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A Comment On The Ratio Of The Maximum And Minimum Dry Density For Sands

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ABSTRACT: This work reveals an interesting feature of sand behaviour: the ratio of the minimum and the maximum dry density, *D*, is about constant. This statement is discussed here on the basis of the analysis of two databases: the database of the "Calibration Chamber" sands and the database of the Danube sands. Results show that although the *D* values are significantly and systematically different for the Calibration Chamber sands and the Danube sands, the coefficients of variation of the *D* values for each sample population is very small in each case. The difference in the characteristic *D* values in each case is explained in terms of differences in the experimental methods used and differences in the geological origin of the soils.

1. INTRODUCTION

An experimental program based on about 33 samples of sandy sediments from the Danube river basin (artificial mixtures of natural soil grains) yielded the results that the ratio of the minimum and maximum dry density was basically constant, although it decreased slightly as the size of the granular fractions decreased (Kabai 1968, 1972, 1974).

In this paper the dry density data of the Danube sand database is statistically compared to the dry density data of the "Calibration Chamber" sand database for 25 sands, presented by Mayne and Kulhawy (1992) and Lunne et al. (1992).

In the Calibration Chamber sand database the minimum and maximum void ratio data for the different sands are recorded. Statistical methods are used to evaluate which of the density-related sample characteristics or their ratios (if any) can be considered identical for the two databases.

2. DENSITY TEST METHODS AND RESULTS

2.1 Danube Sands

In the experimental program of Kabai (1968, 1972, 1974), about 33 continuous sand-silt mixtures, with a diameter range of d_{min} = 0.063 mm and d_{max} = 20 mm, were tested. The data are reproduced in summary in the Appendix (see Table A-1). The grading curve series were defined in terms of the coefficient of uniformity $C_{U,}$ and the grain size parameters d_{max}, d_{10}. The minimum dry density was determined by using a funnel pouring device, to fill a Proctor mold.

The maximum density was determined with a variant of the modified Proctor test (i.e. ASTM D1557 - 09 Standard Test Method for Laboratory Compaction Characteristics of Soil Using Modified Effort 2700 kNm/m^3). The modification (following the results of Leussink and Kutzner, 1962) involved placing an 8 mm thick steel plate on the top of the sample during compaction to prevent the local loosening of the sample.

2.2 Calibration Chamber Sands

The Calibration Chamber sand database considered here was published by Lunne et al (1992), and Mayne and Kulhawy (1992). The data are reproduced in summary in the Appendix (see Table A-2).

The minimum and maximum dry density data for the various sands were assumed to be determined using the following methods:

(i) ASTM D4253 Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table and
(ii) ASTM D4254 Standard Test Methods for Minimum Index Density and Unit Weight of Soils.

3. STATISTICAL TESTS

3.1 Variables

The results of the density tests can be expressed in terms of the void ratio *e*, the dry density ρ_d, the solid volume ratio *s* or its inverse and, the specific volume *v*. The basic definitions are as follows:

$$s = \frac{1}{v} = \frac{\rho_d}{G_s} = \frac{V_s}{V} \quad (1)$$

$$e = \frac{V_v}{V_s} = \frac{1-s}{s} \quad (2)$$

$$s = \frac{1}{1+e} \quad (3)$$

where *V* is the total soil volume, V_s is the volume of solids, V_v is the volume of voids and G_s is the specific gravity. The ratio of the minimum to maximum dry density values, *D*, is given by

$$D = \frac{\rho_{d,\max}}{\rho_{d,\min}} = \frac{s_{\max}}{s_{\min}} = \frac{V_{s,\max}}{V_{s,\min}} \quad (4)$$

where the subscripts max and min refer to the maximum and minimum quantities referred to in Section 2.

3.2 Danube Sands data

In the work of Kabai (1972) the minimum and maximum dry densities are presented for about 33 Danube sand mixtures. From these, the void ratios *e* and the solid volume ratios *s* were computed and their corresponding ratios were determined. The results are shown in the Appendix, in Table A-1.

The mean of the s_{min} and s_{max} values are 0.640 and 0.700, with mean coefficients of variation of 0.070 and 0.072, respectively. The mean of *D* (= s_{min} / s_{max}) is equal to 0.915 with a coefficient of variation *CV*=0.0116, indicating that *D* is practically constant.

