Prediction of Stone Column Bearing Capacity Using Artificial Neural Network Model (ANNs)

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ABSTRACT: In the area of ground improvement, the stone columns (SCs) play a definite role. The ground treatment technique has been demonstrated to be effective in improving the embankments' stability and natural slopes by raising the bearing capacity and decreasing settlements. The objectives of this study are to develop models for predicting the performance of SCs-supported embankment foundations utilizing artificial neural networks (ANN). For the aim of creating ANN models, training, testing, and validation sets comprising 70%, 15%, and 15% of the data, respectively steps were done, making use of available numerical results obtained from the 2D finite element analysis. A dataset including about 200 cases is involved, and the mean square error (MSE) with R-squared value is used as performance metrics of the system. The applied data in ANN models are arranged in the component of 4 input parameters, which cover column diameter *d*, centre-to-centre spacing S, the internal friction angle of columns material ϕ, and embankment high H. Relating to these input parameters, the selected responses were the bearing capacity of the SC (BC) and the safety factor against the stability (SF). Based on the simulated results, an ideal 4-14-1 ANN architecture has been settled for the direct prediction. According to the technique used, the forecasted data from the model had a good agreement with the actual datum, where the high regression coefficient (R2) was equal to 0.995 and 0.891 for BC and SF models, respectively. Furthermore, the relative importance of influential variables is examined, which shows that the column diameter is the most effective parameter in the two study models with a significance score of 32.9%. Finally, the outcomes clearly demonstrated that the ANN method is reliable for modelling and optimizing of the SC behaviour.

KEYWORDS: Stone Column, Bearing Capacity, Safety Factor, and Artificial Neural Network (ANN).

1. INTRODUCTION

The scarcity of good quality land for infrastructural development has increased interest in soil improvement methods. The growing lack of suitable land is exacerbated by the high cost of virgin land and the environmental requirement to site infrastructure away from urban areas. As a result, the location of structures on improved or stabilised soft soils has become an economically viable alternative. In general, soft clays are responsible for the excessive settlement observed in constructed embankments (Indraratna and Redana, 2000). Therefore, engineers are required to improve soft soils or clay grounds to meet the technical project requirements and address the shortage of suitable construction land (Moseley and Kirsch, 2004). Over the years, numerous techniques have been proposed for ground improvement including vibro compaction, soil preloading, ground freezing, grouting, and vibro-replacement stone columns (Alfaro et al., 1994; Moseley and Kirsch, 2004; Kirsch and Bell, 2012).

The use of stone columns (SCs) to improve the ground has become more prominent due to its relative economic advantages over conventional piling methods for less sensitive structural settlements (Sivakumar et al., 2010). In practice, the construction of stone columns involves creating a hole in the ground, which is subsequently filled with granular material. The analytical methods approved for SC designs range from experience based semi-empirical design to finite element analyses. Generally, the design of a reinforced ground-based SC is typically performed in two major stages, namely, the ultimate bearing capacity and the lasting drained settlement. For the SCs design, geotechnical engineers are required to rely on either knowledge or analysis techniques such as finite element method (FEM). This typically safeguards the stability against failure and controls the deformation of the subsoil within the permissible limits. So far, efforts have been made by numerous researchers to investigate the treated ground performance through analytical and experimental studies (Greenwood, 1970; Hughes et al., 1976; Priebe, 1976). Furthermore, Ambily and Gandhi (2007) opine that despite the extensive adoption of SC and advances in construction methods, current design methods are still empirical. Hence, current knowledge of SC design in building codes and published materials is still limited. The definite SC bearing capacity is reliant on its geometry, the material, and the native soil properties. Previous studies have provided extensive information on the use of SCs as the most critical ground improvement technique. Furthermore, current studies examine SC behaviour based on several assumptions that simplify the problem under makeshift settings. The main difference between this work and previous studies is the use of simulation or modeling tools to predicate the bearing capability and the safety factor of the soil-columns system.

