

# The Development of Unsaturated Soil Mechanics at Imperial College, London

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**ABSTRACT:** Saturated soil mechanics is a complex subject because of the particulate form of the solid phase of soil, its interaction with the aqueous water phase and also because soil is a product of nature and so has potentially great variability. When the soil dries such that there is also an air phase it becomes unsaturated and its behaviour is far more complex because of the interface between the air and the water and the volumetric response of the air under changing conditions of pressure and temperature. Additionally the pressure in the water phase becomes negative (tensile) and measuring such pressures has until recently been fraught with problems. As many parts of the world are covered by unsaturated soils, understanding their response would significantly enhance engineering design and analysis. This paper describes the research work done at Imperial College over the past decades to advance our understanding of unsaturated soils. The work is considered under four main headings of theoretical formulations, laboratory experimentation, field studies and numerical analysis research.

## 1. INTRODUCTION

This paper covers four main themes relating to unsaturated soils: theoretical formulations, laboratory experimentation, field studies and the numerical analysis of boundary value problems. Within these themes there will be occasional examples that relate to the behaviour of unsaturated soils but the scope of the paper cannot give a very extensive review in this respect. The intention is to highlight the contributions that have been made by researchers at Imperial College over the past fifty years.

In fully saturated or dry soils changes in shear strength and volume can be conceptually understood and quantified using a single stress state variable, effective stress. A similar approach was attempted for unsaturated soils over a number of years. Professor Alan Bishop at Imperial was one of those who tried to develop an appropriate formulation. Eventually it was realised that this idea could not be extended to the more complex response of unsaturated soils. The various reasons and limitations are explained and discussed.

There are two major differences between the behaviour of saturated and unsaturated soils. One is that in unsaturated soils the pore pressures are usually negative. Measuring negative pore pressures directly is notoriously difficult, primarily because of cavitation in the measuring system and transducers. The second factor is that changes in overall sample volume in saturated soils are simply controlled by the flow of water into or out of the soil. In the case of unsaturated soils the pore fluid (comprising gas and liquid) is compressible. Both of these factors have required extensive developments in laboratory and field monitoring instrumentation and testing and control systems for unsaturated soils. Many advances have been made in the design of instrumentation and experimental apparatus at Imperial.

With the development of testing and measuring systems, an increased understanding of the behaviour of unsaturated soils was gained. In the early years this was mostly in connection with compacted fills. Certain types of response associated with unsaturated soils, e.g. shrinking, collapsing and swelling were identified and frameworks expressing this behaviour developed. More recently the responses of both natural and artificial soils to drying and wetting have been studied and their responses expressed using soil water retention curves.

As a better understanding of the response of unsaturated soils has evolved, conceptual and constitutive models have been developed to simulate their behaviour. These mathematical models then allow boundary value problems to be analysed, for example to investigate the effect of partial saturation on the stability of slopes or the settlement of foundations. In recent years significant advances have been made at Imperial College in the implementation of constitutive models into sophisticated finite element numerical analyses using the Imperial College Finite Element Program (ICFEP). Numerous situations have been analysed, frequently using

parameters determined from laboratory studies performed here and validated against field measurements with which the group has been involved.

Thus the fourth theme is the one that draws on the knowledge and understanding that has been developed over many years, combining them to allow real life boundary value problems to be designed and analysed.

The final part of the paper will summarise what has been done and briefly describe work currently underway and areas for future research.

## 2. THEME 1: THEORETICAL FORMULATIONS

### 2.1 Principle of effective stress for saturated soils

Terzaghi's principle of effective stress (Terzaghi, 1936) represents a milestone in the history of soil mechanics. It is probably the most fundamental underlying principle on which most soil mechanics theories are based. The introduction of the concept of an effective (neutral) stress,  $\sigma'$ , and how this is linked to pore water pressure,  $u_w$ , and total stress,  $\sigma$ , has allowed the science of soil mechanics to be developed. Expressed algebraically, the effective stress principle for saturated soils is given by the following well known expression:

$$\sigma' = \sigma - u_w \quad (1)$$

A *stress state variable* is defined by Fredlund and Rahardjo (1993) as a non-material variable that is required for the characterisation of the stress condition. Terzaghi (1936) recognised that effective stress, which is a stress state variable, for fully saturated soils directly controls changes in shearing resistance and volumetric strains.

$$\frac{\Delta V}{V} = f(\Delta\sigma') \quad \text{and} \quad \Delta\tau_f = g(\Delta\sigma') \quad (2)$$

where for the latter expression  $\Delta\sigma'$  acts normal to the plane being considered (Burland, 1965). The value of the effective stress principle lies in the fact that it enables the behaviour of soils undergoing changes in  $\sigma$  and  $u$  to be related to a single variable. Thus the one stress state variable is sufficient to define most behavioural changes in saturated soils.

Burland (1965) states: 'Effective stress is more than an acknowledgement that the pore pressure has a fundamental influence on soil behaviour. It is based on the rigorous assumption that changes in pore pressure (or some function of it) are equivalent to changes in total pressure as far as mechanical behaviour of the soil is concerned.'

The simplification resulting from the use of the single variable  $\sigma'$  is very great. For example in the laboratory, instead of having to ensure the exact simulation of both  $\sigma$  and  $u_w$ , only the magnitude of the effective stress needs to be considered. Equally the effective stress principle is very useful for obtaining a qualitative understanding of soil behaviour as well as in quantitative predictions.'

## 2.2 Stress state variables for unsaturated soils

In the early days when frameworks of behaviour for unsaturated soil mechanics were being developed, various attempts were made to identify a single stress state variable that would encompass the behaviour of unsaturated soils in the same way that effective stress does for fully saturated (or completely dry) soils. A number of research groups were working, often independently, on this approach.

Aitchinson and Donald (1956) showed that provided the soil remains fully saturated, the soil moisture suction or pressure deficiency in the soil,  $p''$  contributes directly to the effective stress in the soil and the saturated effective stress expression, Eq. (1), becomes:

$$\sigma' = \sigma + p'' \quad (3)$$

In this expression, although  $p''$  is a negative quantity (c.f. negative pore water pressure) it is expressed as a positive value. The problem with Eq. (3) is that once air enters the pore space, the pressure in the pore water no longer acts over the whole cross-sectional area and the expression is no longer valid.

In the late 1950s and early 1960s a number of workers, such as Dr Aitchison at CIRSO in Australia, Professor Jerry Jennings in South Africa and Bishop in the UK, developed more complete formulations that took into account the two-phase nature of the pore fluid in a partly saturated soil. Some of these expressions are very similar. At that time the expression proposed by Bishop (1959) was considered to be the most general as it included a term for the pressure in the gas phase,  $u_a$ . The following is probably the most familiar form of this expression.

$$\sigma' = \sigma + \chi(u_a - u_w) - u_a \quad (4)$$

where  $\chi$  is a quantity that is a function of the degree of saturation,  $S_r$  and the soil type (Bishop and Blight, 1963, give several examples of how  $\chi$  varies with soil type and  $S_r$ ). The component  $(u_a - u_w)$  in Eq. (4) is the matrix suction,  $s$ , which for negative values of pore water pressure encountered in unsaturated soils is a positive quantity.

At that time Bishop at Imperial was leading research closely related to understanding the behaviour of compacted fills with a particular emphasis on embankment dams. A number of MSc dissertations and PhD theses were produced during this time, many with the common theme of testing whether a single expression such as Eq. (4) can be formulated for compacted, and hence in almost all cases unsaturated, soils. Examples of the MSc and PhD theses either directly or indirectly working on unsaturated and compacted soils include: Alpan, 1959; Donald, 1961; Blight, 1961; El-Ruwayih, 1975; Hanson, 1955; Matyas, 1963 and Vaughan, 1965.

Many developments were made during this period, especially of apparatus, measuring devices and testing procedures. Extensive data sets were also compiled, mostly for various compacted natural clays. These data and developments were published in numerous journals and conference proceedings, for example: Bishop, Alpan, Blight and Donald (1960); Bishop and Donald (1961); Bishop and Henkel (1962); Bishop and Blight (1963); Blight (1965); Matyas and Radhakrishna (1968); and later Bishop, Kumapley and El-Ruwayih, (1975); Marinho, Standing and Kugiwama (2003).

Limitations of Eq. (4) to model completely aspects of unsaturated soil response in the same way that effective stress does for saturated soils were recognised from the results of the testing programmes (e.g. see Bishop *et al.*, 1960). Probably though it was the paper by Jennings and Burland (1962) that concisely and firmly challenged the idea of modelling unsaturated soils with a single stress state variable. Professor Jennings at Witswatersrand University had been working on a similar concept (Jennings, 1960) and the paper by him and John Burland followed the latter's research work for his MSc thesis 'The concept of effective stress in partly saturated soils' (Burland, 1961). Even at this time John Burland was to have a significance impact on the subject (e.g. Burland 1964, 1965). He probably did not realise then that he would one day become Professor of Soil Mechanics at Imperial College and continue studying unsaturated soil mechanics for several decades.

Jennings and Burland (1962) showed, using data from oedometer tests performed on air-dried specimens of silt, wetting them at different normal loads and  $S_r$  values, that the single expression, Eq. (4) did not provide an adequate relationship between volume change and effective stress for most unsaturated soils. This was particularly the case for soils below a *critical degree of saturation* which varies according to soil type. This critical degree of saturation is closely linked to the point where air present within the pore fluid changes from being in the form of occluded bubbles within the pore water to continuous phases.

Two key figures from Jennings and Burland (1962) are shown in Figure 1. The test data in Figure 1 are from samples of silt, originally mixed at 35% water content and then allowed to dry until they reached a constant mass before trimming and installing them within rings in an oedometer. Data in the upper part of Figure 1 are from samples loaded to a different applied pressure and soaked (i.e. at 0.1, 2, 4, 8, and 16 t/ft<sup>2</sup>) after which loading was continued. In every case when the partly saturated silt was soaked under constant applied load it underwent settlement, or 'collapsed', even at small values of applied stress, i.e. there is a *decrease* in volume. The collapse is the reverse of the behaviour predicted on the basis of the effective stress principle.

The test data shown in the lower part of Figure 1 are from another series of silt samples prepared in the same way and loaded to 2, 4, 8, and 16 t/ft<sup>2</sup>. When equilibrium had been reached under these applied loads, the samples were soaked and the applied loads were controlled to try to maintain volume in the oedometer. The results offer further evidence that the normal concept of effective stress is not always valid for such soils. If the principle were valid it would be expected that soaking at constant volume would require an increase in applied stress. However, to retain constant volume it is necessary to *reduce* the applied load during soaking.

A variety of results from other tests and field data are presented by Jennings and Burland to substantiate the fact that unsaturated soils cannot be modelled using a single effective stress principle. This is particularly the case for soils below the critical degree of saturation, for which  $\chi$  has no physical significance but is merely a convenient empirical factor (also observed by Bishop *et al.* 1961). In essence it is the form of the pore water (and of course pore air) that is the controlling factor. A single effective stress approach might work if there were only bulk water. It is the presence of the meniscus water that causes the difficulties.

