

# CHAPTER 2

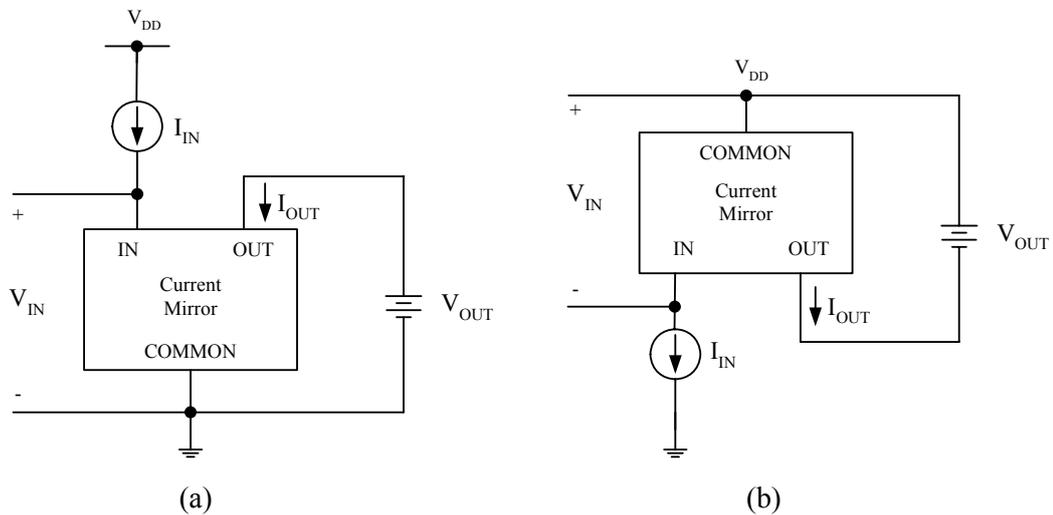
## Active circuit elements

In this chapter, the circuit elements which will be used as major active circuit elements in this thesis are described. The circuit descriptions of the CMOS current mirror, the CMOS current squarer circuit and the CMOS OTA are respectively outlined.

### 2.1 Current-mirror

#### 2.1.1 Circuit Description

Current mirrors have come to be widely used in analog integrated circuits both as biasing elements and as active load devices for amplifier stages. The use of current mirrors in biasing can result in superior insensitivity of circuit performance to variations in power supply and temperature [19], [20].



**Fig. 2.1** Current-Mirror block diagram referenced to  
(a) ground and (b) the positive supply.

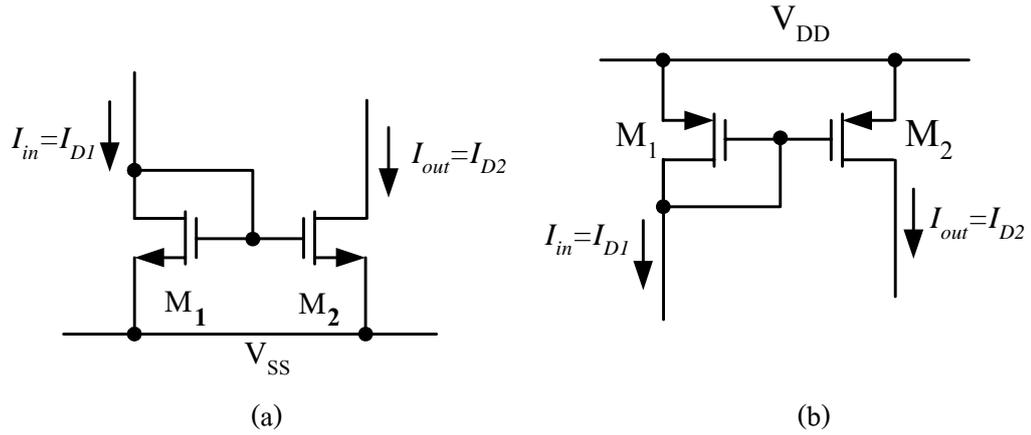
A current mirror is an element with at least three terminals, as shown in Fig. 2.1. The common terminal is connected to a power supply, and the input current source is connected to the input terminal. The output current is ideally equal to the input current multiplied by a desired current gain. If the gain is unity, the input current is simply reflected to the output, leading to the name current mirror. Under ideal conditions, the current-mirror gain is independent of input frequency, and the output current is independent of the voltage between the output and common terminal. Furthermore, the voltage between the input and common terminals is ideally zero because this condition allows the entire supply voltage to appear across the input current source, simplifying its transistor-level design. More than one input and/or output terminals are sometimes used.

For MOS transistor, a commonly used parameter for the characterization of channel length modulation ( $\lambda$ ) is the reciprocal of the Early voltage ( $V_A$ ),  $\lambda = 1/V_A$ . Thus we can include the effect of channel-length modulation in the  $I$ - $V$  characteristics. If one assumes that the transistors operate in the saturation region ( $V_{DS} > V_{GS} - V_T$ ), the value of  $I_D$  in this region can be given by

$$I_D = \frac{\mu_n C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \quad (2.1)$$

where the parameters are:

$\mu_n$	surface mobility of the channel, ( $cm^2 / volt \cdot sec$ )
$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$	gate oxide capacitance density, ( $F/cm^2$ )
$W$	effective channel width, (m)
$L$	effective channel length, (m)
$V_T$	threshold voltage, (volt)
$\lambda$	channel length modulation parameter, ( $volt^{-1}$ )



**Fig. 2.2** Simple MOS current mirrors

(a) NMOS current mirror (b) PMOS current mirror

A simple realization of a CMOS current-mirror is presented in Fig. 2.2. From the circuit

$$V_{GS2} = V_{T2} + \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} (W/L)_2}} = V_{GS1} = V_{T1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} (W/L)_1}} \quad (2.2)$$