Bouassida et al. (2009) adopted design graphs to deduce the bearing capacity of a floating SCs collection. In the study, friction between the footing and soil along the SC distribution was ignored. However, Etezad et al. (2015) established a systematic model of a group of SCs subjected to general shear failure in soft soil. The model is robust enough to estimate the critical bearing capability of the reinforced ground. Other available solutions have been obtained by different researchers in recently published studies. For example, Fattah et al. (2017) developed a formula to predict the bearing ability of floating SCs group mounted on clays of undrained shear strengths ranging from 4 kPa to 25 kPa, diverse diameters of column d and length ratios L/d. The formula was acquired by executing a statistical analysis using the SPSS program based on the data from their empirical work and previous studies. Next, the bearing capability of soft clay strengthened with SCs was successfully calculated using the Morgenstern-Price method of slices (Khalifa et al., 2018). The study utilized the slices technique to predict the ultimate bearing capability of the soil supported by a series of SCs based on an analytical model. In 2018, Ng predicted the formula for ultimate bearing capacity that accounts for the undrained shear strength of the surrounding soil and the friction angle of the SC. In summary, the load-bearing capability of a specific SC is a multifaceted issue, including the relations between the constituent column materials and the adjoining soil. So far, an accurate mathematical solution to calculate the ultimate bearing ability is still lacking (Dheerendra et al., 2013).

Recently, in geotechnical engineering, numerous researchers have published successful usage of alternative traditional forecasting models to simulate complex problems (Haque and Steward, 2020; Kharit et al., 2020; Armaghani et al., 2021; and Barkhordari et al., 2022). Similarly, the NNs have become eloquent tools that permit data to be examined to define the practical connections between the consideration parameters. The NNs are computer systems that are skilled in determining the complicated relationships between several variables or data sets. Many researchers have defined the assembly and procedures of ANNs (Ripley, 2007). A typical ANNs contain numerous processing elements (PEs) or nodes, which are typically in a set of layers, namely: the input, output, and various hidden layers (Figure 1).

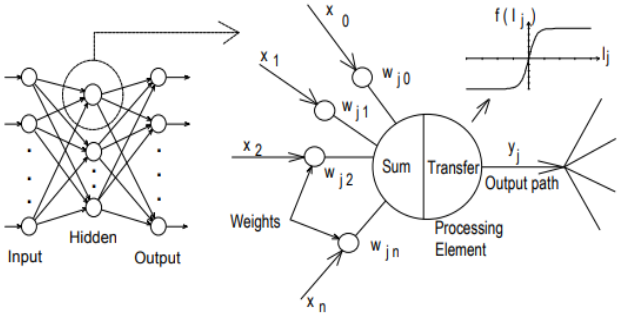


Figure 1 Biological processing elements-neuron (Maier and Dandy 1998)

The ANNs are defined as a class of artificial intelligence (AI) that seeks to imitate human behaviour or neural system (Shahin et al., 2001). In the 1940s, the modern neural network views began and extended to the end of the 1960s with the works of McClloch and Pitts (1943) and Hebb (1949). During the 1980s, research in NNs increased dramatically. Over time, the ANN application has soared into fields such as geotechnical science. Artificial neural networks have successfully adapted to model the behaviour of soils, liquefaction, and earthquakes. Other applications include the characterization of sites, ground-retaining buildings, slope stabilization, subways, and subterranean slots. Lastly, soil bulging, classification, prediction of pile capability, and footing settlement are other uses of ANNs (Al-Ani et al., 2009).

Moreover, ANN has the ability to model the non-linear relationship between a set of input variables and the corresponding outputs without a prior definition of mathematical equations (Maizir et al., 2015). Similarly, The NNs can provide complex input-output mapping, which is easier to use than the classical computational methods. By modelling a system, specialists can predict its future evolution under the influence of different factors. In ANNs, choosing the input variables that have a considerable effect on the outputs is one of the main stages. In current work, ANNs are utilized to forecast the SCs behaviour based on numerical data collected from previous studies. The bearing capacity (BC) and a safety factor (SF) of the system are the targets of the neural network. The efficiency of an ANN can be enhanced by reducing its computational complexity. This is according to the principle that the network's computational complexity is typically influenced by the neurons available in each layer. However, the different between the actual data and predicted data of an ANN model is typically used to measure its performance.

Overall, it is difficult to predict the performance of SCs reinforced grounds comprised of soft clay under embankment load. This is generally attributed to the non-linear relation between the input and output factors. Furthermore, the effects of SC behaviour regarding the ultimate bearing capacity (q_u) are complex and poorly understood at present. Therefore, this study will attempt to enhance the understanding of the behaviour of stabilized SCs based on the ANNs technique.