Therefore it is not possible just to combine  $\sigma$ ,  $u_a$  and  $u_w$  into a single effective stress. The expression suggested by Bishop and the alternative variations by many others is not viable - there can be no satisfactory definition of a single effective stress for unsaturated soils.

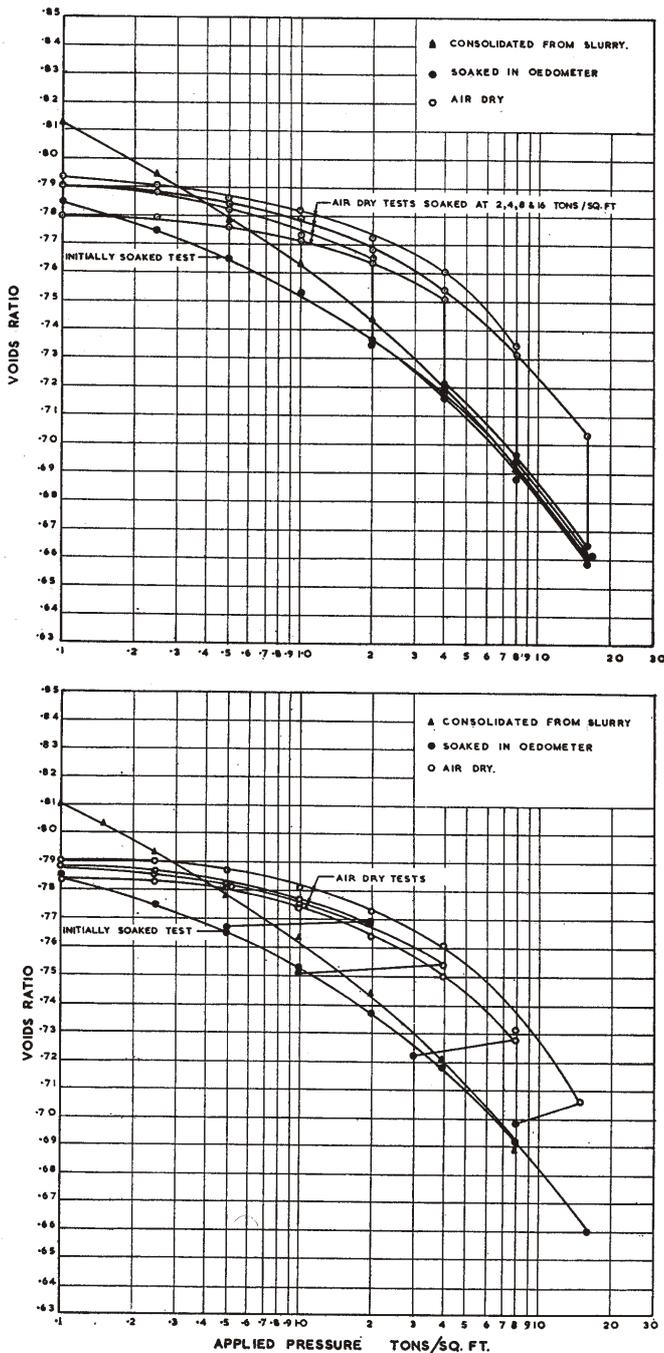


Figure 1. Oedometer curves for air-dried silt. Upper figure: soaked at various constant applied pressures; lower figure: soaked at various constant voids ratio values (Jennings and Burland, 1962).

In practice it is necessary to use two stress state variables. Any pair of the following three will suffice (Fredlund and Morgenstern, 1977).

- (i)  $(\sigma - u_a)$  ('net stress')
- (ii)  $(\sigma - u_w)$  (similar to saturated effective stress)
- (iii)  $(u_a - u_w)$  (matrix suction)

There are three possible combinations. The combination 'net stress' and 'matrix suction' is the most frequently used choice and is considered to be the most satisfactory for use in engineering practice (Fredlund, 1979). The earliest use of these was by Coleman (1962) and Bishop and Blight (1963). Most progress in constitutive modelling of unsaturated soils achieved in the past 30 years has been in terms of net stress and suction.

### 3. THEME 2: LABORATORY TECHNIQUES FOR UNSATURATED SOILS

Broadly there are three headings under which this subject is to be discussed: (i) measurement of changes in sample volume, (ii) measurement of soil suction and (iii) apparatus for the control of sample soil suction.

#### 3.1 Measurement of sample volume changes

As mentioned in the introduction, as unsaturated soils contain air within their pores, measurement of changes in overall sample volume cannot be determined uniquely from water flow in to/out of the sample as it can in the case of saturated soils. Air present within an unsaturated soil can exist in one of three forms: dissolved within the pore liquid (water); in the form of occluded bubbles and as free continuous phases. As the pore pressure decreases air will expand (according to Boyle's Law and Charles' Law) and perhaps come out of solution if dissolved air is present within the water phase (according to Henry's Law). Bubbles of occluded air will increase in size and, depending on the degree of saturation, may eventually coalesce to form continuous phases if these do not already exist. The converse will occur to air present when there is an increase in pore pressure. If the degree of saturation is sufficiently high, an adequate increase in pore pressure may result in the soil becoming fully saturated as any continuous phase is first reduced to occluded bubble form and these eventually compress to such a small size that they fully dissolve within the water phase. Temperature also plays a key role in the magnitude of volume change of air – this is not considered further in this paper (there has been much work in recent years looking at the effects of temperature on the response of unsaturated soils, especially in the context of the possibility of using unsaturated soil liners to contain nuclear waste – there are a number of papers covering this subject in the Géotechnique Symposium in Print on 'Thermal behaviour of the ground' given in the April and May 2009 issues).

Knowing the volume of air present is essential for determining the degree of saturation of the soil, fundamental for anticipating its response. This can be achieved by carefully measuring the overall sample volume and any flow of water in to/out of the sample from the time the sample is first set up in an apparatus. In a triaxial apparatus there are two basic methods of measuring the overall sample volume change.

- (i) Monitoring the changes in volume of the surrounding cell fluid.
- (ii) Measuring combined local axial and radial strains of the sample.

Methods developed at Imperial College adopting the first approach (i) are discussed first.

Professor Bishop, as well as being recognised as one of the foremost pioneers of soil mechanics of his time, was also a brilliant mechanical engineer. Many forms of soil testing apparatus based on his designs are still extensively used. Numerous apparatus, including those for testing unsaturated soils, are described in the classic text book by Bishop and Henkel (1962, first published in 1957). Figure 2, from Bishop and Donald (1961), is an excellent example of one of the complex apparatus developed to test unsaturated soils.

The apparatus involves a double-walled triaxial cell with mercury surrounding the sample in the inner chamber so that air could not diffuse from the soil through the rubber membrane surrounding the sample. Deflections of the inner chamber wall are avoided by having the same pressure either side and overall sample volume changes are determined by monitoring the level of a stainless steel ball floating on the surface of the mercury using a cathetometer. Complex ancillary systems that form part of the overall experimental set-up include a closed-circuit bubble pump and air trap and a volume gauge for use with positive or negative pressures. Mercury was extensively used in soil testing apparatus at these times (both for the above application and applying pressures) –

it is very unlikely that this would be permitted now because of health concerns.

components of suction are explained and discussed to be clear about exactly what is measured and/or controlled.

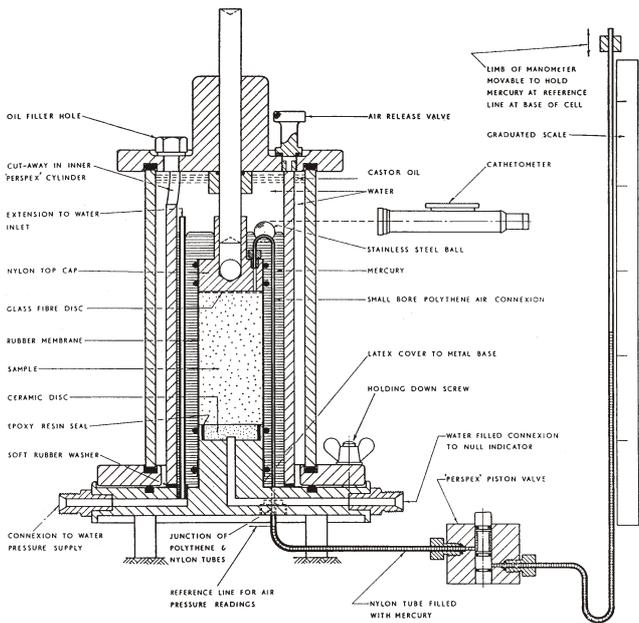


Figure 2. Modified triaxial cell with sample and rubber membrane surrounded with mercury (Bishop and Donald, 1961).

A cruder form of apparatus for measuring overall sample volume changes, developed by Bishop, is described by Egele (1980). The single cell chamber enclosing the sample and cell water is made from steel and measurement of flow in to/out of the cell allows the overall sample volume to be determined, taking into account the cell's compliance. As with conventional triaxial apparatus flow in to /out of the sample is measured using an independent volume gauge with a bubble trap incorporated in to the system. This apparatus is still used for research (now with modern Imperial College volume gauges, the first form of which is described by Maswoswe, 1985) and also for post-graduate teaching (the notorious dissipation test that for many years was run by Professor Peter Vaughan). When using such apparatus for unsaturated soils it is important to surround the sample with a diffusion barrier to avoid air migration in to/out of the sample (this is usually either: butyl rubber, neoprene or metallic foil).

The methodologies used in these two systems are still those principally in modern apparatus used for measuring overall unsaturated sample volume changes using approach (i).

The other approach (ii) is to measure local axial and radial strains and from these calculate the volumetric strain. This can be achieved using instrumentation attached directly to the sample, e.g. LVDTs; Hall-effect transducers; inclinometer devices (electrolevel or strain-gauged pendula) and proximity transducers. Although not directly connected to the study of unsaturated soil mechanics, some of these devices were developed at Imperial College (e.g. Jardine et al., 1984; Ackerley et al, 1987).

The main advantage of measuring local axial and radial strains is that the instruments/ techniques are accurate at small strains. This is particularly important when calculating small-strain stiffness values. The disadvantages are that the measuring systems are inaccurate if slip planes form and the axial and radial strains of only a central gauge length are obtained.

### 3.2 Measurement of soil suction

In this section, only techniques used, developed or modified at Imperial are discussed. A comprehensive review of available techniques is given by Ridley and Wray (1995). Some more recent developments are described in the Géotechnique Symposium in Print on 'Suction in unsaturated soils' (February and March 2003 issues). Before discussing the measurement of soil suction, the

#### 3.2.1 Components and definitions of suction

Suction is the isotropic pressure that the pore water imposes to absorb more water. Soil suction is usually divided into two components, matrix (related to the soil matrix, i.e. the combination of particle type and structural arrangement) and osmotic (due to the concentration of salts in the soil water). The total suction is the sum of the matrix and osmotic suctions.

$$\psi = (u_a - u_w) + \pi = s + \pi \quad (5)$$

where:  $\psi$  = total suction;  $(u_a - u_w) = s$  = matrix suction;  $\pi$  = osmotic suction.

