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Research Article

Nondestructive Evaluation of Three-Dimensional Plastic Strains Based on Nonlinear Inverse Analysis Using Displacements

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Abstract:
In order to achieve a rapid supply of electricity from power plants after an earthquake, the integrity of power plants needs to be assessed more quickly and accurately. Currently, hardness tests are used as a method to evaluate the plastic strain of pipes after a relatively large earthquake, but only the outer surface of the pipes can be measured. Furthermore, it cannot be regarded as a non-destructive test, as the hardness test produces indentations. There is a method to estimate the three-dimensional plastic strain, which is the cause of the displacement, from the non-destructively measured surface displacement by inverse problem analysis. The aim of this study is to apply this method to the non-destructive estimation of plastic strains for whole structure using displacements caused by external forces, and to verify its effectiveness by numerical analysis. Here, the relationship between plastic strains and displacements becomes non-linear when relatively large plastic strain is generated. In order to solve such a non-linear problem, an inverse analysis method using iterative calculations was introduced. In order to verify the principle of the method, a simple three-point bending model is used in this numerical analysis and its plastic strains are estimated using the surface displacements. The numerical results show that the plastic strain distribution can be estimated with relatively high accuracy for large deformation where the relation between plastic strains and displacements becomes non-linear.

Keywords: *Plastic strain, Non-destructive, Inverse problem, Displacement*

1. Introduction

In energy plants that may be subject to large-scale earthquakes, the response to an earthquake is required to be implemented according to the magnitude of the seismic motion and the extent of the earthquake's impact. The post-earthquake equipment integrity assessment guidelines of the Japan Nuclear Technology Institute [1] provide the concept and procedures for inspecting and assessing equipment immediately after an earthquake up to plant restart, and detailed inspections are carried out when the magnitude of the earthquake is relatively large. If the inspection reveals significant damage to the plant's main equipment, or if significant damage is found in the power plant facilities when the seismic ground motion level is relatively large and the vibration period is not relatively high, an integrity assessment and countermeasures based on the results are carried out. The integrity assessment consists of two parts: an assessment based on seismic response analysis and an assessment based on an inspection with an extended scope (hereinafter referred to as an extended inspection). First, the seismic response analysis for the observed earthquake is based on a dynamic analysis using the horizontal and vertical seismic records observed during the earthquake. By

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reproducing the dynamic behavior of the equipment, the stress state of parts that cannot be checked by inspection can be confirmed and the point of maximum stress generation, etc., can be evaluated. On the other hand, extended inspections are carried out by engineers with specialist knowledge by expanding the equipment to a larger area than in priority inspections in order to check whether there is damage to the equipment, the extent of the damage and the cause of the damage. As a result, the equipment that is judged to have a relatively small margin of safety is also measured in terms of dimensions and plastic strain. Based on the results, a damage cause analysis [2] is carried out based on the assessed equipment integrity, and repairs are carried out as necessary. Therefore, in order to quickly determine whether or not a power plant can be restarted after an earthquake, there is a need to establish a more accurate and rapid inspection method for plastic strains.

Currently, the hardness testing methods are used for measuring plastic strains [3, 4], but only the outer surface of pipes can be measured. Moreover, these methods are not non-destructive because they cause indentations. Therefore, a relatively simple nondestructive estimation of the three-dimensional plastic strains would improve the reliability of structural integrity assessment and help in the rapid restoration of power plants after earthquakes.

