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Numerical Modeling of Geosynthetic-Reinforced Earth Structures and Geosynthetic-Soil Interactions

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ABSTRACT: Nowadays geosynthetics have been used as a routine reinforcement in earth structures such as mechanically stabilized earth (MSE) walls, column-supported embankments, soil slopes, and paved/unpaved roads. In those applications, reinforcement mechanisms of the geosynthetics are vaguely described as confinement, interlocking, and load shedding respectively but not fully understood. The uncertainties of the mechanisms have been reflected as overconservativeness, inconsistence and empiricism in current design methods of those applications. Various researches have been widely carried on to investigate the mechanisms of reinforcement of the above mentioned applications, especially the geosynthetic-soil interactions and then quantitatively consider them into design methods**.** Numerical modeling characterized as cost- and time- saving, is preferred in many circumstances. An appropriate modeling strategy is vital to yield reliable results. This paper reviewed and summarized the modeling techniques used to model modular-block MSE walls, reinforced embankments/slopes, and reinforced paved/unpaved roads, which include conventional continuum modeling based on constitutive relationships as well as micro-mechanical modeling based on Newton's law of motion, i.e., modeling the soil mass as an assembly of soil particles governed by universal physics principles. The review of conventional continuum modeling includes constitutive models for soils, geosynthetics and other components (e.g., modular blocks), interface models for contacts between dissimilar materials, and simulation of construction, while the review of the micro-mechanical modeling is extended to the principle of the micro-mechanical modeling and how the micro-mechanical modeling is implemented to model the geosynthetic-soil interaction by using the most popular micro-mechanical scheme-PFC as an example. The objective of this paper is to provide a state-of-art review of the various numerical modeling techniques and consequently promote the usage of numerical modeling in research and practice of geosynthetic-reinforced earth structures.

1. INTRODUCTION

Geosynthetics have been more and more frequently included as reinforcement in the four major types of earth works, i.e., retaining walls, embankments, soil slopes, and paved/unpaved roads. Nowadays, mechanically stabilized earth (MSE) walls, reinforced embankments, slopes, and paved/unpaved roads constitute the majority of the newly constructed earth works compared with their unreinforced counterparts. For instance, FHWA 2001 statistic data indicated that over 700,000 m^2 of MSE walls were constructed in the United States every year, which counted for more than 50% of all types of retaining walls in the US transportation system (Elias and Christopher 2001).

Even though geosynthetics act as reinforcement in all of the above-stated applications, they function differently, that is, stabilizing the earth mass for retaining walls, transferring the load for embankments, and providing confinement for paved/unpaved roads to mitigate rutting and cracking. Many design methods have been proposed to promote the practice on those applications. For MSE walls and reinforced slopes, the design methods are derived from limit equilibrium analysis of force and/or moment (Elias and Christopher 2001). For reinforced embankments, the design methods are based on the tensioned membrane to account for the tension effect as a deformed sheet and soil arching theory if column support is available (Collin 2003). For reinforced unpaved/paved roads, the design methods consider the effect of geosynthetic reinforcement by comparing the field measured performance of the reinforced and unreinforced road sections in experimental studies. Since the mechanism of geosynthetic reinforcement in the above-discussed earth structures has not been fully understood, especially the soil-geosynthetic interaction, various assumptions have been adopted to develop the design methods, which consequently yield over-conservativeness, inconsistence, and/or empiricism design according to practice evidences. A detailed discussion of the current design methods can be found in the sections followed.

Extensive researches have been conducted to either investigate the reinforcement mechanisms or quantify a certain aspect of the reinforcement effects such as stress reduction for reinforced embankments and structure number increase for reinforced paved/unpaved roads, which include field and full-scale tests (e.g., Hatami and Bathurst 2005; Kwon et al. 2009a and 2009b) as well as numerical modeling (e.g., Hatami and Bathurst 2005 and 2006; Huang 2007). Numerical modeling has been increasingly adopted in researches since in addition to their outstanding cost- and time-effectiveness, they possess the following preferable advantages as compared with the field and full-scale tests:

- Flexibility. Variables can be easily fixed or varied to assess their effects. Parametric studies can be easily performed.
- Comprehensive data. The numerical modeling can provide a complete set of data, some of which are difficult or not able to be obtained from instrumentations such as shear stress/strain.
- Efficiency for long-term behavior performance study. The long-term performance is one of the interests for research and practice, e.g., consolidation of reinforced embankments and creep behavior of MSE walls. Given the appearance of geosynthetic in 1970's, valid long-term monitoring data are rare. Numerical modeling can extend the time domain to the point of interest.
- Exclusion of scale effect and external disturbance. Full-scale laboratory tests tend to be influenced by scale, more or less. And field tests are inevitably disturbed by external impacts. These scale effect and external disturbance can be easily excluded from or minimized in the numerical modeling.
- Minimum measurement errors. The experimental data intrinsically possess measurement errors, which is not a problem in numerical modeling.

Considering the above merits of the numerical modeling, numerical modeling plays an important, sometime irreplaceable, role in promoting the research and practice. So far, two approaches have been employed, that is, continuum modeling based on constitutive theories and micro-mechanical modeling based on assembly of soil mass from a collection of individual particles. The numerical modeling based on the continuum approach has been successfully used to simulate all of the above-discussed geosynthetic reinforced earth works. The micro-mechanical modeling is more sophisticated and has been primarily used to investigate the geosynthetic-soil interaction in a micro-scale (i.e., particle scale), since even though different mechanisms (confinement, interlocking, and load shedding) exist for different applications, there is a general agreement that the geosynthetic-soil interaction is a key element of these mechanisms. A down to particle size modeling will definitely allow detailed examination of the interaction. The soil particle simulation up to date is limited to simulating the geosynthetic-soil interaction in a reduced dimension due to the constraint of computational capacity.

Upon the discussed significance of numerical modeling in promoting the research and practice of the four types of earth works, this paper reviews the completed numerical modeling of MSE walls, reinforced embankments and slopes, and reinforced paved/unpaved roads based on the continuum approach and also the completed modeling of geosynthetic-soil interaction based on micro-mechanical simulation.

2. MODELING OF MSE WALL BASED ON CONTINUUM APPROACH

2.1 Introduction of MSE Wall

MSE walls use metal strips, wire meshes or geosynthetics as reinforcement to retain soil mass. Since the advent of MSE walls using geosynthetics in 1970s, they are now constructed routinely as retaining wall structures for a variety of applications ranging from private properties to public facilities (Allen et al. 2002). According to the survey of earth retaining structure practice in the North America, geosynthetic-reinforced MSE walls represented the lowest cost for all wall heights among all types of retaining walls (Yako and Christopher 1988; Koerner and Soong 2001). Besides the economic advantage, MSE walls possess other advantages such as easy construction, good tolerance to differential settlement, and excellent aesthetics. In recent years, MSE walls constructed in the North America are predominantly MSE walls with geosynthetic reinforcement and modular-block facing. This type of MSE walls represents the largest growth in the U.S. due to the availability of dry-cast modular block fabrication (Koerner and Soong 2001). Thus the discussion of MSE walls in this paper will be limited to this type of MSE walls, i.e., geosynehtic-reinforced MSE walls with modular-block facing.

Currently the design methods of the MSE walls in the North America mainly come from the NCMA Design Manual (Collin 1997) and FHWA (Elias and Christopher 2001). These design methods are based on lateral earth pressure and limit equilibrium analyses accompanied by a series of assumptions such as shape and location of the sliding plane and orientation of geosynthetic tensile force (Collin 1997; Elias and Christopher 2001; Christopher et al. 2005). The assumptions induce overconservative design results. For instance, the design module of internal stability is based on limit-equilibrium tied-back wedge methods, which have been proved over-conservative on maximum geosynthetic tensile force by experience and statistical analyses from data collected from full-scale tests and well instrumented walls (Rimoldi 1988; Billiard and Wu 1991; Ochiai et al. 1993; Huang et al. 2009a).