Total suction (sometimes referred to as free energy of the soil water) can be expressed by the thermodynamic relationship involving the partial pressure of the pore-vapour:

$$\psi = -\frac{RT\rho_w}{\omega_v} \ln \frac{u_v}{u_{v,0}} = -\frac{RT\rho_w}{\omega_v} \ln(RH) \quad (6)$$

where:  $\psi$  = soil suction or total suction (kPa);  $R$  = universal (molar) gas constant (8.31432 J/(mol.K));  $T$  = absolute temperature ( $T = t + 273.16$ ) (K);  $t$  = temperature ( $^{\circ}\text{C}$ );  $\omega_v$  = molecular mass of water vapour (18.016 kg/kmol);  $u_v$  = absolute partial vapour pressure of pore water vapour (kPa);  $u_{v,0}$  = absolute saturation pressure of water vapour over a flat surface of pure water at the same temperature (kPa);  $\rho_w$  = density of water (998 kg/m<sup>3</sup> at 20  $^{\circ}\text{C}$ );  $RH$  = relative humidity =  $u_v/u_{v,0}$ .

From Eq. (6) it can be seen that the total suction is a direct function of relative humidity. Both the matrix and osmotic components also independently induce changes in relative humidity (RH reduces as either  $(u_a - u_w)$  or  $\pi$  increases).

Aitchison (1965) provided definitions for the three components of suction which have been used for several decades. They come from the viewpoint of someone versed in physics and chemistry and are not always easy to visualise for geotechnical engineers. Dr Andrew Ridley worked on the measurement of suction, under the supervision of Professor John Burland, at Imperial College. Ridley (1993) points out that suction can be measured in the vapour phase or the liquid phase of the soil pore-fluid. If the soil has no component of osmotic suction (i.e. the soil-water is pure, without salts), the suction measured in both phases should be the same. He set out to relate the components of suction to what is being measured, without changing the underlying concepts. He proposed alternative definitions which provide a different visualisation of soil suction that more correctly represent what is actually measured.

The component of suction measured is directly linked to the technique used to measure it. In particular it depends on where the suction is measured, i.e. whether the sensor is in contact with the liquid or vapour phase of the soil pore-fluid. With a device in contact with the soil, it is matrix suction that is measured as any salts within the soil pore water will have an equal effect on the suction within the transducer as well as the soil. Non-contact techniques provide measurements of total suction as it is the relative humidity that controls the suction in the device which results from both components of suction as discussed above.

Historically it has not been possible to measure reliably negative pore water pressures because of problems with cavitation within the water in both the connecting water lines and the pore pressure transducer itself. Cavitation usually occurs when the free water pressure is at about zero absolute pressure (i.e. -100 kPa gauge pressure). Negative water pressure can therefore be measured up to this point beyond which cavitation occurs and it is not possible to reduce pressure further as the air simply expands according to Boyle's Law. Once cavitation has occurred pore pressure measurements become very unreliable, even in the positive range until the transducer and connecting lines are fully saturated again.

### 3.2.2 Axis-translation technique

The problem with negative pore pressures and cavitation was overcome by the use of the axis-translation method (Hilf, 1956). This technique involves increasing both the air and water pressures ( $u_a$  and  $u_w$ ) and total stresses ( $\sigma$ ) simultaneously so that the stress state variables matrix suction ( $u_a - u_w$ ) and net stress ( $\sigma - u_a$ ) remain unchanged. Usually the soil sample is placed in a robust chamber and the cell pressure surrounding the sample is controlled by air pressure in such a way that as the cell pressure ( $\sigma$ ) increases, so too do the air and water pressures ( $u_a$  and  $u_w$ ) within the sample. The cell pressure is increased until the pore water pressure in the sample becomes positive and can be measured using a standard transducer. Figure 3, taken from Fredlund and Rahardjo (1993), illustrates the idea. Devices such as the pressure-plate or pressure membrane extractor apparatus rely on this technique to provide measurements of soil suction. The technique is also used extensively to control suction during testing on unsaturated soils – this will be discussed briefly in the third section of this Theme 2.

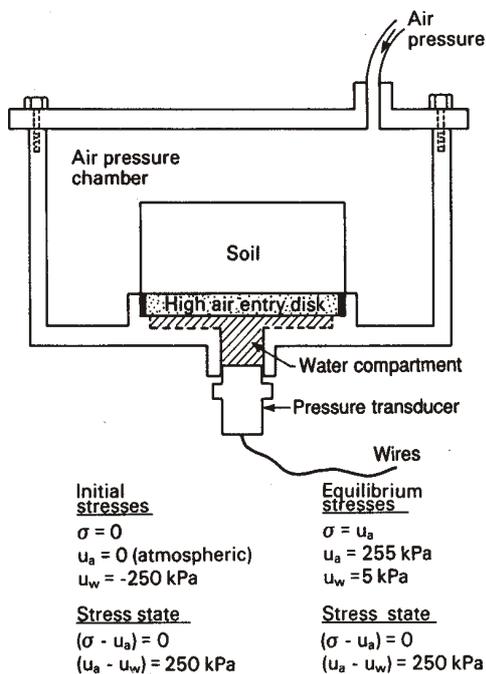


Figure 3. Schematic diagram showing pressure changes associated with the measurement of matrix suction in a pressure plate apparatus (Fredlund and Rahardjo, 1993).

An important feature of the axis-translation apparatus shown in Figure 3 is the *high air-entry porous stone* at the base of the sample. These stones or discs have small pores of relatively uniform size and are commonly manufactured from sintered kaolin. Once the disc is saturated with water, air cannot pass through it due to the surface tension developed on the contractile skin. The difference between the air pressure above the contractile skin and the water pressure below it is the matrix suction,  $u_a - u_w$ . The maximum matrix suction that can be maintained across the surface of the disc is called its air-entry value,  $(u_a - u_w)_d$ , which is a function of the surface tension and the radius of curvature of the contractile skin or the radius of the maximum pore size.

The disc acts as an interface between the unsaturated soil and the pore-water measuring system. Water in the disc acts as a link between the pore-water in the soil and the water in the measuring system, while air cannot pass through the high air-entry stone into the measuring system. This prevents cavitation in the pore pressure transducer and connecting tubes, providing the matrix suction in the sample does not exceed the air-entry value of the stone. The flow of

air through a 1500 kPa air-entry value porous stone, once its air-entry value is exceeded is shown schematically in Figure 4.

The characteristics and capacity of high air-entry porous stones play a crucial role in the measurement of suction and the testing of unsaturated soils. Bishop and Henkel (1962) provide details of various types of stone and their capacities.

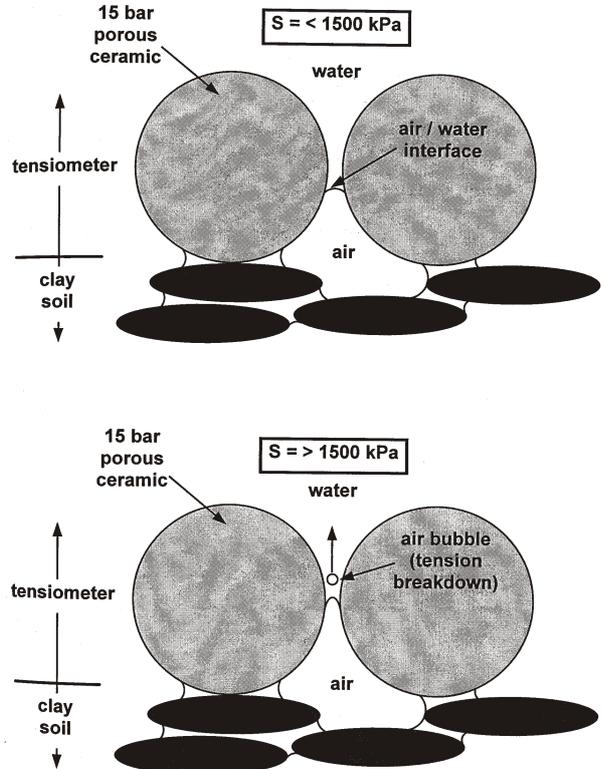


Figure 4. Schematic view of 1500 kPa high air-entry porous stone in contact with a soil sample at suctions above and below its air-entry value. Note that the stone is represented here as being within a tensiometer device (Dineen, 1997).

### 3.2.3 Filter paper technique

The filter paper method has been used extensively for many years at Imperial College. It was originally developed by soil scientists and later adopted by geotechnical engineers to measure suctions in unsaturated soils. The principle is based on the fact that the moisture content of an absorbent material (such as laboratory filter paper) is related to suction in a similar manner to the moisture characteristic curve of the soil. So, if a filter paper is allowed to absorb moisture from a soil specimen, and the amount of moisture taken from the soil is small enough to have negligible effect on the soil suction, when equilibrium is reached, the suction in the filter paper will be equal to the suction in the soil.

The method therefore involves placing a dry filter paper in contact with, or close to, the soil specimen (for measuring matrix or total suction respectively). The filter paper absorbs moisture from the specimen eventually coming into equilibrium. The moisture content of the filter paper is measured and a calibration curve is used to relate that to the suction in the soil.

At Imperial College techniques have been developed to improve the accuracy of measurement using this method as it is very sensitive to small changes in the water content of the filter paper and great skill is needed when making these measurements. Professor Dick Chandler and other researchers have used the technique extensively (e.g. Gutierrez, 1985; Chandler and Gutierrez, 1986; Chandler et al, 1992; Crilly et al, 1992; Crilly and Chandler, 1993; Marinho, 1994; Ridley and Edenmosun 1999).

In more recent years the filter paper method has been used in particular to formulate *Soil Water Retention Curves* (SWRC). The importance of these curves and what they represent has been progressively appreciated in the study of unsaturated soils. Essentially the curve depicts how either degree of saturation,  $S_r$ , volumetric water content,  $\theta_w$  ( $V_w/V$ ) or void ratio,  $e$ , change with suction. Different (hysteretic) curves relate to whether a soil is on a drying or wetting path and whether it is dried from a slurry or a state where the soil has already been dried and wetted. Examples of SWRCs are given in Figure 5, showing primary drying and wetting curves (i.e. from slurry) and scanning curves. Several researchers have developed these curves to help characterise the unsaturated response of artificial and natural soils from slurry, natural or compacted states (Ridley, 1993, Marinho, 1994; Dineen, 1997; Cunningham, 2000; Colmenares, 2002; Melgarejo, 2004; Jotisankasa, 2005). As discussed in Theme 4, they are used extensively as part of constitutive models in the numerical analysis of boundary value problems.

