



Inventory Routing Problem with Vehicle Resource Sharing in the Two-depot and Multi-retailer System

Wisut Supithak* and Titirat Vivithkeyoonvong

Industrial Engineering Department, Faculty of Engineering, Kasetsart University, Bangkok, 10220, Thailand

* Corresponding author. E-mail address: fengwsst@ku.ac.th

Received: 22 March 2021; Revised: 10 June 2021; Accepted: 21 June 2021

Abstract

The research considers the problem of determining vehicle routes and replenishment intervals in the system composed of two depots supplying an inventory item to multiple retailers. Each transportation vehicle leaving a depot can end its route at any depot and, therefore, the vehicle route can be either close loop or open loop. However, the number of vehicles leaving any depot must equal to the number of vehicles arriving the depot. The objective is to minimize the sum of inventory holding cost, delivery setup cost, transportation cost, and vehicle owning cost. A genetic algorithm is developed in order to determine a good solution to the problem when there exists at most one delivery route for each depot. The chromosome representation is designed in such a way that, after decoding, a chromosome can represent both types of vehicle route. To determine the optimal replenishment interval, the concept of economic order interval with joint replenishment is inserted in the genetic algorithm structure. Two numerical experiments are conducted for the performance evaluation. The first experiment is to compare the genetic algorithm solution with the optimal solution obtained from the enumerative search. The result shows that the proposed method can provide optimal solution for 27 out of 30 randomly generated problems. The maximum percentage deviation is 1.79 percent. The second experiment is to compare the genetic algorithm solution with the solution yielded from a three-step heuristic at different levels of number of retailers and vehicle owning costs. According to the experimental result, the proposed method can provide better solution for all 270 randomly generated problems with the average percentage deviation of 10.33 percent. As the number of retailers and the vehicle owning cost increase, the open vehicle route tends to yield better solution than the close vehicle route.

Keywords: Inventory Routing Problem, Open Vehicle Routing, Vehicle Sharing, Genetic Algorithm

Introduction

The vehicle routing problem (VRP) is one of the most critical tasks facing most organizations today. Traditionally, the problem involves determination of vehicle routes in a system consisted of one depot and multiple customers in such a way that the total cost (or total distance) incurred from the vehicle transportation is minimized. Each vehicle must start and end its travelling route at the depot. The problem was first addressed by Dantzig and Ramser (1959) with an application of examining proper routes for gasoline delivery trucks. Since then, the VRP and its variants have gained a lot of attentions among researchers. Seeking for the optimal solution to the VRP is NP-hard (Lenstra & Rinnooy-Kan, 1981). Many heuristics and metaheuristics are developed to determine the good solution to the problem. Comprehensive reviews of the VRP can be found in the works of Braekers, Ramaekers, and Nieuwenhuys (2016) and Modhi, Burhanuddin, and Asaad (2017).

Recently, one of the most widely considered research involving the extension of vehicle routing problem is the inventory routing problem (IRP). The IRP considers the inventory replenishment policy at customer, such as whom to serve and how much to deliver, while making decision on transportation routes. The objective is to minimize total transportation cost such that demands and capacity constraints at customers are not violated. The tutorial paper of IRP is conducted by Bertazzi and Speranza (2012). Viswanathan and Mathur (1997) discussed



a method based on the power of two policy combined with the economic order quantity (EOQ) model to determine which customers to visit and how many units should be delivered at each replenishment period. For the similar problem, Campbell and Savelsbergh (2004) proposed a two phases approach which is to first create a delivery schedule utilizing the integer programming and then construct a set of delivery route employing routing and scheduling heuristics. Serna, Cortes, and Daniela (2015) presented the genetic algorithm (GA) to determine good solution for the IRP in a system composed of multiple depots. In the past few decades, the extension of VRP to the open vehicle routing problem (OVRP) has attracted attention of practitioners and researchers. For the OVRP, the vehicle may not return to the original depot after visiting the last customer of the route or it may return by revisiting all customer in the route with reverse order. The OVRP was first introduced by Sariklis and Powell (2000). The research suggested a heuristic method based on minimum spanning tree with penalties to determine a good solution. Various solving methods relevant to the determination of OVRP solution were discussed by Feiyue, Bruce, and Edward (2007). Gurpreet and Vijay (2014) presented an approach based on an ant colony algorithm for solving the open vehicle routing problem of school buses. The adaptation of the OVRP to the IRP was discussed by Supithak and Supithak (2018). They proposed a method based on cluster-first route-second concept which is to first assign those retailers to be served at each time of replenishment according to the power of two policy and then determine the vehicle routes utilizing the ant colony optimization (ACO).