Besides the conservativeness, current methods have the following major limitations, which hinder the practice of MSE walls (Cai and Bathurst 1995; Christopher et al. 2005; Guler et al. 2007; Huang et al. 2009a):

- Applicable to only simple geometries and difficult to extrapolate to complex geometries, such as multi-tiered walls;
- Limited to uniform backfill materials with specified gradation and difficult to design for non-ideal reinforced fill soils;
- Good for a limit equilibrium state and cannot evaluate deformation;
- Cannot account for interaction, which is important for the integrity of the wall system;
- Cannot evaluate the response under complex loading conditions.

Physical wall tests are the best approach to extend our knowledge and improve current design methods; however, they are costly and time-consuming. As an alternative, numerical modeling including finite-element method (FEM) and finite-difference method (FDM) has been widely used to expand the database of physical wall tests. So far, numerous numerical modeling of MSE walls have been successfully carried on and have achieved satisfactory results. Examples are Karpurapu and Bathurst (1995), Leshchinsky and Vulova (2001), Rowe and Skinner (2001), Ling and Leshchinsky (2003), Hatami and Bathurst (2005), Hatami and Bathurst (2006), Yoo and Song (2006), Guler et al. (2007), Liu and Ling (2007), Huang et al. (2009a) and Liu and Won (2009). The completed numerical modeling of MSE walls has been focused on different aspects of uncertainty such as influence of geosynthetic on failure plane, the distribution of tension within geosynthetic, the influence of foundation yield on deformation and geosynthetic tension, the influence of surcharge loading, the influence of geoysnthetic creep behavior on MSE walls, the performance of multi-tier walls and so on. Apparently, a suitable strategy for numerical modeling of MSE wall is premier, which warrants the reliable and applicable results. Therefore, different from other published papers which summarized the findings disclosed by completed numerical studies (e.g., Hatami and Bathurst 2001), this paper will synthesize modeling strategies used by the completed numerical modeling of MSE walls, i.e., how the MSE wall system was simulated, which is itemized into the following six aspects:

- How the backfill soil was modeled, i.e., constitutive models being used;
- How the MSE wall facing was modeled, i.e., constitutive model(s) for modular-blocks;
- How the geosynthetic reinforcement was modeled;
- How the interfaces were modeled, i.e., the interface between soil and modular-block, the interface between soil and geosynthetic, and the interface between modular-blocks;
- How the construction was modeled, i.e., compaction.

The discussion or summary presented hereafter is limited to numerical modeling of MSE walls subjected to monotonic loading. The numerical modeling of MSE walls under seismic conditions is beyond the scope of this paper.

2.2 Modeling of Backfill Soil – Constitutive Models

According to FHWA specifications, the gradation of the backfill soil for MSE walls should strictly confirm to the following criteria: (1) Fine content (particle size less than 0.075mm) less than 15%; and (2) Plastic Index (PI) less than 6 (Tanyu et al. 2007). Such specification warrants that the backfill soils are of free-drain and granular materials unless other non-standard specification is followed. Considering the essential nature of granular materials, in recent years four soil constitutive models have been successful used to simulate their behavior as MSE wall backfill, i.e., linearly-elastic perfectly-plastic model with Mohr-Coulomb failure criteria (also called Mohr-Coulomb model in some commercial software packages), the Duncan-Chang model and its modified versions, Lade's model, and the generalized plastic model proposed by Ling and Liu (2003).

2.2.1 Linearly-elastic perfectly-plastic model

Linearly-elastic perfectly-plastic model is a linearly elastic model with a Mohr-Coulomb failure criterion. The Mohr–Coulomb failure surface is a cone with a hexagonal cross section in a three-dimensional deviatoric stress space. Either a non-associated or an associated flow rule can be used for plastic strain increment. Even though, due to its simplicity, the linearly-elastic perfectly-plastic model has been widely used in various circumstances to simulate the soil behavior, its application in MSE walls is not as common as the Duncan-Chang model. The main disadvantages of the linearly-elastic perfectly-plastic model in MSE wall modeling are: (1) stress-dependent soil stiffness behavior is not considered and (2) shear-softening or hardening behavior is not included. Different approaches have been used to overcome these two disadvantages. Without stress-dependent stiffness, the main difficulty of using the linearly-elastic perfectly-plastic model in MSE walls would be of selecting a single value for an elastic modulus (Huang el al. 2009a). Huang el al. (2010) used the modulus obtained from triaxial tests with a confined stress equal to that at the wall mid-height and yielded reasonably good results. Fakharian and Attar (2007) accounted for soil strain-softening behavior by varying the frictional angle of backfill soil as a function of plastic strain, i.e., reduced the maximum value at zero plastic strain to a residual value at a large plastic strain.

2.2.2 Duncan-Chang model and its modified versions

As a well-known non-linear stress-dependent stress–strain relationship, the Duncan-Chang model proposed by Duncan et al. (1980) and its different versions have been widely used to simulate MSE walls (for example, Ling and Leshchinsky 2003; Hatami and Bathurst 2005 and 2006; Yoo and Song 2006; Fakharian and Attar 2007; Guler et al. 2007; Huang et al. 2009a). The Duncan-Chang model accounts for both the hyperbolic effect of confining stress and the Mohr-Coulomb failure criterion on the modulus. Although the Duncan-Chang model overpasses the linearly-elastic perfectly-plastic model in representing the stress-dependent behavior, it has its own limitations as well: (1) strain hardening behavior cannot be directly considered; (2) shear dilatancy cannot be taken into account; and (3) the modulus would be zero if the confining stress is zero, which would lead to small modulus and then instability of a numerical model. The first two disadvantages were considered to have an insignificant influence if MSE walls modeled were under their operational conditions (Ling and Leshchinsky 2003). The intrinsic instability induced by the Duncan-Chang model has been tackled in different approaches. For instance, Hatami and Bathurst (2005) ensured the stability of the Duncan-Chang model by limiting the modulus within a certain range.

The linearly-elastic perfectly-plastic model and the Duncan-Chang model have been widely used, since their parameters carry physical meaning and can be readily obtained from simple laboratory tests. Special attention should be paid when soil is simulated by either the linearly-elastic perfectly-plastic model or the Duncan-Chang model. Most of the time an MSE wall is modeled in plane-strain conditions; however, soil parameters may be obtained from triaxial tests. Hence, adjustment may be necessary (Hatami and Bathurst 2005). Empirical relationships proposed by Bolton (1986) and Hanna (2001) have been used to convert the peak friction angle from conventional triaxial tests to the peak plane-strain friction angle. Additionally, considering that the Duncan-Chang model was based on triaxial tests, Boscardin et al. (1990) developed a unique set of input parameters which are applicable to both triaxial and the plane-strain loading conditions to help the original Duncan-Chang model be applied into a much broader area.

2.2.3 Lade’s model

Lade’s model (Kim and Lade 1988) is an elastoplastic constitutive model with single yielding surface for cohesionless geomaterials. In the elastic portion, the elastic modulus is hyperbolically related to the confining stress. The yielding is depicted as a conical surface with a cap. The cone surface is oriented along a hydrostatic line. Lade’s model adopted non-associated flow. The main advantage of Lade’s model is that it can account for stress-dependent soil stiffness and both hardening and softening behavior. Additionally, different from the two constitutive models discussed above, Lade’s model explicitly accounts for the effects of plane-strain conditions and no adjustment is required when MSE walls are modeled two-dimensionally. The disadvantage of Lade’s model is that it is formulated in parameters, many of which lack physical meaning and cannot be obtained from simple testing directly.

Huang et al. (2009a) performed a comparison among the linearly-elastic perfectly-plastic model, the Duncan-Chang model, and Lade’s model on their predictions of MSE wall performance. The comparison verified the adequacy of these three models in simulating MSE walls under working stress conditions.

