ภาคผนวก

ภาคผนวก ก.

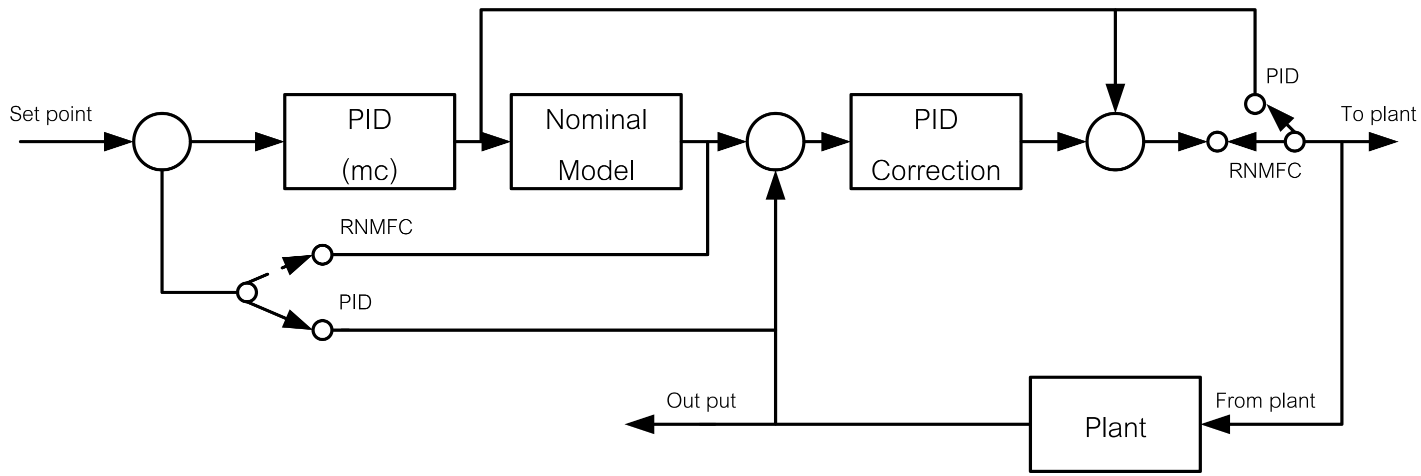
ตารางที่ ก, แสดงบอร์ดวงจรอนาลอก , และรูปโครงงานจริง

ตารางที่ ก. แสดงตัวอย่างตัวคูณจากตาราง t-Distribution

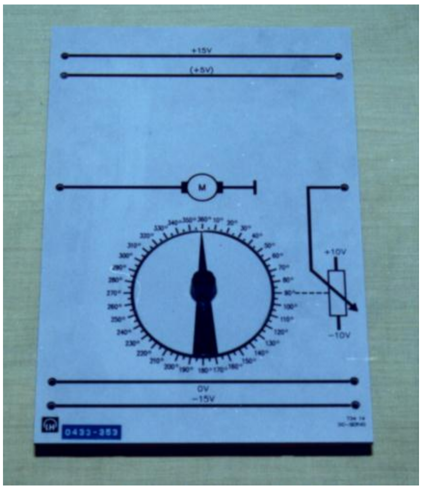
Degree of freedom , V	**% ของข้อมูลที่ต้องการ (Confidence Level)**					
	68.27	**90**	**95**	**95.45**	**99**	**99.73**
1	1.84	6.31	12.71	13.97	63.66	235.80
2	1.32	2.92	4.30	4.53	9.92	19.21
3	1.20	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.60	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
10	1.05	1.81	2.23	2.28	3.17	3.96
30	1.02	1.70	2.04	2.09	2.75	3.27
∞	1.000	1.645	1.960	2.000	2.576	3.000

ภาคผนวก ก.