2. DATABASE USED IN ANN

The ANN is used to forecast the behaviour of SCs supporting embankment highway projects resting on soft clay soil, as shown in Figure 2. The database of about 200 recorded cases from the bearing capacity BC and safety factor SF reported to design SCs was compiled from the reference Gaber 2019. The work was based on 2-D finite element analysis conducted by the Plaxis software to evaluate the performance of the SC under various conditions. 2-D modelling is one of the preferable ways to create the actual situations of this kind of projects. However, the ANN analysis was conducted using Matlab software. A database of a numerical study was selected to create and validate the ANN models.

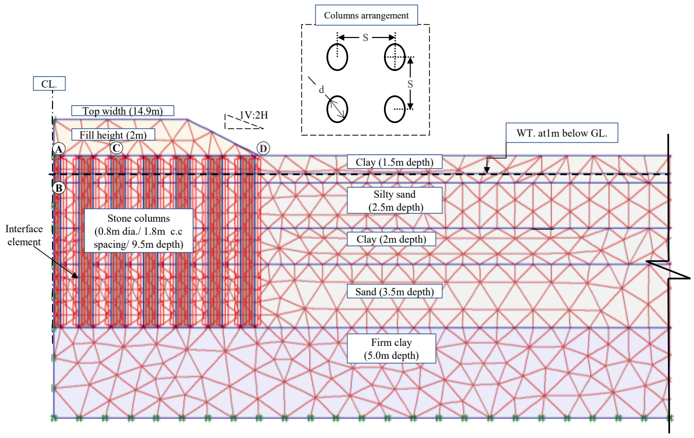


Figure 2 Site cross-section of the case study with geometric characteristics (Gaber 2019)

3. DEVELOPMENT OF ANN MODEL

The procedure for designing ANN models involve the designating the inputs and outputs of the model, along with sorting out the available data. Other requirements include selecting suitable network architecture (NA) as well as enhancing the joining weights. Subsequently, Matlab software was employed to determine the optimal NA using trial and error manner.

3.1 Inputs and Outputs

The parameters selected as inputs for the ANN models are column diameter d, center-to-center interval S, internal friction angle of the SC ϕ, and embankment high H. Next, these four variables were considered to be input model due to their considerable effect on the selected responses: bearing capacity and safety factor. The data selected for this paper is statistically listed in Table 1.

Table 1 Details of model tests program

Parameter	Symbol/Unit	Value Range
Stone column diameter	(d) [m]	0.7-1.2
Spacing between adjacent columns ratio	(S/d) [-]	1.875 - 3.125
Internal friction Angel of SC	ϕ [°]	28 - 45
Height of embankment	H [m]	1.8 - 3.5

3.2 Data Division

The following stage of creating ANN models is splitting the selected data into subsets. Consequently, this data is randomly distributed into triple groups: training, testing, and validation. This technique of division in data is known as "cross-validation technique," which has been applied as the discontinuing criteria in current work. The overall data used for the training set was 70% whereas the twin sets of testing and validation accounted for 30%.

3.3 Network Architecture, Optimization, and Ending Criteria

The hard process in the ANN model creation is to determine the architecture network. The Back-Propagation Neural Networks (BPNN) is considered to be one of the most common ANNs model (Priddy and Keller, 2007). Typically, the BPNN comprises triple inter-connected sets of layers, namely, the input, output, and hidden layers, as indicated in Figure 3. The objective of training through the BPNN is to iteratively alter the neuron weights to ensure error minimization. Hence, the sum of the hidden neurons is essential to the BPNN. Nonetheless, there is no established technique to compute the total number of neurons in the hidden layers. As defined by Nawari et al. (1999), excess hidden layers elongate the training period, however, a limited number of hidden layers traps the learning algorithm in a local minimum. The appropriate number of hidden layer is computed by trial and error, as affected in this paper. Feed-forward networks have been effectively implemented in numerous geotechnical engineering challenges (Shahin et al., 2002). The method is generally adapted to determine the optimum weight arrangement for the feed-forward-based neural network, which is the backpropagation algorithm that was previously highlighted.