2.2.4 Generalized plastic model

Based on the Pastor-Zienkiewicz-Chan model, Ling and Liu (2003) proposed a generalized plasticity model for simulating the behavior of cohesionless soil. The major improvement of the generalized plastic model from the Pastor-Zienkiewicz-Chan model lies in that friction angle, elastic and plastic moduli are depicted as stress-dependent. In addition, it considers the plastic modulus of loading, unloading, and reloading differently; therefore, the generalized plastic model can be used for both static and cyclic loading conditions. Liu and Won (2009) adopted this model to study the long-term performance of MSE walls. Similar to Lade’s model, it contains many parameters lacking of direct physical meaning.

The above-discussed constitutive models are the most popular models, but not all, used in modeling of MSE walls, each of which has its own advantages and disadvantages. The selection of an appropriate constitutive model should be based on understanding of soil behavior. A comprehensive review conducted by Lade (2005) covered more constitutive models and discussed their capabilities and shortcomings, which can be used as a reference during soil constitutive model selection.

2.3 Geosynthetic Reinforcement

Geosynthetic is a term used to describe a wide range of products made from polymeric materials such as polyester, polypropylene, and high-density polyethylene (Koerner 1998). Compared with metal strips or wire meshes, geosynthetics are more favorable in most of the MSE wall applications, since they possess excellent resistance to corrosion. However, geosynthetics are non-linear, elastoplastic, or viscoplastic materials, which demand more sophisticated models to depict their behavior (Walters et al. 2002; Liu and Ling 2005 and 2007; Yoo and Song 2006). Geosynthetics have been modeled as simple linear elastic, non-linear elastic, elasto-plastic, and visco-plastic materials, depending on chemical compositions, loading conditions, and exposure to temperature fluctuations. Commonly, geosynthetics are deemed of zero compressive strength (e.g., Fakharian and Attar 2007). The load-strain response relationship is in reality a tension-strain response.

2.3.1 Linear elastic

In reality, the tension-strain relationship of geosynthetic is never linear. In most MSE wall applications, the maximum strain the geoysnthetic experienced is less than 1.5% (Hatami and Bathurst 2005; Guler et al. 2007). Within such a small range of strain variation, tension-strain relationship of some geosynthetics can be approximated as linear without introducing too much error. Yoo and Song (2006) adopted a constant tensile stiffness for the geosynthetic to model a two-tier MSE wall and yielded satisfactory results. They also disclosed through their study that 10-20% variation of tensile stiffness would not have an evident influence on their results.

2.3.2 Non-linear elastic

Geosynthetic materials are inherently nonlinear compared to steel and other metallic reinforcements (Ling et al. 2001) and some geosynthetics exhibit strongly non-linear load-strain relationships even at a low strain level.

To more accurately model the non-linear tension-strain response, Ling et al. (1995) proposed a hyperbolic elastic model to consider the nonlinearity as a function of applied force. The tangential stiffness of the geosynthetic is expressed in terms of applied force, ultimate strength, and initial stiffness as shown in Eq. 1. This model has been used by Ling and Leshchinsky (2003) for a parametric study of MSE walls.

$$J_{tan} = J_o \left(1 - \lambda \frac{T}{T_f} \right)^2 \quad (1)$$

where J_o = initial tangential stiffness; T_f = ultimate strength, T = applied load, and λ = constant.

Hatami and Bathurst (2005) carried out a series of in-isolation constant rate of strain (CRS) tests under different loading rates. A measured load–strain response at 0.01% strain/min was adopted and was fitted by a parabolic curve in terms of strain as shown in Eq. 2. Then the tangential tensile stiffness varies linearly with strain (Eq. 3). Two constants of the linear equation, i.e., the intercept and slope constant A and B, have to be determined from the test.

$$T = A\varepsilon - B\varepsilon^2 \quad (2)$$

$$J_{tan} = \frac{dT}{d\varepsilon} = A - 2B\varepsilon \quad (3)$$

where T = applied load; ε = strain; and A and B = constants.

It is noteworthy that the above two non-linear tensile stiffness relationships were derived based on in-isolation tests, which did not account for the influence of soil.

2.3.3 Elastic-plastic

The above-discussed hyperbolic model proposed by Hatami and Bathurst (2005) was simply bounded by geosynthetic rupture strength to consider large strain conditions. As a matter of fact, other elastic models can also be used in a similar manner to consider the rupture failure of geosynthetic reinforcement.

Cai and Bathurst (1995) considered that the range of pure elastic strain is very small and negligible for some geosynthetics, i.e., most of strains developed even at a low strain level were not recoverable. Ling et al. (2001) proposed a one-dimensional bounding surface model to simulate the stress-strain hysteresis, which is suitable for monotonic and cyclic loading conditions. The bounding surface model was proposed by Dafalias and Popov (1975) and Krieg (1975) to consider the Bauschinger effect under complex loading conditions especially cyclic loading, which is a simplified version of the multi-surface model. The bounding surface model includes two surfaces: inner surface (yielding surface) and outer surface (bounding surface). The stress state is determined by its distance from the bounds. The plastic modulus is determined by proximity of the surfaces as these surface move and change in size. Within the framework of the bounding surface model, Ling et al.'s one-dimensional bounding surface model with linear bounding lines can be used to consider the loading-unloading behavior of geosynthetic reinforcement.

2.3.4 Visco-plastic

Geosynthetic, composed of large molecule chains, has demonstrated that its mechanical properties are functions of loading rate, duration of loading, and temperature (McGrow et al. 1984; Yeo 1985; Allen and Bathurst 2002; Watlers et al. 2002). Depending on the polymer used to manufacture geosynthetics, some geosynthetics exhibit salient viscous behavior. To predict the long-term performance of MSE walls reinforced by such geosynthetics, a visco-plastic model is necessary to account for the time effect.

Hatami and Bathurst (2006) formulated the isochronous load-strain curves obtained from creep tests into a hyperbolic load-strain-time function as shown in Eq. 4 below. Obviously, the tangential tensile stiffness is a function of time and developed strain. The initial tangential tensile stiffness and ultimate strength are also time-dependent.

$$J_{tan} = \frac{1}{J_o(t)\left[\frac{1}{J_o(t)} + \frac{\eta(t)}{T_f(t)} \varepsilon \right]^2} \quad (4)$$

where $J_o(t)$ = initial tangent stiffness; $\eta(t)$ = scaling function; $T_f(t)$ = load-rupture function for the reinforcement; ε = strain; and t = time.

Liu and Ling (2007) developed a unified viscoplastic bounding surface model to consider time-dependent behavior of geosynthetic reinforcement, which was modified from Ling et al. (1995 and 2001) by including the visco characteristic of geosynthetics. In addition, the viscoplastic bounding surface model ignored the elastic strain and considered nonlinear bounding lines. The model has been evaluated using creep, stress relaxation, monotonic, and cyclic loading test results and has been successfully used by Liu and Won (2009) to investigate the long-term performance behavior of MSE walls under various conditions.

2.4 Modular Block

Modular blocks are typically made of Portland cement mortar, which has compressive strength of at least 28 MPa according to the American Association of State Highway and Transportation Officials (AASHTO) (2002). In MSE wall applications, the compressive load is always much lower than the compressive strength. Therefore, the individual modular blocks are always modeled as linear elastic materials (for example, Ling et al. 1995; Hatami and Bathurst 2005 and 2006; Yoo and Song 2006; Guler et al. 2007; Huang et al. 2010). Since the MSE wall facing is subject to lateral force, which affects the MSE wall facing by shearing and bending, an appropriate simulation of the interaction between individual blocks (interfaces) is more important. The interfaces between individual blocks will be discussed in the following section.

2.5 Interfaces

The complexity of the MSE wall system is largely attributed to the materials of dissimilar properties, including backfill soil, geosynthetic reinforcement, and modular blocks. The interactions between these materials have to be appropriately represented in order to explore the reinforcing mechanisms. Various approaches have been used to simulate the interfaces between backfill soil and geosynthetic, between modular blocks and backfill soil, and between modular blocks.

