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2D invariant image recognition using the reduced coefficient of elliptic Fourier and principal component analysis with neural network

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Abstract

This paper presents the methods to show how to recognize the boundary shape of 2D invariant images using the reduced coefficient of elliptic Fourier descriptors (RCEFDs) and principal component analysis (PCA) with neural networks. The boundary of each image is represented by the amount of arc (harmonic) ellipses and then to compute the coefficient of elliptic Fourier descriptors (CEFDs) to be image codes. The system can save storage size by a reduced number of coefficient codes; using of the three coefficients (a_n, b_n, c_n or a_n, c_n, d_n) instead of four coefficients (a_n, b_n, c_n, d_n).The amount of harmonics which best fit to the boundary of each object is 15 harmonics. For getting a satisfactory recognition rate and for solving the problems that the objects have size variables and orientations, it is necessary to set the group of objects by rotating the original object by different degrees, normalizing the scale to standard size and translating to a suitable situation. In this paper, 100 images are used for training and 100 images are used for testing. In the Training stage, each image is rotated into 36 different angles. The principal component analysis is then applied to find the mean of each object image for storing only instead of storing all member sets into the knowledge base for marking a small-scale knowledge base. In the testing stage, the RCEFDs and PCA method are applied to find the output comparison using classical (PCA) method with the Neural Network method. This proposed method can reduce storage size of Fourier coefficients by 1/4, and then the PCA used storage size to 1/36 of conventional method. The results show that: the recognition rate of PCA with Neural Network method is 94.23%, and the recognition rate of the Neural Network method is 96.20%, respectively.

Keywords: *Elliptic Fourier descriptors, principal component analysis*

1. Introduction

The main problem in 2D invariant image recognition is the possibility that the objects may have size variables and orientations. There are different approaches in this field, such as Combined invariants to blur and rotation using Zernike moment descriptors (Zhu, Liu & Ji, 2010), Membership Matching Score and 3-Layer Matching Search (Pansang, Attachoo & Kimpan, 2005), genetic and neural network (Phokharatkul, Marang & Kimpan, 2000), etc; which have been proposed to solve this problem. In general, the boundary description technique does play an important role in the area of invariant image recognition. In the conventional technique, a boundary is often represented as a straight line segment composed of a polygon object and then using the chain codes (Schalkoff, 1990) to be the features of unknown images. Using the most interesting technique, we can find the boundary based analysis via elliptic Fourier which is represented by the arc of ellipses, because it can work on the boundary function of invariant images.

In our previous work, "Invariant Shape Object Recognition using B-Spline, Cardinal Spline and Genetic Algorithm." (Phokharatkul, Kamnuanchai, & Kimpan, 2006), the boundary of an object can be represented by piecewise polynomial curves, and blending functions are the codes. The recognition rate is good only for the boundary of object being polynomial curves. The recognition time is not fixed depending on the complexity of the unknown image according with Genetic Algorithm. In our previous work, "Invariant

Image Recognition Using Elliptic Fourier Descriptors, and Membership Matching Score" (Phokharatkul, Kamnuanchai & Kimpan, 2007) the boundary of image is represented by the arc of ellipses and elliptic Fourier descriptions and MMS are applied to solve the recognition problem. The research above used large scale knowledge base in which uneconomical resources and large recognition time (sentence fragment). This paper shows the method to reduce the storage size of RCEFDs and PCA. The processing system will be explained in the next section.

2. Objectives

The object of this paper is to recognize the boundary shape extraction, and the reduced storage size of the knowledge base. The boundary of each image is represented by the amount of arc (harmonic) ellipses and then to compute the coefficient of elliptic Fourier descriptors (CEFDs) to image codes. The system can save storage size by a reduced number of coefficient codes, using three coefficients (a_n, b_n, c_n or a_n, c_n, d_n) instead of four coefficients (a_n, b_n, c_n, d_n).The amount of harmonics which best fit the boundary of each object is 15 harmonics. For solving invariant image problem, the mean of the each object is found image by using PCA (sentence fragment).

