## In-situ and laboratory investigations of stress-dependent permeability function and SDSWCC from an unsaturated soil slope

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**ABSTRACT:** Permeability function of an unsaturated soil,  $k(\psi)$ , where  $\psi$  is suction, is a vital hydrogeological property that governs seepage in various geotechnical problems. Owing to considerably long test duration, direct measurement of  $k(\psi)$  is often avoided if at all possible. Instead, numerous semi-empirical predictive equations have been developed to determine  $k(\psi)$  indirectly. However, effects of drying-wetting history and net normal stress are not generally considered, casting doubts on the validity of some semi-empirical predictive equations. In this paper, stress-dependent  $k(\psi)$  and stress-dependent soil-water characteristic curve (SDSWCC) of a decomposed silty clay are investigated under both field and laboratory conditions. To measure effects of drying and wetting on  $k(\psi)$  directly, an *in-situ* one-dimensional (1D) permeability test was carried out using the instantaneous profile method on a saprolitic hillslope in Hong Kong. In the laboratory, a new 1D stress-controllable soil column was developed to determine stress-dependent  $k(\psi)$  and SDSWCC on block samples taken from the same hillslope. Effects of drying-wetting cycle(s) and net normal stress on measured stress-dependent  $k(\psi)$ s and SDSWCCs are explored and analysed. By comparing measured and predicted  $k(\psi)$ s, the predictability of some existing semi-empirical equations is evaluated.

## 1. INTRODUCTION

Permeability function and soil-water characteristic curve (SWCC) are well-recognised as two vital hydraulic properties of an unsaturated soil for transient seepage analyses in various geotechnical and geo-environmental problems. It is generally understood that a permeability function describes the relationship between the coefficient of water permeability, k, and volumetric water content ( $\theta_w$ ) (i.e.,  $k(\theta_w)$ ), whereas a SWCC describes the water retention capability of a soil at a given soil suction,  $\psi$ . A SWCC is often expressed in terms of water content and  $\psi$ . Through a SWCC, a permeability function can also be expressed in terms of  $\psi$  (i.e.,  $k(\psi)$ ).

Although it is well recognised that both  $k(\psi)$  and SWCC of an unsaturated soil depend strongly on its pore size distribution (PSD) which is not often easy to be quantified conveniently, void ratio is hence commonly used as an average parameter for simplicity. Extensive experimental and theoretical studies have demonstrated that the behaviour of an unsaturated soil including changes of void ratio, are generally governed by two independent stress-state variables, namely soil suction and net normal stress (Burland 1965; Fredlund & Morgenstern 1977). In recent decades, research carried out by Ng & Pang (2000a, b), Lloret et al. (2003), Ng & Menzies (2007), Ng & Chen (2008), Natalia et al. (2008) and Romero et al. (2011) explored and revealed the effects of net normal stress and stress paths on SWCCs. A term, stress-dependent SWCC (SDSWCC), was introduced by Ng & Pang (2000a) to illustrate the influence of applied stress on the water retention capability of an unsaturated soil at a given  $\psi$ . It is vital to clarify that the application of net normal stress does not only cause a change in soil density (or void ratio) but it also results in redistribution of PSD. Figure 1(a) compares the measured drying and wetting SDSWCCs of a natural (N) and a recompacted (R) decomposed volcanic soil specimens at zero net normal stress. Before applying any suction, the R-0 specimen (open symbol) was purposely recompacted to the same initial dry density (or void ratio) under the same water content as that of the N-0 specimen (solid symbol). When suction changed, they behaved differently (i.e., possessing different air-entry values (AEV) and sizes of hysteresis loops) since they had different initial PSDs even though their initial void ratios were the same.

On the other hand, when a net normal stress of 40kPa was applied on two other soil specimens (i.e., R-40 and N-40) prepared at the same initial dry density under the same water content (see Figure 1(b)), it was expected that the applied stress would have caused different changes in void ratio and PSD in these two specimens since they had different stress histories (i.e., one was a natural specimen but the other was a recompacted one).



Figure 1 Comparisons of drying and wetting SDSWCCs of natural (N) and re-compacted (R) decomposed volcanic soil loaded at net normal stresses of (a) 0kPa and (b) 40kPa (Ng & Pang 2000*a*)

Microstructural analysis such as mercury intrusion porosimetry (MIP) would be very useful to measure any redistribution of PSDs in these two specimens due to the applied stress. As expected, these two specimens show different absorption rates and size of hysteresis loops. Therefore, the application of net normal stress, which can affect both void ratio and PSD, should not be treated as equivalent to a change of the average parameter, void ratio, only.

Although advancements have been made for measuring SWCC/SDSWCC, existing test setups for measuring  $k(\psi)$  rarely

consider both the effects of net normal stress and suction independently and directly. Thus, both of their influences on  $k(\psi)$  are seldom reported and discussed explicitly.

In view of a considerably long test duration when measuring  $k(\psi)$  in either field or laboratory, some researchers and engineers have shifted their focuses on formulating simplified semi-empirical equations to predict  $k(\psi)$  indirectly (Childs & Collis-George 1950; Mualem 1976; van Genuchten 1980; Fredlund *et al.* 1994). The predictability of some predictive equations on laboratory  $k(\psi)$  has been evaluated by Leong & Rahardjo (1997) and Agus *et al.* (2003). On the contrary, few assessments have been carried out to compare and validate any predicted  $k(\psi)$  against direct field measurements of  $k(\psi)$ . Hence, the accuracy and validity of some semi-empirical equations in predicting field  $k(\psi)$  are not known for sure. Therefore, critical evaluation and assessment of these semi-empirical equations are needed.

In this paper, it is intended: (i) to measure and compare field and laboratory  $k(\psi)$  of a decomposed silty clay directly; (ii) to investigate effects of  $\psi$ , net normal stress and drying-wetting cycle on measured  $k(\psi)$  and SDSWCC; and (iii) to evaluate the predictability of  $k(\psi)$  by some commonly used semi-empirical equations. To fulfil these intentions, an *in-situ* 1D permeability test was carried out to measure field  $k(\psi)$  on a saprolitic hillslope in Hong Kong. Moreover, a new 1D stress-controllable soil column was developed to measure laboratory  $k(\psi)$  of the saprolitic soil under different stress and suction conditions.