2.5.1 Interface between backfill soil and geosynthetic

Geosynthetic has negligible bending stiffness, thus, the interaction between geosynthetic and backfill soil occurs mainly through surface friction and particle interlocking. Leshchinsky and Vulova (2001), Yoo and Song (2006), and Huang et al. (2009a) ignored the interface between geosynthetic and backfill soil and assumed full bonding between them, i.e., no relative movement at the contact surface was allowed. This assumption was argued to have a little effect on the results for geogrid, since pullout tests showed that in many soils the slip occurred in the soil mass and not at the interface between geogrid and soil, if the confining stress was not extremely low (Leshchinsky and Vulova 2001; Yoo and Song 2006).

Mohr-Coulomb slip interface, characterized as a linear spring-slider assembly with slippage governed by the Mohr-Coulomb criterion, is the most commonly used one to represent the interface between geosynthetic and backfill soil (Fakharian and Attar 2007; Liu and Won 2009; Huang et al. 2010). The key of using this interface is to select suitable parameters for interface. The frictional parameters of the interface can be easily derived from friction angle and cohesion of the backfill soil by applying reduction factors (Fakharian and Attar 2007; Huang et al. 2010). However, the linearly elastic (spring) parameters (i.e., shear and normal stiffness) depend on both backfill soil and geosynthetic, and are usually the information being lack of (Huang et al. 2010).

Erban et al. (1992) used a nonlinear horizontal spring to simulate the interface between geosynthetic and backfill soil. In their simulation, the nonlinear load-strain response was approximated by a piecewise function consisting of a few linear lines. And slip was mobilized once the shear stress reached a certain value.

2.5.2 Interface between backfill soil and modular blocks

To model the interface between backfill soil and modular blocks, two similar approaches to what have been used for the interfaces between geosynthetic and backfill soil have been adopted for the interface between backfill soil and modular blocks, i.e., (1) no interface, that is, fully bonded; and (2) Mohr-Coulomb slider (e.g., Hatami and Bathurst 2006; Huang et al. 2009a; Huang et al. 2010). Besides, other approaches have been used as well.

Stiffness interface methods such as Goodman interface elements, proposed by Goodman et al. (1968), have been used to model the interface between backfill soil and modular blocks. Developed by Goodman et al. (1968) to model the rock joints, a Goodman element has been widely used to model the discontinuity of materials, which is a zero-thickness element with constant normal and shear stiffness. The relationship between stress and relative displacement is linear. Katona (1983) incorporated the Mohr-Coulomb failure criterion into Goodman element implementation to allow occurrence of slip at the interface.

A very thin layer of elements has been used to model the interface between backfill soil and modular blocks (Desai et al. 1984; Yoo and Song 2006). The layer was assumed to be elastic with a low shear modulus but high bulk modulus to represent the possible relative movement at the contact surface of backfill soil and modular blocks. The similar approach was also adopted by Liu and Won (2009).

2.5.3 Interface between modular blocks

The full bonding, Mohr-Coulomb slider, and Goodman element have been used to simulate the interfaces between individual modular blocks (e.g, Katona 1983; Cai and Bathurst 1995). The use of the Mohr-Coulomb slider method for the interface between modular blocks is similar to that for geosynthetic-soil and backfill soil-modular block interfaces. For a full bonding approach, the modulus of the modular blocks has to be reduced to account for the stiffness reduction due to discontinuity of the individual blocks (Yoo and Song 2006; Huang et al. 2010), that is, modeling the modular blocks as a whole continuum zone but with a reduced modulus. The difficulty exists on how to consider the reduction since the reduction varies from case to case. Yoo and Song (2006) and Huang et al. (2010) used equivalent bending stiffness to consider the reduction.

2.6 Construction Simulation

The MSE walls are constructed by sequential placement of modular blocks, geosynthetic layers, and backfill soil from the bottom to the top. During the process, compaction is exercised to meet the relative density requirement. The backfill soil compaction has two effects: (1) increasing the lateral earth pressure (Duncan et al. 1991); and (2) reducing Poisson's ratio (Hatami and Bathurst 2005). Compaction-induced stresses during construction may not be explicitly considered in the analysis (e.g., Rowe and Skinner 2001; Huang et al. 2010).

If the compaction effect is to be included in the modeling, only equivalent static pressure was used to represent the compaction effect regardless the compaction methods (Gotteland et al. 1997; Hatami and Bathurst 2006; Guler et al. 2007). The studies completed by Duncan et al. (1991) and Filz et al. (2000) can be used to estimate the equivalent pressure.

Besides the constitutive modeling of backfill soil, geosynthetic, and modular block, and the modeling of the interfaces and compaction discussed previously, some other details are needed as well, such as the connection between geosynthetic and modular blocks, the boundary conditions, and the use of a small strain or large strain mode in numerical implementations. These details may have significant influence on numerical results, but vary from case to case; therefore, they will not be discussed herein.

3. MODELING OF REINFORCED EMBANKMENTS AND SLOPES BASED ON CONTINUUM APPROACH

3.1 Introduction of Reinforced Embankments and Slopes

Embankments have been built over columns with or without geosynthetic reinforcement. As the advent of geosynthetics as a cheap and durable reinforcement material, geoysnthetic-reinforced column-supported (GRCS) embankments become more and more popular. In the GRCS embankment system, the load over the low strength and high compressible soil is transferred to competent columns by soil arching and membrane effects (Collin 2003). The design methods were proposed based on different soil arching and membrane theories, which are derived from limit equilibrium analyses and specific assumptions such as the shape of the deformed geosynthetic layer and the shape of the soil arching (Terzaghi 1943; Hewllet and Randolph 1988; Giroud et al. 1990). Large deviations have been found among these design methods (Naughton and Kempton 2005). To improve the understanding of the GRCS embankment, many studies have been conducted, for example, Forsman et al. (1999), Han and Gabr (2002), Chew et al. (2004), Filz and Smith (2005), and Huang (2007). Numerical modeling has been widely performed, since besides its time- and cost-saving benefits, it is very hard to study the GRCS embankment system in laboratory due to the presence of columns and involvement of groundwater.

MSE wall and GRCS embankment systems have some similarities but also some differences. The similarities are apparent, which allow the soil constitutive models, geosyntheic models, and interface models for soil and geosynthetic used in MSE wall simulations to be readily used in GRCS simulations as well. Meanwhile, the GRCS embankment system differs from the MSE wall in three major aspects: (1) The embankment fill is less selective than MSE wall backfill; (2) The embankment is bounded by slopes but not modular blocks; and (3) The subgrade soil is usually of lower strength and higher compressibility and usually has high clayey content, which is usually saturated. As a result, the strategy of simulating GRCS embankment may be different from that of MSE walls.

3.2 Modeling of Embankment Fill and Foundation Soil

The embankment fill is not specified as strictly as MSE wall backfill by manuals, standards or specification. In practice, the embankment fill can have a much wider range of soil types than MSE wall backfill. Consequently, besides the linearly-elastic perfectly-plastic model (e.g., Han et al. 2007; Liu et al. 2007; Huang et al. 2009b) and Duncan-Chang model (Han and Gabr 2002), other constitutive models have been used to simulate GRCS embankments.

The Modified Cam-Clay (MCC) model has been used to model soft foundation clays (e.g., Huang et al. 2006; Oztoprak and Cinicioglu 2006; Liu et al. 2007; Smith and Filz 2007). MCC was proposed by Roscoe and Burland (1968) originally for re-molded clays based on the critical state soil mechanics. The MCC model in reality is an elasto-plastic hardening model with exponential elasticity prior to yielding and its yielding surface is described by ellipses. And associated flow rule is adopted. MCC has been one of the most widely used constitutive models for clayey soils.

An elasto-plastic hardening-soil model, proposed by Schanz et al. (1999), was used by Pham and White (2007) to simulate the foundation soil. The Schanz et al. hardening model is a nonlinear, elasto-plastic model. The elastic modulus is hyperbolically dependent on confining stress. Since Mohr-Coulomb criteria are used as the yielding rule, the yield surface is a capped hexagonal yield surface. Non-associated flow is assumed on the hexagonal yielding surface but an associated flow rule is used on the cap. All the model parameters can be obtained from conventional lab testing. Jenck et al. (2007) used the two-mechanism elastoplastic model with isotropic hardening to simulate embankment fill, which was originally proposed by Cambou and Jafari (1987) to simulate the granular soils. The model was based on nonlinear elasticity and two mechanisms of plasticity: one for isotropic and one for deviatoric stresses. Different hardening mechanisms were considered in isotropic and deviatoric yielding as well. More importantly, it incorporated a surface to allow the consideration of dilation behavior of granular materials.