Until now, inverse problem analysis [5] using the eigenstrain method [6-8] estimates the eigenstrains that is the cause of the residual stress. Another method is to estimate the three-dimensional residual stress distribution over the entire structure by obtaining the eigenstrain from the displacements before and after welding using inverse analysis and input the estimated eigenstrains onto a finite element model [9-11]. The eigenstrains estimated here are the in-elastic strains to express the displacements and residual stresses associated with welding. This in-elastic strains are considered to consider the strains in the liquid weld metal. However, plastic strains in compression occurring after cooling is not taken into account [12]. Therefore, the authors proposed a technique to evaluate the creep strain of a turbine from its contour information when creep deformed in a certain direction without a melting process [13, 14]. However, this inverse problem method can only be solved for smaller creep strain, as it assumes that creep strain is linearly proportional to displacement. As the creep strain increases, displacement does not vary linearly with creep strain and the estimation accuracy decreases. The authors have therefore developed a non-linear inverse analysis method for such large deformation problems to evaluate the three-dimensional creep strains from the surface displacements [15]. This method enables nondestructive evaluation of creep strains even when relatively large creep strains occur. In this study, this method is applied to estimate the three-dimensional plastic strains generated by the input of loads from surface displacements. The aim is to evaluate the effectiveness of the method by numerical analysis. The present study aims to verify the basic validity of the method and shows that the plastic strain distribution over the entire structure can be estimated from the surface displacement of a single plastic deformation caused by a three-point bending load.

2. Theory of Three-Dimensional Plastic Strain Estimation Using Displacements

2.1 Formulation in Linear Problems [15]

Assume that plastic strain of all components occurs $\varepsilon_x^*, \varepsilon_y^*, \varepsilon_z^*, \gamma_{xy}^*, \gamma_{yz}^*, \gamma_{zx}^*$ in each element in a finite element model with q elements and l nodes. The problem of determining the plastic strains from the displacements is considered. The plastic strain vector $\{\varepsilon^*\}$ and displacement vector $\{u\}$ can be described as in Eqs. (1) and (2), respectively.

$$\{u\} = \left\{u_{x1}, \cdots, u_{xl,} u_{y1}, \cdots, u_{yl}, u_{z1,} \cdots, u_{zl}\right\}^{\mathrm{T}} \tag{1}$$

$$\{\varepsilon^*\} = \left\{\varepsilon_{x1}^*, \cdots, \varepsilon_{xq}^*, \varepsilon_{y1}^*, \cdots, \varepsilon_{y\mathrm{q}}^*, \varepsilon_{z1}^*, \cdots, \varepsilon_{z\mathrm{q}}^*, \gamma_{xy1}^*, \cdots, \gamma_{xyq}^*, \gamma_{yz1}^*, \cdots, \gamma_{yz\mathrm{q}}^*, \gamma_{zx1}^*, \cdots, \gamma_{zx\mathrm{q}}^*\right\}^{\mathrm{T}} \tag{2}$$

In the case of linear problems, the relationship between plastic strain and displacement is as follows:

$$[R]\{\varepsilon^*\} = \{u\} \tag{3}$$

where elastic response matrix $[R]$ is a relationship between plastic strain and displacement. Therefore, the inverse analysis to estimate the plastic strain from the displacement is as follows:

$$[R]^+\{u\} = \{\varepsilon^*\} \tag{4}$$

where the subscript + denotes the Moor-Penrose matrix of $[R]$ [16]. In the actual measurement, measurement errors $\{u_{\mathrm{err}}\}$ occurs, so the displacements are given by $\{u_{\mathrm{m}}\}$, as describe as Eq. (5):

$$\{u_{\mathrm{m}}\} = [R]\{\varepsilon^*\} + \{u_{\mathrm{err}}\} \tag{5}$$

Therefore, the inverse problem to estimate plastic strains $\{\varepsilon^*_{\mathrm{est}}\}$ can be described as Eq. (6):

$$[R]^+\{u_{\mathrm{m}}\} = \{\varepsilon^*_{\mathrm{est}}\} \tag{6}$$

2.2 Non-Linear Method for Estimate Relatively Large Plastic Strains

In linear problems, the displacement changes constantly with respect to the change in plastic strain, but as the plastic strain increases, the displacement is no longer proportional to plastic strain, as shown by the red line in Fig. 1. This is due to the change in stiffness, and the displacements are generally obtained based on the updated Lagrangian method [17]. In this case, the problem becomes non-linear and conventional inverse analysis methods are less accurate in their estimation. As an example of non-linearity, for example, the relationship between force and displacement applied to a spring also changes if the spring is plastically deformed.

