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THESIS

INPUT/OUTPUT LINEARIZATION CONTROL WITH TWO-DEGREE-OF-FREEDOM STRUCTURE FOR UNCERTAIN NONLINEAR PROCESSES

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A Thesis Submitted in Partial Fulfillment of
the Requirements for the Degree of
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In chemical processes, the process uncertainty naturally appears in the process parameters, the process measurements, and the unmeasured disturbance. The uncertainty leads to the deterioration in robustness and control performance that may result in the off-specification of a product quality and unsafe condition during operation.

This work takes advantage of two-degree-of-freedom control for improving robustness of the control system that setpoint tracking and disturbance rejection can be handled independently. Input/output (I/O) linearization technique is selected to provide the setpoint tracking ability. For disturbance rejection, the high-gain technique is used to compensate the effect of the uncertainty. Two types of control structure are proposed for (i) the uncertain processes without time-delay and (ii) the uncertain processes with input time-delay. The control performance of both control methods under servo and regulatory tests are evaluated through numerical simulation with various chemical processes. Furthermore, both proposed control scheme are compared with the existing methods developed by Wu *et al*. (2001) and Hu and Rangaiah (1999) for the processes without and with time-delay, respectively. The results show that the control methods successfully force and maintain the outputs at the desired setpoints and give better performance comparing with other methods.

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Pisit Sukkarnkha
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C_A	=	Concentration of *A* (mol L^{-1}, kmol m^{-3})
$C_{A.0}$	=	Concentration of *A* in the first feed stream
$C_{A.1}$	=	Concentration of *A* in the first reactor (kmol m^{-3})
$C_{A.2}$	=	Concentration of *A* in the second reactor (kmol m^{-3})
$C_{A.i}$	=	Concentration of *A* in the feed stream (mol L^{-1})
C_B	=	Concentration of *B* (mol L^{-1}, kmol m^{-3})
$C_{B.1}$	=	Concentration of *B* in the first reactor (kmol m^{-3})
$C_{B.2}$	=	Concentration of *B* in the second reactor (kmol m^{-3})
C_C	=	Concentration of *C* (mol L^{-1}, kmol m^{-3})
$C_{C.0}$	=	Concentration of *C* in the second feed stream (kmol m^{-3})
$C_{C.2}$	=	Concentration of *C* in the second reactor (kmol m^{-3})
C_D	=	Concentration of *D* (mol L^{-1}, kmol m^{-3})
C_p	=	Heat capacity of the coolant (J g^{-1} K^{-1})
D	=	Vector of unmeasured disturbance
$\mathcal{D}$	=	Differential operator
E_a	=	Activation energy (kJ $kmol^{-1}$)
E_1	=	Activation energy of the first reaction (kJ $kmol^{-1}$)
E_2	=	Activation energy of the second reaction (kJ $kmol^{-1}$)
F	=	Feed flowrate (m^3 hr^{-1}, L min^{-1})
F_1	=	Flowrate of the first feed stream (m^3 s^{-1})
F_2	=	Flowrate of the second feed stream (m^3 s^{-1})
F_{j1}	=	Flowrate of the first cooling stream (m^3 s^{-1})

LIST OF ABBREVIATIONS (Continued)

F_{j2}	=	Flowrate of the second cooling stream (m^3 s^{-1})
f, h	=	Vectors of nonlinear functions
ΔH	=	The change in enthalpy (J mol^{-1})
ΔH_1	=	The change in enthalpy of the first reaction (kJ kmol^{-1})
ΔH_2	=	The change in enthalpy of the second reaction (kJ kmol^{-1})
I	=	Identity matrix
J	=	Jacobian matrix
K	=	Tuning parameters of the disturbance rejection controller
k_0	=	Arrhenius factor (min^{-1}, hr^{-1})
k_{01}	=	Arrhenius factor of the first reaction (s^{-1})
k_{02}	=	Arrhenius factor of the second reaction (s^{-1})
k_1	=	Rate constant of $A \rightarrow B$ (min^{-1})
k_2	=	Rate constant of $B \rightarrow C$ (min^{-1})
k_3	=	Rate constant of $2A \rightarrow D$ (mol L^{-1} min^{-1})
m	=	Numbers of outputs
n	=	Numbers of state variables
Q	=	Heat removal rate of cooling system (K hr^{-1})
R	=	Gas constant (kJ kmol^{-1} K^{-1})
r_i	=	Relative order between the controlled output y_i and manipulated inputs
T	=	Reactor temperature (K)
T_0	=	Temperature in the first feed stream (K)
T_1	=	Temperature of the first reactor (K)

LIST OF ABBREVIATIONS (Continued)

T_2	=	Temperature of the second reactor (K)
T_c	=	Coolant temperature (K)
T_i	=	Temperature in the feed stream (K)
T_{j1}	=	Jacket temperature of the first reactor (K)
T_{j2}	=	Jacket temperature of the second reactor (K)
T_{ji1}	=	Inlet jacket temperature of the first reactor (K)
T_{ji2}	=	Inlet jacket temperature of the second reactor (K)
t	=	Time (min, hr)
UA_1	=	Heat transfer coefficient of the first reactor (kJ s^{-1} K^{-1})
UA_2	=	Heat transfer coefficient of the second reactor (kJ s^{-1} K^{-1})
u	=	Vectors of disturbance-free manipulated inputs
$\tilde{u}$	=	Vectors of manipulated inputs
V	=	Volume of the reactor (m^3, L)
V_1	=	Volume of the first reactor (m^3)
V_2	=	Volume of the second reactor (m^3)
V_{j1}	=	Volume of the first cooling jacket (m^3)
V_{j2}	=	Volume of the second cooling jacket (m^3)
x	=	Vector of state variables
$\hat{x}$	=	Vector of estimated state variables
x^*	=	Vector of predicted state variables
y	=	Vector of controlled outputs
$\hat{y}$	=	Vector of estimated outputs
y^*	=	Vector of predicted outputs
y_{sp}	=	Vector of desired setpoints

LIST OF ABBREVIATIONS (Continued)

γ	=	Specific heat of reaction (K.m^3 $kmol^{-1}$)
δ	=	Vector of estimated disturbances
ε	=	Tuning parameters of the setpoint tracking controller
θ	=	Time-delay (min)
ζ	=	Zero dynamics
λ	=	Eigenvalues
v	=	Vector of compensated setpoint
ρ	=	Density of coolant (g L^{-1})
Φ, Ψ	=	Vector of nonlinear functions

INPUT/OUTPUT LINEARIZATION CONTROL WITH TWO-DEGREE-OF-FREEDOM STRUCTURE FOR UNCERTAIN NONLINEAR PROCESSES

INTRODUCTION

The presence of uncertainty makes the difficulty for system analysis and controller design, especially for model-based control that a process model is used in controller synthesis. The control performance of the model-based control strongly depends on the quality of the model. However, there is a quite difficult task to develop a highly accurate model for uncertain processes. The process parameters, such as kinetic constants, physical properties, and transfer coefficients, are obtained from the laboratory and pilot plant. The obtained values may be deviate from a commercial-scale reactor due to differences in mixing condition, volume ratio, heat transfer, and mass transfer. The quality of instruments is also a cause of uncertainty appearance. Sometimes, the range of measuring device may not give good resolution of process monitoring. The sensitivity of the sensor is also another issue. These mentioned uncertainties can lead to deterioration of robustness and control performance, which may result in the off-specification of product quality and unsafe operation.

Input/output (I/O) linearization technique is a type of model-based control. This approach is based on a transformation of coordination to create a linear relationship between input and output responses (Kravaris and Chung, 1987). The advantages of I/O linearization technique are a few numbers of tuning parameters in the control system and the guarantee of closed-loop stability. However, I/O linearization cannot be directly applied for uncertain processes. For handling the uncertainty problem, it needs to be treated with special design technique, for example, multi-objective H_2/H_∞ approach (Kolavennu *et al*., 2000; Palanki *ei al*., 2003) and μ synthesis (Sampath *et al*., 1998; Elisante *et al*., 2004). Although there are many existing control methods of I/O linearization for controlling uncertain processes, the practical implementation of those controllers is quite difficult due to the requirement of real-time optimization. In addition, the trade-off between control performance and robustness is still the other concerned problem.

Two-degree-of-freedom (2DOF) control is an effective method for handling uncertainties problems, especially for the input disturbance. The 2DOF control structure consists of two controllers that one is used for setpoint tracking, and the other is used for disturbance rejection. The main advantage of the 2DOF scheme is that command input and disturbance responses can be set independently from each other in the control law (Fujimoto and Kawamura, 1995). Over the past decade, the 2DOF control system for uncertain processes has been received considerable attention in many research works. They developed the 2DOF control method in Laplace transform domain. There are only a few research works developed in a nonlinear system. Wright and Kravaris (2005, 2006) proposed the 2DOF control structure for an unstable process in the continuous and discrete-time control systems that perform the zero-pole cancellation. However, it has not concerned for a time-delay system. Time-delay exists due to transport phenomena, time-consuming information processing in measurement devices, and calculation of the control input. It also degrades the control performance and the robustness. Therefore, time-delay need to be considered in the controller design.

This work presents two new control methods based on the 2DOF control structure for uncertain nonlinear system with and without time-delay. The input/output (I/O) linearization and high-gain techniques are employed to handle the processes with unmeasured input disturbances and parametric uncertainties. In the development of the control system without time-delay, the open-loop observer is applied to estimate the disturbance-free state responses for the setpoint tracking controller and calculate the disturbance-free output responses for the disturbance rejection controller. For the development of the control system in the second case, the state predictor with time-delay compensator is used instead of the state estimator to provide the information of future values of disturbance-free states and outputs.

OBJECTIVES

The objective of this research is to develop the control systems for uncertain processes with and without time-delay governed by I/O linearization control technique with 2DOF control structure.

Scopes of thesis

The control systems are developed based on the 2DOF control structure for the processes in the presences of unmeasured input disturbance and parametric uncertainty. The control structure is designed for both systems with and without time-delay. The developed control systems include the setpoint tracking controller, disturbance rejection controller, and the specific disturbance-free state estimators for the cases of the process with and without time-delay. The setpoint tracking controller is designed by using I/O linearization technique, and the disturbance rejection controller is designed by using a high-gain technique. To illustrate the performance, the proposed control method is applied through various types of chemical reactor with unmeasured input disturbances and parametric uncertainties.

Impact of results

This research developed the new control methods for uncertain nonlinear with and without time-delay processes. The presented methods are capable of providing good tracking and disturbance rejection performance. It is also easy to implement to the real application because of a few tuning parameters. With the result of controller design, the developed method is an efficient tool for operating uncertain time-delay systems under maintaining the desired condition and optimal operating costs.

LITERATURE REVIEW

1. Uncertainties in Chemical Processes

The influence of uncertainty is an important factor in the controller design because it may cause degradation of control performance leading to closed-loop instability. The uncertainty naturally occurs in chemical processes due to a complexity of reaction, interactions between unit operation, and quality of instruments. Generally, it can be classified into two categories: parametric uncertainty and unmeasured disturbance.

1.1 Parametric uncertainty

The parametric uncertainty is an uncertainty that relates to the physical properties, kinetic constants, and transfer coefficients of the model (Pistikopoulos, 1995). In practice, the information of process parameters is usually obtained from laboratory-scale experiments. The values of the parameters may be inaccurate, when the pilot plant is scaled up to the commercial level. Sometimes, the assumptions that some process parameters are considered to be constant values are incorrect because most chemical processes are characterized by having time-varying parameters.

1.2 Unmeasured disturbance

Unmeasured disturbance is an uncertainty that occurs in manipulated input and/or controlled output during process operation. There are many reasons that the unmeasured disturbance can be occurred, for example, the fluctuation in the process streams, the friction in control valves, and the sensitivity of measuring devices (Pistikopoulos, 1995).

2. Nonlinear Model-based Control for Uncertain Process

Many chemical processes require high performance control strategies due to increasing demands on product quality, safety, and environmental management. The configuration of traditional linear feedback controllers like proportional-integral (PI) and proportional-integral-derivative (PID) may not give satisfactory results because of nonlinearity of processes.

Nonlinear model-based control (NMBC) is the control technique that a mathematic model of process is integrated into control system. NMBC becomes an interesting method for controlling nonlinear process because it has ability to handle the process nonlinearity directly in the control law. The typical configuration of NMBC controller is shown in Figure 1. NMBC can be categorized into 3 main techniques: differential geometric control (DGC), model predictive control (MPC), and Lyapunov-based control (LBC) (Panjapornpon *et al*., 2006).

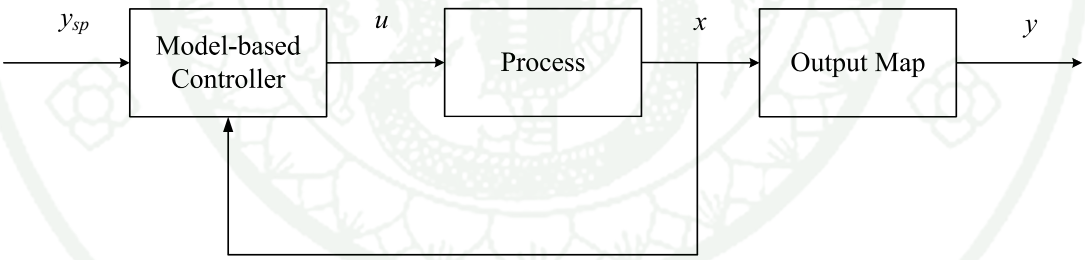

Figure 1 The typical configuration of model-based controller

2.1 Differential geometric control (DGC)

The DGC is related to nonlinear transformations on the states and manipulated inputs. It provides an input/output closed-loop linear dynamical behavior (Kravaris and Chung, 1987; Isidori, 1989). One of the main contributions of the differential geometric approach is input/output (I/O) linearization technique (Kravaris and Soroush, 1990). The advantage of this technique is to guarantee the closed-loop stability. It also has a few tuning parameters compared with other techniques. However, the I/O linearization technique cannot be directly applied for the processes with uncertainty. In order to control the uncertain processes, a special technique is required in the formulation of I/O linearization. For example, the I/O linearization was designed with input constraint developed with Lyapunov method (Kravaris and Palanki, 1988; Hashimoto *et al*., 1996). Tyner *et al*. (1999) presented the adaptive control method for handling the uncertain processes. Other methods for compensation of uncertain processes are the use of special designs of the feedback compensator such as H_2/H_∞ synthesis (Kolavennu *et al*., 2000; Palanki *ei al*., 2003) and μ-synthesis (Sampath *et al*., 2002; Elisante *et al*., 2004). The compensator provides the proper setpoint that can be stabilized the process under I/O linearizing controller.

2.2 Model predictive control (MPC)

The MPC is a powerful control strategy due to its ability to handle the constraint problem. The basic concept of the MPC is that the future control actions are calculated based on the solution of an optimal control problem. Although the MPC is efficient technique, it requires many efforts in computation to solve optimization online. Moreover, general MPC does not guarantee close-loop stability. For control uncertain processes, Mhaskar *et al.* (2005) proposed a robust hybrid predictive control structure. This technique was achieved by switching between MPC and Lyapunov-based bounded robust controller. The Lyapunov-based controller is used as bounded controller that can be switched in at any time to maintain the closed-loop stability in the event that MPC fails to yield a control moves or lead to instability. In addition, another method is to use min-max robust MPC (Limon *et al*., 2006). The objective function of this method is able to determine the optimal control action that minimizes the worst performance objective for uncertainty.

2.3 Lyapunov-based control (LBC)

Lyapunov theory is a method to analyze the stability of the system. There are two dominant methods of Lyapunov stability, which are indirect and direct methods. In the indirect method, the eigenvalues of the Jacobian matrix at the steady state condition is analyzed. If the real parts of all eigenvalues are less than zero (in the left side of the complex plane), it can be said that this process is stable at this steady state condition. The direct method is to determine the stability properties of a process by constructing a Lyapunov function as real positive definite for the process. To design LBC, it can be achieved by following the direct method of Lyapunov stability analysis. Although LBC guarantees the stability in spite of controlling at an unstable steady state of the open-loop, it is difficult to find the proper Lyapunov function.

3. The Two-degree-of-freedom Control Structure

Degree-of-freedom generally expresses the number of independent parameters required to specify the configuration of a system. For two-degree-of-freedom (2DOF) control, the control architecture consists of setpoint tracking controller and disturbance rejection controller. The characteristics of this control structure are that the setpoint tracking and the disturbance rejection controllers can be set independently (Fujimoto and Kawamura, 1995). For the control system based on 1DOF, if the setpoint (y_{sp}) and the disturbance (D) vary in similar manner, the controller can be chosen that error (e) is satisfactory. The controller may not handle the process when dynamic behaviors of y_{sp} and D are different. For example, one expects step changes, and another occurs in ramp changes. The controller has to be chosen either for good response to steps or good response to ramps or else some compromise has to be found. Therefore, another control has to be applied to the control system for adding more DOF control (Morari and Zafiriou, 1989). The 2DOF control structure based on feedback control configuration is shown in Figure 2.

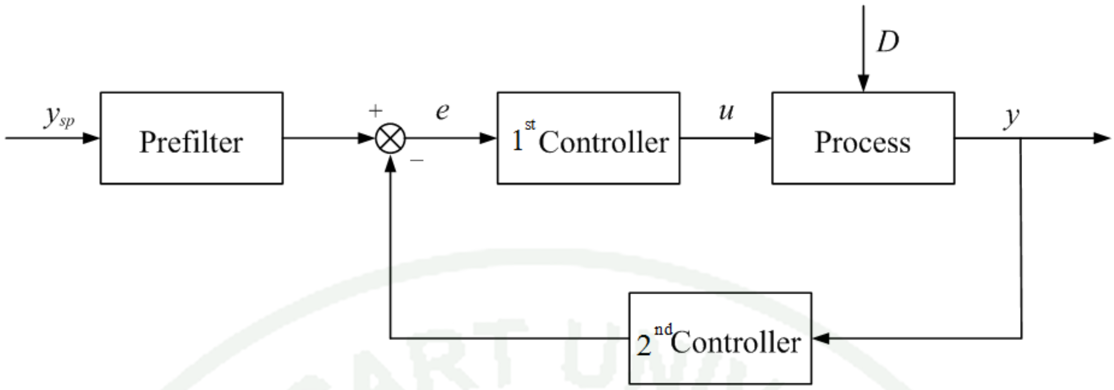

Figure 2 2DOF control structure based on feedback control configuration

In addition, 2DOF control structure based on internal model control (IMC) is also regularly used as shown in Figure 3. The desired process model is introduced into the design of control system for completely eliminating the offset errors caused by uncertainties. In the case of the open-loop unstable processes, the 2DOF control structure based on IMC cannot be implemented directly that some modified IMC methods of 2DOF control (Tan *et al*., 2003) or the 2DOF structure based on feedback control have to be applied instead. However, Wright and Kravaris (2005, 2006) presented the 2DOF structure based on IMC for unstable processes. The control configuration allows replacing unstable poles, while the Luenberger observer is used to eliminate uncertainty.

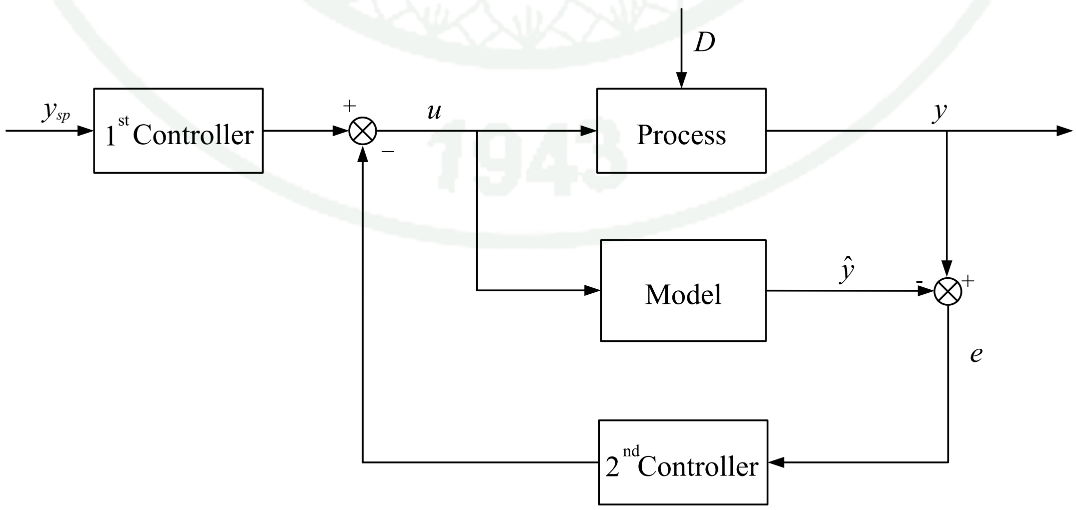

Figure 3 2DOF control structure based on internal model control

4. Control Structure for Time-delay

Time-delay is the property of physical system by which the response to an applied force is delayed in its effect. Whenever material, information, or energy is physically transmitted from one place to another, there is a delay associated with the transmission (Zhong, 2006). The presence of time-delay degrades the control performance and the robustness of the controller. The first time-delay compensator is Smith predictor designed for linear system (Smith, 1957). The classical Smith predictor is shown in Figure 4 where $C(s)$, $P(s)$, and $Z(s)$ are controller, delay-free plant, and Smith predictor, respectively. The Smith predictor for nonlinear system was proposed by Kravaris and Wright (1989) and Huang and Wang (1992). The key concept of Smith predictor is to estimate the future states for controller. Furthermore, a number of 2DOF structures with time-delay compensator have been proposed to handle uncertain time-delay processes. There are developed based on linear system (Zhang *et al*., 1998; Tan *et al*., 2003, 2010; Lu *et al*., 2005; Tsai and Tung, 2010).

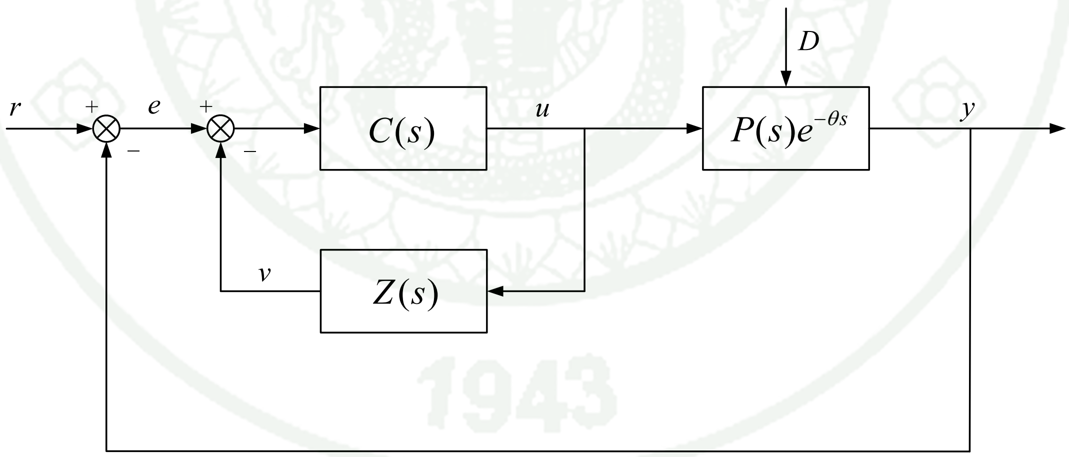

Figure 4 Classical Smith predictor

MATERIALS AND METHODS

Materials

1. Personal Computer with Intel Core 2 Quad CPU at 2.67 GHz, 4.00 GB of RAM running on Microsoft Windows XP Professional x86 (32-bit) Edition

2. Software
   2.1 MATLAB Version R2009a (The MathWorks, Inc.)
   2.2 Wolfram Mathematica Version 7.0.0 (Wolfram Research, Inc.)

Methods

1. The Development of the Control System for Uncertain Processes without Time-delay

The proposed control system is developed and procedures are summarized in a flow diagram as shown in Figure 5. First of all, the desired steady-state should be analyzed the stability of open-loop and zero dynamics to identify that I/O linearization technique can be used. Next, the feedback controller is designed by using the I/O linearization technique for tracking the desired setpoint. The disturbance rejection is developed to eliminate effect of uncertainty despite of the model mismatch or the unmeasured disturbances by using high-gain feedback technique. Then, the open-loop observer is used to design the disturbance-free state estimator for estimating information of the disturbance-free states and outputs. Finally, the control system is applied to the chemical reaction processes by using the simulation technique in order to illustrate the control performance of the developed control system. The details of each step are shown in the following sections.

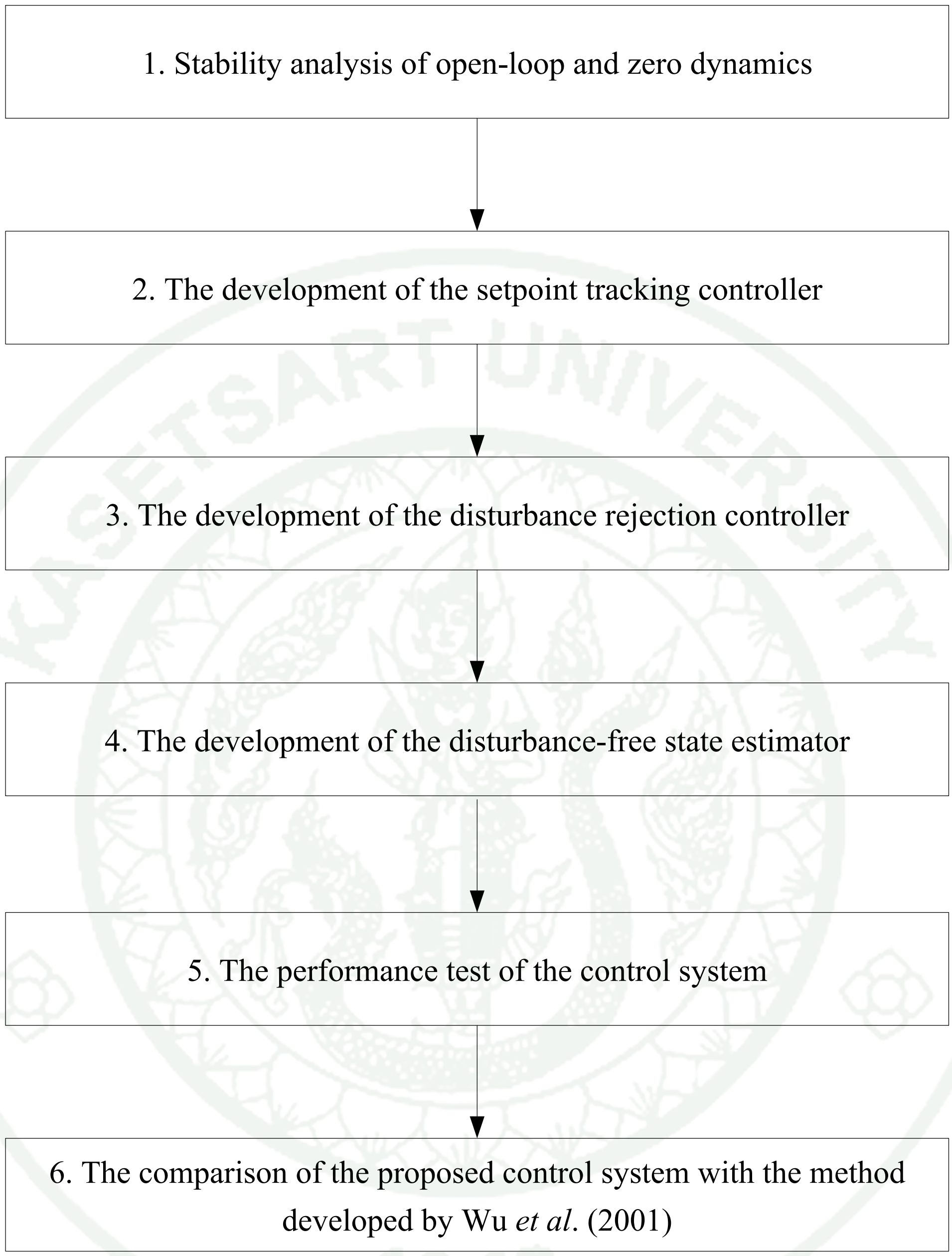

Figure 5 The procedure of the control system for uncertain processes without time-delay

1.1 Mathematical preliminaries

Consider the general class of process model as a nonlinear system of the form:

$$\dot{x} = f(x,u)$$
$$y = h(x) \tag{1}$$

where $x \in R^n$, $u \in R^m$,and $y \in R^m$ denote the vectors of state variables, manipulated inputs, and controlled outputs, respectively, and f and h are assumed to be smooth function.

1.1.1 Relative order

For nonlinear processes in Equation (1), $r_1,...,r_m$, denote the relative orders of the controlled output $y_1,...,y_m$, with respect to the manipulated inputs, where r_i is the smallest integer for which $\partial[d^{r_i} y_i / dt^{r_i}]/\partial u \neq 0$. The following notation is used:

$$\begin{aligned} y_i &= h_i(x) \\ \frac{dy_i}{dt} &= h_i^1(x) \\ &\vdots \\ \frac{d^{r_i-1} y_i}{dt^{r_i-1}} &= h_i^{r_i-1}(x) \\ \frac{d^{r_i} y_i}{dt^{r_i}} &= h_i^{r_i}(x,u) \end{aligned} \tag{2}$$

The following assumptions are made:

1) The relative orders, $r_1,...,r_m$, are definite.

2) The characteristic matrix of the process is non-singular on X × U, which is $\frac{\partial}{\partial u} h^r(x,u) \neq 0$.

3) The process is locally controllable and observable on X × U.

In case that the controlled output, y, does not have a finite relative order ($r = \infty$), the manipulated output, u, will not affect the controlled output, y.

1.1.2 Stability analysis

The stability analysis of an open-loop model is evaluated by Lyapunov's indirect method. To analyze the stability, the eigenvalues of the Jacobian matrix are evaluated around the desired equilibrium pairs (x_{ss},u_{ss}). The eigenvalues are obtained by:

$$|J - \lambda I| = 0 \tag{3}$$

where J is the Jacobian matrix, which $J = \left[\frac{\partial f(x,u)}{\partial x}\right]_{x_{ss},u_{ss}}$, λ is the vector of eigenvalues, and I is the identity matrix.

The character of a steady state can be summarized as follows (Haddad and Chellaboina, 2008):

i) Stable node, if both eigenvalues are real and negative.

ii) Unstable node, if both eigenvalues are real and positive.

iii) Saddle node, if the eigenvalues are real with opposite signs.

iv) Stable focus, if the eigenvalues are complex conjugate with negative real parts.

v) Unstable focus, if the eigenvalues are complex conjugate with positive real parts.

1.1.3 Zero dynamics

The zero dynamics is analyzed to investigate the influence on control system design. The process with unstable zero dynamics exhibits the inverse response behavior (non-minimum phase process), which is one of the limitations in the controller design.

Referring to the inverse process, the nonlinear dialog of the minimum-phase property is related to the stability of the dynamic process:

$$\begin{aligned} \dot{\zeta}_1 &= F_1\left(y, \dot{y}, \ldots, y^{(r-1)}, \zeta_1, \ldots, \zeta_{n-r}\right) \\ &\vdots \\ \dot{\zeta}_{n-r} &= F_{n-r}\left(y, \dot{y}, \ldots, y^{(r-1)}, \zeta_1, \ldots, \zeta_{n-r}\right) \end{aligned} \tag{4}$$

where ζ is the states of zero dynamics

In the case that the output is constrained to a constant value (setpoint, y_{sp}), the zero dynamics is analyzed to determine the stability of the inverse process:

$$\begin{aligned} \dot{\zeta}_1 &= F_1\left(y_{sp}, 0, \ldots, 0, \zeta_1, \ldots, \zeta_{n-r}\right) \\ &\vdots \\ \dot{\zeta}_{n-r} &= F_{n-r}\left(y_{sp}, 0, \ldots, 0, \zeta_1, \ldots, \zeta_{n-r}\right) \end{aligned} \tag{5}$$

If the eigenvalues of the dynamics in (5) lies in the left-half of the complex plane, the process is said to be stable zero dynamics or minimum phase at the desired equilibrium pairs (x_{ss},u_{ss}).

1.2 Setpoint tracking controller

The setpoint tracking controller is designed by using I/O linearization technique. Let us request a linear response of the following form for each output:

$$
\begin{aligned}
(\varepsilon_1 \mathcal{D} + 1)^{r_1} \; y_1 &= y_{sp,1} \\
&\vdots \\
(\varepsilon_m \mathcal{D} + 1)^{r_m} \; y_m &= y_{sp,m}
\end{aligned} \tag{6}
$$

Where $\mathcal{D} = d / dt$ is the differential operator, $y_{sp,1}, \ldots, y_{sp,m}$ are the desired setpoints, $\varepsilon_1, \ldots, \varepsilon_m$ are tuning parameters that adjust the speed of the responses of the outputs, $y_1, \ldots, y_m$, respectively. By substituting the time derivatives of the outputs defined in Equation (2), one obtains:

$$
\begin{aligned}
h_1(x) + \binom{r_1}{1} \varepsilon_1 h_1^1(x) + \cdots + \binom{r_1}{r_1} \varepsilon_1^{r_1} h_1^{r_1}(x,u) &= y_{sp,1} \\
&\vdots \\
h_m(x) + \binom{r_m}{1} \varepsilon_m h_m^1(x) + \cdots + \binom{r_m}{r_m} \varepsilon_m^{r_m} h_m^{r_m}(x,u) &= y_{sp,m}
\end{aligned} \tag{7}
$$

where $\begin{pmatrix} a \\ b \end{pmatrix} = \dfrac{a!}{(b-a)!}$

The closed-loop responses of the outputs in Equation (7) can present in the compact form:

$$
\begin{aligned}
\Phi_1(x,u) &= y_{sp,1} \\
&\vdots \\
\Phi_m(x,u) &= y_{sp,m}
\end{aligned} \tag{8}
$$

By solving the Equation (8) for *u*, the static feedback controller can be obtained in following form:

$$
u = \Psi(x, y_{sp}) \tag{9}
$$

Note that the feedback controller in (9) is limited that the process under consideration must be open-loop stable and minimum phase (stable zero dynamics).

1.3 Disturbance rejection controller

The disturbance rejection controller is used to force the process outputs to approach the estimated disturbance-free outputs. The disturbance rejection controller is constructed based on high-gain technique as the following equation:

$$\begin{aligned} \delta_1 &= K_1\left(y_1 - \hat{y}_1\right) \\ &\vdots \\ \delta_m &= K_m\left(y_m - \hat{y}_m\right) \end{aligned} \tag{10}$$

where $\delta_1,...,\delta_m$ are the estimated disturbances of the outputs $y_1,...,y_m$, $\hat{y}_1,...,\hat{y}_m$ are the estimates of disturbance-free process outputs obtained from the disturbance-free state estimator, and $K_1,...,K_m$ is the tuning parameters that should be selected for stabilizing the uncertain processes.

1.4 Disturbance-free state estimator

The setpoint tracking controller and the disturbance rejection controller require the disturbance-free responses of states and outputs. Thus, the state estimator is applied in the control scheme to provide that information for both controllers by constructing the open-loop observer. The dynamics of the disturbance-free state estimator is described by following equation:

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x},u) \\ \hat{y} &= h(\hat{x}) \end{aligned} \tag{11}$$

where $\hat{x}$ is the vector of estimated states and $\hat{y}$ is the vector of estimated outputs.