3.3 Modeling of Columns

In the GRCS embankment system, many types of columns have been used such as concrete piles, stone columns, rammed aggregate piers, deep mixed columns, timber piles and so on. Since the column type and column properties vary, different constitutive models have been used to simulate the columns in the GRCS embankment systems. As the columns are often much more competent as compared with the foundation soil and failure of the columns is unlikely to occur, a purely elastic model has been often used for columns (e.g., Huang et al. 2005). When the failure of the columns becomes possible, a linearly-elastic perfectly-plastic model and Schanz et al.'s hardening model have been used to model the columns (e.g., Liu et al. 2007; Huang et al. 2009b; Pham and White 2007).

3.4 Modeling of Consolidation

Different from MSE walls, GRCS embankments often are constructed over soft clay with high ground water tables. The construction of the embankment will generate excess pore water pressure, which dissipates during and after construction. Hence, the embankment behavior is time-dependent. Numerical modeling has been conducted to couple the hydraulic (consolidation) and mechanical process (Hossain et al. 2006; Huang et al. 2006; Huang 2007; Huang et al. 2009b). Typically, the coupling was implemented in a quasi-static scenario based on Biot's theory and Darcy's law. The modeling of consolidation can incorporate at least two tasks, which cannot be fulfilled if consolidation modeling is not included: (1) The behavior of GRCS embankments is framed in a time domain and the post-construction performance can be evaluated; and (2) The stress and deformation calculation is accurate, since the real-time effective stress is used to detect the occurrence of failure within soil mass (Huang 2007).

The numerical modeling of geosynthetic reinforced soil slope (GRS) is less complicated than that of MSE walls and GRCS embankments, because only soil and geosynthetic involve in the simulation. Thus the modeling of GRSS does not warrant any modeling techniques other than what being used for MSE walls and GRCS embankments. No further discussion on the modeling of GRSS will be provided in this paper. It is noteworthy that the discussion herein on the simulation of MSE walls, GRCS embankments and GRS slopes are limited to operational conditions. Simulation of failure or excessive deformation of these earth works may turn into different modeling strategy.

4. MODELING REINFORCED PAVED/UNPAVED ROADS ON CONTINUUM APPROACH

4.1 Introduction of Reinforced Paved/Unpaved Roads

Geosynthetic-reinforcements are used in paved/unpaved roads for two main purposes: (1) increasing the bearing capacity of subgrade and (2) reinforcing the unbound base. Although geosynthetics are more often used for the first purpose, most of the previous numerical studies focused on the modeling of reinforced bases, which is generally considered more important in roadway design analyses. Geogrid- and geocell are commonly used geosynthetic reinforcements, both of which can provide lateral constraint to the unbound base materials. Typical cross-sections of geosynthetic-reinforced flexible pavements are shown in Fig. 1. The lateral constraint effect will not only reduce the lateral movement of the base materials, but also stiffen the base course and reduce the vertical stress transferred to the soft subgrade. Obviously, the overall behavior of the geosynthetic-reinforced base depends on the characteristics of both geosynthetics and the reinforced base materials.

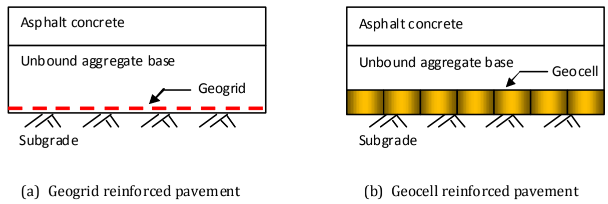

Figure 1. Typical cross-sections of the geosynthetic-reinforced flexible pavements

Traditional design methods of paved and unpaved roads consider the effects of the geosynthetics by introducing a traffic benefit ratio (TBR) or base course reduction ratio (BCR) (Perkins and Edens 2002). These ratios are often derived by comparing the field measured performances (e.g., the permanent deformation developed on the top of the road) of the reinforced and unreinforced road sections in experimental studies. Therefore the design basis is largely empirical. In order to understand the mechanisms and develop more analysis-based design methods for the geosynthetic-reinforced base course, numerical analyses were performed by many researchers. The objectives of most numerical modeling can be depicted as to simulate the interaction between different geosynthetics and unbound materials and predict the response (i.e., stress and strain) and/or performance (i.e., rutting, cracking, etc.) of reinforced roadways. To be more specific, numerical modeling techniques are used in these studies to characterize the interaction between different geosynthetics and unbound materials and predict the response (i.e., stress and strain) and/or performance (i.e., rutting, cracking, etc.) of reinforced roadways.

4.2 Modeling of Paved/Unpaved Roads Based on Continuum Approach

4.2.1 Constitutive model for base/subbase materials

Even though the base/subbase materials vary according to the regional specifications, for example, in the U.S. different state transportation agents specify different requirements for acceptable materials for base/subbases. Generally the base/subbase materials are of granular materials with small or no fine contents. Considering such nature, the constitutive models used in simulating paved/unpaved roads are the models being proved effective to capture the behavior of cohesionless soils.

In order to simulate the stiffening of unbound base materials under lateral constraint, stress dependency behavior of unbound base materials have to be considered in the constitutive model. If only the pavement response under a monotonic load is concerned, a nonlinear elastic model can be used for unbound base materials. In the new mechanistic-empirical pavement design guide developed under NCHRP Project 1-37A (Applied Research Associates Inc. 2004), an isotropic nonlinear elastic model (shown in Eq. 5) was adopted in the FEM mechanistic model:

$$M_r = k_1 p_a \left(\frac{\theta}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} \tag{5}$$

where θ = the bulk stress; τ_{oct} = the octahedral shear stress ($\tau_{oct} = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \;/\; 3$); k_1, k_2, and k_3 = dimensionless parameters. Perkins (2004) and Yang (2010) used this model in their numerical simulations for geogrid-reinforced flexible pavements and for geocell-reinforced unpaved road sections, respectively.

When stress dependent-models are used for the reinforced unbound bases, an initial stress condition will affect the modulus of the unbound base and thus affect the calculated pavement response. It has long been realized that compaction induces additional residual lateral stress in the soil. Due to the lateral constraint effect of the geosynthetics, more lateral stress will be developed within an "influence zone" in the unbound base materials both above and below the location of the geosynthetics. This phenomenon has been proved by numerical simulation and field test results (Kwon and Tutumluer 2009). Perkins (2004) and Kwon et al. (2009a) both emphasized the importance of considering the compaction-induced horizontal stress within the reinforced base course. In Perkins' model, an isolated compaction module was run first to calculate the residual stress induced by compaction. The residual stress was estimated indirectly by shrink the geocell by 1% strain horizontally. Kwon et al. (2009a) directly assigned a certain initial horizontal stress to the base within material in proximity of the geogrid to consider the compaction effect. Yang (2010) modified an analytical compaction model proposed by Duncan and Seed (1986) to estimate the compaction-induced horizontal stress in the geocell-reinforced layer. This model correlated the compaction-induced horizontal stress with the compaction pressure and some basic soil properties.

Additional lateral stress can also accumulate, as Perkins (2004) suggested, as the permanent tensile strain in geosynthetics increases with the number of load passes. He developed two traffic modules (Traffic I and II) to calculate the residual stress in the base after a certain number of wheel passes. Yang (2010) suggested that the modulus increase of the reinforced base material induced by the permanent strain in the geosynthetics can be considered by using a set of equivalent resilient modulus parameters. He verified this method by calculating the resilient modulus of a geocell-reinforced soil from another study. Good agreement was found between the calculated and measured resilient modulus results.