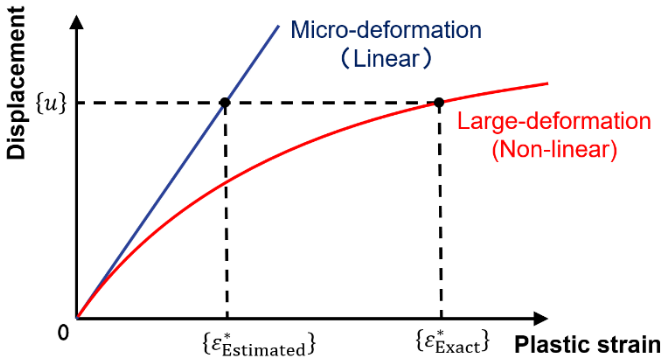

Fig. 1. Differences between linear and non-linear problems.

Therefore, an iterative calculation technique that can evaluate the exact plastic strains have been proposed even in non-linear problem [15]. The method first uses the estimated plastic strain $\{\varepsilon^*_0\}$ to perform a large deformation analysis to obtain the displacements $\{u_0\}$. The plastic strain that reproduces the difference between the measurable displacement $\{u\}$ and the displacement $\{u_0\}$ is obtained by inverse analysis and added to the estimated plastic strain value to update the plastic strain value. The plastic strain $\{\varepsilon^*_1\}$ is used to perform another large deformation analysis to obtain the displacements, and the process is repeated until the measurable displacements are approached [13]. In this study, the number of iterations is n.

3. Numerical simulation

3.1 Evaluation of Estimation Accuracy

To validate the effectiveness of the method, the accuracy of this method was assessed numerically. Specifically, a numerical analysis was carried out by applying a load to the structure, and the resulting plastic strain distribution was obtained as the correct solution. In order to verify the effectiveness of the method in principle, the correct plastic strain was obtained by performing a large deformation analysis of a relatively simple beam in three-point bending. Next, the correct displacement was reproduced by thermally expanding or contracting the element by the amount of the exact plastic strains. The plastic strains were estimated using the surface displacements by inverse problem analysis, and the evaluation accuracy of the method was assessed by comparing with the exact plastic strains.

3.2 Analytical Condition and Finite Element Model

A simplified model used to verify the principle of the method is shown in Fig. 2. The model assumes a carbon steel for piping, with Young's modulus of 200 GPa, Poisson's ratio of 0.3. The dimensions of the model are axial length 1000 mm, thickness 40 mm and width 40 mm. The number of nodes and elements are 99 and 40 respectively. The coordinates *x*, *y* and *z* are in the width, thickness and axial directions respectively. At the nodes at both axial ends (z = 0 mm and z = 1000 mm) at y = -20 mm, the displacements of the *y*-directional component were constrained so that no rotational or rigid body motion occurred. Correct plastic strain was induced in the eight elements shown in blue in Fig. 2 by applying a line load of 70 kN at y = 20 mm and z = 500 mm. It results in the same plastic strain in all four elements shown in Group A in Fig. 2, and similarly in the four elements in Group B. The FEM software ANSYS used in this numerical analysis is not capable of obtaining the relationship between the displacement and the shear component of the plastic strain. Therefore, when obtaining the correct plastic strain distribution by 3-point bending, the yield stress ratio of the shear component was set relatively high so that the plastic strain of the shear component does not occur. However, by using other finite element analysis tools, e.g. Abaqus, the shear component can also be estimated.

