathbf{z}_t) - \frac{\boldsymbol{\Lambda}_t'\Omega\boldsymbol{\Lambda}_t}{2} - \boldsymbol{\Lambda}_t'\Omega^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1} \qquad (6)$$

The term $(\bar{r} + \boldsymbol{\gamma}'\mathbf{z}_t)$ is the real short rate. It can vary over time with a set of K latent factors $\mathbf{z}_t' = [z_{1,t}, \dots, z_{K,t}]$. The real short rate is constant if $\boldsymbol{\gamma}' = [\gamma_1, \dots, \gamma_K]$ zero vector.[3] Vector $\boldsymbol{\Lambda}_t\Omega^{\frac{1}{2}}$ is time-varying risk premiums.

$$\boldsymbol{\Lambda}_t = \boldsymbol{\lambda} + \beta\mathbf{z}_t. \qquad (7)$$

Vector $\boldsymbol{\lambda}' = [\lambda_1,\dots,\lambda_K]$ and matrix $\beta = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1K} \\ \vdots & \ddots & \vdots \\ \beta_{K1} & \cdots & \beta_{KK} \end{bmatrix}$. The risk premium for factor K is constant if vector $[\beta_{k1}, \dots, \beta_{kK}]$ is zero. $\boldsymbol{\varepsilon}_{t+1}' = [\varepsilon_{1,t+1}, \dots, \varepsilon_{K,t+1}]$ are Gaussian shocks of factors zt+1. Their mean vector is zero and their covariance matrix is $\boldsymbol{\Omega} = \begin{bmatrix} \sigma_1^2 & 0 \dots & 0 \\ 0 & \ddots & \vdots \\ \vdots & & 0 \\ 0 & \dots 0 & \sigma_K^2 \end{bmatrix}$. Factors $\mathbf{Z}_{t+1}$ follow a VAR(1) process in eq. (8).

$$\mathbf{z}_{t+1} = \varphi\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}. \qquad (8)$$

Coefficient matrix $\varphi = \begin{bmatrix} \varphi_{11} & 0 \dots & 0 \\ \varphi_{21} & \varphi_{22}\ 0 \dots & \vdots \\ \vdots & \ddots & 0 \\ \varphi_{K1} & \varphi_{K2} \ \cdots & \varphi_{22} \end{bmatrix}$ is lower triangular. The VAR structure is assumed because of three reasons. One, the structure allows unconditional correlations among the latent factors. Two, in an affine model mean-reverting factors ensure mean-reverting real and nominal yields. Cairns (2004) pointed out that mean reversion was one of the desired properties of an interest rate model. And three, the model relates inflation linearly with the factors. So, the model's inflation is mean-reverting. This property is consistent with the inflation targeting policy being implemented by many countries including Thailand.

When Gaussian shocks are assumed, the model's inflation, real yields and nominal yields can be negative. Negative inflation, real yields and, especially, nominal yields are not consistent with stylized facts in some countries. Previous studies such as Chen et al. (2010) assumed different processes to ensure their positivity. This study argues that Gaussian shocks can be used in the analysis. Negative inflations are observed in a short horizon for example for a one-month horizon in Thailand (to be reported below). Negative real short yields were found in the countries such as the U.S.A. for example by Ang et al. (2008). Although negative nominal yields are uncommon, the Gaussian shocks can still be consistent if the probability of negative yields in the model is low.

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[3] In Joyce et al. (2010), γ_k is one or zero depending on whether the real short rate does or does not vary with factor $Z_{k,t}$. In this study, I allow γ_k to be a real value and test for its significance.

This study acknowledges the observation made by Ang et al. (2008) that interest rates and inflations can switch regimes. A regime-switching latent factors are more appropriate than a fixed structure in eq. (8). However, the objective of this study is to propose a technique to estimate daily real yields and inflation expectations. A single-regime model suffices to demonstrate the working of my technique. In addition, the study considers Thailand as the sample country. The sample period is short from March 1, 2001 to August 30, 2013 and is after Thailand adopted the inflation-targeting policy. The possibility of and the effects from regime switching, if there are any, should be small.

Because the logged nominal pricing kernel m^*_{t+1} is $m_{t+1}-\pi_{t+1}$, from eq. (8) it must equal

$$m^*_{t+1} = -(\bar{r} + \boldsymbol{\gamma}'\mathbf{z}_t) - \frac{\boldsymbol{\Lambda}_t'\Omega\boldsymbol{\Lambda}_t}{2} - \boldsymbol{\Lambda}_t'\Omega^{\frac{1}{2}}\boldsymbol{\varepsilon}_{t+1} - \pi_{t+1}. \tag{9}$$

2.4 The Pricing

Following Duffie and Kan (1996), Joyce et al. (2010) derived the solutions for the real and nominal yields as affine functions of latent factors in eqs. (10) and (11).

$$y_t^{n,R} = -\frac{1}{n}\{A_n + \mathbf{B}_n'\mathbf{z}_t\} \tag{10}$$

$$y_t^{n,N} = -\frac{1}{n}\{A_n^* + \mathbf{B}_n^{*\prime}\mathbf{z}_t\}, \tag{11}$$

where $A_0 = A^*_0 = 0.00$ and $\mathbf{B}_0 = \mathbf{B}^*_0$ are (Kx1) zero vectors. Coefficients $\mathbf{A}_{n>0}$ and $\mathbf{A}^*_{n>0}$ and vectors $\mathbf{B}_{n>0}$ and $\mathbf{B}^*_{n>0}$ are determined sequentially with respect to the systems of equations (12).

$$A_n = -\bar{r} + A_{n-1} - \mathbf{B}_{n-1}'\Omega\boldsymbol{\lambda} + \frac{1}{2}\mathbf{B}_{n-1}'\Omega\mathbf{B}_{n-1} \tag{12.1}$$

$$\mathbf{B}_n' = -\boldsymbol{\gamma}' + \mathbf{B}_{n-1}'(\varphi - \Omega\beta) \tag{12.2}$$

and

$$A_n^* = -\bar{r} - \mu_\pi + A_{n-1}^* - \mathbf{B}_n^{*\prime}\Omega\boldsymbol{\lambda}^* + \frac{1}{2}\mathbf{B}_{n-1}^{*\prime}\Omega\mathbf{B}_{n-1}^* + \frac{\sigma_1^2}{2} + \sigma_1^2\lambda_1 \tag{12.3}$$

$$\mathbf{B}_n^{*\prime} = -(\boldsymbol{\gamma}' + \varphi_1) + \mathbf{B}_{n-1}^{*\prime}(\varphi - \Omega\beta) + \boldsymbol{\iota}'\Omega\beta, \tag{12.4}$$

where $\varphi_1 = \varphi[_{11}\ 0 \ldots 0]$ and $\boldsymbol{\iota}' = [1 \quad 0\ldots \quad 0]$. μ_π is the unconditional mean of the inflation. The specifications (12.3) and (12.4) are specific to the perfect correlation assumption of factor $Z_{1,t}$ with inflation πt. Modification needs to be made under a different assumption for πt.