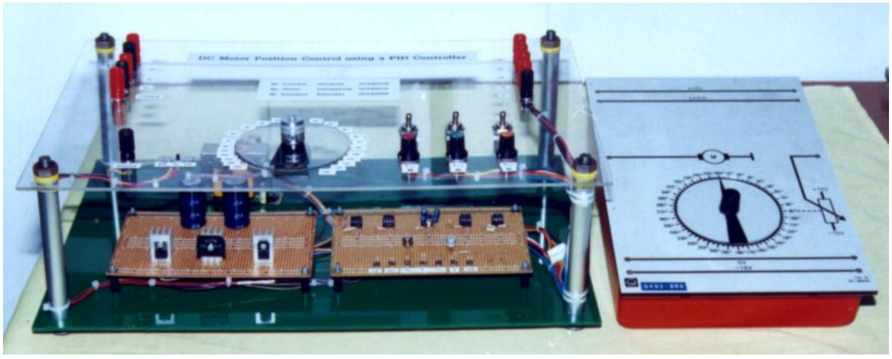
บอร์ดวงจรอนาลอก



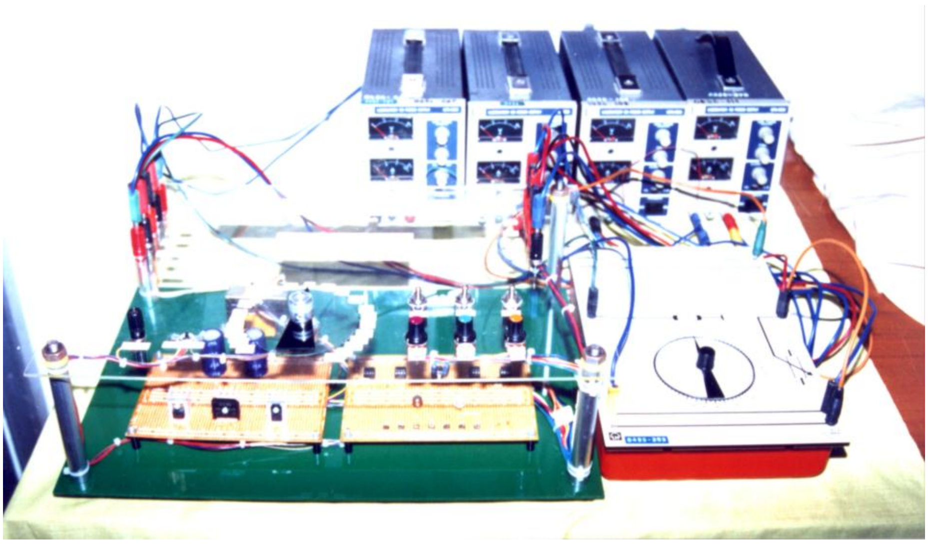
รูปที่ ก.1 แสดงบอร์ดวงจรอนาลอก



รูปที่ ก.2 แสดงภาพถ่ายของมอเตอร์ที่ใช้ในการทดลองจริง



รูปที่ ก.3 แสดงภาพถ่ายของชุดทดลองและมอเตอร์ที่ใช้ในการทดลองจริง



รูปที่ ก.4 แสดงภาพถ่ายการต่อวงจรของชุดทดลองและมอเตอร์ที่ใช้ในการทดลองจริง

ภาคผนวก ข.

ผลงานวิจัยที่ได้รับการตีพิมพ์เผยแพร่

1. K. Witheephanich, W. Piyarat, **R. Keteruksa**, and V. Tipsuwanporn, "**Robustly Stabilizing Controller Design for Networked Control Systems Considering Plant Uncertainties**," Mechatronics & Robotics '04, pp. 50-55, Germany, September 13-15, 2004.
2. K. Witheephanich, W. Piyarat, V. Tipsuwanporn, and **R. Keteruksa**, "**Robust Controller Synthesis for Robustness Property Enhancement**," 25th Electrical Engineering Conference (EECON-25), pp. 26-30, Thailand, November 21-22, 2002.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September 13 - September 15, 2004
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Robustly Stabilizing Controller Design for Networked Control Systems Considering Plant Uncertainties

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***Abstract*— There is increasing interest in controlling systems over communication networks. Using a Robust Model Following Control (RMFC) scheme, this paper proposes robustly stabilizing loop controller designed and then added to local location of a distributed system to cope with the plant uncertainties. A systematic design methodology is established and it is linked to the conventional control system design. The robust loop controller designs for differently distributed architectures are shown to illustrate the benefits of the proposed technique with a number of simulation examples.**

I. INTRODUCTION

In recent years, there has been increasing interest in controlling systems over communication networks because communication networks [1] are among the fastest-growing areas in engineering. These systems are frequently modeled from the control point of view as time-delay systems because of the inherent propagation delays; see, for example, [2, 3]. These delays are crucial to the system stability and the quality-of-service (QoS). As is well known, the presence of delays makes the control design and system analysis much more complicated. For details on the control of communication networks, see [2, 3, 4, 5] and the references therein.

Continuous-time networked control system models were considered by [6, 7, 8, 9, 10]. Goktas et al. used a modified Pade approximation and considered the network delay as an uncertainty. They designed a robust controller to compensate for the uncertain delay in an ATM network. Kim et al. used a Lyapunov approach to obtain the maximum allowable delay bound for the stability of a network delayed system. A scheduling algorithm for determining the sampling rate and allocating bandwidth was also provided. Walsh et al. also adopted the Lyapunov approach on a continuous-time model to obtain the maximum allowable transfer interval and to analyze the stability of the closed-loop system. They further analyzed the impact of different scheduling algorithms on the maximum transfer allowable transfer interval. Nevertheless, only a conservative delay bound is obtained. The impact of delay variance on control performance is discussed in these works, but is not formally characterized.

It is obvious from these references that the robust stability analysis of time-delay systems is not well established and has become a very active research field in recent years. Current efforts can be divided into two categories: delay-dependent stability criteria [11, 12, 13] and delay-independent stability criteria [14]. In addition, one concept that emerged in the context of controller design for a distributed architecture is that of the controller to be "split" in two components [15]. One is local to the plant and this component affects the plant directly. The other portion of the control signal is generated at a remote location and reaches the plant after some communication delay. However, one can immediately notice that no explicit design methodology is currently available to simultaneously design local system control structure that meets the robust stability for the range of plant uncertainties. Therefore, in this paper, we further explore the idea of control with networks and propose a technique of designing local control structure, robust loop controller, connected to each other by a network. This technique uses the fundamental of model following control scheme that can be applied with the other developed techniques to network-based control.

Adaptive Model Following Control (AMFC) is proven to be an effective and preferred control strategy [16, 17]. Although adaptive model following control theory has been well developed, the simplification of control structure and the relaxation of strict assumptions still need further research. In recent years, many efforts have been made to the selection of appropriate reference model and the elimination of requirement for full state information which is usually difficult to acquire [18]. Thus, in this paper, a robust control approach based on the adaptive model following concept called Robust Model Following Control (RMFC) is proposed to design the local control configuration. The approach is quite different from and much simpler than existing AMFC schemes.

The paper is organized as follows. In section II we present RMFC scheme and in section III present controller architecture. Numerical examples are given in section IV.

II. ROBUST MODEL FOLLOWING CONTROL SCHEME

Perfect model following is an ideal case of model following control systems. In fact, due to parameter variations and load disturbance in the real plant, the asymptotic model following based on adaptive ideas is more practical. Generally, an AMFC system includes a reference model which prescribes the dynamic behavior of the model and reflects the expected performance of the controlled plant, and an adaptation mechanism which generates a correction signal to force the plant to follow the model.

Since it is not easy to select an appropriate reference model which matching conditions of AMFC scheme, one may wonder why not to directly use a nominal model of the plant as a reference model. This question has in fact motivated the development of the underlying RMFC. Certainly, the direct use of the nominal model of the plant as reference model requires great revolution of present architectures of AMFC schemes.