3. Materials and Methods

3.1 Processing system

The image recognition system is shown in Figure 1 and involves three stages. The first stage (preprocessing stage) involves gray scale conversion, binary image conversion, smoothing (use an erosion-dilation filter to reduce noise, hairlines or grains of sand) and normalized scale to standard size and edge detection for reducing storage size of knowledge base respectively. We applied an edge detection algorithm (Anil, 1989) and contour flowing algorithm (Richard & Peter, 1973) for segmentation of the object image. The close contour of each object image was forced by edging the binary image and the extruded contour into the boundary position of object images respectively (Phokharatkul & Kimpan, 2002). In the second stage (Recognition stage), the edge of 2D-invariant images were created into the form of 4 CEFDs along their contours. Then, the 4 CEFDs are reduced to 3 CEFDs. The CEFDs are the feature code of images, and these codes are kept in the knowledge base. PCA is used to compress the data for the small-scale knowledge base. Figure 2 shows an example color image, binary image, edge and the representation of the CEFDs image. In this paper, the training data set is composed of the original images which are normalized to standard size, and their rotated data with rotating in different angles (examples 200 original data, rotating in 10, 32, 57, 80, 132, 140, 280 degrees, totally 1,600 images). The testing data set is comprised of rotating data in different degrees (examples 100 original data, rotating 5, 12, 30, 42, 75, 120, 150, 240 degrees, totally 800 images).

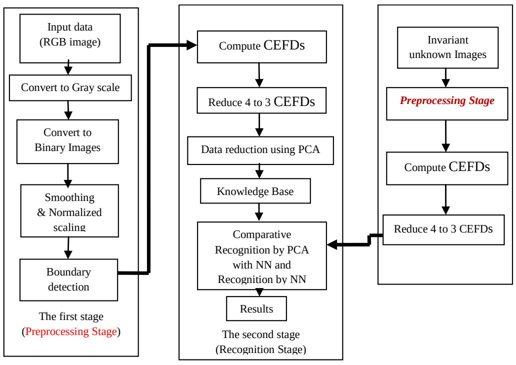


Figure 1 Flow diagram of the image recognition system

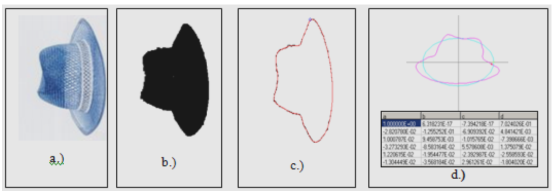
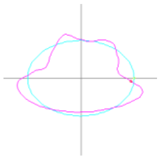


Figure 2 The example of images: (a) Original color image data; (b) Binary image; (c) Edge of image; (d) CEFDs of image.

3.2 Image data from Elliptic Fourier

The boundary shapes (edge) of 2D invariant images are represented by the amount of elliptic arcs, and CEFDs are the feature code of images. The elliptic arcs and CEFDs can be computed by the elliptic Fourier descriptor method (Kuhl & Giardina, 1982). In this paper, we approximated the shape of the boundary image by 100, 20, 15, and 10 harmonics (ellipses) for each image, which are invariant in size, rotation and shift of starting point along the contour. From the empirical testing, the 15 harmonics give information leading to satisfactory results. The example of the coefficient of the hat image a_n, b_n, c_n and d_n which it has 60 values as shown in Figure 3.



(a)

No. of Harmonic	a_n	b_n	c_n	d_n
1	1.0000000e+00	8.3690103e-16	4.4021882e-17	6.9487308e-01
2	2.2125556e-02	1.1233913e-01	7.7054166e-02	-5.2317174e-03
3	1.8848915e-02	5.7623630e-03	-1.6365245e-02	-9.2669010e-03
4	2.9274279e-02	9.0073036e-02	-5.9136493e-03	-8.6093386e-03
5	1.9343765e-02	-1.9097825e-02	-2.4319455e-02	-2.4229543e-02
6	1.0307504e-02	3.6551464e-02	-3.0982774e-02	2.1401435e-02
7	1.0346986e-02	-9.4417890e-03	9.9471447e-03	4.7794848e-03
8	8.8173203e-04	7.5534102e-03	-3.5780761e-03	-3.9040759e-03
9	-2.6176281e-03	1.2062285e-03	-4.6481248e-03	-5.1113144e-03
10	2.8501018e-03	2.0390606e-03	1.1748126e-03	4.4974193e-03
11	-4.2920410e-03	1.1606642e-03	1.1748126e-03	3.1093330e-04
12	-8.9644339e-04	4.5203411e-05	6.5275935e-04	8.6600208e-04
13	-3.4634827e-03	4.0289209e-03	2.6175390e-03	-1.7306717e-03
14	-2.2161524e-03	-1.2924915e-03	9.1775960e-04	-3.7833557e-03
15	-1.6673036e-03	6.3668450e-03	-1.5201086e-03	9.2037666e-04

(b)