3. MODEL ESTIMATION

3.1 Measurement Equations

Because factors z_t are latent, the econometrician will have to relate them with observed variables. In this study, I consider inflation and nominal yields because these variables are observed in most countries. The measurement equations for day t are given by

$$\begin{bmatrix} \pi_t \\ -n_1 y_t^{n_1,N} \\ \vdots \\ -n_H y_t^{n_H,N} \end{bmatrix} = \begin{bmatrix} \mu_\pi \\ A_{n_1}^* \\ \vdots \\ A_{n_H}^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\iota}' \\ \mathbf{B}_{n_1}^{*\prime} \\ \vdots \\ \mathbf{B}_{n_H}^{*\prime} \end{bmatrix} \mathbf{z}_t + \begin{bmatrix} 0 \\ \omega_{n_1,t} \\ \vdots \\ \omega_{n_H,t} \end{bmatrix}. \quad (13)$$

$y_t^{nh,N}$ is the daily nominal yield with an n_h-day maturity. I follow Piazzesi (2010) to impose a month of 21 trading days. So, n_h is 21h and 252h days for h-month and h-year maturities respectively. $\omega_{nh,t}$ is the measurement error due to, for example, bid-ask spreads and zero-curve interpolation. Inflation in eq. (13) ensures its dynamic is consistent with the determining factors of real and nominal yields.

I assume factor $z_{1,t}$ is correlated perfectly with inflation in order to simplify the model structure. The first factor then can be interpreted as being inflation factor. The perfect correlation assumption is not restrictive. The factors are latent. When the first factor is inflation, the remaining factors can be rotated so that the fit of the model remains unchanged.

The measurement equations of the observed variables in (13) and the transition equations of the latent factors in (8) can be estimated recursively by the Kalman filter. The technique is common but quite difficult to use especially in a highly non-linear model such as the one in this study. Moreover, because inflation is reported monthly, daily estimation by the conventional Kalman filter is not possible. The objective to estimate daily real yields and expected inflations cannot be satisfied. To proceed, Harvey (1989, pp. 309-312) suggested the filter had to be modified. But the estimation by the modified filter is even more difficult and complicated.

3.2 A Linear Projection of Latent Variables

I propose an alternative technique to estimate the model on a daily basis even if inflation is reported monthly. Latent factors $\mathbf{z}_t$ can be projected linearly by a set of observed information variables $\mathbf{q}'_t = [q_{0,t} = 1, q_{1,t}, \ldots, q_{\eta-1,t}]$. The projection equation is given by

$$\mathbf{z}_t = \mathbf{b}'\mathbf{q}_t + \mathbf{v}_t, \quad (14)$$

where $\mathbf{b}' = \begin{bmatrix} b_{1,0}, b_{1,1}, \ldots, b_{1,\eta-1} \\ \vdots \\ b_{K,0}, b_{K,1}, \ldots, b_{K,\eta-1} \end{bmatrix}$ is the matrix of projection coefficients and

$\mathbf{v}'_t = [v_{1,t},\ldots,v_{K,t}]$ are projection errors. The linear projection approach follows Mishkin (1981) who estimated unobserved real yields by information variables. When I substitute $\mathbf{b}'\mathbf{q}_t+\mathbf{v}_t$ for $\mathbf{z}_t$ in eq. (13) and collect terms, I obtain eq. (15.1).

$$\begin{bmatrix}\pi_t\\ -n_1 y_t^{n_1,N}\\ \vdots\\ -n_H y_t^{n_H,N}\end{bmatrix} = \begin{bmatrix}\mu_\pi\\ A^*_{n_1}\\ \vdots\\ A^*_{n_H}\end{bmatrix} + \begin{bmatrix}\boldsymbol{\iota}'\\ \mathbf{B}^{*\prime}_{n_1}\\ \vdots\\ \mathbf{B}^{*\prime}_{n_H}\end{bmatrix}\mathbf{b}'\mathbf{q}_t + \begin{bmatrix}v_{1,t}\\ \omega_{n_1,t}+\mathbf{B}^{*\prime}_{n_1}\mathbf{v}_t\\ \vdots\\ \omega_{n_H,t}+\mathbf{B}^{*\prime}_{n_H}\mathbf{v}_t\end{bmatrix} \tag{15.1}$$

$$= \begin{bmatrix} b_{1,0}+\mu_\pi & b_{1,1} & \ldots & b_{1,\eta-1}\\ A^*_{n_1}+\mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_1 & \ldots & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_K\\ & \vdots & & \\ A^*_{n_H}+\mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_1 & \ldots & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_K\end{bmatrix}\mathbf{q}_t + \begin{bmatrix}v_{1,t}\\ \omega_{n_1,t}+\mathbf{B}^{*\prime}_{n_1}\mathbf{v}_t\\ \vdots\\ \omega_{n_H,t}+\mathbf{B}^{*\prime}_{n_H}\mathbf{v}_t\end{bmatrix} \tag{15.2}$$

$$= \boldsymbol{\alpha}'\mathbf{q}_t + \mathbf{u}_t. \tag{15.3}$$

$\mathbf{b}'_{q-1}$ is column q of coefficient matrix $\mathbf{b}'$. Eq. (15.2) rearranges the coefficient vectors and matrices in eq. (15.1) by noticing that $q_{0,t} = 1$. I define $\mathbf{u}_t = \begin{bmatrix}v_{1,t}\\ \omega_{n_1,t}+\mathbf{B}^{*\prime}_{n_1}\mathbf{v}_t\\ \vdots\\ \omega_{n_H,t}+\mathbf{B}^{*\prime}_{n_H}\mathbf{v}_t\end{bmatrix}$ and

$$\boldsymbol{\alpha}' = \begin{bmatrix} b_{1,0}+\mu_\pi & b_{1,1} & \ldots & b_{1,\eta-1}\\ A^*_{n_1}+\mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_1 & \ldots & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_K\\ & \vdots & & \\ A^*_{n_H}+\mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_1 & \ldots & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_K\end{bmatrix}$$

so that eq. (15.3) is in a familiar regression format.