This research considers the integration of OVRP in the IRP with vehicle resource sharing. The distribution system composes of two depots supplying an inventory item to multiple retailers. Any transportation vehicle can end its route at any depot which create the solution space comprised of both close and open vehicle routes. Nonetheless, for each replenishment interval, the number of vehicles entering any depot must equal to the number of vehicles leaving the depot. The objective is to determine vehicle routes and inventory replenishment interval in such a way that the sum of inventory holding cost, delivery setup cost, transportation cost, and vehicle owning cost is minimized.

Problem Statements

Problem Notations

The following notations are applied throughout the paper.

- TTC* total annual cost (dollars/year)
- HC* annual holding cost (dollars/year)
- SC* annual delivery setup cost (dollars/year)
- TC* annual transportation cost (dollars/year)
- VC* annual vehicle owning cost (dollars/year)
- m* number of depots in the system
- n* number of retailers in the system
- C* item cost (dollars/unit)
- f* holding cost fraction
- D_j* annual demand of retailer *j* (units/year); for $j = 1, 2, \dots, n$
- Q_j* replenishment quantity of the retailer *j* (units); for $j = 1, 2, \dots, n$



- S_i delivery setup cost incurred at the depot i for each replenishment trip (dollars/replenishment); for $i = 1, 2, \dots, m$
- L_i total distances of the route starting at depot i (kilometer); for $i = 1, 2, \dots, m$ number of replenishments made by depot i (replenishments/year); for $i = 1, 2, \dots, m$
- T_i time between replenishments made by depot i (year); for $i = 1, 2, \dots, m$
- M_i number of replenishments per year made by the depot i (year); for $i = 1, 2, \dots, m$
- V_i number of vehicles owned by the depot i ; for $i = 1, 2, \dots, m$
- k vehicle owning cost (dollars/vehicle/year)
- r fuel consumption cost (dollars/kilometer)
- X_{ij} binary variable; for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$
 $X_{ij} = 1$, if the retailer j is replenished by supplier i ; otherwise $X_{ij} = 0$
- Q_{max} vehicle capacity (units)
- N number of parent chromosomes
- p_c probability of crossover
- p_m probability of mutation
- TC_p annual total cost of the chromosome p ; for $p = 1, 2, \dots, 2N$
- F_p fitness value of the chromosome p ; for $p = 1, 2, \dots, 2N$
- P_p probability that the chromosome p will be selected; for $p = 1, 2, \dots, 2N$

Problem Characteristics

There is a distribution system consisted of two depots and multiple retailers. Each retailer has its own constant demand rate for an item which must be satisfied by one of the two depots. Any transportation vehicle starting the route at a depot can end its route at any depot and, hence, the delivery route can be either close loop or open loop. In order to balance the number of vehicles of both depots, for each replenishment interval, the number of vehicles departing any depot must equal to the number of vehicles entering the depot. All vehicles have equally limited capacity and owning a vehicle incurs cost to the system. The objective is to determine proper delivery routes and replenishment intervals in order to minimize the total annual cost (TTC) composed of inventory holding cost (HC), delivery setup cost (SC), transportation cost (TC) and vehicle owning cost (VC). The total annual cost calculation is shown in equation (1).

$$TTC = HC + OC + SC + VC \quad (1)$$

The inventory holding cost of the system is the sum of inventory holding cost at all retailers. The calculation of annual inventory holding cost is shown in equation (2).