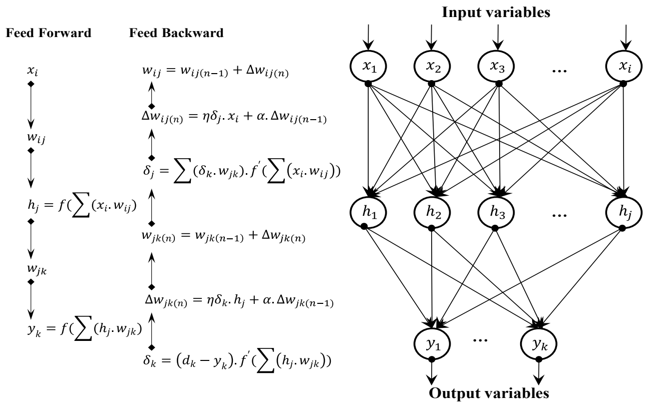


Figure 3 Back-propagation algorithm

However, the decision to terminate the training procedure is accomplished with the ending criteria. The training sets are utilized to regulate the connecting weights, whereas the testing set scales the model's capacity to popularize. Furthermore, the model performance is tested at various steps using this set during the training procedure. The process is terminated when the error of the testing set begins increasing (Shahin et al., 2002).

3.4 Model Validation

Once the model training phase is successfully completed, the performance of the trained model is validated using the validation data that has not been utilized in building the sample. The evaluation of the performance of ANN models can be performed by:

- Determination Coefficient (R^2): It is used to measure the relation between the observed (desired) and predicted (calculated) data:

$$R^2 = \frac{\sum_{k=1}^{n}(d_k - \bar{D})^2}{\sum_{k=1}^{n}(y_k - \bar{D})^2} \tag{1}$$

where, d_k is the desired output value, y_k is the predicted output, $\bar{D}$ is the mean of the desired output and *n* is the amount of data.

- Mean- Square Error (MSE): It is the most widespread index for the error and has the benefit that large slip obtains more consideration than smaller variants.

$$\text{MSE} = \frac{1}{n}\sum_{k=1}^{n}(y_k - d)^2 \tag{2}$$

The determination coefficient (R^2) and the mean square error (MSE) was utilized as the fitness purpose in this study.

4. MODELLING PROCESS

In this study, the training was performed through the backpropagation algorithm since the BPNN yields precise forecasts to any uninterrupted purpose with adequate neurons. The objective of the ANN pattern is to obtain the best training datum that responds to testing. The variety of sizes of the hidden nodes must be carefully carried out to obtain a reliably good model. Subsequently, the ANN model is developed to ascertain the total hidden layers and hidden nodes in each hidden layer. The most favoured option is lower hidden nodes in a network (Alsugair and Al-Qudrah, 1998). The reason is that there is an improved capability for oversimplification along with fewer issues with the fittings. However, the performance of the networks may be impaired if the nodes are not sufficient to capture the underlying behaviour of the data. Therefore, in this paper, the trial and error (t&e) technique was selected to enhance the numeral of the hidden layers and the nodes in the hidden layer. The MSE value is the objective of the analysis, and its significance fluctuates with the correlation and determination coefficients (R and R^2, respectively) of the results tested (Park and Cho, 2010). In principle, the R^2 value defines the input value that contributed to computing the objective output value.

The output data of an ANN could be turned back to the real data amount (un-normalized step). Similarly, the resulting output forecast by the result of the ANN could then be compared with the output targeted from the result of the FE analysis. This is described as the process of testing and validation.

5. RESULTS AND DISCUSSIONS

5.1 ANN Performance

5.1.1 ANN Training and Calibration

The statistical performance of the ANN model was examined for 55%, 60%, 70%, and 80% training set and 45%, 40%, 30%, and 20% testing validation sets, as summarized in Table 2. The purpose of training is to discover a set of connection weights that will cause the lowest MSE in the less probable period (Hagan et al., 1996).

Table 2 Details of various training sets

Training-testing validation sets [%]	Correlation coefficient [R]	Mean Square Error [MSE]
55-45	0.9110	0.00908
60-40	0.9679	0.00419
70-30	0.9983	0.00111
80-20	0.9905	0.00154

The results indicate that the low training data set for the process can be fitted easily, although the model cannot be used to produce reliable results. Moreover, the use of higher training data sets, such as 80%, complicates the process without yielding a good fit. In addition, the optimal training sets for reliable modelling time training was 70% of the available data. The values were observed when the MSE is minimum, and R is maximum. Therefore, the training-testing set (70-30)% was adopted to predict the behaviour of SC using the ANN method.

5.1.2 Learning and Selected Activation Function of ANN Model

The feasible prediction capability for each model was obtained by classifying the data into triple sets, namely, training, testing, and validation. The three networks utilize the network training purpose trainlm, which updates the weight and bias values based on the Levenberg-Marquardt optimization. The trainlm is typically recommended as the quickest, first-choice, back-propagation toolbox supervised algorithm, despite its high memory requirements. The training mechanically rests when the generalization no longer improves, as shown by the higher MSE of the validation models (Matlab, R2018b). In the current study, two ANN-based architectures were established (one model per output parameter) with two transfer functions, namely, tan-sigmoidal and linear transfer functions.