Fig. 1 shows the principle diagram of the proposed RMFC scheme. The block nominal model denotes a mathematical description of the plant in the nominal case and acts as a reference model in the system. The model controller is designed from the nominal model and it can be any type of controller. The model controller has two tasks. One is to ensure a desired reference output trajectory y_m for the plant to follow which satisfies all performance specifications required by the designer, and the other is to provide part of control input for the plant. The correction mechanism generates an additional correction signal so that the output of the real plant can still follow the desired reference output trajectory in the presence of modeling errors.

In principle, the correction mechanism can be of any type as long as it guarantees asymptotic model following. As aforementioned [16], the adaptation law developed by means of hyperstability theory is very complicated and quite time-consuming in the implementation and it may also introduce poorly matched initial values of controller parameters. In order to reduce the computation in the correction mechanism, well-developed conventional control techniques may be employed. A simple PID correction mechanism is here adopted.

A. Asymptotic model following conditions

Assume that the plant parameters are unknown but constant or slow time-varying. The input-output relationship of the plant can then be approximated by a transfer function with a variable parameter vector. The reference model, the plant and the PID correction mechanism are represented in terms of transfer functions as shown in Fig. 2, where $G_m(s)$, $G_p(s,v)$ and $G_a(s)$ denote the transfer functions of the reference model, plant and PID correction mechanism respectively, and v is the parameter vector of the plant.

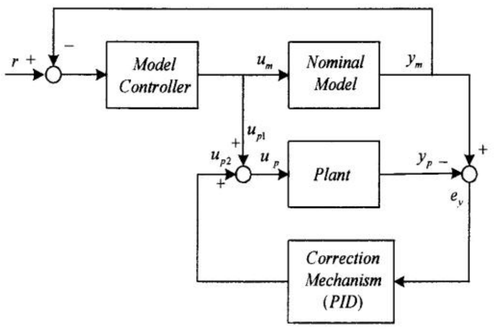

Fig. 1 Principle block diagram of RMFC scheme.

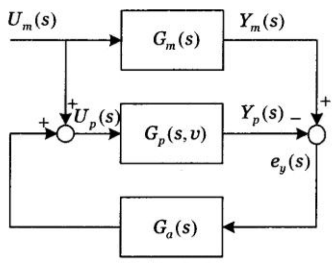

Fig. 2 Simplified diagram of RMFC scheme in terms of transfer functions.

Asymptotic model following conditions for the proposed RMFC scheme are stated as follows:

Theorem: The plant output $y_p(t)$ asymptotically follows the model output $y_m(t)$ if

(1) the model and the plant do not include any poles in the right half plane and on the imaginary axis with an exception at the origin,

(2) the closed loop system of the plant is stable if the same PID correction mechanism is taken as a feedback controller,

(3) there is a limit on the output of the model controller when the time approaches infinity, i.e.

$$\lim_{t\to\infty} u_m(t) = k, \tag{1}$$

where k is an arbitrary constant.

Proof:

From Fig. 2, it follows that

$$Y_m(s) = G_m(s)U_m(s), \tag{2}$$

$$Y_p(s) = G_p(s,v)[U_m(s) + G_a(s)e_y(s)], \tag{3}$$

$$e_y(s) = Y_m(s) - Y_p(s) \tag{4}$$

Hence, the transfer function of the generalized output error $e_y(s)$ to the model input $U_m(s)$ is

$$\frac{e_y(s)}{U_m(s)} = \frac{G_m(s) - G_p(s,v)}{1 + G_p(s,v)G_a(s)} \tag{5}$$

Condition (1) implies that

$$D_m(s) = s^n D_m^{'}(s), \tag{6}$$

$$D_p(s,v) = s^n D_p^{'}(s,v), \tag{7}$$

where n is integer equal to or larger than zero (note that the plant and its nominal model are supposed to have the same number of poles located at the origin) and $D_m^{'}(s)$ and $D_p^{'}(s,v)$ are stable polynomials.

Equation (5) can then be expressed in terms of polynomials stated above as

$$\frac{e_y(s)}{U_m(s)} = \frac{D_a(s)[D_p^{'}(s,v)N_m(s) - D_m^{'}(s)N_p(s,v)]}{D_m^{'}(s)[D_p(s,v)D_a(s) + N_p(s,v)N_a(s)]} \tag{8}$$

The closed loop system formed by the plant and the same PID correction mechanism as the feedback controller is shown in Fig. 3. Its closed loop transfer function can be written as

$$\frac{Y^{'}(s)}{r^{'}(s)} = \frac{N_p(s,v)N_a(s)}{D_p(s,v)D_a(s) + N_p(s,v)N_a(s)} \tag{9}$$

Because $D_m^{'}(s)$ is a stable polynomial, it can be seen from (8) and (9) that the dynamics of the generalized output error e_y in the RMFC scheme has the same stability property as the closed loop system formed by the plant and the same PID correction mechanism as shown in Fig. 3. If the closed loop system as shown Fig. 3 is stable, the dynamics of the generalized output error e_y in the RMFC scheme must be stable.

From (8), it follows that

$$\lim_{s\to 0} se_y(s) = \lim_{s\to 0} \frac{D_a(s)[D_p^{'}(s,v)N_m(s) - D_m^{'}(s)N_p(s,v)]}{D_m^{'}(s)[D_p(s,v)D_a(s) + N_p(s,v)N_a(s)]} sU_m(s) \tag{10}$$

Considering condition (3), one can rewrite (10) as

$$\lim_{s\to 0} se_y(s) = \lim_{s\to 0} \frac{D_a(s)[D_p^{'}(s,v)N_m(s) - D_m^{'}(s)N_p(s,v)]}{D_m^{'}(s)[D_p(s,v)D_a(s) + N_p(s,v)N_a(s)]} \lim_{s\to 0} sU_m(s) \tag{11}$$

For the PID-type correction mechanism

$$u_{p2} = k_{pa}\left(e_y + \frac{1}{T_{ia}}\int_0^t e_y d\tau + T_{da}\frac{de_y}{dt} \right) \tag{12}$$

That is, $D_a(s) = T_{ia}(s)$ .Hence

$$\lim_{s\to 0} se_y(s) = 0 \tag{13}$$

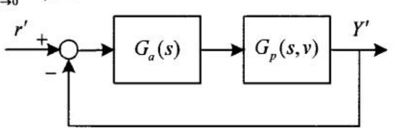

Fig. 3 Feedback control system of plant with PID correction mechanism as controller.