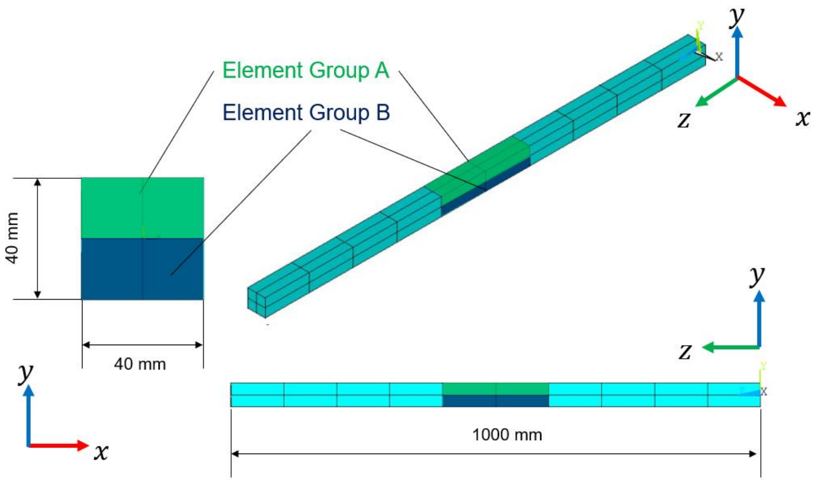

Fig. 2. Simplified FEM model to verify the principle of this method.

In this study, the problem was simplified and the analysis was carried out under the condition that it is known in advance that plastic strain occurs only in the eight elements shown in red in Fig. 2. This means that the number of unknown estimators is 24, as the plastic strains of the three directional components *x*, *y* and *z* are unknown in each of the eight elements. On the other hand, the measurement information used for the inverse analysis is the displacements of the three directional components at x = 20 mm and -20 mm at y = 20 mm, so the total number of measurement information is 66. Non-destructive measurements of displacements are possible using the digital image correlation method [18] and/or laser displacement transducers such as the Keyence WM-3500.

3.3 Application of Conventional Method

First, Table 1 shows the estimation accuracy of 3D plastic strain estimated from surface displacement in the micro-deformation and large-form problems in the absence of measurement errors. The axial component, which is the dominant plastic strain in three-point bending, is shown here as a representative example. The results show that for small deformation problems, the estimation is relatively accurate. Since the cause and effect relationship is linear in micro-deformation problems, the elastic response matrix of the obtained by the linear relationship can be used to correctly estimate the plastic strain. On the other hand, the estimation accuracy decreased in large deformation problem due to changes in the causal relationships, resulting in less accurate estimation. Note that these estimated values in Table 1 are only presented as deterministic values, as no measurement error is assigned.

Table 1: Estimated plastic strains in linear and non-linear problems compared with the exact strains.

Component and element	Exact	Estimated (Micro-deformation)	Estimated (Large-deformation)
ε_x^* in group A	1.687826×10^{-4}	1.687826×10^{-4}	1.722910×10^{-4}
ε_x^* in group B	-1.705638×10^{-4}	-1.705638×10^{-4}	-1.553715×10^{-4}
ε_y^* in group A	1.675003×10^{-4}	1.675003×10^{-4}	1.666321×10^{-4}
ε_y^* in group B	-1.746392×10^{-4}	-1.746392×10^{-4}	-1.777506×10^{-4}
ε_z^* in group A	-3.362829×10^{-4}	-3.362829×10^{-4}	-3.362551×10^{-4}
ε_z^* in group B	3.452030×10^{-4}	3.452030×10^{-4}	3.440200×10^{-4}

3.4 Estimation of Relatively Large Plastic Strains by Iterative Calculation Methods

The plastic strain distribution was evaluated by this iterative calculation in non-linear problem. Comparisons between the correct plastic strain and the estimated values are shown in Tables 2 and 3 and Figs. 3 and 4. The results show the effectiveness of the proposed method. Note that although the results are shown here for ε_z^*, which has the largest value among the plastic strain components, the trend was the same for the other directional components and converged to the correct value.

Table 2: Estimated and exact values in Group A.

