EXPERIMENTAL INVESTIGATION OF ARBITRARY-ORIENTATION CONE-BEAM X-RAY TOMOGRAPHY

T. Chanwimalueang , M. Sangworasil and C. Pintavirooj

Department of Electronics, Faculty of Engineering,
Research Center for Communications Technology (ReCCIT)
King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand
Email: Kpchucha@kmitl.ac.th

Abstract— **X-ray Computed Tomography is a technique to reconstruct an image of trans-axial slab of the object from a series of x-ray radiographs taken at a prior-known angle. Sequences of x-ray radiographs are served as two-dimensional projection data for a 3D tomography. The most popular Feldkamp Algorithm which is based on Filtered Backprojection (FBP) approaches has shown to perform well for 3D reconstruction. In the case of limited view, however, Feldkamp Algorithm suffers from star artifact. In these scenarios, an algebraic reconstruction technique such as the Simultaneous Algebraic Reconstruction Technique (SART) is engaged for reconstructing tomograms. Conventional x-ray computed tomography was implemented on a c-arm x-ray apparatus where the x-ray source and detector is capable of rotating to capture radiograph at any specific angle. The implementation of conebeam -geometry reconstruction algorithm, however, requires that the center location of the detector is accurately identified. Any slightly-missed alignment of the x-ray source or the detector could result in the error of the position of the center and hence the error in reconstructed image. Consequently, x-ray radiography tomography is normally implemented on a c-arm x-ray apparatus where the correct orientation of x-ray tube with respect to x-ray detector is achievable. The aim of this paper is to implement the x-ray tomography on a non c-arm x-ray apparatus where the x-ray source can be in any orientation with respect to x-ray detector. To determine the orientation, we take the radiograph of the reference transparent object, say the plastic box, of which the coordinate of the landmark, say the corner point, is known. The shadowgram of the box is analyzed to extract the coordinate of landmark image and to determine the orientation matrix using classical direct linear transform method (DLT). Once the orientation is known, modified conebeam tomography is performed to derived 3D reconstruction volumetric data. The experimental results demonstrated the potential of such method.**

Keywords— **Image Reconstruction, Backprojection, SART, Volume Rendering, and Radiograph**

I. Introduction

Three-dimensional (3D) visualization, including surface rendering and volume rendering, has been studied extensively for medical application for the past several years and applied to such various medical applications as volume measurement, surgical planning, automated image-guided surgery, and telepresence surgery. The 3D visualizations are well established as clinical tool for CT imaging [1-2]. Visualization techniques for MRI and PET have been explored in [3-4]. Recently, special techniques have been purposed for multi-modal image which is the combination of PET and MR data [5-6]. 3D-image reconstruction for 3D ultrasonic data has been investigated in [7-8] to visualize the left ventricle of heart and the mitral valve.

3D visualization from X-ray radiographs has drawn a lot of attention from many researchers [9-10]. The ultimate goal is to derive the stack of cross section from image reconstruction algorithm followed by rendering the stack of cross-sectional images, so-called volumetric data, to provide the 3D model. The scheme to inverse the projections to a cross section can be categorized into 2 classes, the transformation method [11] such as the FBP and the algebraic formulation [12] such as the ART. It is proved by [13] that, for the same limited number of projections, the algebraic formulation give the better result of cross section than that of the transformation method. It has been shown that the algebraic reconstruction is suitable for the X-ray radiography which has limited projections caused by the X-ray overdose problem [10].

Image reconstruction from projection used in [9], and [10] is based on the assumption that the beam geometry is parallel. In practice, however, the assumption is acceptable only in the case where the distance between x-ray tube and film (or detector) is relatively high. If this is not the case i.e. for (C-ARM x-ray Apparatus), cone-beam geometry must be applied. The implementation of cone-beam -geometry reconstruction algorithm, however, requires that the center location of the detector is accurately identified. Any slightly-missed alignment of the x-ray tube or the detector could result in the error of the position of the center and hence the error in reconstructed image. Consequently, x-ray radiography tomography is normally implemented on a c-arm x-ray apparatus where the correct orientation of x-ray tube with respect to x-ray detector is achievable. The aim of this paper is to implement the x-ray tomography on a non c-arm x-ray apparatus where the x-ray tube can be in any orientation with respect to x-ray detector.