If the transistors are identical,  $(W/L)_2 = (W/L)_1$ , and therefore we get the relation of the currents

$$I_{out} = I_{D2} = I_{D1} \quad (2.3)$$

Eqn. (2.3) shows that the current that flows in the drain of  $M_1$  is mirrored to the drain of  $M_2$ . Using Kirchhoff's Current Law, eqn. (2.3) yields

$$I_{out} = I_{D2} = I_{in} \quad (2.4)$$

Thus for identical device operating in the saturation region with infinite output resistance, the gain of the current mirror is unity. This result holds when the gate currents are zero; that is, eqn. (2.4) is at least approximately correct for DC and low-frequency AC currents. As the input frequency increases, however, the gate currents of  $M_1$  and  $M_2$  increase because each transistor has a nonzero gate-source capacitance. In practice, the devices need not be identical. Then from eqns. (2.2) and (2.4),

$$\begin{aligned}
I_{out} &= \frac{(W/L)_2}{(W/L)_1} \left( \frac{V_{GS2} - V_{T2}}{V_{GS1} - V_{T1}} \right)^2 \cdot \frac{1 + \lambda V_{DS2} \frac{\mu_{n2} C_{ox2}}{\mu_{n1} C_{ox1}}}{1 + \lambda V_{DS1} \frac{\mu_{n2} C_{ox2}}{\mu_{n1} C_{ox1}}} I_{D1} \\
&= \frac{(W/L)_2}{(W/L)_1} \left( \frac{V_{GS2} - V_{T2}}{V_{GS1} - V_{T1}} \right)^2 \cdot \frac{1 + \lambda V_{DS2} \frac{\mu_{n2} C_{ox2}}{\mu_{n1} C_{ox1}}}{1 + \lambda V_{DS1} \frac{\mu_{n2} C_{ox2}}{\mu_{n1} C_{ox1}}} I_{in} \quad (2.5)
\end{aligned}$$

Then, if the effect of source and load impedances is first neglected, the current-mirror large signal equation shows current gain ( $A_i$ ) proportional to the aspect ratios of transistors  $M_1$  and  $M_2$ , with certain additional dependencies

$$A_i = \frac{I_{out}}{I_{in}} = \frac{(W/L)_2}{(W/L)_1} \left( \frac{V_{GS2} - V_{T2}}{V_{GS1} - V_{T1}} \right)^2 \cdot \frac{1 + \lambda V_{DS2} \frac{\mu_{n2} C_{ox2}}{\mu_{n1} C_{ox1}}}{1 + \lambda V_{DS1} \frac{\mu_{n2} C_{ox2}}{\mu_{n1} C_{ox1}}} \quad (2.6)$$

From this equation, we found that the process variation of the channel width  $W$ , channel length  $L$ , mobility  $\mu_n$ , and oxide thickness  $t_{ox}$  can produce linear gain error.

If the effect of channel-length modulation can be neglected ( $\lambda = 0$ ), assume the process parameters such as  $V_p$ ,  $\mu_n$ ,  $C_{ox}$ , of MOS transistor are matched and  $V_{DS2} \cong V_{DS1}$ . The eqn. (2.5) can be rewritten as

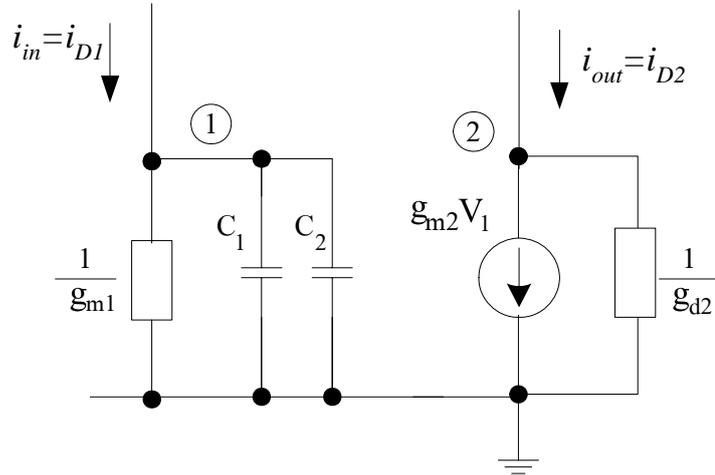
$$I_{out} = \frac{(W/L)_2}{(W/L)_1} I_{D1} = \frac{(W/L)_2}{(W/L)_1} I_{in} \quad (2.7)$$

and the current gain ( $A_i$ ) in eqn. (2.6) can be rewritten as

$$A_i = \frac{I_{out}}{I_{in}} = \frac{(W/L)_2}{(W/L)_1} \quad (2.8)$$

Eqn. (2.8) shows that the gain of the current mirror can be larger or smaller than unity because the transistor sizes can be ratioed. To ratio the transistor sizes, either the widths ( $W$ ) or the lengths ( $L$ ) can be made unequal in principle.

Due to finite input and output impedances have a significant effect on the current mirror gain accuracy. The small signal equivalent circuit of the current mirror will be used to analyze the input impedance, the output impedance and the current gain.



**Fig. 2.3** Small signal equivalent circuit of NMOS current mirror in Fig. 2.2

From Fig. 2.3, the small-signal input impedance of the current mirror depends on the transconductance of the input transistor  $M_1$

$$r_{in} \approx \frac{1}{g_{m1}} = \frac{1}{\sqrt{2\mu_{o1}C_{ox1} \frac{W_1}{L_1} I_{D1}}} \quad (2.9)$$

where  $g_{m1}$  is the transconductance of transistor  $M_1$ , and the small-signal output impedance depends on the drain-source conductance of the output transistor  $M_2$ , accordingly

$$r_{out} \approx \frac{1}{g_{d2}} = \frac{1}{\lambda I_{D2}} \quad (2.10)$$

where  $g_{d2}$  is the drain-source conductance of the transistor  $M_2$ .

The relation between the input current ( $I_{in}$ ) and the output current ( $I_{out}$ ) of the basic current mirror can be written as

$$I_{out} = \frac{g_{m2}}{g_{m1}} I_{in} \quad (2.11)$$

For the high frequency response of the simple current mirror, from Fig. 2.3 the transfer function of input current and output current ( $i_{out}(s)/i_{in}(s)$ ) can be expressed in eqn. (2.12)

$$\frac{i_{out}(s)}{i_{in}(s)} = \frac{g_{m2}}{g_{m1}} \cdot \frac{1}{1 + \frac{s(C_1 + C_2)}{g_{m1}}} \quad (2.12)$$

where  $C_i = C_{gsi} + C_{gdi}$

$C_{gsi}$  is the gate-source capacitance of transistor  $M_i$ , (F)

$C_{gdi}$  is the gate-drain capacitance of transistor  $M_i$ , (F)

The basic current mirror characteristics from eqns. (2.5) to (2.12) show that the current gain can be controlled by the channel width (W) and/or channel length (L) of the MOS transistor.