$$HC = \frac{1}{2} Cf \sum_{i=1}^m \sum_{j=1}^n (X_{ij} D_j T_i) \quad (2)$$

The delivery setup cost is the fixed cost incurred when a trip is initiated at the depot and assumed to be independent of the travelling distance. Some examples of delivery setup cost are driver cost, vehicle preparation cost, and those costs associated with delivery document preparation. Equation (3) demonstrates the calculation of the annual delivery setup cost.

$$SC = \sum_{i=1}^m M_i S_i \quad (3)$$

The cost of fuel consumed by the delivery vehicle is considered as the transportation cost. This cost is dependent on the travelling distance. The annual system transportation cost is the sum of transportation costs acquired by all depots in a year and can be calculated as follows.

$$RC = r \sum_{i=1}^m M_i L_i \tag{4}$$

Owning a vehicle can incur cost to the system. This cost can be viewed as the cost of vehicle depreciation. The calculation of annual vehicle owning cost is demonstrated in equation (5).

$$VC = k \sum_{i=1}^m V_i \tag{5}$$

The research investigates the case when there exists at most one delivery route originating at each depot and all retailers have equal time between replenishments. The problem characteristic is illustrated in the following example.

Problem Example

Given that the system composes of two depots, A and B, supplying an inventory item to eight retailers. The annual demands of eight retailers are 3,200 1,400 1,900 1,600 2,700 2,900 2,900 and 4,500 units. The item cost is assumed to be 180 dollars per unit. Holding cost fraction is 0.20. Any delivery trip initiates the fixed setup cost at the depots A and B of 50 dollars and 80 dollars, respectively. All delivery vehicles have the same capacity of 500 units. The fuel consumption rate is 0.5 dollars per kilometer. The vehicle depreciation cost is expected to be 20,000 dollars per year. The locations of both depots and all retailers along with distance matrix associated with them are illustrated in figure 1.

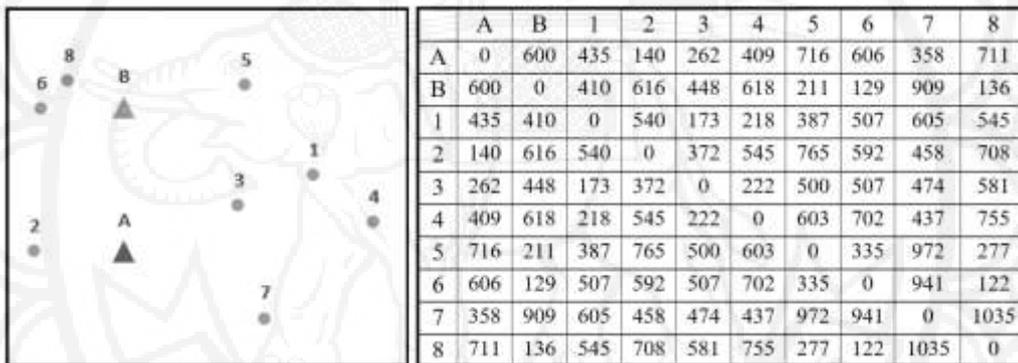


Figure 1 Locations of depots (nodes A and B) and retailers (nodes 1 to 8)

One possible solution (may not be optimal) is to have two open routes with replenishment interval of 10 days each. The first route is A → 2 → 6 → 8 → B with the total distance of 990 kilometers and the second route is B → 5 → 1 → 4 → 3 → 7 → A with the total distance of 1,870 kilometers. Since there are two open delivery routes, only one vehicle may be used. Given that there are 360 working days in a year, the two delivery routes and the calculation of replenishment quantity of each retailer are illustrated in figure 2.