From conditions (2) and (3) and Eq. (8), there must exist a limit on the generalized output error $e_y(t)$ when t approaches infinity. Therefore,

$$\lim_{t\to\infty} e_y(t) = \lim_{s\to 0} se_y(s) = 0 \tag{14}$$

That is, asymptotic model following is achieved.

III. CONTROLLER ARCHITECTURE

The manner in which a control algorithm will be implemented for a particular application gives rise to a corresponding system architecture. Consider the case where one decides to implement a feedback controller in a centralized manner. Specifically, all measurement data is collected at a specific location and the control action is computed at this location with all available sensor data. The actuation signal is then directly given to the plant. This gives rise to the "standard" unity feedback configuration (i.e., system architecture) depicted in Fig. 4. In these diagrams sensor dynamics are not explicitly shown but are assumed to be included in the "plant" block.

An architecture that more appropriately addresses the constraints found in systems controlled over networks is shown in Fig. 5 [10,19]. In this situation there is a delay associated with measurements (represented by the delay block Δ_1) and a different delay associated with the transfer of the controller output to the plant (represented by the delay block Δ_2). The delay blocks Δ_1, Δ_2 represent communication delays. All information obtained from measurements is available to the controller (albeit delayed), the control computation is done in a centralized manner and the controller output reaches the plant after some delay.

For networked controlled systems in the context of plant uncertainties one can also regard systems where the architecture shown in Fig. 6 may be appropriate. Such systems have the feature that a robust loop controller is added to local location in order to cope with the plant uncertainties at the same time. The model controller can be considered as "remote", receives a delayed measurement (through Δ_1) from the nominal model output and delivers a delayed command signal to the plant (through Δ_2). This architecture will definitely have better "robustness" characteristics with respect to plant uncertainties when compared with an implementation based on Fig. 5. The next section draws attention to this architecture by considering two motivating examples.

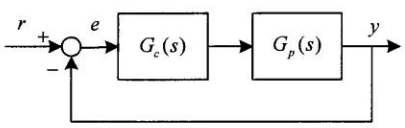

Fig. 4 A centralized control architecture.

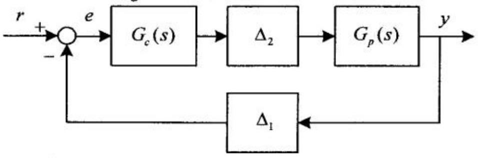

Fig. 5 A Networked Control System.

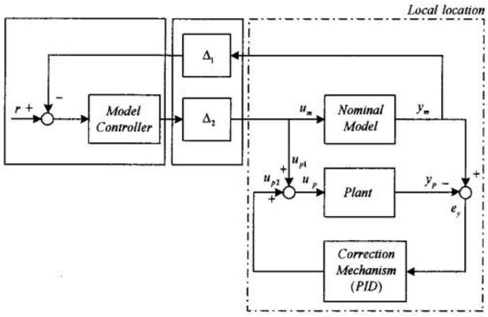

Fig. 6 Block diagram of control over networks with RMFC scheme.

IV. Numerical Examples

A. Distributed architecture [10, 19] with RMFC

Consider a distributed control system as in Fig. 6 where we assume that the delay lies in some interval $0 \le \tau \le 1\,sec.$ Consider the example of a nominal model of the plant, dc motor modeled using the second order transfer function [22]:

$$G_m(s) = \frac{\omega(s)}{V_a(s)} = \frac{1/K_b}{(\tau_1 s+1)(\tau_2 s+1)} = \frac{0.02513}{(8.21*10^{-3}s+1)(0.322*10^{-3}s+1)} \quad (15)$$

After the nominal model of the plant is determined, the model controller is designed in the approach proceeding in two steps. As previously mentioned, any design technique can be used to construct the model controller. First, a PID controller $G_{c1}(s)$ is designed for a system without the presence of a delay using root locus method in this paper [20]. The transfer function of the designed PID controller is expressed as

$$G_{c1}(s) = 41.354\left(1+\frac{1}{8.53*10^{-3}s}+3.1*10^{-4}s\right) \quad (16)$$

Second, the following controller $G_{c2}(s)$ cascaded wit $G_{c1}(s)$ is modified employing loopshaping technique [21] to handle delays:

$$G_{c2}(s) = \frac{5.287*10^{-5}s^3+0.1706s^2+20s}{0.000169s^4+0.5286s^3+82.41s^2+2469s+0.1228} \quad (17)$$

As stated in section II, the design of the PID controller part is equivalent to the design of a PID feedback controller of the plant in the worst case of parameter variations. The transfer function of the plant in the considered worst case is here evaluated as

$$G_p(s) = \frac{0.08}{(0.03284s+1)(1.288*10^{-3}s+1)} \quad (18)$$

The PID control part can then be designed also using root locus method as

$$G_a(s) = 7.606\left(1+\frac{1}{0.038s}+0.077s\right) \quad (19)$$

In Fig. 7 we show Simulink simulations of the closed loop system with transport delay $\tau = 1\,sec.$ There are two step responses with no plant uncertainties shown in Fig. 7(a) one for the distributed architecture without RMFC and one for the distributed architecture with RMFC. Whereas there are two step responses with the worst case of parameter variations shown in Fig. 7(b) one for the distributed architecture without RMFC and one for the distributed architecture with RMFC. The results indicate that significant robustness characteristic enhancement with respect to plant uncertainties in the speed response by proposed technique is achieved.