## 2. RESEARCH HILLSLOPE IN HONG KONG

## 2.1 Description of the site

Figure 2 shows the location and overview of the study area for conducting in-situ permeability test and full-scale field monitoring. The test site is located at a sloping hillside above the North Lantau Expressway, near the International Airport on Lantau Island, Hong Kong. The main planar face of the blunt ridge forming the study area faces roughly north to northwest at an average slope gradient of

280 and overlooks the Tung Chung Eastern Interchange. The natural terrain is moderately to densely vegetated. It forms a blunt ridgeline located between a major stream channel on its northeastern side and a shallow topographic valley to the south and west. At the midportion of the study area, the topography itself formed a very slight bowl-shaped depression. As revealed from field mapping and trial trench explorations, some prominent features of a landslide body including main scarp, lateral tension cracks and thrust features were identified. The landslide mass appears to undergo a retrogressive slab-type movement with limited mobility (Leung *et al.* 2011).

#### 2.2 Ground profile and properties

According to GEO (1994), the site was underlain by undivided rhyolite lava and tuff of the Lantau Formation. A ground investigation was conducted in the vicinity of the test plot comprising trial trench 1 (TT1), trial trench 2 (TT2) and trial pit 1 (TP1). Measured soil profiles and properties are shown in Figure 3. The ground consists of about 1m of loose colluvial deposit accumulated through the action of gravity. The colluvium is thus anticipated to have large interpores, while its thickness may vary from place to place. A layer of about 2m of completely decomposed tuff (CDT) is then successively encountered, and some relict joints with silty clay infill are identified. The CDT at this site is a saprolitic soil, which is defined as Grade IV and V materials according to the six-fold weathering grade classification by GCO (1988), and is commonly found in Hong Kong. It was described as extremely to moderately weak, light grey, dappled light brown, completely decomposed coarse ash tuff with occasional angular and subangular fine gravel. The decomposed tuff overlies moderately and slightly decomposed tuffs (MDT/SDT) which are classified as rock (GCO 1988) at further depths. On the other hand, measured water content (by mass) of the ground decreases with depth whereas both dry density and DPT-N value increases with depth, as expected in a typical unsaturated weathered-rock ground profile. Moreover, drillhole records reveal that the groundwater table (GWT) is located at about the depth of 3m.



Figure 2 Location and overview of the study area (modified from Ng et al. 2011)



\* DPT dynamic probe tests carried out according to GCO (1987); = number of blows per 100 mm penetration. Similar to the standard BS EN ISO 22476-2:2005 DPL, the hammer mass is 10 kg, except that the height of fall is modified to be 300 mm instead of 100 mm by the GCO (1987).

- (1) Firm, light brown, slightly sandy silt with occasional angular to sub-angular fine to coarse gravel and sub-angular cobbles of moderately decomposed tuff (colluvium).
- (2) Very weak (Grade V), light grey, dappled light brown, completely decomposed coarse ash crystal tuff (clayey silt with occasional Angular to

sub-angular fine gravel). Joints are closely spaced, rough planar, extremely narrow and dipping at 10°, 20°, 30° and 40° to the horizontal.

(3) Moderately strong (Grade III), grey, dappled brown, moderately decomposed fine ash crystal tuff.

Figure 3 Soil profiles and properties revealed by ground investigation at test plot (from TT1, TT2 and TP1) (Ng et al. 2011)

Block samples of colluvium and CDT were taken in the vicinity of the test plot. As determined by the sieve and the hydrometer analysis (BSI 1990), the colluvium sample is found to have a wide range of particle-size distributions, suggesting heterogeneity nature of the soil in the field. On the contrary, the sand, silt and clay content of the CDT sample are 35%, 40% and 25%, respectively. Based on the measured particle-size distributions and Atterberg limit, both soil types are described as inorganic silty clay of low to medium plasticity (CL) in accordance to the Unified Soil Classification System (ASTM 2000). Some measured index properties of colluvium and CDT are summarized in Table 1.

Table 1 Index properties of colluvium and CDT (modified from Leung *et al.* 2011)

Index properties	Colluvium	CDT
In-situ water content (%)	20.1	17.3
<i>In-situ</i> dry density (g/m <sup>3</sup> )	1.50	1.6
Maximum dry density (g/m <sup>3</sup> )	1.58	1.67
Optimum water content (%)	15.2	20
Liquid limit (%)	41	34
Plastic limit (%)	17	20
Plasticity index (%)	24	14
Gravel content (≥2mm, %)	0 – 35	0
Sand content $(2mm - 63\mu m, \%)$	20 - 25	35
Silt content $(2 - 63\mu m, \%)$	30 - 50	40
Clay content (≤2µm, %)	15 – 25	25
Specific gravity	2.73	2.68

### 3. THEORETICAL BACKGROUND - INSTANTANEOUS PROFILE METHOD

In order to measured  $k(\psi)$  of CDT directly and timely in field and laboratory, the Instantaneous Profile Method (IPM; Watson 1966), which is a transient-state method, is adopted. During a 1D flow process, both water content and pore-water pressure (PWP) profiles of an unsaturated soil are measured instantaneously to determine water flow rate and hydraulic gradient. By using the Darcy's law,  $k(\psi)$  at any location and time instant can hence be determined. The theoretical calculation procedures are described in the following paragraphs.