1.5 Control system

To ensure offset-free response of the closed-loop system when there are process model mismatch and/or load disturbance in the system, a control system should have integral action. Due to lack in integral action of the state feedback in (9), the feedback compensator is added to eliminate the accumulation of error by adjusting the desired setpoint to the form:

$$v = y_{sp} - (y - \hat{y})$$

where $v \in R^m$ is the vector of compensated setpoints. Hence, the controller with the compensation is denoted by:

$$u = \Psi(\hat{x}, v) \tag{12}$$

By combining the state feedback of (12), estimated disturbances (10), and the disturbance-free states (11), the feedback control system can be described by:

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}, u) \\ u &= \Psi(\hat{x}, y_{sp} - (y - h(\hat{x}))) \\ \delta &= K(y - h(\hat{x})) \\ \tilde{u} &= u - \delta \end{aligned} \tag{13}$$

where $\tilde{u}$ is the vector of compensated inputs. The schematic diagram of the developed control system is shown in Figure 6.

1.6 Tuning criteria

There are two sets of tuning parameters in the control system, the setpoint tracking controller tuning parameters and the disturbance rejection controller tuning parameters. The total number of the tuning parameters is greater than the number of controlled outputs in double. The optimal tuning parameters are obtained by following procedures. First, the control system has to test with a perfect model without the presence of uncertainty. The tuning parameters of the disturbance rejection controller (*K*) are set to be zero, and the setpoint tracking controller tuning parameters (ε) are adjusted until the controller can regulate the process. Note that the values of ε must be greater than zero ($\varepsilon > 0$). A smaller value of ε gives faster responses of output. However, the unsuitable selection of ε can lead to the oscillation of closed-loop output response or closed-loop instability. Now, the control system is ready to apply for the uncertain process. The tuning parameters of the disturbance rejection controller (*K*) are subsequently adjusted to reduce the effect of uncertainty. The tuning parameters, *K,* need to choose to be a positive value when direct control action requires, and vice versa. The magnitude of these tuning parameters varies with the error between the controlled output and the estimated output. Finally, both tuning parameters should be adjusted to achieve the fastest output response and meet the design criteria.

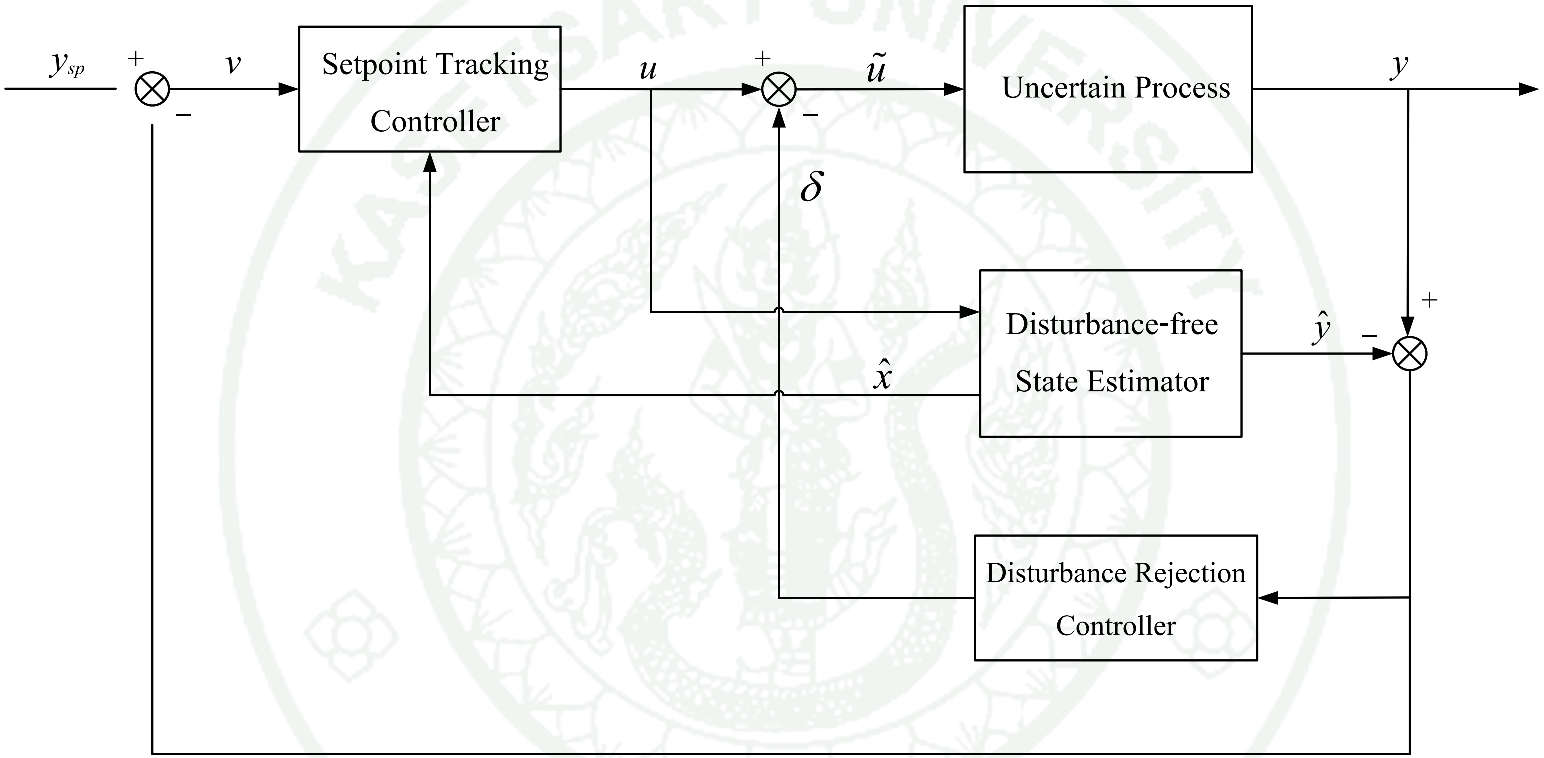

Figure 6 Schematic diagram of the developed control system for uncertain processes without time-delay

2. The Development of the Control System for Uncertain Time-delay Processes

The developed structure in Figure 6 cannot handle a time-delay system. In this section, the proposed system is then reformed by substituting the pervious state estimator with state predictor. The procedure of controller design for time-delay system is summarized in the flow diagram as shown in Figure 7. First of all, the stability of open-loop and zero dynamics of the desired steady-state should be analyzed to identify that I/O linearization technique can be used or not. Next, the setpoint tracking controller is formulated by using the I/O linearization technique for time-delay system. The disturbance rejection is developed by using high-gain feedback technique. Then, the open-loop observer is employed in the design of disturbance-free state predictor. Finally, the control system is applied to the various examples in order to evaluate the control performance. The details of each step are given in the following sections.

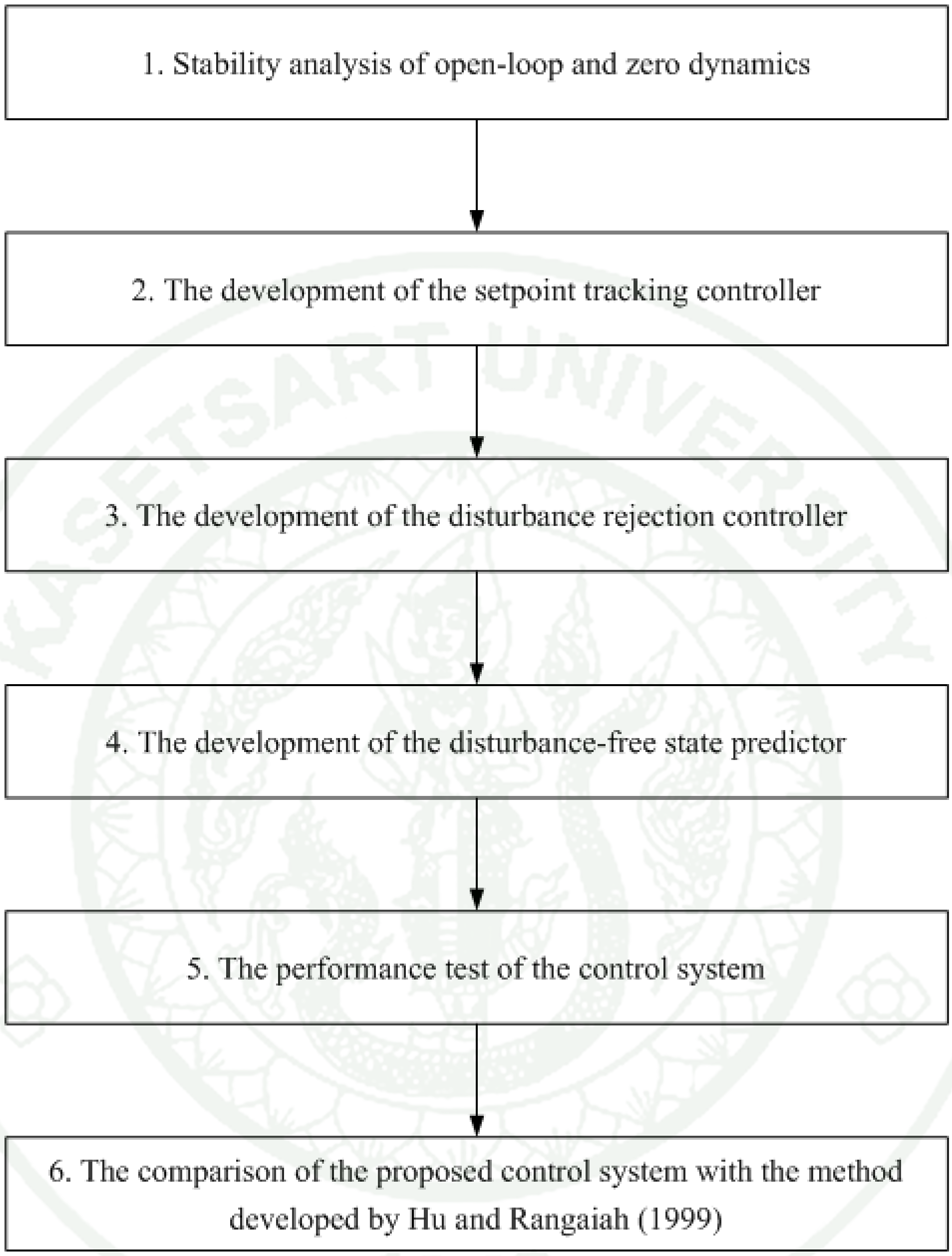

Figure 7 The procedure of the control system for uncertain time-delay processes

2.1 Mathematical preliminaries

Consider the nonlinear system with input time-delay of the following form:

$$\dot{x}(t) = f(x(t), u(t-\theta))$$
$$y(t) = h(x(t)) \tag{14}$$

where $\theta \in R^m$ denotes the input time-delay.

For nonlinear input time-delay processes in Equation (14), the following notation will be used:

$$\begin{aligned} y_i(t) &= h_i(x(t)) \\ \frac{dy_i(t)}{dt} &= h_i^1(x(t)) \\ &\vdots \\ \frac{d^{r_i-1}y_i(t)}{dt^{r_i-1}} &= h_i^{r_i-1}(x(t)) \\ \frac{d^{r_i}y_i(t)}{dt^{r_i}} &= h_i^{r_i}(x(t), u(t-\theta)) \end{aligned} \tag{15}$$

The following assumptions are made:

1) The relative orders, $r_1, \ldots, r_m$, are definite.

2) The characteristic matrix of the process is non-singular on X × U, which is $\frac{\partial}{\partial u} h^r(x(t), u(t-\theta)) \neq 0$.

3) The process (14) is locally controllable and observable on X × U.

2.2 Setpoint tracking controller

The setpoint tracking controller is designed by using I/O linearization control technique. From the closed-loop trajectory in Equation (6), it can be rewritten in the form:

$$
\begin{aligned}
h_1(x(t)) + \binom{r_1}{1}\varepsilon_1 h_1^1(x(t)) + \cdots + \binom{r_1}{r_1}\varepsilon_1^{r_1} h_1^{r_1}(x(t),u(t-\theta)) &= y_{sp,1}(t-\theta) \\
&\vdots \\
h_m(x(t)) + \binom{r_m}{1}\varepsilon_m h_m^1(x(t)) + \cdots + \binom{r_m}{r_m}\varepsilon_m^{r_m} h_m^{r_m}(x(t),u(t-\theta)) &= y_{sp,m}(t-\theta)
\end{aligned} \tag{16}
$$

The closed-loop responses of the outputs in Equation (14) can present in the compact form:

$$
\begin{aligned}
\Phi_1(x(t),u(t-\theta)) &= y_{sp,1}(t-\theta) \\
&\vdots \\
\Phi_m(x(t),u(t-\theta)) &= y_{sp,m}(t-\theta)
\end{aligned} \tag{17}
$$

By solving the Equation (17) for *u*, the static feedback controller can be obtained in following form:

$$u(t-\theta) = \Psi(x(t), y_{sp}(t-\theta)) \tag{18}$$

where all quantities in the right-hand side function are at time *t*. The above equation can be rewritten as:

$$u(t) = \Psi(x(t+\theta), y_{sp}(t)) \tag{19}$$

As described in Section 1.5 that Equation (19) cannot provide offset-free responses, and I/O technique requires information of future values of disturbance-free states. Therefore, the controller is improved to:

$$u(t) = \Psi(x(t+\theta), v(t)) \tag{20}$$

where $v(t) = y_{sp}(t) - (y(t) - \hat{y}(t))$

Note that the feedback controller in (20) is limited that the process under consideration must be open-loop stable and minimum phase (stable zero dynamics).

2.3 Disturbance rejection controller

To design the disturbance rejection controller for time-delay system, the high-gain technique is also applied, which needs information of disturbance-free output to compare with actual outputs. The disturbance rejection controller is constructed as the following equation:

$$\begin{aligned} \delta_1(t) &= K_1\left(y_1(t) - \hat{y}_1(t)\right) \\ &\vdots \\ \delta_m(t) &= K_m\left(y_m(t) - \hat{y}_m(t)\right) \end{aligned} \tag{21}$$

2.4 Disturbance-free state predictor

The dynamics of open-loop state observer is described by following equation:

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t), u(t-\theta)) \\ \hat{y}(t) &= h(\hat{x}(t)) \end{aligned} \tag{22}$$

The controller in Equation (20) requires the future values of disturbance-free states, which cannot be implemented directly with the state estimator in Equation (19). Therefore, prediction model is constructed by removing time-delay shown in Equation (20).

$$\begin{aligned} \dot{x}^*(t) &= f(x^*(t), u(t)) \\ y^*(t) &= h(x^*(t)) \end{aligned} \tag{23}$$

By comparison of Equation (18) and (19), it follows $x^*(t) = \hat{x}(t+\theta)$. If the state predictor is initialized, then, $x^*(0) = \hat{x}(\theta)$.

2.5 Control system

By combining the state feedback of (20), the estimated disturbances (21), and the disturbance-free state predictor (23), the feedback control system can be described by: $u(t) = \Psi(x(t+\theta), v(t))$

$$\begin{aligned} \dot{x}^* &= f(x^*, u) \\ u(t) &= \Psi(x(t+\theta), v(t)) \\ \delta &= K(y - \hat{y}) \\ \tilde{u} &= u - \delta \end{aligned} \tag{24}$$

The schematic diagram of the developed control system is shown in Figure 8. Note that the disturbance rejection controller in (21) essentially requires the present values of estimated outputs. Therefore, the state predictors are appropriately delayed to generate the estimated outputs, $\hat{y}$, for the disturbance rejection controller.

2.6 Tuning criteria

The tuning steps for system with time-delay are similar to the procedure in Section 1.6. First, the control system has to test with a delay model without the presence of uncertainty. The disturbance rejection tuning parameters (K) are set to be zero. The setpoint tracking controller tuning parameters (ε) are then adjusted until the controller can regulate the process. Remind that the values of ε must be greater than zero ($\varepsilon > 0$). The smaller value of the tuning parameter provides the faster the responses of output. However, the unsuitable selection of the tuning parameter values can lead to the oscillation of closed-loop output response or closed-loop instability. Next, the control system will be employed for uncertain time-delay process. The disturbance rejection controller tuning parameters are consequently adjusted to reduce the effect of uncertainty. The tuning parameters, K, should to adjust to be positive and negative values when the control actions are direct and inverse, respectively. The magnitude of these tuning parameters varies with the error between the controlled output and the estimated output. Finally, these tuning parameters are adjusted again to achieve the optimal output response and meet the design criteria.

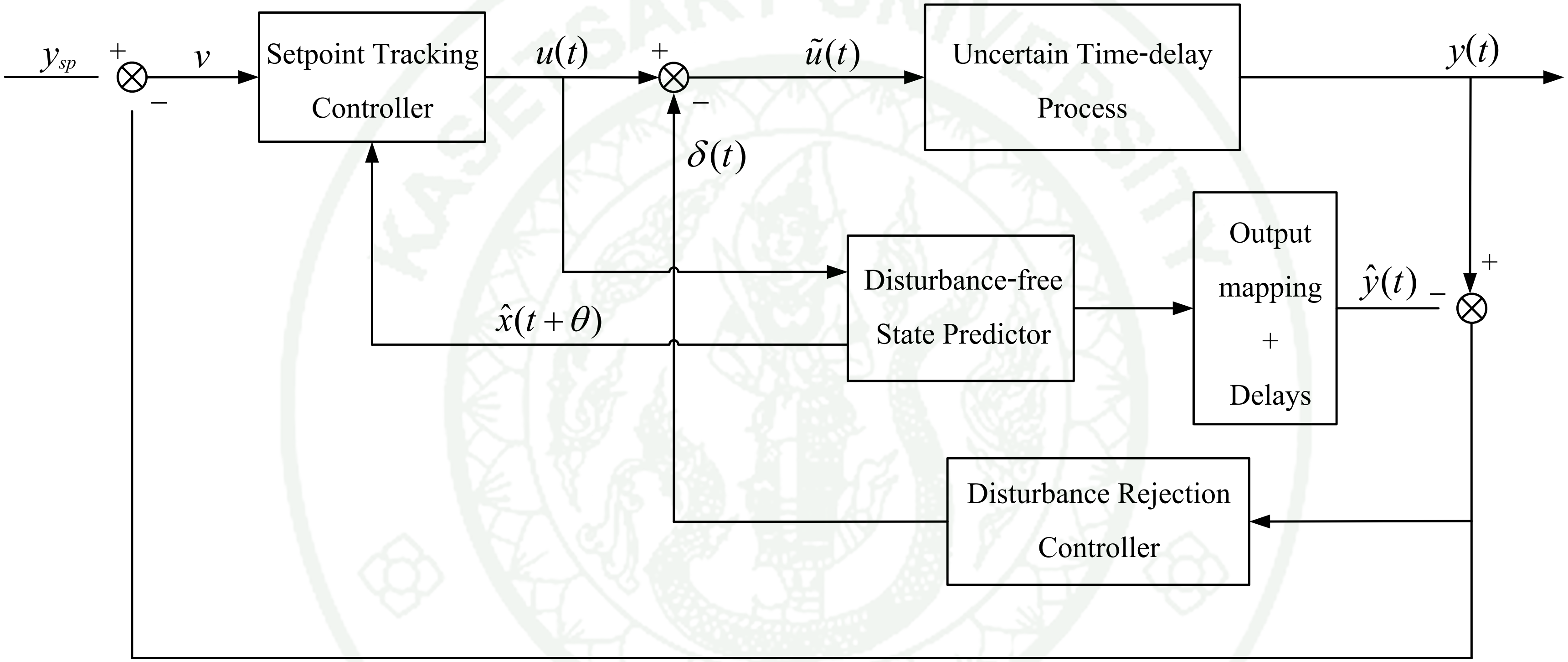

Figure 8 Schematic diagram of the developed control system for uncertain time-delay processes

3. The Performance Test of the Control System

The applications for the presented control technique are illustrated through various simulations of the chemical reaction processes, which are a single continuous stirred tank reactor (CSTR) and two CSTRs connected in series. The feedback controller equations are solved by Mathematica software while the simulations are carried out by using MATLAB software. The control performance test is divided into two types, the servo and the regulatory. The servo performance is tested by adjusting the input setpoint to the new value, while the regulatory performance is tested by introducing the uncertainty to the process. In this work, the full-state measurement is available, and the uncertainty is considered in unmeasured input disturbance and random noise parametric uncertainty.

4. The Comparison of the Developed Control System Performance

The proposed control method is compared with the existing controllers to guarantee the performance of the developed control system. The presented structure without time-delay is compared with the 2DOF structure with P-controller proposed by Wu *et al*. (2001). For the structure with time-delay, the internal model control with time-delay compensator presented by Hu and Rangaiah (1999) is selected to compare the performance.

RESULTS AND DISCUSSION

The illustrative examples for uncertain processes without and without time-delay are shown in this section. The single-input-single-output (SISO) continuous stirred tank reactor (CSTR) with various reactions and multiple-input-multiple-output (MIMO) two CSTRs are used as the illustrative examples. The proposed control method is evaluated under the servo and the regulatory performance and also compared with the method of Wu *et al.* (2001) and the method of Hu and Rangaiah (1999).

1. Illustrative Examples for Uncertain Processes without Time-delay

1.1 Single-input-single-output reactor with isothermal Van de Vusse reaction

1.1.1 Process description

Consider a CSTR shown in Figure 9. The reaction is isothermal Van de Vusse reaction, which consists of two decomposition reactions of A taking place in parallel.

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C \qquad\qquad 2A \xrightarrow{k_3} D$$

The reactor volume and physical parameters are assumed to be constant and the reactor is operated under the perfect mixing condition. The mathematical model of the reactor can be expressed by

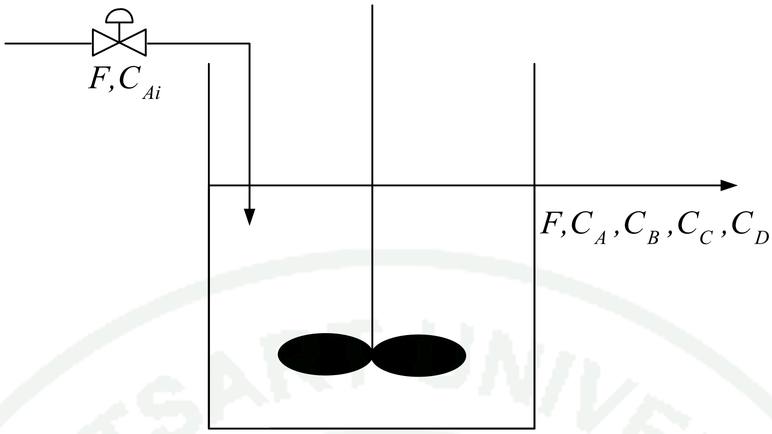

Figure 9 Schematic of the continuous stirred tank reactor

$$\frac{dC_A}{dt} = -k_1 C_A - k_3 C_A^2 + (C_{Ai} - C_A)\frac{F}{V}$$
$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B - C_B \frac{F}{V}$$

where C_A and C_B are the concentration of *A* and *B*, respectively. In this study, it is desired to control C_B at the setpoint by manipulating the dilution rate (*F/V*). The values of the process parameters are given in Table 1.

Table 1 Parameter values for the example of SISO reactor with isothermal Van de Vusse reaction

Symbol	Quantity	Value
$C_{A,i}$	Concentration of *A* in feed stream	10 mol L^{-1}
k_1	Rate constant of $A \rightarrow B$	5/6 min^{-1}
k_2	Rate constant of $B \rightarrow C$	5/3 min^{-1}
k_3	Rate constant of $2A \rightarrow D$	1/6 mol L^{-1} min^{-1}

Source: Aufderheide and Bequette (2003)

By applying the proposed control method in Equation (13) with the relative order of output, $r = 1$, the equations of the control system are shown in Appendix B.

The servo and the regulatory performance are tested in the following subsection with the initial condition $C_{A,ss}$ = 8.2 mol L^{-1}, $C_{B,ss}$ = 0.586 mol L^{-1}, and $(F/V)_{ss}$ = 10 min^{-1}, and the tuning parameters ε = 0.6 and K = -120.

1.1.2 Servo performance

The process starts at the initial condition, and it is desired to control at the setpoint, y_{sp} = 0.99 mol L^{-1} corresponding to $C_{A,ss}$ = 6.7 mol L^{-1}, $C_{B,ss}$ = 0.99 mol L^{-1}, and $(F/V)_{ss}$ = 4 min^{-1}. The open-loop dynamic behavior at the steady-state condition is analyzed by eigenvalues of Jacobian matrix of both open-loop process and zero dynamics, which are shown in Table 2. Figure 10 depicts the closed-loop responses of the controlled output, state variable, and the manipulated input under the servo test. The simulation results show that the controller can force C_B to the desire setpoint.

Table 2 The open-loop dynamics behavior analysis of the example of SISO reactor with isothermal Van de Vusse reaction

Steady-state pair	Eigenvalues of Jacobian matrix		Dynamics behavior	Condition
$[C_{A,ss}, C_{B,ss}, (F/V)_{ss}]$	Process	Zero		
[8.2, 0.586, 10]	-11.67, -13.57	-11	SMP	IC
[6.7, 0.99, 4]	-5.67, -7.07	-4.26	SMP	SP

Note: SMP = Stable minimum phase, IC = Initial condition, SP = Desired setpoint

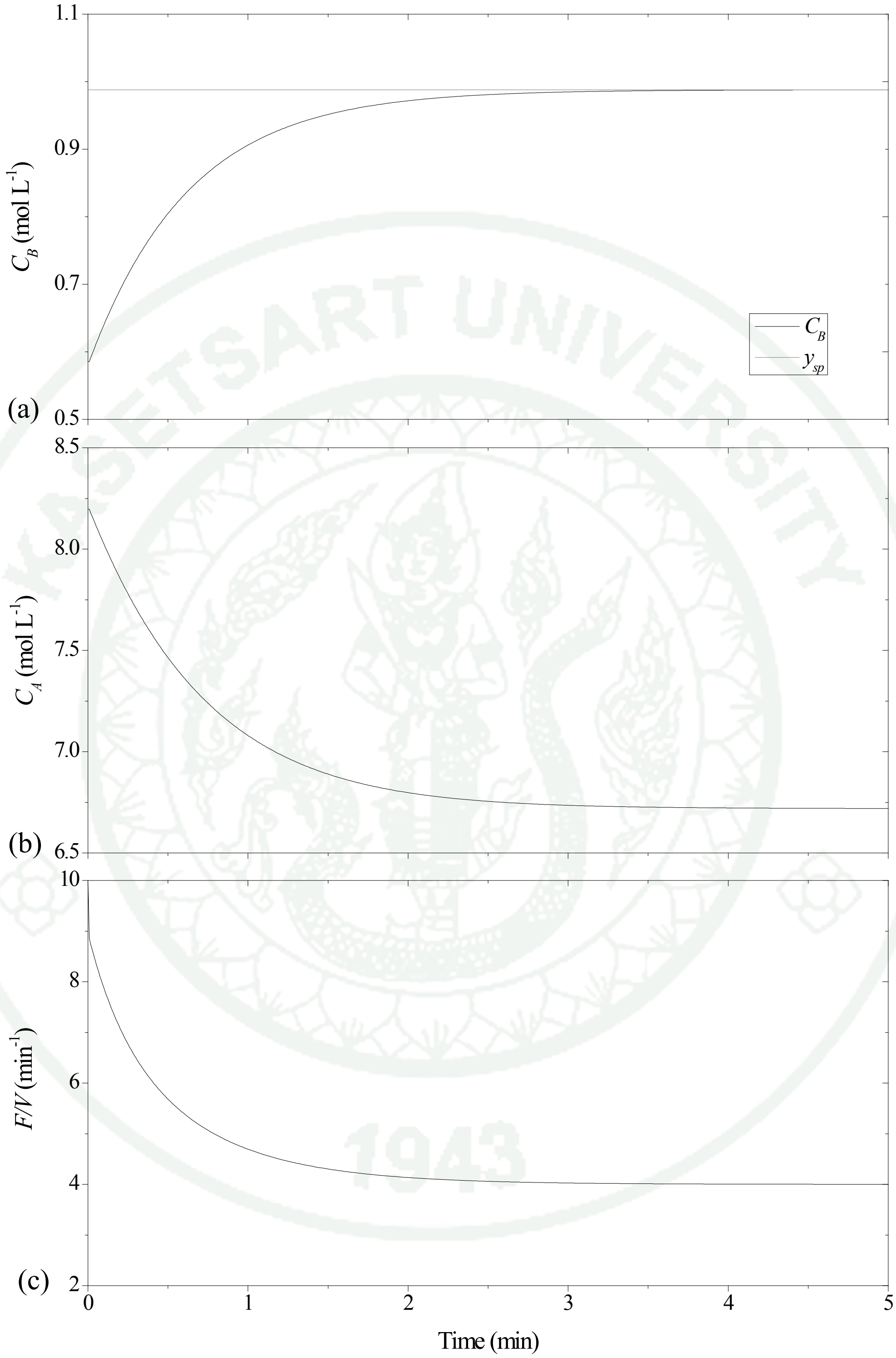

Figure 10 Closed-loop responses of (a) C_B, (b) C_A, and (c) F/V under the servo test

1.1.3 Regulatory performance

The proposed control technique is applied through the CSTR with uncertainty to control the concentration of *B* at desired setpoint. To investigate the control performance, a step change of unmeasured disturbance in manipulated input and random noise of parametric uncertainty in k_1 are considered. Under the given conditions, the capability of the disturbance rejection controller has been tested, thus, the proposed 2DOF structure and the feedback controller of I/O linearization with IMC structure (1DOF) are compared.

1.1.3.1 Unmeasured disturbance in *F/V*

The process is initially at y_{sp} = 0.586 mol L^{-1} and maintains at that setpoint throughout the process. +1 min^{-1} step change of the unmeasured disturbance in the manipulated input, *F/V*, is introduced at t = 0. Figure 11 shows the responses of the controlled output, the state variable, and the manipulated input in the presence of unmeasured disturbance, respectively. The simulation results show that the proposed method can eliminate for the unmeasured disturbance and offers the better responses, which are smaller overshoot and spending less time for compensation.

1.1.3.2 Parametric uncertainty in k_1

The process starts at y_{sp} = 0.586 mol L^{-1} and maintains at that setpoint throughout the process. ± 20% random noise parametric uncertainty in rate constant k_1 is introduced at t = 0. Figure 12 shows the responses of the controlled output, the state variable, and the manipulated input, respectively, in the presence of parametric uncertainty. As shown in Figure 12, the proposed method provides better responses of controlled output with small oscillation compared with I/O linearization with IMC structure.