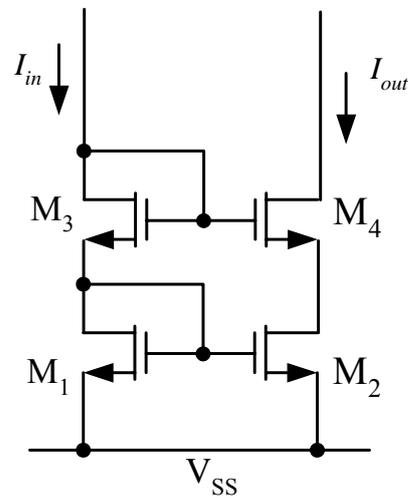
From eqn. (2.12) the cut-off frequency of the basic current mirror in Fig. 2.2 which depending on one pole can be expressed as eqn. (2.13)

$$f_{-3dB} \cong \frac{g_{m1}}{2\pi(C_1 + C_2)} \quad (2.13)$$

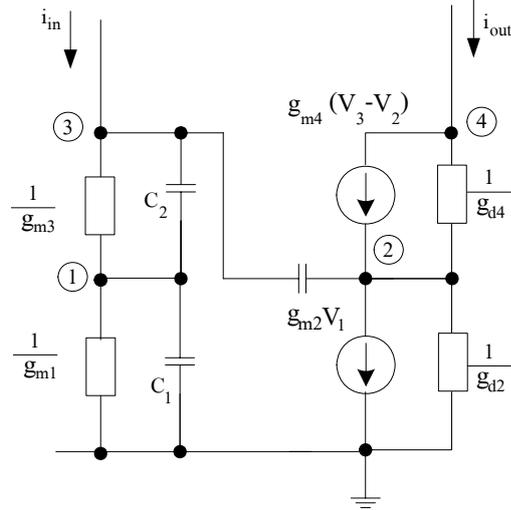
Eqn. (2.13) shows that the dominant pole of the basic current mirror is normally situated at quite high frequencies because of the low value of  $g_{m1}$ . Thus, the current mirror operates well up to high frequencies.

In modern sub-micron CMOS processes the  $g_m/g_{ds}$  ratio is less than 100 and consequently results in not a significant gain error. This gain error is usually reduced by increasing the output impedance using different cascode mirror topologies rather than the simple two transistor current-mirror. In this case, however, the drain-source voltages  $V_{DS1} \neq V_{GS}$  gain error as expressed in eqn. 2.6 may result. Moreover, these drain voltages are signal dependent and thus the channel length modulation  $\lambda$ , which continuously increases as progressively shorter channel length devices are

introduced, can additionally produce significant amount of distortion. Therefore, topological improvements must be made in the current-mirror in order to maintain the drain-source voltages  $V_{DS1}$  and  $V_{DS2}$  as equal as possible. This is achieved by using the current-mirror topologies rather than the simple current-mirror or the cascode current-mirror that shows in Fig. 2.4. The additional current-mirror constructed of transistors  $M_3$  and  $M_4$  is added on top of the original current mirror in order to force the drain voltage  $V_{DS2}$  of the transistor  $M_2$  equal to  $V_{DS1} = V_{DS2} = V_{GS}$  [19]. Unfortunately, this reduces significantly the input and output voltage ranges and thus the minimum supply voltage of the circuit is quite high.



**Fig. 2.4** A simple four transistor cascode current mirror.



**Fig. 2.5** Small signal equivalent circuit of cascode current mirror in Fig. 2.4

From the small signal equivalent circuit in Fig. 2.5, the relation between the input current ( $i_{in}$ ) and the output current ( $i_{out}$ ) of the cascode current mirror in Fig 2.3 can be written as

$$i_{out} = \frac{g_{m2}g_{m4}}{g_{m1}g_{m3}} i_{in} \quad (2.14)$$

The input impedance of the cascode current mirror can be written as

$$r_{in} = \frac{g_{m1} + g_{m3}}{g_{m1}g_{m3}} \quad (2.15)$$

and the output impedance can be written as

$$r_{out} = r_{d2} + r_{d4} + g_{m4}r_{d2}r_{d4} \cong g_{m4}r_{d2}r_{d4} \quad (2.16)$$

where  $r_{di}$  is the drain-source resistance of the MOS transistor  $M_i$

The frequency response of the cascode current mirror in Fig. 2.5 can be expressed as

$$\frac{i_{out}(s)}{i_{in}(s)} = \frac{g_{m4}}{g_{m1}} \left[ \frac{1}{\frac{C_1(C_2 + C_3)}{g_{m1}g_{m2}}s^2 + \frac{((C_1 + C_2)(g_{m1} + g_{m2}) + C_{gs4}g_{m2})}{g_{m1}g_{m2}}s + 1} \right] \quad (2.17)$$

Eqn. (2.17) shows that the cascode current mirror has a complex pole. Therefore, its bandwidth of the cascode current mirror is less than the basic current mirror. However, the cascode current mirror has higher output resistance and less error in current  $i_{out}$ .

**Table 2.1** Characteristics summary of the basic current mirror and cascode current mirror

Current mirror	Input resistance ( $r_{in}$ )	Output resistance ( $r_{out}$ )	Frequency response $i_{out}(s)/i_{in}(s)$
Basic Current mirror	$r_{in} = 1/g_{m1}$	$r_{out} = 1/g_{d2}$	$\frac{i_{out}(s)}{i_{in}(s)} = \frac{g_{m2}}{g_{m1}} \left( \frac{1}{1 + \frac{S(C_1 + C_2)}{g_{m1}}} \right)$
Cascode Current mirror	$r_{in} = \frac{g_{m1} + g_{m3}}{g_{m1}g_{m3}}$	$r_{out} = r_{d2} + r_{d4} + g_{m4}r_{d2}r_{d4}$ $\cong g_{m4}r_{d2}r_{d4}$	$\frac{i_{out}(s)}{i_{in}(s)} = \frac{g_{m4}}{g_{m1}} \left[ \frac{1}{\frac{C_1(C_2 + C_3)}{g_{m1}g_{m2}}s^2 + \frac{((C_1 + C_2)(g_{m1} + g_{m2}) + C_{gs4}g_{m2})}{g_{m1}g_{m2}}s + 1} \right]$