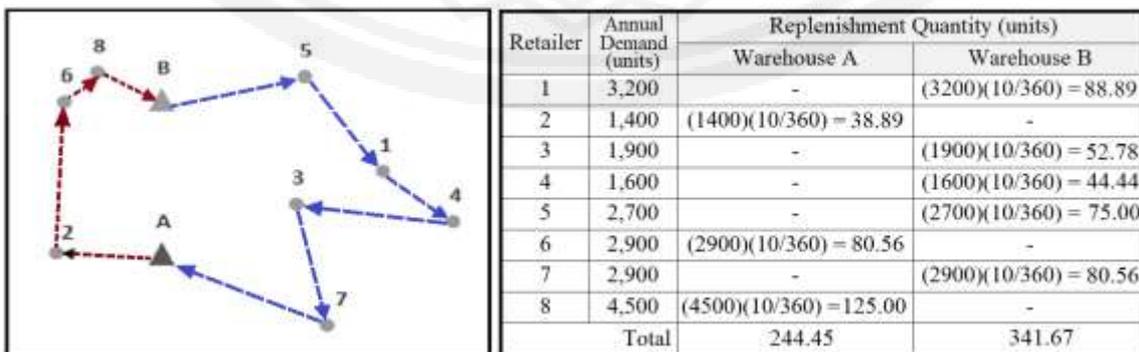


Figure 2 Two open vehicle routes and replenishment quantity of each retailer



All relevant costs of the solution previously mentioned can be calculated as follows.

$$HC = \left(\frac{1}{2}\right)(180)(0.2)(244.45 + 341.67)$$

$$= 10,550.16 \text{ dollars per year}$$

$$SC = (50)\left(\frac{360}{10}\right) + (80)\left(\frac{360}{10}\right)$$

$$= 4,680 \text{ dollars per year}$$

$$TC = (5)\left(\left(\frac{360}{10}\right)990 + \left(\frac{360}{10}\right)1,870\right)$$

$$= 514,800 \text{ dollars per year}$$

$$VC = (1)(200,000)$$

$$= 20,000 \text{ dollars per year}$$

$$TTC = (10,550.16 + 4,680 + 514,800 + 20,000)$$

$$= 550,030.16 \text{ dollars per year}$$

Methods

Genetic Algorithm

A genetic algorithm is developed to determine a good solution to the research problem in a reasonable amount of time. Figure 3 illustrates the genetic algorithm procedure employed in this study. The structure of genetic algorithm can be explained as follows.

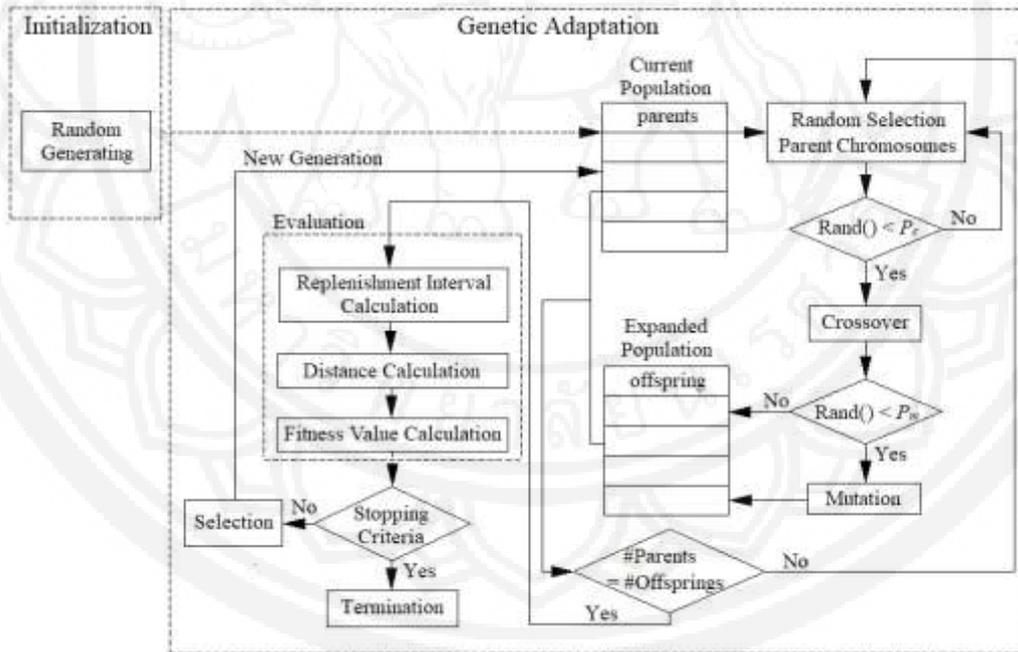


Figure 3 GA procedure

1. Chromosome Representation and Initialization

To explore the solution space composed of open vehicle routes and close vehicle routes, the chromosome encoding is designed such that a chromosome can represent both types of vehicle route. Here, each chromosome composes of $n + 1$ genes. All chromosome genes are nonnegative integer numbers taking values from “0” to



“*n*”. They are all different from each other. Number “0” is used to distinguish vehicle routes and numbers from “1” to “*n*” represent retailers in the system.