Figure 3 Coefficient of the image

$$l_p = \sum_{i=1}^{P} \Delta l_i \qquad (1)$$

$$x_p = \sum_{i=1}^{P} \Delta x_i \qquad (2)$$

and

$$y_p = \sum_{i=1}^{P} \Delta y_i \qquad (3)$$

By the definition of parameter t as $2\pi l / L$, $t \in [0,2\pi]$, l is the length from starting point to a certain point along the boundary shape, the Fourier descriptors may be written in matrix form as

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{n=1}^{\infty} \underbrace{\begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}}_{M_n} \begin{bmatrix} \cos nt \\ \sin nt \end{bmatrix} \qquad (4)$$

where M_n is the *n-th* harmonic ellipses, a_n, b_n, c_n, and d_n are the elliptic Fourier descriptor, $n = 1, 2, 3, \ldots \infty$ are define as

$$a_n = \frac{L}{2n^2\pi^2} \sum_{p=1}^{P} \frac{\Delta x_p}{\Delta l_p} \left\{ \cos\left(\frac{2n\pi\pi_p}{L}\right) - \cos\left(\frac{2n\pi\pi_{p-1}}{L}\right) \right\} \qquad (5)$$

$$b_n = \frac{L}{2n^2\pi^2} \sum_{p=1}^{P} \frac{\Delta x_p}{\Delta l_p} \left\{ \sin\left(\frac{2n\pi\pi_p}{L}\right) - \sin\left(\frac{2n\pi\pi_{p-1}}{L}\right) \right\} \qquad (6)$$

$$c_n = \frac{L}{2n^2\pi^2} \sum_{p=1}^{P} \frac{\Delta y_p}{\Delta l_p} \left\{ \cos\left(\frac{2n\pi\pi_p}{L}\right) - \cos\left(\frac{2n\pi\pi_{p-1}}{L}\right) \right\} \qquad (7)$$

$$d_n = \frac{L}{2n^2\pi^2} \sum_{p=1}^{P} \frac{\Delta y_p}{\Delta l_p} \left\{ \sin\left(\frac{2n\pi\pi_p}{L}\right) - \sin\left(\frac{2n\pi\pi_{p-1}}{L}\right) \right\} \qquad (8)$$

where $\Delta x_p = (x_p - x_{p-1})$, $\Delta_p = (y_p - y_{p-1})$, (9)

$$\Delta l_p = \sqrt{\Delta x_p^2 + \Delta y_p^2},\ L = \sum_{p=1}^{P} \Delta l_p,\ l_p = \sum_{i=1}^{p} \Delta l_i \qquad (10)$$

P is the total number of points along the boundary.

If the geometric center of the boundary shape moves to the a_0, c_0 position, the translation problem can be solved. However, the coefficients a_n, b_n, c_n and d_n depend upon both the selected starting point along the boundary and the rotation angle of shape, but not on translation. To derive a set of descriptors that are invariant with respect to starting point and rotation variant, we move the starting point back on the semi major axis of ellipses M_n (shifting Ψ_n) and rotate the ellipses so that their axis are aligned with the axis of the coordinate system (rotation by ϕ). Form matrix M_n in equation (4), the phase shift Ψ_n is computed by the equation below.

$$\psi_n = \psi_1 = \frac{1}{2}\tan^{-1}\left[\frac{2(a_1 b_1 + c_1 d_1)}{a_1^2 + b_1^2 - c_1^2 - d_1^2}\right]$$

The orientation ϕ of the boundary shape can be computed from Ψ_1 by:

$$\varphi = \tan^{-1} = \frac{-c_1 \sin\psi_1 + d_1 \cos\psi_1}{a_1 \cos\psi_1 + b_1 \sin\psi_1}$$

The description method in this section is a general theory where the conventional methods do not pertain to the reduction of storage size. Thus, this section will be proposed by the reduction technique in section 3.3 and 3.4, respectively.

3.3 Reduced coefficient of Elliptic Fourier (RCEF): Reduction codes

This paper has a new hypothesis for the data reduction, by reducing the number of elliptic Fourier coefficients. The coefficients a_n, b_n, c_n, and d_n are the features that represent the boundary of a harmonic (an ellipse). Two methods have been proposed to reduce coefficient feature codes.