The mean of the e_{max} and e_{min} values are 0.569 and 0.436, with coefficients of variation of 0.192 and 0.234, respectively. The mean of the e_{min} / e_{max} is equal to 0.760 with a coefficient of variation *CV*=0.060.

3.3 The Calibration Chamber sand data

In this work, the minimum and maximum void ratio data presented for about 25 sands were used to compute the s_{min} and s_{max} values, the ratio of these *D* and the ratio of the minimum and maximum void ratios. The results are shown in the Appendix, in Table A-2.

The mean of the s_{min} and s_{max} values are 0.536 and 0.652, with the coefficients of variation of 0.045 and 0.039, respectively. The mean of D ($=s_{min} / s_{max}$) is equal to 0.822 with a coefficient of variation CV=0.024. As was found for the data of Kabai (CV=0.0116), this value indicates that the s ratio is practically constant for the Calibration Chamber sands.

The mean e_{max} and e_{min} values are 0.870 and 0.536, with coefficients of variation of 0.097 and 0.110, respectively. The mean of the e_{min} / e_{max} is equal 0.617 with a coefficient of variation of CV=0.061.

3.4 Comparing the mean values and variances

It can be seen from the data in Tables 1 and 2 that the means of the minimum and maximum dry densities, expressed in terms of the solid volume ratio s, are larger for the Danube sands than for the Calibration Chamber sands, and the variances are also larger. The opposite is true for the mean density data expressed in terms of the void ratio. Similarly, the mean of D ($=s_{min} / s_{max}$) is larger for the Danube sands, than for the calibration chamber sands. However, the reverse is true for the coefficient of variation of the D.

The means (x) and the standard deviations (σ) of the variables s_{min}, s_{max} and D for the two groups of data were compared using some statistical tests (Réth※áti, 1985).

Table 1. Results of the statistical evaluation for the 33 Danube sands

	s_{min} [-]	s_{max} [-]	D [-]	e_{max} [-]	e_{min} [-]	e ratio [-]
Mean	0.640	0.700	0.915	0.569	0.436	0.760
σ	0.045	0.050	0.011	0.109	0.102	0.045
CV	0.070	0.072	0.012	0.192	0.234	0.060

Table 2. Results of the statistical evaluation for the 25 different Calibration Chamber sands

	s_{min} [-]	s_{max} [-]	D [-]	e_{max} [-]	e_{min} [-]	e ratio [-]
Mean	0.536	0.652	0.822	0.870	0.536	0.617
σ	0.024	0.026	0.020	0.084	0.059	0.038
CV	0.045	0.039	0.024	0.097	0.110	0.061

The F-test was used to evaluate the null hypothesis that the s and D parameters of the two data sets have the same variance.
The statistic of the F-test is:

$$F = \frac{\sigma_1^2}{\sigma_2^2}, \sigma_1 > \sigma_2 . \qquad (5)$$

The u' test and t' tests were used to test the null hypothesis that the s and D parameters of the two data sets have the same means with the data having possibly equal and unequal variances, respectively. The u' statistic for unequal sample sizes and unequal variance is as follows:

$$u' = \frac{x_1 - x_1}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad (6)$$

The t' statistic for unequal sample sizes ($n_1 \neq n_2$) and equal variance ($\sigma_1 = \sigma_2$) is as follows:

$$t' = \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 2)}{n_1 + n_2}} \frac{x_1 - x_1}{\sqrt{n_1 \sigma_1^2 + n_2 \sigma_2^2}} \qquad (7)$$

The complete results can be seen in Table A-3 in the Appendix. In each case, the null hypotheses were rejected:

- The null hypothesis of the F test was rejected on a level of p=0.01 (having a threshold of 1.89).
- The null hypothesis of the u test was rejected on a level of p=0.001 (having a threshold of 3.29 for n>100).
- The null hypothesis of the t' test was rejected on a level of p=0.001 (having a threshold of 2.66 for n=60).

3.5 The linear correlation between the density variables

Despite confirming (in the previous section) that the two sand sample populations have different density and ratio statistics, there is a surprising linearity in the relationship between the minimum and the maximum dry density variables within each sample database. This is evident from Figure 1 and Figure 2 where the maximum and minimum solid volume ratio and maximum and minimum void ratio values are plotted against each other (respectively). These results imply that a linear relationship exist between the minimum and the maximum dry density data, and it is consistent with the fact that the coefficients of variation of the ratio of dry density in terms of s (i.e. D) or in terms of e (i.e. e_{min} / e_{max}) are relatively small.