B. Split-controller architecture [15] with RMFC

Consider a distributed controller system from [15] as in Fig. 8 and the example of a nominal model of the plant:

$$G_m(s) = \frac{10}{s^2+s} \quad (20)$$

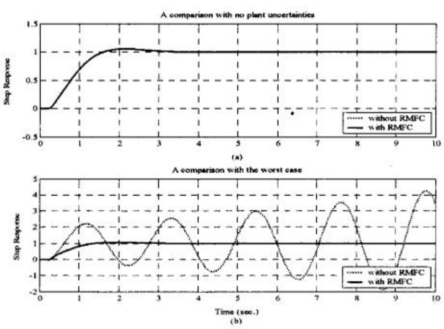

Fig. 7 A comparison (a) with no plant uncertainties (b) with the worst case.

Djaferis proposed such system that can allow the controller to be split in two components. One is a local controller $G_{c1}(s)$ and the other $G_{c2}(s)$ can be considered as remote. Clearly, this architecture was effective for better robustness characteristics with respect to delays when compared with an implementation based on Fig. 5.

Therefore, in this example, the RMFC scheme is applied with the split-controller architecture to demonstrate the benefits of the proposed technique that is very flexible to other architectures. Again, after the nominal model of the plant is determined, the model controller used in this example is components $G_{c1}(s)$ and $G_{c2}(s)$ derived from [15]. Then, the transfer function of the plant in the considered worst case is here evaluated as

$$G_p(s) = \frac{9.7}{(5s^2 + s)} \tag{21}$$

The PID control part can then be designed also using root locus method as

$$G_a(s) = 682.995\left(1 + \frac{1}{6829.95s} + 0.053s\right) \tag{22}$$

In Fig. 9 we show Simulink simulations of the closed loop system with transport delay $\tau = 1\ sec.$ There are three step responses with no plant uncertainties shown in Fig. 9(a) one for the distributed architecture without RMFC, one for the split-controller architecture without RMFC, and one for the split-controller architecture with RMFC. Whereas there are three step responses with the worst case of parameter variations shown in Fig. 9(b) one for the distributed architecture without RMFC, one for the split-controller architecture without RMFC, and one for the split-controller architecture with RMFC. The results indicate that significant robustness characteristic improvement with respect to plant uncertainties in the response by proposed technique is accomplished.

V. Conclusions

In this paper, the robust stability of the systems in the presence of plant uncertainties controlled over the network is further considered using the RMFC scheme. The RMFC methodology is derived. Then, the robust loop controller based on the proposed technique is designed in order to add to the local location of differently distributed architectures. It is revealed that the proposed method is very effective in the robustness characteristic enhancement of the distributed control systems in the presence of plant uncertainties.

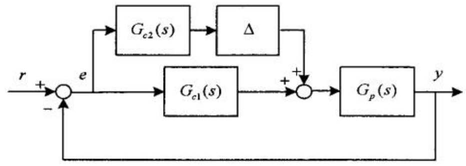

Fig. 8 A split-controller architecture.

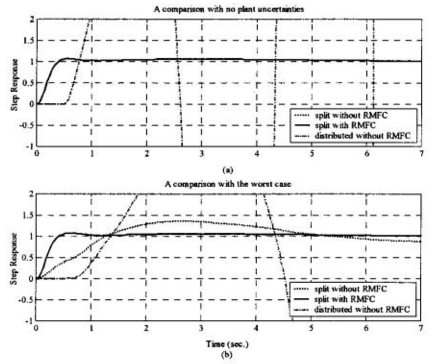

Fig. 9 A comparison (a) with no plant uncertainties (b) with the worst case.

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กลับไปสารบัญหมวด

Robust Controller Synthesis for Robustness Property Enhancement

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Abstract

The control system design problem is the ability to synthesize controller that achieve robust stability and robust performance. This paper explains the Finite Inclusions Theorem (FIT) by the procedure namely FIT synthesis developed for synthesizing robustly stabilizing controller for systems with parametric uncertainty. The basic problem is to determine that all the polynomials in a given family is Hurwitz. By FIT, it can prove this problem from only approximate knowledge of the location of a finite number of polynomial value sets at appropriately chosen frequencies.

The procedure can be done in two steps. First, an initial robustly stabilizing controller constructed by any techniques with respect to an initial region of uncertainty. Second, the FIT is applied to enlarge the uncertainty region for which robust stability can be achieved. In the process of expanding the region of uncertainty, the controller is redesigned. Results from example show that the controller synthesized by FIT synthesis is better than by H_∞ synthesis with parametrically uncertain system as well as satisfied the objectives for considerably larger range of uncertainties.

Keywords: robust controller, systems with parametric uncertainty, robustness property enhancement.

1. Introduction

Modeling uncertainties affect the system in many different ways: degradation of the system performance and destabilization are the two most important adverse effects. In this paper we deal with the linear time-invariant single-input single-output problem of controller synthesis for guaranteed robust stability and performance when the system includes parameter uncertainty [1]. One method of incorporating uncertainty in system models is by using a parametric description [1,2]. Recent development in the analysis of systems with parametric uncertainty has been implied by Kharitonov's theorem [3].

Let $\phi(s,a)$ be the closed loop characteristic polynomial with coefficients that depend on the vector parameter **a**, which lies in a some set: $\Omega_a = \{\ a \in R^k \,|\, a_i^- \le a_i \le a_i^+ ,\ 1 \le i \le k\}$, $a_i^- < 0$, $a_i^+ > 0$, $1 \le i \le k$, of a unity feedback configuration. A recent result, the Finite Inclusions Theorem (FIT) [2], shows that robust stability of a polynomial family can be determined from the location of a finite number of the value set $\phi(j\omega,\Omega_a)$ at appropriately chosen frequencies.

We will first state the FIT and present an iterative synthesis algorithm based on FIT. Controller parameters improved at each iteration are computed by solving linear inequalities [6]. First of all there is a trade off between conservativeness of the solution and the number of the inequalities to be solved by a computer that can be controlled by the designer. The controller order may be fixed ahead of time, so we can avoid high order controllers usually generated by H_∞. An example will be used to demonstrate the procedure.