Figure 4 shows two arbitrary profiles of volumetric water content (VWC),  $\theta_w$ , and hydraulic head at elapsed time  $t = t_1$  and  $t_2$  when an unsaturated soil column is subjected to 1D downward flow under some controlled boundary conditions. Volumetric water content and PWP at  $z_A$  (Row A),  $z_B$  (Row B),  $z_C$  (Row C) and  $z_D$  (Row D) can be measured by using different types of instruments. Measured VWC profiles may be extrapolated to the surface and the bottom of the soil column for water flow rate determination. Considering the 1D mass continuity, water flow rate at any arbitrary depth  $z_B$ ,  $v_{zB,tave}$ , can be expressed as:

$$v_{zB,tave} = -\frac{d}{dt} \int_{zB}^{ze} \theta_w(z,t) \cdot dz + v_{ze,tave}$$
(1)

where  $\theta_w(z, t)$  is VWC profile as a function of depth z at specific time t; dt is time interval between the two measurements (i.e.,  $t_2 - t_1$ );  $v_{ze,tave}$  is the outflow rate at the bottom boundary for average elapsed time  $t_{ave} = (t_1+t_2)/2$ . The first term at the right hand side of Eq. (1) physically means the water storage between depths  $z_B$  and  $z_e$  upon transient flow within dt. This particular term can be calculated by determining the area bounded by VWC profiles at  $t_1$  and  $t_2$ geometrically as shown in Figure 4. As a result,  $v_{zB,tave}$  in Eq. (1) can hence be expressed as:

$$v_{zB,tave} = -\frac{\Delta V}{t_2 - t_1} + v_{ze,tave}$$
<sup>(2)</sup>

where  $\Delta V$  is the shaded area between the  $\theta_w(z, t)$  at  $t = t_1$  and  $t_2$ .

On the other hand, by estimating the slope of a hydraulic head profile which is the summation of measured PWP head profile and gravitational head profile, hydraulic head gradient,  $i_{zB,tave}$ , at any depth  $z_B$  for any average elapsed time  $t_{ave}$  can be determined by:

$$i_{zB,tave} = \frac{1}{2} \left( \frac{dh_{zB,t1}}{dz} + \frac{dh_{zB,t2}}{dz} \right)$$

$$= \frac{1}{4} \left[ \left( \frac{h_{zA,t1} - h_{zB,t1}}{z_A - z_B} + \frac{h_{zB,t1} - h_{zC,t1}}{z_B - z_C} \right) + \left( \frac{h_{zA,t2} - h_{zB,t2}}{z_A - z_B} + \frac{h_{zB,t2} - h_{zC,t2}}{z_B - z_C} \right) \right]$$
(3)

where  $h_{zi,ij}$  is hydraulic head at depth  $z_i$  (i = A, B, C and D) for elapsed time  $t_i$  (j = 1, 2).

By using the Darcy's law, permeability,  $k_{zB,tave}$ , at any depth  $z_B$  for any average elapsed time  $t_{ave}$  can hence be calculated by dividing the water flow rate by the corresponding hydraulic head gradient:

$$k_{zB,tave} = -\frac{v_{zB,tave}}{i_{zB,tave}} \tag{4}$$

#### 4. FIELD AND LABORATORY MEASUREMENTS OF PERMEABILITY FUNCTIONS

## 4.1 Field measurements – Instantaneous Profile (IP) test

## 4.1.1 Test setup and instrumentation

A one-dimensional *in-situ* permeability test, using the IPM and so thereafter called (IP) test (Ng et al. 2011), was conducted near the toe of the hillslope (see solid circle in Figure 2). The test aimed to measure *in-situ* SDSWCCs and  $k(\psi)$ s of the ground directly and to investigate the effect of wetting-drying cycle on the two hydraulic properties. Figure 5 illustrates a cross-section of the experimental setup. Before instrument installation, a flat test plot of 3.5m x 3.5m was first formed by cutting into the ground at the test location. To achieve the 1D vertical water flow assumption for the IPM (see Eq. (1)), a 3m deep and 1.2m wide trench was excavated at the uphill side of the test plot to install a polythene sheeting. It aimed to act as a cut-off to minimise any possible lateral groundwater flow and recharge from up-slope during the test. The trench was then backfilled after the sheeting installation. Subsequently, a circular steel test ring which is 3m in diameter was installed at the ground surface and embedded 100mm into the flattened plot to retain water for the IP test.



Figure 4 Arbitrary profiles of volumetric water content,  $\theta_w$ , and hydraulic head, *h*, at elapsed time,  $t = t_1$  and  $t_2$ , along a one-dimensional soil column (Ng & Leung 2011)

Ten jet fill tensiometers (JFT\_1 to JFT\_10) were installed at 0.36, 0.77, 0.95, 1.17, 1.54, 1.85, 2.13, 2.43, 2.6 and 2.99m depths to measure negative PWP directly while four time-domain reflectometers (TDR\_1 to TDR\_4) were installed at 0.84, 1.85, 2.5 and 3.59m depths to measure VWC indirectly. Owing to the possibility of cavitation, the measuring range of each JFT was from 0 to -90kPa of PWP. On the other hand, laboratory calibrations of each TDR were conducted on colluvium and CDT samples taken from the site. The results revealed that the maximum deviation from each calibration curve was within 2% for VWC ranged from 10 to 40% for both colluvium and CDT.

#### 4.1.2 Test programme and procedures

The test procedures were divided into 4 stages, consisting of two cycles of wetting-drying phases. For each wetting phase, a water level of 0.1m was ponded on the flatten ground surface inside the test ring for 4 days. The water level was checked and refilled to the same level every 12 hours. On the other hand, the test plot was allowed to dry under both natural evaporation and internal drainage for each drying phase. More details of the test programme and procedures are given by Ng *et al.* (2011).



Figure 5. Arrangement of instruments for the IP test (Ng et al. 2011)

#### 4.2 Laboratory measurements using 1D soil column

## 4.2.1 Development of the new 1m high stress-controllable soil column

In order to investigate the effects of suction, net normal stress and drying-wetting cycle(s) on  $k(\psi)$  timely and economically, it is desirable to conduct permeability tests in the laboratory. Based on the IPM, a 1m high stress-controllable soil column (SC) was designed to control net normal stress and boundary drainage conditions and to measure suction when determining  $k(\psi)$  (Ng & Leung 2011). The general layout of the SC is shown in Figure 6. It consists of a 1m high acrylic hollow cylinder, a constant head water supply system and a loading system. The inner and outer diameters of the hollow cylinder are 150 and 160mm, respectively (i.e., 5mm in thickness). The aspect ratio (i.e., height-to-diameter ratio) of this SC is designed to be greater than 6. This is to ensure 1D flow conditions for determining  $k(\psi)$  using the IPM (see Eq. (1)). This is consistent with the approach reported by many researchers that the aspect ratio of most soil column are usually greater than 5 when studying 1D infiltration problems (Chapuis et al. 2006; Choo & Yanful 2000; Li et al. 2009; Watson 1966; Wendroth et al. 1993; Yang et al. 2004).