Other researchers found that cross-anisotropy has to be considered in the constitutive model for unbound bases. Barksdale et al. (1989) compared a nonlinear elastic-plastic model and a cross-anisotropic elastic model in his numerical model for geogrid reinforced flexible pavements. The comparison showed that a cross-anisotropic elastic model for the base material generated much better estimation of the pavement responses. Kwon et al. (2009a and 2009b) also used a cross-anisotropic model for the unbound base in their modeling of geogrid-reinforced flexible pavements.

The above mentioned mechanistic models can be used to predict the pavement responses under a monotonic wheel load well. Empirical damage models are needed to transfer the pavement responses to long-term performances such as rutting. Alternatively, the rut depth of the paved and unpaved roads can be predicted by incorporating advanced plasticity models into the numerical model for the unbound base and subbase materials.

Saad et al. (2006) and Howard and Warren (2009) performed dynamic modeling of geogrid reinforced flexible pavements, where a dynamic wheel load (as a function of time) is applied instead of a static wheel load. Saad et al. (2006) used the Drucker-Prager model for the unbound base and applied a triangular-wave wheel load. They qualitatively compared the numerical results from the model with those from the previous literature and found fairly good agreements. Howard and Warren (2009) used a nonlinear elasto-plastic model for the unbound base material and applied a Haversine load pulse to the model. Overall, the numerical model agreed reasonably well with the field test data. Nazzal et al. (2010) developed a modified critical state two-surface model for simulating the behavior of unbound materials under cyclic loading. However, the numerical result of accumulated rutting under cyclic loads was not compared with test data.

4.2.2 Constitutive model for geosynthetics

Since mostly geogrid and geocell have been used to reinforce paved/unpaved roads and properly designed geosynthetics often work under a small amount of tensile strain (less than 2%), linear elasticity is often assumed for both geogrid and geocell materials. However, different from MSE walls, the geogrid used in paved/unpaved roads is a biaxial geogrid and cannot be simply modeled in a plane-strain mode. Most of the biaxial geogrid products in the market have different stiffness in the machine direction (MD) and the cross-machine direction (XD). Perkins and Edens (2002 and 2003) used an orthotropic linear elastic model to simulate the geogrid in a three-dimensional numerical model. For use in axisymmetric models, Perkins (2004) later proposed a method to convert the orthotropic linear elastic properties to the equivalent isotropic linear elastic properties.

The interface models used for the contact surface between geosynthetic-base/subbase materials have not gone beyond what have been discussed in the section of MSE walls.

5. MICROMECHANICAL MODELING OF GEOSYNTHETIC-SOIL INTERACTION

5.1 Introduction to Micromechanical Modeling

Micromechanical modeling of materials constitutes the study of force transmission via contact points of interacting particles and the local kinematics (translational as well as spin velocity) of these particles. Micro-mechanical modeling has been a powerful tool for studying the behavior of granular materials in a micro-scale. The earlier research conducted on granular materails can be traced back to Coulomb, who deduced the law of friction for granular materials in 1785. The statics and kinematics of granular materials have been extensively investigated in physics, chemical engineering, and geotechnical engineering. The micro-mechanical modeling has progressed in three broad subdivisions: micro-structural continuum approach, Discrete Element Method (DEM, also called distinct element method), and experimental approach.

In the micro-structural continuum approach, contact forces are averaged over a sufficient volume (Elementary Representative Volume, ERV) to define a continuum-equivalent stress tensor. This approach assumes the known geometry (shape, location, and orientation) of the particles. Using this micro-structural continuum approach, a closed-form stiffness tensor of a random packing was obtained (Chang and Misra 1989 and 1990; Misra 1991). Using a similar micro-structural continuum approach, Bathurst and Rothenburg (1990) expressed an angle of friction at a macro level in terms of the coefficient of contact orientations under a large strain.

In the DEM approach, modeling is based on the representation of materials as an assembly of disks and spheres. The mechanical response of the material is obtained by solving Newton's equation of motion for each particle. The particles are modeled as rigid bodies interacting at soft contacts. The force-deformation relationship at the contact points is modeled using contact laws. The resulting differential equations are often solved using explicit numerical schemes, for example, the Particle Flow Code (PFC). An experimental verification of the DEM could use an optically sensitive material where the contact forces of interacting particles can be determined. Cundall and Strack (1979) performed tests on an assembly of discs the contact forces among the particles and their displacements and rotations were independently verified. A comparison of the results from the DEM and the photo-elastic disc tests indicated that the numerical model may be used to replace tests on photo-elastic discs. The initial comparisons were qualitative and relied on the visual comparison of the contact forces. Nonetheless, the correspondence of the force vectors between the numerical and photo-elastic experiments laid a foundation for the DEM as a potential tool in research. Since then significant research has been done to understand the fundamental properties of granular materials with a particular emphasis on strain localization and shear band formation at the particle level (Gardiner and Tordesillas 2005, Pena et al. 2007; Wang et al. 2007; Zhang and Thornton 2007).

5.2 Framework of Micromechanical Modeling of Geosynthetic-Soil Interactions

The interaction between geosynthetic and soil is a key factor in the performance improvement of the geosynthetic-reinforced structures. The DEM can supplement the understanding of the mechanisms and is an ideal tool for studying the interaction between geosynthetic and soil (Konietzky et al. 2004). Traditional laboratory tests (i.e. direct shear tests, triaxial tests, and pullout tests) as well as geosynthetic-reinforced structures have been modeled using DEM techniques to study theses mechanisms. In these formulations, the interaction between soil particles is mainly evaluated using a linear contact stiffness model, and a slip and separation model.

5.2.1 Contact stiffness model

A linear contact and the Hertz-Mindlin contact models are mainly used to calculate the shared forces at contacts. In the linear model, a total normal contact force is based on a normal stiffness via a total normal displacement. Similarly, an incremental shear force is related to a shear tangential stiffness via an incremental shear displacement (Cheng 2004). The normal and shear tangential stiffness are computed from the stiffness of the two contacting particles (A and B) assuming that they act in series (Eqs. 6 and 7).

$$K^n = \frac{k_n^{[A]} k_n^{[B]}}{k_n^{[A]} + k_n^{[B]}} \qquad (6)$$

$$K^s = \frac{k_s^{[A]} k_s^{[B]}}{k_s^{[A]} + k_s^{[B]}} \qquad (7)$$

where K^n = effective normal stiffness; K^s = effective shear contact stiffness; $k_n^{[A]}$ and $k_n^{[B]}$ = normal stiffness of particles A and B respectively; and $k_s^{[A]}$ and $k_s^{[B]}$ = shear stiffness of particles A and B respectively.

The forces and displacements are linearly related by the constant stiffness in the linear model. In the Hertz-Mindlin model, the particle shear modulus and Poisson's ratio are specified and a non-linear force displacement relationship governs the contact behavior. The normal compressive force increases linearly with an overlap, and there is no limit on the force (Itasca 2004).

5.2.2 Bonding models

A contact bond or a parallel bond can be used to model the tensile strength of the geosynthetic. The contact bonds act over a vanishingly small area and can be envisioned as a pair of elastic springs at a point of glue. No slippage occurs at the contact bond though the particles can rotate. The parallel bond can resist both the forces and moments. The contact bond serves as a cut-off tool for the tensile normal force and the contact shear force. The contact bond breaks when either of these two forces exceeds the prescribed bond strength. The normal tensile force is limited to its bond strength (Itasca 2004).

5.2.3 Slip and separation model

The slip and separation model is mutually exclusive to a contact bond. In absence of the contact bond, the slip model limits the maximum shear force between the two interacting particles using a simple frictional law. In other words, slippage between two particles can occur only when the particles are not bonded or the bond is already broken. Slippage will occur if the shear force exceeds the maximum shear resistance given by Eq. 8:

$$F_{\max}^s = \mu \left| F^n \right| \qquad (8)$$

where $F_{\max}^s$ = the maximum allowable shear resistance; μ = the minimum coefficient of friction of two interacting particles; and F^n = the normal component of contact forces at the contact points.