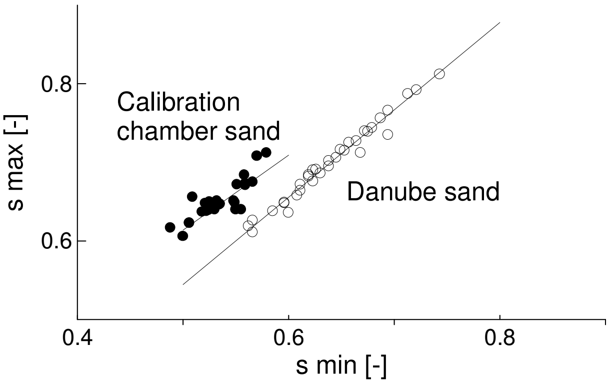

Figure 1 The linearity of the s_{min} - s_{max} relation for 33 Danube sands and 25 Calibration Chamber sands

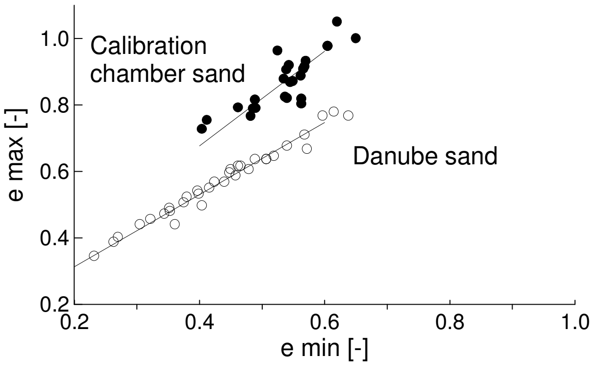

Figure 2 The linearity of the e_{min} - e_{max} relation, 33 Danube sands, 25 different Calibration Chamber sands

This linearity can be quantified using the Pearson correlation. The Pearson correlation R is obtained by dividing the covariance of the two variables by the product of their standard deviations. The Pearson correlation is +1 in the case of a perfect positive (increasing) linear relationship (correlation) and −1 in the case of a perfect decreasing (negative) linear relationship, since the following relationship is met:

$$R(x, y) = 1 \leftrightarrow x = ax + b \qquad (8)$$

The Pearson correlation is some value between −1 and 1 in all other cases, indicating the degree of linear dependence between the variables. The parameters of the best fit linear function are as follows (E denotes expected value which can be approximated by the mean value):

$$a = \sigma(y) / \sigma(x), \; b = E(y) - aE(x) \qquad (9)$$

As the Pearson correlation approaches zero there is less of a linear relationship. The closer the coefficient is to either −1 or 1, the stronger the linear correlation between the variables. If the variables are independent, Pearson's correlation coefficient is 0, but the converse is not true, the correlation coefficient detects only linear dependencies between two variables.

According to Table 3, the Pearson coefficient of correlation is equal to 0.99 for Danube sands indicating that both the s_{min} - s_{max} and e_{min} - e_{max} relations are linear and the slopes of the lines are nearly equal to 1. The Pearson coefficients of correlation are smaller for Calibration Chamber sands – being equal to 0.84 (for *s*) and 0.85 (for *e*) – still indicating a strong linear correlation.

Table 3. The linear correlation between the s_{min} - s_{max} and e_{min} - e_{max} variables for 33 Danube sands and 25 Calibration Chamber sands

	Danube sands		C. Chamber sands	
relation	*s*	*e*	*s*	*e*
a, slope of line	1.11	1.08	1.07	1.42
b, offset of line	-0.01	0.10	0.08	0.11
Pearson correlation, *R*	0.99	0.99	0.85	0.84

3.6 The dependence on the grain size

There is some information available on the grain size available from the Danube sand database (see Table A-1 in Appendix) This allows propositions that the *s* and *e* values and their max/min ratios may be independent of the maximum grain size d_{max} to be tested. In regard to the ratios of *s* and *e* (represented in Figures 3 and 4 and in Table 4) it is apparent that this proposition is acceptable for the *s* ratio only (Figure 3 and Table 4).