The regression is linear in information variables. But it is highly nonlinear in the parameters. Eq. (15.3) is important. All the regressors and regressants are observed. Now, the econometrician can use simple regressions for the estimation.

3.3 The Estimation Equations

Eq. (15.3) is the model for the day. Although nominal yields are reported daily, inflation is reported monthly. To proceed, eq. (15.3) needs be adjusted to align with the monthly observation of inflation data. Let d_T be the number of trading days in month T. Summing eq. (15.3) for all day t in month T gives

$$\begin{bmatrix}\sum_{t=1}^{d_T}\pi_t\\ -n_1\sum_{t=1}^{d_T} y_t^{n_1,N}\\ \vdots\\ -n_H\sum_{t=1}^{d_T} y_t^{n_H,N}\end{bmatrix} = \boldsymbol{\alpha}^{\mathbf{T}}\sum_{t=1}^{d_T}\mathbf{q}_t + \sum_{t=1}^{d_T}\mathbf{u}_t \tag{16}$$

Variables $\sum_{t=1}^{d_T} \pi_t$, $\sum_{t=1}^{d_T} y_t^{n_1,N}$,…, $\sum_{t=1}^{d_T} y_t^{n_H,N}$ and $\sum_{t=1}^{d_T} \mathbf{q}_t$ are observed on a monthly basis. Because $\sum_{t=1}^{d_T} \pi_t$ is the sum of daily inflation, by definition it is monthly inflation. The nominal yields and information variables are available daily, so their sums for the month can be computed in a straightforward way. $\sum_{t=1}^{d_T} \mathbf{u}_t$ is the sum of regression errors in the month.

Eq. (16) enables the econometrician to estimate the model from monthly inflation and aggregate nominal yields. Because the parameters in eq. (16) are the ones from the daily model, the resulting estimates are daily. Daily real yields and expected inflations can be inferred from these estimates and information variables for the days. This aggregation technique to estimate the daily parameters from monthly data is similar to that in Adrian et al. (2013).

3.4 The Regressions

I use the nonlinear seemingly unrelated regression estimation (SURE) technique to estimate eq. (16). SURE does not require Gaussian shocks. The SURE estimates are consistent and efficient due to the information in correlated shocks. The procedure is two-step. In step 1, I estimate the covariance matrix of the shocks $\sum_{t=1}^{d_T} \mathbf{u}_t$ Because the model is linear in the information variables, the covariance matrix can be estimated conveniently by a linear SURE regression. In step 2, I use nonlinear SURE to estimate the parameters embedded in eq. (16). I assume the covariance matrix of the shocks is the one from the first step.

It is important to note that t$\sum_{t=1}^{d_T} \mathbf{u}_t$ are the sums of all daily shocks in month T. Although the daily shocks have constant variances, their monthly aggregates do not because each month has different numbers of trading days. To correct for heteroscedasticity, the monthly variables for month T is weighted by $\sqrt{d_T}$.

3.5 Discussion of Related Literature

Because the proposed technique is based on SURE, the estimation is computationally fast compared to the traditional Kalman filtering technique. Recently alternative computationally-fast estimation techniques to have been proposed in the literature. Joslin et al. (2011) propose a two-step estimation technique. They assume an N-factor model in which N yields are priced without errors and the remaining yields are priced with small errors. In the first steps the parameters to govern the dynamics of N factors are estimated from a VAR of those N yields. These parameters and the N yields are then used in the second step to recover the remaining structural parameters from the remaining yields. Hamilton and Wu (2012) make the same assumption as do Joslin et al. (2011) about pricing errors and regress all the sample yields on the current and lagged N yields to obtain reduced-form parameters. Noticing that the resulting estimators are functionally related with the model parameters, they recover all the model parameters from the reduced-form parameters by a minimum-chi-square estimation. Adrian et al. (2013) assume observed factors and estimate the parameters by liner regressions. Their technique is similar to that of Fama and Macbeth (1973). Because the estimation in each step is linear regression, the computation is extremely fast.

As opposed to the three studies, my technique does not assume that factors are observed or that certain yields are priced without errors. It maintains the factors are latent and the pricing suffers measurement errors. The linear projection to extract the information about the unobserved factors in a pricing model with measurement errors should be more realistic.

4. THE DATA

4.1 Samples and Data Sources

I apply the technique to estimate the model for Thailand. The sample period is from March 1, 2001 to August 30, 2013. Because this period is not very long and is after May 2000--the point at which the Bank of Thailand adopted the inflation targeting policy, the possibility of and effects from structural change should be small. The nominal yield data are daily for 1-month, 3-month, 6-month and 1-year up to 10-year maturities, with one-year increments, from the Thai Band Market Association (Thai BMA). Although the Thai BMA zero-coupon curve expands the maturities up to 48 years, I employ the yields of up to 10-year maturity because of low trading liquidity of loner-term bonds. The inflation is logged monthly inflation, computed using the headline consumer price index from the Bureau of Trade and Economic Indices, Ministry of Commerce. Panel 1.1 in Table 1 reports the descriptive statistics of inflation and nominal yields.

The average inflation is 2.6804%. It is within the 0.0-to-3.5 percent band being monitored by the Bank of Thailand. The inflation varied each month and it was negative at times. Realized negative inflation supported the Gaussian assumption for factor z1t in eqs. (8) and (13). The term structure of average nominal yields has a normal shape, while the volatility structure is inverted. The normal term structure is similar to the ones found for the U.S.A. by Jian and Yan (2009) and the U.K. by Joyce et al. (2010). But the volatility structures in the U.S.A. and the U.K. are normal. Thailand's inverted volatility term structure is probably because long-termed bonds are less liquid. On a no-trading day, the yields of these bonds are quoted yields from dealers who interpolate today's yields from yesterday's yields.