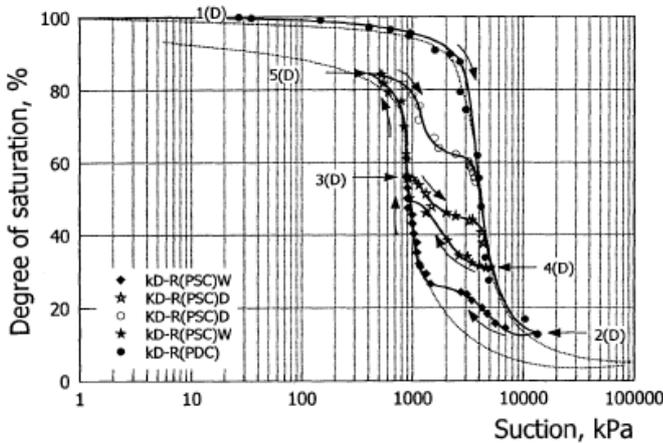


Figure 5 Examples of soil water retention curves for kaolin (from Melgarejo, 2004).

### 3.2.4 Imperial College tensiometer (suction probe)

Ridley and Burland (1993) developed a new miniature tensiometer capable of measuring pore water suctions greater than 100kPa. The essential components of the latest version of this device are shown in Figure 6. It consists of a 1500 kPa high air-entry porous ceramic stone with a very narrow clearance (the reservoir) between it and the sensing element which is a strain-gauged diaphragm. The ceramic and reservoir are filled with de-aired water which is then subjected to a pressure of about 4000 kPa. The high pressure dissolves any remaining free air and drives the air-water interfaces far into any microscopic crevices. When not in use the tensiometer should be kept under high pressure. The IC tensiometer is capable of reliably measuring matrix suctions of up to about 1500 kPa with a response time of a few minutes. An example of the response of the device when placed in contact with a soil and then subsequently placed in water is shown in Figure 7. Ridley and Burland (1995) showed that the limiting suction is controlled by the air-entry value of the ceramic. At suctions close to the air-entry value, air does eventually enter the reservoir and the tension in the water breaks down. At lower suctions, laboratory measurements can be made over a number of days before tension break down takes place.

This is a good point at which to mention the significant contribution of the technicians in the Soil Mechanics Section at Imperial College. Currently they are Messrs. Steve Ackerley, Alan Bolsher, Graham Keefe and Duncan Parker. Without the support and ideas from the technicians many of the developments described here would not be possible. In particular Steve Ackerley has made significant contributions to or been responsible for many of the ideas and developments, especially regarding the suction probe.

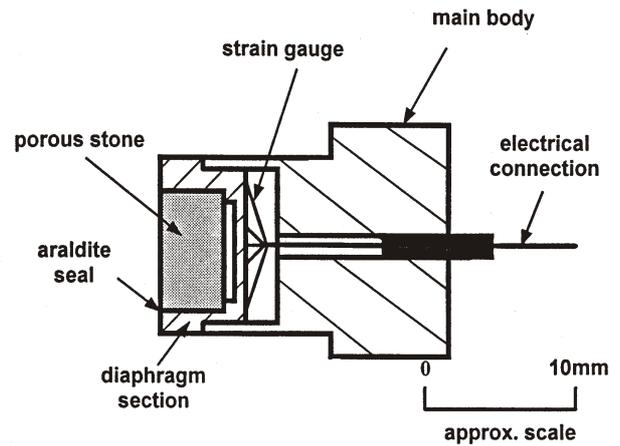


Figure 6. Details of the Imperial College tensiometer (suction probe) (Ridley and Burland, 1996).

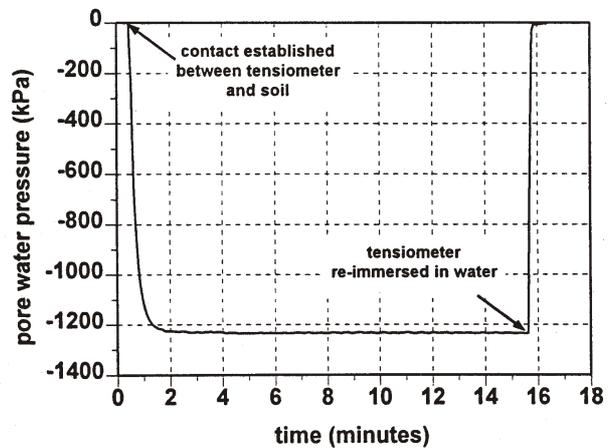


Figure 7 Response of the IC tensiometer to contact with soil followed by immersion in water (Dineen, 1997).

This development has radically enhanced the measurement and control of suction over a significant range of suctions encountered in geotechnical engineering (see Ridley, Dineen, Burland & Vaughan 2003). It is now possible to test unsaturated soils with negative pore water pressures, i.e. in the same condition they exist in situ without the need to apply the axis-translation technique. Also suction can be measured directly and quickly after taking samples from boreholes or in the laboratory. Monroy *et al.* (2012) describe further advances.

Following this original work a number of other similar devices have been developed by other researchers (e.g. see Géotechnique SIP on 'Suction in unsaturated soils'). It still remains a challenge at higher suction values (i.e. in excess of 1500 kPa). The direct measurement of osmotic suction has not been achieved with much success: attempts that have been made are not discussed here (refer to Fredlund and Rahardjo, 1993 and Ridley and Wray, 1995).

### 3.3 Apparatus for the control of soil sample suction

Laboratory equipment for testing unsaturated soils generally has the same form as for saturated soils, e.g. triaxial apparatus and oedometer, but with modifications for taking the air phase and suction into account. In particular it is the problems of avoiding cavitation, measuring suction and overall sample volume accurately that have to be overcome. Three methods of controlling  $u_a$  and  $u_w$  have been devised to avoid these problems:

- axis-translation technique;
- osmotic method of suction control;
- control of relative humidity (using an air-water regulation system).

### 3.3.1 Axis-translation technique

This technique has been explained and discussed already (see Section 3.2.2 and Figure 3). It has been used extensively since the 1950s and was used in the early days at Imperial. The apparatus described by Bishop and Donald (1961) discussed in the section on volume change measurements used the axis-translation technique.

Many modern triaxial apparatus still use the method with water pressure being applied at the base of the sample (via a high air-entry stone) and air pressure controlled through the upper sample platen. The extensive series of tests performed by Wheeler and Sivakumar (1993, 1995) to formulate their elasto-plastic constitutive model were performed using such an apparatus with a double walled-cell to facilitate measurement of overall sample volume. At Imperial College the technique is used with pressure-plate and pressure membrane extractor apparatus. These work in exactly the same way as described earlier (see Figure 3 for a broad idea of the set-up) the difference between them being that the former uses a high air-entry porous stone and the latter a cellulose membrane that can sustain much higher suctions while allowing water to flow through its pores but not air.

There are some drawbacks to the axis-translation technique. From a practical point of view, the use of elevated air pressures requires robust apparatus. The most limiting factor is that the air phase within the sample has to be continuous. If occluded bubbles are present they simply compress, resulting in a pore pressure gradient and irreversible volume changes. Therefore an upper limit of 80% saturation is often suggested for the axis-translation technique (but this value depends on the nature of the soil being tested).

### 3.3.2 Osmotic method of suction control

The method of osmotic suction control has been used effectively by a number of researchers since the 1960s. In particular Professor Pierre Delage and his team at ENPC, Paris made several developments and improvements to apparatus applying suction using the technique. It essentially relies on the principle of osmosis which is illustrated schematically in Figure 8.

Dineen (1997), who also worked with Professor Burland, explains that osmosis is the term used to describe the flow of a solvent (a liquid capable of dissolving a substance) into a solution (a liquid containing dissolved substance and solvent) or for the flow of a dilute solution into a more concentrated solution when separated by an impermeable membrane that is permeable to the solvent and wholly impermeable to the solute (the dissolved substance). Strictly the flow should be one of solvent only to be termed osmosis, if some of the solute is transferred the process is one of diffusion. The transfer of solvent results in an increase in pressure on the solution side of the membrane. This excess pressure will increase and ultimately prevent the flow of solvent into the solution. The pressure at this equilibrium state is termed the osmotic pressure, as shown in Figure 8 ( $h = f(c) = \text{osmotic pressure head}$ ).

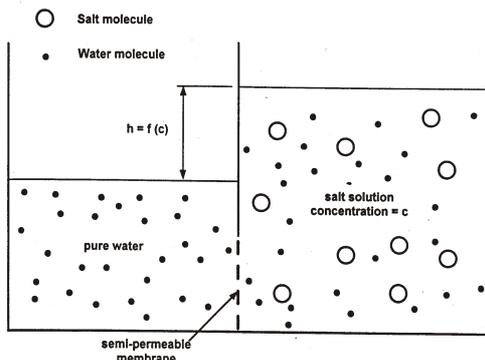


Figure 8. Schematic diagram showing principle of osmosis (Dineen, 1997).

In applying the osmotic potential to control pore-water suction, a restrained semi-permeable membrane is used which separates the solvent (i.e. the soil water) from the solute (a salt solution used to impose the suction). In theory the pressure developed across the membrane should be equivalent to the osmotic potential of the solution. However, in practice the suction developed may differ from the theoretical value. This is one of the drawbacks of the method when it is not possible to measure the suction directly.

Delage *et al.* (1987) developed one of the first osmotic suction-controlled triaxial apparatus. Polyethylene glycol (commonly denoted PEG) was the material used to form a concentrated solution with which to generate the osmotic potential. This was circulated, using a peristaltic pump, to the platens either end of the sample on which semi-permeable membranes were glued. Later Delage *et al.* (1992) went on to develop an oedometer apparatus which was also controlled by the same principle, as shown in Figure 9. In this apparatus there was a closed system for the solution circulation with a graduated capillary tube to monitor changes in the overall solution volume, allowing the movement of moisture into and out of the sample to be determined.

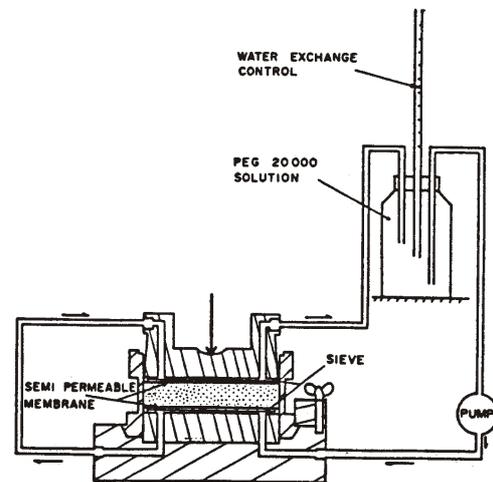


Figure 9. Osmotic suction-controlled oedometer (Delage *et al.*, 1992).

The main drawbacks in these methods were that suctions were not measured directly (reliance on the osmotic potential from the solution) and the loss of water from the sample through evaporation during the circulation of the solute. Dineen (1997) developed a new osmotically-controlled oedometer that addressed these issues, as shown in Figure 10. The suction within the sample was measured directly using three IC suction probes instead of relying on the theoretical relationship involving the concentration of the PEG solution. Also the water inflow from the sample was measured by mass instead of volume, thus avoiding temperature effects and providing a greatly improved resolution in the measurement of volume change (Dineen and Burland, 1995). In addition to these modifications the confining ring of the oedometer was designed to allow radial stresses to be determined. This worked by having four thin-walled diaphragms machined into the confining ring to which strain gauges are attached. A pressure compensating system is used to keep the strain gauges at a null position to ensure  $K_0$  conditions. The pressure required to achieve this is assumed to represent the radial stress within the sample.