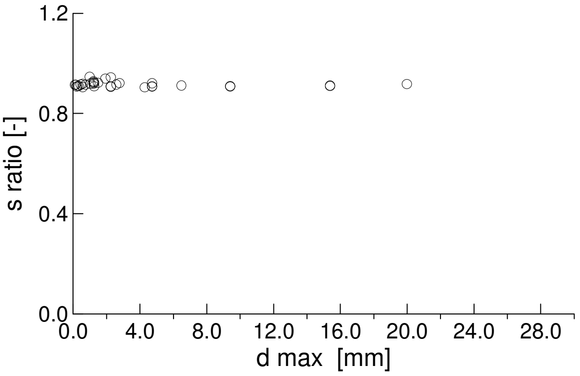

Figure 3 The s_{min} / s_{max} ratio with the maximum grain diameter for 33 Danube sands

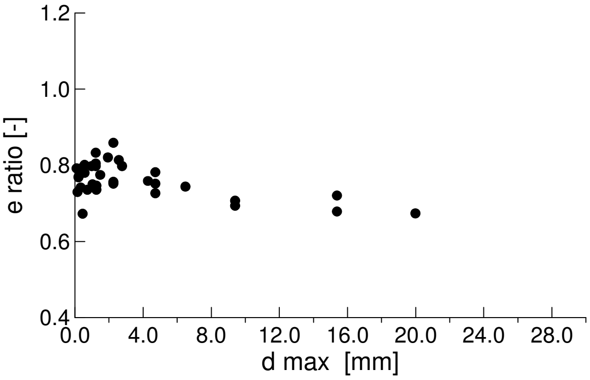

Figure 4 The e_{min} / e_{max} ratio with the maximum grain diameter for 33 Danube sands

Table 4. Pearson coefficients of correlation, *R,* between *s, e* ,their ratios and d_{max} for 33 Danube sands

D	s_{max}	s_{min}	*e* ratio	e_{max}	e_{min}
-0.20	0.57	0.55	-0.58	-0.57	-0.58

If we consider the *s* and *e* variables separately with respect to d_{max}, (Figures 5 and 6 and Table 4) then it is found that the s_{min} and s_{max} increase with d_{max} (Figure 5). The opposite is true in terms of the e_{min} and e_{max} (Figure 6) due to their functional relationship with variable *s*. If *s* increases then the numerator decreases, the denominator increases in Equation (2) and *e* decreases.

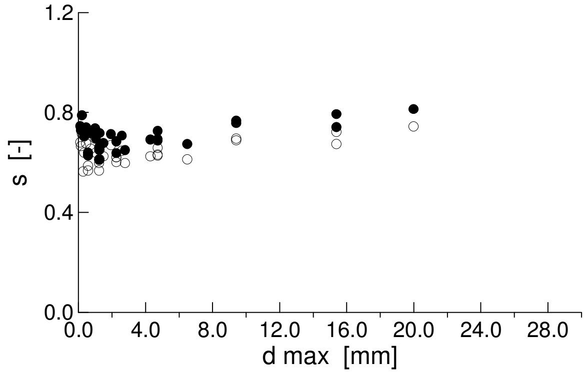

Figure 5 The s_{min} and s_{max} with the maximum grain diameter for 33 Danube sands

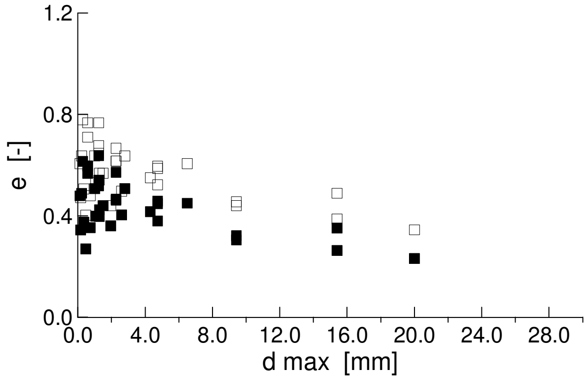

Figure 6 The e_{min} and e_{max} with the maximum grain diameter for 33 Danube sands

4. DISCUSSION

4.1 Similarities between the databases

The results of the statistical analyses indicate that the ratios of the minimum and the maximum dry density are practically constant for the sands in each database, since the coefficient of the variation is very small. The coefficient of variation of the *e* ratio is equal to 0.061 for the Calibration Chamber sands; 0.060 for the Danube sands; the coefficient of variation of the *s* ratio is equal to 0.024 for the Calibration Chamber sands; 0.0116 for the Danube sands.