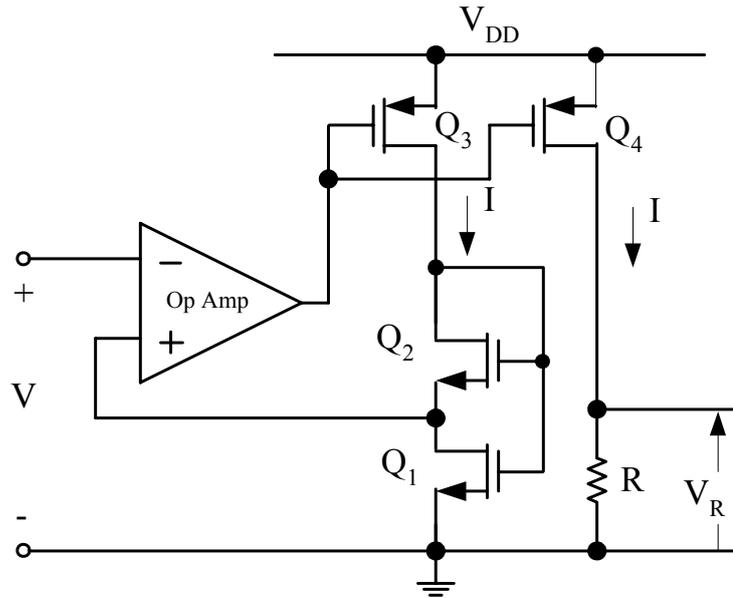
## 2.2 CMOS current squarer circuit

Squarer circuit is one of the useful basic circuit functions in the amplitude domain, the need for which arises quite often. They are essential to the extraction of the exact RMS value of a signal, and in power measurement in general.

Recently, the realization of squarer circuits have been proposed for various techniques [7], [21]-[30]. For [21]-[23], the circuits are designed by using the operational amplifier (op-amp) where the circuit designs with transistors that are operated in the saturation and the triode region. The output current is the function of their input voltage squared. On the other hand, the works in [24] - [30] are designed by using only MOS transistors. These circuits are function as a voltage-

mode squarer circuit. The circuits are designed by using square-law characteristic. All of MOS transistor are biased in the saturation region. Finally, the [7] is a current-mode squarer circuit.

Some example of the proposed squarer circuits will be outlined as follow.



**Fig. 2.6** Squarer circuit [21]

The squarer circuit that shown in Fig. 2.6 was proposed by *I.M. Filanovsky and H.P. Baltes* [21]. The circuit structure include an operational amplifier (op-amp) and nested transistor pair ( $Q_1$  and  $Q_2$ ) where transistor  $Q_1$  operates in triode region. When the input voltage ( $V$ ) is applied to the op-amp, the op-amp output voltage will force the transistor  $Q_3$  and the current  $I$ , which will flow through the transistors  $Q_1$  and  $Q_2$  can be calculate as follow.

The transistor  $Q_2$  will be pinch-off and transistor  $Q_1$  will be forced into the triode region of operation. The gate-source voltage  $V_{GS2}$  of the transistor  $M_2$  will be

$$V_{GS2} = V_{TN} + \sqrt{\frac{2I}{k_2}} \quad (2.18)$$

where  $V_{TN}$  is the threshold voltage of n-channel transistor and  $K_2 = \mu_n C_{ox} (W/L)_2$  from the other side, due to the triode operation of  $Q_3$ , the current  $I$  will satisfy the equation

$$I = K_1 (V_{GS1} - V_{TN})V - \frac{K_1 V^2}{2} \quad (2.19)$$

where  $K_1 = \mu_n C_{ox} (W/L)_1$ . Besides,

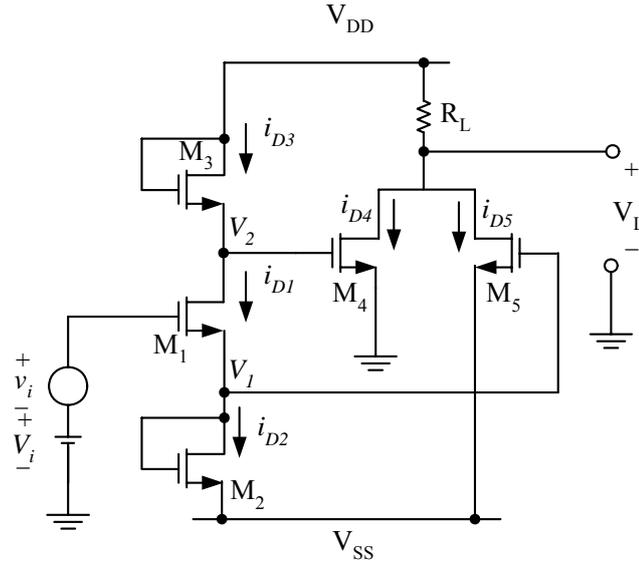
$$V_{GS1} = V + V_{GS2} \quad (2.20)$$

substituting (2.18) and (2.20) into (2.19), one can find that

$$I = K_1 \left[ \sqrt{(K_1/K_2)} \sqrt{(K_1/K_2) + 1} + (K_1/K_2) + 1/2 \right] V^2 \quad (2.21)$$

Transistors  $Q_3$  and  $Q_4$  are used to copy the current  $I$  to the output of the squaring circuit. Eqn. (2.21) shows that this circuit can operate as squarer function. The output current is proportional to the square of the input voltage. Due to this technique requires the operational amplifier, the circuit performance depends on the performance of the operational amplifier. Such as, the bandwidth is limited, since the cut-off frequency of the open-loop gain of the op-amp is typically in the range of 10-100Hz.

Fig. 2.7 shows the voltage-mode squarer circuit that proposed by *O.A. Seriki and R.W. Newcomb* [28]. The circuit is designed by the used of CMOS transistors. The circuit is function as voltage-mode squarer circuit.