Schreiner (1988) also developed an oedometer where radial stresses could be measured but this relied on a pressure cell installed flush with the internal surface of the confining ring. The results from his research on swelling clays are mentioned in Theme 4.

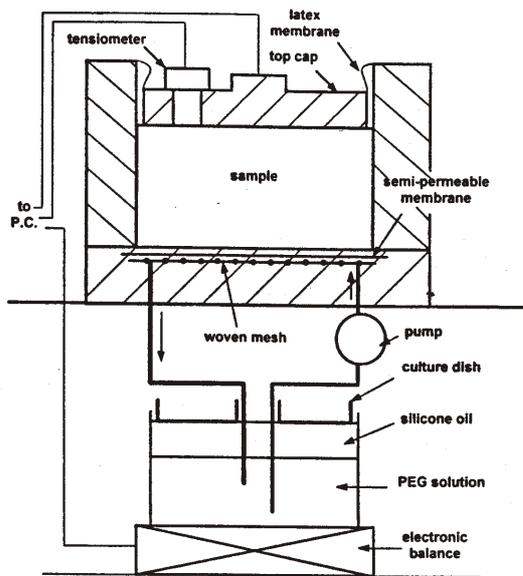


Figure 10 Improved osmotic suction-controlled oedometer (Dineen, 1997).

Colmenares (2002) was the first to use the newly modified suction-controlled oedometer in his research on compacted sand/bentonite mixtures. Unfortunately the granular nature of the soil mixture led to problems with the contact between the pore fluid in the sample and the osmotic solution, separated by the semi-permeable membrane. Colmenares in fact used the apparatus but with the osmotic system replaced by a self-compensating mercury control system to impose suction (as originally conceived by Bishop and Henkel, 1962) although this could only impose suctions up to about 100kPa. A schematic diagram of the apparatus is given in Figure 11 to show the modification (although this is not an osmotic system it seems logical to include the figure here).

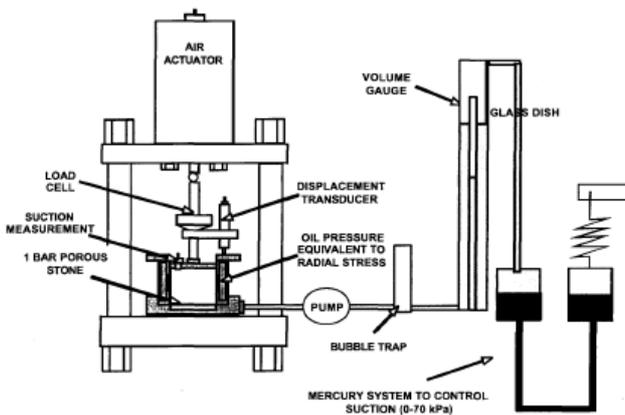


Figure 11 Oedometer with mercury suction control system (Colmenares, 2002).

A successful series of tests on compacted London Clay samples was performed by Monroy (2006) as part of his research. Monroy mentions further modifications that were made to the apparatus that again reflect the ingenuity of the technicians, Steve Ackerley in particular. It was Steve who had the idea to place the assembly on the weighing scale so that mass rather than volume of water coming out of the sample was monitored (see Figure 10). Equally Monroy (2006) recalls that there were problems of leaks from the edges of the semi-permeable membrane when the solution was being pumped against it. Once again Steve had the idea of pulling the solution through rather than pumping it thus avoiding the positive pressure gradient that caused this effect. Following this work Monroy *et al.*

(2007) discuss the suitability of the osmotic technique for the long-term testing of partly saturated soils. Examples of results from his test programme, showing the influence of suction on wetting under load of the compacted London Clay samples are shown in Figure 12 (Monroy *et al.*, 2010). It can be seen that as suction increases the loading paths travel further beyond the normal compression line for a saturated soil (full details are given in the cited paper).

Cunningham (2000) originally intended to use the osmotic system with a semi-permeable membrane but struggled to find a suitable successful combination of PEG solution and membrane type for his long-term tests (although this issue was solved eventually by Monroy, 2006). Controlling matrix suction was not reliable and although no definitive reason was established, it was thought that a progressive degeneration of the cellulose membrane was the primary cause. As result of this deficiency Cunningham went on to devise a system where suction in the sample was controlled by circulating dry air at its boundaries

### 3.3.3 Air-water regulation system to control relative humidity

Cunningham's apparatus is shown in Figure 13. It can be seen that relatively dry air is circulated at a controlled rate through a porous disc at the base of the sample. As the air has a lower relative humidity than the sample, moisture is drawn from the sample and suction is generated within it. By controlling the flow of air being circulated the suction can be regulated which is monitored by IC suction probes at the top and near the base of the sample.

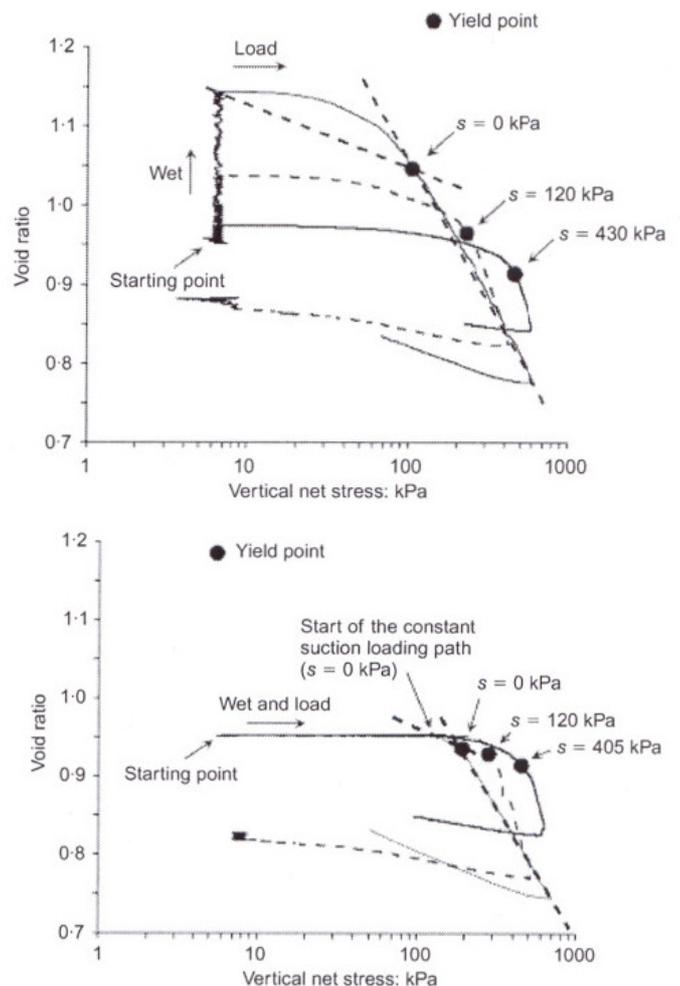


Figure 12. Response of compacted London Clay samples, expressed as void ratio versus vertical net stress, during wetting at (a) nominal load (upper plot) and (b) constant volume followed by loading-unloading at constant suction (lower plot) (Monroy *et al.*, 2010).

Some typical test results are shown in Figure 14 from Cunningham *et al.* (2003). The suction control achieved is excellent. It is noted that the full period of the test including inducing the suction was about one week and that if the same were to have been achieved with an osmotic-controlled system it would have taken about 2 months. Only drying tests were performed at this stage.

Jotisankasa (2005) improved Cunningham's apparatus by incorporating a relative humidity sensor into the line exiting the sample through which dry air had been flushed, thus allowing the amount of moisture extracted from the sample to be determined.

Another significant development to the apparatus used to investigate the mechanical behaviour of an unsaturated compacted silty clay was to provide a wetting system so that a more complete idea of the material's response could be investigated. The system is shown in Figure 15 and involves circulating water and air via the upper platen of the sample. The mass of moisture imparted to the sample was deduced by careful measurements of the volume of water in the supply tube, accounting for other water within the system. Thus with the new systems described here, samples can be dried or wetted in a controlled manner and the resulting changes in suction measured and controlled. Local instrumentation for measuring axial and radial strains was also incorporated to calculate overall volume changes (and accurate stiffness determinations). Further details of both the drying and wetting systems are given in Jotisankasa *et al.* (2007b). Some test results from Jotisankasa's research using the new apparatus are shown in Figure 16. The control of stress paths under given suction values is demonstrated and the experimental data used to define the load collapse yield curve (see also Jotisankasa *et al.*, 2007a).

As with all laboratory work there is a constant need to monitor carefully test apparatus, solve problems and overcome environmental effects. During the research work of Jotisankasa the suction probes at one stage started to behave erratically. This was caused by small leaks into the back of the probe from the cell water and was overcome by using glycerine as a cell fluid.

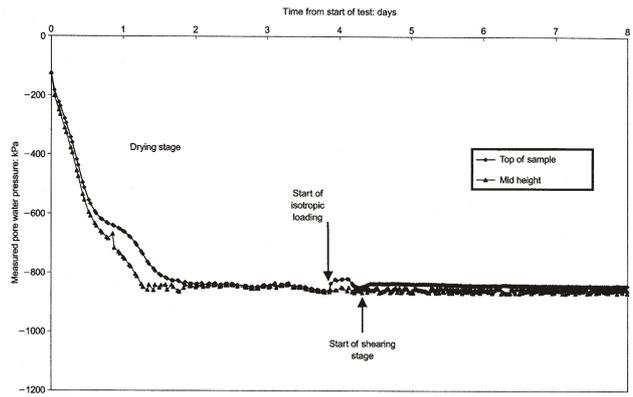


Figure 14. Measured suction versus time for constant-suction shearing test with  $s = 850$  kPa (Cunningham *et al.*, 2003).

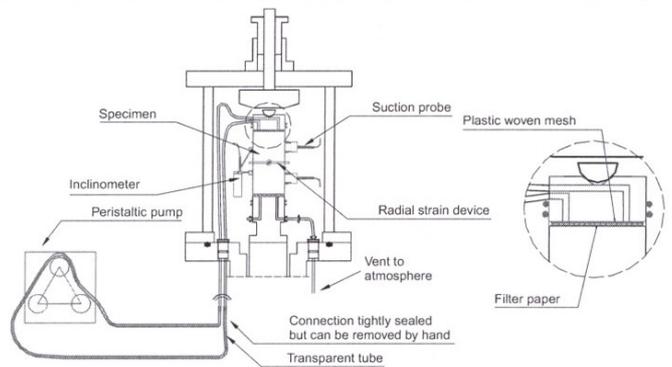


Figure 15. Schematic layout of wetting system for suction controlled triaxial apparatus (Jotisankasa *et al.*, 2009).