2. Formulation

Theorem 1 *(Robust stability) Let* $p(s,a) = \sum_{j=0}^{n} p_i(a)s^j$, $a \in \Omega_a$, $n \ge 0$ *and* $p_i : \Omega_a \to \mathbb{C}$. *Further, Let* $\Gamma \subset \mathbb{C}$ *be a closed Jordan curve such that int* Γ *is convex. Then for all* $a \in \Omega_a$, $p(s,a)$ *is of degree n and has all its roots in int* Γ *if there exists* $m \ge 1$ *intervals* $(c_k, d_k) \subset \mathbb{R}$ *and a real* ω_k, $1 \le k \le m$ *such that:*

$$0 \le \omega_1 \le \omega_2 \le \omega_3 \le \ldots \le \omega_m$$

$$\frac{-\pi n}{2} - \pi \le c_1 < d_1 \le -\frac{\pi n}{2} + \pi$$

$$\frac{\pi n}{2} - \pi \le c_m < d_m \le \frac{\pi n}{2} + \pi$$

$$\forall\ 1 \le k \le m \ \ \max\{d_k - c_{k+1}, d_{k+1} - c_k\} \le \pi$$

$$\forall\ 1 \le k \le m \ \ p(j\omega_k, \Omega_a) \in \{re^{j\theta} \,|\, r > 0, c_k < \theta < d_k\}$$

Let $C(s) = \dfrac{n_c(s)}{d_c(s)}$ is a proper controller. The sensitivity transfer function is given by $S(s,a) = (1 + P(s,a)\ C(s))^{-1}$ where $\phi(s,a) = d_c(s)d_p(s,a) + n_c(s)n_p(s,a)$ is a closed loop characteristic polynomial. For robust asymptotic tracking one has design specification: Find a controller $C(s)$ that robustly stabilizes the loop and guarantees that $|S(j\omega,a)| < |W_1(j\omega)|^{-1}$, $\forall \omega \in [0,\infty)$ and $a \in \Omega_a$. $W_1(s) = \dfrac{n_w(s)}{d_w(s)}$ is a given strictly proper, stable, minimum phase, transfer function which weights the sensitivity transfer function. This requirement can be equivalently stated as

$$|W_1(j\omega)S(j\omega,a)| < 1 \quad \forall \omega \in [0,\infty),\ a \in \Omega_a$$

After several steps [2] one can show that a necessary and sufficient condition for robust asymptotic tracking is the stability of the polynomial family

$$\psi(s,a,r,\alpha) = re^{j\alpha} n_w(s)d_c(s)d_p(s,a) + d_w(s)\phi(s,a)$$

for all $\alpha \in [0,2\pi)$, $r \in [0,1]$ and $a \in \Omega_a$. Let $\Omega_q = \{q = (a,r,\alpha) \mid a \in \Omega_a, r \in [0,1], \alpha \in [0,2\pi)\}$.
Stability of $\psi(s,a,r,\alpha)$ is necessary as well. In view of Theorem 1 and the above discussion we have

Theorem 2 *(Robust performance) A degree q proper controller $C(s)$ with monic denominator robustly stabilizes the family of degree $\bar{n}$ plants $P(s,a) = \frac{n_p(s,a)}{d_p(s,a)}$ and makes $|W_1(j\omega)S(j\omega,a)| < 1$, $\forall \omega \in [0,\infty)$ and $a \in \Omega_a$ if for some $m \geq 1$ there exist real ω_k, c_k, d_k, $c_k < d_k$, $1 \leq k \leq m$ such that:*

$$0 \leq \omega_1 \leq \omega_2 \leq \omega_3 \leq \ldots \leq \omega_m$$

$$\frac{-\pi n}{2} - \pi \leq c_1 < d_1 \leq -\frac{\pi n}{2} + \pi$$

$$\frac{\pi n}{2} - \pi \leq c_m < d_m \leq \frac{\pi n}{2} + \pi$$

$$\forall\ 1 \leq k \leq m \quad \max\{d_k - c_{k+1}, d_{k+1} - c_k\} \leq \pi$$

$$\forall\ 1 \leq k \leq m \quad \psi(j\omega_k, \Omega_q) \in \{re^{j\theta} \mid r > 0, c_k < \theta < d_k\}$$

3. An algorithm for robust performance synthesis

From theorem 2, let the order q controller be parameterized as

$$C(s) = \frac{x_{2q+1}s^q + x_{2q}s^{q-1} + \ldots + x_{q+1}}{s^q + x_q s^{q-1} + x_{q-1}s^{q-2} + \ldots + x_1} = \frac{n_c(s)}{d_c(s)}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_{2q+1}) \in \mathbb{R}^d$ and $d = 2q+1$. Robust performance will be achieved, if the polynomial family

$$\psi(s,a,b) = bn_w(s)d_c(s)n_p(s,a) + d_w(s)\phi(s,a)$$

is simultaneously stabilized for $a \in \Omega_a$ and **b** in the unit disc. Since **b** takes values in the unit disc the value sets for $\phi(s,a)$ will not be polygonal. A FIT based algorithm can be computationally enhanced, if these value sets are polygons at each frequency. This can be easily achieved, if some polygonal overbound is employed for the unit disc. This can be done arbitrarily closely by using higher order polygonal overbounds. Here we will use the simplest one (see Fig. 1), the unit square Ω_{bs} where the complex number $\mathbf{b} = b_1 + jb_2$

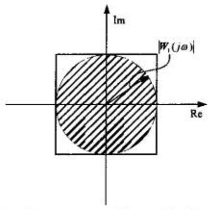


Fig. 1. Square overbound of a disc.

Another (better) type of overbounding is an octagon overbounding.

3.1 Algorithm

STEP 1 Let $\mathbf{x}^{(1)} \in \mathbb{R}^d$ and $\Omega_{ab}^{(1)} \subset \Omega_{ab}$ be until $\phi(s, \Omega_{ab}^{(1)}, x^{(1)})$ is stable and set j:= 1.