Table 1: Data Descriptions

Panel 1.1: Descriptive Statistics

Variables	Average	Max	Min	Std.	Skew.	E. Kurt.	JB Stat.
Inflation	2.6804%	25.8264%	-36.7878%	6.6873%	-1.2803	9.2715	578.2402***
1M	2.4260%	[illegible]33%	0.7799%	1.0915%	0.6027	-0.3174	198.0985***
3M	2.4968%	5.053[illegible]%	0.7981%	1.0767%	0.5817	-0.3078	184.6539***
6M	2.5987%	5.2136%	0.8633%	1.0643%	0.5423	-0.3619	166.6587***
1Y	2.7271%	5.3154%	0.9314%	1.0585%	0.5171	-0.4169	158.5104***
2Y	3.01[illegible]2%	5.5432%	1.1781%	1.0499%	0.5662	-0.3152	176.1810***
3Y	[illegible]460%	5.8372%	1.3491%	1.0056%	0.5616	-0.1891	165.3990***
4Y	[illegible]937%	6.1637%	1.4515%	0.9443%	0.4743	-0.0377	114.9058***
5Y	3.7237%	6.3980%	1.5680%	0.9260%	0.4121	-0.0992	87.8464***
6Y	3.9504%	6.6710%	1.7383%	0.8942%	0.3239	-0.1741	57.3600***
7Y	4.1527%	6.7853%	1.8978%	0.8694%	0.2844	-0.2531	49.4308***
8Y	4.3061%	6.8614%	2.0604%	0.8875%	0.2759	-0.4829	68.5419***
9Y	4.4184%	6.9546%	2.2364%	0.9191%	0.3041	-0.5455	85.0894***
10Y	4.5586%	7.1884%	2.4839%	0.9458%	0.3368	-0.5930	102.6925***

Note: The statistics for inflation is monthly, while those for nominal yields are daily. *** = Significance at a 99% confidence level.

Panel 1.2: Principal Component Analysis of Nominal Yields

Principal Component	Contribution	Accumulated Contribution
1	77.5967%	77.5967%
2	20.3197%	97.9163%
3	1.6444%	99.5608%
4 and Over	0.4392%	100.0000%

None of Thailand's nominal yields were negative. But these stylized facts do not refute the Gaussian assumption for the latent factors. The assumption is still valid if the probability of negative nominal yields is small. Finally, the Jarque-Bera tests reject the normality hypothesis for the inflation and nominal yields. These test results support the use of SURE in the estimation because SURE does not require Gaussian errors.

In a multifactor model, the exact number of factors is unknown. It must be proposed by researchers. In the past, two to four factors were chosen. Their reasons varied. Ang et al. (2008) chose three factors. They argued that three factors had been used often in order to match term structure dynamics. Joyce et al. (2010) acknowledged Joyce at al. (2012) that two factors sufficed to explain real yields in the U.K. Yet, they added two more factors to improve the fit. Chen et al. (2010) conducted a principal component analysis (PCA) to help determine the number of factors. They found that one and two factors could explain 97.26% and 98.61% of the variation in U.S. real and nominal yields. Hence they used a two-factor model in their study.

The behavior of real and nominal yields in different markets can differ. The numbers of factors can differ too. Using the number that fits one market for another may over- or under-parameterized the model. I follow Chen et al. (2010) to conduct a PCA to help identify the number of factors. PCA ensures that the chosen number is supported by the data and particular to the market being considered. The findings are report in Panel 1.2. I find that the first two factors can explain 97.92% of the variation. The third factor adds only 1.64%, while factors 4 and over contribute marginally. These findings lead me to choose a two-factor model.

4.2 Information Variables

I project the latent factors by a set of observed information variables so that all the variables are observed and the model can be estimated by simple regressions. The choice of information variables is important. They must be able to project the latent variables. If not, coefficient matrix α' is zero and the model parameters cannot be inferred. I use $\eta = 5$ variables in the projection. The first is a constant. The remainders are 1-day lagged Bjork-Christensen (1999) beta shape factors. As Khanthavit (2013) reported, these factors could predict Thailand's nominal term structure accurately.

To check for projection ability, I regress daily nominal yields on daily information variables and regress monthly inflation and monthly-aggregate nominal yields on monthly-aggregate information variables. From eqs. (14) and (15) if the information variables are able project the latent factors, the regression coefficients must be significant. The results are in Table 2. I find that the coefficients for the nominal yields are highly significant both in the daily and monthly regressions. For inflation, the coefficients for beta shape factors 3 and 4 are significant at a 90% confidence level. Based on these results, I conclude that the chosen information variables have the ability to project the latent factors.

It is noted that the R^2's for nominal yields are very high. All are over 99%. The high R^2's and also highly significant coefficients can be explained by Khanthavit's (2013) observation that the nominal yields and beta shape factors were long-memory, near-I(1) variables. So, the results were similar to the ones from co-integration regressions.

5. EMPIRICAL RESULTS

5.1 Parameter Estimates

The parameter estimates are reported in Table 3. Firstly, I'd like to direct attention to the expected inflation μ_π. The estimate is 2.46% per year, which is close to the sample average of 2.68%. Secondly, the autocorrelation coefficient φ_{11} of daily inflation is 0.0179. The positive autocorrelation is consistent with the positive 0.3332 coefficient from an AR(1) regression of monthly inflation. But its implied monthly level of 0.000854 is not close. The difference is probably due to the fact that the 0.000854 level is jointly determined by the inflation and aggregate nominal yields, while the 0.3332 level is by inflation alone. Thirdly, the projection coefficients $b_{(k=1,2),(q=0,1,...,4)}$ are significant thereby once again ensuring the projection ability. Fourthly, the parameters are comparable with the ones found for the U.K. by Joyce et al. (2010). Our λ and β's are very large, while our 's are very small. Finally, the average real short rate in this study of 0.0048% per year is small. In the following section, the small real short rate explains a limited role of the average expected 1-day real yield in the variation of nominal yields.