In this SC, the top and the bottom boundary flow conditions can be controlled and measured. On the top of the SC, a 150mmdiameter and 20mm-thick, circular, perforated, stainless-steel plate is placed so that any vertical stress and boundary flux can be applied independently during a test. When a soil column is subjected to surface ponding upon wetting process, constant head infiltration can be achieved and the corresponding infiltration rate can be measured by a constant head water supply system. The system adopts a similar principle to that of a Mariotte's bottle (McCathy 1934). To allow for uniform drainage at the bottom of a soil column, another circular perforated plate which can be completely filled with water to form a water compartment is placed. A valve is installed at the outlet of drainage line to control bottom boundary flux condition.



Figure 6. Experimental setup of the 1m high stresscontrollable 1D soil column

Regarding net normal stress, a vertical load can be applied using a pneumatic actuator. Any applied load is transmitted to the centre of the stainless steel plate through a loading ram to soil column. The magnitude of an applied load is recorded by a load cell. Based on the cavity expansion theory, it is estimated that the radial strain of the 1m high acrylic cylinder due to a vertical load of 100kPa is less than 0.02%. This value is 60% smaller than the radial strain of 0.05% required by the Japanese Geotechnical Society (JGS-0525) for satisfactory  $K_0$  loading conditions in a triaxial apparatus (JGS 1999).

To account for effects of friction at the acrylic-soil interface and self-weight on vertical stress ( $\sigma_{\nu}$ ') distributions due to an applied load, an effective stress finite element (FE) analysis was conducted. A model column was loaded under  $K_0$  condition and its base was drained. Since vacuum grease was pasted around the inner wall of the acrylic cylinder before test, the coefficient of acrylic-grease-soil interface friction is estimated to be 0.02 (Powrie 1986). As expected, owing to the presence of friction at the soil-wall interface, computed  $\sigma_{\nu}$ ' decreased linearly with depth at an applied stress. Under vertical stress of 80kPa, the maximum reduction of  $\sigma_{\nu}$ ' due to friction is less than 12kPa (Ng & Leung 2011).

#### 4.2.2 Instrumentation and calibration

In this research, four pairs of miniature-tip tensiometers and thetaprobe soil moisture probes were used to measure PWP and VWC, respectively. There were installed at 100mm (Row A), 125mm (Row B), 600mm (Row C) and 725mm (Row D) below the surface of a soil column. Each pair of tensiometer and theta-probe is purposely aligned at the same elevation. This arrangement aims to verify measurements from each other and to obtain relationships between measured PWP and VWC (i.e., SDSWCC).

Prior to installation, all tensiometers and theta-probes were calibrated. Each tensiometer was fully saturated with de-aired water and its response time was checked to ensure that it was free of plugging. Each tensiometer was re-saturated immediately if water in its plastic tube cavitated, as indicated by sudden drops of measured PWP to 0kPa, and/or any bubble was observed in the tube during a test.

The working principle of a theta-probe is to measure the difference in the amplitude of a standing wave, which primarily depends on the apparent dielectric constant of a soil. The VWC could then be determined through a soil-specific calibration curve indirectly. For simplicity, one-to-one linear and polynomial relationships between VWC and square root of dielectric constant are often adopted (Topp *et al.* 1980; White *et al.* 1994). However, any increase in net normal stress would increase the bulk density of a soil and hence cause a decrease in the mobility of water dipoles (or increase dielectric constant) (Skierucha 2000). Based on Skierucha (2000), the polynomial empirical calibration equation proposed by Topp *et al.* (1980) is therefore modified and adopted in this study. The modified calibration equation is expressed as follows:

$$\theta_{w} = \frac{1.07 + 6.4V - 6.4V^{2} + 4.7V^{3} - (a_{1} + a_{2}\rho)}{(a_{3} + a_{4}\rho)}$$
(5)

where *V* is output voltage from each theta-probe;  $a_1$ ,  $a_2$   $a_3$  and  $a_4$  are calibration coefficients; and  $\rho$  is soil bulk density.

Each of the four theta-probes installed at Rows A to D was calibrated against three dry densities,  $\rho_d$ s, of 1550, 1600 and 1620kg/m<sup>3</sup> and four VWCs of 0, 20, 35, 40 and 45%. Figure 7 shows the calibration curves for the theta-probe at Row A at the three different  $\rho_d$ s. Each curve is obtained by best-fitting all calibration data using Eq. (5). The goodness-of-fit ( $R^2$ ) of each calibration curve is better than 0.997. It is evident that, using a single set of calibration coefficients, the three calibration curves overlap each other. This means that the range of  $\rho_d$  calibrated has negligible influence on VWC measurements. Moreover, the maximum difference between actual (by oven-drying) and measured (via calibration curve) VWC is ±5%, consistent to the value stated

by the manufacturer (Delta-Devices Ltd. 1999). Similar calibration results are found for the other three theta-probes at Rows B, C and D.



## 4.2.3 Test material and sample preparation

The test material used for the 1D soil column test was CDT, which was sampled from the test site in Lantau Island. The measured index properties of CDT are summarized in Table 1.

Prior to specimen compaction, a thin layer of vacuum grease was pasted around the inner wall of the 1m high cylinder to minimize any preferential flow path and interface friction. A stainless steel plate was then placed at the bottommost of the cylinder and was filled with de-aired water. To allow for uniform drainage, a water saturated filter paper was placed on the top of the plate. Each soil column in this study has diameter of 150mm and height of 900mm and was compacted by the moist tamping method. The targeted gravimetric water content (GWC) and  $\rho_d$  of each soil column was divided into 20 layers. For better contact, the top surface of each soil layer was scarified before the compaction of the successive layer. After the compaction, another stainless steel plate was placed on the top of each soil column.



Figure 8. Measured dry density profiles by soil sampling (Ng & Leung 2011)

A trial compaction was conducted to examine the uniformity of a soil column, which was prepared using the above procedures. The measured  $\rho_d$  profile is shown in Figure 8 (open circles). It can be seen that measured  $\rho_d$  increases slightly with depth and the maximum deviation from the targeted  $\rho_d$  of 1552kg/m<sup>3</sup> is less than 1.2%. This small variation provided evidence that friction along the acrylic-grease-soil interface during compaction was negligible.