Number of iterations n	Estimated plastic strains
0	-3.362551×10^{-4}
1	-3.362826×10^{-4}
2	-3.362829×10^{-4}
3	-3.362829×10^{-4}
4	-3.362829×10^{-4}
5	-3.362829×10^{-4}
6	-3.362829×10^{-4}
7	-3.362829×10^{-4}
8	-3.362829×10^{-4}
Exact	-3.362829×10^{-4}

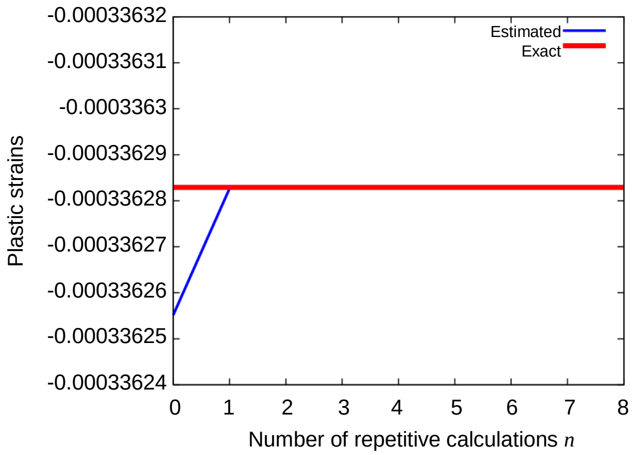

Fig. 3. Estimated and exact strains in Group A.

Table 3: Estimated and exact values in Group B.

Number of iterations n	Estimated plastic strains
0	3.440200×10^{-4}
1	3.451950×10^{-4}
2	3.452030×10^{-4}
3	3.452030×10^{-4}
4	3.452030×10^{-4}
5	3.452030×10^{-4}
6	3.452030×10^{-4}
7	3.452030×10^{-4}
8	3.452030×10^{-4}
Exact	3.452030×10^{-4}

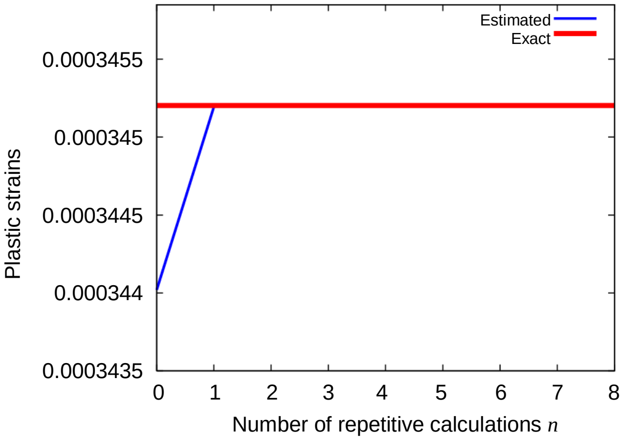

Fig. 4. Estimated and exact strains in Group B.

4. Conclusion

A method for estimating the plastic strains for whole structure by inverse analysis using the surface displacement is proposed for rapid power restoration after a relatively large earthquake. Plastic strains, such as those occurring in power plants, are relatively large, so relationship between plastic strain and displacement become non-linear. In such cases, the plastic strain distribution cannot be correctly estimated by the conventional inverse analysis method based on the assumption of a micro-deformation problem. Therefore, this study attempted to estimate the plastic strains for whole structure using a non-linear inverse analysis method that estimates the creep strains in turbine blade from the surface displacement. To assess the effectiveness of the method, surface displacements in a three-point bending simulation of a simple model were used to estimate based on the plastic strains, and the three-dimensional plastic strains were estimated using the surface displacements and compared with the exact values. The results showed that the estimated values of plastic strain converged to the correct values after repeated calculations using this method, and finally agreed with the correct plastic strains.

Future work is aimed at making it possible to estimate three-dimensional plastic strain even when plastic deformation occurs more than once. In addition, a method that enables relatively accurate estimation even when measurement errors are taken into account will be investigated.

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