Table 2: Tests for Projection Ability of Information Variables

Variables	Monthly Data						Daily Data					
	Constant	Beta F. 1	Beta F. 2	Beta F. 3	Beta F. 4	R^2	Constant	Beta F. 1	Beta F. 2	Beta F. 3	Beta F. 4	R^2
Inflation	0.0002	-0.0028	[illegible]	0.0027*	-0.0069*	0.0223	[redacted]					
1M	0.0000	-0.0834***	-0.0817***	-0.0022***	-0.0793***	0.9992	0.0000	-0.0834***	-0.0816***	-0.0023***	-0.0792***	0.9986
3M	0.0000	-0.2498***	-0.2311***	-0.0178***	-0.2144***	0.9995	0.0000	-0.2496***	-0.2310***	-0.0177***	-0.2144***	0.9989
6M	0.0000	-0.4986***	-0.4253***	-0.0633***	-0.3698***	0.9986	0.0000	-0.4985***	-0.4253***	-0.0632***	-0.3700***	0.9979
1Y	0.0004	-1.0050***	-0.7667***	-0.1885***	-0.6096***	0.9989	0.0004***	-1.0047***	-0.7669***	-0.1879***	-0.6104***	0.9980
2Y	-0.0005	-1.9992***	-1.1881***	-0.5689***	-0.7648***	0.9981	-0.0004***	-2.0001***	-1.1898***	-0.5667***	-0.7689***	0.9967
3Y	-0.0004	-2.9839***	-1.4033***	-0.9329***	-0.7718***	0.9983	-0.0005***	-2.9818***	-1.4066***	-0.9290***	-0.7782***	0.9962
4Y	-0.0029***	-3.9139***	-1.4839***	-1.1803***	-0.7763***	0.9983	-0.0030***	-3.9105***	-1.4856***	-1.1784***	-0.7781***	0.9955
5Y	0.0047***	-5.0992***	-1.6560***	-1.3638***	-0.8986***	[illegible]	0.0043***	-5.0921***	-1.6612***	-1.3575***	-0.9072***	0.9931
6Y	0.0025**	-6.0478***	-1.6639***	-1.4865***	-0.8743***	0.9965	[illegible]	-6.036***	-1.6663***	-1.4815***	-0.8786***	0.9921
7Y	-0.0048***	-6.8711***	-1.6379***	-1.5335***	-0.8292***	0.9944	[illegible]	-6.8659***	-1.6459***	-1.5243***	-0.8437***	0.9894
8Y	-0.0005	-7.9966***	-1.8089***	-1.5360***	-1.0176***	0.9976	-0.0011**	[illegible]	-1.8065***	-1.5333***	-1.0165***	0.9934
9Y	0.0028	-9.0818***	-1.8073***	-1.7198***	-0.9086***	0.9968	0.0020***	[illegible]	[illegible]	-1.7124***	-0.9168***	0.9928
10Y	-0.0028	-9.8984***	-1.5238***	-1.8332***	-0.5623***	0.9971	-0.0039***	-9.8756***	-1.5243***	-1.8261***	-0.5681***	0.9934

Note: * and *** = Significance at 90% and 99% confidence levels. In the monthly data case, the statistics are weighted least squares coefficients of the monthly inflation and aggregate nominal yields on the monthly aggregate information variables. In the daily data case, the statistics are ordinary least squares coefficients.

Table 3: Parameter Estimates

Parameters	Value
$\bar{r} \times 25200$	0.0048***
γ_1	-0.7986***
γ_2	-0.0004***
λ_1	-59.0315***
λ_2	38.6664***
β_{11}	4932.5540***
β_{12}	-620.7170***
β_{21}	-2[illegible]5.23[illegible]1***
β_{22}	[illegible]0239.[illegible]04***
φ_{11}	0.[illegible]179***
φ_{21}	-1.074[illegible]***
φ_{22}	0.9[illegible]23***
σ_1	0.0002***
σ_2	0.0027***
$\mu_\pi \times 25200$	2.4620***
$b_{1,0}$	-0.0002***
$b_{1,1}$	0.0034***
$b_{1,2}$	-0.0130***
$b_{1,3}$	0.0218***
$b_{1,4}$	-0.0335***
$b_{2,0}$	0.2599***
$b_{2,1}$	-10.2587***
$b_{2,2}$	-8.8686***
$b_{2,3}$	-2.0555***
$b_{2,4}$	-6.9794***

Note: *** = Significance at a 99% confidence level.

5.2 Specification Tests

I follow Ang et al. (2008) to conduct specification tests for the model. If the model fits, the moments of sample and fitted nominal yields should not differ. Comparison of the means, standard deviations, skewnesses and excess kurtoses are in Table 4. The numbers in the first lines are for fitted yields and those in the second lines are their deviations from the sample moments. Significance is based on the White (2000) procedure.

Table 4: Specification Tests

Maturity	Descriptive Statistics			
	Mean	Std.	Skew.	E. Kurt
1M	2.4317	1.0884	0.6103	-0.3509
	0.0057	-0.0031	0.0076	-0.[illegible]35
3M	2.4833	1.1351	0.5028	-0.4670
	-0.0136	0.0584	-0.0789	-0.[illegible]91
6M	2.5697	1.1313	0.4800	-0.4943
	-0.0290	0.0670	-0.0623	-0.1325
1Y	2.7382	1.0934	0.4696	-0.5069
	0.0111	0.0349	-0.0474	-0.0899
2Y	3.0480	1.0083	0.46[illegible]	-0.5128
	0.0328	-0.0416	-0.1015	-0.1976
3Y	3.3237	0.9291	0.463[illegible]	-0.5147
	0.0776	-0.0765	-0.0984	-0.3256
4Y	3.5693	0.8578	0.[illegible]624	-0.5156
	0.0756	-0.0865	-0.0119	-0.4779
5Y	3.7886	0.7937	0.462	-0.5161
	0.0649	-0.1323***	0.0499	-0.417
6Y	3.9847	0.7362	0.4617	-0.5165
	0.0343	-0.1581***	0.1378	-0.3424
7Y	4.1606	0.[illegible]845	0.4615	-0.5167
	0.0079	-0.1[illegible]49***	0.1771	-0.2637
8Y	4.3185	0.[illegible]381	0.4614	-0.5169
	0.0124	-0.2494***	0.1855	-0.0340
9Y	4.4607	0.5962	0.4613	-0.5170
	0.0423	-0.3229***	0.1572	0.0284
10Y	4.5890	0.5584	0.4612	-0.5171
	0.0304	-0.3875***	0.1243	0.0758

Note: *** = Significance at a 99% confidence level. The statistics in the upper lines are the moments of fitted yields and the ones in the lower lines are the deviations from sample moments.

The deviations are small and not significant for all the moments and maturities, except for the standard deviations of 5-year and longer yields. The significance of standard deviations was also reported for most specifications of the Ang et al. (2008) model. With respect to the small number of significant cases and when compared and contrast with the ones reported by previous study, I conclude that the two-factor model satisfactorily fit Thailand's nominal yields.

5.3 Daily Real Yields and Expected Inflations

The estimation of daily real yields and expected inflations is successful. In Panel 5.1 of Table 5, the term structure of Thailand's real yields is time varying. Its average has a normal shape. The averages for 1- and 3-month maturities are negative but rising. They turn positive for a 6-month maturity and over. In Panel 5.2, the expected inflations are more volatile for short horizons, while those for long horizons do not vary much. The average structure is flat. A flat shape is expected due to a small autocorrelation of daily inflation.