The training data for each network model was comprised of 70% of the learning matrix that includes training samples. The MSE between the output and the target (desired response) was minimized by performing the training, validation and testing processes of multilayer perceptron neural network MLP-NN sample. At the target training epoch, the MSE convergence curve of each training model was determined as displayed in Figure 4.

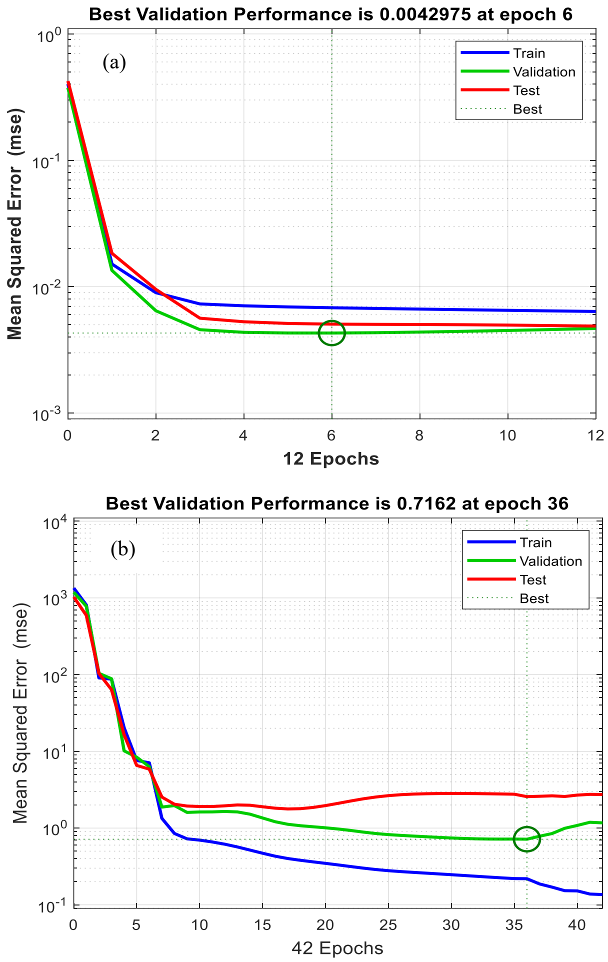


Figure 4 Network convergence during training ANN models (a) SF model, (b) BC model

The results are realistic due to the similar characteristics of the test set error and the validation set error. Likewise, no significant overfitting was observed. The training was stopped where the greatest validation performance occurred after 6 and 36 iterations for SF and BC models, respectively. Also, the figure demonstrates that the network MSE starts at a high value but declines rapidly, indicating the network learning process is active. Overall, the plots show that the networks are learning.

5.1.2 Optimum Neurons Number

The selection of neurons in the hidden layer is a significant characteristic of artificial intelligence techniques. Typically, when an inadequate number of neurons are designated, the network is not incapable of modelling complex data, thereby resulting in a poor fit. However, there is currently no mathematical (official) method to suitably determine the "optimal set" of important factors of a neural network. Consequently, the trial-error method was selected to perform this task in the current study. Therefore, the optimum arrangement and architecture were determined through the Feed-Forward network samples. The growing numbers of neurons in the hidden layer were randomly selected from 1 to 30 neurons. Next, these were investigated to determine the best number of nodes in the hidden stratum based on the lowest observed errors (Lek et al., 1996). Therefore, the (t&e) process for choosing the optimal number of neurons for a definite ANN architecture was performed for each model. Table 3 presents the findings of the MSE and R values obtained for better visualization. The comparisons indicate that the best number of neurons operating in the hidden layer of the neural network is 14. This is because 14 neurons in the hidden layer result in a lower MSE value and the biggest R (regression) value.

In the framework of this study, the MSE and R were adopted to establish that the proposed model can consistently provide high accuracy during the entire period. Therefore, the use of the two indices guarantees consistent levels of error while providing the potential to examine the model for hidden data during the period of testing.