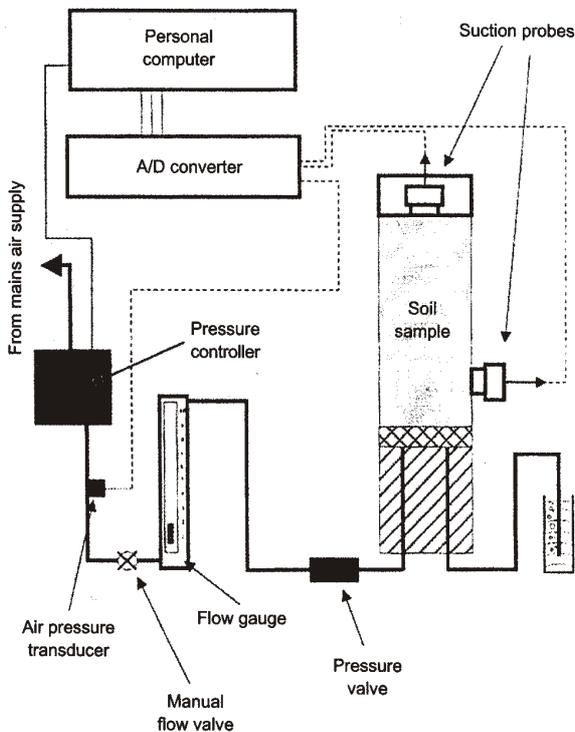


Figure 13. Schematic diagram of configuration of air-circulation suction-control system (Cunningham *et al.*, 2003).

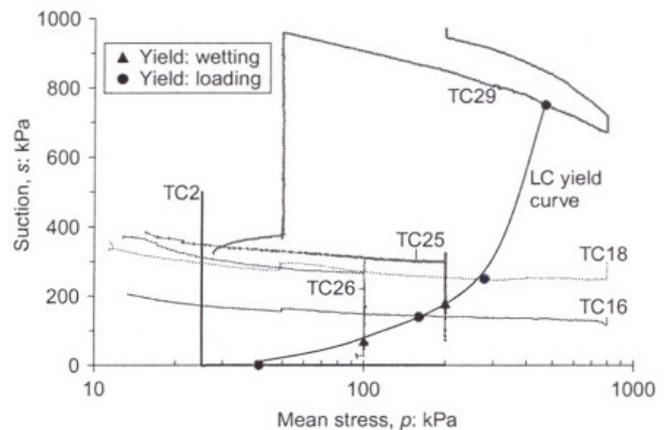


Figure 16. Test results from apparatus shown in Figure 15: compression and wetting tests under isotropic loading conditions (Jotisankasa *et al.*, 2009).

**4. THEME 3: FIELD TECHNIQUES FOR UNSATURATED SOILS**

A number of developments have been made in field measurements in unsaturated soils. As noted earlier, much of the research into unsaturated soils was originally initiated by Professor Bishop's and subsequently Professor Vaughan's work on compacted fills, embankments and embankment dams.

Most compacted fills are unsaturated and so have negative pore water pressures which cannot be measured with conventional piezometers. Cut slopes also often have negative pore pressures, especially initially after they have been cut. For a full understanding and control of engineered fills and cut slopes it is essential to know about the magnitude and changes in pore water pressures.

Development in the measurement of negative pore water pressures (suctions) is therefore the focus of this theme. Most other field monitoring techniques are the same as for saturated soils. Two techniques for the field measurement of suction are described and these are based on those developed for the laboratory. Before discussing these, a tensiometer device able to measure matrix suctions to about 100 kPa is described. Comparisons with the other two devices can then be made.

**4.1 Measurement of negative pore pressures**

**4.1.1 Standard tensiometers**

A tensiometer essentially comprises a sealed reservoir (a water filled pipe) with a porous ceramic filter at the base that is in direct contact with the soil and a measuring device at the top (e.g. Bourdon gauge, manometer or pressure transducer). It measures negative pore water pressure in the soil by allowing water to be extracted from the sealed reservoir, through the ceramic filter and into the soil, until the stress holding water in the tensiometer is equal to the stress holding the water in the soil, (i.e. the soil suction). At equilibrium, no further flow of water will occur between the soil and the tensiometer. The suction will then manifest itself in the reservoir as a tensile stress in the water and can be measured using a pressure measuring instrument.

The maximum pore suction that can be measured using these types of devices is usually limited to about 100 kPa because air bubbles form in the reservoir either by passing through the porous ceramic stone or by being drawn out of microscopic defects in the walls of the reservoirs (i.e. by cavitation). SoilMoisture Equipment Corporation manufactures different versions of these devices, some with a capability for flushing the reservoir without removing the tensiometer from the ground. One of their devices is shown in Figure 17.

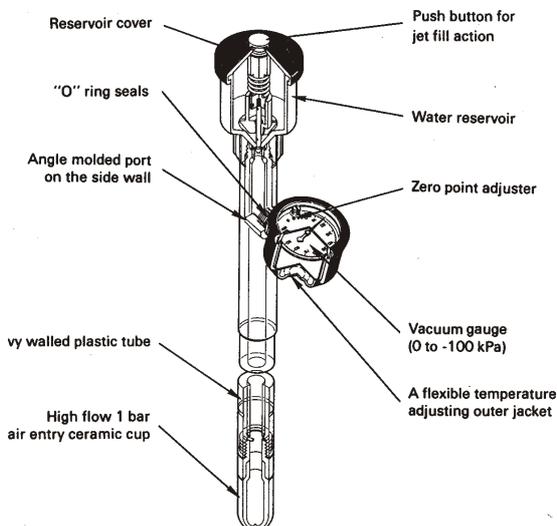


Figure 17. Jet-fill tensiometer manufactured by SoilMoisture Equipment Corporation (from Fredlund and Rahardjo 1993).

**4.1.2 Filter paper device**

Crilly *et al.* (1991) describe a device based on the filter paper technique which allows much higher suctions to be measured. A schematic section through the system is given in Figure 18, showing both closed and open modes (a detailed description of the various components, A to P, can be found in the original paper).

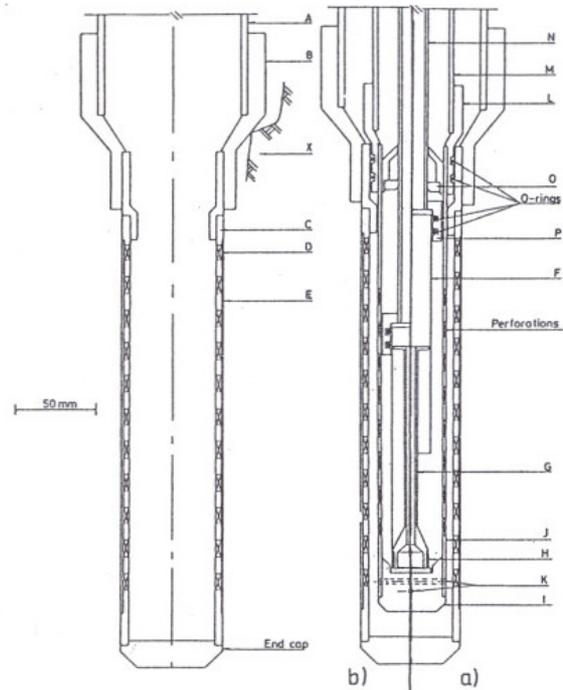


Figure 18. Filter paper suction probe: outer lining (left-hand side) and inner assembly (right-hand side – (a) open, (b) closed). (Crilly *et al.*, 1991).

Essentially the filter paper, wrapped around an aluminium guide rod (G) is contained within an inverted glass test tube (F) that is installed at the base of a 150 mm diameter borehole and allowed to equilibrate with the humidity of the surrounding soil which is exposed at slotted areas (D) along the outer casing (C) in contact with the soil. An elaborate sealing system was used so that the humidity within the enclosed volume was representative of that in the soil. It was decided that non-contact measurements of suction should be made because of the uncertainty in ensuring good contact between the filter paper and the base of the borehole. It is therefore total suction that is measured. A number of field trials were made to test the sealing system, the effect of number of papers and the time to allow for equilibration of moisture within the sealed atmosphere and the filter paper. When ready to take the measurement a plunger (N) is activated to seal the open end of the test tube (H) containing the filter paper so that it can be retrieved from the borehole and the mass of the filter weighed accurately.

The filter paper probe was used at a number of sites in the UK and also Kenya. Some results from this type of device are shown in Figure 19. Four filter paper probes were installed in each side of a test pit divided into two halves and filled with compacted London Clay, one half at its natural water content while the other had been left to dry out before compacting ( $w = 23\%$  and  $18\%$  respectively). Initially the filter papers were extracted every week for a five week period but there was a large scatter in the results because of insufficient equilibration. The data shown in Figure 19 are from measurements taken every two weeks. The data indicate suctions up to about 5 MPa and that both dry and wet sides were drying (this was confirmed independently). Scatter between the four measurements is generally very reasonable (especially given the expected variability of a compacted clay).

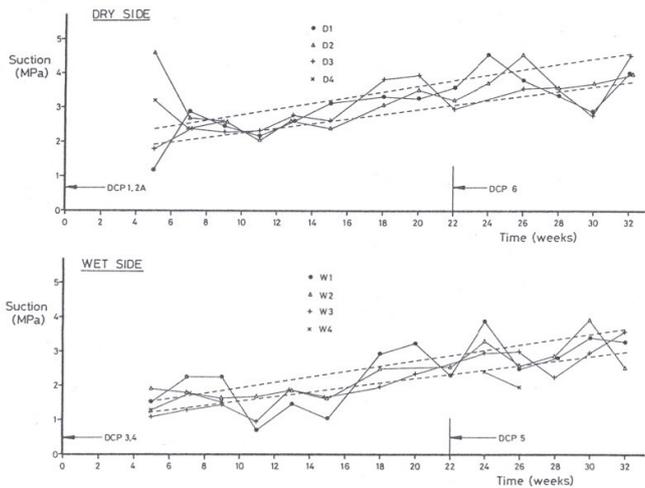


Figure 19. Results from field monitoring of suction using a filter paper suction probe within a test pit divided into two zones where the soil compacted was ‘dry’ (upper figure) and ‘wet’ (lower figure) (Crilly *et al.*, 1991).

**4.1.3 In-situ IC probe tensiometer**

Ridley and Burland (1996) describe how the suction probe discussed earlier (Figure 6) has also been utilised for in-situ measurements of matrix suction. As with its use in the laboratory a key factor in successful measurements is the contact between the face of the probe and the soil. A careful procedure was therefore developed to install the suction probe at the base of the borehole – the sequence of forming the hole is shown in Figure 20. Once the hole is formed (with a smaller secondary hole with carefully cut flat base) the suction probe is lowered on a spring-loaded mounting and the spring depresses until the face of the probe is in contact with the soil.