These results imply linear correlations between the minimum and the maximum dry density data. The Pearson coefficient of correlation was equal to 0.99 for the Danube sands indicating that both the s_{min} - s_{max} and e_{min} - e_{max} relations are linear, the slope of the lines are nearly equal to 1. The Pearson coefficients of correlation were smaller for the Calibration Chamber sands (0.84 for *s* and 0.85 for *e*) but still indicating significant linear correlations.

The larger coefficient of variation values (and the smaller Pearson coefficient of correlation values) for the Calibration Chamber sands can primarily be attributed to different geological origins, as discussed in the following section.

4.2 Differences in the databases

In comparing the Calibration Chamber sands and the Danube sands, it is found that whilst the *D* ratio is almost constant for each sand, the loosest - densest dry density data for each sand are statistically, significantly different.

The primary reason for this is likely to be differences in the density testing methods. The Danube sands were tested on the basis of the Hungarian Standard with some modifications, and the Calibration Chamber sands were tested on the basis of the ASTM standard in most of the cases. In regard to the minimum density testing methods, the molds have different sizes in the two procedures. In regard to the maximum density testing methods, it is accepted that the ASTM modified Proctor dry density (which was used by Kabai) and the ASTM maximum dry density are about the same (Poulos and Hed, 1974). However, Kabai applied a steel plate element on the top of the sample. It is possible that this element – placed to prevent the local loosening of the sample - may cause some additional vibratory compaction effect in the special version of the modified Proctor test used by Kabai (1972).

Closer inspection of the data in Figure 1 suggests that the s_{max} values of the Calibration Chamber sands lie within (although at the lower end of) the values for the Danube sands. However, the s_{min} values of the calibration sands lie below the range of values obtained for the Danube sands. This suggests that the s_{min} estimates for the Calibration Chamber results are consistently lower, and that the minimum dry density test method used for the Calibration Chamber sands was able to produce consistently lower relative densities. There is also some suggestion that the maximum dry density test method used for the Danube sands was able to produce generally greater relative densities, consistent with the expected influence of the steel plate used in these tests.

Despite the dependence of the measured values of maximum and minimum dry density on the method used, it is expected that particular methods should give systematically different values, which are proportionally different in a consistent way. It follows then, that whilst this should affect the absolute value of the minimum and maximum measurements, it will have little effect on the ratio between them. Hence, it is likely that if the two groups of sands had been tested using exactly the same methods, then a single linear correlation could be found.

It is also likely, however, that the geological characteristics of the particles in each of the groups are significantly different and that this could have some systematic effect on the outcomes. In particular, it is apparent that there is much greater variation in the Calibration Chamber database results, which include sands from many different geological environments, that are each likely to be composed of grains of particular mineral assemblages. By contrast, the Danube sands are artificially produced blends of natural grains which all have the same geological provenance, and accordingly, their results are significantly more consistent.

4.3 The dependence on the grain size

The linear correlation between s_{min} and s_{max} or e_{min} and e_{max} may imply that the ratios of these are practically independent of the maximum grain size d_{max}. According to the results, this assumption is reasonably acceptable only for *D* (the *s* ratio in Figure 3). The values s_{min} and s_{max} increase slightly with d_{max} but the opposite is true for e_{min} or e_{max} (see Figure 6) due to their functional relationship with *s*.

These findings are in agreement with a dry density function suggested by Imre et al. (2009), constructed from the data of Lőrincz (1986). The s_{min} is given in terms of the ΔS entropy increment and S_0 base entropy, which can be interpreted in terms of the uniformity coefficient C_U (or the number of fractions in the soil, *N*) and the mean grain diameter d_{mean}, respectively. The s_{min} function increases with S_0 (or mean grain diameter d_{mean}) for a specified ΔS (or uniformity coefficient C_U or the fraction number *N*).

5. CONCLUSIONS

The minimum and the maximum dry density data of the Calibration Chamber sands and the Danube sand are significantly different. This difference is primarily attributable to the density testing methods (i.e. Hungarian standard based methods and ASTM based methods).

However, within each database, the ratio of the minimum and the maximum dry density is practically constant since the coefficient of variation of *D* is very small. There is a strong linear correlation between the measured minimum and maximum dry density values in both databases.