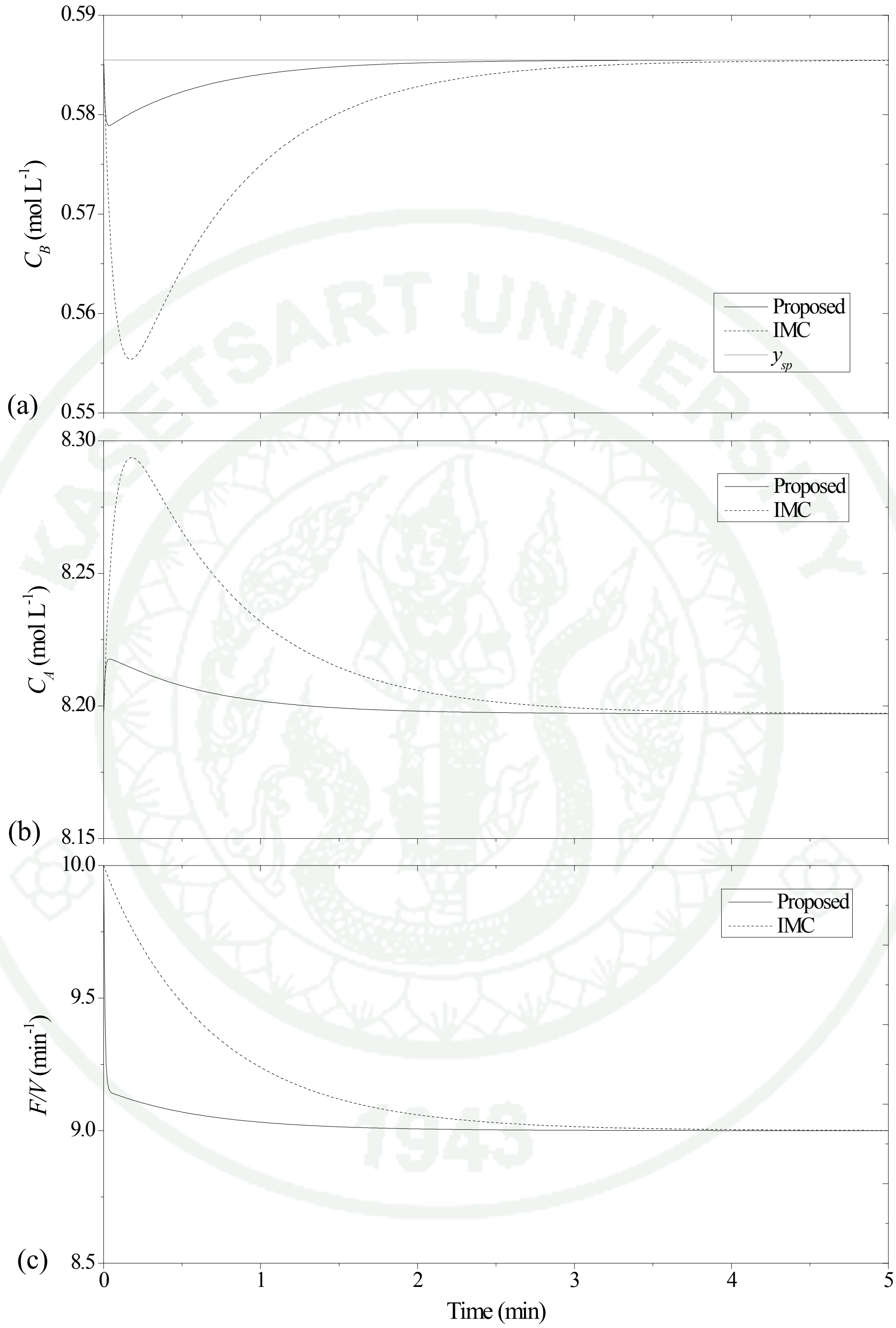

Figure 11 Closed-loop responses of (a) C_B, (b) C_A, and (c) F/V under the case of unmeasured disturbance in F/V in SISO CSTR with isothermal Van de Vusse reaction

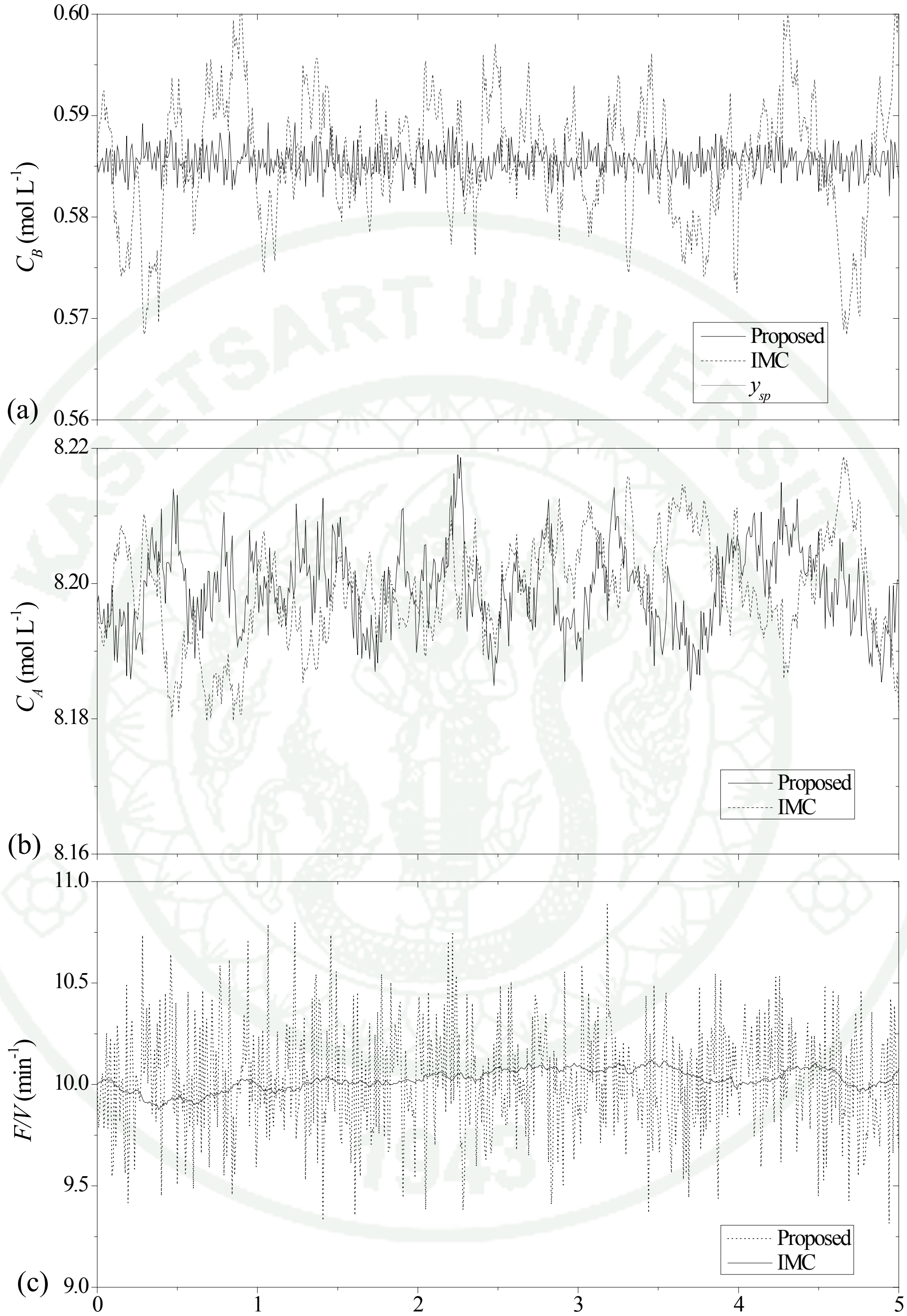

Figure 12 Closed-loop responses of (a) C_B, (b) C_A, and (c) F/V under the case of parametric uncertainty in k_1 in SISO CSTR with isothermal Van de Vusse reaction

1.2 Multiple-input-multiple-output two reactors in series

1.2.1 Process description

The proposed control methodology is now applied to MIMO system shown in Figure 13. The process consists of two jacketed CSTRs in series, where reaction $A \rightarrow B$ takes place in the first reactor, and reaction $B + C \rightarrow D$ takes place in the second reactor. Both reactions are exothermic, irreversible reaction. The inlet jacket temperature of both reactors and the feed flow rate of C in the second reactor can be manipulated to control outlet reactor temperature of each reactor and outlet concentration of C from the second reactor. The mathematical model is then given by:

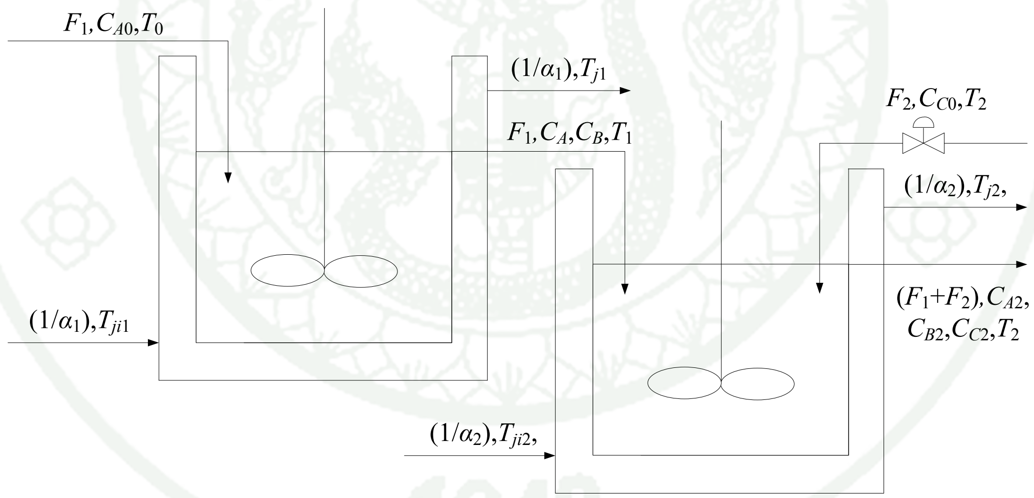

Figure 13 Schematic of two continuous stirred tank reactors in series

$$\frac{dT_1}{dt} = \frac{F_1}{V_1}(T_0 - T_1) - \frac{UA_1}{V_1 \rho C_p}(T_1 - T_{j1}) + \frac{(-\Delta H_1)}{\rho C_p} k_{01} C_{A1} \exp\left(-\frac{E_1}{RT_1}\right)$$

$$\frac{dC_{A1}}{dt} = \frac{F_1}{V_1}(C_{A0} - C_{A1}) - k_{01} C_{A1} \exp\left(-\frac{E_1}{RT_1}\right)$$

$$\frac{dC_{B1}}{dt} = \frac{F_1}{V_1} C_{B1} + k_{01} C_{A1} \exp\left(-\frac{E_1}{RT_1}\right)$$

$$\frac{dT_{j1}}{dt} = \frac{F_{j1}}{V_{j1}}(T_{ji1} - T_{j1})$$

$$\frac{dT_2}{dt} = \frac{F_1}{V_2}(T_1 - T_2) - \frac{UA_2}{V_2 \rho C_p}(T_2 - T_{j2}) + \frac{(-\Delta H_1)}{\rho C_p} k_{01} C_{A2} \exp\left(-\frac{E_1}{RT_2}\right) +$$

$$\frac{(-\Delta H_2)}{\rho C_p} k_{02} C_{B2} C_{C2} \exp\left(-\frac{E_2}{RT_2}\right)$$

$$\frac{dC_{A2}}{dt} = \frac{F_1}{V_2} C_{A1} - \frac{F_1 + F_2}{V_2} C_{A2} - k_{01} C_{A2} \exp\left(-\frac{E_1}{RT_2}\right)$$

$$\frac{dC_{B2}}{dt} = \frac{F_1}{V_2} C_{B1} - \frac{F_1 + F_2}{V_2} C_{B2} + k_{01} C_{A2} \exp\left(-\frac{E_1}{RT_2}\right) - k_{02} C_{B2} C_{C2} \exp\left(-\frac{E_2}{RT_2}\right)$$

$$\frac{dC_{C2}}{dt} = \frac{F_2}{V_2} C_{C0} - \frac{F_1 + F_2}{V_2} C_{C2} - k_{02} C_{B2} C_{C2} \exp\left(-\frac{E_2}{RT_2}\right)$$

$$\frac{dT_{j2}}{dt} = \frac{F_{j2}}{V_{j2}}(T_{ji2} - T_{j2})$$

where the value used for each parameter is given in Table 3. The process is operated under the proposed control system. The state variables, controlled outputs, and manipulated inputs are defined as $x = [T_1\ C_{A1}\ C_{B1}\ T_{j1}\ T_2\ C_{A2}\ C_{B2}\ C_{C2}\ T_{j2}]^T$, $y = [T_1\ T_2\ C_{C2}]^T$ and $u = [T_{ji1}\ T_{ji2}\ F_2]^T$, respectively.

By applying the proposed control method in Equation (13) with the relative orders of outputs, $\{r_1 = 2, r_2 = 2, r_3 = 1\}$, the equations of the control system are shown in Appendix C.

Table 3 Parameter values for the example of MIMO two jacketed CSTRs in series

Symbol	Quantity	Value
V_1	Volume of the first reactor	1 m^3
V_2	Volume of the second reactor	1.5 m^3
UA_1	Heat transfer coefficient of the first reactor	1.4 kJ s^{-1} K^{-1}
UA_2	Heat transfer coefficient of the second reactor	1.75 kJ s^{-1} K^{-1}
F_{j1}/V_{j1}	The first feed flowrate of coolant per volume	300 s
F_{j2}/V_{j2}	The second feed flowrate of coolant per volume	300 s
$-\Delta H_1$	The change in enthalpy of the first reaction	46,000 kJ $kmol^{-1}$
$-\Delta H_2$	The change in enthalpy of the second reaction	20,000 kJ $kmol^{-1}$
k_{01}	Arrhenius factor of the first reaction	1.8×10^7 s^{-1}
k_{02}	Arrhenius factor of the second reaction	2.0×10^6 kmol m^{-3} s^{-1}
E_1	Activation energy of the first reaction	67,000 kJ $kmol^{-1}$
E_2	Activation energy of the second reaction	60,000 kJ $kmol^{-1}$
C_{A0}	Concentration of *A* in the first feed stream	7.5 kmol m^{-3}
C_{C0}	Concentration of *C* in the second feed stream	20 kmol m^{-3}
ρ	Density of coolant	1,000 kg m^{-3}
C_p	Heat capacity of coolant	4.2 kJ kg^{-1} K^{-1}
T_0	Temperature in the first feed stream	300 K
R	Gas constant	8.345 kJ $kmol^{-1}$ K^{-1}
F_1	Flowrate of the first feed stream	6.5×10^{-4} m^3 s^{-1}

Source: Wright and Kravaris (2003)

The servo and the regulatory performance are tested in the following subsection with the initial condition $T_{1,ss}$ = 311.7 K, $C_{A1,ss}$ = 6.36 kmol m^{-3}, $C_{B1,ss}$ = 1.14 kmol m^{-3}, $T_{j1,ss}$ = 310 K, $T_{2,ss}$ = 384.4 K, $C_{A2,ss}$ = 0.17 kmol m^{-3}, $C_{B2,ss}$ = 0.65 kmol m^{-3}, $C_{C2,ss}$ = 0.28 kmol m^{-3}, and $T_{j2,ss}$ = 345.8 K. The corresponding initial values of three inputs are $T_{ji1,ss}$ = 310 K, $T_{ji2,ss}$ = 345.8 K, and $F_{2,ss} = 2.2\times 10^{-4}$ m^3 s^{-1}.

The sets of tuning parameters are {ε_1 = 700, ε_2 = 500, ε_3 = 800} and {K_1 = 50, K_2 = 30, K_3 = 0.003}.

1.2.2 Servo performance

The process starts at the initial condition, and it is desired to control the outputs at the setpoint, $y_{sp,1}$ = 344, $y_{sp,2}$ = 381.8, and $y_{sp,3}$ = 0.48. The corresponding steady-state and the open-loop dynamics behavior condition are analyzed by eigenvalues of Jacobian matrix of both open-loop process and zero dynamics, which are shown in Table 4. Figure 14 shows the responses of controlled outputs. Figures 15 and 16 depict the responses of state variables. Figure 17 demonstrate the responses of manipulated inputs under the servo test. As seen in the Figures, that the controller can drive the temperatures of both reactors and concentration of *C* in the second reactor to each desire setpoint.

1.2.3 Regulatory performance

The proposed control technique is applied through the process with uncertainty to control the temperatures of both reactors and concentration of *C* in the second reactor at each desired setpoint. To evaluate the control performance, step change of unmeasured disturbances in manipulated input and random noise of parametric uncertainties are considered. Under the given condition, the capability of the disturbance rejection controller has been tested, thus, the proposed 2DOF structure and a feedback controller developed by using I/O linearization with IMC structure (1DOF) are compared.

Table 4 The open-loop dynamics behavior analysis of the example of MIMO two jacketed reactors in series

Steady-state pair [$T_{1,ss}$, $C_{A1,ss}$, $C_{B1,ss}$, $T_{j1,ss}$, $T_{2,ss}$, $C_{A2,ss}$, $C_{B2,ss}$, $C_{C2,ss}$, $T_{j2,ss}$, $T_{ji1,ss}$, $T_{ji2,ss}$, $F_{2,ss}$]	Eigenvalues of Jacobian matrix		Dynamics behavior	Condition
	Process	Zero		
[311.7, 6.36, 1.14, 310, 384.4, 0.17, 0.65, 0.28, 345.8, 310, 345.8, 2.2×10^{-4}]	-0.016, -0.012, -0.003, -0.003, -7.4×10^{-4}, -6.5×10^{-4}, -5.8×10^{-4}, $-5.38\times10^{-4}\pm(1.63\times10^{-4})i$	-6.5×10^{-4}, -7.67×10^{-4}, -1.47×10^{-3}, -1.59×10^{-2}	SMP	IC
[344, 2.48, 5, 322.6, 381.8, 0.07, 0.46, 0.48, 350, 322.6, 350, 2.41×10^{-4}]	-0.014, -0.011, -0.003, -0.003, -7.2×10^{-4}, -6.5×10^{-4}, -5.9×10^{-4}, $-2.62\times10^{-4}\pm(5.37\times10^{-4})i$	-6.5×10^{-4}, -1.96×10^{-3}, -5.36×10^{-4}, -1.38×10^{-2}	SMP	SP

Note: SMP = Stable minimum phase, IC = Initial condition, SP = Setpoint

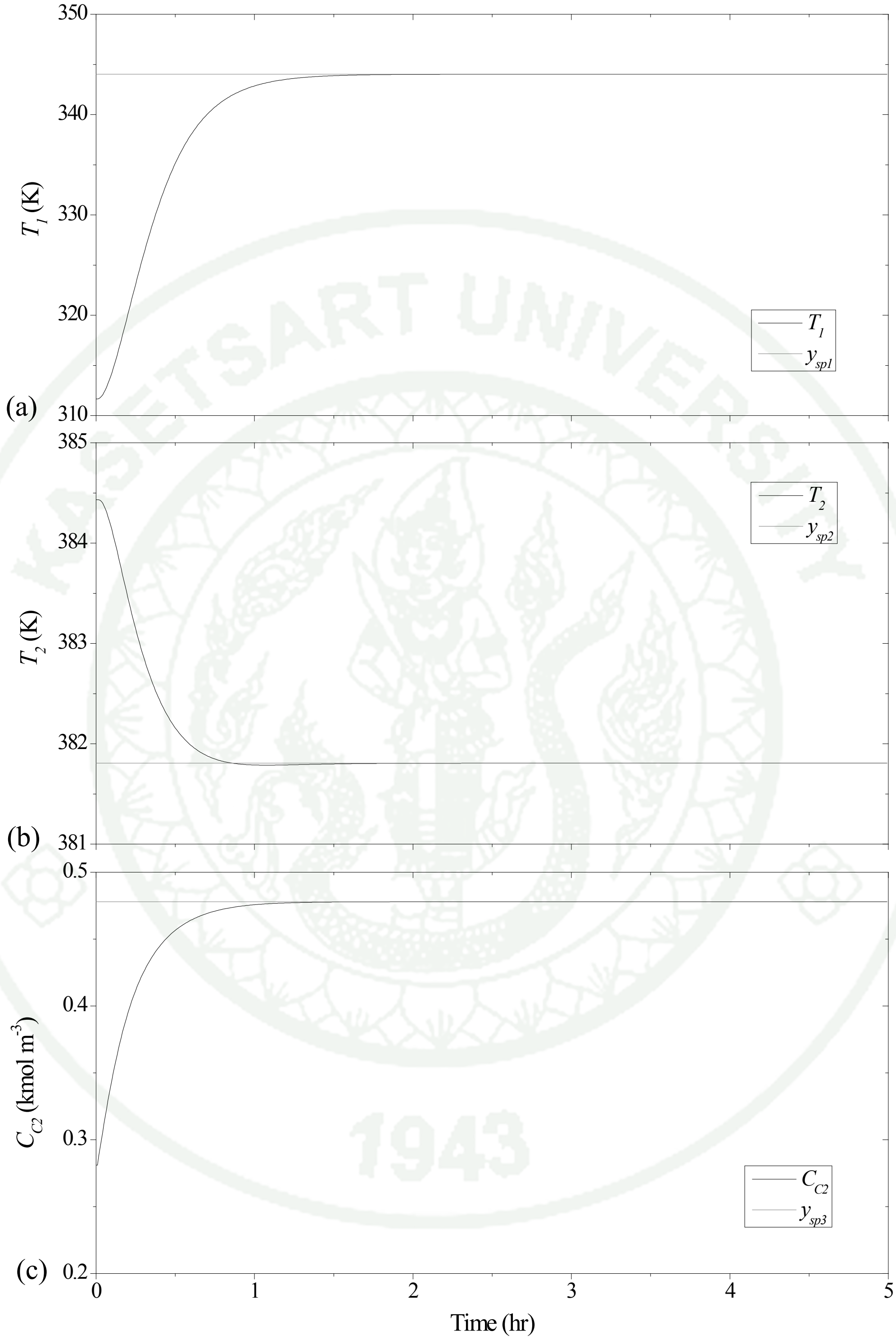

Figure 14 Closed-loop responses of (a) T_1, (b) T_2, and (c) C_{C2} under the servo test in MIMO two CSTRs

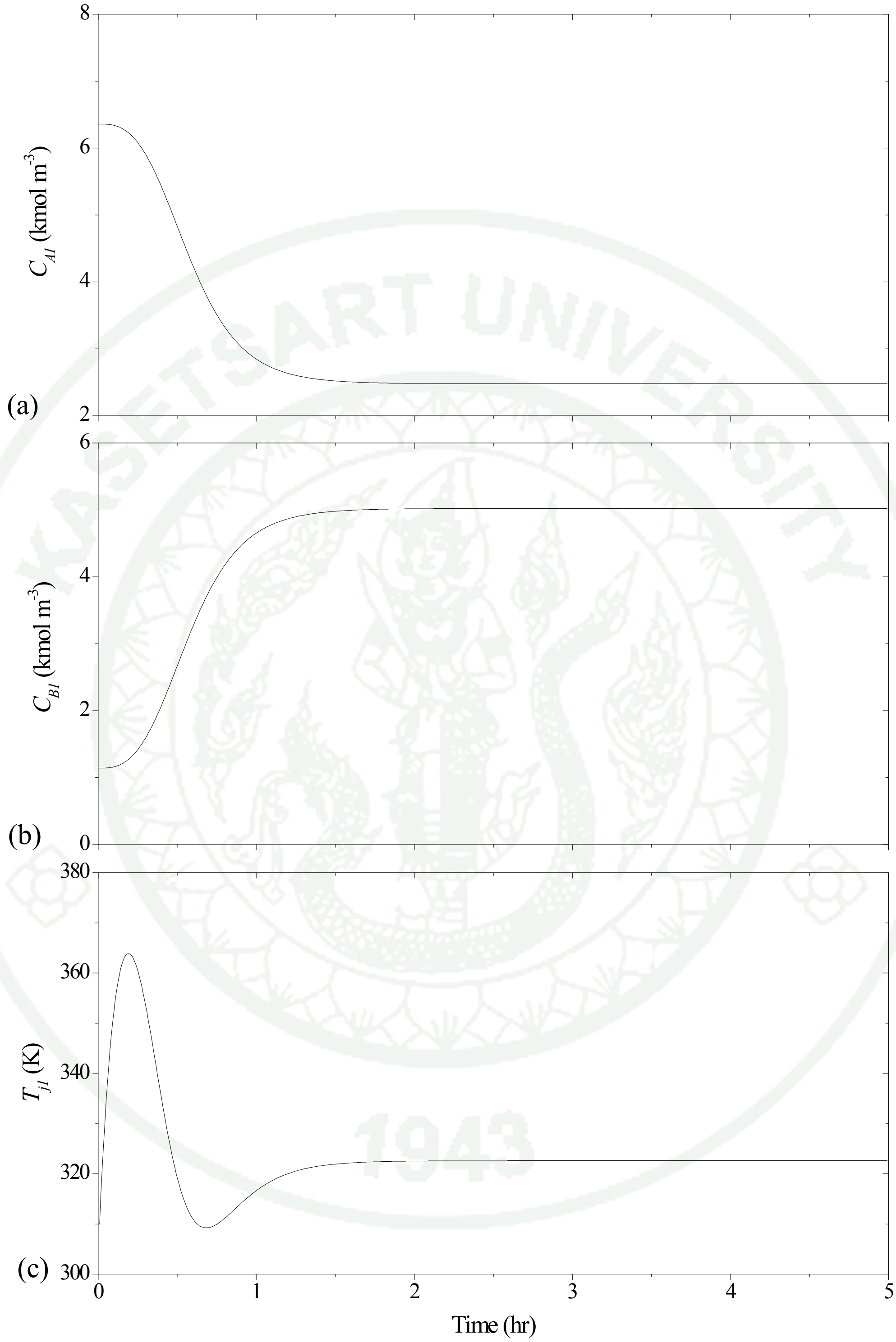

Figure 15 Closed-loop responses of (a) C_{A1}, (b) C_{B1}, and (c) T_{j1} under the servo test in MIMO two CSTRs

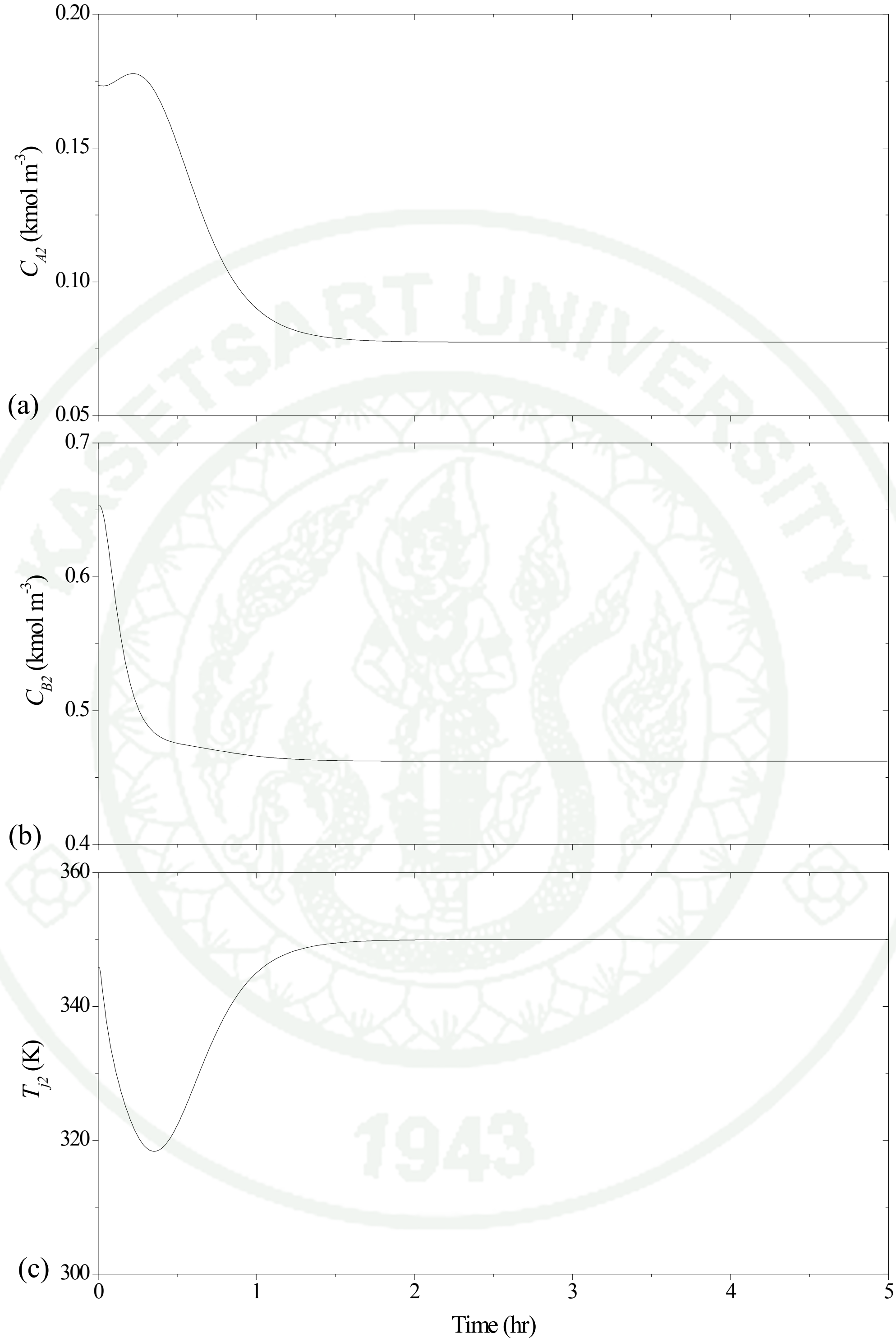

Figure 16 Closed-loop responses of (a) C_{A2}, (b) C_{B2}, and (c) T_{j2} under the servo test in MIMO two CSTRs

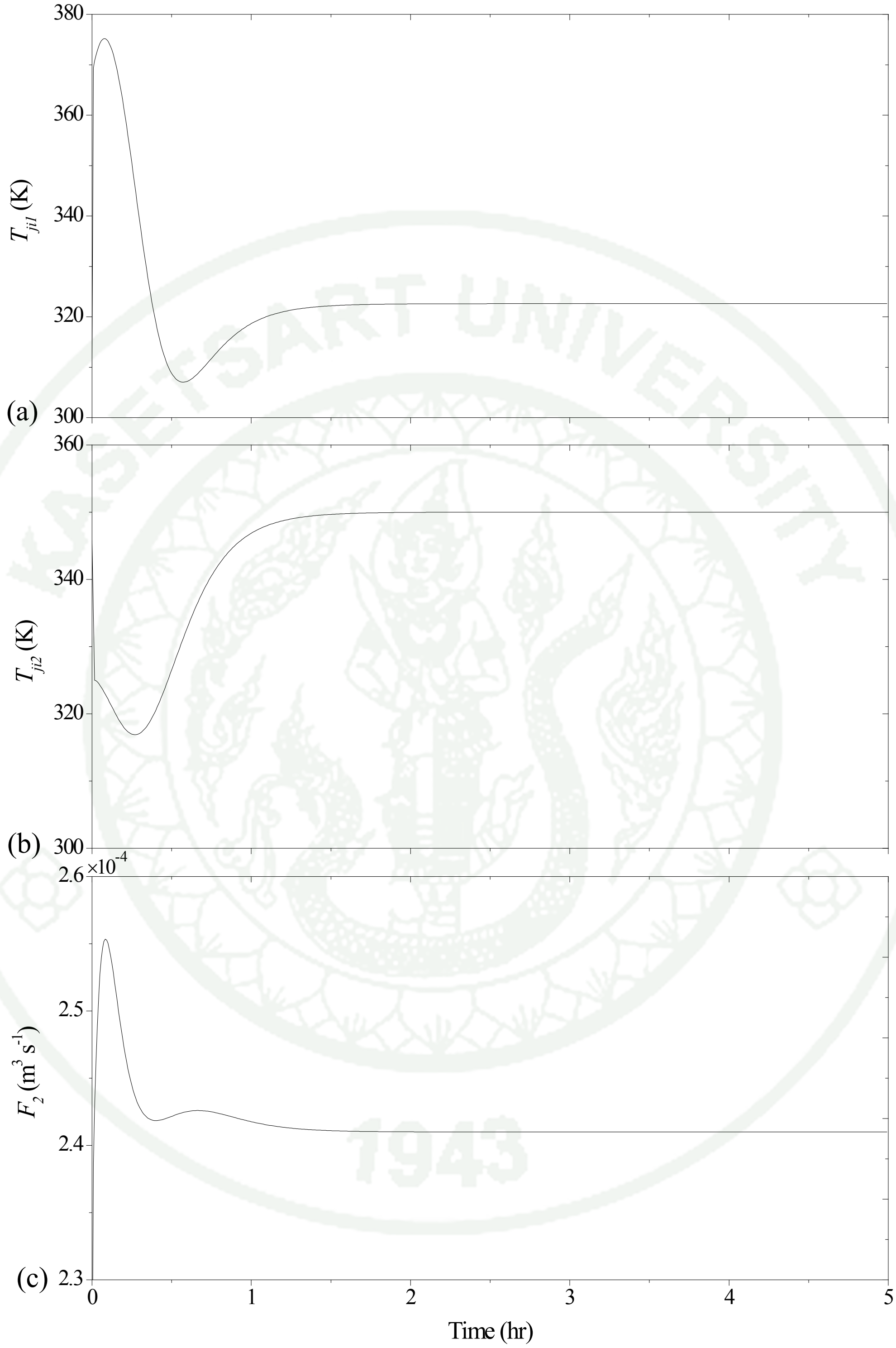

Figure 17 Closed-loop responses of (a) T_{ji1}, (b) T_{ji2}, and (c) F_2 under the servo test in MIMO two CSTRs

1.2.3.1 Unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2

The process is initially at $y_{sp,1}$ = 311.7, $y_{sp,2}$ = 384.4, and $y_{sp,3}$ = 0.28, and maintains at that setpoint throughout the process. +5 K, +2 K, and $+2\times10^{-5}$ m^3 s^{-1} step change of the unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2, respectively, are considered at t = 0. Figure 18 shows the responses of controlled outputs. Figures 19 and 20 depict the responses of state variables. Figure 21 demonstrates the responses of manipulated inputs in the presence of unmeasured disturbances. The simulation results indicate that the disturbance rejection controller can compensate for all unmeasured disturbances and gives the better responses compared with I/O linearization with IMC structure, in which the output responses show small overshoot with a less time to reach the setpoints. In Figure 21, the responses of manipulated inputs show the oscillation because the controller maintains the controlled outputs at the setpoint all the time.

1.2.3.2 Parametric uncertainty in ΔH_1 and ΔH_2

The process is initially at $y_{sp,1}$ = 311.7, $y_{sp,2}$ = 384.4, and $y_{sp,3}$ = 0.28. To test the robustness of the proposed control method, ± 20% random noise parametric uncertainties in the heat of reactions (ΔH_1 and ΔH_2) are introduced at t = 0. Figure 22 shows the responses of controlled outputs. Figures 23 and 24 depict the responses of state variables, and Figure 25 demonstrates the responses of manipulated inputs in the presence of parametric uncertainties. The simulation results show that both methods can maintain the controlled outputs around the desired setpoints. However, the proposed method provides the better performances, which are the smaller oscillation of the output responses compared with IMC.