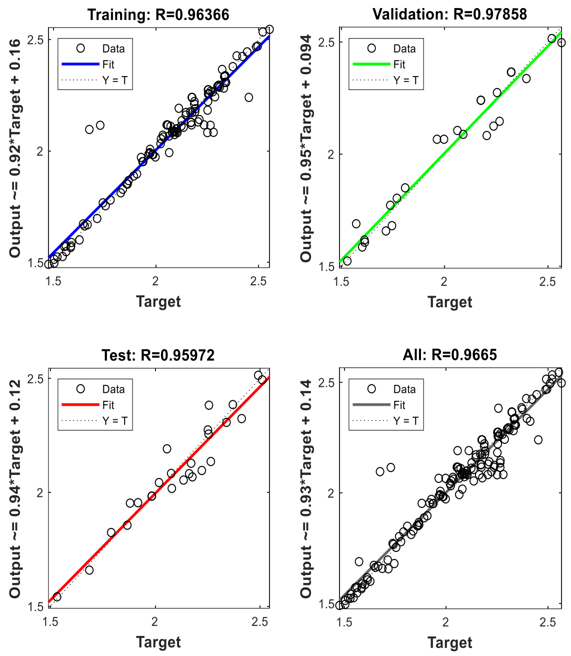
Table 3 NN performance indices versus the number of neurons

MODEL	SF		BC	
Neurons Number	**MSE**	**R**	**MSE**	**R**
1	0.00876	0.92	3.03	0.985
3	0.00377	0.9467	3.317	0.9912
5	0.003	0.9811	1.804	0.9933
7	0.001848	0.983	1.298	0.9961
10	0.00213	0.9866	1.6749	0.9929
13	0.000905	0.9918	1.3616	0.9902
14	**0.000749**	**0.9939**	**1.1245**	**0.9978**
15	0.00237	0.9806	1.2732	0.99656
17	0.00387	0.976	1.9316	0.9933
19	0.00319	0.974	1.569	0.99433
23	0.003962	0.95948	1.2152	0.9968
25	0.004823	0.96318	1.3211	0.9951
27	0.002545	0.9793	1.4649	0.9946
30	0.00569	0.9786	1.3145	0.9948

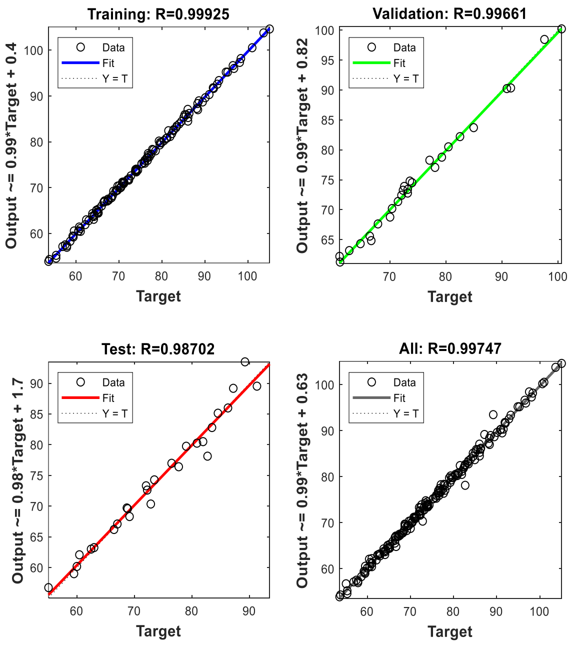
Interestingly, changing the number of hidden network neurons can greatly affect the prediction performance. The outcomes show that the prediction performance increased with a growing number of hidden neurons (up to 14) with a consistent increase in R but decreased MSE for both models. However, a further increase in hidden neurons in the network did not enhance the prediction performance but lowered the process. For instance, the best arrangement of the suggested statistical indices for estimating the predicting model for SF, when the ANN architecture had 14 neurons was MSE = $7.5*10^{-4}$ and R = 0.9939. After numerous network training runs, it was observed the two layers neural network model with 14 neurons in the single hidden layer (4:14:1) yields the best-predicted result with quick convergence based on the Feed Forward backpropagation network.

5.2 Prediction of the Behavior of Stone Column Using ANN Model

The MLP-ANN method was applied to predict SC behaviour by testing the main parameters (outputs). These include the bearing capability of each column (BC) and the safety factor (SF). The outcomes of the training and testing stages of the designated network are presented in Figure 5. The figures display the calculated and predicted parameters for the best network models, which yield the most accurate prediction. In general, the predictive aptitude of the ANN was sufficient for all the entire parameters for the period of training, testing, and validation. The observed correlation coefficient values (R) ranged from 0.959 to 0.997, which indicates high model accuracy. The maximum error percentage displayed as the model performance in Figure 6, represents the variance between the calculated and forecasted data. The plots of ANN indicate that when the zone of the data range is weak in prediction, the error level increases, resulting in a mismatch of the output-input results. For instance, the charts in Figure 6a clearly indicate the mismatch between the calculated and prediction data in specific points around the data of the number 150, which led to show a higher error amount.