5.4 Inflation Premiums

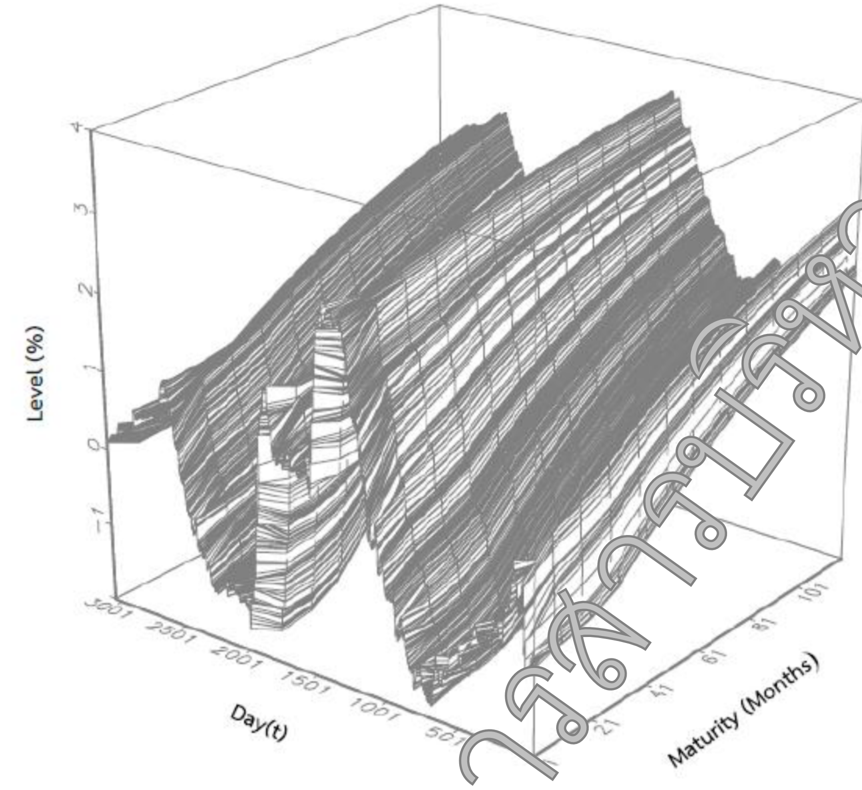
Nominal yields are real yields plus expected inflation plus inflation premiums. For some reasons, researchers, practitioners and regulators assume zero inflation premiums at times. For example, Mill and Wang (2006) defined real yields by the difference between nominal yields and expected inflation in their study of real yields in Asian markets. And the Bank of Thailand defined real yields in a similar fashion.

Inflation premiums need not be zero. In this study, I compute inflation premiums for Thailand by subtracting the real-yield and expected-inflation estimates from the sample nominal yields. The premiums are reported in Panel 5.3. The premiums for short maturities are positive and those for long maturities are negatives. The inverted shape is different from a normal shape in the U.S.A. (Ang et al. (2008)) and a humped shape in the U.K. (Joyce et al. (2010)).

I test for zero inflation premiums and reject the hypotheses for all the maturities. Significant inflation premiums imply that the estimates of Thailand's real yields based on a zero-premium assumption are biased downward for short maturities and biased upward for long maturities.

Table 5: Daily Term Structures

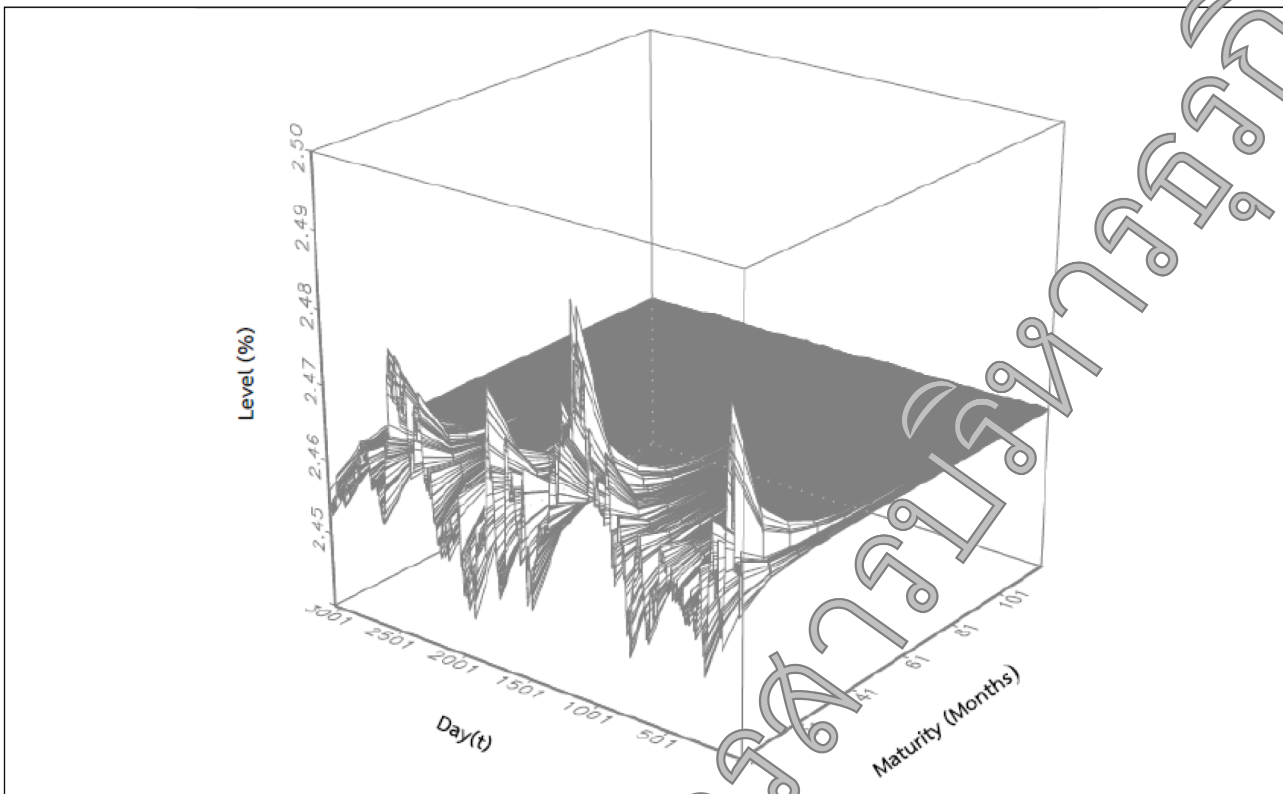
Panel 5.1: Real Yields



Maturity	Average	Max	Min	Std.
1M	-0.0727***	2.6757	-1.8494	1.1365
3M	-0.0188	2.7628	-1.9543	1.1847
6M	0.0714***	2.8680	-1.8994	1.1804
1Y	0.2472***	2.9596	-1.6759	1.1406
2Y	0.570[illegible]	3.0754	-1.2111	1.0518
3Y	0.8577***	3.1674	-0.7862	0.9692
4Y	[illegible]	3.2466	-0.4048	0.8947
5Y	1.3424***	3.3162	-0.0633	0.8279
6Y	1.5470***	3.3778	0.2427	0.7679
7Y	[illegible].7303***	3.4328	0.5173	0.7140
8Y	1.8950***	3.4820	0.7642	0.6655
9Y	2.0432***	3.5261	0.9866	0.6218
10Y	2.1770***	3.5658	1.1873	0.5824