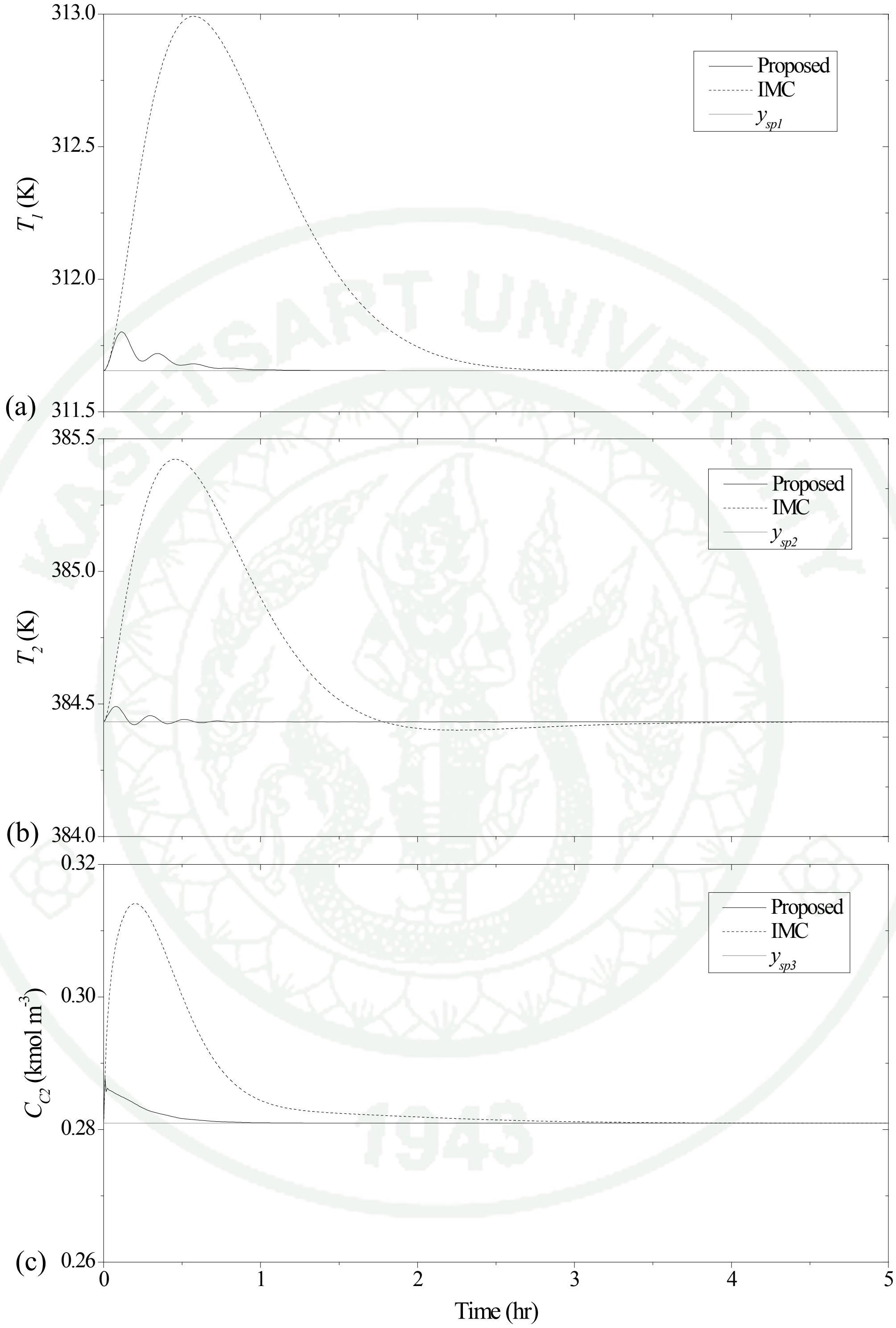

Figure 18 Closed-loop responses of (a) T_1, (b) T_2, and (c) C_{C2} under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

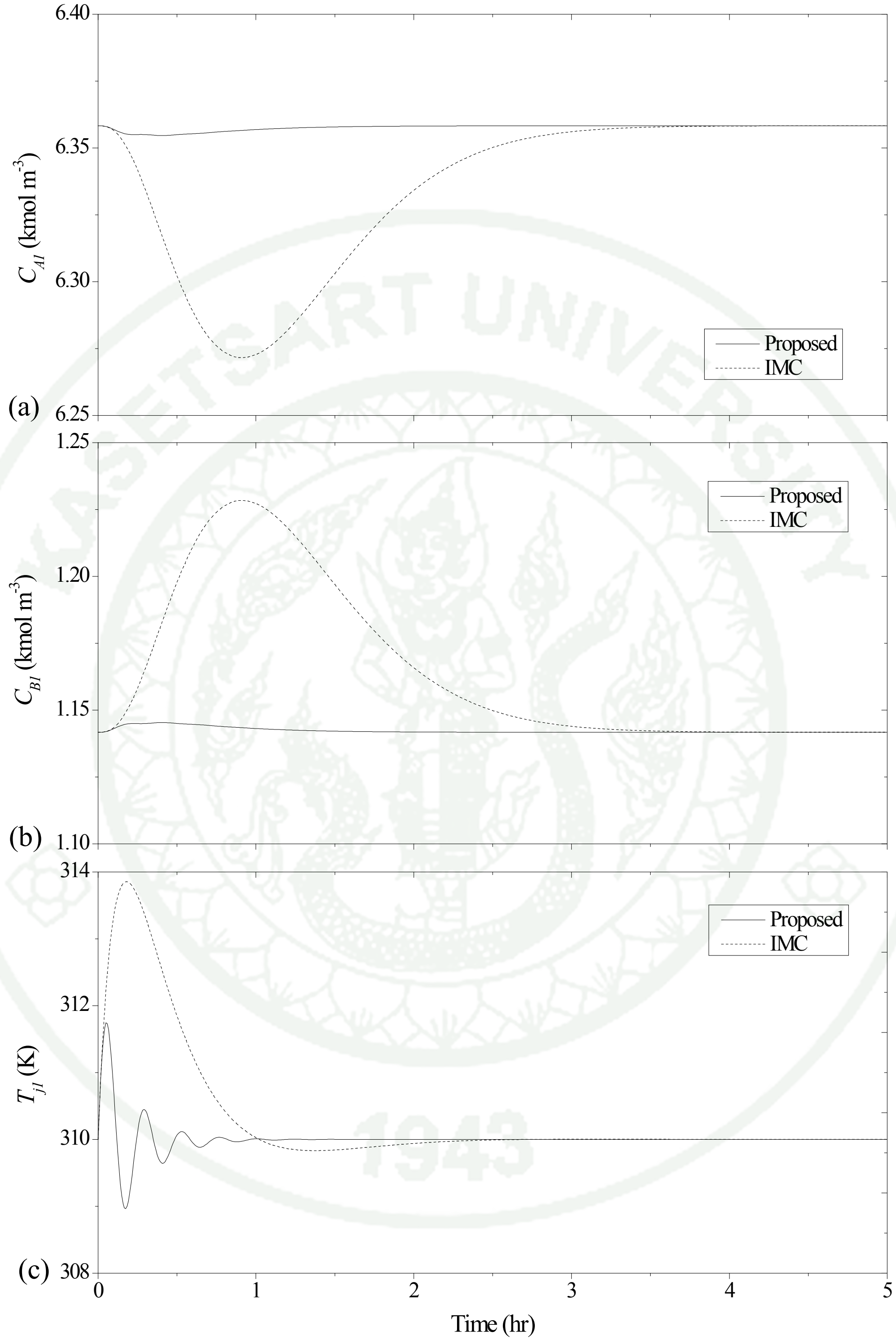

Figure 19 Closed-loop responses of (a) C_{A1}, (b) C_{B1}, and (c) T_{j1} under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

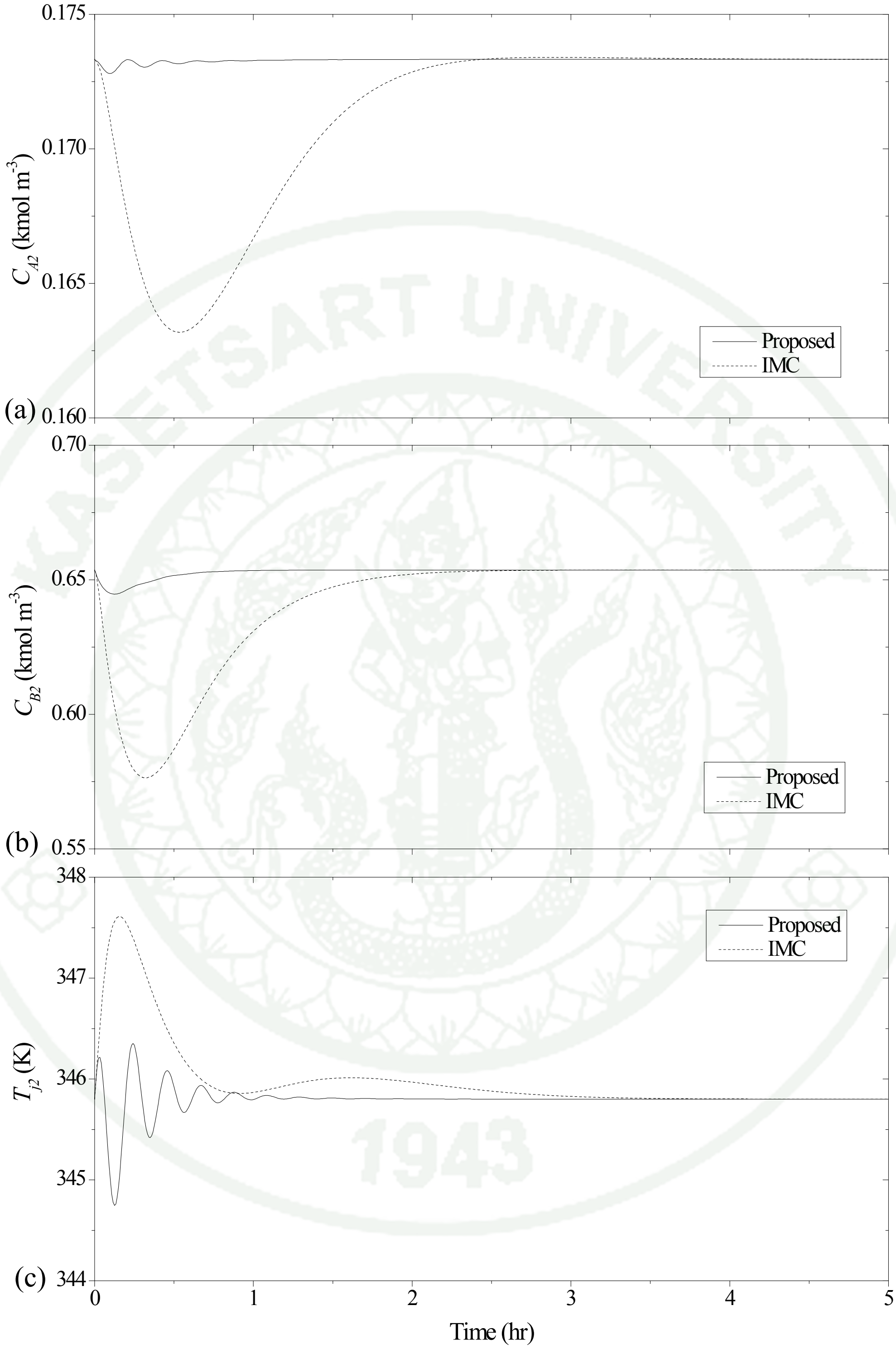

Figure 20 Closed-loop responses of (a) C_{A2}, (b) C_{B2}, and (c) T_{j2} under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

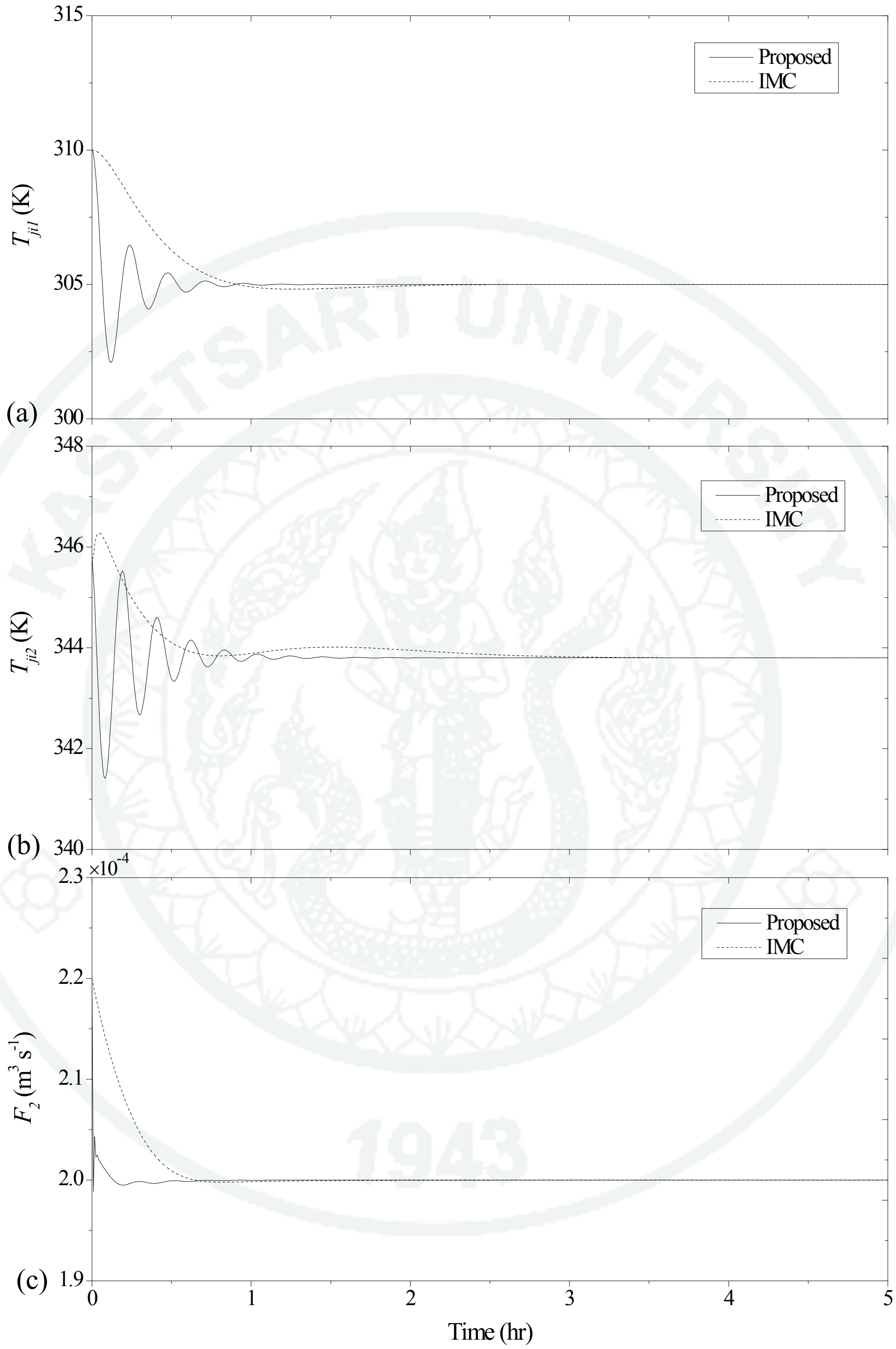

Figure 21 Closed-loop responses of (a) T_{ji1}, (b) T_{ji2}, and (c) F_2 under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

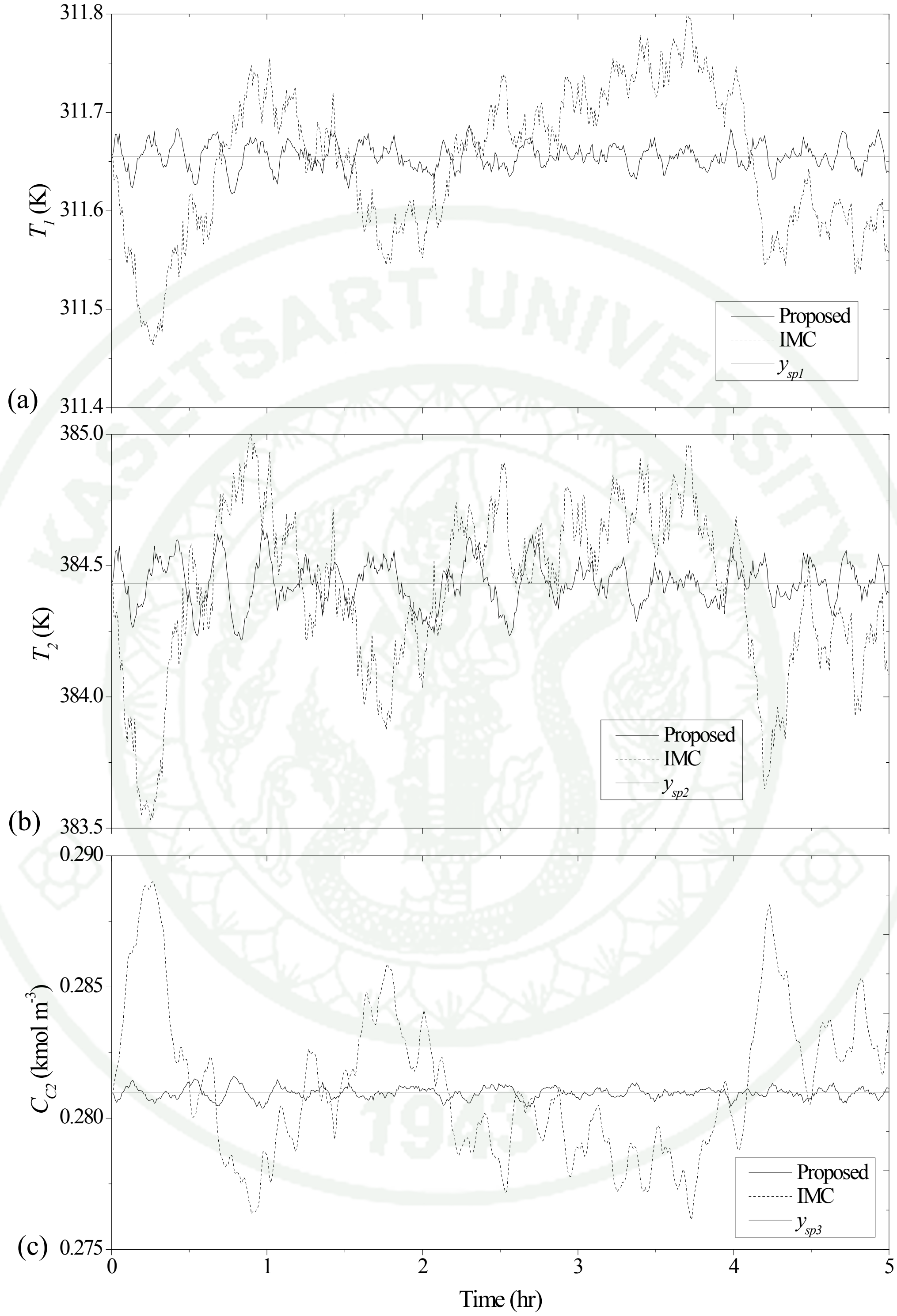

Figure 22 Closed-loop responses of (a) T_1, (b) T_2, and (c) C_{C2} under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

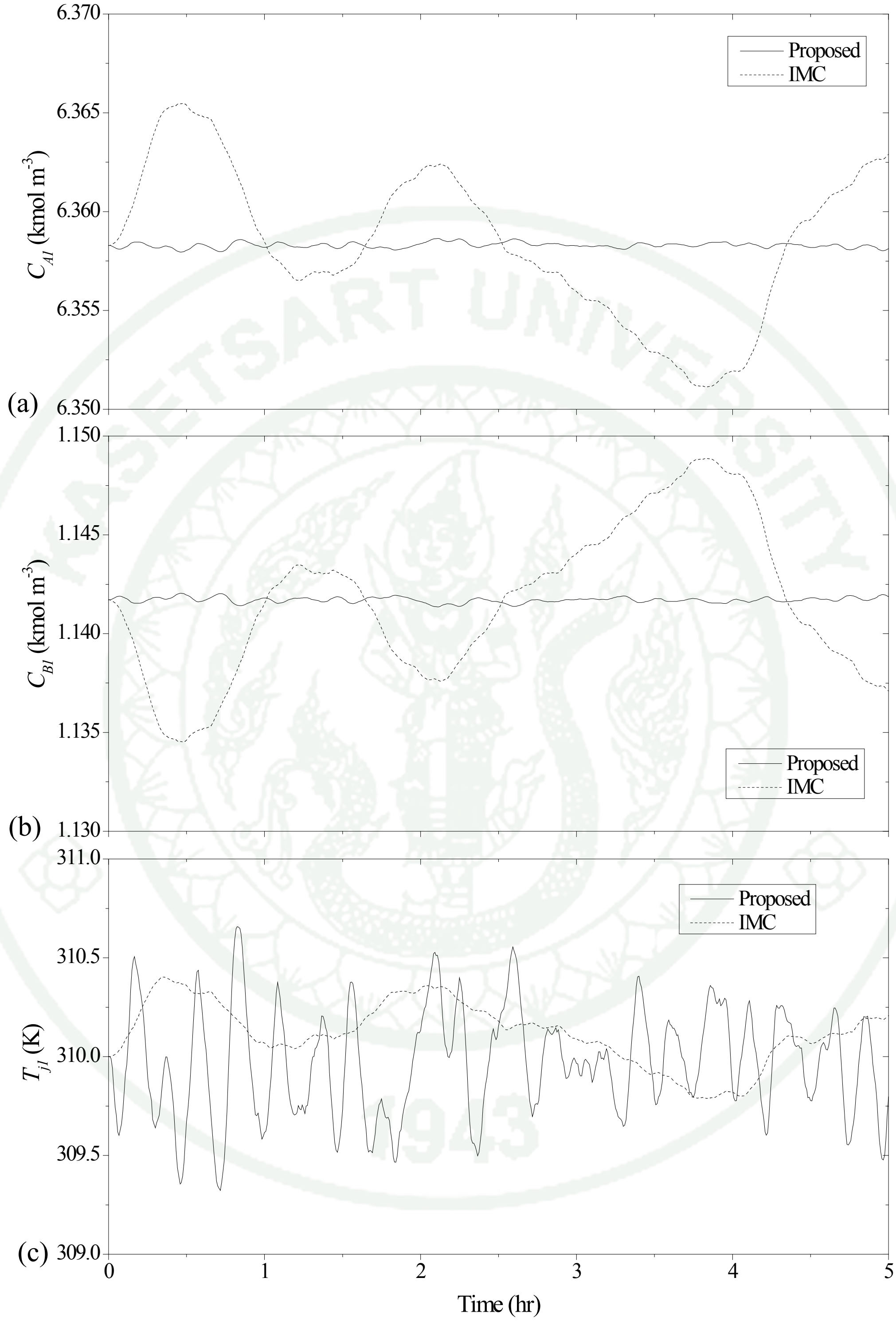

Figure 23 Closed-loop responses of (a) C_{A1}, (b) C_{B1}, and (c) T_{j1} under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

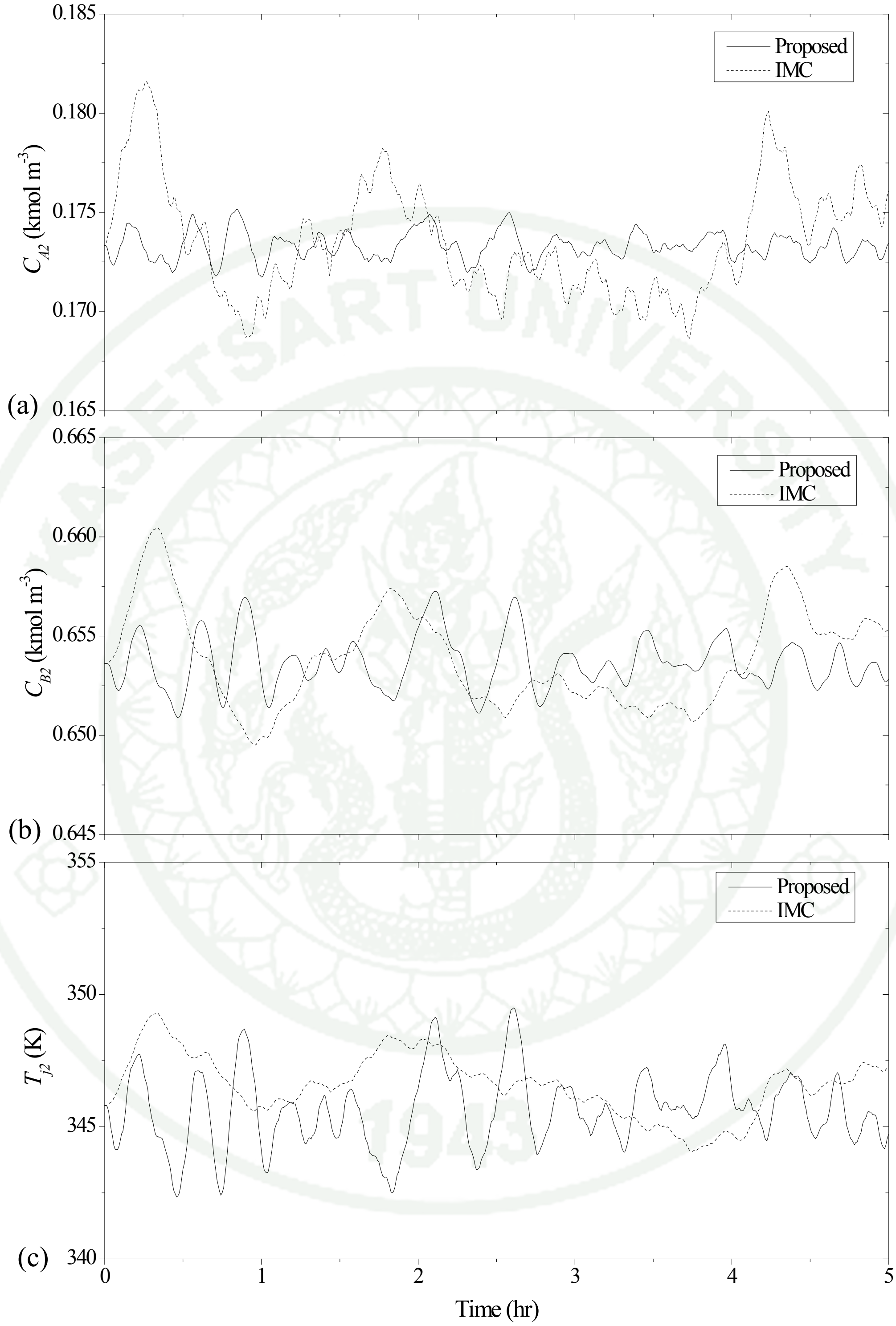

Figure 24 Closed-loop responses of (a) C_{A2}, (b) C_{B2}, and (c) T_{j2} under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

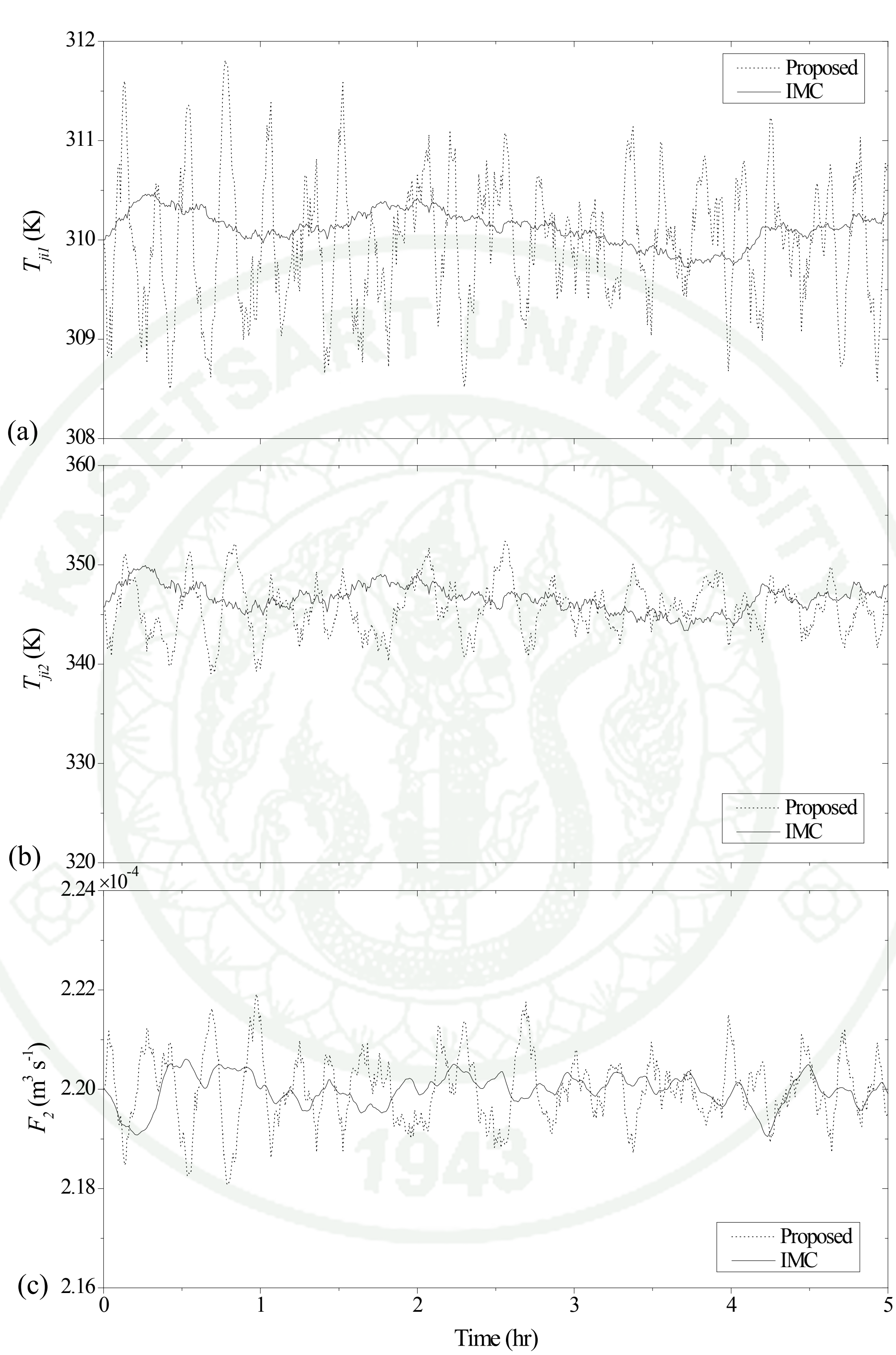

Figure 25 Closed-loop responses of (a) T_{ji1}, (b) T_{ji2}, and (c) F_2 under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

1.3 Comparison of performance with a 2DOF structure with P-controller

To compare the control performance of the proposed control scheme, the 2DOF control structure with P-controller proposed by Wu *et al.* (2001) shown in Figure 26 is considered, where $G(s)$ and $\hat{G}(s)$ denote the process plant and nominal model. $Y_{sp}(s)$, $Y(s)$, $U(s)$, $\Delta(s)$, $\tilde{U}(s)$, $D(s)$ are the Laplace transform of the setpoint (y_{sp}), output (y), input from the feedback controller (u), estimated disturbance (δ), compensated input ($\tilde{u}$), and load disturbance (D). The constant gains, K_C and K_F, are employed for the setpoint tracking and load disturbance rejection. To illustrate performance, the application of the chemical reactor is proposed in the following section.

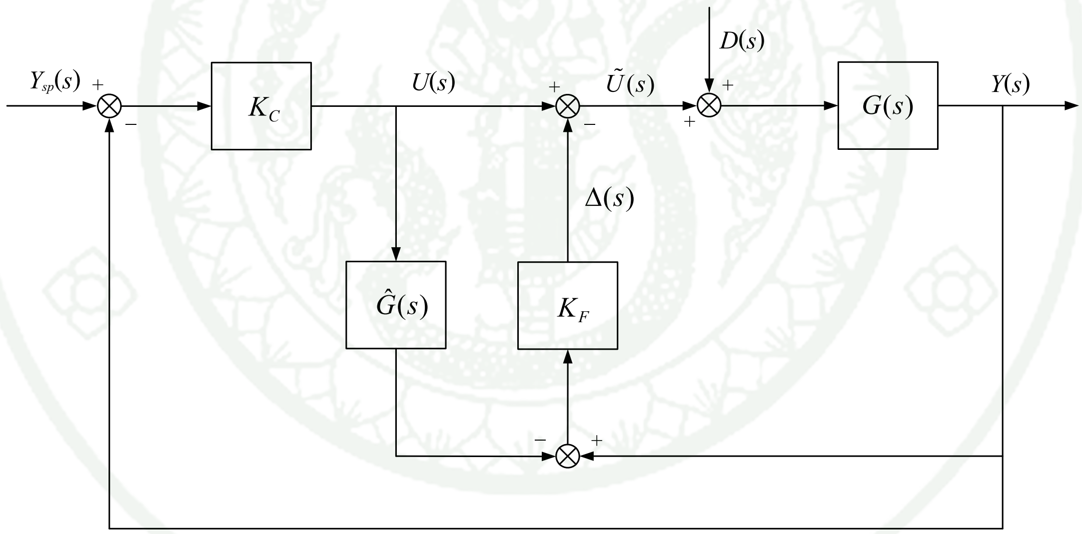

Figure 26 Schematic diagram of 2DOF structure proposed by Wu *et al.* (2001)

1.3.1 Process description

The simple case of CSTR with constant heat removal is applied and shown in Figure 27. The reaction is an exothermic, irreversible, first-order reaction of $A \rightarrow B$ and takes place in the liquid phase. The reactor volume and physical parameters are assumed to be constant and the reactor is operated in perfect mixing condition. The equations that govern the system are:

$$\frac{dC_A}{dt} = -k_0 \exp\left(-\frac{E_a}{RT}\right)C_A + (C_{Ai} - C_A)\frac{F}{V}$$
$$\frac{dT}{dt} = \gamma k_0 \exp\left(-\frac{E_a}{RT}\right)C_A + (T_i - T)\frac{F}{V} + Q$$

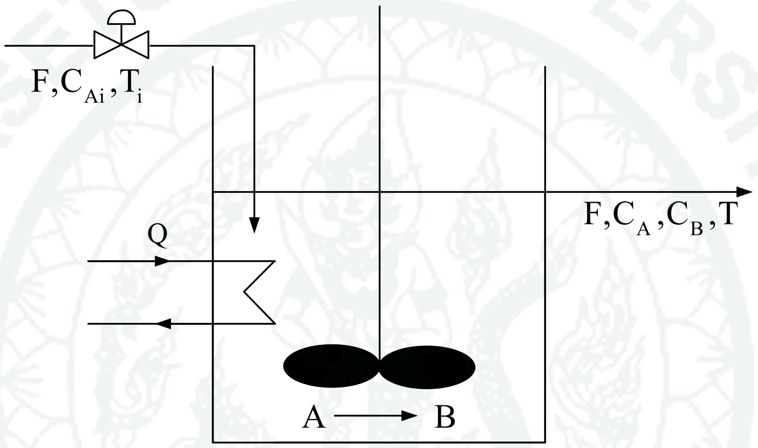

Figure 27 Schematic of continuous stirred tank reactor with a constant heat removal

where C_A is the concentration of A and T is the reactor temperature. The objective is to control T by manipulating the feed flowrate, F. The values of the process parameters and nominal operating conditions are given in Table 5.

By applying the proposed control method in Equation (13) with the relative order of output, $r = 1$, the equations of the control system are shown in Appendix D.

The control performance is tested in the following subsection with the initial condition $C_{A,ss}$ = 11.1 kmol m^{-3}, T_{ss} = 273.2 K, and F_{ss} = 0.3 m^3 hr^{-1}. The tuning parameters of the proposed scheme are chosen as ε = 0.7 and K = 0.6. For comparing with Wu's control method, the constant gains K_C and K_F are selected to be

0.04 and 0.6, respectively. The process is transformed to Laplace domain as following:

$$G(s) = \frac{267.62s + 875.53}{s^2 + 5.11s + 6.33}$$

Table 5 Parameter values for the example of SISO reactor with a constant heat removal

Symbol	Quantity	Value
$C_{A,i}$	Concentration of *A* in feed stream	12 kmol m^{-3}
T_i	Temperature in feed stream	300 K
k_0	Arrhenius factor	1.8×10^{12} hr^{-1}
E_a/R	Activation energy/gas constant	8,100 K
Q	Heat removal rate of cooling system	-90.7 K hr^{-1}
V	Volume of reactor	0.1 m^3
γ	Specific heat of reaction	3.9 K.m^3 $kmol^{-1}$

1.3.2 Control performance

In this case, the servo and the regulatory performances are tested simultaneously due to the similarity in the design of the disturbance rejection method. The process is initially at initial condition. First, the desired setpoint of the process is $y_{sp,1}$ = 283.3 K ($C_{A,ss}$ = 10.2 kmol m^{-3},T_{ss} = 283.3 K, and F_{ss} = 0.38 m^3 hr^{-1}), then the setpoint changes to $y_{sp,2}$ = 293.9 K ($C_{A,ss}$ = 8.4 kmol m^{-3}, T_{ss} = 293.9 K, and F_{ss} = 0.45 m^3 hr^{-1}) at t = 6 hr. The open-loop dynamic behavior at the steady-state condition is analyzed by eigenvalues of Jacobian matrix of both open-loop process and zero dynamics, which are shown in Table 6. The performance and the robustness are tested by adding +0.02 m^3 hr^{-1} step disturbance in the inlet flow rate. Figure 28 depicts the closed-loop responses of the controlled output, state variable, and the manipulated input under the servo test. The simulation results show that the proposed method can drive the concentration of *B* to the desire setpoint without offset, while the

Wu's method cannot achieve. Moreover, the responses of proposed method in Figure 28 (c) are more smooth than Wu's method.