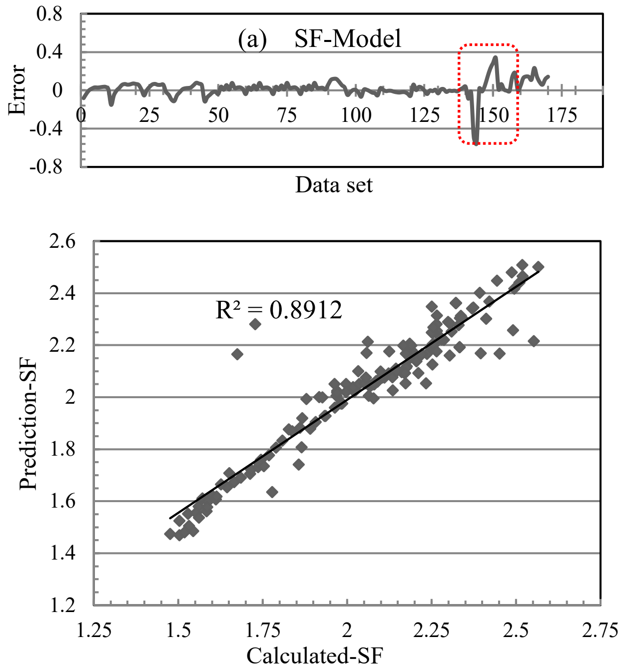


(a)



(b)

Figure 5 ANN model correlation coefficients for the three-phase network with the arrangement (4:14:1); (a) SF model (b) BC model



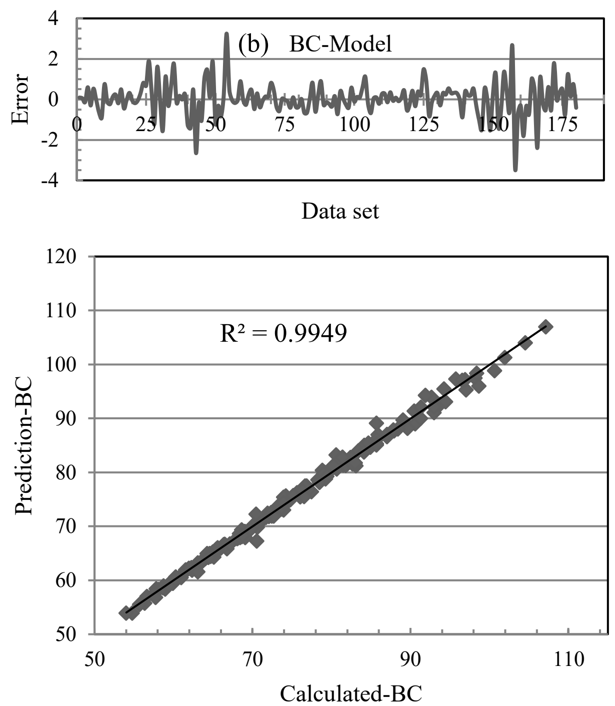


Figure 6 Test results of selected optimal NN models: (a) SF model (b) BC model

5.3 Relation Importance of Input (RI)

Garson (1991) suggested a modest procedure to identify the vital relationship between the input variables through the connection weights of the training network model. The techniques analyse the training network and its connected weights. Therefore, the important inputs required to predict the ANNs output were determined through the connection weight method. Therefore, the linked weight technique adds the hidden output and input-hidden products and connected weights from each input to output neuron for the entire input variables (Olden et al., 2004). The relative importance of input variable RI is given by the relation:

$$RI_i = \frac{\sum_{j=1}^{m} w_{ij} w_{jk}}{\sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} w_{jk}} \times 100\% \tag{3}$$

$$\mathrm{i} = 1,2,3, \ldots\ldots, \mathrm{n} \text{ and } \mathrm{j} = 1,2,3, \ldots\ldots, \mathrm{m} \tag{4}$$

where, RI_i is the relative importance (%) of the variable i in the input layer on the output variable; j is the index number of the hidden node; w_{ij} is the connection weight between input variable i and hidden node j and w_{jk} is the connection weight between hidden node j and the output node k.