As an example of chromosome representation, the list of [2 3 1 4 6 0 8 5 7] can be decoded to two different solutions. The first solution composes of two close vehicle routes, which are (A → 2 → 3 → 1 → 4 → 6 → A) and (B → 8 → 5 → 7 → B). The second solution composes of two open vehicle routes, which are (A → 2 → 3 → 1 → 4 → 6 → B) and (B → 8 → 5 → 7 → A). The concept of joint inventory replenishment policy is applied to determine the proper inventory replenishment interval for each vehicle route of each solution. The solution with lower total cost is designated as the result of chromosome decoding. In the chromosome initialization step, there are *N* chromosomes being randomly generated.

2. Crossover

Among several types of crossover operators, the uniform order-based crossover is recognized to be best fit for sequencing problems (Cheng, Gen, and Tosawa, 1995; Supithak and Plongon, 2011). Therefore, this type of crossover is deployed in the research. The following example illustrates the characteristics of uniform order-based crossover.

Consider two parent chromosomes and a bit string having the same number of genes as the parents.

Parent 1	5	3	1	4	0	6	2	7	8
Parent 2	7	2	0	5	1	8	4	3	6
Binary Template	0	1	1	0	1	0	1	1	0

The child 1 (or child 2) is first constructed by copying genes from Parent 1 (or Parent 2) at those positions that the bit string contains the value of “1” (or “0”). The result is shown below.

Child 1	-	3	1	-	0	-	2	7	-
Child 2	7	-	-	5	-	8	-	-	6

The operator then copies those genes, from Parent 2 (or Parent 1), which are not included in Child 1 (or Child 2) and paste them to the available positions of Child 1 (or Child 2) with the same order as appear in the Parent 2 (or Parent 1). The complete child chromosomes can be demonstrated as follows.

Child 1	5	3	1	8	0	4	2	7	6
Child 2	7	3	1	5	4	8	0	2	6

To perform the crossover operation, two parent chromosomes are randomly selected from the current population. Each pair of selected parent chromosomes has equal probability of crossover occurrence (p_c). In order to evaluate if the crossover operation should be performed, a real random number within the range of [0, 1] is generated. The crossover occurs only when the value of random number generated is lower than the p_c value.

3. Mutation

The random exchanging mutation is applied in the research. Two genes of a chromosome are randomly selected to exchange their positions. All children created from the crossover operation have equal probability of mutation (p_m). Here, a real random number within the range of [0, 1] is generated. The random exchanging mutation only occurs to the child when the value of random number is less than the p_m value. Otherwise, the child is not mutated.



4. Evaluation

Chromosome evaluation is conducted to determine the quality of each chromosome. A chromosome with better quality has higher chance to be selected as a member of the next generation. The total cost mentioned in equation (1) is used to represent the quality of each chromosome. The transportation cost incurred from each vehicle route can be determined directly according to the sequence of retailers as appearing in the chromosome representation. Nonetheless, in order to calculate the inventory holding cost, it is necessary that the proper inventory replenishment interval must be determined. Since all retailers have the same replenishment interval, the replenishment intervals made by both depots are equal ($T_i=T_1=T_2$). Without the vehicle capacity limitation, the replenishment interval made by each depot can be calculated according to the economic order interval with joint replenishment policy as shown in equation (6).

$$T_i = \sqrt{\frac{2 \sum_{i=1}^2 (S_i + rL_i)}{cf(\sum_{j=1}^n D_j)}} \quad (6)$$