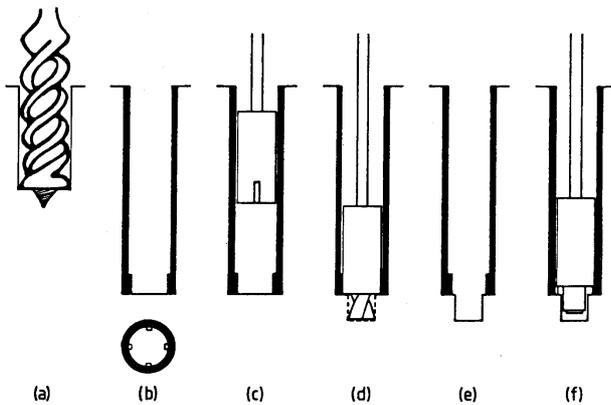


Figure 20. Drilling procedure for installing the in-situ IC tensiometer (Ridley and Burland, 1996).

Results from tests using the new system installed in a large 1.5 m diameter container partly filled with compacted London Clay are shown in Figure 21a. The results of two tests are shown. In the first test the excess water on the high air-entry stone was removed, resulting in a higher suction (lower pore water pressure) once the probe was installed, but after a short while the suction increased and stabilised. In the second test the excess water was left and a more rapid equilibration occurred. Both results ultimately converged to the same value of -570 kPa water pressure. A sample of the same London Clay material was mixed to the same water content as that in the container and compacted using Proctor apparatus and measured using the same tensiometer (Figure 21b). The measured water pressure stabilised at -560 kPa after a short time showing

excellent agreement with the result from the in-situ measurement (the results were also broadly confirmed using the laboratory filter paper technique which gave an average suction of 490kPa).

As mentioned earlier, much of the field research was instigated by a need to gain a better understanding of embankment dams and cut slopes. Professor Peter Vaughan was deeply involved with the design, analysis and construction of a number of dams internationally (Vaughan, 1994). Some of the embankment dams that he was involved with in the UK are Balderhead, Carsington, Cow Green, Empingham and Selsel dams. He was also involved with the short- and long-term response of cut slopes (e.g. Vaughan and Walbanke, 1973). In the early 1990s he became involved with investigating the stability of London Underground Limited’s earth embankments and cuttings. Many of the embankments are formed from uncompacted ‘clods’ of London Clay (from the cuttings) and the behaviour of the matrix of clods with soft surrounding clay complicates the understanding of the behaviour of the fill. Closely associated with the clods is the nature of the air present, i.e. whether it is in the form of occluded bubbles or continuous phases (Vaughan, 2003). These factors complicate the interpretation of measured pore pressures. The devices mentioned above, in particular the standard and in-situ IC probe tensiometers, were frequently used in the field monitoring undertaken. These studies often illustrated the important influence of climate and vegetation on pore pressures and hence deformations and the stability of these earth structures (Marsland *et al.*, 1998; Rodriguez, 1998; Ridley *et al.*, 2004a and b; see also Crilly and Driscoll, 2000 for the effects of trees on pile response).

The need to understand better the mechanisms taking place within embankments and cut slopes led to the development of new constitutive models that could be implemented in numerical analyses programs, in particular the ever increasingly sophisticated Imperial College Finite Element Program (ICFEP). This is the next theme to be covered.

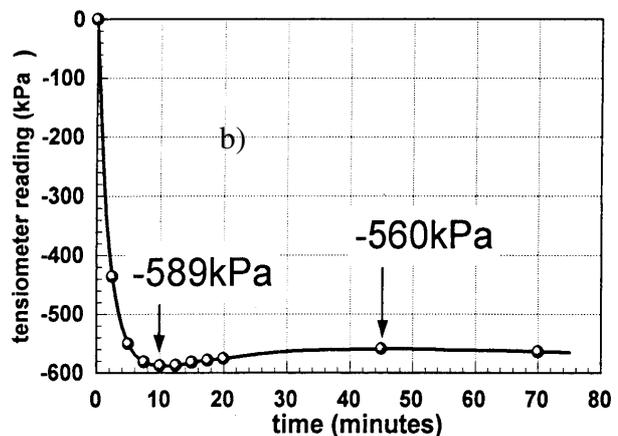
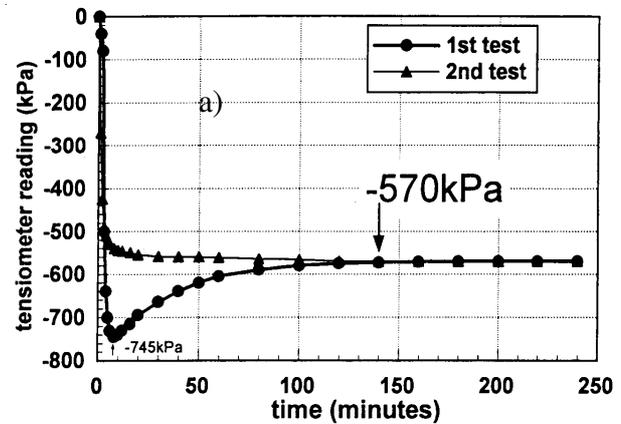


Figure 21. Comparative measurements using tensiometer with (a) in situ set-up and (b) laboratory mode (Ridley and Burland, 1996).

## 5. THEME 4: NUMERICAL ANALYSIS OF BOUNDARY VALUE PROBLEMS

Constitutive modelling of unsaturated soils has developed rapidly in the past two decades. As the capability of testing has improved so it has been possible to develop a better overall understanding of the behaviour of unsaturated soils. Strength and yield stresses are usually increased because of suction, but as discussed earlier volume changes do not follow what might be expected with saturated soils. Unsaturated soils may swell or collapse depending on factors such as the degree of saturation, stress path followed (in particular whether the soils is being wetted or dried), nature of the soil and the magnitude of applied load.

The early work supervised by Professor Bishop led to a number of PhD theses and publications that described the increase in strength and yield stress with partial saturation (e.g. Bishop and Blight, 1963) and this work has been greatly enhanced and substantiated in recent years (e.g. Fredlund *et al.*, 1978; Toll, 1988 & 1990; Wheeler and Sivakumar 1993). This early work also led to a better understanding of volume change with partial saturation (e.g. Matyas, 1963 and the subsequent work by Matyas and Radhakrishna, 1968).

Burland and Ridley (1996) drew on this earlier work and made use of two further more recent sets of Imperial College research data (supervised by Professor Burland) to illustrate the volumetric behaviour of an expansive clay and a clayey sand (Maswoswe, 1985 and Schreiner, 1988). Maswoswe (1985) investigated stress paths experienced during collapse due to wetting of a loose partly saturated soil under one-dimensional ( $K_0$ ) conditions in the triaxial apparatus using the axis-translation technique and local instrumentation for measuring small axial and radial strains. The soil tested was a low-plasticity clayey sand typical of many natural collapsible soils (see Maswoswe *et al.*, 1992). Schreiner (1988) investigated differences between three experimental procedures for predicting the swell of partly saturated clay (see also Schreiner and Burland, 1991). The material used was compacted high plasticity black cotton clay from Kenya, which was tested in an oedometer with lateral stress measurement and suction control using the axis-translation technique.

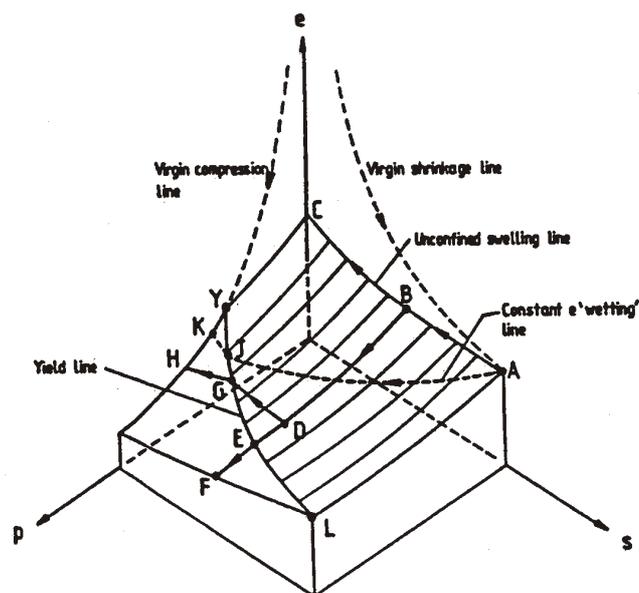


Figure 22. State paths in  $e, p, s$  space - for a compacted soil (Burland and Ridley, 1996)

It is concluded from this research work that the behaviour of collapsible soils and expansive compacted clays is highly stress path dependent and is dominated by the influence of suction changes on the soil fabric. There is a strong influence of microfabric on volume change behaviour as has been explained by a number of researchers (e.g. Burland, 1965; Alonso *et al.*, 1987) and a greater understanding has developed with improved imaging technology (Monroy *et al.*, 2010).

It has been possible to formulate models based on a large number of research test programmes. A conceptual model for volume change under varying suctions and mean net stress is shown in Figure 22. Of particular importance is the yield line (LEGJY), often referred to as the load-collapse yield line, beyond which large irreversible plastic strains occur. It should be noted that the model shown is based on a compacted sample that is wetted. The line AC represents an SWRC wetting path (see Figure 5), while the 'virgin shrinkage line' represents the SWRC drying path. Also in the  $e - p$  plane (void ratio - mean net stress) where suction  $s$  is zero, the path YKH represents part of the virgin compression line.

Alonso *et al.* (1990) describe a constitutive model for unsaturated soils, which is often referred to now as the Basic Barcelona Model (BBM). This model has been modified in a number of ways but its basis largely remains the same.

At Imperial College, ICFEP, written by Professor David Potts and continually updated by him and Dr Lidija Zdravkovic, has been used for many years to analyse boundary value problems. Prof Potts had worked closely with Prof Vaughan assessing mechanisms of failure in cuttings and embankments using fully saturated soil mechanics constitutive models (e.g. Potts *et al.*, 1990; Dounias *et al.*, 1996; Potts *et al.*, 1997). The next logical development within ICFEP was to implement a constitutive model for unsaturated soils and the first model coded was based on the BBM.

Georgiadis (2003) was the first researcher working with Prof Potts and Dr Zdravkovic on the implementation of two new generalised constitutive models for partly and fully saturated soils. The implementation and performance of the models were checked through a series of single element analyses and the results are compared with analytical and experimental data. Having validated the model and implementation a boundary value problem involving shallow and piled foundations was analysed (see Georgiadis *et al.*, 2003 & 2005).

2003 was a good year for PhD production in the numerical analysis of unsaturated soils. Smith (2003) concentrated on slope stability and in particular taking into account the cyclic nature of seasonal wetting (i.e. by infiltration) and drying. He developed models for the fully coupled behaviour of unsaturated soils and formulated a basic model of the soil water retention curve (SWRC). An example of how different portions of the SWRC represent degrees of drying in the numerical modelling is shown in Figure 23. With these formulations he went on to analyse the Tung Chung residual soil slope in Hong Kong for which there are extensive rainfall and pore pressure monitoring data. His analyses demonstrated that the variation of pore water pressure is very sensitive to the relationship between suction and the degree of saturation (i.e. the SWRC). Also important is the influence of suction on permeability and accurately reproducing the true nature of the rain event.