II. FELDKAMP CONE-BEAM TOMOGRAPHY

The goal of image reconstruction is to obtain an image f(x,y) of a cross section of the object from these projections. The algorithm can also be classified based on the geometry of the beam into parallel-beam, fan-beam and cone-beam tomography. While the projection data for the parallel-beam and fan-beam tomography is a 1D vector, the projection data for the cone-beam tomography is a 2D array. The well-known algorithm for cone-beam tomography is called Feldkamp cone-beam tomography [11]. The Feldkamp algorithm based on a 3D filtered backprojeciton. The generalized cone beam image reconstruction is expressed as follows:

$$g(x,y,z) = \frac{1}{2}\int_0^{2\pi} \frac{D^2}{(D-s)^2} \int_{-\infty}^{\infty} R(p,\zeta,\beta) h\left(\frac{Dt}{D-s} - p\right) \frac{D}{\sqrt{D^2 + p^2 + \zeta^2}} \, dp d\beta \quad (1)$$

where $g(x, y, z)$ is a reconstructed volumetric data, D is the distance between the source and the Z-axis, β is the source rotation angle relative to the z axis, $R(p,\zeta,\beta)$ is the cone-beam projection data, $t = x\cos\beta - y\sin\beta$, $s = -x\sin\beta - y\sin\beta$ and (p,ζ) is the coordinate system of the detector plane. The generalized cone-beam reconstruction can be divided into the following steps:

1. Obtain the weighted projection data

$$R'(p,\zeta,\beta) = \frac{D}{\sqrt{D^2 + p^2 + \zeta^2}} R(p,\zeta,\beta)$$

2. Filter the weighted data

$$Q(p,\ \zeta,\ \beta) = R'(p,\ \zeta,\ \beta) * h(p)$$

3. Weight and backproject the filtered data

$$g(x,y,z) = \frac{1}{2}\int_0^{2\pi} \frac{D^2}{(D-s)^2} Q(p,\zeta,\beta) d\beta$$

III. X-RAY TUBE ORIENTATION DETERMINATION

A digital x-ray radiograph undergoes a linear transformation from the 3D projective space to the 2D projective space which can be described by the equation:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = K[I_3 \mid O_3] \begin{bmatrix} R & -T \\ O_3^T & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (2)$$

or $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ where $M = [KR \mid -KRT]$

A scene point [x y z 1] ^{T} is expressed in a world Euclidean co-ordinate system (X, Y, Z) (see figure 2) and [u v w] T is the image of a scene point in pixel unit in x-ray tube Euclidean co-ordinate axes (u, v, w).

Matrix K in equation (2) is the x-ray tube calibration matrix

$$K = \begin{bmatrix} fa & 0 & u_0 & 0 \\ 0 & fb & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

Where f is the focal length of the projection, a and b is the conversion factor from physical unit to pixel unit and (u0, v0) is the principal point of the projection.

Matrix R and T contributes to the extrinsic parameter of the x-ray tube. Rotation matrix R expresses three elementary rotation of x-ray tube Euclidean co-ordinate axes (u, v, w) with respect to the world Euclidean Coordinate system (X,Y,Z)- rotation along x, y and z are termed pan, tilt and roll respectively. Translation vector t gives three elements of the translation of the origin of the world co-ordinate system with respect to x-ray tube co-ordinate system. For the general cone-beam tomography, the world coordinate system is assigned as the coordinate system of the object and assumed to be aligned in the same orientation of the x-ray tube coordinate system except translated by some translation. To obtain M, observe each known point X=[x y z 1]T and its corresponding 2D image point [u v w]T yield an equation

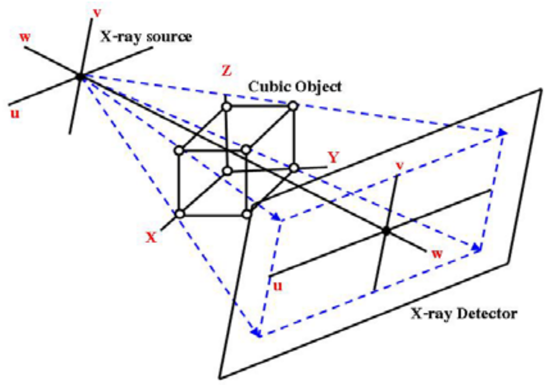

Fig. 1 X-ray tube coordinate system (u, v, w) and World coordinate system (X, Y, Z)