## 4.2.4 Test programme and procedures

In total, three tests were carried out to investigate the influences of net normal stress and drying-wetting cycle on  $k(\psi)$  of CDT. A drying-wetting cycle was applied on a compacted soil column loaded at targeted vertical net normal stresses of 0kPa (SC0), 40kPa (SC40) and 80kPa (SC80). The testing procedure involved four stages; (a) saturation, (b) pre-consolidation, (c) evaporation and (d) ponding, and is summarised below.

Each compacted soil column was first subjected to bottom-up saturation by opening the bottom valve (see Figure 6). The saturation stage was considered to be completed when all four tensiometers recorded zero reading for at least 24 hours.

After saturation, a required vertical load was then applied on the top of each soil column via the loading ram for pre-consolidation purpose. The load was ramped up at 0.09kN/day (or 5kPa/day) until a targeted value was reached. The slow applied loading rate aimed to minimise any generation of excess PWP. The process was considered to be completed when (i) all tensiometers recorded zero readings and; (ii) change of water outflow rate was less than 15cm<sup>3</sup>/day for at least 24 hours. This was equivalent to an average GWC change of 0.06%/day.

After pre-consolidation, each pre-consolidated soil column then underwent an evaporation stage. The bottom valve was closed to achieve zero boundary flux condition while the surface of each soil column was allowed to evaporate naturally. This stage was terminated when the tensiometer at Row A recorded a PWP of -80kPa to minimise the effects of cavitation.

After the evaporation stage, ponding process was carried out subsequently. About 50mm of constant head ponding was applied and controlled on each column surface using the constant head water supply system. The bottom valve of the soil column was re-opened to drain water. The mass of infiltrated water was recorded continuously to determine infiltration rate (i.e.,  $v_{ze,tave}$  in Eq. (2)).

During both the evaporation and ponding stages, variations of PWP and VWC profiles with time were continuously monitored along each soil column. At the end of each test, the soil column was unloaded and eight specimens were sampled along the soil column to determine its final GWC and  $\rho_d$ . Figure 8 compares the measured  $\rho_d$  profile after each test (solid symbols). Owing to the applied loads at the pre-consolidation stage, the measured  $\rho_d$  increased along the depth of each soil column for both SC40 and SC80. It is evident that each  $\rho_d$  profile is fairly uniform. The average measured  $\rho_d$  for SC0, SC40 and SC80 were 1567, 1601 and 1627kg/m<sup>3</sup>, respectively. Among the eight soil specimens measured, the maximum deviation from the average  $\rho_d$  was less than 1% for all the three tests. The relatively small variation of  $\rho_d$  along each soil column implies that soil volume changes during the four stages of a test are limited.

#### 5. STRESS-DEPENDENT PERMEABILITY FUNCTIONS

Based on measured variations of VWC and PWP profiles, drying and wetting  $k(\psi)$  of the ground in the field (i.e., IP test) and each soil column in the laboratory (i.e., 1D soil column test) can be determined using the IPM (refer to Eqs. (1) to (4)). In the following discussion, influences of net normal stress, soil suction and drying-wetting cycle on  $k(\psi)$  of three CDT filled columns are explored first. In each laboratory test, soil homogeneity, top and bottom flux boundary conditions and applied stress level are well-defined and controlled. Field and laboratory measured  $k(\psi)$  of decomposed tuff are then compared to investigate and highlight any observed discrepancy

### 5.1 Stress dependency of unsaturated permeability

#### 5.1.1 Permeability-suction relationship

Figures 9(a), (b) and (c) show the laboratory measured permeability function, which relates permeability to its corresponding matric suction,  $k(\psi)$ , for SC0, SC40 and SC80, respectively. By taking into account the effects of friction at the acrylic-soil interface and selfweight of a CDT specimen in the FE analysis, an average vertical net normal stress of 4, 39 and 78kPa are adopted to report the stress applied in the SC0, SC40 and SC80, respectively. Saturated permeability,  $k_s$ , at each net normal stress is also shown in Figure 9 for comparison. The  $k(\psi)$  measured at the four rows in each test are denoted using four different symbols separately. It is evident that the overall trends of both drying and wetting  $k(\psi)$  are consistent among the four rows for a given average vertical net normal stress. Moreover, at a given suction, the maximum variation of  $k(\psi)$  is less than half an order of magnitude at four different depths. This means that the interface friction and soil self-weight do not appear to influence the investigation of stress-dependency of  $k(\psi)$ significantly.

At a given applied average vertical net normal stress, the measured drying  $k(\psi)$  decreased log-linearly as matric suction increased. The reduction of drying  $k(\psi)$  at constant zero vertical net normal stress was up to two orders of magnitude as matric suction increased from 0 to 80kPa (Figure 9(a)). An increase in matric suction may essentially induce more air bubbles into each soil column. These induced air bubbles block some hydraulic paths and hence increase the tortuosity of the water flow paths and reduce the permeability. When the average vertical net normal stress was increased from 4 to 78kPa, the maximum reduction in drying  $k(\psi)$  at a matric suction of 6kPa was about one order of magnitude (see Figures 9(a), (b) and (c)). Moreover, at a given change of matric suction from 4kPa to 80kPa, the decreasing rate of drying  $k(\psi)$ reduced with an increase in average vertical net normal stress, particularly when the average vertical net normal stress was increased from 39 to 78kPa. A summary of the estimated decreasing rate of each measured drying  $k(\psi)$  is given in Table 2. When compared to experimental data reported by Li et al. (2009), the decreasing rate of  $k(\psi)$  of a silty clay at zero stress was estimated to be  $2.17(\log_{10}\text{m/s}) (\log_{10}\text{kPa})^{-1}$ . The observed stress dependency of the drying  $k(\psi)$  in this study is likely attributed to the substantial increase of  $\rho_d$  of the soil column (see Figure 8(a)) and also possible redistribution of PSD. This is consistent to the microstructure analysis of a Boom clay carried out by Romero et al. (1999).