The influence of seasonal climatic changes, especially rainfall infiltration into the ground and the influence of vegetation on its abstraction and consequently on the pore pressure regime were topics that Nyambayo (2003) concentrated on. His work was driven by the London Underground railway embankment studies that Prof Vaughan was working on. During wet winter months the vegetation does not require much water and evaporation and evapotranspiration are low. The converse is the case during hot dry summer periods (these can be limited in the UK!). At the time there were no unified design profiles for winter and summer pore pressures. A non-linear root uptake model was developed and

implemented into ICFEP. This model uses meteorological data as input data and predicts pore pressure regimes from them. The model was validated against field measurements, providing reasonable predictions of both pore pressures and ground movements (Nyambayo and Potts, 2010).

Nyambayo (2003) also investigated the effect of desiccation cracking on permeability and developed a smeared crack permeability model to simulate the increased permeability. Analyses using the root uptake model confirmed that seasonal changes induce strain-softening that can lead to progressive failure. With the new model it was also possible to consider the influence of the depth of roots and their influence on ground movements, suggesting that vegetation management can help control seasonal movements in embankments.

The SWRC plays a crucial role in understanding cyclic seasonal ground responses. Melgarejo (2004) developed SWRCs for a number of soils experimentally and then went on to model them using existing theoretical expressions. At this point only monotonic SWRCs were implemented in ICFEP.

Recently Tsiamposi (2011) describes a far more advanced SWRC that she has formulated and implemented into ICFEP. This model defines the relationship between degree of saturation, or water content and suction and accounts for its hysteretic nature. It also incorporates the effect of specific volume, thus making it a three-dimensional SWRC which is referred to as a Soil Water Retention Surface, SWRS and is shown in Figure 24. Results from this research have recently been submitted for publication (Tsiamposi *et al.*, 2012). Another significant advance that she has made involves the modelling of over-consolidated soils. A new surface has been introduced to replace the yield and plastic potential on the dry side of critical state as given by former constitutive models in order to avoid the overestimation of peak deviator stresses. These new formulations have been applied to boundary value problems involving the behaviour of unsaturated slopes under seasonal changes of suction and the stability of slopes in over-consolidated unsaturated soils.

**6. A BRIEF SUMMARY OF FORMER AND CURRENT RESEARCH**

This paper has attempted to cover the main research developments made at Imperial College since the late 1950s into the behaviour, monitoring, modelling and analysis of unsaturated soils. It begins by explaining the differences and difficulties encountered with unsaturated soils compared with those that are fully saturated. Initially a single all-encompassing stress state variable was sought but the response of unsaturated soils cannot be modelled so simply.

A much better understanding of the behaviour of unsaturated soils has been formed from both laboratory and field studies. It is felt that, hopefully not immodestly, that many important contributions have been made by former and current teams of researchers at Imperial College. In particular the development of the suction probe was a major breakthrough that has radically changed monitoring and testing methodologies in the laboratory and the field. There is still a strong emphasis on laboratory investigations.

Perhaps though, the greatest advances recently have been with the implementation of more complete constitutive models for unsaturated soils in numerical analyses and their use in solving boundary value situations using ICFEP. Of course laboratory test data are still required to supply representative parameters for the constitutive models and to provide frameworks of behaviour against which the implemented models can be validated. Equally field Most existing constitutive models are based on laboratory testing programmes that have been achieved using the axis-translation technique where pore pressures are positive. Research is currently underway by Fauzilah Ismail at Imperial to compare results from tests performed using equivalent stress state parameters (e.g. matrix suction) and stress paths but with in one case with the pore water pressure being positive (using the axis-translation technique) and the

other with it negative (using data from Jotisankasa, 2005). The intention is to check whether there are any appreciable differences in behaviour and resulting parameters derived from the tests.

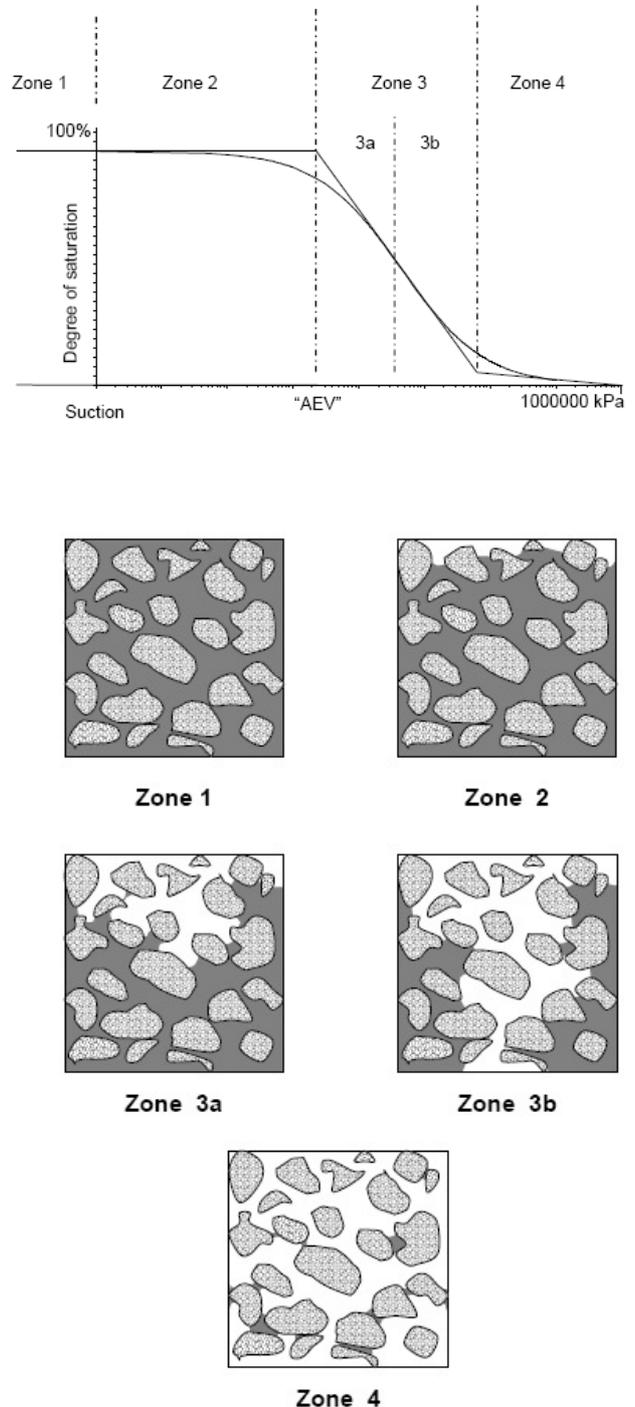


Figure 23. The conceptual zonal model adopted by Smith (2003) in terms of the SWRC and degree of drying of the soil shown schematically (Smith, 2003).

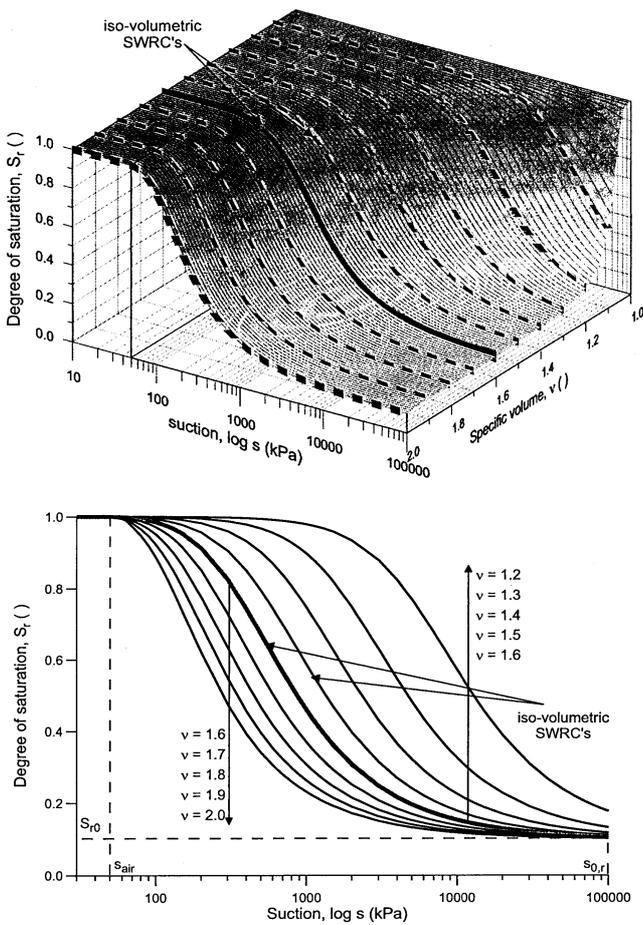


Figure 24 Three-dimensional Soil Water Retention Surface (SWRS) at constant specific volume (iso-volumetric SWRCs) and a projection in the  $s$ - $S_r$  plane of iso-volumetric SWRCs (Tsiampousi, 2011).

Another factor with the existing constitutive models is that most of them are based on testing compacted artificial soils such as kaolin or mixtures of kaolin, silt and sand. Little complete comprehensive data exist from testing of natural unsaturated soils. There was fortunately an opportunity to take block samples of brickearth from beneath St Paul's Cathedral in London in 2007. This soil has a natural degree of saturation of about 70% and so three sets of tests are being performed by Fadzillah Khirradi with the soil either unsaturated, artificially saturated or saturated and reconstituted. By comparing the results from these suites of tests it should be possible to isolate independently the effects of the natural soil structure and the effects of partial saturation. It is planned that a sufficient data set will be collected to provide a full set of parameters for a basic constitutive model (e.g. the BBM).

Other natural soils are also being investigated for example a gypsiferous soil from Libya (by Osama Sigutri) and very plastic expansive soils from Sudan (Khalid Al Haj). These too will provide representative parameters for natural soils. The gypsiferous soil has necessitated new developments with the laboratory testing and appropriate pore fluids to use to avoid dissolving the gypsum present. These four researchers are being supervised by the author.

These studies will be fed back into the numerical analysis research to complete the loop between laboratory, field and numerical studies.

The final brief point to make is that the subject of unsaturated soils has been taught for many years on the MSc Soil Mechanics course (the author currently teaches this module). The course links very well with another course dealing with earthworks and embankments (taught for many years by Professor Vaughan). Thus engineers are gradually gaining a better awareness of the subject of unsaturated soil mechanics and we are continually getting closer to being able to understand and model response accurately to improve our engineering design and analysis.

## 7. ACKNOWLEDGEMENTS

I would like to express many thanks to my colleagues who have discussed various aspects of this paper and read and checked through it: Professor John Burland, Professor David Potts and Dr Lidija Zdravkovic. Dr Andrew Ridley and Mike Crilly helped also with the supply of information. I would also like to acknowledge my gratitude to my colleagues above and to former colleagues, in particular the late Professor Peter Vaughan, and Dr Angus Skinner for their thoughts and inspiration.

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