In the research, each vehicle has limited capacity, which restrict the length of joint replenishment interval. The joint replenishment interval with vehicle capacity limitation can be determined by equation (7).

$$T_i = \min \left(\sqrt{\frac{2 \sum_{i=1}^2 (S_i + rL_i)}{cf(\sum_{j=1}^n D_j)}}, \frac{Q_{max}}{\sum_{j=1}^n X_{1j} D_j}, \frac{Q_{max}}{\sum_{j=1}^n X_{2j} D_j} \right) \quad (7)$$

To compare the quality of a specific chromosome with other chromosomes of the same generation, it is necessary that the fitness value of each chromosome must be determined. The fitness value is calculated in such a way that those chromosomes having low total costs have high fitness values and, therefore, they have more chances to be selected as a member of the new generation. Given that TC_p is the total cost of chromosome p , equations (8) and (9) demonstrate the calculation of chromosome fitness value and probability of each chromosome being selected, respectively. Note that there are $2N$ chromosomes in each generation.

$$F_p = \frac{1}{TC_p} \quad (8)$$

$$P_p = \frac{f_p}{\sum_{k=1}^{2N} f_k} \quad (9)$$

5. Selection

The selection is accomplished on the current enlarged population (parents + offspring) to generate a new population for the next generation. The elitist method along with roulette wheel selection are applied to perform this step. Here, the best fitted chromosome from the current generation is reproduced in the new generation first and then the roulette selection is performed. The concept of roulette wheel selection is to reproduce the next generation in such a way that a better chromosome has larger chance to be reproduced. Each individual chromosome is assigned a slice of a circular roulette wheel. The size of a slice depends on the fitness value. The basic roulette selection spins the wheel N times in order to create N chromosomes for the new generation.

Experimental Results and Discussion

To evaluate the performance of purposed GA discussed previously, the procedures are coded in the Blocks IDE (C Language) and run on the Intel Core i5, CPU 1.70GHz, RAM 4.00GB, 64-bit Operating System. Two stopping criteria are implemented in the heuristic. The first criterion terminates the procedure when the number of generations reaches 10,000. The second criterion stops the procedure when the improvement of the



solution is less than 0.01% after 300 consecutive generations. The search is ended when any of the two criteria is met. In order to construct problems for the numerical experiment, the annual demand of each retailer is randomly generated from a discrete uniform random number of [1000, 5000]. The location of each retailer is created using Cartesian coordinate system in which the values of horizontal axis and vertical axis are randomly generated from discrete uniform random numbers [0, 800] and [0, 1800], correspondingly. The two warehouses are located at (400, 600) and (400, 1200). The proper probabilities of crossover and mutation are determined by running the GA on thirty problems consisting of two-warehouse and eight-retailer at different percentages of crossover ($P_c = 0.6, 0.7, \text{ and } 0.8$) and mutation ($P_m = 0.1, 0.2, \text{ and } 0.3$). It has been found that the combination of ($P_c = 0.7, P_m = 0.1$) provided the best result and, hence, this combination has been selected as parameters for further study.

Two experimental studies are conducted. The first experiment is to compare the solution obtained from the GA with the optimal solution determined by the enumerative method on 30 randomly generated problems of two depots and eight retailers. From the study result, the GA can provide optimal solution for 27 out of 30 problems with the maximum deviation percentage of 1.79 percent. The second experiment is to evaluate the performance of the GA at different number of retailers and vehicle owning costs. Note that the vehicle owning cost is of interest because this factor may affect the type of transportation route in the problem solution. The experiment is conducted on three levels of number of retailers (5, 8, and 12) and three levels of vehicle owning cost per year (10000, 20000, and 40,000). There are totally 270 problems created from 30 replications of 9 treatment combinations. The proposed GA solution is compared with the solution obtained from a three-step heuristic. The heuristic is to first assign retailers to each warehouse according to the work of Olivera, Enayatifar, Sadaei, Guimaraes, and Potvin (2016) and then apply the greedy algorithm to search for the proper transportation route of each cluster. The last step is to determine the inventory replenishment interval using the concept of EOI with joint ordering. The percentage deviation value, serving as the response variable, is calculated as shown in equation (10).

$$\%Dev = \frac{TC_{HEU} - TC_{GA}}{TC_{HEU}} \times 100 \quad (10)$$

where, TC_{HEU} is the total cost obtained from the three-step heuristic and TC_{GA} is the total cost yielded from the proposed GA. According to the experimental result, the proposed GA provides better solution (lower total annual cost) for all 270 problems with the average percentage deviation of 10.33 percent. The average computational time for the GA is approximately 9.88 seconds.