No. of Harmonic	a_n	b_n	c_n
1.	1.0000000e+00	8.3690103e-16	4.4021882e-17
2.	2.2125556e-02	1.1233913e-01	7.7054166e-02
3.	1.8848915e-02	5.7623630e-03	-1.6365245e-02
4.	2.9274279e-02	9.0073036e-02	-5.9136493e-03
5.	1.9343765e-02	-1.9097825e-02	-2.4319455e-02
6.	1.0307504e-02	3.6551464e-02	-3.0982774e-02
7.	1.0346986e-02	-9.4417890e-03	9.9471447e-03
8.	8.8173203e-04	7.5534102e-03	-3.5780761e-03
9.	-2.6176281e-03	1.2062285e-03	-4.6481248e-03
10.	2.8501018e-03	2.0390606e-03	-2.8772377e-03
11.	-4.2920410e-03	1.1606642e-03	1.1748126e-03
12.	-8.9644339e-04	4.5203411e-05	6.5275935e-04
13.	-3.4634827e-03	4.0289209e-03	2.6175390e-03
14.	-2.2161524e-03	-1.2924915e-03	9.1775960e-04
15.	-1.6673036e-03	6.3668450e-03	-1.5201086e-03

(a)

No. of Harmonic	a_n	c_n	d_n
1.	1.0000000e+00	4.4021882e-17	6.9487308e-01
2.	2.2125556e-02	7.7054166e-02	-5.2317174e-03
3.	1.8848915e-02	-1.6365245e-02	-9.2669010e-03
4.	2.9274279e-02	-5.9136493e-03	-8.6093386e-03
5.	1.9343765e-02	-2.4319455e-02	-2.4229543e-02
6.	1.0307504e-02	-3.0982774e-02	2.1401435e-02
7.	1.0346986e-02	9.9471447e-03	4.7794848e-03
8.	8.8173203e-04	-3.5780761e-03	-3.9040759e-03
9.	-2.6176281e-03	-4.6481248e-03	-5.1113144e-03
10.	2.8501018e-03	-2.8772377e-03	4.4974193e-03
11.	-4.2920410e-03	1.1748126e-03	3.1093330e-04
12.	-8.9644339e-04	6.5275935e-04	8.6600208e-04
13.	-3.4634827e-03	2.6175390e-03	-1.7306717e-03
14.	-2.2161524e-03	9.1775960e-04	-3.7833557e-03
15.	-1.6673036e-03	-1.5201086e-03	9.2037666e-04

(b)

Figure 4 Reduced coefficient of the image:
(a) Coefficients a_n, b_n and c_n (b) Coefficients a_n, c_n and d_n

In the first method, the major axis of CEFDs is fixed, meaning that both a_n and b_n are fixed, and then c_n or d_n can be selected. In the second method, the minor axis of CEFDs is fixed, meaning that both c_n and d_n are fixed, and then a_n, or b_n can be selected. Using this hypothesis, the example data are shown in the Figure 4 which shows that the number of features can be reduced from 60 features (4x15 harmonics) to 45 features (3x15 harmonics) which is enough information to process in the next section. RCEFs are only codes for the recognition system, but are of little interest for the reconstruction of images

3.4 Principal Component Analysis (PCA)

The advantage of this principal component analysis is that it gives satisfactory recognition rate for small-scale knowledge base. The original images generated to other images by mathematical rotation to various angles for setting a group of each object. To then find the mean of a group image by computing eigenvector according to the following equation (11), (12), (13) and keeping only the eigenvector that gives maximum eigenvalue into the knowledge base, which can be the average image of the group object.

Let $p[i,j]$ be the group image of each object which get from coefficient of the Figure 3 or Figure 4 as

$$p[i,j] = \begin{bmatrix} a_{1,1} a_{1,2} a_{1,3} \cdots a_{1,n} \\ a_{2,1} a_{2,2} a_{2,3} \cdots a_{2,n} \\ \vdots \\ a_{m,1} a_{m,2} a_{m,3} \cdots a_{m,n} \\ c_{1,1} c_{1,2} c_{1,3} \cdots c_{1,n} \\ c_{2,1} c_{2,2} c_{2,3} \cdots c_{2,n} \\ \vdots \\ c_{m,1} c_{m,2} c_{m,3} \cdots c_{m,n} \\ d_{1,1} d_{1,2} d_{1,3} \cdots d_{1,n} \\ d_{2,1} d_{2,2} d_{2,3} \cdots d_{2,n} \\ \vdots \\ d_{m,1} d_{m,2} d_{m,3} \cdots d_{m,n} \end{bmatrix} \quad (11)$$

where j = 1,2,3,…n : number of members in a group and i = 1,2,3,…m : number of harmonics.