Figure 9. Laboratory measured  $k(\psi)$  of compacted CD1 at an average vertical net normal stress of (a) 4kPa, (b) 39kPa and (c) 78kPa (Ng & Leung 2011)

Table 2. Summary of estimated decreasing and increasing rates of  $k(\psi)$  and  $k(\theta_w)$  for each test (Ng & Leung 2011)

Test identity	$k(\psi)$		$k(\theta_w)$	
	*Decreasing rate	*Increasing rate	*Decreasing rate	*Increasing rate
	$(\log_{10} m/s) \cdot (\log_{10} kPa)^{-1}$		(log <sub>10</sub> m/s)	
SC0	1.392	0.982	8.730	15.544
SC40	1.116	0.869	14.507	27.881
SC80	0.628	0.734	14.818	26.138

\*each decreasing/increasing rate is determined by averaging the constant portion of a drying/wetting curve

On the other hand, at a given average vertical net normal stress, the average wetting  $k(\psi)$  was always lower than the average drying one by about a half order of magnitude at any matric suction considered in the three tests (see Figures 9(a), (b) and (c)). The observed consistent lower wetting  $k(\psi)$  at all stress levels is likely because the VWCs along wetting SDSWCC were less than that along the drying one at any given suctions, possibly leading to lesser hydraulic flow paths and hence smaller wetting  $k(\psi)$ . Measured drying and wetting SDSWCCs are discussed later. As summarised in Table 2, when average vertical net normal stress was increased from 4 to 78kPa, the estimated increasing rate of wetting  $k(\psi)$ reduced relatively slowly when compared to the reduction in the decreasing rate of drying  $k(\psi)$ . Moreover, remarkable hysteresis loop is identified between the drying and the wetting  $k(\psi)$  at each average vertical net normal stress. The loop size bounded by each pair of drying and wetting  $k(\psi)$  seems to reduce with an increase in average vertical net normal stress (see Figures 9(a), (b) and (c)). This may be because the "ink-bottle" effect is less pronounced for a denser soil column which is likely to have a smaller average PSD (Ng & Pang 2000a).

#### 5.1.2 Permeability-VWC relationship

A permeability function can also be expressed in another form by relating the measured permeability to its corresponding VWC,  $k(\theta_w)$ . Figure 10 shows the measured  $k(\theta_w)$  at average vertical net normal stresses of 4, 39 and 78kPa. In responses to the decreases of VWC upon evaporation, measured drying permeability showed log-linear decrease at any applied vertical load consistently (see solid symbols). When the average vertical net normal stress was increased from 4 to 39kPa, the estimated decreasing rate of drying permeability almost doubled at a given change of VWC. However, there is negligible increase in the decreasing rate of the  $k(\theta_w)$  when the applied vertical stress was increased further from 40 to 80kPa. The estimated decreasing rate of each drying  $k(\theta_w)$  is summarised in Table 2.

Upon ponding, measured wetting permeability at a given average vertical net normal stress increased with an increase in VWC but at a higher rate than that of the corresponding drying  $k(\theta_w)$ . Moreover, it can be observed that average drying permeability at any stress level is higher than average wetting permeability at low VWC consistently. However, when VWC increased to a higher value, the drying permeability becomes lower. The critical VWCs where the observed trend reversed for SC0, SC40 and SC80 are about 25, 28 and 32%, respectively. The maximum difference between average drying and wetting permeability is less than a half order of magnitude.

When the applied vertical stress was increased from 0 to 80kPa, the maximum reduction in permeability (either drying or wetting) is up to two orders of magnitude at VWC of 30%. This implies that when the soil column subjected to different average vertical net normal stresses, any changes of its PSD might cause different distributions of water and hydraulic flow paths even at the same VWC.

## 5.2 Comparisons between field and laboratory measured $k(\psi)$ s of decomposed tuff

Figure 11 compares  $k(\psi)$  of CDT measured in the field and in the laboratory. Considering the overburden stress at 1.85m depth (i.e., ~37kPa), laboratory measurement made at average vertical net normal stress of 39kPa is selected for comparison. A range of  $k_s$  (from 3 x 10<sup>-6</sup> to 9 x 10<sup>-6</sup>m/s), which was obtained from constanthead tests in boreholes and double-ring infiltration tests in the midlevels of Hong Kong Island (GCO 1982), is also shown for reference (shaded area).

It is evident that the average of all field measured  $k(\psi)$ s during the two wetting-drying cycles (see dotted line) remains nearly constant, which is close but less permeability than the lower bound of  $k_s$  determined by GCO (1982) in the mid-levels of Hong Kong Island. The measured  $k(\psi)$ s by the IP test are consistent with negligible changes of VWCs measured at 1.85m depth during the two wetting-drying cycles, as reported by Ng *et al.* (2011).

On the other hand, surprisingly, the field measured  $k(\psi)$ s along the two wetting paths (open diamonds and triangles) are higher than that of drying paths (solid diamonds and triangles) by about one and half orders of magnitude. For suctions ranging between 0.1 and 5kPa, the field  $k(\psi)$  varies from 3 x 10<sup>-6</sup> to 1 x 10<sup>-4</sup>m/s along the wetting paths, and from 4 x  $10^{-7}$  to 3 x  $10^{-6}$  m/s along the drying paths. The observed higher wetting  $k(\psi)$ s in the field was also found in Guelph loam presented by Brook & Corey (1964) and Glendale clay loam reported by Dane & Wierenga (1975). It is revealed from the IP test that measured VWCs along drying and wetting paths at 1.85m depth were comparable (Ng et al. 2011). Similar drying and wetting  $k(\psi)$  might thus be anticipated. In contrast, laboratory measurements demonstrate that the drying  $k(\Psi)$  (solid circles) of the CDT specimen is higher than the wetting one (open circles) at any suction. This appears to oppose to the field measurements that wetting  $k(\psi)$  is generally higher.