**Fig. 2.7** Squarer circuit proposed by O.A. Seriki and R.W. Newcomb [2.8]

This circuit is a two-stage arrangement. The first stage which consisting of equal transistors  $M_1$ ,  $M_2$  and  $M_3$  (NMOS) is a linear amplifier, where the output voltages  $V_1$  and  $V_2$  are measured with respect to ground, having their signal components the negative of each other. In the second stage, which is the squaring function, the current  $i_L$  that flowing through  $R_L$  is the sum of the drain currents  $i_{D4}$  and  $i_{D5}$  of the two equal transistors  $M_4$  and  $M_5$ . These currents being squares of the inputs when the transistors are properly biased. For the circuit analysis, assuming the transistors are biased to operate in the square-law region, the drain current  $i_D$  of an n-channel MOS transistors is related to its gate-to-source voltage,  $V_{GS}$ , by

$$i_D = K(V_{GS} - V_T)^2 \quad (2.22)$$

Since the current  $i_{D1}$ ,  $i_{D2}$  and  $i_{D3}$  flowing through the transistors are the same, the gate-to-source voltage  $V_{GS1}$ ,  $V_{GS2}$  and  $V_{GS3}$  are equal. Since  $V_{GS1} + V_{GS2} = 2V_{GS2} = 2V_{GS3} = V_i + V_{SS} + v_i$ , Kirchoff's voltage law shows that the voltages  $v_1$  and  $v_2$  are given by

$$v_1 = V_{DD} - V_{GS3} = V_{DD} - \frac{1}{2}(V_i + V_{SS}) - \frac{1}{2}v_i \quad (2.23)$$

$$v_2 = V_{GS2} - V_{SS} = \frac{1}{2}(V_i - V_{SS}) + \frac{1}{2}v_i \quad (2.24)$$

Hence, a signal  $v_i$  fed into the input of the first stage divided equally, but with opposite signs, between  $v_1$  and  $v_2$  to form the signal components of the voltages which control the gates of transistors  $M_4$  and  $M_5$  of the second stage.

The current  $i_L$  flowing through the resistor  $R_L$  is the sum of the drain current  $i_{D4}$  and  $i_{D5}$ . The total out put voltage,  $v_L$ , is given by

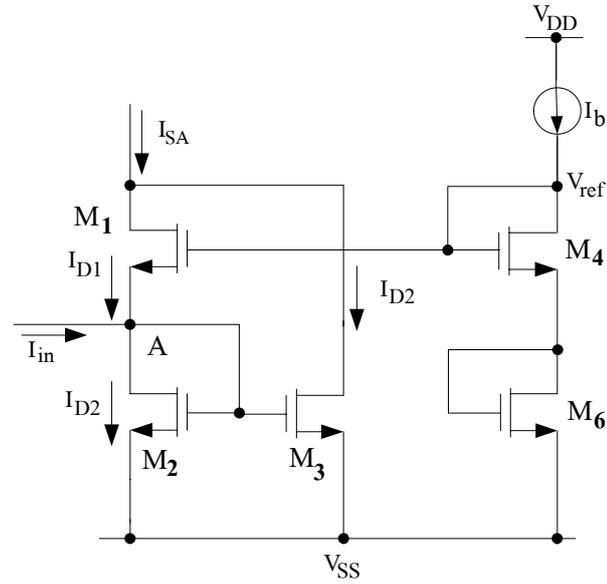
$$v_L = V_{oo} - KR_L(a_2 - a_1)v_i - \frac{1}{2}KR_Lv_i^2 \quad (2.25)$$

where  $V_{oo}$  is the dc bias component,  $V_{oo} = V_{DD} - KR_L(a_1^2 + a_2^2)$ ,  $a_1 = V_{DD} - \frac{1}{2}V_{SS} - \frac{1}{2}V_i - V_T$  and  $a_2 = \frac{1}{2}V_{SS} + \frac{1}{2}V_i - V_T$ .

From eqn. (2.25), since this second term is not desired in the output of a squaring circuit, it contributes to the amount of distortion or error introduced by the circuit. The last term is an undistorted reproduction of the square of the input signal and this is the desired output signal. The second term can be reduced or eliminated completely by satisfying the condition  $a_2 - a_1 = 0$ . We can found that this circuit can function as voltage squaring circuit.

The circuit is shown in Fig. 2.8 that has been proposed in [7], is the current mode squarer circuit. This work is the one of the method that realized by using the square law characteristic of MOS transistor where all transistors are biased in saturation region. The circuit structure is simple and provides good accuracy over a wide range of input currents. Therefore, in this thesis, the current squarer circuit as shown in Fig. 2.8 has been used for designing the proposed true rms to dc converter. The circuit structure composes of two current mirrors and the output current is the function of the square input current and DC bias current. This current squarer circuit description will be outlined in this section.

### 2.2.1 Circuit Description



**Fig. 2.8** CMOS current squarer circuit

The operation of the CMOS current squarer circuit of the Fig. 2.8 is based on the square law characteristic of MOS transistors biased in the saturation region [31], [32]. Transistors  $M_1$  through  $M_3$  function as a current squarer, where  $M_4$  and  $M_6$  and the current source  $I_b$  formed as the current-controlled bias circuit. The input signal current  $I_{in}$  is injected into point  $A$ . From the squarer circuit in Fig. 2.8, consider the transistor  $M_1$ ,  $M_3$ ,  $M_4$  and  $M_6$ , we will obtain

$$V_{GS4} + V_{GS6} = V_{GS1} + V_{GS3} \quad (2.26)$$

Assume all MOS transistors are biased in the saturation region, the square-law behavior for the current –voltage relationship of the MOS transistor in saturation is assumed as

$$I_D = K(V_{GS} - V_T)^2 \quad (2.27)$$

where  $V_{GS}$  can be expressed as

$$V_{GS} = \sqrt{\frac{I_D}{K}} + V_T \quad (2.28)$$

from eqns. (2.26) and (2.28), set  $V_{T1} = V_{T2} = V_{T4} = V_{T6} = V_T$  and  $K_1 = K_2 = K_4 = K_6 = K$  and from circuit analysis  $I_{D4} = I_{D6} = I_b$ , therefore

$$2\sqrt{I_b} = \sqrt{I_{D1}} + \sqrt{I_{D3}} \quad (2.29)$$

squaring the eqn. (2.29) given

$$4I_b = I_{D1} + 2\sqrt{I_{D1}I_{D2}} + I_{D2} \quad (2.30)$$

From routine circuit analysis, the relation between  $I_{D1}$ ,  $I_{D2}$  and  $I_{in}$  can be expressed as