Table 6 The open-loop dynamics behavior analysis of the example of SISO reactor with a constant heat removal

Steady-state pair $[C_{A,ss}, T_{,ss}, F_{ss}]$	Eigenvalues of Jacobian matrix		Dynamics behavior	Condition
	Process	Zero		
[11.1, 273.2, 0.3]	-3, -2.11	-3.27	SMP	IC
[10.2, 283.3, 0.38]	-3.8, -1.72	-4.77	SMP	SP1
[8.4, 293.9, 0.45]	-4.5, -0.5	-10.87	SMP	SP2

Note: SMP = Stable minimum phase, IC = Initial condition, SP1 = The first setpoint, SP2 = The second setpoint

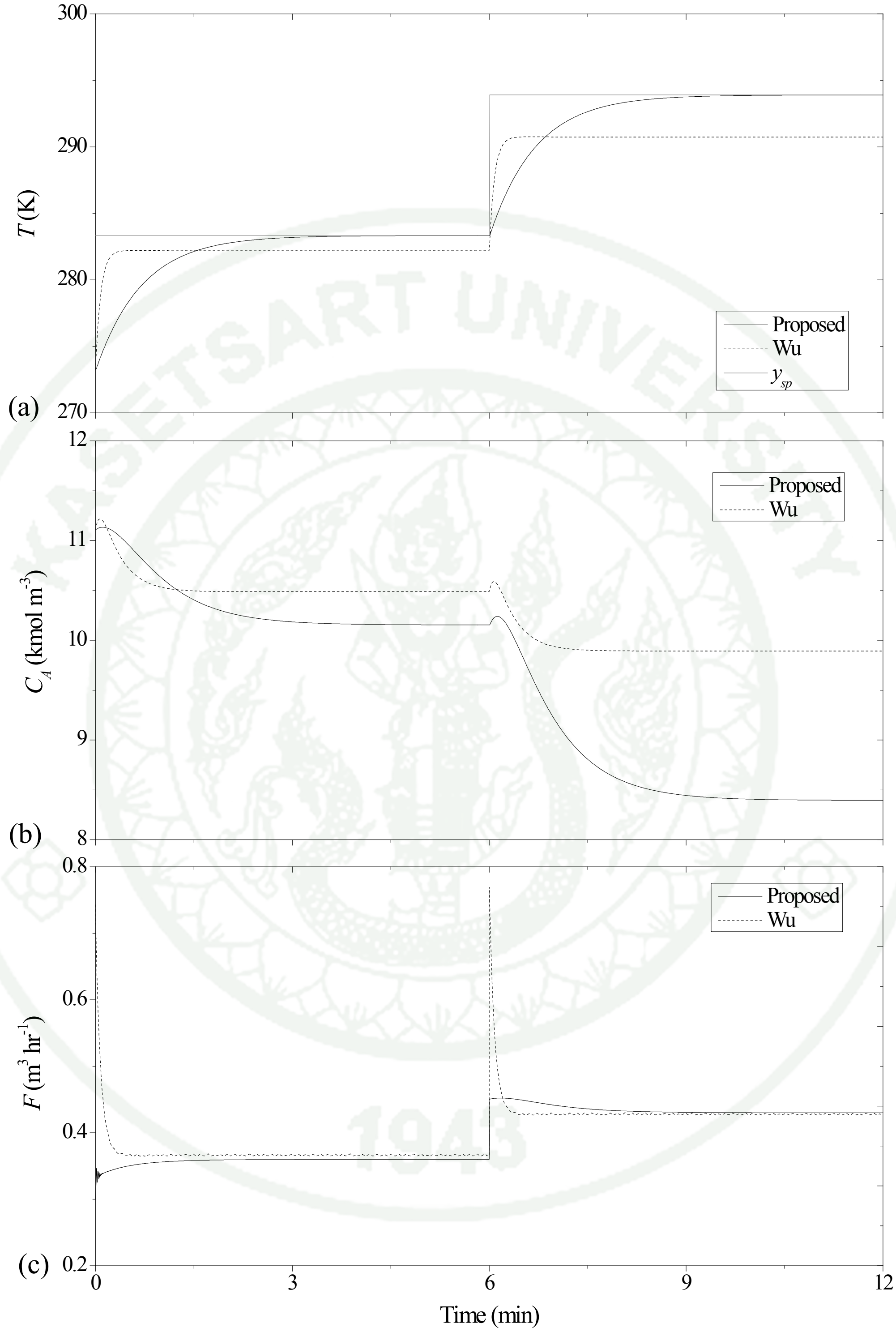

Figure 28 Closed-loop responses of (a) T, (b) C_A, and (c) F in the case of comparison with Wu's method

2. Illustrative Examples for Uncertain Time-delay Processes

2.1 Single-input-single-output reactor with constant heat removal and the presence of time-delay

2.1.1 Process description

The developed control methodology is now applied to the case of continuous stirred tank reactor (CSTR) with constant heat removal shown in Figure 27 in Section 1.3.1. An exothermic irreversible reaction of $A \rightarrow B$ takes place in the liquid phase. The reactor volume and physical parameters are assumed to be constant and the reactor is operated in perfect mixing condition. Furthermore, there is time-delay (θ) in the inlet feed flow rate (F). The governing equations of the system are:

$$\frac{dC_A}{dt} = -k_0 \exp\left(-\frac{E_a}{RT}\right) C_A + (C_{Ai} - C_A)\frac{F(t-\theta)}{V}$$

$$\frac{dT}{dt} = Q + \gamma k_0 \exp\left(-\frac{E_a}{RT}\right) C_A + (T_i - T)\frac{F(t-\theta)}{V}$$

where C_A is the concentration of A and T is the reactor temperature. The values of the process parameters and nominal operating conditions are given in Table 5 (Section 1.3). The control objective is to control the reactor temperature (y = T) with time-delay (θ = 900 s) by manipulating the feed flowrate ($u = F$).

By applying the proposed control method in Equation (24) with the relative order of output, $r = 1$, the equations of control system is shown in Appendix E.

The servo and the regulatory performance are tested in the following subsection with the initial condition $C_{A,ss}$ = 10.15 kmol m^{-3}, T_{ss} = 283.3 K, and F_{ss} = 0.38 m^3 hr^{-1}. The following tuning parameters are used ε = 0.7 and K = 0.03.

2.1.2 Servo performance

The process starts at initial condition, and it is desired to control the output at the setpoint, y_{sp} = 293.9 K, which corresponds to $C_{A,ss}$ = 8.4 kmol m^{-3}, T_{ss} = 293.9 K, and F_{ss} = 0.45 m^3 hr^{-1}. The open-loop dynamic behavior at the steady-state condition is analyzed by eigenvalues of Jacobian matrix of both the open-loop process and the zero dynamics, which are shown in Table 6 (Section 1.3). Under the given condition, the proposed control method with and the IMC method are compared to evaluate the capability of time-delay compensation. Figure 29 depicts the closed-loop responses of the controlled output, state variable, and the manipulated input under the servo test. As seen in Figure 29, the results show that the proposed method can drive the reactor temperature to the desire setpoint, while IMC cannot maintain stability.

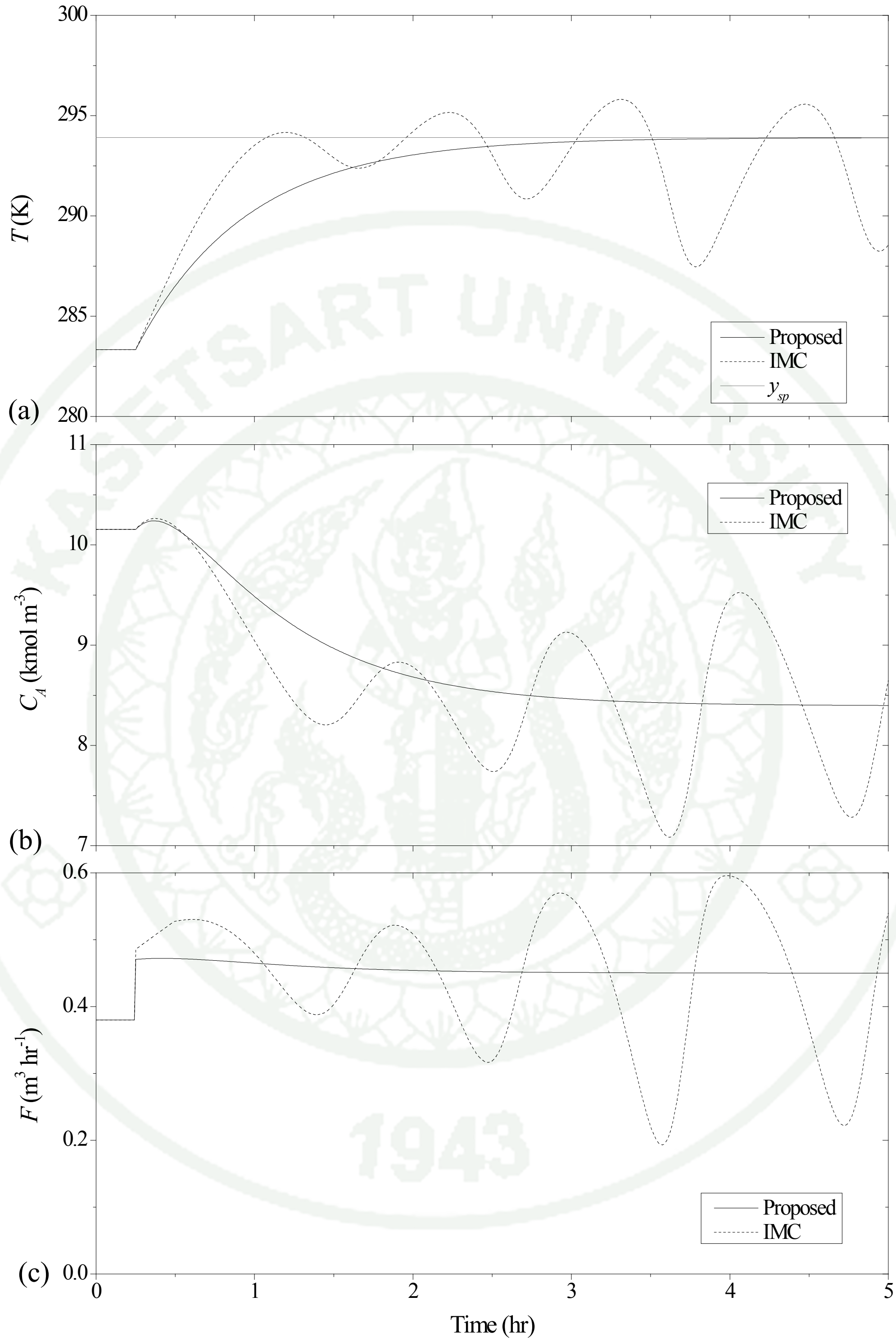

Figure 29 Closed-loop responses of (a) T, (b) C_A, and (c) F under the servo test in the SISO CSTR with the heat removal

2.1.3 Regulatory performance

The proposed control technique is applied through the CSTR with uncertainty to control the reactor temperature at desired setpoint. To investigate the control performance, a step change of unmeasured disturbance in manipulated input and random noise of parametric uncertainty are considered.

2.1.3.1 Unmeasured disturbance in *F*

The process is initially at y_{sp} = 293.9 K and maintains at the given setpoint throughout the process. +0.02 m^3 hr^{-1} step change of the unmeasured disturbance in the manipulated input, *F*, is introduced at the beginning of the test. Figure 30 shows the responses of the controlled output, state variable, and the manipulated input in the presence of unmeasured disturbance. The simulation results clearly show that the disturbance rejection controller can compensate for the unmeasured disturbance, although there is time-delay in the process.

2.1.3.2 Parametric uncertainty in k_0

The process starts at y_{sp} = 293.9 K and maintains at that setpoint throughout the process. ± 20% random noise parametric uncertainty in rate constant k_0 is introduced at t = 0. Figure 31 shows the responses of the controlled output, state variable, and the manipulated input in the presence of parametric uncertainty. The simulation results show that the disturbance rejection controller can reduce the effect of uncertainty in spite of time-delay in the process.

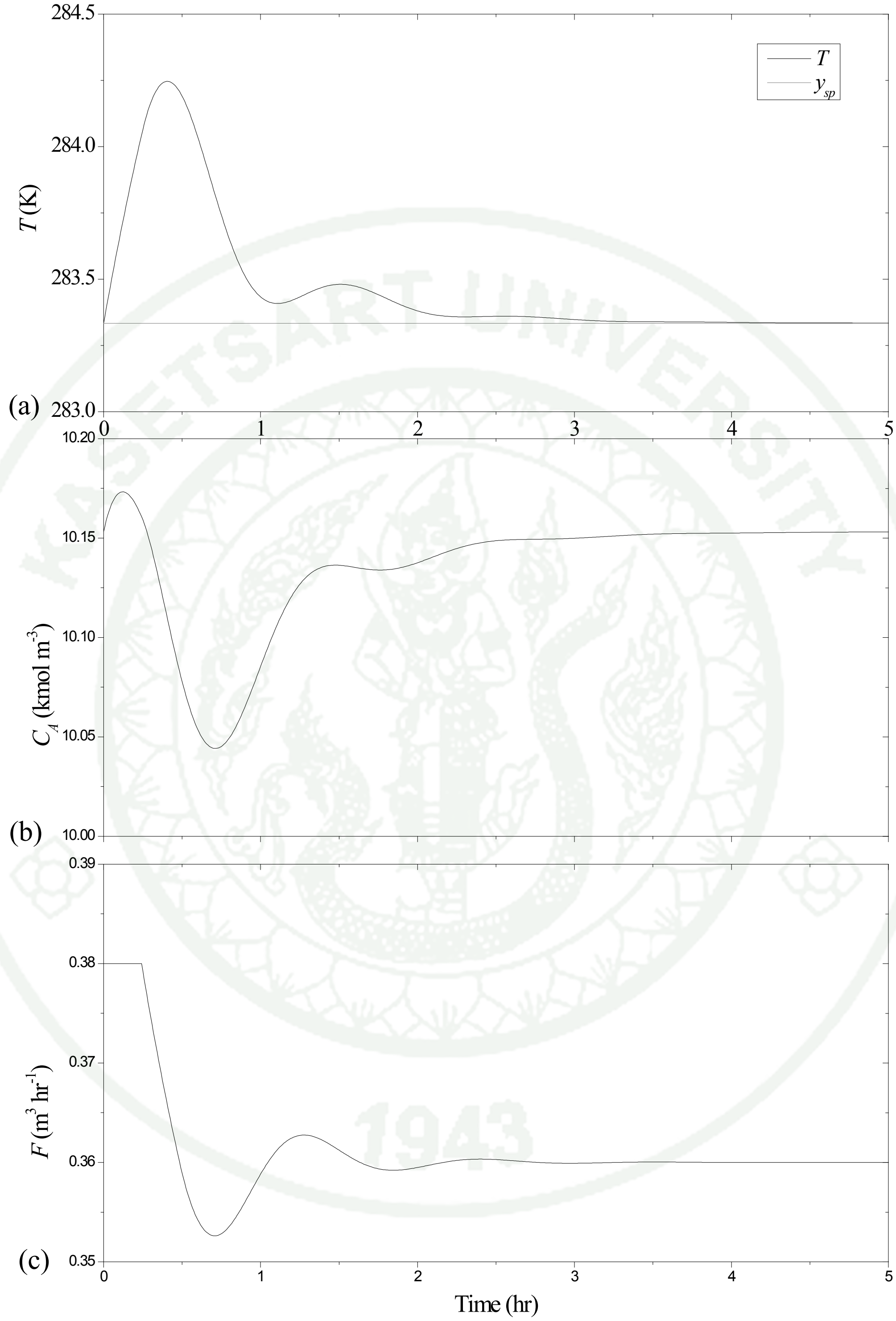

Figure 30 Closed-loop responses of (a) T, (b) C_A, and (c) F under the case of unmeasured disturbance in F in SISO CSTR with constant heat removal

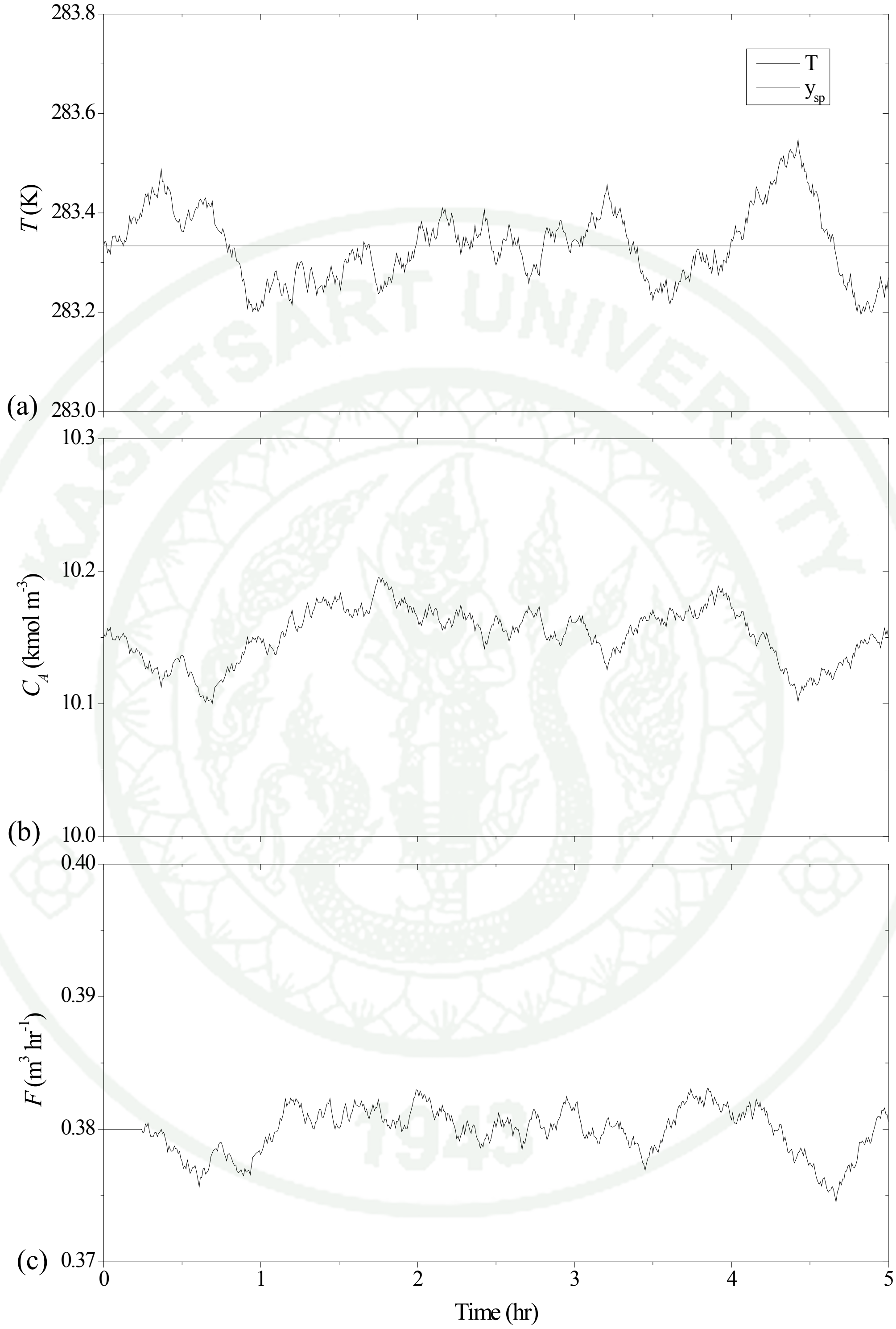

Figure 31 Closed-loop responses of (a) T, (b) C_A, and (c) F under the case of parametric uncertainty in k_0 in SISO CSTR with constant heat removal

2.2 Multiple-input-multiple-output two reactors in series with the presence of time-delay

2.2.1 Process description

Another example is to apply the proposed control methodology with MIMO system. The same reactor of the previous example in Section 1.2 is now considered as the time-delay system. The dynamic model of this system with input time-delay (θ) is:

$$\frac{dT_1}{dt} = \frac{F_1}{V_1}(T_0 - T_1) - \frac{UA_1}{V_1 \rho C_p}(T_1 - T_{j1}) + \frac{(-\Delta H_1)}{\rho C_p} k_{01} C_{A1} \exp\left(-\frac{E_1}{RT_1}\right)$$

$$\frac{dC_{A1}}{dt} = \frac{F_1}{V_1}(C_{A0} - C_{A1}) - k_{01} C_{A1} \exp\left(-\frac{E_1}{RT_1}\right)$$

$$\frac{dC_{B1}}{dt} = \frac{F_1}{V_1} C_{B1} + k_{01} C_{A1} \exp\left(-\frac{E_1}{RT_1}\right)$$

$$\frac{dT_{j1}}{dt} = \frac{F_{j1}}{V_{j1}}(T_{ji1}(t-\theta_1) - T_{j1})$$

$$\frac{dT_2}{dt} = \frac{F_1}{V_2}(T_1 - T_2) - \frac{UA_2}{V_2 \rho C_p}(T_2 - T_{j2}) + \frac{(-\Delta H_1)}{\rho C_p} k_{01} C_{A2} \exp\left(-\frac{E_1}{RT_2}\right) + \frac{(-\Delta H_2)}{\rho C_p} k_{02} C_{B2} C_{C2} \exp\left(-\frac{E_2}{RT_2}\right)$$

$$\frac{dC_{A2}}{dt} = \frac{F_1}{V_2} C_{A1} - \frac{F_1 + F_2(t-\theta_3)}{V_2} C_{A2} - k_{01} C_{A2} \exp\left(-\frac{E_1}{RT_2}\right)$$

$$\frac{dC_{B2}}{dt} = \frac{F_1}{V_2} C_{B1} - \frac{F_1 + F_2(t-\theta_3)}{V_2} C_{B2} + k_{01} C_{A2} \exp\left(-\frac{E_1}{RT_2}\right) - k_{02} C_{B2} C_{C2} \exp\left(-\frac{E_2}{RT_2}\right)$$

$$\frac{dC_{C2}}{dt} = \frac{F_2(t-\theta_3)}{V_2} C_{C0} - \frac{F_1 + F_2(t-\theta_3)}{V_2} C_{C2} - k_{02} C_{B2} C_{C2} \exp\left(-\frac{E_2}{RT_2}\right)$$

$$\frac{dT_{j2}}{dt} = \frac{F_{j2}}{V_{j2}}(T_{ji2}(t-\theta_2) - T_{j2})$$

where the value used for each parameter is given in Table 3 (Section 1.2). The process is operated under the proposed control system, which the state variables, controlled outputs, and manipulated inputs are defined as $x = [T_1\ C_{A1}\ C_{B1}\ T_{j1}\ T_2\ C_{A2}\ C_{B2}\ C_{C2}\ T_{j2}]^T$, $y = [T_1\ T_2\ C_{C2}]^T$ and $u = [T_{ji1}\ T_{ji2}\ F_2]^T$, respectively. The tested time-delay are appeared in the inputs, which the values of time-delay are θ_1 = 1,440 s, θ_2 = 1,080 s, and θ_3 = 900 s.

By applying the proposed control method in Equation (24) with the relative order, $\{r_1 = 2, r_2 = 2, r_3 = 1\}$, the equations of control system are shown in Appendix F.

The servo and regulatory performance are tested in the following subsection with the initial condition T_1 = 311.65 K, C_{A1} = 6.358 kmol m^{-3}, C_{B1} = 1.14 kmol m^{-3}, T_{j1} = 310 K, T_2 = 384.43 K, C_{A2} = 0.173 kmol m^{-3}, C_{B2} = 0.654 kmol m^{-3}, C_{C2} = 0.281 kmol m^{-3}, and T_{j2} = 345.8 K. The corresponding initial values of three inputs are T_{ji1} = 310 K, T_{ji2} = 345.8 K, and $F_2 = 2.2 \times 10^{-4}$ m^3 s^{-1}, and the sets of the tuning parameters are $\{\varepsilon_1 = 700,\ \varepsilon_2 = 500,\ \varepsilon_3 = 800\}$ and $\{K_1 = 0.6, K_2 = 0.4, K_3 = 0.001\}$.

2.2.2 Servo performance

The process starts at initial condition, and it is desired to control the outputs at the setpoints, $y_{sp,1}$ = 344, $y_{sp,2}$ = 381.8, and $y_{sp,3}$ = 0.48. The corresponding steady-state and the open-loop dynamics behavior condition is analyzed by eigenvalues of Jacobian matrix of both open-loop process and zero dynamics, which are shown in Table 4 (Section 1.2). Under the given condition, the proposed control method and IMC method are compared to evaluate the capability of time-delay compensation. Figures 32-35 demonstrate the closed-loop responses of controlled outputs, state variables, and manipulated inputs under the servo test. The results show that the proposed method can drive the temperatures of both reactors and concentration of C in the second reactor to each desire setpoints, but IMC cannot maintain stability.

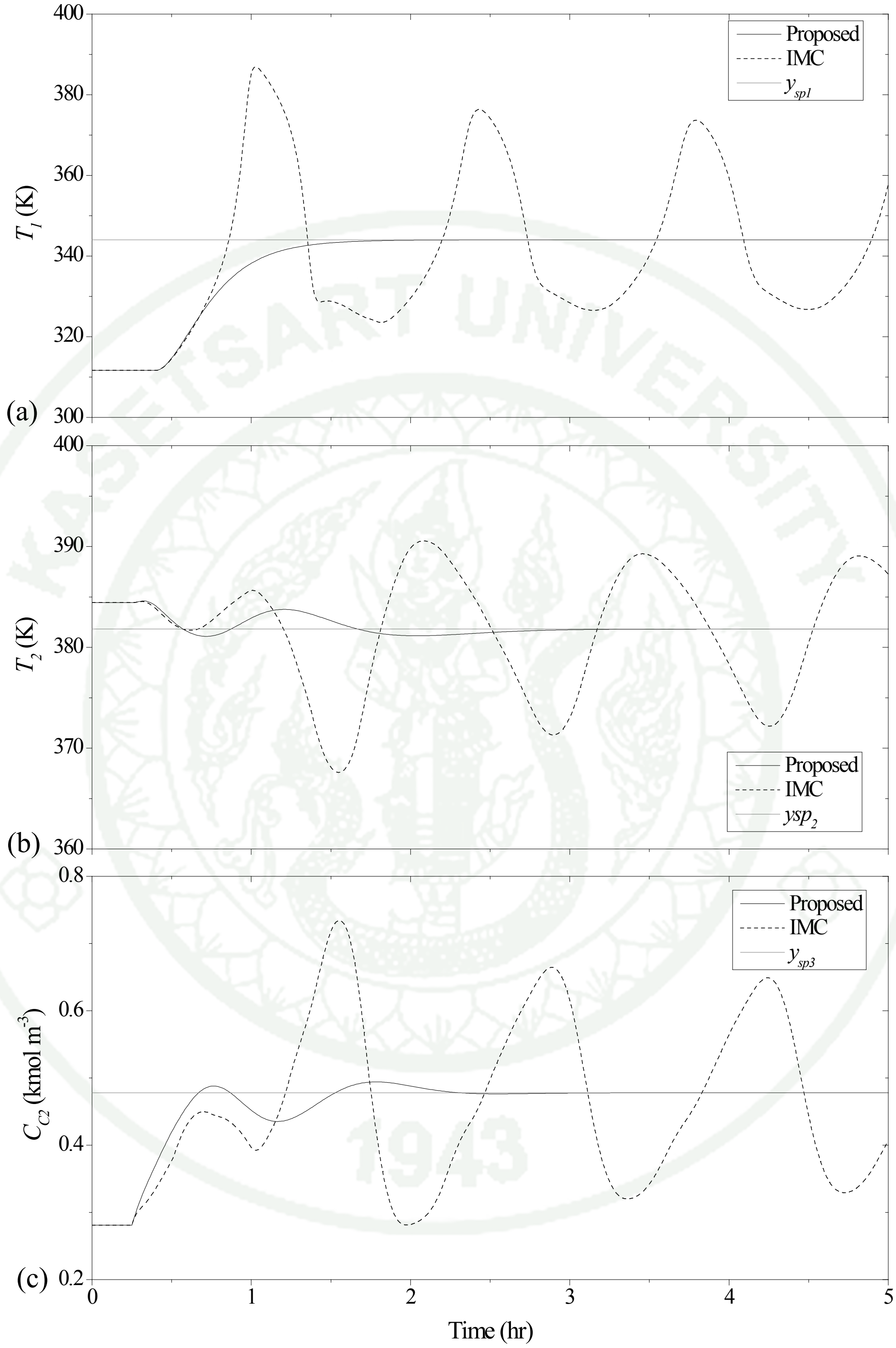

Figure 32 Closed-loop responses of (a) T_1, (b) T_2, and (c) C_{C2} under the servo test in MIMO two CSTRs

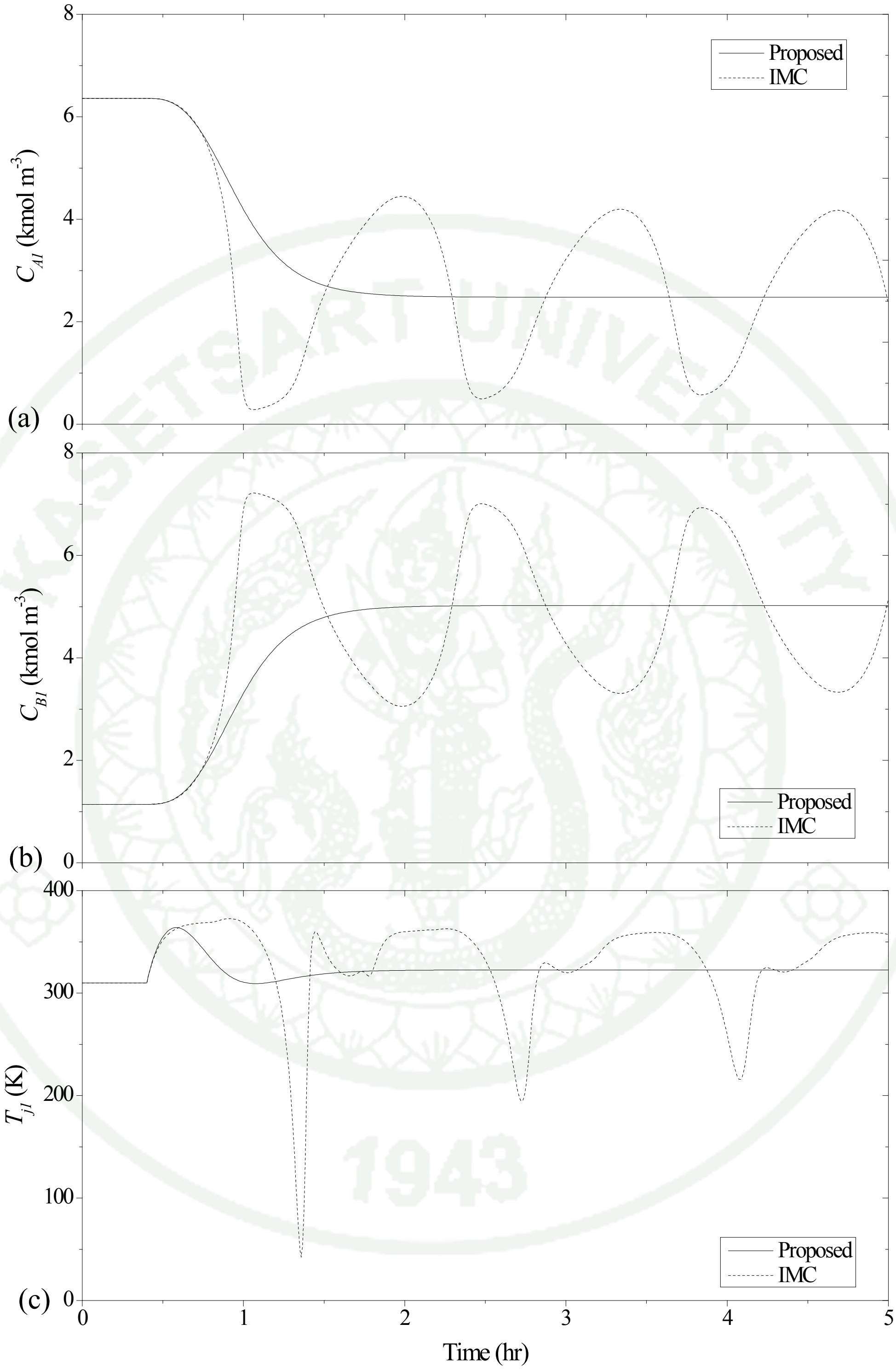

Figure 33 Closed-loop responses of (a) C_{A1}, (b) C_{B1}, and (c) T_{j1} under the servo test in MIMO two CSTRs

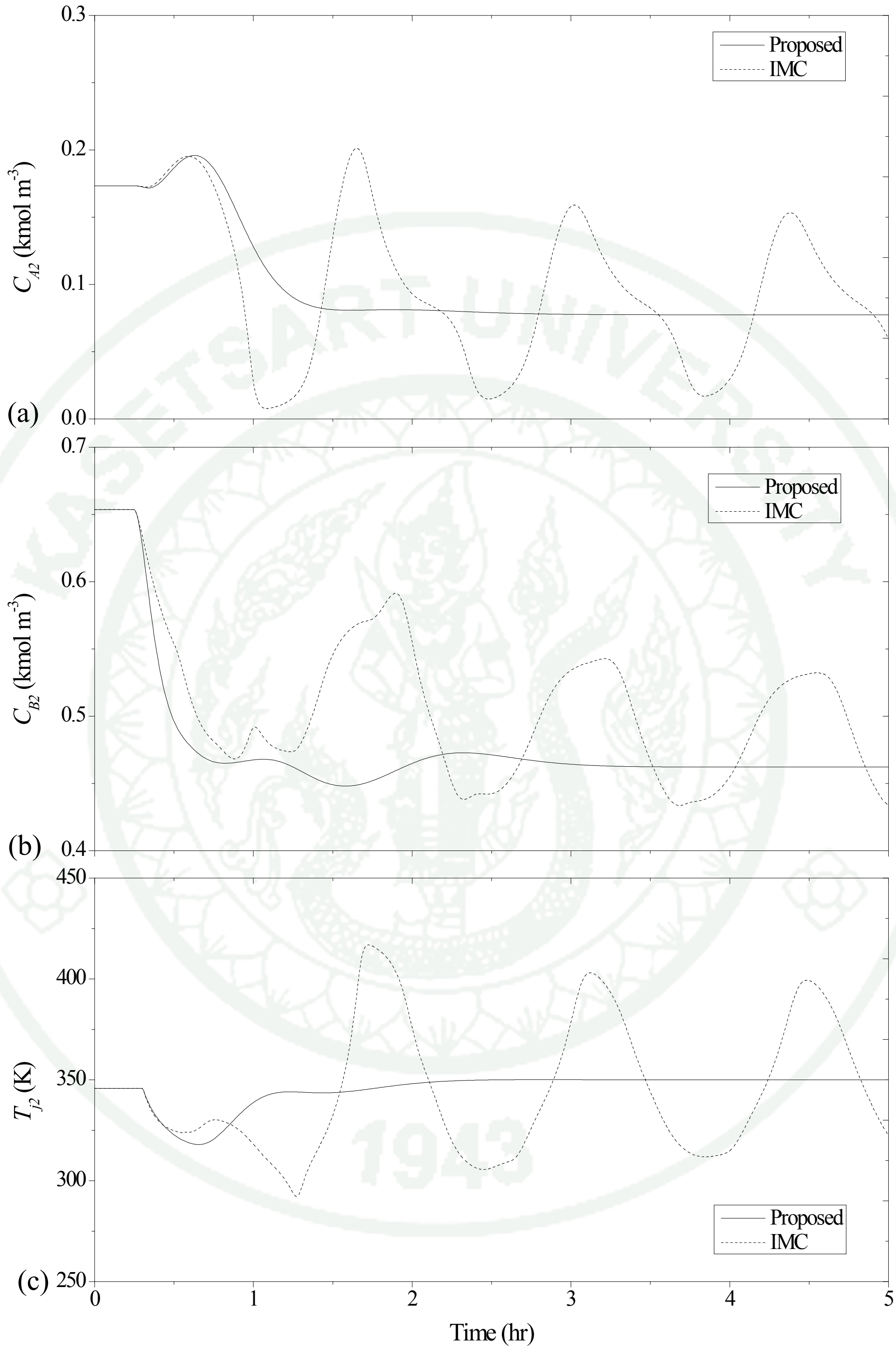

Figure 34 Closed-loop responses of (a) C_{A2}, (b) C_{B2}, and (c) T_{j2} under the servo test in MIMO two CSTRs

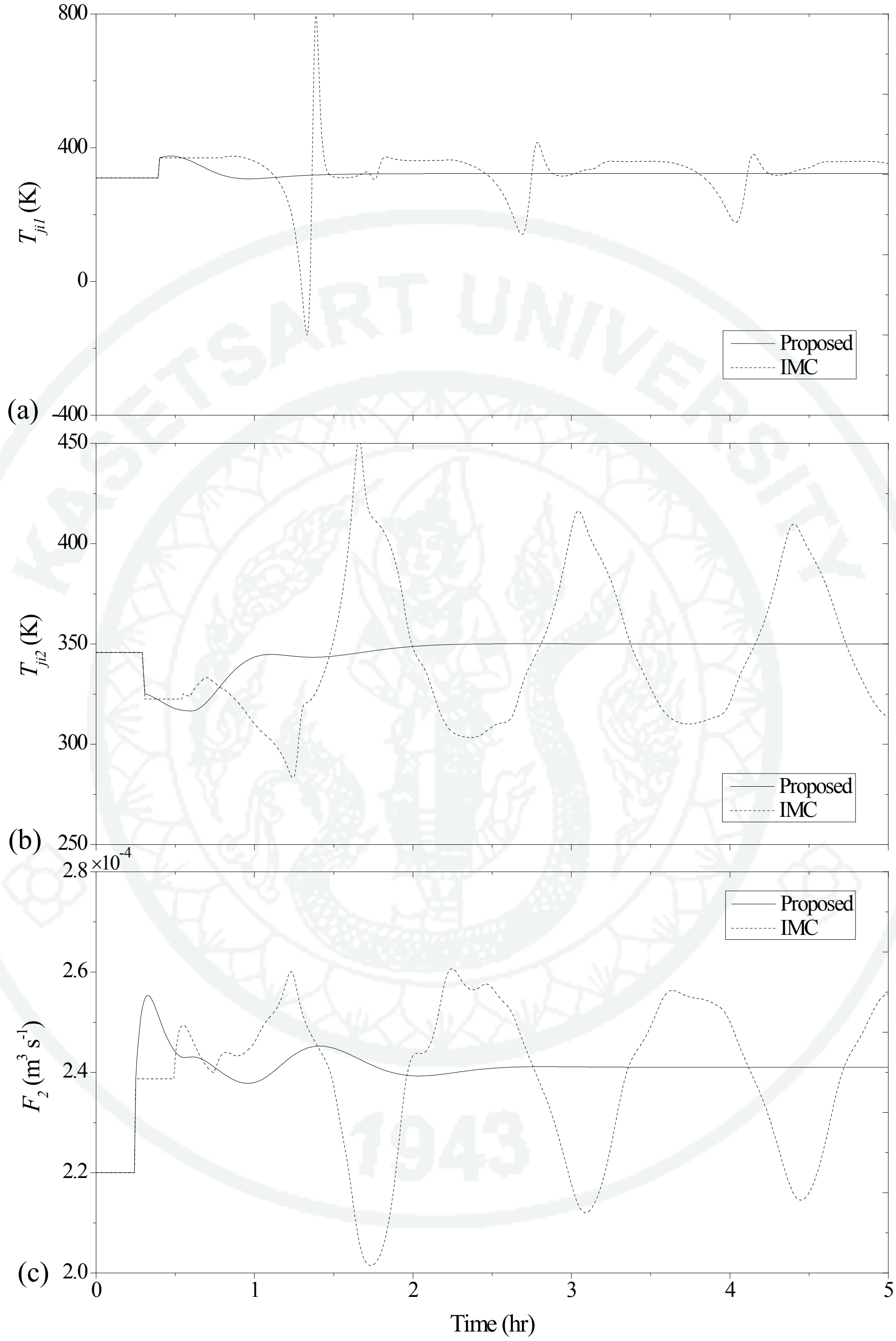

Figure 35 Closed-loop responses of (a) T_{ji1}, (b) T_{ji2}, and (c) F_2 under the servo test in MIMO two CSTRs

2.2.3 Regulatory performance

The proposed control technique is applied through the process with uncertainty to control the temperatures of both reactors and concentration of *C* in the second reactor at each desired setpoints. A step change of unmeasured disturbances in manipulated input and random noise of parametric uncertainties are applied to evaluate the control performance.