Note: *** = Significance at a 99% confidence level. Day (t=1) is March 1 , 2001 and Day (t=3060) is August 30, 2013

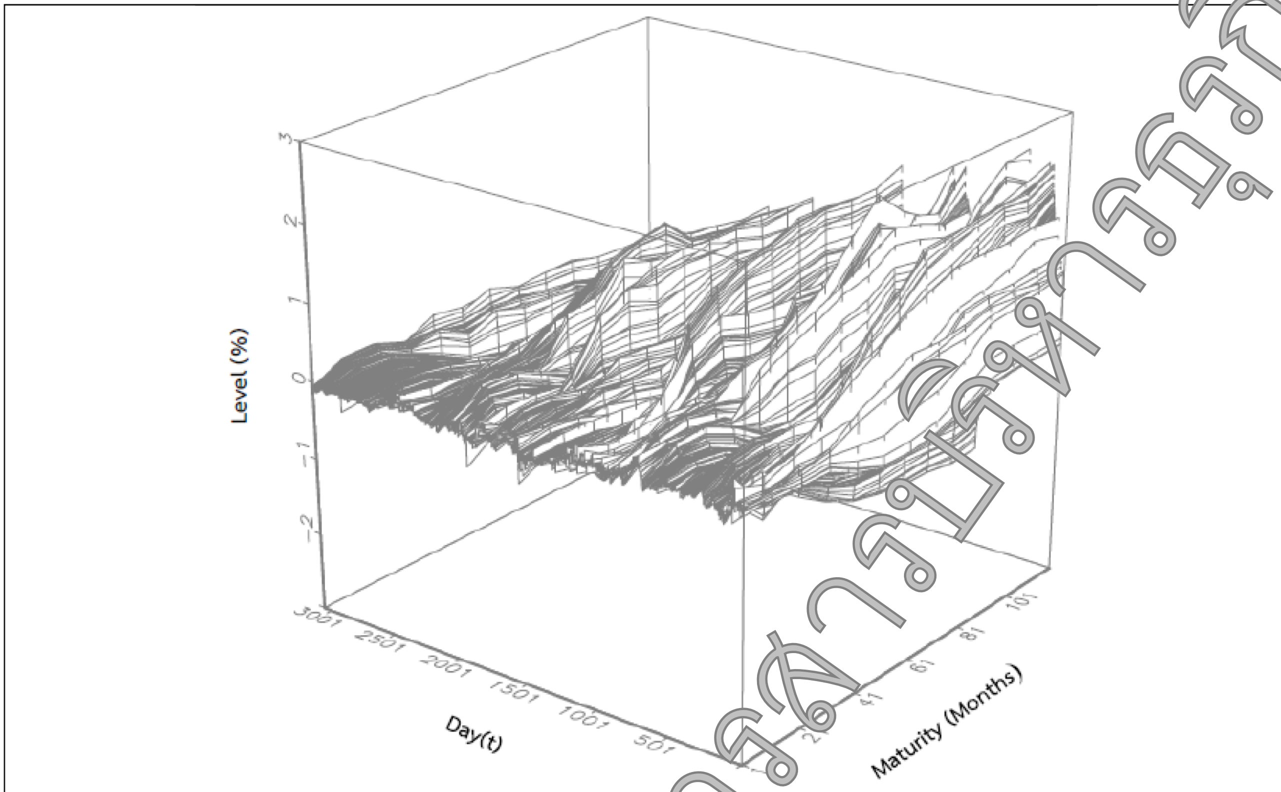
Panel 5.2: Expected Inflations



Maturity	Average	Max	Min	Std.
1M	2.4617***	2.4906	2.4454	0.0077
3M	2.4619***	2.4715	2.4565	0.0026
6M	2.4619***	2.4667	2.4592	0.0013
1Y	2.4620***	2.4644	2.4606	0.0006
2Y	2.4620***	2.4632	2.4613	0.0003
3Y	2.4620***	2.4628	2.4615	0.0002
4Y	2.4620***	2.4626	2.4616	0.0002
5Y	2.4620***	2.4625	2.4617	0.0001
6Y	2.4620***	2.4624	2.4617	0.0001
7Y	2.4620***	2.4623	2.4618	0.0001
8Y	2.4620***	2.4623	2.4618	0.0001
9Y	2.4620***	2.4622	2.4618	0.0001
10Y	2.4620***	2.4622	2.4618	0.0001

Note: *** = Significance at 99% confidence level. Day (t=1) is March 1 , 2001 and Day (t=3060) is August 30, 2013

Panel 5.3: Inflation Premiums



Maturity	Average	Max	Min	Std.
1M	0.0371***	0.4962	-0.4446	0.1006
3M	0.0538***	0.4361	-0.3546	0.1573
6M	0.0654***	0.4742	-0.3315	0.1620
1Y	0.0180***	0.6479	-0.5332	0.1742
2Y	-0.0170***	1.0771	-0.8724	0.3254
3Y	-0.0736***	1.3813	-1.1197	0.4671
4Y	-0.0821***	1.6536	-1.3033	0.5814
5Y	-0.0808***	1.9798	-1.4887	0.6803
6Y	-0.0586***	2.0082	-1.6576	0.7538
7Y	-0.0396***	2.0744	-1.7572	0.8131
8Y	-0.0508***	2.0990	-1.8848	0.8737
9Y	-0.0868***	2.3838	-2.0282	0.9213
10Y	-0.0803***	2.4159	-1.9876	0.9835

Note: *** = Significance at a 99% confidence level. Day (t=1) is March 1 , 2001 and Day (t=3060) is August 30, 2013