Moreover, the s_{min} and s_{max} values increase slightly with the maximum grain diameter d_{max} in the tested Danube sand database. This result is in agreement with a previously constructed dry density function (Imre et al. 2009). The *s* ratio is basically independent of the maximum grain diameter d_{max}. This result means that s_{min} can be estimated from s_{max} and vice versa.

Further research is suggested on the ratio of the minimum and the maximum dry density as a function of the grading, the geological origin and the density testing methods, and especially, a comparison of the density testing standards and the frequently used density testing methods.

6. REFERENCES

Imre, E, Lőrincz, J, Trang, Q.P, Fityus, S., Pusztai, J, Telekes, G, Schanz, T. (2009) “Some dry density transfer function for sands.” KSCE Journal of Civil Engineering 13(4), pp257-272.

Kabai, I. (1968) “The compactibility of sands and sandy gravels”. Acta Technica Acad. Sci. Hung., pp113-124.

Kabai, I. (1972) “Relationship between the grading curve and the compactibility”. Doctoral thesis, Technical University of Budapest, Hungary (in Hungarian).

Kabai, I. (1974) “The effect of grading on the compactibility of coarse grained soils”. Periodica Polytechnica, 18(4), pp255-275.

Leussink, L.H. and Kutzner, C. (1962) “Laboratoriumversuch zur Ferstellung der dichtesten Lagerung korniger Erdstoffe.“ Verüöff. Des Inst. f. Bodenmech. Techn. Hochschule Frid. Karlsruhe, Germany.

Lőrincz, J. (1986) “Grading entropy of soils.” Doctoral Thesis, Technical Sciences, Technical University of Budapest, Hungary.

Lunne, T., Robertson, P.K. and Powell, J.M. (1992) Cone Penetration Testing. Blackie Academic & Professional.

Mayne P.W. and Kulhawy F.H. (1992) “Calibration Chamber database and boundary effects corrections for CPT data.” Proc. On Calibration Chamber Testing, Potsdam, Elsevier, New York.

Poulos, S.J. and Hed, A. (1974) “Density Measurements in a Hydraulic Fill. Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils.” ASTM STP523. pp402-424.

Réthági, L. (1985). Valószínűségelméleti megoldások a geotechnikában. Akadémiai Kiadó, Budapest, pp393.

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APPENDIX

Table A-1 Danube sand data (n=33)

Series	d_{10} [mm]	d_{max} [mm]	C_U [-]	□$_{min}$ [t/m^3]	s_{min}[-]	s_{max} [-]	s ratio [-]	e_{max} [-]	e_{min} [-]	e ratio [-]
B0	0.145	0.290	1.3	1.49	0.562	0.619	0.908	0.779	0.615	0.790
		0.580	2	1.55	0.585	0.638	0.917	0.710	0.568	0.800
		1.240	3	1.61	0.608	0.658	0.923	0.646	0.519	0.804
		1.270	5	1.72	0.649	0.716	0.907	0.541	0.397	0.735
		9.420	10	1.84	0.694	0.766	0.906	0.440	0.305	0.693
		15.400	15	1.91	0.721	0.792	0.910	0.387	0.263	0.678
		20.000	30	1.97	0.743	0.812	0.916	0.345	0.232	0.673
B1	0.29	0.580	1.3	1.5	0.566	0.626	0.904	0.767	0.597	0.779
		1.240	2	1.58	0.596	0.649	0.918	0.677	0.540	0.797
		2.270	3	1.64	0.619	0.682	0.907	0.616	0.466	0.756
		4.740	5	1.74	0.657	0.725	0.906	0.523	0.380	0.726
B2	0.58	1.240	1.3	1.5	0.566	0.611	0.927	0.767	0.638	0.832
		2.270	2	1.59	0.600	0.636	0.943	0.667	0.572	0.858
		4.740	3	1.66	0.626	0.691	0.907	0.596	0.448	0.751
		15.400	5	1.78	0.672	0.740	0.908	0.489	0.352	0.720
B3	1.24	2.27	1.3	1.64	0.619	0.684	0.905	0.616	0.462	0.751
		4.74	2	1.67	0.630	0.686	0.919	0.587	0.458	0.781
		9.42	3	1.82	0.687	0.756	0.908	0.456	0.322	0.706
C0	4.74	2.79	1.3	1.58	0.596	0.648	0.920	0.636	0.507	0.797
		1.28	2	1.62	0.611	0.664	0.921	0.568	0.424	0.746
		0.36	4	1.69	0.638	0.702	0.908	0.506	0.375	0.741
		0.17	6	1.76	0.664	0.727	0.913	0.472	0.344	0.729
		0.115	8	1.8	0.679	0.744	0.913	0.606	0.479	0.791
C1	4.74	1.5	2.3	1.65	0.623	0.676	0.921	0.568	0.440	0.774
		1.05	3.2	1.69	0.638	0.695	0.918	0.532	0.399	0.749
		0.75	4.2	1.73	0.653	0.715	0.913	0.480	0.353	0.735
		0.47	6.1	1.79	0.675	0.739	0.914	0.402	0.270	0.672
		0.22	11.4	1.89	0.713	0.787	0.906	0.636	0.489	0.768
C2	19.5	6.5	2.4	1.62	0.611	0.672	0.910	0.606	0.450	0.743
		4.3	3.3	1.65	0.623	0.690	0.903	0.550	0.416	0.758
		2.6	5	1.71	0.645	0.706	0.914	0.497	0.404	0.813
		1.95	6.5	1.77	0.668	0.712	0.938	0.440	0.361	0.820
		1	10.5	1.84	0.694	0.735	0.945	0.636	0.507	0.797