2.2.3.1 Unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2

The process is initially at $y_{sp,1}$ = 311.7, $y_{sp,2}$ = 384.4, and $y_{sp,3}$ = 0.28, and maintains at that setpoint throughout the process. +5 K, +2 K, and $+2\times10^{-5}$ m^3 s^{-1} step change of the unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2, respectively, are considered at $t = 0$. Figure 36 shows the responses of controlled outputs, while Figures 37 and 38 depict the responses of state variables, and Figure 39 demonstrate the responses of manipulated inputs in the presence of the unmeasured disturbances. As seen in the Figures, they indicate that the disturbance rejection controller can compensate for all unmeasured disturbances, although there is time-delay in the process.

2.2.3.2 Parametric uncertainty in ΔH_1 and ΔH_2

The process is initially at y_{sp} = [311.7, 384.4, 0.3]. To test the robustness of the proposed control method, ± 20% random noise parametric uncertainty in the change of enthalpy of both reactions (ΔH_1 and ΔH_2) are introduced at the beginning. Figures 40-43 demonstrate controlled outputs, state variables, and manipulated inputs in the presence of parametric uncertainties. The simulation results show that the disturbance rejection controller can reduce the effect of uncertainty to maintain the outputs at the desired setpoints in spite of the time-delay in the process.

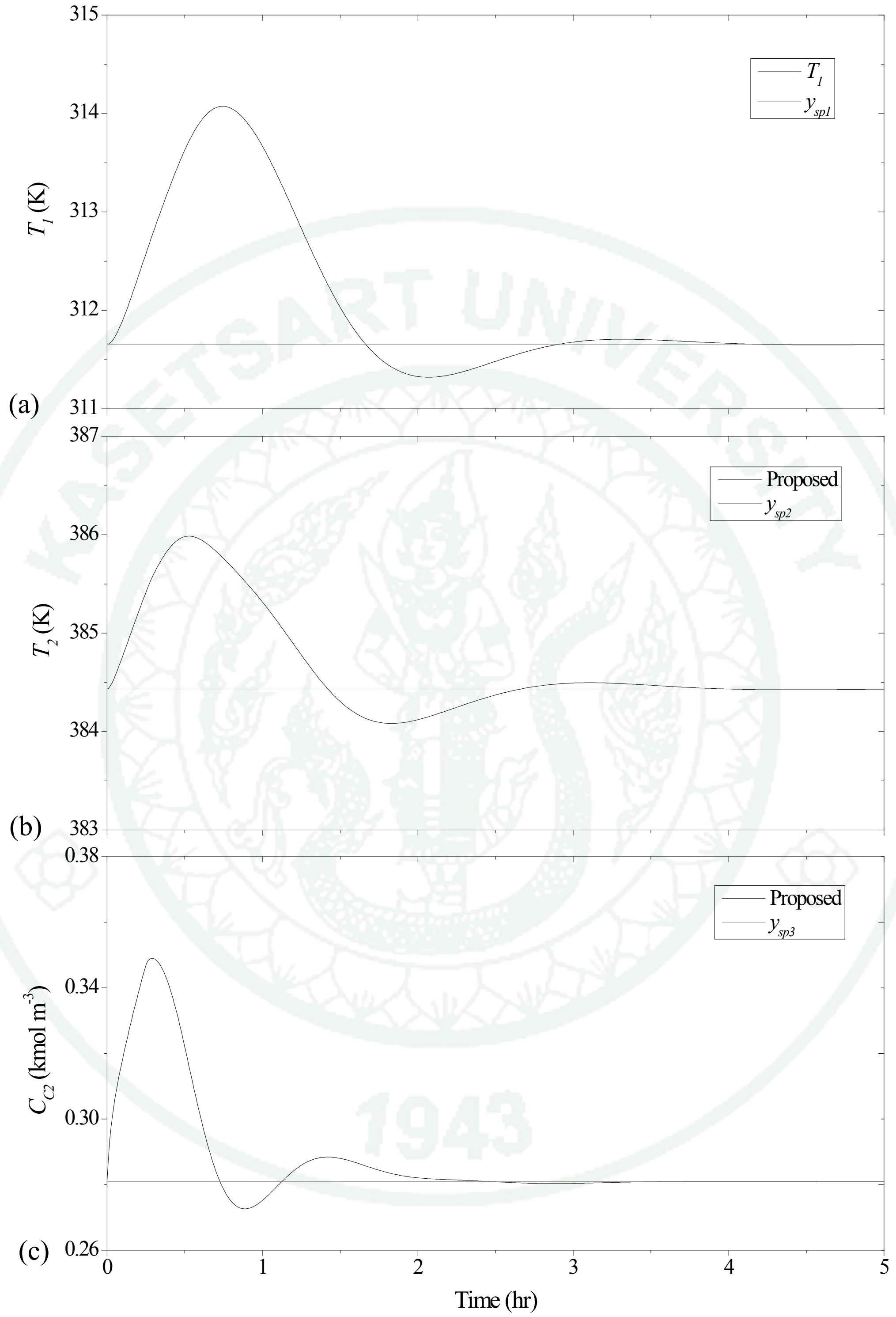

Figure 36 Closed-loop responses of (a) T_1, (b) T_2, and (c) C_{C2} under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

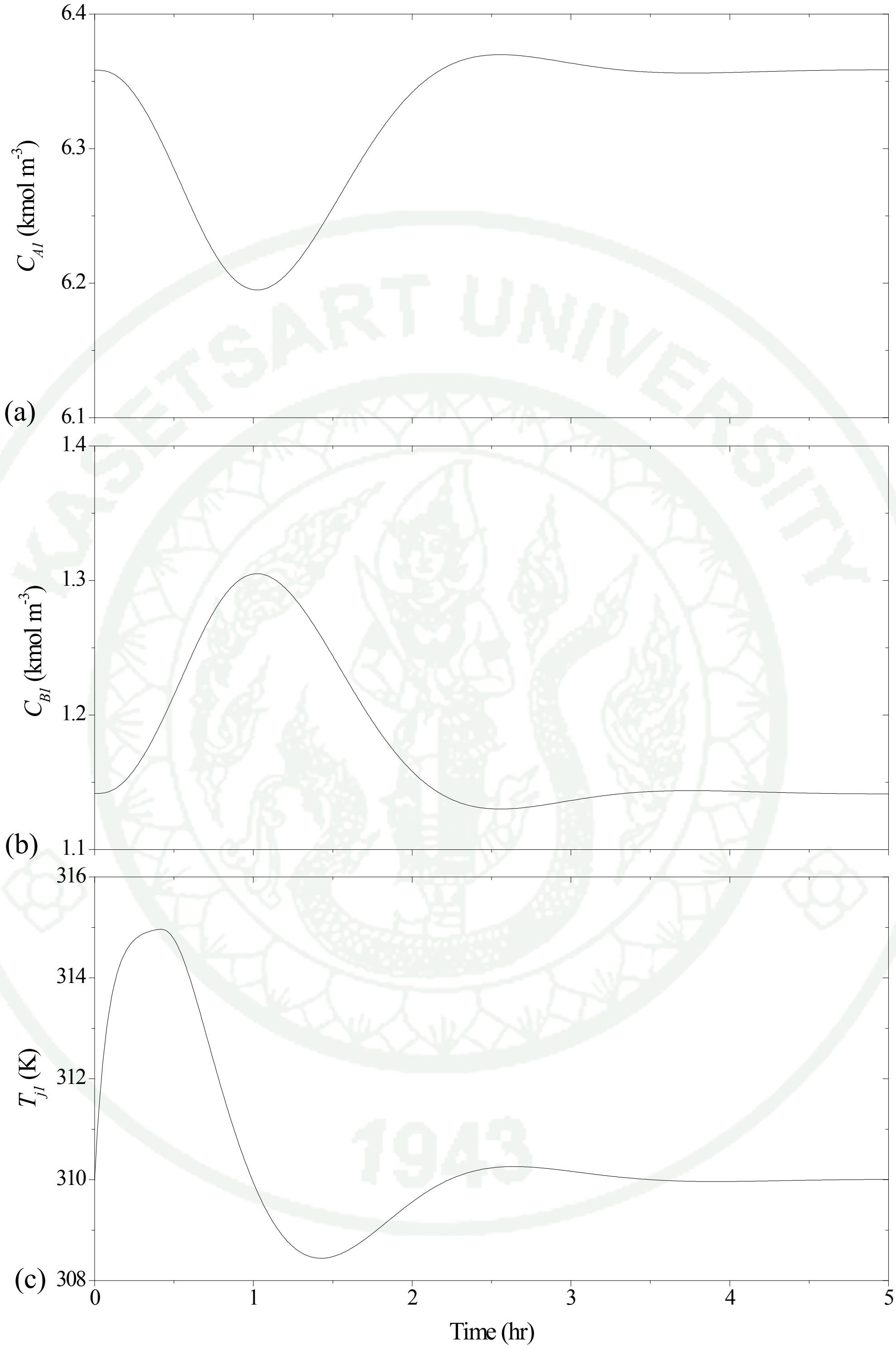

Figure 37 Closed-loop responses of (a) C_{A1}, (b) C_{B1}, and (c) T_{j1} under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

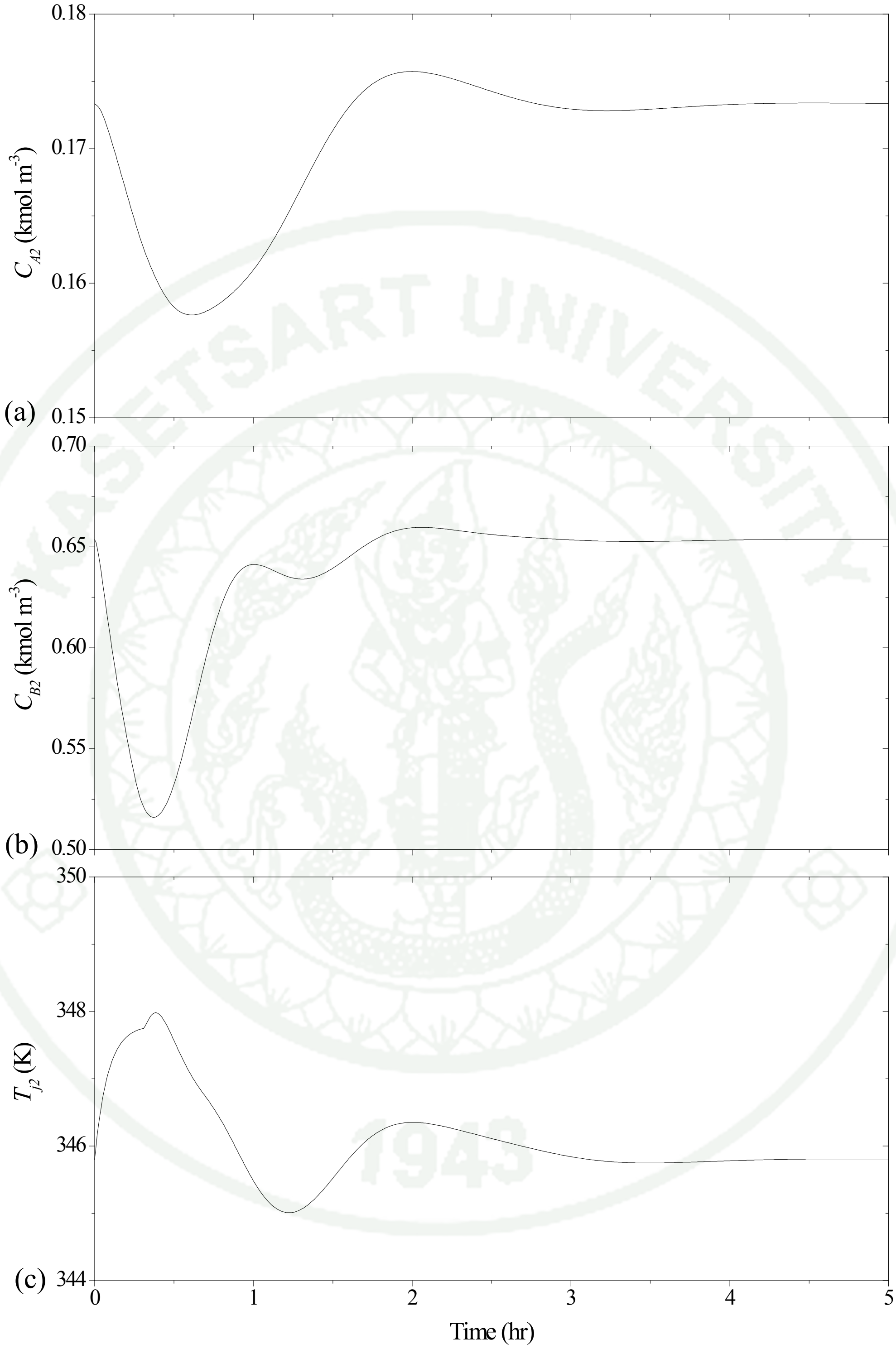

Figure 38 Closed-loop responses of (a) C_{A2}, (b) C_{B2}, and (c) T_{j2} under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

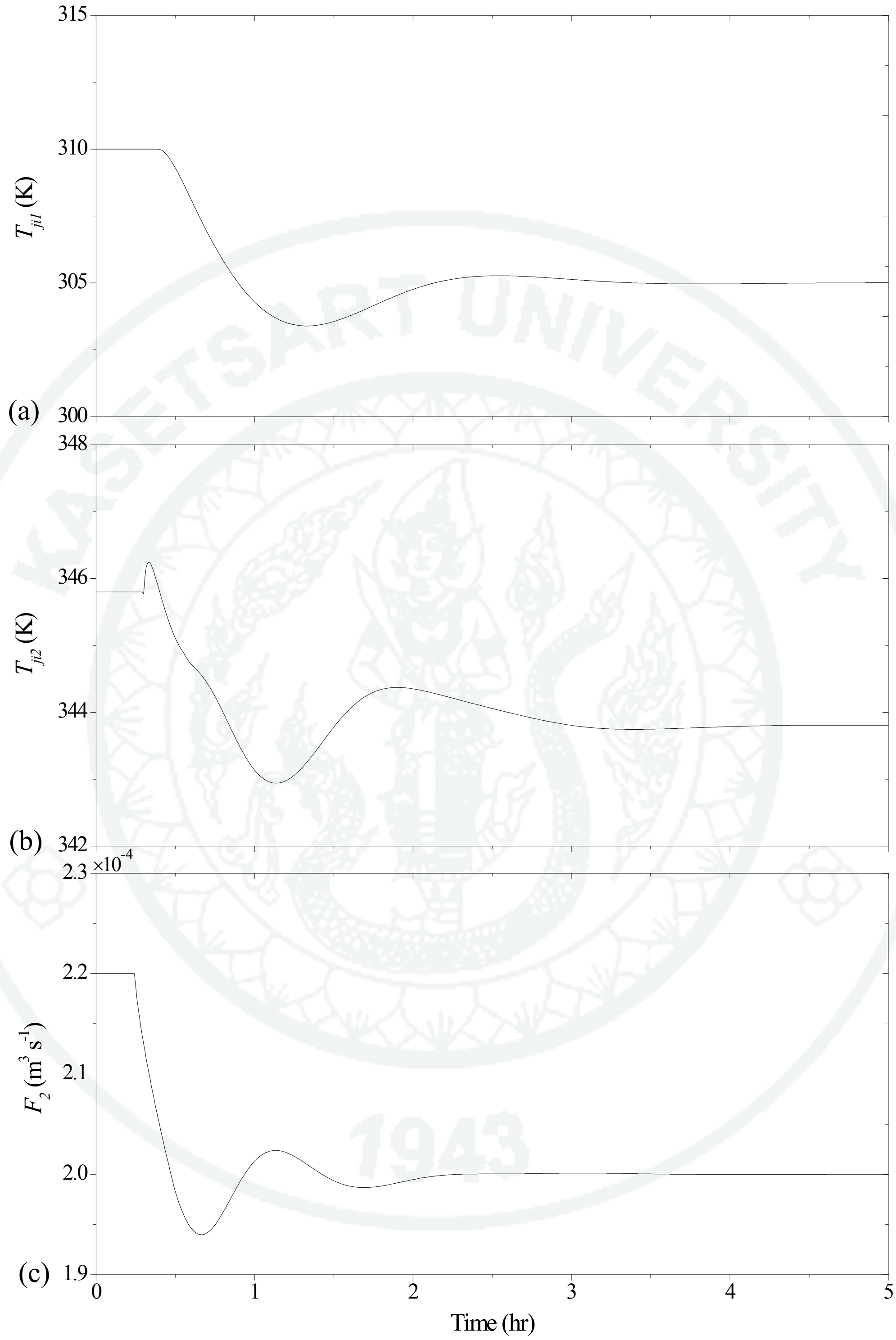

Figure 39 Closed-loop responses of (a) T_{ji1}, (b) T_{ji2}, and (c) F_2 under the case of unmeasured disturbances in T_{ji1}, T_{ji2}, and F_2 in MIMO two CSTRs

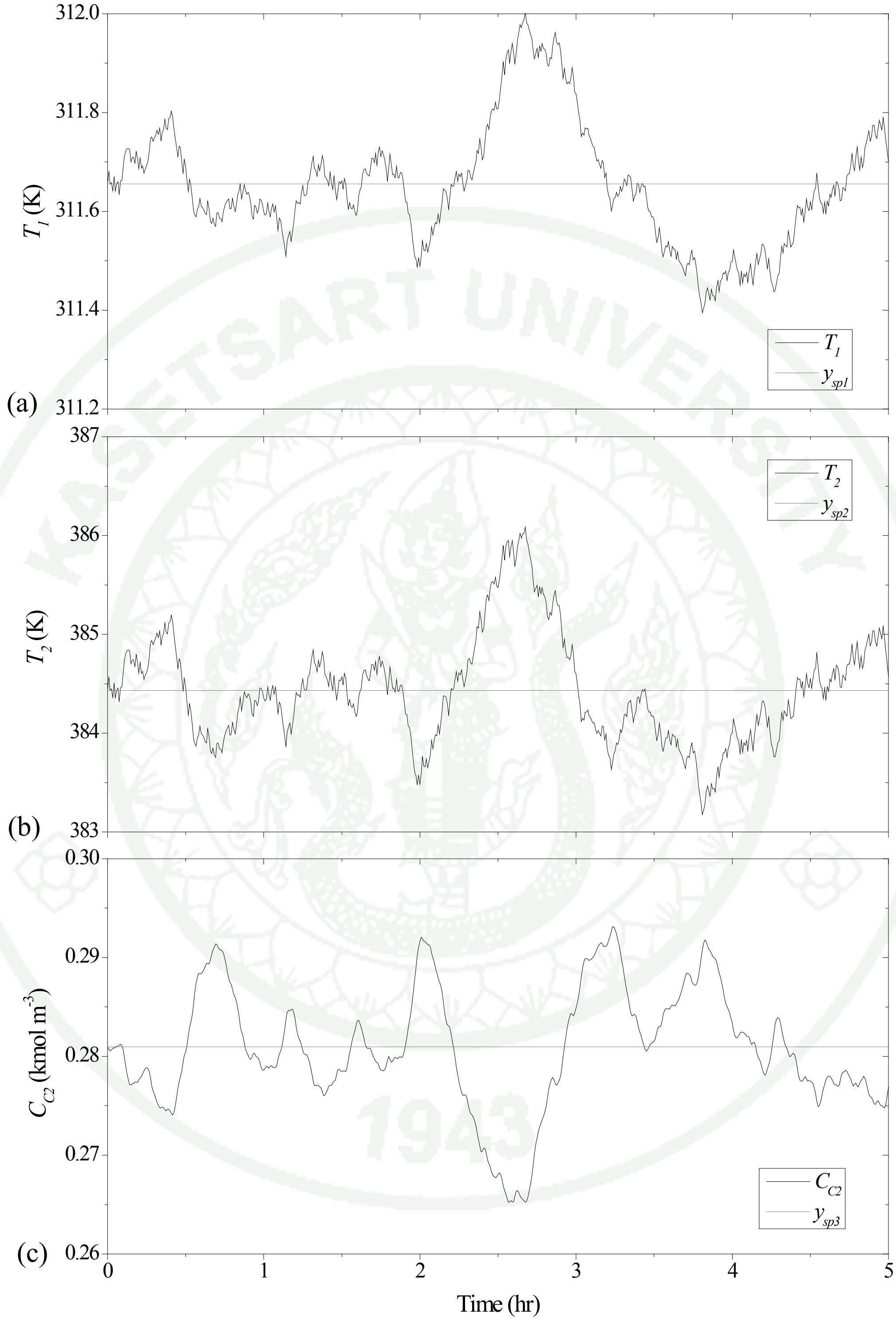

Figure 40 Closed-loop responses of (a) T_1, (b) T_2, and (c) C_{C2} under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

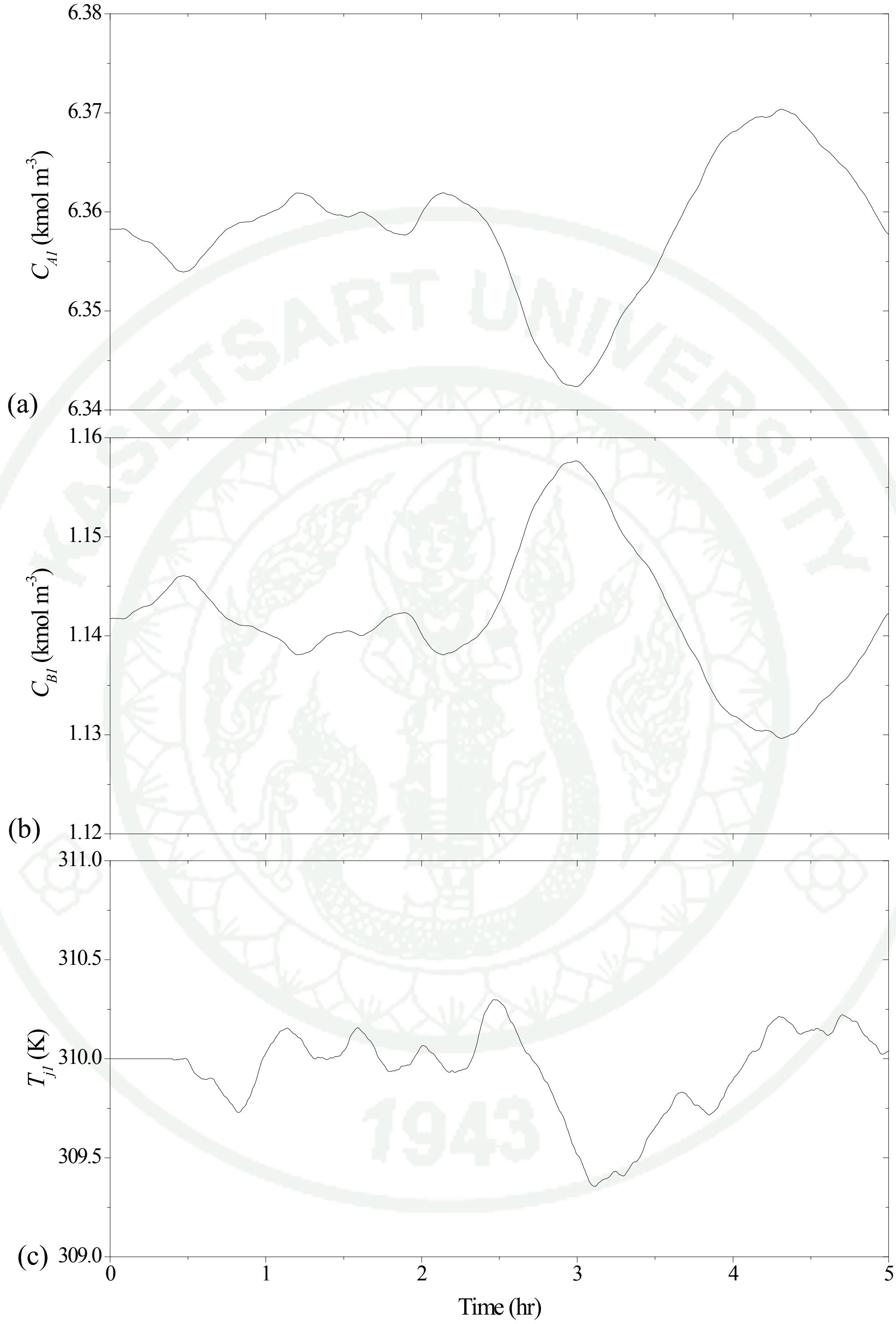

Figure 41 Closed-loop responses of (a) C_{A1}, (b) C_{B1}, and (c) T_{j1} under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

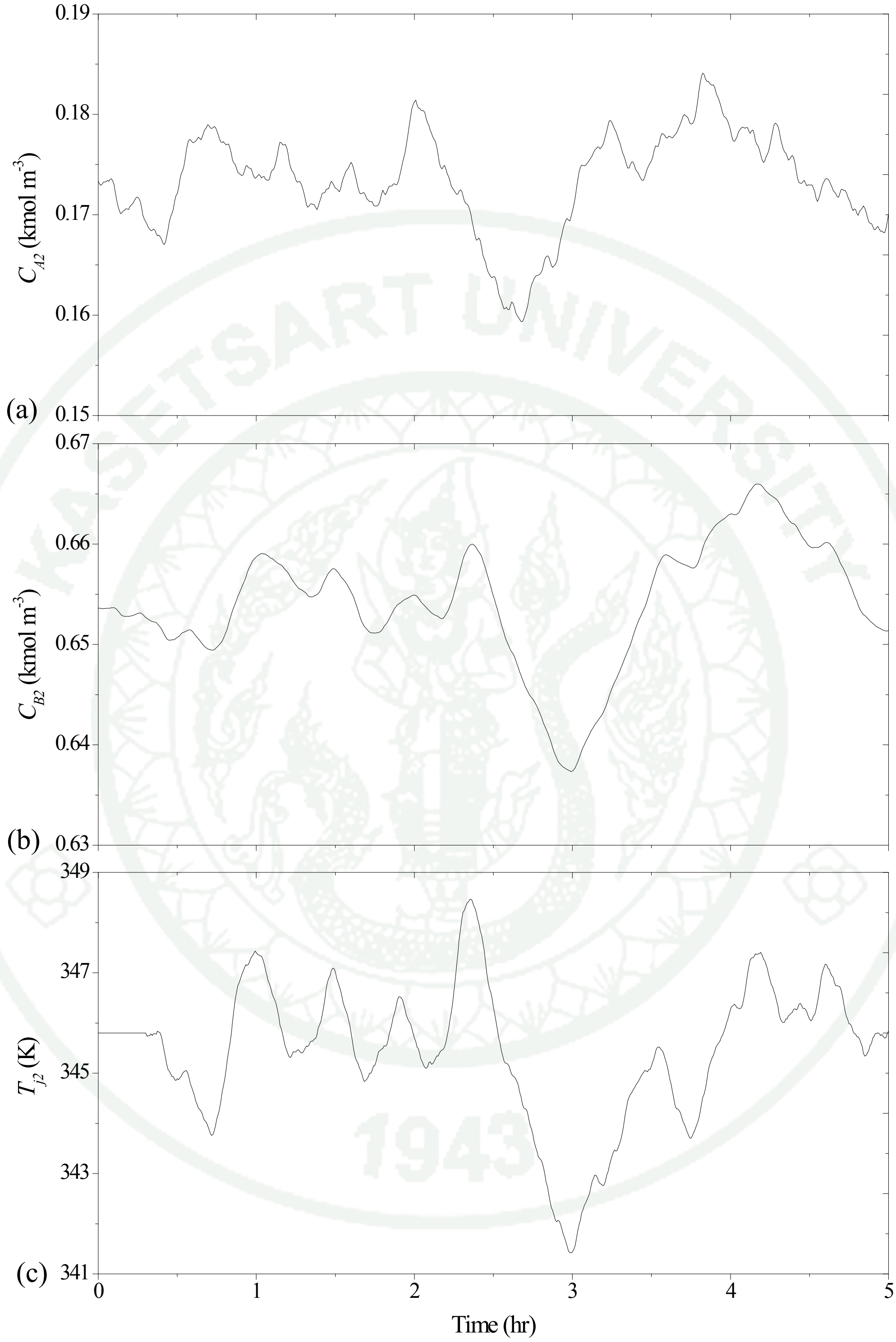

Figure 42 Closed-loop responses of (a) C_{A2}, (b) C_{B2}, and (c) T_{j2} under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

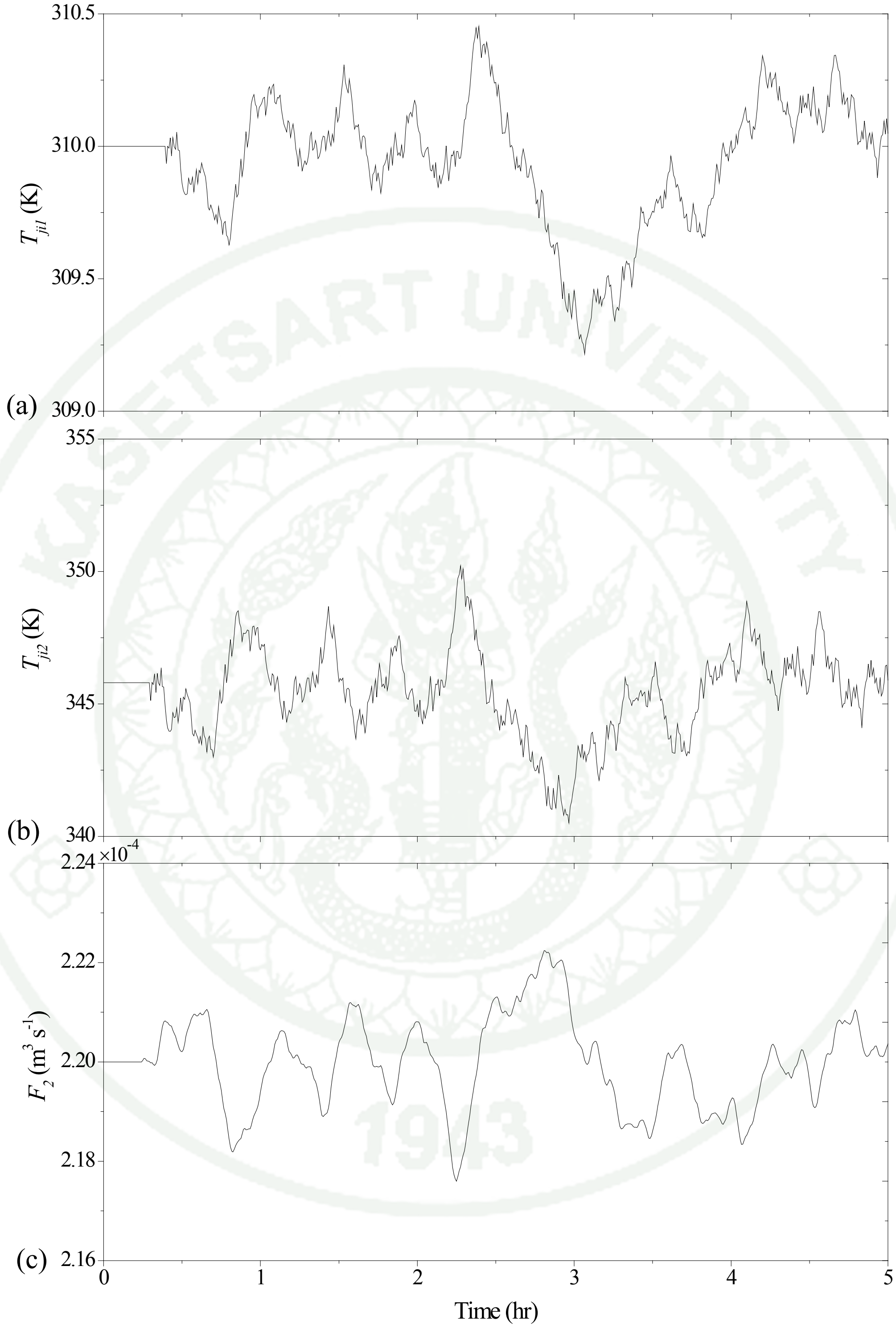

Figure 43 Closed-loop responses of (a) T_{ji1}, (b) T_{ji2}, and (c) F_2 under the case of parametric uncertainties in ΔH_1 and ΔH_2 in MIMO two CSTRs

2.3 Comparison of performance with internal model control with time-delay compensator

Hu and Rangaiah (Hu and Rangaiah, 1999) proposed internal model control with time-delay compensator. The schematic diagram of Hu's method is shown in Figure 44. The nonlinear controller is developed based on exact linearization, and the feedback compensator with the rate limiter is used to avoid aggressive control action.