If n such points are available, A will be of size 2n x 12. To solve for M, perform Singular Value Decomposition (SVD) of A to derive $A=UDV^T$. The last column of V is the solution for M. To separate extrinsic parameter, observe that M can be written as $M=[A\,|\,b]$ then $T=-A^{-1}b$. To determine R, we decompose A into a product of two matrices K and R using QR decomposition

$$\begin{bmatrix} x & y & z & 1 & 0 & 0 & 0 & 0 & -ux & -uy & -uz & -u \\ 0 & 0 & 0 & 0 & x & y & z & 1 & -vx & -vy & -vz & -v \\ & & & & & & \vdots & & & & & \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ \vdots \\ m_{34} \end{bmatrix} = 0$$

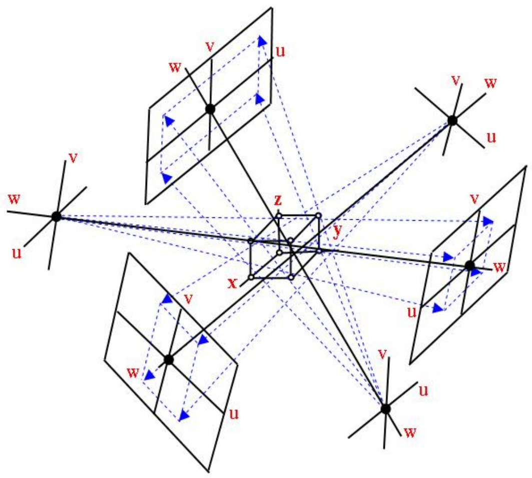

Fig. 2 Calibration setup for three orientation of x-ray tube.

or $AM=0$ (4)

In this paper, a shadowgram of standard box where the coordinate of the corner is known is analyzed to determine the internal and external parameter of the x-ray-tube model.

IV. Experiments and results

The process of Feldkamp cone-beam tomography with x-ray tube in arbitrary orientation is divided into 2 steps; (i) calibration step and (ii) modified reconstruction step.

In the first step, a series of radiograph of reference object with x-ray tube in arbitrary orientation are captured (shown in figure 3). The reference object is a cube. The original of world coordinate is the lower corner of the cube and hence the corner coordinate of cube is easily determined... In each position of the x-ray tube, the 3D coordinates with respect to the world coordinate and their correspond image coordinates with reference to image coordinate system (which passes through the center of image) are used to compute the projection matrix M as described in the previous section. In the next step, the reference cube is then replaced with object to be reconstructed. The series of radiograph of the object are then captured with the same series of x-ray tube position. These radiographs are served as projection data for modified Feldkamp cone-beam tomography. In the modified Feldkamp cone-beam tomography, the 3D reconstruction volume is reoriented using the extracted rotation matrix R such that the Z axis of world co-ordinate system is aligned with the w axis of x-ray tube co-ordinate system (see Figure 2). The normal process of Feldkamp is then performed on the reoriented volumetric data to derive a stack of cross-section images.

Fig. 3 Experiment setup for three orientation of pig bone with rotating platform.

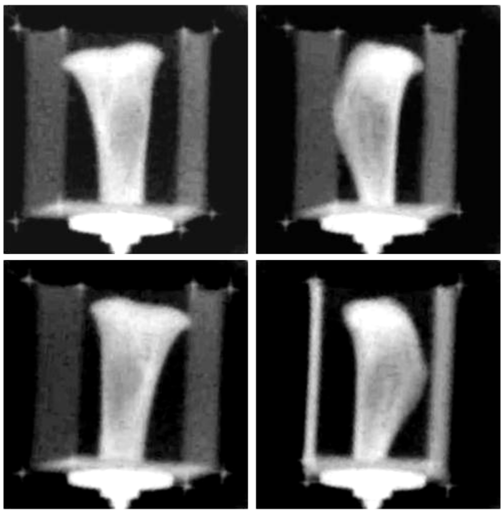

Fig. 4 Sample of projection data for arbitrary x-ray tube orientation.