To determine the main effects of number of retailers (Factor A) and vehicle owning cost (Factor B) on the performance of GA, the percentage deviation is used to conduct the Kruskal-Wallis test for multiple comparisons. The results are shown in figure 4.

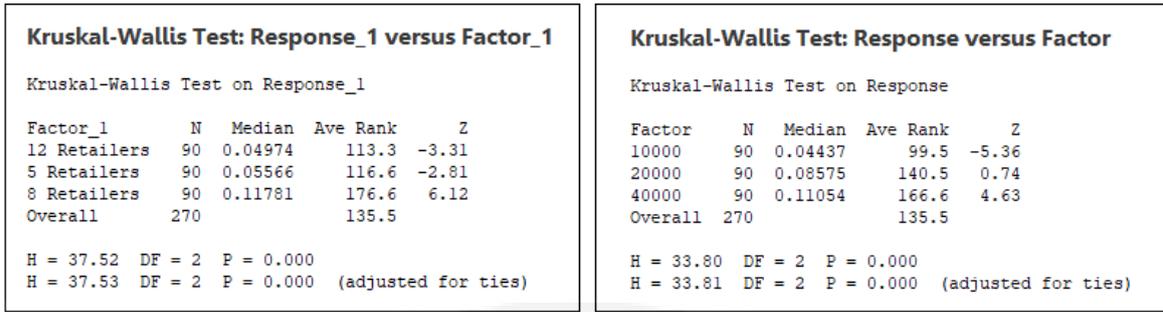


Figure 4 Test results of the main effect of factors A and B

From figure 4, the main effects of factors A and B are found to be significant (at the 0.05 level of significance). The Wilcoxon Signed Rank method is applied to determine the confidence interval on percentage deviation at each level of each factor. The results are illustrated in figure 5. Consider the factor A, the percentage deviation at the level of $n = 8$ retailers is largest and significantly different from other levels. For the factor B, the percentage deviation at the level of $k = 10,000$ is smallest and significantly different from other levels. The advantage of the GA is more pronounced at higher level of vehicle owning cost. Figure 6 demonstrates percentages of open vehicle route solution and close route solution, at different level of factors A and B, obtained from the GA. It can be seen that as the number of retailers and vehicle owning cost increase, the open vehicle route tends to yield better solution than the close vehicle route. The result emphasizes the benefit of applying open vehicle routing strategy, which allows depots to share their transportation vehicles, to those distribution systems having large number of retailers and high vehicle owning costs.



Figure 5 Confidence intervals on the percentage deviation at different levels of factors A and B

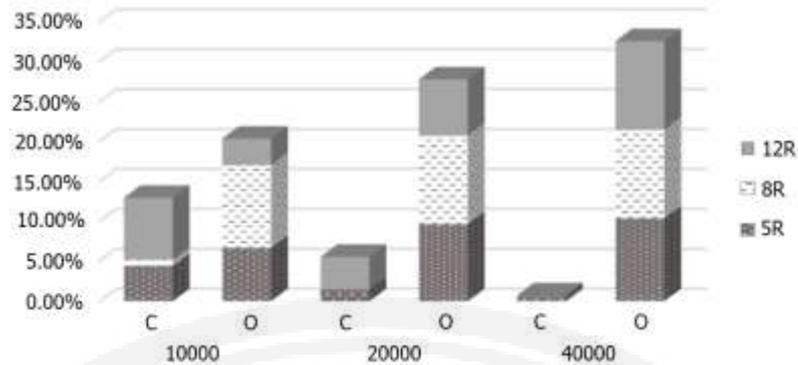


Figure 6 Percentages of open vehicle route solution and close vehicle route solution