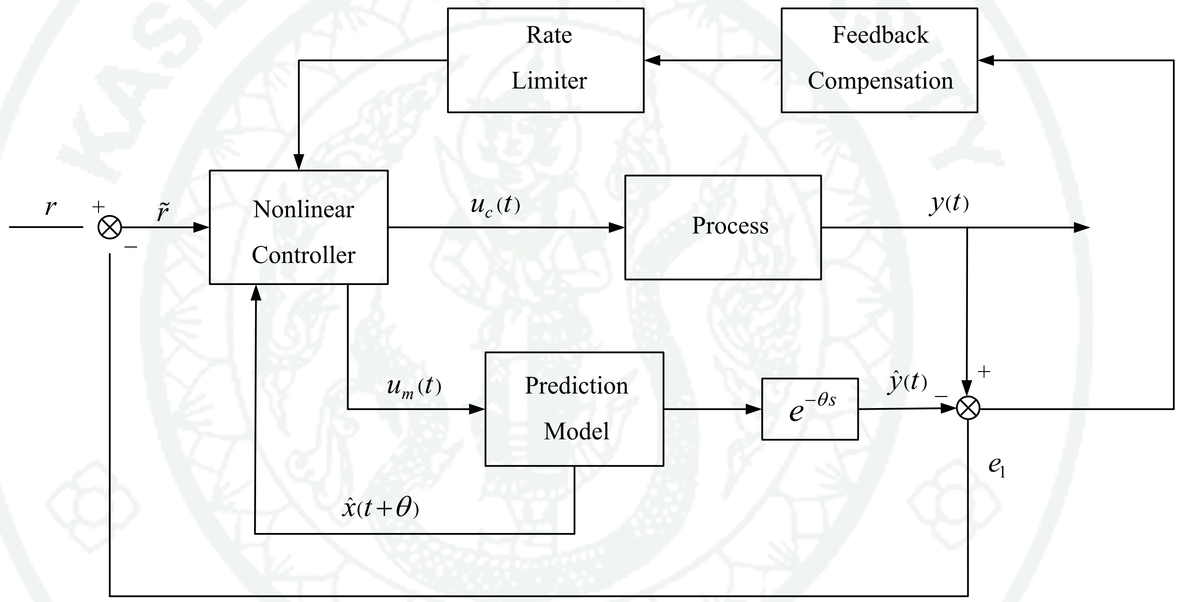

Figure 44 Schematic diagram of the method proposed by Hu and Rangaiah (1999)

2.3.1 Process description

To demonstrate the control performance, both control methods are applied to the SISO CSTR with cooling jacket example, where a first order reaction of $A \rightarrow B$ is occurred. There is a time-delay (θ) in the manipulated coolant temperature (T_C). The mathematical model described for this system is then given by

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Ai} - C_A) - k_0 \exp\left(-\frac{E_a}{RT}\right)C_A$$

$$\frac{dT}{dt} = \frac{F}{V}(T_i - T) + \frac{(-\Delta H)}{\rho C_p} k_0 \exp\left(-\frac{E_a}{RT}\right)C_A + \frac{UA}{V\rho C_p}(T_C(t-\theta) - T)$$

where C_A is the concentration of A and T is the reactor temperature. The coolant temperature, T_C, can be manipulated to control T at the desired setpoint. The tested time-delay is considered at $\theta = 0.1$ min and the values of process parameters are given in Table 4.

By applying the proposed control method in Equation (24) with the relative order of output, $r = 1$, the equations of control system are shown in Appendix G.

Table 7 Parameter values for the example of SISO reactor with a cooling jacket

Symbol	Quantity	Value
V	Volume of reactor	100 L
$-\Delta H$	The change in enthalpy	50,000 J mol^{-1}
k_0	Arrhenius factor	7.2×10^{10} min^{-1}
E/R	Activation energy/gas constant	8,750 K
C_{Ai}	Concentration of A in feed stream	1 mol L^{-1}
T_i	Temperature in feed stream	350 K
ρ	Density of cooling water	1,000 g L^{-1}
C_p	Heat capacity of cooling water	0.239 J g^{-1} K^{-1}
θ	Time-delay	0.2 min
F	Feed flow rate	100 L min^{-1}

Source: Hu and Rangaiah (1999)

The servo and the regulatory performance are tested in the following subsection with the initial condition $C_{A,ss}$ = 0.1 kmol L^{-1}, T_{ss} = 383.8 K, and $T_{ji,ss}$ = 309.9 K, and the tuning parameters ε = 0.5 and K = 2. In the simulation, the performance of the designed control system is compared with the Hu's method and the feedback IMC without time-delay compensator.

2.3.2 Servo performance

The process starts at initial condition, then, +10 K and -10 K step changes in the setpoint take place in the servo test. Figure 45 depicts the closed-loop responses of the controlled output under the servo test, which Figure 45 (a) and (b) demonstrates the responses for +10 K and -10 K step changes in setpoint, respectively. The corresponding steady-state and the open-loop dynamics behavior condition are analyzed by eigenvalues of Jacobian matrix of both open-loop process and zero dynamics, which the analysis results are shown in Table 8. In Figure 45, the proposed method gives the satisfactory control performance. It can force the controlled output to the setpoint faster with the less oscillation than Hu's method, while IMC cannot handle the process.

Table 8 The open-loop dynamics behavior analysis of the example of SISO reactor with a cooling jacket

Steady-state pair	Eigenvalues of Jacobian matrix		Dynamics behavior	Condition
[$C_{A,ss}$, $T_{,ss}$, $T_{C,ss}$]	Process	Zero		
[0.1, 383.8, 309.9]	-0.95 ± 4.35i	-10	SMP	IC
[0.06, 393.8, 320.6]	-4.55 ± 4.6i	-17.1	SMP	SP1
[0.17, 373.8, 302.1]	-0.94 ± 2.55i	-5.9	SMP	SP2

Note: SMP = Stable minimum phase, IC = Initial condition, SP1 = The first setpoint, SP2 = The second setpoint

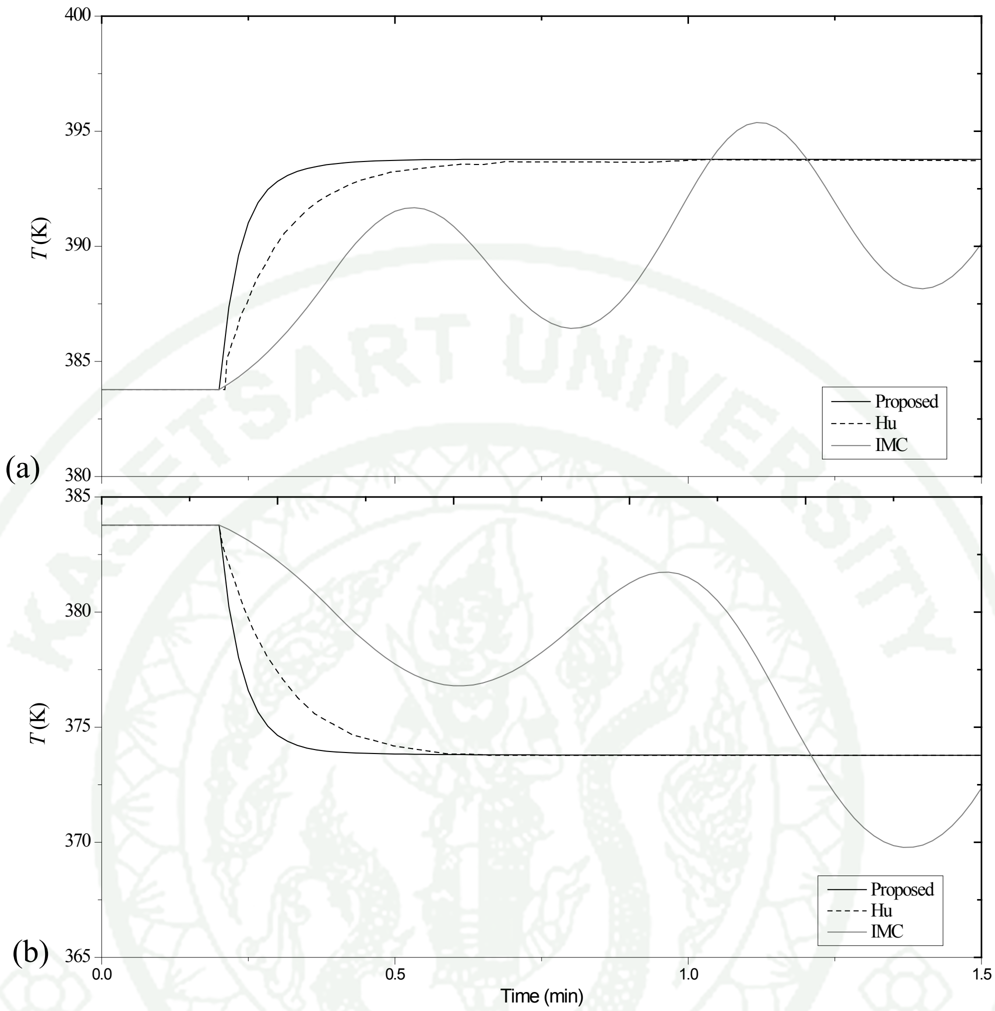

Figure 45 Closed-loop responses of *T* under the servo test compared with Hu's method (a) +10 K and (b) -10 K step change in setpoint

2.3.3 Regulatory performance

The proposed control technique is applied through the CSTR with uncertainty to control the reactor temperature at desired setpoint. To investigate the control performance, a step change of unmeasured disturbance in feed flowrate and error in parametric uncertainty θ are considered.

2.3.3.1 Unmeasured disturbance in *F*

The process is initially at y_{sp} = 383.8 K and maintains at the setpoint throughout the process. +10% and -10% step changes of the unmeasured disturbance in the feed flowrate, *F*, which each case is introduced at $t = 0$. Figures 46 and 47 show the responses of the controlled output and the manipulated input in the presence of +10% and -10% disturbances in *F*, respectively. The results indicate that the proposed method successfully maintains the output at the desired setpoint, and gives the fast and smooth responses, while there are oscillation for Hu's method and IMC.

2.1.3.2 Unmeasured disturbance in *F* and parametric uncertainty in θ

The process starts at y_{sp} = 383.8 K and maintains at that setpoint throughout the process. In this case, the control methods are tested under -10% unmeasured disturbance in *F* and parametric uncertainty in θ (±20%), and they are introduced at $t = 0$. Figure 48 (a) shows the responses of the controlled output under -10% disturbance in *F* and +20% modeling error in θ, while Figure 48 (b) depicts the responses of the controlled output under -10% disturbance in *F* and -20% error in θ. The simulation results indicate that the proposed method efficiently handles the disturbances and error in θ within 1 hr for both cases. Hu's method completely compensates at $t = 3$ hr for +20% and $t = 5$ hr for -20% error in θ. IMC succeeds at $t = 5$ hr for +20% and more than 5 hr for -20% error in θ.

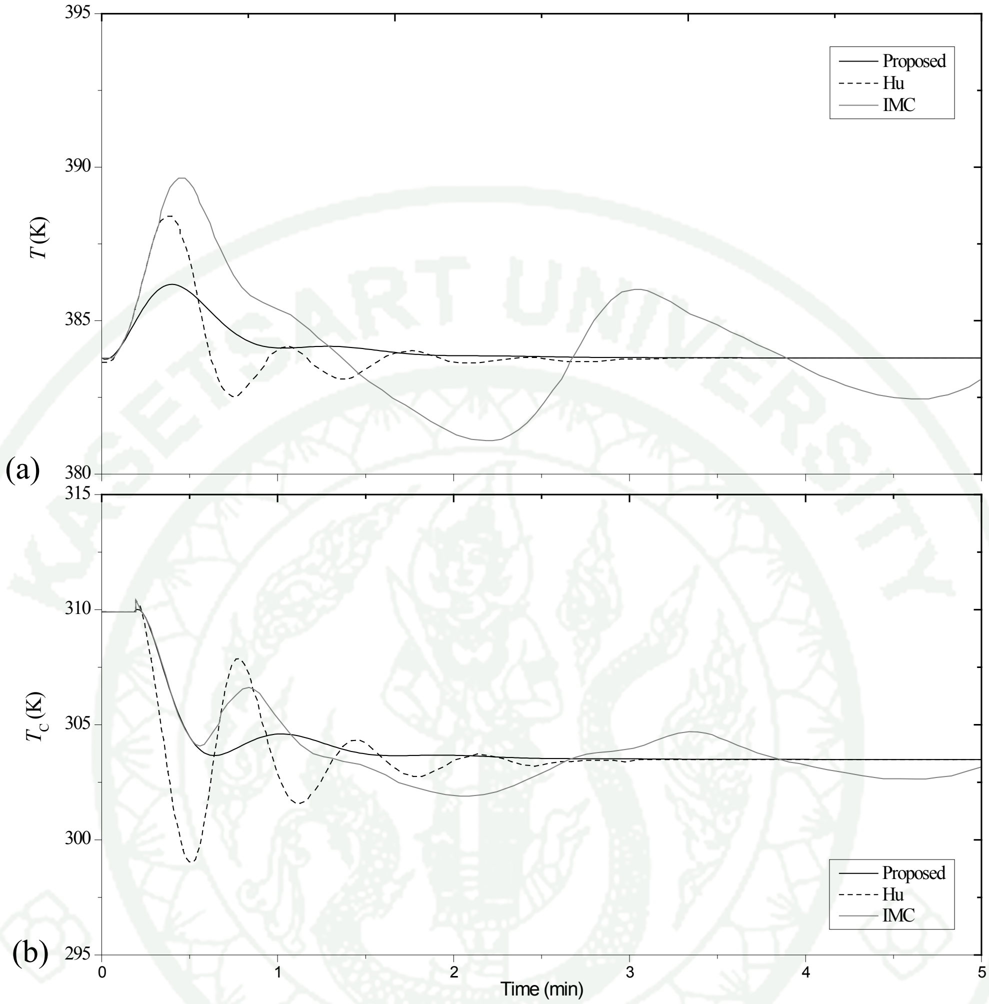

Figure 46 Closed-loop responses of (a) T and (b) T_c under the case of +10% unmeasured disturbance in F compared with Hu's method

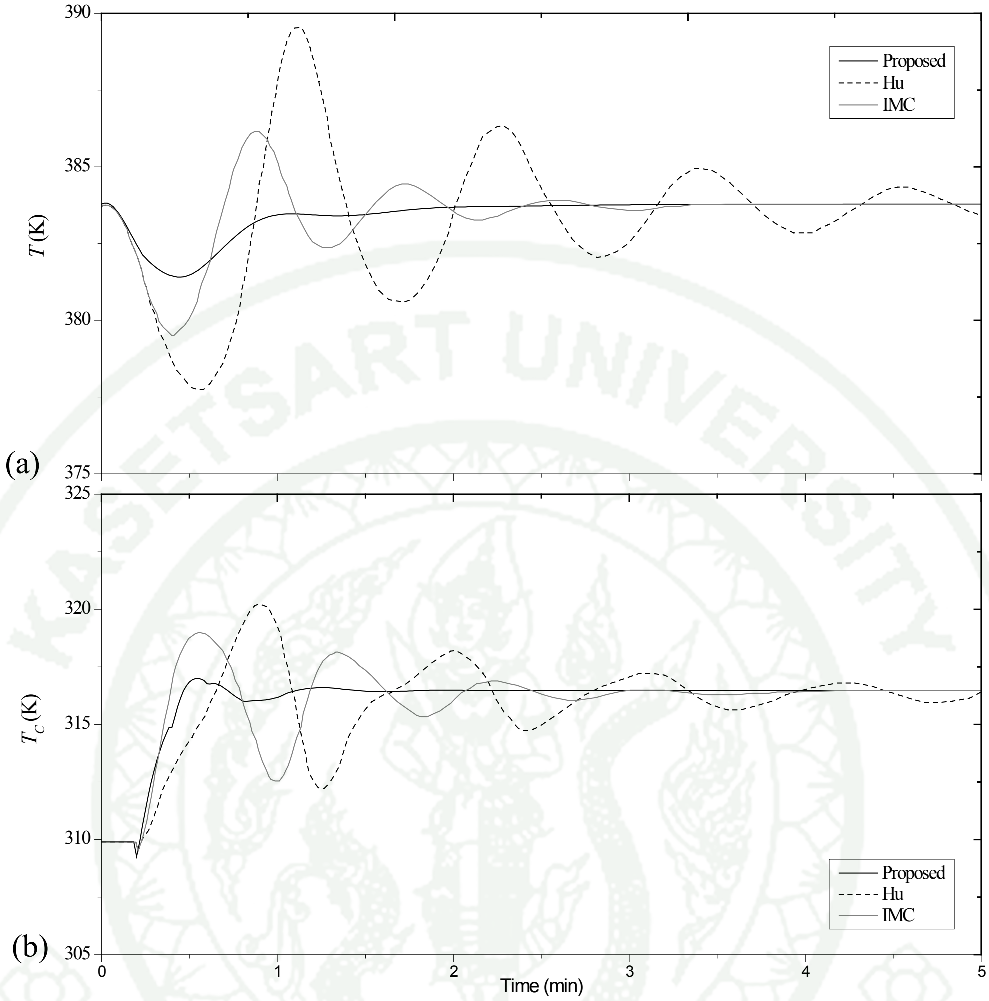

Figure 47 Closed-loop responses of (a) T and (b) T_c under the case of -10% unmeasured disturbance in F compared with Hu's method

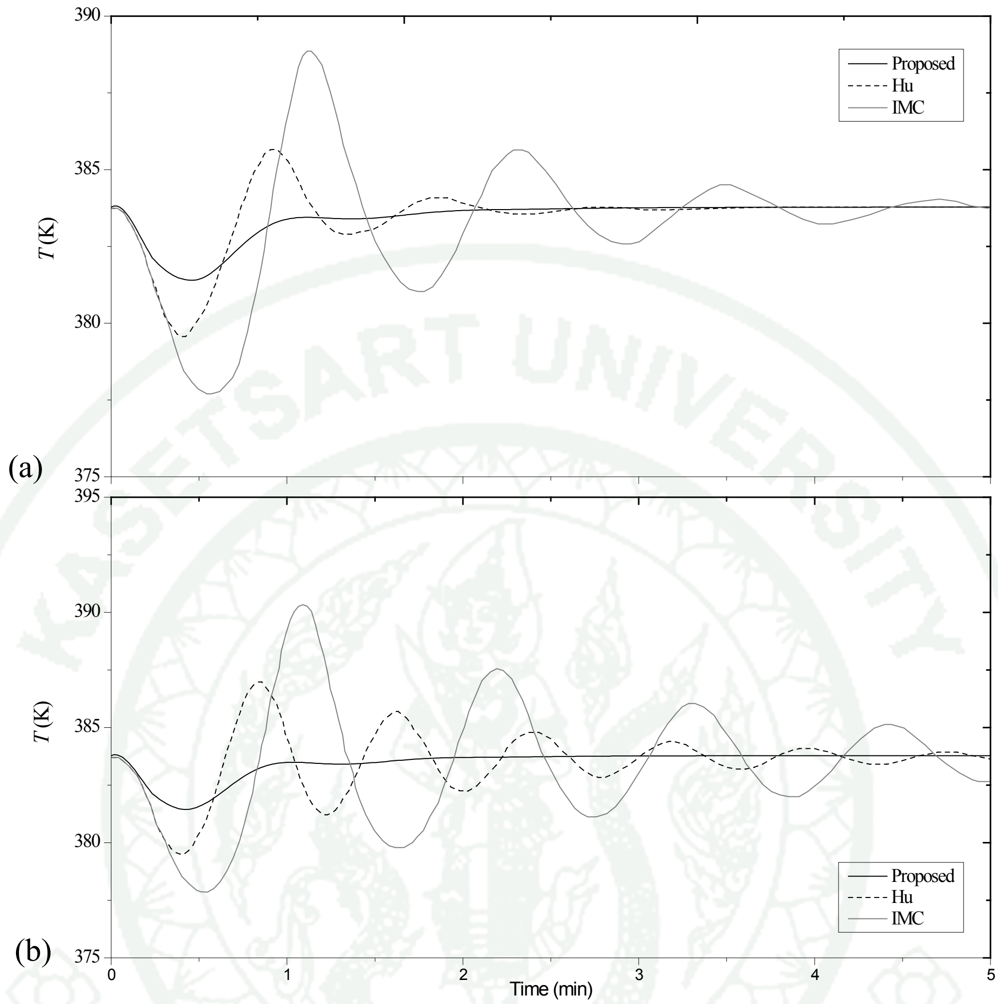

Figure 48 Closed-loop responses of *T* for -10% unmeasured disturbance in *F* under the case of (a) +20% and (b) -20% compared with Hu's method

CONCLUSION AND RECOMMENDATION

Conclusion

This work presented two control methods for the uncertain processes with and without time-delay. Both proposed methods took the advantages of the I/O linearization technique and 2DOF control structure to handle setpoint tracking and disturbance rejection problems. The concept of the open-loop observer was employed to design the state estimator and the state predictor for developed schemes with and without time-delay compensator, respectively. Moreover, the addition of feedback signal was used to ensure the offset-free of the closed-loop output responses in the presence of load disturbances, effects of approximation, model-process mismatch, and other sources of modeling errors.

The illustrative examples were used to demonstrate the performances of the proposed methods. The simulation was studied under servo and regulatory problems. The results show that both proposed control schemes can successfully operate the reactors at the desired setpoints. The effects of unmeasured disturbance and parametric uncertainties were compensated by disturbance rejection controller to maintain the closed-loop stability. Moreover, the results showed that the 2DOF structure provided more effectiveness of disturbance rejection compared with the IMC structure (1DOF). Since another DOF was added into the control structure for reducing the effect of uncertainties on the controlled output, the responses of the proposed method are approached to the setpoint faster than the responses of IMC structure.

There are many advantages that can be obtained from the proposed control method. First, the developed controller can be applied to the chemical process, not only in the illustrative examples, with uncertainty and time-delay. Second, the tuning parameters in the control system can be used to adjusting the speed of the controller. Consequently, the controller can force the output response to reach the desired output setpoint faster. Moreover, the developed control system has less tuning parameters when compared with the existing controllers.

Recommendation

In this research work, the developed control method requires the information of the controlled outputs to compensate the uncertainty. The adequate additional measurement sensors are needed in order to provide the estimated disturbances. Furthermore, the developed control technique is focused on the class of stable open-loop and minimum phase (stable zero dynamics) processes. Actually, there are many industrial processes that exhibit non-minimum phase and/or open-loop unstable behavior. Sometimes, some information of process parameters is unavailable. Moreover, an implementation of the proposed method to a real process should be considered in future research.

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APPENDICES

Appendix A

The example of the stability analysis of open-loop system and zero dynamics

To analyze the open-loop stability of the isothermal Van de Vusse reaction process, the equilibrium pairs ($C_{A,ss}$ = 8.2 mol L^{-1}, $C_{B,ss}$ = 0.586 mol L^{-1}, and $(F/V)_{ss}$ = 10 min^{-1}) is studied. Let $x = \begin{bmatrix} C_A & C_B \end{bmatrix}^T$, $u = F/V$, and $y = C_B$. The component balances of x_1 and x_2 are in the form of:

$$\begin{aligned} \frac{dx_1}{dt} &= -k_1 x_1 - k_3 x_1^2 + (C_{Ai} - x_1)u \\ \frac{dx_2}{dt} &= k_1 x_1 - k_2 x_2 - x_2 u \\ y &= x_2 \end{aligned} \tag{A.1}$$

The Jacobian matrix of the process in (A.1) is described as:

$$J = \begin{bmatrix} -k_1 - 2k_3 x_1 - u & 0 \\ k_1 & -k_2 - u \end{bmatrix}_{x_{ss}, u_{ss}} \tag{A.2}$$

The eigenvalues of the Jacobian matrix in (A.2) can be obtained as following steps.

$$J - \lambda I = \begin{bmatrix} -k_1 - 2k_3 x_1 - u & 0 \\ k_1 & -k_2 - u \end{bmatrix}_{x_{ss}, u_{ss}} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{A.3}$$

$$\begin{aligned} |J - \lambda I| &= 0 \\ \left(-k_1 - 2k_3 x_{1,ss} - u_{ss} - \lambda\right)\left(-k_2 - u_{ss} - \lambda\right) &= 0 \end{aligned} \tag{A.4}$$

The eigenvalues are -11.67 and -13.57. Therefore, this equilibrium point is open-loop stable.

To describe the zero dynamics of the process, the equilibrium pairs ($C_{A,ss}$ = 8.2 mol L^{-1}, $C_{B,ss}$ = 0.586 mol L^{-1}, and $(F/V)_{ss}$ = 10 min^{-1}) is also considered. The relative order of this process is r = 1, and the number of zero dynamics is equal to one. The state variables are set as following:

$$\begin{aligned} x_1 &= \zeta \\ x_2 &= y_{sp} \end{aligned} \tag{A.5}$$

Consider the derivative term of output at the equilibrium point.

$$\begin{aligned} x_2 &= y_{sp} \\ k_1\zeta - k_2 y_{sp} - y_{sp}u_{ss} &= 0 \\ u_{ss} &= \left(k_1\zeta - k_2 y_{sp}\right) / \, y_{sp} \end{aligned} \tag{A.6}$$

By substituting (A.6), the zero dynamics of the Van de Vusse reaction model are:

$$\frac{d\zeta}{dt} = -k_1\zeta - k_3\zeta^2 + \frac{\left(C_{Ai} - \zeta\right)\left(k_1\zeta - k_2 y_{sp}\right)}{y_{sp}} \tag{A.7}$$

The eigenvalues of the zero dynamics is -11. Therefore, this equilibrium point is stable zero dynamics (minimum phase).

Appendix B

The formulation of control system for SISO CSTR with isothermal Van de Vusse reaction without time-delay

The formulation of the control system for the process described in Section 1.1 is as follows:

The disturbance-free state estimator

$$\begin{aligned}\frac{d\hat{x}_1}{dt} &= -k_1\hat{x}_1 - k_3\hat{x}_1^2 + (C_{Ai} - \hat{x}_1)u \\ \frac{d\hat{x}_2}{dt} &= k_1\hat{x}_1 - k_2\hat{x}_2 - \hat{x}_2 u \\ \hat{y} &= \hat{x}_2\end{aligned} \tag{B.1}$$

The feedback compensator

$$v = y_{sp} - (y - \hat{y}) \tag{B.2}$$

The setpoint tracking controller

The equation of the setpoint tracking controller is briefly described as following. The relative order of this process can be analyzed from Equation (2).

$$\begin{aligned}y &= C_B \\ \frac{dy}{dt} &= \frac{dC_B}{dt} \\ &= k_1 C_A - k_2 C_B - C_B \frac{F}{V}\end{aligned}$$

In this case, the relative is r = 1. The implementation of Equation (6) for containing the integral action takes the form of:

$$(\varepsilon\mathcal{D} + 1)^r \hat{y} = v$$

By substituting the time derivative of output with r = 1, then obtains:

$$(\varepsilon\mathcal{D}+1)\hat{x}_2 = v$$
$$\varepsilon\mathcal{D}\hat{x}_2 + \hat{x}_2 = v$$
$$\varepsilon\left(k_1\hat{x}_1 - k_2\hat{x}_2 - \hat{x}_2 u\right) + \hat{x}_2 = v$$

Therefore, the equation of the setpoint tracking controller is:

$$u = \left((-v+\hat{x}_2)/\varepsilon \ + \ k_1\hat{x}_1 - k_2\hat{x}_2\right)/\hat{x}_2 \tag{B.3}$$

The disturbance rejection controller

$$\delta = K(y-\hat{y}) \tag{B.4}$$

Therefore, the manipulated input for uncertain process is the form of:

$$\tilde{u} = \left((-v+\hat{x}_2)/\varepsilon \ + \ k_1\hat{x}_1 - k_2\hat{x}_2\right)/\hat{x}_2 - K(y-\hat{y}) \tag{B.5}$$

Appendix C

The formulation of control system for MIMO two CSTRs in series without time-delay

The formulation of the control system for the process described in Section 1.2 is as follows:

The disturbance-free state estimator

$$\begin{aligned}
\frac{d\hat{x}_1}{dt} &= \frac{F_1}{V_1}(T_0 - \hat{x}_1) - \frac{UA_1}{V_1\rho C_p}(\hat{x}_1 - \hat{x}_4) + \frac{(-\Delta H_1)}{\rho C_p}k_{01}\hat{x}_2 \exp\left(-\frac{E_1}{R\hat{x}_1}\right) \\
\frac{d\hat{x}_2}{dt} &= \frac{F_1}{V_1}(C_{A0} - \hat{x}_2) - k_{01}\hat{x}_2 \exp\left(-\frac{E_1}{R\hat{x}_1}\right) \\
\frac{d\hat{x}_3}{dt} &= \frac{F_1}{V_1}\hat{x}_3 + k_{01}\hat{x}_2 \exp\left(-\frac{E_1}{R\hat{x}_1}\right) \\
\frac{d\hat{x}_4}{dt} &= \frac{F_{j1}}{V_{j1}}(u_1 - \hat{x}_4) \\
\frac{d\hat{x}_5}{dt} &= \frac{F_1}{V_2}(\hat{x}_1 - \hat{x}_5) - \frac{UA_2}{V_2\rho C_p}(\hat{x}_5 - \hat{x}_9) + \frac{(-\Delta H_1)}{\rho C_p}k_{01}C_{A2} \exp\left(-\frac{E_1}{R\hat{x}_5}\right) \\
&\quad + \frac{(-\Delta H_2)}{\rho C_p}k_{02}\hat{x}_7\hat{x}_8 \exp\left(-\frac{E_2}{R\hat{x}_5}\right) \qquad \text{(C.1)} \\
\frac{d\hat{x}_6}{dt} &= \frac{F_1}{V_2}\hat{x}_2 - \frac{F_1 + u_3}{V_2}\hat{x}_6 - k_{01}\hat{x}_6 \exp\left(-\frac{E_1}{R\hat{x}_5}\right) \\
\frac{d\hat{x}_7}{dt} &= \frac{F_1}{V_2}\hat{x}_3 - \frac{F_1 + u_3}{V_2}\hat{x}_7 + k_{01}\hat{x}_6 \exp\left(-\frac{E_1}{R\hat{x}_5}\right) - k_{02}\hat{x}_7\hat{x}_8 \exp\left(-\frac{E_2}{R\hat{x}_5}\right) \\
\frac{d\hat{x}_8}{dt} &= \frac{u_3}{V_2}C_{C0} - \frac{F_1 + u_3}{V_2}\hat{x}_8 - k_{02}\hat{x}_7\hat{x}_8 \exp\left(-\frac{E_2}{R\hat{x}_5}\right) \\
\frac{d\hat{x}_9}{dt} &= \frac{F_{j2}}{V_{j2}}(u_2 - \hat{x}_9) \\
\hat{y}_1 &= \hat{x}_1 \\
\hat{y}_2 &= \hat{x}_5 \\
\hat{y}_3 &= \hat{x}_8
\end{aligned}$$

The feedback compensator

$$
\begin{aligned}
\nu_1 &= y_{1,sp} - (y_1 - \hat{y}_1) \\
\nu_2 &= y_{2,sp} - (y_2 - \hat{y}_2) \\
\nu_3 &= y_{3,sp} - (y_1 - \hat{y}_3)
\end{aligned} \tag{C.2}
$$