$$I_{D1} = I_{D2} - I_{in} \quad (2.31)$$

or

$$I_{D2} = I_{D1} + I_{in} \quad (2.32)$$

From eqns. (2.30) and (2.31), we will obtain

$$2\sqrt{I_{D2}^2 - I_{in}I_{D2}} = 4I_b + I_{in} - 2I_{D2} \quad (2.33)$$

By squaring eqn. (2.33), the drain current of transistor  $M_2$ , ( $I_{D2}$ ) can be written as

$$I_{D2} = \frac{16I_b^2 + 8I_bI_{in} + I_{in}^2}{16I_b} \quad (2.34)$$

Eqn. (2.34) can be rewritten as

$$I_{D2} = \frac{(4I_b + I_{in})^2}{16I_b}, \quad |I_{in}| \leq 4I_b \quad (2.35)$$

From eqns. (2.29) and (2.34), the drain current of transistor  $M_1$ , ( $I_{D1}$ ) can be written as

$$I_{D1} = \frac{16I_b^2 - 8I_b I_{in} + I_{in}^2}{16I_b} \quad (2.36)$$

or

$$I_{D1} = \frac{(4I_b - I_{in})^2}{16I_b}, \quad |I_{in}| \leq 4I_b \quad (2.37)$$

The unity gain positive current mirror  $CM_1$ , formed by  $M_2$  and  $M_3$ , reflects the current  $I_{D2}$  in order to add with the current  $I_{D1}$ . Then, from eqns. (2.35) and (2.36), the summation of the currents  $I_{D1}$  and  $I_{D2}$  or  $I_{SA} = I_{D1} + I_{D2}$  becomes

$$I_{SA} = I_{D1} + I_{D2} \quad (2.38)$$

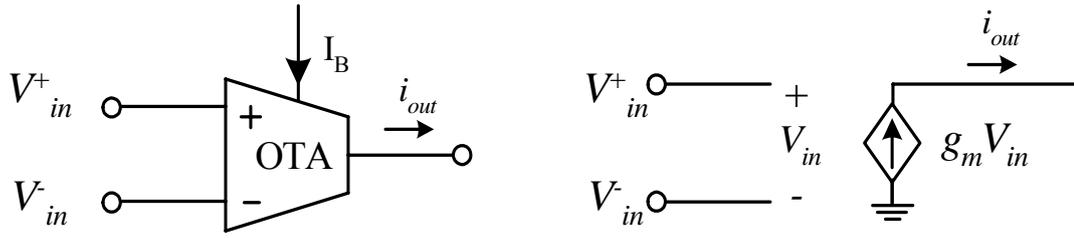
Replace  $I_{D1}$  and  $I_{D2}$  in eqn. (2.32) with  $I_{D1}$  and  $I_{D2}$  from eqns. (2.34) and (2.36) respectively,  $I_{SA}$  can be rewritten as

$$I_{SA} = 2I_b + \frac{I_{in}^2}{8I_b} \quad (2.39)$$

We can see that  $I_{SA}$  consists of the signal current which is the squaring of the input signal  $I_{in}$  and the DC current  $2I_b$ . If the DC current  $2I_b$  can be compensated, the circuit will be functioned as a squarer/divider circuit, which can be used as a basic cell to realize rms-to-dc converter by the implicit computation method in this thesis.

### 2.3 A balanced CMOS OTA

An operational transconductance amplifier (OTA), also called a transconductance element or a transconductance, is a device that converts voltage inputs to current outputs (or a voltage controlled current source) such that  $i_{out} = g_m V_{in}$ . The transconductance gain  $g_m$  can usually be varied over a wide range by adjusting an external DC bias current  $I_b$ .



**Fig. 2.9** Ideal model of the operational transconductance amplifier (OTA)

In Fig. 2.9, the small-signal model for an ideal OTA is seen to be a differential input voltage-controlled current source (VCCS), with infinite input and infinite output impedances. An OTA is a voltage controlled current source, more specially the term “operational” comes from the fact that it takes the difference of two voltages as the input for the current conversion. The ideal transfer characteristic is therefore

$$i_{out} = g_m (v_{in}^+ - v_{in}^-) \quad (2.40)$$

or, by taking the pre-computed difference as the input,

$$i_{out} = g_m v_{in} \quad (2.41)$$

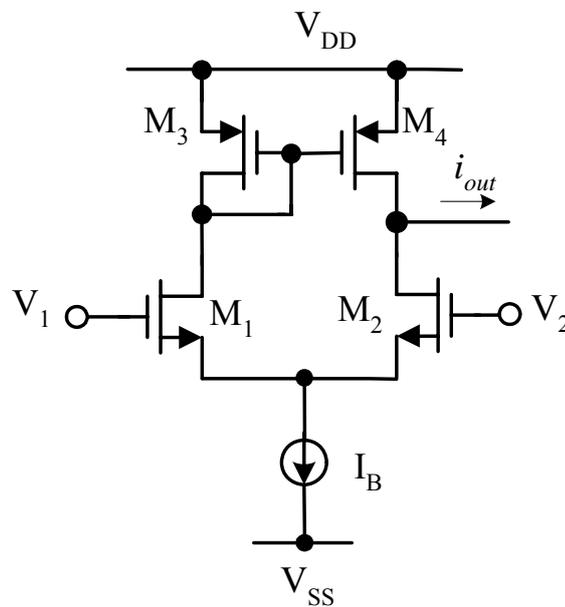
with the ideally constant transconductance  $g_m$  as the proportional factor between the two. In practice the transconductance is also a function of the input differential voltage, as we will later see.

In the literature, many design techniques have been proposed for realizing the operational transconductance amplifier. For CMOS-based OTA, a well-known practical linearization technique for transconductors is source degeneration [33]-[35] and by using cross-coupled differential pairs operating in saturation, e.g. [36]-[40]. Some of these realizations are not intended for low-power usage, others have problems with the body effect and/or channel length-modulation. In addition an interesting approach of a low-power linear OTA working in the non-saturation region has been proposed in [41]. However a double supply voltage and matching of a

p-type and n-type OTA is needed. As a matter of principle the transconductances for positive and negative input voltages do not match exactly.