STEP 2 Determine $m^{(j)} \geq 1$ sectors $S_k^{(j)}$, $1 \leq k \leq m^{(j)}$ and frequencies $\omega_k^{(j)}$ along the $j\omega$ axis until $\phi(\omega_k^{(j)}, Ext\Omega_{ab}^{(j)}, x^{(j)}) \subset S_k^{(j)}$. By FIT, $\phi(s, \Omega_{ab}^{(j)}, x^{(j)})$ is stable. Each $\omega_k^{(j)}$ should roughly center (angularly) the set $\phi(\omega_k^{(j)}, Ext\Omega_{ab}^{(j)}, x^{(j)})$ in $S_k^{(j)}$.

STEP 3 Choose a slightly larger set $\Omega_{ab}^{(j+1)} \supset \Omega_a^{(j)}$. First this should affect the **b** parameters. When (if) $\Omega_b^{(j)} \supset \Omega_{bs}$, the enlargement in the **b**-direction terminates and the enlargement in the **a**-direction commences.

STEP 4 Compute a new vector of controller parameters $x^{(j+1)}$ until $\phi(j\omega_k^{(j)}, Ext\Omega_{ab}^{(j+1)}, x^{(j+1)}) \subset S_k^{(j)}$ for all k. Note, this is equivalent to solving a system of linear inequalities in $x^{(j+1)}$. If no solutions exist to this system of inequalities, return to step 3 and choose a smaller $\Omega_{ab}^{(j+1)}$.

STEP 5 Let $j := j+1$, and if $\Omega_{ab}^{(j)} \supset \Omega_{ab}$, stop; otherwise, go to Step 2.

4. Example

Consider the parametrically uncertain system. We now want to find a controller which also satisfies a robust performance objective. The following system is nominal plant

$$P_0(s) = \frac{s-1}{(s-0.1)(s+5)}$$

with multiplicative uncertainty

$$\Delta_m(s,a) = \frac{a_1(0.5s^3 + 2.22s^2 + 40s + 1.5) + a_2(0.5s^3 + 48.31s^2 + 10s - 3.5)}{(s+1)(s+5)(s+20)}$$

where $P(s,a) = P_0(s)(1+\Delta_m(s,a))$, $\mathbf{a} = (a_1, a_2)$ lies is the rectangle $\Omega_a = \{(a_1,a_2) \mid -\alpha < a_1 < \alpha,\ -\alpha < a_2 < \alpha\}$. The positive parameter α regulates the size of the rectangle. The performance objectives are robust stability and robust asymptotic tracking where the bound on the sensitivity transfer function is generated by

$$W_1^{-1}(s) = 0.25(20s+1)^2$$

We would like to achieve these objectives for as large a value of α as possible. As mentioned in the introduction we would like to compare FIT synthesis with other methodologies and in this paper we choose H_∞. The following steps show the procedure of H_∞ synthesis.

Consider the transfer function of a family plant with multiplicative uncertainty $P(s) = P_0(s)(1+\Delta W_2(s))$ where $W_2(s)$ is some given minimum phase stable transfer function which weights the complementary sensitivity transfer function $T(s)$ and Δ is any arbitrary stable and proper transfer function with $|\Delta(j\omega)| \le 1$, for all $\omega \in [0,\infty)$. A necessary and sufficient condition for robust stability is $|W_2(j\omega)T(j\omega)| < 1$, for all $\omega \in [0,\infty)$. This elegant characterization of robust stability allows for controller synthesis as a solution to a Nevanlina-Pick method [4,5].

A bound $W_2(j\omega)$ for the magnitude of $\Delta_m(j\omega,a_1,a_2)$ can be generated from

$$W_2(s) = \frac{5(10s+1)(s+1)(0.02s+1)}{(s+1)(s+5)(s+20)}$$

A necessary and sufficient condition for robust stability and performance for the loop with norm bounded uncertainty (assuming nominal stability) is

$$\left\| |W_1(j\omega)S(j\omega)| + |W_2(j\omega)T(j\omega)| \right\| < 1 \quad \forall\ \omega \in [0,\infty)$$

In our case we want to find the largest α for which robust stability and robust performance can be achieved. This was accomplished by multiplying $W_2(s)$ by α, solving the corresponding H_∞ problem. The controller for the largest α was found to be

$$n_{c\infty}(s) = -9.10966263142966 4e+8s^5 - 2.74640568945091 5e+10s^4 - 2.09025376928553 8e+11s^3 - 4.86053433552497 7e+11s^2 - 6.90111330308956 0e+10s - 3.17257475311388 4e+9$$

$$d_{c\infty}(s) = s^6 + 3.13023974286752 8e+6s^5 + 5.10295505970227 8e+9s^4 + 2.29679010346894 3e+11s^3 + 3.66722901801894 5e+11s^2 + 3.49522489822975 3e+10s + 8.59483178412574 9e+8$$

for the parameter set:

$$\Omega_a = \{a \in R^2 \mid a_1 \in [-3.1289, 3.1289],\ a_2 \in [-3.1289, 3.1289]\}$$

Figure 2 and 3 show the plots of the magnitude $W_1(j\omega)$ against $S(j\omega)$ and the magnitude of $W_2(j\omega)$ against $T(j\omega)$ respectively.

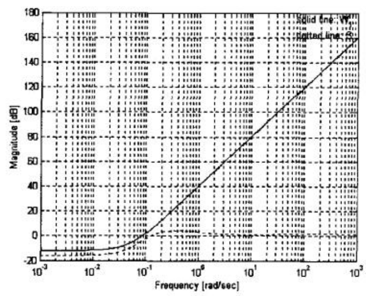


Fig. 2. the plot of the magnitude of $W_1(j\omega)$ against $S(j\omega)$.

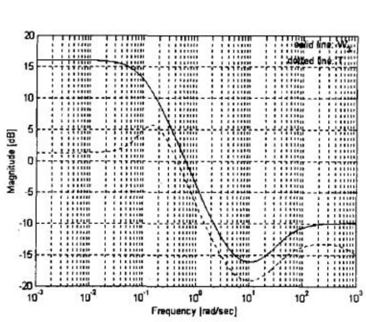


Fig. 3. the plot of the magnitude of $W_2(j\omega)$ against $T(j\omega)$.