Obviously, for suctions less than 10kPa, the field  $k(\psi)$ s are always higher than the laboratory ones by up to two orders of magnitude, though the *in-situ* and laboratory CDT were subjected to similar overburden stress of 40kPa. The higher field  $k(\psi)$ s may be as



Figure 10. Laboratory measured  $k(\theta_w)$  of CDT at an average vertical net normal stress of 4kPa, 39kPa and 78kPa (Ng & Leung 2011)

expected because *in-situ* geological features like cracks, fissures and rootlets, which were not taken into account in the laboratory test, could result in preferential subsurface flow (Basile *et al.* 2006).

## 6. STRESS-DEPENDENT SOIL WATER CHARACTERISTIC CURVE

A SDSWCC describes water retention capability of an unsaturated soil at given soil suction and net normal stress (Ng & Pang 2000*a*). By relating measured VWC to its corresponding matric suction, drying and wetting SDSWCCs can be obtained at different depths in the IP test and at different applied loads in the 1D soil column test.

#### 6.1 Effects of stress level and drying-wetting cycle

To better understand the soil-water characteristics of CDT, measured results from the 1D column test, where homogeneity and boundary conditions of each soil column were well-controlled, are discussed first. Figure 12 shows the measured drying and wetting SDSWCCs at three different average vertical net normal stresses. At any of the three stress levels, the measured results at the four rows appear to be consistent with each other and the maximum variation of VWC at a given suction is less than 4%. These suggest that interpretation of stress-dependent soil-water characteristic would less likely to be influenced by the effects of friction at the acrylic-soil interface and self-weight of the CDT column.

Along the drying path, the measured VWCs appear to start decreasing significantly when matric suction reached AEV at each stress level. The AEV is estimated to increase from 2 to 5.1kPa as the average vertical net normal stress was increased from 4 to 78kPa. Similar increasing trends of AEV due to increases of  $\rho_d$  were also observed from drying SWCC measurements of tills (Vanapalli et al. 1999) and kaolinitic-illitic clays (Romero & Vaunat 2000). When each soil column continued to dry under natural evaporation, the increases of suction resulted in a log-linear reduction of VWC. The estimated desorption rate is found to be reduced by nearly 50% when the average vertical net normal stress was increased from 4 to 39kPa. The estimated desorption rate appears not to reduce further as the vertical stress was increased to 78kPa. Similar stressdependency of estimated adsorption rate can be observed from the wetting SDSWCCs. The observed trends of the AEV and the desorption and absorption rates indicate that the water retention capability of CDT increases with increasing net normal stress. As shown in Figure 8, the average  $\rho_d$  of the soil column is found to increase from 1567 to 1627kg/m<sup>3</sup> as the applied vertical stress is increased from 0 to 80kPa. The measured 5% increase of the  $_d$  likely results in a smaller average soil PSD (Romero *et al.* 1999; Ng & Pang 2000*a*).

At a given stress level, the VWCs along the wetting path are always less than that along the drying path, exhibiting a remarkable hydraulic hysteresis. The observed hysteretic behaviour may be due to the geometric nonuniformity of individual pores or "ink-bottle" effects (Hillel 1998) and the difference of contact angle between advancing (drying) and receding (wetting) meniscus (Hillel *et al.* 1972). In additions, air bubbles may have been entrapped in the soil column under transient flow conditions (Mohamed & Sharma 2007). By estimating the area between each drying and wetting SDSWCC, the hysteresis loop size seems to decrease with increasing stress level. Reductions of hysteresis loop sizes due to an increase of net normal stresses and a decrease of void ratios were also measured and observed by Romero *et al.* (1999), Ng & Pang (2000*a*) and Miller *et al.* (2008).

# 6.2 Comparisons between field and laboratory measured SDSWCCs

#### 6.2.1 Colluvium

Figure 13(a) compares the field and laboratory measured SDSWCCs of colluvium. Since drying and wetting SDSWCCs of colluvium were not measured using SC, a 1D stress-controllable volumetric pressure plate extractor (PP; Ng & Pang 2000*a*) is used, instead. Drying and wetting SDSWCCs of a natural colluvium sampled from the hillslope were measured at zero stress condition (i.e., PP0; solid and dash lines) and are used to compare those measured at 0.36m in the field. It can be seen that the laboratory measured SDWCCs appear to capture the overall field situation satisfactorily in the first wetting–drying cycle. The AEVs estimated from both the field and laboratory tests are 1kPa. Moreover, their desorption rates appear to be comparable for suction ranged between the AEV and 6kPa. A consistent reduction of desorption rates is observed at matric suction of 6kPa. In addition, remarkable hydraulic hysteretic loops are observed from both tests.

Considering the fact that the ground has been subjected to countless wetting-drying cycles in the past, it is not surprising to find that the *in-situ* SDSWCCs obtained from the first wetting-drying cycle (circle symbols) are comparable to those from the second cycle (triangle symbols) (Ng & Pang 2000*a*). Relatively speaking, the consistency between the laboratory and field SDSWCCs obtained in the second wetting-drying cycle is not as



Figure 11. Comparisons between *in-situ* and laboratory measured  $k(\psi)$  of CDT (data from Ng *et al.* 2011 and Ng & Leung 2011)

good as that measured in the first one, especially at low suctions. This is attributed mainly to the smaller hysteretic loop obtained from the second cycle than that obtained in the first cycle for the laboratory specimen (dotted line). Given the consistent *in-situ* SDSWCCs between the two cycles (data points), it seems to suggest that there might have been a change in PSD of the laboratory specimen after the first cycle, leading to different measurements being obtained during the second cycle. It is reported that the size of the hysteretic loop can be reduced significantly in a virgin compacted decomposed volcanic between the first and the second wetting–drying cycles (Ng & Pang 2000*a*), possibly due to collapse of the soil structure during the first wetting cycle. Similarly, it may not be unreasonable to expect a collapse in the soil structure in natural specimens if it has been disturbed during sampling. Using Young-Laplace equation,

$$u_a - u_w = \frac{2T_s}{R_s} \tag{6}$$

where  $T_s$  is surface tension and  $R_s$  is radius of curvature of the



Figure 12. Laboratory measured SDSWCC of compacted CDT at an average vertical net normal stress of (a) 4kPa, (b) 39kPa and (c) 78kPa (Ng & Leung 2011)

meniscus, and assuming a zero contact angle, the radius of curvature may be considered analogous to the average pore radius of the soil specimen. Assuming that the  $T_s$  is equal to 72mN/m at 25 degree, the calculated pore radii are 1.4mm at matric suction of 0.1kPa and 0.072mm at suction of 2kPa. It is postulated that these relatively large pores at low suctions might have collapsed after the first wetting–drying cycle, leading to different laboratory SDSWCCs in the second cycle.