When the two interacting particles are separated by a finite distance, the normal force does not exist and this model is not implemented (Itasca 2004).

The simulation of geosynthetic requires at least a contact bond model in addition to the contact stiffness and the slip and separation models (Konietzky et al. 2004; McDowell et al. 2006; Bhandari et al. 2009a; Bhandari and Han 2010). The simulation of planar geosynthetic such as geotextile is straightforward; a string of particles connected by a contact bond can be generated to represent the geotextile. The bond strength between the particles characterizes the ultimate tensile strength of the geotextile. In additional to the contact bond, contact stiffness, and slip and separation models, the simulation of planar geosynthetic such as geogrid requires parallel bond between the particles to simulate a bending stiffness of the geogrid. It should be noted that a two-dimensional model cannot represent an actual geometry of the geogrid; however can be useful to study the interaction mechanisms. For example, a string of interconnected large and small particles arranged in such a way to represent the geogrid aperture size can capture the aggregate-geogrid interlock mechanisms (Bhandari et al, 2009a). A three-dimensional model is required to study the interaction mechanisms between soil and cellular reinforcement such as geocell.

Compared with continuum modeling, micro-mechanical modeling is relatively new and the modeling approach is still in development and consequently its application is not as wide as continuum modeling. The number of particles required to simulate even a simple real engineering problem can easily approach tens of millions. Using the current DEM approach and computational capacity, modeling routine engineering problems can be daunting. However, the usage of the micro-mechanical modeling in studying the engineering problems in general and geosynthetic-reinforced earth structures in particular will become more and more common. The micro-mechanical modeling has been successfully used to investigate soil-geosynthetic interactions. Even though the past studies are mostly qualitative in nature, they are vital to our understanding of soil-geosynthetic interactions which cannot be easily studied using continuum modeling. Some of the representative completed studies and their findings will be discussed in the following section.

5.3 Examples of Completed Studies

5.3.1 Pullout test

The microscopic parameters of geosynthetic and soil can be calibrated using pull-out and other simple test data. A biaxial compression test and a geogrid pull-out test in a horizontal plane were used to derive the microscopic properties of soils and the geogrid-soil interface in PFC^{3D} to study the geogrid-anchorage mechanisms (Chareyre and Villard 2003). Using the calibrated parameters, Chareyre and Villard (2003) simulated the force-displacement curves for the geogrid anchorage in sand and silt using PFC3D. Large fluctuations in the force were observed from the numerical results when the geogrid was anchored in sand, but the force-displacement curve was smooth with constant periodicity in force fluctuations when the geogrid was anchored in silty sand. The anchorage failure mechanism was qualitatively captured by the PFC^{3D} in both cases. Similar pull-out tests with different anchorage shapes of geotextiles were simulated numerically (Villard and Chareyre 2004). Villard and Chareyre (2004) considered an L- and a V-shaped anchorage in their laboratory models. The analytical, laboratory, and numerical models derived consistent anchorage failure mechanisms of the geotextile-reinforced slopes. Villard and Chareyre (2004) concluded that the failure of the geotextile-soil interface was dominant for cohesive soil while the instability of anchoring soil mass as well as the interface failure was dominant for cohesionless soil.

The interlocking effect of geogrid helps mobilize the intrinsic capacity of aggregates which depends upon the aperture size of geogrid and the diameter of aggregates. Using the PFC^{3D} model of laboratory pull-out tests, McDowell et al. (2006) demonstrated that the ratio of aperture size to aggregate diameter played an important role in the peak strength of the geogrid-reinforced aggregate and its corresponding displacement as shown in Fig. 2. For a tested sample with 40-mm diameter aggregates, the optimum interlock was observed at a ratio of 1.4. The distribution of the average shear force in the model showed that the interlocking effect of the geogrid was confined to a relatively narrow thickness of 10 cm below and above the geogrid. The ratio of the peak shear and normal forces at the end of the test to its initial values reached 10. This study clearly demonstrated the confinement effect of geogrids on aggregates (Fig. 2). A similar conclusion was drawn by Konietzky *et al.* (2004). For aggregates with a particle size distribution between 0.6 mm to 20 mm, a punched-drawn geogrid had a confinement influence zone of 10 cm on either side of the geogrid. Beyond this influence zone, the contact forces in aggregates did not change significantly.

5.3.2 Triaxial test

Konietzky et al. (2004) modeled triaxial tests using PFC^{3D} to study the confinement zone of a geogrid. The back analysis of triaxial tests conducted for the geogrid-reinforced aggregates showed that the resulted strength increase cannot be attributed solely to the tensile stress of the geogrid. The difference on the deviatoric stress between the reinforced and unreinforced samples to produce the same deviatoric strain was larger than the mobilized tensile stress of the geogrid. The additional strength of the reinforced sample must have been derived from the confinement effect of the geogrid. Subsequent studies by McDowell et al. (2006) showed the importance of aggregate shape on the peak strength and dilatancy behavior of railway ballasts. McDowell et al. (2006) also investigated the effect of three layers of geogrids on the displacement of the model as compared with one layer of geogrid reinforcement. The numerical results showed that the single-layer geogrid-reinforced aggregate had 50% more axial and radial displacements compared to the three-layer geogrid-reinforced aggregate. This comparison indicated the benefit of multiple layers of geogrids on reinforcing the aggregates.

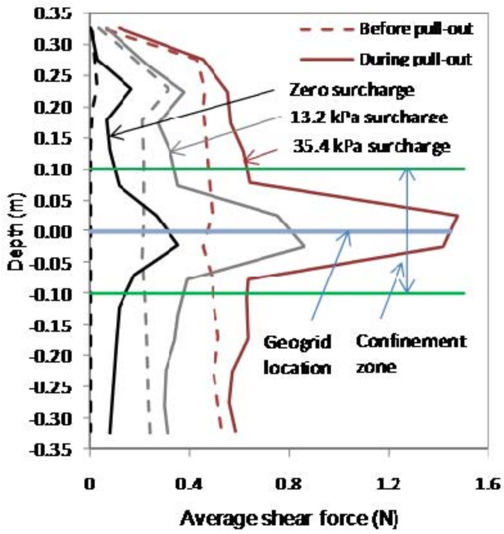

Figure 2. Average shear force before and at end of the pull-out test (modified from McDowell et al. 2006)

5.3.3 Application of PFC^{2D} in geosynthetic-reinforced pile-supported embankments

In many geotechnical engineering applications, soil arching is an ubiquitous phenomenon and has been studied for the past seven decades since Terzaghi (1943). The details of soil arching and the governing factors can be found in (Giroud, J.P., et al. 1990; McKelvey 1994; Han and Gabr 2002; Jenck et al. 2007; Chen et al. 2008). Several governing differential equations have been proposed based on experimental observations at a macro level which led to a lack of consensus method for designing a structure that inolves soil arching. Understanding this phenomenon at a microscale can improve the design methods. Soil arching can be studied at microscale using Photoelastic Discrete Simulation (PDS) and or the DEM simulation. To observe the soil arching at a micro level, PDSs were successfully used in a laboratory (Tien 2001). Tien (2001) concluded that soil arching develops at a small movement (2 mm) of the trapdoor and particle translations contribute to the soil arching formation more than particle rotations.

Recently, research has focused on a coupled finite-discrete model of the geosynthetic-reinforced embankment over piles. The coupled finite-discrete model accommodates the fibrous structure of geosynthetics including the tensioned membrane effect. Le Hello and Villard (2009) proposed a specific three node Finite Element Model (FEM) to simulate the geosynthetic. The proposed FEM also depended on the Newton second law of motion and can be incorporated into the DEM computation. Meanwhile, the discrete model of the embankment fill evaluates the force transfer between the particles at the particle contact levels which is the key mechanism of soil arching (Le Hello and Villard 2009; Villard et al., 2009). The interactions between the geosynthetic and embankment fill materials occur at contact points. Specific contact laws govern those interactions. Le Hello and Villard (2009) observed that the stiffness of the geosynthetic had a minimum influence on the load transfer mechanism but governed the maximum geosynthetic deformation. This finding signifies that the load transfer to the pile caps due to soil arching develops at small particle movement in the embankments. The displacement vector of the particles in the embankments indicated a triangular shape of soil arching. Bhandari et al. (2009b) also reported the triangular shape of soil arching.