The following steps show the procedure of FIT synthesis. For this example the polynomial $\psi(s,a,b)$ that needs to be robustly stabilized is given by

$$\psi(s,a,b) = d_c(s)d_p(s,a,b) + n_c(s)n_p(s,a,b)$$

where

$$n_p(s,a,b) = (2000a_1 + 2000a_2 + 4000)s^6 + (7080a_1 - 195040a_2 + 100400)s^5 + (151813a_1 + 213721a_2 + 406010)s^4 - (138870.8a_1 + 31164.1a_2 + 60150)s^3 - (210222a_1 - 9183.1a_2 + 409010)s^2 - (985a_1 - 1265a_2 + 40250)s - 15a_1 + 35a_2 - 1000$$

$$d_p(s,a,b) = 4000s^7 + 124000s^6 + (40b + 1019970)s^5 + (1236b + 2899069)s^4 + (10076b + 1992319)s^3 + (27980b - 22005)s^2 + (17100b - 15725)s - 2000b - 500$$

The Algorithm requires that an initial stabilizing controller be entered. That is

$$C(s) = \frac{(-0.106719s^3 - 3.201557s^2 - 24.011677s - 53.359282)}{(s^2 + 48.307927s + 62.054492)(\frac{s}{10000} + 1)}$$

4.1 Square overbounding – third order controller

The resulting controller is

$$n_{cFIT}(s) = 4.86275020e+4s^3 - 1.76974310e+5s^2 - 3.24824016e+5s - 1.95099408e+6$$

$$d_{cFIT}(s) = s^3 + 1.00023156\,e+4s^2 + 4.83127819\,e+5s + 5.26571259\,e+5$$

for the parameter set:

$\Omega_a = \{a \in R^2 | a_1 \in [-0.4837, 0.4837]\ ,\ a_2 \in [-0.4837, 0.4837]\}$.

Figure 4 shows the most critical value sets for this case.

4.2 Square overbounding - high order controller

In this case we used a proper sixth order initial controller matching the structure of the one obtained from H_∞. The resulting controller is

$$n_{cFIT}(s) = 2.8190150e+7s^5 + 1.6039105e+7s^4 - 3.5156129e+7s^3 - 8.6797967e+7s^2 - 1.3577211e+7s - 6.2714599e+5$$

$$d_{cFIT}(s) = s^6 + 1.0025699e+6s^5 + 5.1835484e+7s^4 + 9.5681291e+7s^3 + 6.9692153e+7s^2 + 6.2775291e+6s + 1.5128244e+5$$

for the parameter set:

$\Omega_a = \{a \in R^2 | a_1 \in [-1.9649, 1.9649]\ ,\ a_2 \in [-1.9649, 1.9649]\}$.

Figure 5 shows the most critical value sets for this case.

4.3 Octagon overbounding – high order controller (sector size $7\pi/8$)

In this case we increase the sector size to $7\pi/8$ and employ octagon overbounding. The resulting controller is

$$n_{cFIT}(s) = -1.08510509e+9s^5 - 4.00821736e+10s^4 - 1.34430894e+11s^3 - 5.5969669e+11s^2 - 8.27020644e+10s - 3.36460074e+9$$

$$d_{cFIT}(s) = s^6 + 3.13058617e+6s^5 + 4.74938641e+9s^4 + 2.74266126e+11s^3 + 4.37415000e+11s^2 + 4.16333429e+10s + 1.02302322e+9$$

for the parameter set:

$\Omega_a = \{a \in R^2 | a_1 \in [-3.7992, 3.7992], a_2 \in [-3.7992, 3.7992]\}$.

Figure 6 shows the most critical value sets for this case.

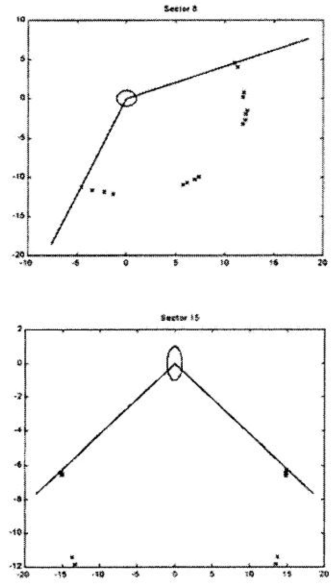


Fig. 4. Value sets in sectors 8,15 for positive frequencies.

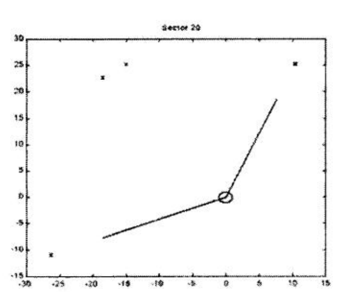
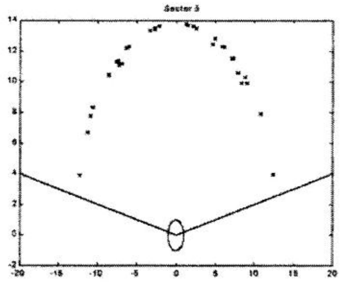


Fig. 5. Value sets in sectors 10,20 for positive frequencies.



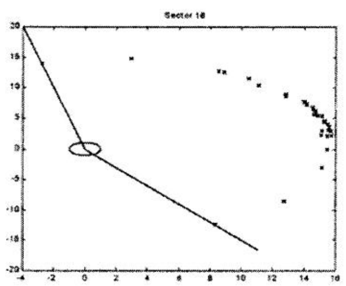


Fig. 6. Value sets in sectors 5,18 for positive frequencies.

5. Conclusion

The synthesized robust controller based on the Finite Inclusions Theorem has been proposed in this paper. In example this method provides good results when compared with H_∞ synthesis. The controller can also be synthesized to satisfy the desired specifications.

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