Table 4 shows the connection weight values extracted from artificial neurons of the NN based SF model. The tabulated weights are derived from the NN toolbox of Matlab software.

Table 4 Connection weights (*w1*& *w2*) of the SF model

Neurons	**connection weight between the input and hidden layers (*w1*)**				**connection weight between hidden and output layers (*w2*)**
	Input				**Output**
	H [m]	**S [m]**	**ϕ [°]**	**D [m]**	**BC**
1	0.01412	1.71606	1.4073	2.1146	-0.09456
2	-0.7076	1.47332	-4.474	0.1621	0.301981
3	-0.8334	-1.04308	1.6045	-0.286	0.075503
4	-0.0714	2.21330	-1.278	-0.363	0.478747
5	0.76577	-0.05212	0.4518	-0.288	0.7599
6	-0.0554	-5.97209	-0.500	-4.735	0.157276
7	2.26267	0.56177	2.5379	-0.819	-0.06715
8	-0.9083	-0.53481	3.2314	-2.448	0.121887
9	0.03746	-2.74432	-3.324	1.2501	0.157884
10	-0.1965	-0.4357	0.1129	0.2802	-1.66731
11	0.98633	-0.9605	0.8444	1.8427	0.250637
12	-0.9517	2.82052	-1.811	-1.367	0.169143
13	-0.0859	-1.90339	1.7275	0.7909	0.404076
14	2.14330	0.30729	1.2232	-0.212	-0.21625

The effect of each parameter on the SC behaviour was identified based on the relative importance RI of the input parameters of the ANN sample. The sensibility analysis was performed on the NN based on performance evaluation. The process of assessment was based on the connected weights of input and output stratus of the NN. The RI of each parameter was related to the others in the model and presented in Figure 7.

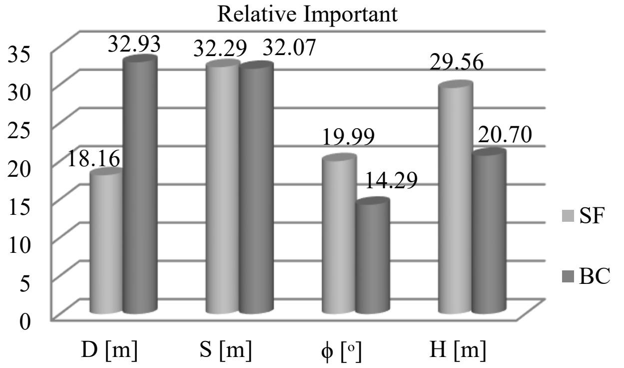


Figure 7 RI of input parameters in ANN models

The results show that the impact of all parameters is effective in predicting the SC behaviour ranging from 14 to 32% based on the examination of the RI of the inputs. In general, the plot indicates that the most effective inputs were *S* and *d*, which are related to the column number and area replacement ratio (As) for both models. This result is in agreement with (Kardani et al., 2020) when they estimated the bearing capability of columns in cohesionless ground using optimised machine learning methods. Similarly, (Saxena, and Roy 2022) observed that the spacing and column diameter are two critical variables that influence the strength of the stone columns system. This was expected since the input variables were selected carefully according to the outcomes of FE analysis.

6. CONCLUSIONS

The work objective was to test the capacity of AI approaches to forecasting the behaviour of SCs. To achieve this, ANN is developed with various quantities of the overall data collected. Once about 70% training is completed, the training is satisfactorily conducted through the feed forward-backwards propagation neural network. Based on the upshots acquired, the following conclusions were drawn:

- The ANN method attained a high level of accuracy in modelling the behavioural variables of SCs. The predicted values are compared with the real values resulting in deviation with ± 3.5% only.
- The pattern with 14 neurons in the hidden stratum demonstrated to be the 'best' pattern for the 4-14-1 ANN design to obtain the bearing ability and safety factor of the system.
- The simulated and forecast values are found to be in close agreement, the coefficient of efficiency (R) attained in the testing phase being more than 0.95 in both investigated models. This shows that the established ANN models are able of forecasting the bearing ability and factor of safety of stone columns with highly acceptable accuracy.
- Upshots of variable importance analysis point out that the column diameter and c/c spacing are the most effective parameter (score = 0.329 and 0.322) for bearing capacity and safety factor estimation of SC, respectively.

7. ACKNOWLEDGMENTS

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