The setpoint tracking controller

$$
\begin{aligned}
u_1 = {} & \exp(-2E_1 / (R\hat{x}_1))(\alpha_1 C_p \varepsilon_1 k_{01} R V_1^2 (-\Delta H_1)\hat{x}_1^2 \hat{x}_2 - \alpha_1 E_1 \varepsilon_1^2 k_{01}^2 V_1^2 (-\Delta H_1)^2 \hat{x}_2^2 + \\
& \exp(-2E_1 / (R\hat{x}_1)) R\hat{x}_1^2 (\alpha_1 C_p \rho (\varepsilon_1^2 F_1 T_0 (C_p F_1 \rho + UA_1) - 2C_p \varepsilon_1 F_1 \rho T_0 V_1 + \\
& C_p \rho v_1 V_1^2) - \alpha_1 (\varepsilon_1 UA_1 + C_p \rho (\varepsilon_1 F_1 - V_1))^2 \hat{x}_1 + \varepsilon_1 UA_1 (\alpha_1 \varepsilon_1 (C_p F_1 \rho + UA_1) + \\
& C_p (-2\alpha_1 + \varepsilon_1) \rho V_1) \hat{x}_4 - \alpha_1 \exp(-2E_1 / (R\hat{x}_1)) \varepsilon_1 k_{01} V_1 (-\Delta H_1)(-E_1 \varepsilon_1 \\
& (C_p F_1 \rho + UA_1) \hat{x}_1 \hat{x}_2 + R\hat{x}_1^2 (C_{A0} C_p \varepsilon_1 F_1 \rho - (2C_p \varepsilon_1 F_1 \rho + \varepsilon_1 UA_1 - 2C_p \rho V_1) \hat{x}_2) + \\
& E_1 \varepsilon_1 \hat{x}_2 (C_p F_1 \rho T_0 + UA_1 \hat{x}_4)))) / (C_p \varepsilon_1^2 R UA_1 V_1 \hat{x}_1^2)
\end{aligned} \tag{C.3-1}
$$

$$
\begin{aligned}
u_2 = {} & (\alpha 2 C_p \rho V_1 \;/ (\varepsilon_2^2 UA_2))(v_2 - \varepsilon_2^2 F_1^2 T_0 / (V_1 V_2) + (\exp(E_1 / (R\hat{x}_1) + 2(E_1 + E_2) / \\
& (R\hat{x}_5))(\alpha_2 \varepsilon_2^2 \exp(E_1 (2\hat{x}_1 + \hat{x}_5) / (R\hat{x}_1 \hat{x}_5)) k_{02}^2 V_1 V_2^2 (-\Delta H_2) \hat{x}_7 \hat{x}_8 (-E_2 (-\Delta H_2) \\
& \hat{x}_7 \hat{x}_8 + C_p R \rho \hat{x}_5^2 (\hat{x}_7 + \hat{x}_8)) + \exp(2E_2 / (R\hat{x}_5))(-\alpha_2 \varepsilon_2^2 \exp(E_1 / (R\hat{x}_1)) \\
& E_1 k_{01}^2 V_1 V_2^2 (-\Delta H_1)^2 \hat{x}_6^2 + \alpha_2 \varepsilon_2^2 C_p \exp(E_1 (\hat{x}_1 + \hat{x}_5) / (R\hat{x}_1 \hat{x}_5)) F_1 \rho \hat{x}_1 \\
& (\exp(E_1 / (R\hat{x}_5)) R (-2C_p \rho V_1 V_2 + \varepsilon_2 (UA_2 V_1 + UA_1 V_2 + C_p F_1 \rho (V1 + V2))) \\
& \hat{x}_5^2 - \varepsilon_2 E_1 k_{01} V_1 V_2 (-\Delta H_1) \hat{x}_6) - \exp(E_1 (2\hat{x}_1 + \hat{x}_5) / (R\hat{x}_1 \hat{x}_5)) R\hat{x}_5^2 \\
& (\alpha_2 \varepsilon_2^2 C_p F_1 \; \rho UA_1 V_2 \hat{x}_4 + \alpha_2 V_1 (\varepsilon_2 (C_p F_1 \; \rho + UA_2) - C_p \rho V_2)^2 \hat{x}_5 - \\
& \varepsilon_2 UA_2 V_1 (\alpha_2 \varepsilon_2 (C_p F_1 \rho + UA_2) + (-2\alpha_2 + \varepsilon_2) C_p \rho V_2) \hat{x}_9) + \\
& \alpha_2 \varepsilon_2 k_{01} V_1 \; V_2 \; (-\Delta H_1)(\varepsilon_2 C_p R \rho \hat{x}_5^2 (-\exp(2E_1 / (R\hat{x}_5)) F_1 \hat{x}_2 + \\
& \exp(E_1 / (R\hat{x}_1)) k_{01} V_2 \hat{x}_6) - \exp(E_1 (\hat{x}_1 + \hat{x}_5) / (R\hat{x}_1 \hat{x}_5))(\hat{x}_5 \\
& (\varepsilon_2 C_p F_1 R \rho \hat{x}_2 \hat{x}_5 - (\varepsilon_2 E_1 (C_p F_1 \rho + UA_2) + R (2\varepsilon_2 C_p F_1 \rho + \varepsilon_2 UA_2 - 2C_p \rho V_2 + \\
& \varepsilon_2 C_p \rho u_3) \hat{x}_5) \hat{x}_6) + \varepsilon_2 E_1 UA_2 \hat{x}_6 \hat{x}_9)))) - \alpha_2 \varepsilon_1 \exp((E_1 / \hat{x}_1) + ((E_1 + E_5) / \hat{x}_5) / R)
\end{aligned} \tag{C.3-2}
$$

$$k_{02}V_1V_2(-\Delta H_2)\left(\varepsilon_2\, k_{01}V\hat{x}_6\left(C_pR\rho\hat{x}_5^2+(E_1+E_2)(-\Delta H_1)\,\hat{x}_7\right)\hat{x}_8\,+\right.$$
$$\exp\left(E_1/(R\hat{x}_5)\right)\left(\varepsilon_2C_pR\rho u_3\hat{x}_5^2\hat{x}_7\left(C_{C0}-2\hat{x}_8\right)+\hat{x}_8\left(\varepsilon_2C_pF_1R\rho\hat{x}_3\hat{x}_5^2+\hat{x}_7\right.\right.$$
$$\left(\varepsilon_2C_pE_2F_1\rho\hat{x}_1\;+\hat{x}_5\left(-\varepsilon_2E_2\left(C_pF_1\rho+UA_2\right)-R\left(3\varepsilon_2C_pF_1\rho+\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\left.\varepsilon_2UA_2-2C_p\rho V_2\right)\hat{x}_5\right)+\varepsilon_2E_2UA_2\hat{x}_9\right)\right)\right)\right)\right)/\left(\alpha_2C_p^2R\rho^2V_1V_2^2\hat{x}_5^2\right)\right)$$

$$u_3=\left(V_2v_3+\left(\varepsilon_3F_1-V_2+\varepsilon_3\exp\left(-E_2/(R\hat{x}_5)\right)k_{02}V_2\hat{x}_7\right)\hat{x}_8\right)/\left(\varepsilon_3\left(C_{C0}-\hat{x}_8\right)\right) \qquad \text{(C.3-3)}$$

where $\alpha_1=(F_{j1}/V_{j1})$ and $\alpha_2=(F_{j2}/V_{j2})$

The disturbance rejection controller

$$\begin{aligned}\delta_1&=K_1(y_1-\hat{y}_1)\\ \delta_2&=K_2(y_2-\hat{y}_2)\\ \delta_3&=K_3(y_3-\hat{y}_3)\end{aligned} \qquad \text{(C.4)}$$

Therefore, the manipulated input for uncertain process is the form of:

$$\tilde{u}_1=\exp\left(-2E_1/(R\hat{x}_1)\right)\left(\alpha_1C_p\varepsilon_1k_{01}RV_1^2(-\Delta H_1)\hat{x}_1^2\hat{x}_2-\alpha_1E_1\varepsilon_1^2k_{01}^2V_1^2(-\Delta H_1)^2\hat{x}_2^2+\right.$$
$$\exp\left(-2E_1/(R\hat{x}_1)\right)R\hat{x}_1^2\left(\alpha_1C_p\rho\left(\varepsilon_1^2F_1T_0\left(C_pF_1\rho+UA_1\right)-2C_p\varepsilon_1F_1\rho T_0V_1+\right.\right.$$
$$\left.C_p\rho v_1V_1^2\right)-\alpha_1\left(\varepsilon_1UA_1+C_p\rho\left(\varepsilon_1F_1-V_1\right)\right)^2\hat{x}_1+\varepsilon_1UA_1\left(\alpha_1\varepsilon_1\left(C_pF_1\rho+UA_1\right)+\right.$$
$$\left.C_p\left(-2\alpha_1+\varepsilon_1\right)\rho V_1\right)\hat{x}_4-\alpha_1\exp\left(-2E_1/(R\hat{x}_1)\right)\varepsilon_1k_{01}V_1(-\Delta H_1)\left(-E_1\varepsilon_1\right. \qquad \text{(C.5-1)}$$
$$\left(C_pF_1\rho+UA_1\right)\hat{x}_1\hat{x}_2+R\hat{x}_1^2\left(C_{A0}C_p\varepsilon_1F_1\rho-\left(2C_p\varepsilon_1F_1\rho+\varepsilon_1UA_1-2C_p\rho V_1\right)\hat{x}_2\right)+$$
$$\left.\left.\left.E_1\varepsilon_1\hat{x}_2\left(C_pF_1\rho T_0+UA_1\hat{x}_4\right)\right)\right)\right)/\left(C_p\varepsilon_1^2RUA_1V_1\hat{x}_1^2\right)-K_1(y_1-\hat{y}_1)$$

$$
\begin{aligned}
\tilde{u}_2 = &\Big(\alpha 2C_p\rho V_1 \ \ /\left(\varepsilon_2^2 UA_2\right)\Big)\Big(v_2 - \varepsilon_2^2 F_1^2 T_0 / \left(V_1 V_2\right) + \Big(\exp\left(E_1 / (R\hat{x}_1) + 2(E_1 + E_2)/\right.\\
&\left.(R\hat{x}_5)\right)\Big(\alpha_2\varepsilon_2^2 \exp\left(E_1(2\hat{x}_1 + \hat{x}_5)/(R\hat{x}_1\hat{x}_5)\right)k_{02}^2 V_1 V_2^2 (-\Delta H_2)\,\hat{x}_7\hat{x}_8(-E_2(-\Delta H_2)\\
&\hat{x}_7\hat{x}_8 + C_p R\rho\hat{x}_5^2(\hat{x}_7 + \hat{x}_8)\Big) + \exp\left(2E_2/(R\hat{x}_5)\right)\Big(-\alpha_2\varepsilon_2^2\exp\left(E_1/(R\hat{x}_1)\right)\\
&E_1 k_{01}^2 V_1 V_2^2(-\Delta H_1)^2\hat{x}_6^2 + \alpha_2\varepsilon_2^2 C_p \exp\left(E_1(\hat{x}_1 + \hat{x}_5)/(R\hat{x}_1\hat{x}_5)\right)F_1\rho\hat{x}_1\\
&\Big(\exp\left(E_1/(R\hat{x}_5)\right)R\Big(-2C_p\rho V_1V_2 + \varepsilon_2\left(UA_2V_1 + UA_1V_2 + C_pF_1\rho(V1+V2)\right)\Big)\\
&\hat{x}_5^2 - \varepsilon_2 E_1 k_{01}V_1V_2(-\Delta H_1)\hat{x}_6\Big) - \exp\left(E_1(2\hat{x}_1 + \hat{x}_5)/(R\hat{x}_1\hat{x}_5)\right)R\hat{x}_5^2\\
&\Big(\alpha_2\varepsilon_2^2 C_pF_1\ \rho UA_1V_2\hat{x}_4 + \alpha_2V_1\Big(\varepsilon_2\left(C_pF_1\ \rho + UA_2\right) - C_p\rho V_2\Big)^2\hat{x}_5 -\\
&\varepsilon_2UA_2V_1\Big(\alpha_2\varepsilon_2\left(C_pF_1\rho + UA_2\right) + \left(-2\alpha_2 + \varepsilon_2\right)C_p\rho V_2\Big)\hat{x}_9\Big) +\\
&\alpha_2\varepsilon_2k_{01}V_1\ V_2\ (-\Delta H_1)\Big(\varepsilon_2C_pR\rho\hat{x}_5^2\Big(-\exp\left(2E_1/(R\hat{x}_5)\right)F_1\hat{x}_2 +\\
&\exp\left(E_1/(R\hat{x}_1)\right)k_{01}V_2\hat{x}_6\Big) - \exp\Big(E_1\left(\hat{x}_1 + \hat{x}_5\right)/(R\hat{x}_1\hat{x}_5)\Big)\Big(\hat{x}_5\\
&\Big(\varepsilon_2C_pF_1R\rho\hat{x}_2\hat{x}_5 - \Big(\varepsilon_2E_1\left(C_pF_1\rho + UA_2\right) + R\Big(2\varepsilon_2C_pF_1\rho + \varepsilon_2UA_2 - 2C_p\rho V_2 +\\
&\varepsilon_2C_p\rho u_3\Big)\hat{x}_5\Big)\hat{x}_6\Big) + \varepsilon_2E_1UA_2\hat{x}_6\hat{x}_9\Big)\Big)\Big) - \alpha_2\varepsilon_1\exp\Big(\left(E_1/\hat{x}_1\right) + \Big(\left(E_1 + E_5\right)/\hat{x}_5\Big)\\
&/R\Big)k_{02}V_1V_2(-\Delta H_2)\left(\varepsilon_2\ k_{01}V\hat{x}_6\left(C_pR\rho\hat{x}_5^2 + (E_1 + E_2)(-\Delta H_1)\,\hat{x}_7\right)\hat{x}_8 +\right.\\
&\exp\left(E_1/(R\hat{x}_5)\right)\Big(\varepsilon_2C_pR\rho u_3\hat{x}_5^2\hat{x}_7\left(C_{C0} - 2\hat{x}_8\right) + \hat{x}_8\left(\varepsilon_2C_pF_1R\rho\hat{x}_3\hat{x}_5^2 + \hat{x}_7\right.\\
&\Big(\varepsilon_2C_pE_2F_1\rho\hat{x}_1\ + \hat{x}_5\Big(-\varepsilon_2E_2\left(C_pF_1\rho + UA_2\right) - R\Big(3\varepsilon_2C_pF_1\rho +\\
&\varepsilon_2UA_2 - 2C_p\rho V_2\Big)\hat{x}_5\Big) + \varepsilon_2E_2UA_2\hat{x}_9\Big)\Big)\Big)\Big)\Big)\Big)\Big/\Big(\alpha_2C_p^2R\rho^2V_1V_2^2\hat{x}_5^2\Big)\Big) - K_2\left(y_2 - \hat{y}_2\right)
\end{aligned}
\qquad \text{(C.5-2)}
$$

$$
\begin{aligned}
\tilde{u}_3 = &\Big(V_2v_3 + \Big(\varepsilon_3F_1 - V_2 + \varepsilon_3\exp\left(-E_2/(R\hat{x}_5)\right)k_{02}V_2\hat{x}_7\Big)\hat{x}_8\Big)\Big/\Big(\varepsilon_3\left(C_{C0} - \hat{x}_8\right)\Big)\\
&-K_3\left(y_3 - \hat{y}_3\right)
\end{aligned}
\qquad \text{(C.5-3)}
$$

Appendix D

The formulation of control system for SISO CSTR with constant heat removal without time-delay

The formulation of the control system for the process described in Section 1.3 is as follows:

The disturbance-free state estimator

$$\begin{aligned}\frac{d\hat{x}_1}{dt} &= -k_0 \exp\left(-\frac{E_a}{R\hat{x}_2}\right)\hat{x}_1 + (C_{Ai} - \hat{x}_1)\frac{u}{V} \\ \frac{d\hat{x}_2}{dt} &= Q + \gamma k_0 \exp\left(-\frac{E_a}{R\hat{x}_2}\right)\hat{x}_1 + (T_i - \hat{x}_2)\frac{u}{V} \\ \hat{y} &= \hat{x}_2\end{aligned} \tag{D.1}$$

The feedback compensator

$$v = y_{sp} - (y - \hat{y}) \tag{D.2}$$

The setpoint tracking controller

$$u = \left((v - \hat{x}_2)/\varepsilon \; - \; \gamma k_0 \exp\left(-\frac{E_a}{R\hat{x}_2}\right)\hat{x}_1 - Q \right)\frac{V}{(T_i - \hat{x}_2)} \tag{D.3}$$

The disturbance rejection controller

$$\delta = K(y - \hat{y}) \tag{D.4}$$

Therefore, the manipulated input for uncertain process is the form of:

$$\tilde{u} = \left((v - \hat{x}_2)/\varepsilon \; - \; \gamma k_0 \exp\left(-\frac{E_a}{R\hat{x}_2}\right)\hat{x}_1 - Q \right)\frac{V}{(T_i - \hat{x}_2)} - K(y - \hat{y}) \tag{D.5}$$

Appendix E

The formulation of control system for SISO CSTR with constant heat removal with time-delay

The formulation of the control system for the process described in Section 2.1 is as follows:

The disturbance-free state predictor

$$\begin{aligned}
\frac{dx_1^*}{dt} &= -k_0 \exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^* + (C_{Ai} - x_1^*)\frac{u}{V} \\
\frac{dx_2^*}{dt} &= Q + \gamma k_0 \exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^* + (T_i - x_2^*)\frac{u}{V} \\
y^* &= x_2^*
\end{aligned} \tag{E.1}$$

where $x^*(t) = \hat{x}(t+\theta)$

The feedback compensator

$$v = y_{sp} - (y - \hat{y}) \tag{E.2}$$

The setpoint tracking controller

$$u(t) = \left((v - x_2^* / \varepsilon \; - \; \gamma k_0 \exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^* - Q\right)\frac{V}{(T_i - x_2^*)} \tag{E.3}$$

The disturbance rejection controller

$$\delta = K(y - \hat{y}) \tag{E.4}$$

Therefore, the manipulated input for uncertain process is the form of:

$$\tilde{u} = \left((v - x_2^* / \varepsilon \; - \; \gamma k_0 \exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^* - Q\right)\frac{V}{(T_i - x_2^*)} - K(y - \hat{y}) \tag{E.5}$$

Appendix F

The formulation of control system for MIMO two CSTRs
in series with time-delay

The formulation of the control system for the process described in Section 2.2 is as follows:

The disturbance-free state estimator

$$
\begin{aligned}
\frac{dx_1^*}{dt} &= \frac{F_1}{V_1}(T_0 - x_1^*) - \frac{UA_1}{V_1\rho C_p}(x_1^* - x_4^*) + \frac{(-\Delta H_1)}{\rho C_p}k_{01}x_2^* \exp\left(-\frac{E_1}{Rx_1^*}\right) \\
\frac{dx_2^*}{dt} &= \frac{F_1}{V_1}(C_{A0} - x_2^*) - k_{01}x_2^* \exp\left(-\frac{E_1}{Rx_1^*}\right) \\
\frac{dx_3^*}{dt} &= \frac{F_1}{V_1}x_3^* + k_{01}x_2^* \exp\left(-\frac{E_1}{Rx_1^*}\right) \\
\frac{dx_4^*}{dt} &= \frac{F_{j1}}{V_{j1}}(u_1 - x_4^*) \\
\frac{dx_5^*}{dt} &= \frac{F_1}{V_2}(x_1^* - x_5^*) - \frac{UA_2}{V_2\rho C_p}(x_5^* - x_9^*) + \frac{(-\Delta H_1)}{\rho C_p}k_{01}C_{A2} \exp\left(-\frac{E_1}{Rx_5^*}\right) + \\
&\quad \frac{(-\Delta H_2)}{\rho C_p}k_{02}x_7^*x_8^* \exp\left(-\frac{E_2}{Rx_5^*}\right) \qquad \text{(F.1)} \\
\frac{dx_6^*}{dt} &= \frac{F_1}{V_2}x_2^* - \frac{F_1 + u_3}{V_2}x_6^* - k_{01}x_6^* \exp\left(-\frac{E_1}{Rx_5^*}\right) \\
\frac{dx_7^*}{dt} &= \frac{F_1}{V_2}x_3^* - \frac{F_1 + u_3}{V_2}x_7^* + k_{01}x_6^* \exp\left(-\frac{E_1}{Rx_5^*}\right) - k_{02}x_7^*x_8^* \exp\left(-\frac{E_2}{Rx_5^*}\right) \\
\frac{dx_8^*}{dt} &= \frac{u_3}{V_2}C_{C0} - \frac{F_1 + u_3}{V_2}x_8^* - k_{02}x_7^*x_8^* \exp\left(-\frac{E_2}{Rx_5^*}\right) \\
\frac{dx_9^*}{dt} &= \frac{F_{j2}}{V_{j2}}(u_2 - x_9^*) \\
y_1^* &= x_1^* \\
y_2^* &= x_5^* \\
y_3^* &= x_8^*
\end{aligned}
$$

The feedback compensator

$$\begin{aligned} \nu_1 &= y_{1,sp} - (y_1 - \hat{y}_1) \\ \nu_2 &= y_{2,sp} - (y_2 - \hat{y}_2) \\ \nu_3 &= y_{3,sp} - (y_1 - \hat{y}_3) \end{aligned} \tag{F.2}$$

The setpoint tracking controller

$$\begin{aligned} u_1 = {} & \exp\left(-2E_1/(Rx^*_1)\right)\left(\alpha_1 C_p \varepsilon_1 k_{01} R V_1^2(-\Delta H_1) x^{*2}_1 x^*_2 - \alpha_1 E_1 \varepsilon_1^2 k_{01}^2 V_1^2 (-\Delta H_1)^2 x^{*2}_2 + \right. \\ & \exp\left(-2E_1/(Rx^*_1)\right) R x^{*2}_1 \left(\alpha_1 C_p \rho \left(\varepsilon_1^2 F_1 T_0 \left(C_p F_1 \rho + UA_1\right) - 2C_p \varepsilon_1 F_1 \rho T_0 V_1 + \right.\right. \\ & \left. C_p \rho v_1 V_1^2\right) - \alpha_1 \left(\varepsilon_1 UA_1 + C_p \rho \left(\varepsilon_1 F_1 - V_1\right)\right)^2 x^*_1 + \varepsilon_1 UA_1 \left(\alpha_1 \varepsilon_1 \left(C_p F_1 \rho + UA_1\right) + \right. \\ & \left. C_p\left(-2\alpha_1 + \varepsilon_1\right)\rho V_1\right) x^*_4 - \alpha_1 \exp\left(-2E_1/(Rx^*_1)\right)\varepsilon_1 k_{01} V_1 (-\Delta H_1)\left(-E_1 \varepsilon_1 \right. \\ & \left(C_p F_1 \rho + UA_1\right) x^*_1 x^*_2 + R x^{*2}_1 \left(C_{A0} C_p \varepsilon_1 F_1 \rho - \left(2C_p \varepsilon_1 F_1 \rho + \varepsilon_1 UA_1 - 2C_p \rho V_1\right) x^*_2\right) + \\ & \left.\left.\left. E_1 \varepsilon_1 x^*_2 \left(C_p F_1 \rho T_0 + UA_1 x^*_4\right)\right)\right)\right) / \left(C_p \varepsilon_1^2 R UA_1 V_1 x^{*2}_1\right) \end{aligned} \tag{F.3-1}$$

$$\begin{aligned} u_2 = {} & \left(\alpha 2 C_p \rho V_1 \; / \left(\varepsilon_2^2 UA_2\right)\right)\left(v_2 - \varepsilon_2^2 F_1^2 T_0 / \left(V_1 V_2\right) + \left(\exp\left(E_1/(Rx^*_1) + 2(E_1 + E_2)/\right.\right.\right. \\ & \left.(Rx^*_5)\right)\left(\alpha_2 \varepsilon_2^2 \exp\left(E_1(2x^*_1 + x^*_5)/(Rx^*_1 x^*_5)\right) k_{02}^2 V_1 V_2^2 (-\Delta H_2) x^*_7 x^*_8 (-E_2(-\Delta H_2) \right. \\ & \left. x^*_7 x^*_8 + C_p R \rho x^{*2}_5 (x^*_7 + x^*_8)\right) + \exp\left(2E_2/(Rx^*_5)\right)\left(-\alpha_2 \varepsilon_2^2 \exp\left(E_1/(Rx^*_1)\right)\right. \\ & E_1 k_{01}^2 V_1 V_2^2 (-\Delta H_1)^2 x^{*2}_6 + \alpha_2 \varepsilon_2^2 C_p \exp\left(E_1(x^*_1 + x^*_5)/(Rx^*_1 x^*_5)\right) F_1 \rho x^*_1 \\ & \left(\exp\left(E_1/(Rx^*_5)\right) R\left(-2C_p \rho V_1 V_2 + \varepsilon_2 \left(UA_2 V_1 + UA_1 V_2 + C_p F_1 \rho (V1 + V2)\right)\right)\right. \\ & \left. x^{*2}_5 - \varepsilon_2 E_1 k_{01} V_1 V_2 (-\Delta H_1) x^*_6\right) - \exp\left(E_1(2x^*_1 + x^*_5)/(Rx^*_1 x^*_5)\right) R x^{*2}_5 \\ & \left(\alpha_2 \varepsilon_2^2 C_p F_1 \; \rho UA_1 V_2 x^*_4 + \alpha_2 V_1 \left(\varepsilon_2 \left(C_p F_1 \; \rho + UA_2\right) - C_p \rho V_2\right)^2 x^*_5 - \right. \\ & \left. \varepsilon_2 UA_2 V_1 \left(\alpha_2 \varepsilon_2 \left(C_p F_1 \rho + UA_2\right) + \left(-2\alpha_2 + \varepsilon_2\right) C_p \rho V_2\right) x^*_9\right) + \\ & \alpha_2 \varepsilon_2 k_{01} V_1 \; V_2 \; (-\Delta H_1)\left(\varepsilon_2 C_p R \rho x^{*2}_5 \left(-\exp\left(2E_1/(Rx^*_5)\right) F_1 x^*_2 + \right.\right. \\ & \left. \exp\left(E_1/(Rx^*_1)\right) k_{01} V_2 x^*_6\right) - \exp\left(E_1\left(x^*_1 + x^*_5\right)/(Rx^*_1 x^*_5)\right)\left(x^*_5 \right. \\ & \left(\varepsilon_2 C_p F_1 R \rho x^*_2 x^*_5 - \left(\varepsilon_2 E_1 \left(C_p F_1 \rho + UA_2\right) + R\left(2\varepsilon_2 C_p F_1 \rho + \varepsilon_2 UA_2 - 2C_p \rho V_2 + \right.\right.\right. \\ & \left.\left.\left.\left.\left.\left. \varepsilon_2 C_p \rho u_3\right) x^*_5\right) x^*_6\right) + \varepsilon_2 E_1 UA_2 x^*_6 x^*_9\right)\right)\right) - \alpha_2 \varepsilon_1 \exp\left(\left(E_1 / x^*_1\right) + \left(\left(E_1 + E_5\right)/x^*_5\right)/R\right) \end{aligned} \tag{F.3-2}$$

$$k_{02}V_1V_2(-\Delta H_2)\left(\varepsilon_2\, k_{01}Vx^*{}_6\left(C_pR\rho x^{*2}_5+(E_1+E_2)(-\Delta H_1)\,x^*{}_7\right)x^*{}_8+\right.$$
$$\exp\left(E_1\,/\,(Rx^*{}_5)\right)\left(\varepsilon_2C_pR\rho u_3x^{*2}_5x^*{}_7\left(C_{C0}-2x^*{}_8\right)+x^*{}_8\left(\varepsilon_2C_pF_1R\rho x^*{}_3x^{*2}_5+x^*{}_7\right.\right.$$
$$\left(\varepsilon_2C_pE_2F_1\rho x^*{}_1+x^*{}_5\left(-\varepsilon_2E_2\left(C_pF_1\rho+UA_2\right)-R\left(3\varepsilon_2C_pF_1\rho+\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\left.\varepsilon_2UA_2-2C_p\rho V_2\right)x^*{}_5\right)+\varepsilon_2E_2UA_2x^*{}_9\right)\right)\right)\right)\right)\Big/\left(\alpha_2C_p^2R\rho^2V_1V_2^2x^{*2}_5\right)\Big)$$

$$u_3=\left(V_2v_3+\left(\varepsilon_3F_1-V_2+\varepsilon_3\exp\left(-E_2\,/\,(Rx^*{}_5)\right)k_{02}V_2x^*{}_7\right)x^*{}_8\right)\Big/ \left(\varepsilon_3\left(C_{C0}-x^*{}_8\right)\right) \tag{F.3-3}$$

where $\alpha_1=(F_{j1}\,/\,V_{j1})$ and $\alpha_2=(F_{j2}\,/\,V_{j2})$

The disturbance rejection controller

$$\begin{aligned}\delta_1&=K_1(y_1-\hat{y}_1)\\ \delta_2&=K_2(y_2-\hat{y}_2)\\ \delta_3&=K_3(y_3-\hat{y}_3)\end{aligned} \tag{F.4}$$

Therefore, the manipulated input for uncertain process is the form of:

$$\begin{aligned}\tilde{u}_1=&\exp\left(-2E_1\,/\,(Rx^*{}_1)\right)\left(\alpha_1C_p\varepsilon_1k_{01}RV_1^2(-\Delta H_1)x^{*2}_1x^*{}_2-\alpha_1E_1\varepsilon_1^2k_{01}^2V_1^2(-\Delta H_1)^2x^{*2}_2+\right.\\ &\exp\left(-2E_1\,/\,(Rx^*{}_1)\right)Rx^{*2}_1\left(\alpha_1C_p\rho\left(\varepsilon_1^2F_1T_0\left(C_pF_1\rho+UA_1\right)-2C_p\varepsilon_1F_1\rho T_0V_1+\right.\right.\\ &\left.\left.C_p\rho v_1V_1^2\right)-\alpha_1\left(\varepsilon_1UA_1+C_p\rho\left(\varepsilon_1F_1-V_1\right)\right)^2x^*{}_1+\varepsilon_1UA_1\left(\alpha_1\varepsilon_1\left(C_pF_1\rho+UA_1\right)+\right.\right.\\ &\left.\left.C_p\left(-2\alpha_1+\varepsilon_1\right)\rho V_1\right)x^*{}_4-\alpha_1\exp\left(-2E_1\,/\,(Rx^*{}_1)\right)\varepsilon_1k_{01}V_1(-\Delta H_1)\left(-E_1\varepsilon_1\right.\\ &\left(C_pF_1\rho+UA_1\right)x^*{}_1x^*{}_2+Rx^{*2}_1\left(C_{A0}C_p\varepsilon_1F_1\rho-\left(2C_p\varepsilon_1F_1\rho+\varepsilon_1UA_1-2C_p\rho V_1\right)x^*{}_2\right)+\\ &\left.\left.\left.E_1\varepsilon_1x^*{}_2\left(C_pF_1\rho T_0+UA_1x^*{}_4\right)\right)\right)\right)\Big/\left(C_p\varepsilon_1^2RUA_1V_1x^{*2}_1\right)-K_1(y_1-\hat{y}_1)\end{aligned} \tag{F.5-1}$$

$$\tilde{u}_2 = \Big(\alpha 2C_p\rho V_1 \;/\big(\varepsilon_2^2 UA_2\big)\Big)\Big(v_2 - \varepsilon_2^2 F_1^2 T_0 \,/\, (V_1V_2) + \Big(\exp\Big(E_1 \,/\, (Rx^*_1) + 2(E_1 + E_2)\,/\,(Rx^*_5)\Big)\Big(\alpha_2\varepsilon_2^2 \exp\Big(E_1(2x^*_1 + x^*_5)\,/\,(Rx^*_1x^*_5)\Big)k_{02}^2V_1V_2^2(-\Delta H_2)\,x^*_7x^*_8(-E_2(-\Delta H_2)\,x^*_7x^*_8 + C_pR\rho x^{*2}_5(x^*_7 + x^*_8)\Big) + \exp\Big(2E_2\,/\,(Rx^*_5)\Big)\Big(-\alpha_2\varepsilon_2^2 \exp\Big(E_1\,/\,(Rx^*_1)\Big)E_1k_{01}^2V_1V_2^2(-\Delta H_1)^2\,x^{*2}_6 + \alpha_2\varepsilon_2^2C_p \exp\Big(E_1(x^*_1 + x^*_5)\,/\,(Rx^*_1x^*_5)\Big)F_1\rho x^*_1\Big(\exp\Big(E_1\,/\,(Rx^*_5)\Big)R\Big(-2C_p\rho V_1V_2 + \varepsilon_2\Big(UA_2V_1 + UA_1V_2 + C_pF_1\rho(V1+V2)\Big)\Big)x^{*2}_5 - \varepsilon_2E_1k_{01}V_1V_2(-\Delta H_1)x^*_6\Big) - \exp\Big(E_1(2x^*_1 + x^*_5)\,/\,(Rx^*_1x^*_5)\Big)Rx^{*2}_5\Big(\alpha_2\varepsilon_2^2C_pF_1\,\rho UA_1\,V_2x^*_4 + \alpha_2V_1\Big(\varepsilon_2\Big(C_pF_1\,\rho + UA_2\Big) - C_p\rho V_2\Big)^2x^*_5 - \varepsilon_2UA_2V_1\Big(\alpha_2\varepsilon_2\Big(C_pF_1\rho + UA_2\Big) + \big(-2\alpha_2 + \varepsilon_2\big)C_p\rho V_2\Big)x^*_9\Big) + \alpha_2\varepsilon_2k_{01}V_1\,V_2\,(-\Delta H_1)\Big(\varepsilon_2C_pR\rho x^{*2}_5\Big(-\exp\Big(2E_1\,/\,(Rx^*_5)\Big)F_1x^*_2 + \exp\Big(E_1\,/\,(Rx^*_1)\Big)k_{01}V_2x^*_6\Big) - \exp\Big(E_1\Big(x^*_1 + x^*_5\Big)\,/\,(Rx^*_1x^*_5)\Big)\Big(x^*_5\Big(\varepsilon_2C_pF_1R\rho x^*_2x^*_5 - \Big(\varepsilon_2E_1\Big(C_pF_1\rho + UA_2\Big) + R\Big(2\varepsilon_2C_pF_1\rho + \varepsilon_2UA_2 - 2C_p\rho V_2 + \varepsilon_2C_p\rho u_3\Big)x^*_5\Big)x^*_6\Big) + \varepsilon_2E_1UA_2x^*_6x^*_9\Big)\Big)\Big) - \alpha_2\varepsilon_1 \exp\Big(\Big(E_1\,/\,x^*_1\Big) + \Big(\Big(E_1 + E_5\Big)\,/\,x^*_5\Big)/R\Big)k_{02}V_1V_2(-\Delta H_2)\big(\varepsilon_2\,k_{01}Vx^*_6\Big(C_pR\rho x^{*2}_5 + (E_1 + E_2)(-\Delta H_1)\,x^*_7\Big)x^*_8 + \exp\Big(E_1\,/\,(Rx^*_5)\Big)\Big(\varepsilon_2C_pR\rho u_3x^{*2}_5x^*_7\Big(C_{C0} - 2x^*_8\Big) + x^*_8\Big(\varepsilon_2C_pF_1R\rho x^*_3x^{*2}_5 + x^*_7\Big(\varepsilon_2C_pE_2F_1\rho x^*_1 + x^*_5\Big(-\varepsilon_2E_2\Big(C_pF_1\rho + UA_2\Big) - R\Big(3\varepsilon_2C_pF_1\rho + \varepsilon_2UA_2 - 2C_p\rho V_2\Big)x^*_5\Big) + \varepsilon_2E_2UA_2x^*_9\Big)\Big)\Big)\Big)\Big)\Big)\Big)\,/\,\Big(\alpha_2C_p^2R\rho^2V_1V_2^2x^{*2}_5\Big)\Big) - K_2(y_2 - \hat{y}_2) \tag{F.5-2}$$

$$\tilde{u}_3 = \Big(V_2v_3 + \Big(\varepsilon_3F_1 - V_2 + \varepsilon_3 \exp\Big(-E_2\,/\,(Rx^*_5)\Big)k_{02}V_2x^*_7\Big)x^*_8\Big)\,/\,\Big(\varepsilon_3\Big(C_{C0} - x^*_8\Big)\Big) - K_3(y_3 - \hat{y}_3) \tag{F.5-3}$$

Appendix G

The formulation of control system for SISO CSTR with
the cooling jacket with time-delay

The formulation of the control system for the process described in Section 3.3 is as follows:

The disturbance-free state estimator

$$\begin{aligned}\frac{dx_1^*}{dt} &= \frac{F}{V}(C_{Ai}-x_1^*)-k_0\exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^* \\ \frac{dx_2^*}{dt} &= \frac{F}{V}(T_i-x_2^*)+\frac{(-\Delta H)}{\rho C_p}k_0\exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^*+\frac{UA}{V\rho C_p}(u-x_2^*) \\ y^* &= x_2^*\end{aligned} \tag{G.1}$$

where $x^*(t)=\hat{x}(t+\theta)$

The feedback compensator

$$v = y_{sp}-(y-\hat{y}) \tag{G.2}$$

The setpoint tracking controller

$$u(t)=x_2^*+\left(\frac{v-x_2^*}{\varepsilon}-\frac{F}{V}(T_i-x_2^*)-\frac{(-\Delta H)}{\rho C_p}k_0\exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^*\right)\left(\frac{V\rho C_p}{UA}\right) \tag{G.3}$$

The disturbance rejection controller

$$\delta = K(y-\hat{y}) \tag{G.4}$$

Therefore, the manipulated input for uncertain process is the form of:

$$\tilde{u}=x_2^*+\left(\frac{v-x_2^*}{\varepsilon}-\frac{F}{V}(T_i-x_2^*)-\frac{(-\Delta H)}{\rho C_p}k_0\exp\left(-\frac{E_a}{Rx_2^*}\right)x_1^*\right)\left(\frac{V\rho C_p}{UA}\right)-K(y-\hat{y}) \tag{G.5}$$

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