Jenck et al. (2009) presented a two-dimensional small-scale model study as well as discrete and continuum-based numerical modeling of the pile-supported granular platform. Their study showed that the DEM model closely predicted the behavior of the platform, for example, the efficiency (ratio of the load shared by piles to the total load of the platform), as compared with the continuum model even though both approaches over-predicted the experimental value. Furthermore, they pointed out that the stress and strain in the continuum model are coupled while the discrete model has less restriction on the stress and strain relationship. As a consequence, the increased friction angle of the platform material increased the efficiency and reduced the settlement in the continuum model. However, the reduction in the settlement in the discrete model was insignificant.

Han et al. (2010) modeled an unreinforced and a geogrid-reinforced embankment to investigate the differences between the unreinforced and the reinforced embankments on stress and displacement development. Figure 3 shows the contact force distribution for the unreinforced and reinforced embankments at the development of soil arching. The contact forces were plotted on the same scale. Black lines indicated compressive contact forces and red lines indicated tensile contact forces. The thickness of lines represented the relative magnitude of contact forces. The weight of the embankment portion between the pile caps transferred to the adjacent pile caps due to the reorientation of contact forces. Therefore, the compressive contact forces were concentrated over the pile caps for both unreinforced and reinforced embankments. The remaining weight of the embankment portion between the pile caps transferred to the underlying compressible soil in the unreinforced embankment. The geogrid transferred the remaining weight of the embankment portion between the pile caps to the pile caps through the cable action of the geogrid in case of the reinforced embankment. A negligible amount of forces transferred to the compressible soil as the geogrid barely rested over the compressible soil. Furthermore, it is evident that the contact forces at the mid-span of the embankments were increasingly aligned on a horizontal direction towards the top of the embankments. Monitoring the orientation of particle-particle contacts and forces, Han et al. (2010) concluded that the embankment load was transferred to the pile caps by reorientation of principal stresses.

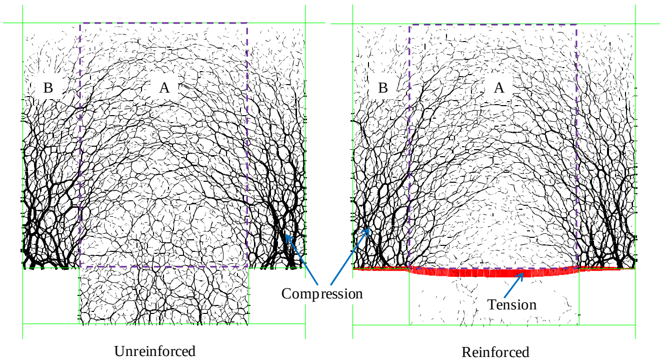

Figure 3. Contact force distribution at the development of soil arching (Han et al. 2010)

Han et al. (2010) also presented the displacement vectors of the soil particles in the unreinforced and reinforced embankments (Fig. 4). The same magnitude of the maximum displacement was used for these two figures for comparison. A triangular shape delineated a zone of larger displacements from the zone of smaller displacements in either case. Denser and longer displacement vectors illustrated larger settlements in the unreinforced embankment than those in the reinforced embankment. A gap between the displacement vectors and the top wall was distinctly observed for the unreinforced embankment that indicated the full height of the embankment was affected by the particle movement resulting from the compression of the compressible soil. The geogrid reduced the overall displacements in the reinforced embankment.

5.3.4 Application of PFC^{2D} in geosynthetic-reinforced roadways

Geosynthetics are increasingly used for subgrade improvement by placing the reinforcement at the interface between base and subgrade or base reinforcement by placing the reinforcement within the base course for roadway applications, such as unpaved and paved roads and railroads. The benefit of the geosynthetic in the improvement of roadways depends on the interaction of the geosynthetic and the soil while the interaction itself depends on properties of soil (subgrade and base), geosynthetic, and the location of geosynthetic in the roadways section. By summarizing the findings of previous experimental research, Perkins and Ismeik (1997) concluded that the proper location of the geosynthetic within the base course depended on the magnitude of the applied load and the strength of the subgrade layer.

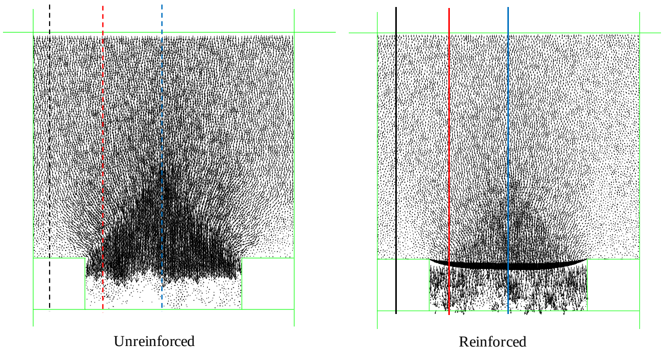

Figure 4. Displacement vectors for unreinforced and reinforced embankments (Han et al. 2010)

The DEM simulation of geosynthetic-soil interaction tests have been used to study the effect of geosynthetic stiffness, geosynthetic placement depth, and magnitude of the applied load on the performance of geosynthetic-reinforced roadways (Bhandari 2010; Bhandari and Han 2010). Using the DEM model (Fig. 5), Bhandari and Han (2010) concluded that the development of tensile stresses in the geotextile helped improve the performance of the reinforced section. The geotextile placed at a deeper depth had small tensile stresses and was less effective in minimizing the surface deformation caused by the cyclic wheel load as compared with the geotextile placed at a shallower depth. Furthermore, the geotextile placement distributed the contact forces to a wider area compared to the unreinforced roadways sections as illustrated in Fig. 6 (Bhandari et al. 2008). The DEM analysis also demonstrated that a low tensile strain developed in the geotextile under cyclic wheel loading. An increase in the stiffness of the geotextile showed a marginal improvement of the performance, particularly when the geotextile was placed at a deeper depth (Bhandari and Han 2010).

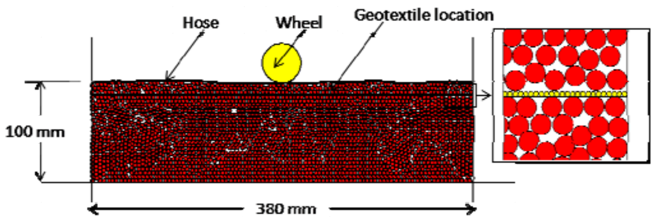

Figure 5. DEM model of APA test simulation (Bhandari and Han 2010)

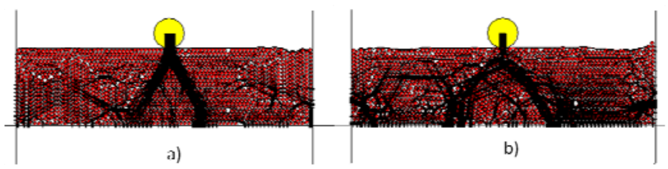

Figure 6. Contact force distribution for a) unreinforced b) geotextile-reinforced roadways (Bhandari et al. 2008)

6. CONCLUSION REMARKS

Numerical modeling has been playing an important role in the study and practice of geosynthetic-reinforced structures, which primarily includes continuum and micro-mechanical approaches. Continuum approach has been successfully used to investigate almost every aspect of the four discussed earth structures. In that course, various constitutive models for soils, geosynthetics, and interfaces have been developed. The selection of the suitable models should consider the materials' intrinsic properties. The recent developed micro-mechanical modeling has been increasingly used to study the interaction of soil-geosynthetic. Even though it is still in a preliminary stage, numerous successful examples are available in the literature, which provide references for application and development. Summarily, numerical modeling is a powerful tool to study the geosynthetic-reinforced earth structures.

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