The a_{ij}, c_{ij}, and d_{ij} are the reduced coefficient of the elliptic from Figure 3 or Figure 4.

The next step of PCA is to compute the covariant matrix $(C[i,i])$ which will be a square matrix by following the equation:

$$C[i,i] = p[i,j] * p[i,j]^T \quad (12)$$

The final step of PCA is to compute eigenvalue and eigenvector by the following equation:

$$CV = \lambda V \quad (13)$$

where V is eigenvector, and λ is eigenvalue.

PCA computes the average image (mean) by the following algorithm and stores them into the knowledge base.

So, the systems that have eigenvector V $[i,1]$ represents the group image of each object.

In data transformation, the input vector V $[i,k]$ is computed from $p[i,j]$ in equation (11) using PCA as show in Figure 5.

$$[\mathbf{P}]_{ixj} \Rightarrow [\mathbf{C}]_{ixi} \Rightarrow [\mathbf{V}]_{ix1}$$

Figure 5 The example of eigenvector $[V]_{ix1}$ computation

where i is the number of coefficient of elliptic,
j is the number of various image of each object.

In supervised training, the input training is built from $[V]_{ix1}$ into $[V]_{ixk}$, and set the diagonal output matrix $[T]_{kxk}$ as shown in Figure 6.

$$\begin{bmatrix} V_{1,1} & V_{1,2} & V_{1,3} & \cdots & V_{1,k} \\ V_{2,1} & V_{2,2} & V_{2,3} & \cdots & V_{2,k} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ V_{i,1} & V_{i,2} & V_{i,3} & \cdots & V_{i,k} \end{bmatrix} \Rightarrow \begin{bmatrix} \text{Neural} \\ \text{Network} \\ \text{System} \end{bmatrix} \Rightarrow \begin{bmatrix} \text{OutPut} \\ \text{Computation} \end{bmatrix}_{kxk}$$

Figure 6 The system input/output of the neural system

where *i* is the number of reduce coefficient elliptic,
k is the number of image target.

In testing, the input testing $[V]_{ix1}$ computes from $[P]_{ix1}$ of an unknown image. Thus, the output of unknown computations from the input testing multiply with the weights of the neural network. The procedure of eigenvector computation as follow:

Step 1: compute the average image $\left(\overline{X}\right)$

Step 2: compute the centralized $\left(\tilde{X}_i \leftarrow X_i - \overline{X}\right)$

Step 3: compute the covariance

$$C \leftarrow \lambda\left[\left(\tilde{X}_i - \overline{X}\right)\left(\tilde{X}_i - \overline{X}\right)^T\right]$$

Step 4: compute the Eigenvector (V) and Eigenvalue (λ)

Step 5: store training data projected into eigenspace

3.5 Artificial Neural Network (ANN)

Scholars Rumelhart, McClelland and his colleagues proposed a multilayer feedforward network MFNN (Mutltilayer Feedforward Neural Networks) in the back propagation learning algorithm, called BP networks (Back Propagation Network) learning algorithm. BP network is a nonlinear differentiable function weight training multilayer feedforward networks. In order to identify the object image, the design of the network has 45 input nodes and 100 output nodes, the number of nodes for hidden layer behind the selection in detail. Target error is 0.0001, from input layer to hidden layer activation function using the S-tangent function tansig, from the hidden layer to the output layer activation function using the S-logarithmic function logsig. This is because the function of the output in the range [0,1], meets the requirements of the network output.

4. Results

4.1 Data sets for experiment

Two hundred pictures of training set and testing set were selected from 1,000 general pictures as the sample in Figure 5 from the data base of the Amsterdam library of object images (ALOI) (Geusebroek, et al, 2005). The Training images are created using equation (11) which contains the original and 7 different rotated images (such as 10, 32, 57, 80, 112, 140, and 280 degrees) for each image. This paper created the knowledge base using the 100 type images which rotated different angles as already mentioned above. These images were then kept (800 images) in the knowledge base of the system. The testing image set consists of the unknown rotated image of 100 type images. Then each type of image is rotated into 8 different degrees (such as 5, 12, 30, 42, 75, 120, 150, and 240 degree), totaling 100 images for testing stage.