In this thesis, a balanced CMOS OTA has been used for realizes the transconductor which can be electronically and linearly tuned. The circuit structure is composed of a  $V/I$  converter (or differential amplifier) and three current mirrors. The circuit description will be described. Firstly a simple CMOS OTA will be introduced.

In Fig. 2.10 a simple CMOS OTA is shown, the simple CMOS OTA consists of a self-bias MOS transistor differential stage with active load. Transistors  $M_1$  and  $M_2$  form a matched transistor pair. Transistors  $M_3$  and  $M_4$  have equal W/L, as well. All current levels are determined by current source  $I_B$ , half of which flows through  $M_1$  and  $M_3$ , with the other half flow through  $M_2$  and  $M_4$ .



**Fig. 2.10** A simple CMOS OTA

At low frequency, the transconductance of the OTA is given by  $G_m = g_{m1} = g_{m2}$ . It is also given by

$$G_m = \sqrt{2I_B K} \quad (2.42)$$

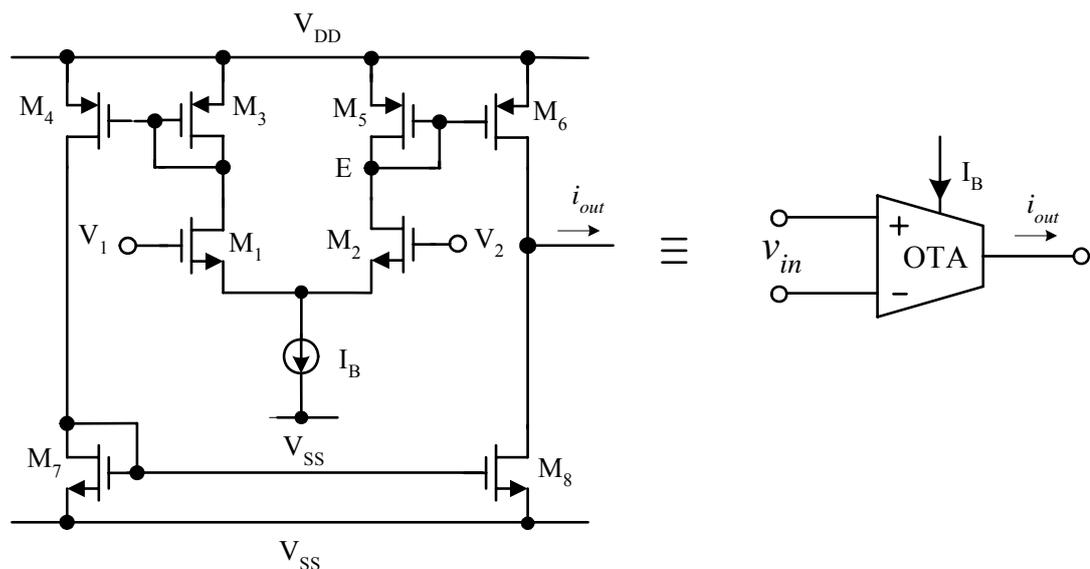
in which  $K = \mu_n C_{ox} W / 2L$  is the transconductance parameter for MOS transistor in saturation. Also note that the current  $I_B$  in eqn.(2.42) is the total current, which is twice the current through one transistor.

For the structure of this simple CMOS OTA, their input stage consists of a differential pair loaded by a current mirror. This load is thus very asymmetrical due to only  $M_3$  is connected as a diode that causes  $V_{DS3}$  fixed, whereas  $V_{DS4}$  is varied up to load. Therefore, the DC current which flows through  $M_1, M_3$ , and the other one flowing through  $M_2, M_4$  are unbalance. From this result, the symmetrical CMOS OTA or balanced CMOS OTA is preferred for the circuit design in thesis, which provides the same load to both input devices.

### Simple balanced CMOS OTA

The circuit schematic of the balanced CMOS OTA with symmetrical input stage is given in Fig. 2.11. The differential pair is formed by the input transistors  $M_1$  and  $M_2$ . They drive their output currents in two transistors,  $M_3$  and  $M_5$ , which are connected as diodes. They are the inputs of two current mirrors with current multiplication factor (current mirror ratio). The output current of transistor  $M_4$  is then mirrored once more in current mirror  $M_7, M_8$ , with the unity gain current mirror, as indicated. In addition, for the balanced OTA structure, current mirrors constructed of transistor  $M_3$ - $M_4$  and  $M_5$ - $M_6$  will force the drain voltage  $V_{DS1}$  of transistor  $M_1$  equal to  $V_{DS2}$  of transistor  $M_2$ .

#### 2.3.1 Circuit Descriptions of a balanced CMOS OTA



**Fig. 2.11** Schematic diagram of a balanced CMOS OTA

For the purpose of the following analysis, we will assume that all MOS devices operate in the saturation region. This means that the transistor drain current  $I_D$  is characterized by a square-law model as

$$I_D = \begin{cases} K(V_{GS} - V_T)^2 & , \text{ for } V_{GS} > V_T \\ 0 & , \text{ for } V_{GS} \leq V_T \end{cases} \quad (2.43)$$

where the transconductance parameter  $K = \mu_n C_{ox} W/2L$ ,  $\mu_n$  is the mobility of the carrier,  $C_{ox}$  is the gate-oxide capacitance per unit area,  $W$  is the effective channel width,  $L$  is the effective channel length, and  $V_{GS}$  and  $V_T$  are the gate-to-source and threshold voltages, respectively.