Conclusion

The research proposed a genetic algorithm to determine the proper vehicle routes and replenishment interval in the system composing of two depots supplying an inventory item to multiple retailers. For the delivery, each vehicle can end its route at any depot. However, for any depot, number of vehicles leaving the depot must equal to number of vehicles entering the depot. The objective is to minimize the sum of inventory holding cost, delivery setup cost, transportation cost, and vehicle owning cost. The chromosome representation is designed such that, after decoding, a chromosome can represent both open and close vehicle routes. In order to determine the proper replenishment interval of each retailer, the concept of economic order interval with joint replenishment is inserted in the structure of GA. According to the experimental result obtained from 30 random generated problems of 2 depots and 8 retailers, the GA can provide optimal solution for 27 out of 30 problems. To evaluate the performance of GA at different levels of number of retailers and vehicle owning costs, the solution obtained from the GA is compared with the solution yielded from a three-step heuristic. It has been found that the GA can provide better solutions for all 270 random generated problems. At high levels of number of retailers and vehicle owning cost, the open vehicle route tends to provide better solution than the close vehicle route.

References

- Bertazzi, L., & Speranza, M. G. (2012). Inventory routing problems: an introduction. *EURO Journal on Transportation and Logistics*, 1(4), 307–326.
- Braekers, K., Ramaekers, K., & Nieuwenhuysen, I. V. (2016). The vehicle routing problem: State of the art classification and review. *Computers & Industrial Engineering*, 99, 300–313.
- Campbell, A. M., & Savelsbergh, M. W. P. (2004). A decomposition approach for the inventory-routing problem. *Transport Science*, 38(4), 488–502.
- Cheng, R., Gen, M., & Tosawa, T. (1995). Minmax earliness/tardiness scheduling in identical parallel machine system using genetic algorithms. *17th International Conference on Computers and Industrial Engineering*, 29, 513–517.
- Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management Science*, 6(1), 80–91.
- Feiyue, L., Bruce, G., & Edward, W. (2007). The open vehicle routing problem: Algorithms, large-scale test problems, and computational results. *Computers & Operations Research*, 34(10), 2918–2930.



- Gurpreet, S., & Vijay D. (2014). Open Vehicle Routing Problem by Ant Colony Optimization. *International Journal of Advanced Computer Science and Applications*, 5(3), 63–68.
- Lenstra, J. K., & Rinnooy-Kan, A. H. G. (1981). Complexity of vehicle routing and scheduling problems. *Networks*, 11(2), 221–227.
- Modhi, L. M., Burhanuddin, M. A., & Asaad, S. H. (2017). Review paper in vehicle routing problem and future research trend. *International Journal of Applied Engineering Research*, 12, 12279–12283.
- Olivera, F. B., Enayatifar, R., Sadaei, S. J., Guimaraes, F. G., & Potvin, J. Y. (2016). A cooperative coevolutionary algorithm for the Multi- Depot Vehicle Routing Problem. *Expert Systems with Applications*, 43, 117–130.
- Sariklis, D., & Powell, S. (2000). A heuristic method for the open vehicle routing problem. *The Journal of the Operational Research Society*, 51(5), 564–573.
- Serna, M. D. A., Cortes J., & Daniela G. S. (2015). Modeling the inventory routing problem (IRP) with multiple depots with genetic algorithms. *IEEE Latin America Transactions*, 13(12), 3959–3965.
- Supithak, A., & Supithak, W. (2018). Determination of inventory replenishment policy with the open vehicle routing concept in a multi-depot and multi-retailer distribution system. *Engineering and Applied Science Research*, 45(1), 23–31.
- Supithak, W., & Plongon, K. (2011). Memetic algorithm for non-identical parallel machines scheduling problem with earliness and tardiness penalties. *International Journal of Manufacturing Technology and Management*, 22(1), 26–38.
- Viswanathan, S., & Mathur, K. (1997). Integrating routing and inventory decisions in one-warehouse multi-retailer multiproduct distribution system. *Management Science*, 43(3), 294–312.