Figure 7 The example images for training set

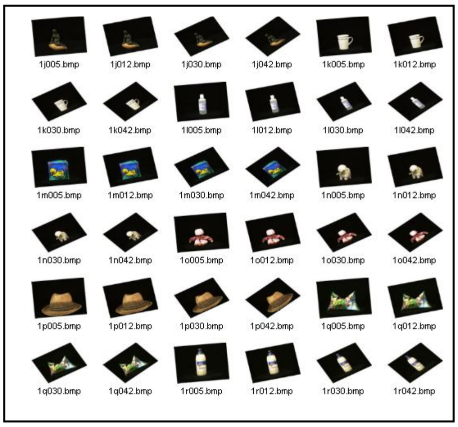


Figure 8 The example images for testing set

4.2 The results of experiment

In the testing stage, the RCEFDs and PCA method are applied to find the output comparing classical (PCA) method with Neural Network method. PCA is used to create the knowledge base by keeping only the average image of a group image, inn order to find an average image (eigenvector with maximum eigenvalue). The testing image set is the same in the conventional knowledge base. The recognition results of system are shown in Table 1 and 2, respectively.

Table 1 The recognition results using PCA knowledge base

No. of coefficient of EFDs per image	No. of Harmonic of EFDs per image	Average Recognition Rate (%)
4 coefficients (a_n, b_n, c_n, d_n)	100 harmonics	90.31
	20 harmonics	90.98
	15 harmonics	91.94
	10 harmonics	80.77
3 coefficients (a_n, b_n, c_n) or 3 coefficients (a_n, c_n, d_n)	100 harmonics	90.10
	20 harmonics	90.88
	15 harmonics	91.84
	10 harmonics	80.50
Total		88.35

Table 2 The recognition results using PCA compared with Neural Network, ANN knowledge base, 3 values of coefficients and 15 harmonics

No. of member rotated images	Average Recognition Rate (%) by PCA with Neural Network	Average Recognition Rate (%) by ANN
8 (random/angle)	91.84	91.95
36 (10 degree/angle)	94.23	96.20
72 (5 degree/angle)	97.45	98.15

5. Discussion

The recognition rates depend on the number of harmonics that can best fit within the shape of the boundary. It also depends upon the number of members in the group of images. According to Table 1, it shows that 15 harmonics give the best recognition rate. From the experimental results, the recognition time depends on the storage size of the system. The small storage size gives a small recognition time. Table 2 shows that the recognition rate is higher than the recognition rate in Table 1. This is because of the increase the number of the training sets from 100 images to 450 images, and to 900 images.

6. Conclusion

The technique in this paper was only used for the boundary images. It cannot be used for recognizing the content of images. The information loss of rotated image is an interesting problem in the recognition system for solving in future work.

7. References

Anil, K. (1989). Fundamentals of Digital Image Processing. *Prentice-Hall International Englewood Cliffs*, NJ, 342-425.

Geusebroek, J. M., Burghouts, G. J. & Smeulders A. W. M. (2005) The Amsterdam library Of object images.*International Journal of Computer Vision*, 13-122.

Kuhl, F. P. & Giardina, C. R. (1982). Elliptic Fourier features of a closed contour. *Comp Graph. Image Proc*, 236-258.

Pansang, S., Attachoo, B. & Kimpan, C. (2005). Invariant Rang Image Multi-Pose Face Recognition using Gradient Face, Membership Matching Score and 3-Layer Matching Search. *IEICE TRANS.INF.& SYST*., Vol. *E88-D*, 188-179.

Phokharatkul, P., Marang, U. & Kimpan, C. (2000). Recognition of Printed Thai Characters Using Elliptic Fourier Descriptors and Genetic Neural Networks. *Proceedings Asia-Pacific Symposium on Broadcasting and Communications*, 406-411.

Phokharatkul, P. & Kimpan, C. (2002). Handwritten thai character recognition using Fourier descriptors and genetic neural network. *Comput.Intell*, 270–293.

Phokharatkul, P., Kamnuanchai, S. & Kimpan, C. (2006). Invariant Shape Object Recognition Using B-Spline, Cardinal Spline and Genetic Algorithm. *WSEAS on Signal Processing*, 40-46.

Phokharatkul, P., Kamnuanchai, S. & Kimpan, C. (2007). Invariant Image Recognition Using Elliptic Fourier Descriptors and Membership Matching Score. Proc, *International Workshop on Advanced Image Technology*, 26.

Richard, D. & Peter, E. (1973) Contour Following. Pattern Classification and Scene Analysis. *Wiley-Interscience*, 290–293.

Zhu, H., Liu, M. & Ji, H. (2010). Combined invariants to blur and rotation using Zernike moment descriptors. *Pattern Analysis Appliations*, Sringer, *13*, 309-319.