5.5 Composition of Nominal Yields

Nominal yields equal real yields plus expected inflations plus inflation premiums, while real yields equal average expected 1-day real yields plus real premiums[4]. It is interesting to examine how much these variables contribute to nominal yields. To analyze the composition, note that the variance $V\left(y_t^{n,N}\right)$ of n-period nominal yield is

$$\begin{aligned} V\left(y_t^{n,N}\right) &= Cov\left(\frac{1}{n}E_t\left(\sum_{j=1}^{n} y_{t+j-1}^{1,R}\right), y_t^{n,N}\right) \\ &+ Cov\left(-\frac{1}{n}\sum_{j=2}^{n} Cov_t\left(\sum_{s=1}^{j-1} m_{t+s}, m_{t+j}\right), y_t^{n,N}\right) \\ &+ Cov\left(\frac{1}{n}E_t\left(\sum_{j=1}^{n} \pi_{t+j}\right), y_t^{n,N}\right) \\ &+ Cov\left(\left(-\frac{1}{2}V_t\left(\sum_{j=1}^{n} \pi_{t+j}\right) + Cov_t\left(\sum_{j=1}^{n} m_{t+j}, \sum_{j=1}^{n} \pi_{t+j}\right)\right), y_t^{n,N}\right). \quad (17) \end{aligned}$$

The terms on the right hand side are the covariances of nominal yield with average expected 1-day real yield, real premium, expected inflation and inflation premium respectively. Dividing eq. (17) by $V\left(y_t^{n,N}\right)$ gives percentage shares of the four variables in the nominal yield's variation. These shares are effectively the slope coefficients from linear regressions of the variables on nominal yield. They are reported in Table 6.

Table 6: Composition of Nominal Yields

Maturity	Average Expected 1-Day Rate	Real Premium	Expected Inflation	Inflation Premium
1M	-49.1417***	152.8965***	0.1731***	-3.9278***
3M	-20.8195***	130.3169***	0.0730***	-9.5705***
6M	-10.6717***	121.0262***	0.0457***	-10.4002***
1Y	-5.3513***	112.0598***	0.0276***	-6.7361***
2Y	-2.594[illegible]***	97.9669***	0.0167***	4.6107***
3Y	-1.6849***	87.3385***	0.0120***	14.3344***
4Y	[illegible]***	77.1448***	0.0091***	24.0592***
5Y	-0.8695***	63.8417***	0.0068***	37.0209***
6Y	-0.6366***	51.9715***	0.0055***	48.6597***
7Y	-0.4569***	40.4410***	0.0045***	60.0114***
8Y	-0.3180***	29.9809***	0.0034***	70.3337***
9Y	-0.2309***	22.8772***	0.0028***	77.3509***
10Y	-0.1458***	15.0450***	0.0023***	85.0985***

Note: *** = Significance at a 99% confidence level.

The movement of nominal yields is principally driven by real premiums and inflation premiums. For short-termed yields, real premiums contribute the most. Their percentage shares fall when maturities are lengthened. For the 10-year nominal yield, the share of real premium falls to 15% while that of inflation premium rises to 85%. Average expected 1-day real yields and expected inflations contribute little. These results are expected due to the small size and low volatility of the average expected 1-day real yields and the low volatility of expected inflations.

[4] The results for average expected 1-day real yields and real premiums can be obtained from the author upon request.

5.6 Hypothesis Tests

It is interesting to ask whether or not Thailand's real short rate and risk premiums are time-varying. To answer these questions, consider eqs. (6) and (7). If the real short rate and risk premiums are *not* time varying, the coefficient vector $\boldsymbol{\gamma}$ and matrix β must be zero. From Table 2, the estimates for $\boldsymbol{\gamma}$ and β are significant. The findings lead me to conclude that Thailand's real short rate and risk premiums are time-varying.

In the model, the covariance $Cov_t(E_t(\pi_{t+2}), \bar{r} + \boldsymbol{\gamma}'\mathbf{z}_{t+1})$ between expected inflation $E_t(\pi_{t+2})$ and real short rate $(\bar{r} + \boldsymbol{\gamma}'\mathbf{z}_{t+1})$ equals $\gamma_1 \varphi_{11} \sigma_1^2$. This statistics has an important implication. The Mundell-Tobin effect predicts a negative covariance, while the Taylor effect predicts a positive one. To test for the Mundell-Tobin effect *versus* the Taylor effect for Thailand, I compute the covariance from the parameter estimates. I find it equal to -3.7994*e*-10 and significant. The statistics supports the Mundell-Tobin effect, meaning that in Thailand the public holds less in money balances and more in other assets in response to inflation. As a result, nominal yields will rise less than one-for-one with expected inflation.

Finally, I test for the Fisher hypothesis which imposes zero inflation premiums. The hypothesis is popular among researchers and practitioners. It offers convenience. Because inflation premiums are unobserved, the hypothesis allows researchers and practitioners to disregard the premiums in their analyses. In Panel 5.3, the average inflation premiums range from -8.68 to 6.54 basis points. Although the levels are small, they are significantly different from zero. The Fisher hypothesis is rejected.

6. CONCLUSION

Real yields and expected inflation provide important information for the trading of securities and the monitoring of the economy. In this study, I propose an approach to estimate the term structures of real yields and expected inflations on a daily basis. The approach is applied for Thailand. Using the data from March 1, 2001 to August 30, 2013, I find that the term structure of average real yields has a normal shape, while that of expected inflations is flat. Inflation premiums are significantly different from zero, hence the Fisher hypothesis is rejected. The movement in nominal yields are principally driven by real premiums and inflation premiums. The real premiums have a larger share to explain short-termed yields, while the inflation premiums do long-termed yields.

The model has one more important application. The chosen information variables are 1-day lagged beta shape factors and these factors are observed daily. The estimation based on today's factors offers tomorrow's real yields and expected inflations. Hence, the model is *ex ante* and can be used to construct trading strategies for the next day.

Although the proposed technique can estimate daily real yields and expected inflations successfully, at least two extensions can be made. One, the technique relies on monthly aggregate nominal yields to align with monthly inflation. The aggregation may average out important information in daily nominal yields. Using daily nominal yields rather than monthly aggregate yields should be more efficient and enhance accuracy. Two, the technique considers inflation so that its movement can be linked with that of nominal and real yields. But if the model is correct, it must be able capture this link with or without inflation in the estimation. The estimation based on nominal yields alone is less complex but should perform equally well. I leave these two extensions for future research.

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