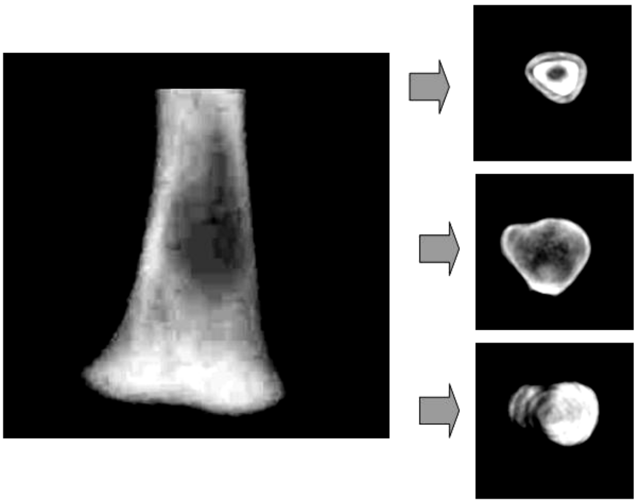

Fig. 5 Sample of reconstruction image

Figure 4 shows a sample of 2D projection data taken with x-ray tube in arbitrary orientation. Observe that the projection of ellipsoidal shape projection data is distorted to some certain degree depending upon the tilt angle of the x-ray tube with respect to axes of world coordinate system. The samples of reconstructed image in some slices are shown in figure 5. Evidently, reconstruction result is very satisfactory.

V. Conclusions

The concept of x-ray computed tomography with x-ray tube in arbitrary orientation is presented in this paper. The purposed methods exploit the idea of camera modeling to extract the orientation of the 3D reconstructed volumetric data with respect to the x-ray tube coordinate system. The Feldkamp cone-beam is then implemented on the reoriented volumetric data to derive a stack of cross-section image. The result and a potential can be use for a real application.

References

1. D.C. Hemmy, et. al., "Three-dimensional Reconstruction of Craniofacial Deformation using Computed Tomographic," Neurosurgery, vol. 13, pp. 534-541, 1983
2. W.G. Totty, and N.W. Vannier, "Analysis of Complex Masculoskeleton Anatomy using Three-dimensional Surface Reconstruction," Radiology, vol. 150, pp. 173-177, 1984.
3. L. Axel, et. al., "Three-dimensional Display of NMR Cardiovascular Images," Comput. Assist. Tomography, vol.7, pp. 172-174, 1983.
4. N.W. Vannier, et. al., "Three-dimensional Magnetic Resonance Imaging of Congenital Heart Diseases," Radiographics, vol. 8, no. 5, pp. 857-871, 1988.
5. D.N. Levin, et. al., "Integrated Three-dimensional Display of MR and PET Images of the Brain," Radiology, vol. 172, pp. 783-789, 1989.
6. N.J. Valentino, "Volume Rendering of Multimodal Image: Application of MRI and PET image of the Human Brain," *IEEE Trans. Medical Imaging*, vol. 10, no. 4, pp. 554-561, 1991.
7. R. Pini, et. al., "Echocardiographic Three-dimensional Visualization of the Heart," *NATO ASI Series*, vol. F60, 3D imaging in Medicine, Hohne, K. H. et. al., Eds., Berlin: Springer-Verlag, 1990.
8. M. Verlande, et. al., "3D Reconstruction of the Beating Left Ventricle and Mitral Valve based on Multiplanar tee," in Proc. Computers in Cardiology. New York: IEEE Computer Society Press, 1991.
9. C. Pintavirooj, C. Ninkaew, M. Sangworasil, and K. Hamamoto, "3D Visualization from Radiograph," ISCIT2001, pp. 307-310, Nov. 2001.
10. P. Ungpinitpong, C. Pintavirooj, P. Leartprasert, and M. Sangworasil, "Improved 3D Visualization from X-Ray Radiograph Using Algebraic Reconstruction Technique," ISCIT2002, Oct. 2002.
11. A.C. Kak, "Tomographic Imaging with Diffracting and Non-Diffracting Sources," Array Signal Processing, S. Haykin, Ed. Eaglewood Cliffs, NJ: Prentice-Hall, 1985.
12. S. Kaczmarz, "Angenaherte Auflosung Von Systemen Linearer Gleichungen," Bull. Acad. Pol. Sci. Lett. A, vol. 6-8A, pp. 355-357, 1937.
13. H. Guan, and R. Gordon, "Computed Tomography using Algebraic Reconstruction Techniques (ART) with Different Projection Access Schemes: A Comparison Study Under Practical Situations," Phys. Med. Biol., No.41, pp.1727-1746, 1996.

.