If we let  $V_{in}$  is the differential input voltage ( $V_{in} = V_1 - V_2$ ),  $i_{out}$  is the output current and  $I_B$  is the bias current. Let us assume that  $M_1$  and  $M_2$  are perfectly matched and the current mirrors have unity current gain. By using eqn. (2.43), the differential output current of the circuit in Fig. 2.11 can be given by [42]

$$\begin{aligned} i_{out} &= i_2 - i_1 \\ &= \sqrt{2I_B K} \cdot V_{in} \cdot \sqrt{1 - \frac{KV_{in}^2}{2I_B}}, \quad \text{for } -\sqrt{\frac{I_B}{K}} \leq V_{in} \leq \sqrt{\frac{I_B}{K}} \end{aligned} \quad (2.44)$$

The transconductance gain ( $g_m$ ) of the MOS coupled pair can be derived by taking the derivative of (2.44) with respect to  $V_{in}$ , yielding

$$g_m = \left. \frac{di_{out}}{dV_{in}} \right|_{V_{in}=0} = \sqrt{2I_B K}, \quad \text{for } -\sqrt{\frac{I_B}{K}} \leq V_{in} \leq \sqrt{\frac{I_B}{K}} \quad (2.45)$$

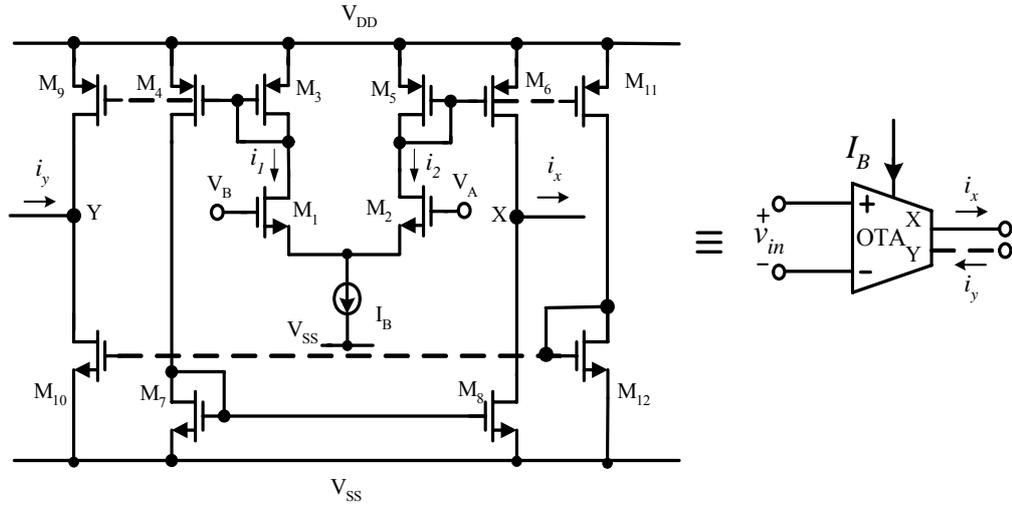
From the eqn. (2.44), the current  $i_{out}$  can be rewritten as

$$i_{out} = g_m V_{in} = \sqrt{2I_B K} \cdot V_{in} \quad (2.46)$$

Eqn. (2.45) shows that the transconductance gain ( $g_m$ ) of the OTA can be varied by the bias current  $I_B$ , but in the form of square root function. In addition, in order to operate in the low distortion range, the input voltage  $V_{in}$  should be in the range of [20]

$$V_{in} \leq \left| \sqrt{I_B / K} \right| \quad (2.47)$$

Moreover, Fig. 2.12 is a multiple current output or fully differential OTA. As shown in dashed-line the multiple current outputs OTA can be implemented by current replicas using current mirrors formed by transistors  $M_9$ - $M_{12}$ . However, the method that uses two OTAs in parallel input terminals to produce multiple outputs can also be used [43].



**Fig. 2.12** Schematic diagram of the fully differential CMOS OTA

Where  $v_{in}$  is the differential input voltage ( $v_{in} = V_A - V_B$ ),  $I_x$  is the output current and  $I_B$  is the bias current. The differential output current of the circuit of Fig. 2.12 can be given by

$$i_x = i_2 - i_1 = \sqrt{2I_B K} \cdot v_{in} \cdot \sqrt{1 - \frac{Kv_{in}^2}{2I_B}}, \quad \text{for } -\sqrt{\frac{I_B}{K}} \leq v_{in} \leq \sqrt{\frac{I_B}{K}} \quad (2.48)$$

The transconductance gain ( $g_m$ ) of the fully differential OTA can be derived by taking the derivative of (2.48) with respect to  $v_{in}$ , yielding

$$g_m = \left. \frac{di_x}{dv_{in}} \right|_{v_{in}=0} = \sqrt{2I_B K} \quad (2.49)$$

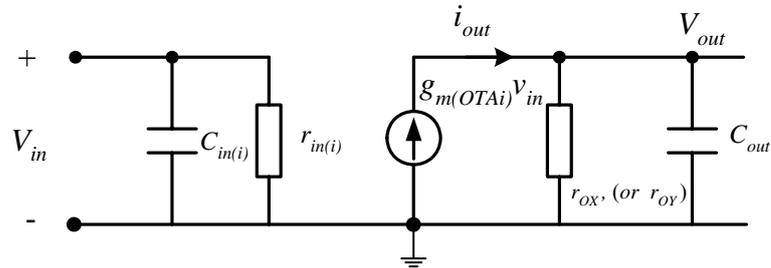
Thus we can obtain the following equation for a small signal;

$$i_x = g_m v_{in} = \sqrt{2I_B K} \cdot v_{in} \quad (2.50)$$

Eqn. (2.49) shows that the transconductance gain ( $g_m$ ) of the OTA can be varied by the bias current  $I_B$ , but in the form of a square root function. It should be noted that the transconductance gain ( $g_m$ ) of the balance CMOS OTA shown in Fig. 2.11 will provide low harmonic distortion if the input voltage is limited in the range of

$$g_m = \sqrt{2I_B K}, \quad \text{for } -\sqrt{\frac{I_B}{K}} \leq v_{in} \leq \sqrt{\frac{I_B}{K}} \quad (2.51)$$

The effect from the non-ideality can be studied through the use of the OTA small signal model that shown in the Fig. 2.13,



**Fig. 2.13** Small signal model of a balanced CMOS OTA

where  $r_{in(i)}$  is the input resistance of OTAi,  $r_{OX}$  or  $r_{OY}$  is the different output resistance at port X or port Y,  $C_{in(i)}$  is the input capacitance of OTAi and  $C_{out(i)}$  is the output capacitance of OTAi [20],[25],[26]. The differential output resistance  $r_{OX}$  is approximately equal to the output resistance of the current mirror  $M_5$ - $M_6$  in parallel with the output resistance of the current mirror  $M_7$ - $M_8$  or  $r_{OX}$  approximate  $1/(g_{d6}+g_{d8})$  and , similarly,  $r_{OY}$  approximate  $1/(g_{d9}+g_{d10})$