Table A-2 Calibration Chamber sand data (continued on next page)

	Raw data			Computed data			
	e_{max} [-]	e_{min} [-]		s_{min} [-]	s_{max} [-]	s ratio [-]	e ratio [-]
Erksak	0.963	0.525	subrounded	0.509	0.656	0.777	0.545
Hilton Mines	1.05	0.62	angular	0.488	0.617	0.790	0.590
Edgar	0.919	0.543	subangular	0.521	0.648	0.804	0.591
S, Oakleigh	0.754	0.412	subangular	0.570	0.708	0.805	0.546
Hokksund	0.906	0.539	angular	0.525	0.650	0.807	0.595
Toyoura	0.977	0.605	subangular	0.506	0.623	0.812	0.619
Toyoura	0.977	0.605	subangular	0.506	0.623	0.812	0.619
S, Oakleigh Fine	0.932	0.57	subangular	0.518	0.637	0.813	0.612
Earlston	0.727	0.404	subrounded	0.579	0.712	0.813	0.556
Frankston	0.792	0.462	subangular to rounded	0.558	0.684	0.816	0.583
Hokksund	0.878	0.535	subangular to angular	0.532	0.651	0.817	0.609
Medium Ticino	0.915	0.568	subrounded to angular	0.522	0.638	0.819	0.621
Leighton Buzzard	0.815	0.489	subrounded	0.551	0.672	0.820	0.600
Lone Star 60	0.908	0.566	subround, to subangular	0.524	0.639	0.821	0.623
Hostun fine	1	0.65	subangular	0.500	0.606	0.825	0.650
Ottawa	0.868	0.545	well-rounded	0.535	0.647	0.827	0.628
Oostershelde	0.887	0.562	rounded	0.530	0.640	0.828	0.634
Reid-Bedford	0.871	0.549	subangular	0.534	0.646	0.828	0.630
Ottawa 90	0.789	0.486	rounded	0.559	0.673	0.831	0.616
Leighton Buzzard	0.79	0.49	subrounded	0.559	0.671	0.832	0.620
Lone Star 2	0.766	0.482	subround, to subangular	0.566	0.675	0.839	0.629
Lone Star 30	0.824	0.537	subround, to subangular	0.548	0.651	0.843	0.652
Monterey 0	0.82	0.54	subrounded	0.549	0.649	0.846	0.659
Lanchester	0.818	0.563	subangular	0.550	0.640	0.860	0.688
Monterey 0/30	0.803	0.563	subround, to subangular	0.555	0.640	0.867	0.701

Table A-3 Results of the statistical tests for the probability variables related to s

	s_{min} [-]		s_{max} [-]		R (s_{min}/s_{max}) [-]	
test	statistic [-]	degrees of freedom [-]	statistic [-]	degrees of freedom [-]	statistic	degrees of freedom [-]
t'	58.057	56	24.148	56	116.158	56
u'	11.418	100<	4.761	100<	21.149	100<
F test	7.033	32	8.510	32	21.997	24