#### 6.2.2 Decomposed tuff

Figure 13(b) shows the *in-situ* SDSWCCs of CDT at 1.85m depth. The SDSWCCs measured from the laboratory 1D soil column test at similar stress level (i.e., 39kPa; SC40 in Figure 12(b)) are also illustrated for comparison. For clarity, only trend lines are shown for the laboratory SDSWCCs. It is evident that, during the two wetting-drying cycles, the field measured VWCs remain nearly unchanged for suctions less than 5kPa (see data points). This means that the hydraulic hysteresis is negligible. Moreover, the SDSWCCs obtained from the first and second wetting-drying cycles appear to nearly overlap each other. Since the ground has been subjected to countless wetting and drying cycles in the past, the field measured SDSWCCs may probably be referred to scanning curves.



Figure 13. Comparisons between field and laboratory measured SDSWCCs of (a) Colluvium at 0.36m depth and (b) CDT at 2.99m depth (data from Ng *et al.* 2011 and Ng & Leung 2011)

In contrast, a remarkable hysteresis loop is observed between laboratory measured drying (solid line) and wetting (dash line) SDSWCC of the CDT specimen. Moreover, it appears that the laboratory SDSWCCs envelop the field measured ones. Unlike the field condition, the compacted CDT specimen experienced only one drying-wetting cycle after specimen saturation (see test procedures in Section 4.2.4). This seems to suggest that the laboratory measured SDSWCCs may be referred to primary curves.

### 7. EVALUATIONS OF EXISTING SEMI-EMPIRICAL PREDICTIVE EQUATIONS

#### 7.1 Commonly used equations for predicting $k(\psi)$

With the availability of field and laboratory measured  $k(\psi)$  of both colluvium and CDT, it is possible to make direct comparisons and to verify predictions of  $k(\psi)$  by some semi-empirical equations. In this section, the predictability of three commonly used semi-empirical equations proposed by Mualem (1976) (ML<sub>P</sub>), van Genuchten (1980) (VG<sub>P</sub>) and Fredlund *et al.* (1994) (FL<sub>P</sub>), is evaluated. Every predictive equation is derived by integrating a fitted/predicted SWCC using a statistical model. The predictive equations by ML<sub>P</sub>, VG<sub>P</sub> and FL<sub>P</sub> can be, respectively, expressed as:

$$k_{r}(\psi) = (a_{b}/\psi)^{2+2.5n_{b}}$$
<sup>(7)</sup>

$$k_{r}(\boldsymbol{\psi}) = \frac{\left\{1 - (\alpha \boldsymbol{\psi})^{n_{r}-1} \left[1 + |\alpha \boldsymbol{\psi}|^{n_{r}}\right]^{m_{r}}\right\}^{2}}{\left[1 + |\alpha \boldsymbol{\psi}|^{n_{r}}\right]^{n_{r}/2}}$$
(8)

$$k_{r}(\boldsymbol{\psi}) = \frac{\int\limits_{\ln(\boldsymbol{\psi})}^{b} \frac{\theta_{w}(e^{y}) - \theta_{w}(\boldsymbol{\psi})}{e^{y}} \theta_{w}'(\boldsymbol{\psi}) dy}{\int\limits_{\ln\boldsymbol{\psi}_{AEY}}^{b} \frac{\theta_{w}(e^{y}) - \theta_{s}}{e^{y}} \theta_{w}'(\boldsymbol{\psi}) dy}$$
(9)

where

$$\theta_{w}(\psi) = \theta_{r} + C(\psi) \frac{(\theta_{s} - \theta_{r})}{\left\{ \ln \left[ e + \left( \frac{\psi}{a} \right)^{n} \right] \right\}^{m}}$$
(10)

and

$$C(\psi) = \frac{\ln(1 + \psi/C_r)}{\ln[1 + (100000/C_r)]}$$
(11)

where  $k_r(\psi)$  is relative coefficient of permeability, which is defined as the ratio of  $k(\psi)$  over  $k_s$ .  $\theta_s$  and  $\theta_r$  are saturated and residual VWC, respectively;  $b = \ln (1\ 000\ 000)$ ; y is dummy variable for integration; and  $\psi_{AEV}$  is AEV; and  $C_r$  is a constant related to matric suction corresponding to residual water content. The parameters,  $a_b$  and  $n_b$ in Eq. (7),  $\alpha$ ,  $n_v$  and  $m_v$  in Eq. (8) and a, n and m in Eq. (10), are fitting coefficients of a semi-empirical SWCC fitting equation proposed by Brook & Corey (1964) (BC<sub>F</sub>), van Genuchten (1980) (VG<sub>F</sub>) and Fredlund & Xing (1994) (FX<sub>F</sub>), respectively. In particular, VG<sub>F</sub> restricts a condition that  $m_v$  is equal to 1- (1 /  $n_v$ ).

It should be noted that both Eqs (7) and (8) were derived using the same statistical model proposed by Mualem (1976) (ML<sub>S</sub>) but integrating over a SWCC fitting equation proposed by BC<sub>F</sub> and VG<sub>F</sub>, respectively. On the other hand, Eq. (9) is formulated by integrating a SWCC fitting equation FX<sub>F</sub> (i.e., Eq. (10)) using a statistical model proposed by Childs & Collis-George (1950) (CCG<sub>S</sub>).

## 7.2 Assumption and application of the predictive and the fitting equations

It is illustrated in the previous sections that both field and laboratory measured  $k(\psi)$ s may be influenced by drying-wetting cycles and stress levels significantly. Before predicting  $k(\psi)$ , it is vital to review and understand fundamental assumptions made implicitly in each of the three semi-empirical predictive equations (i.e., Eqs (7) to (9)).

The predictability of each semi-empirical equation primarily depends on the formulation of its corresponding statistical model and a SWCC fitting equation. In using a statistical model and a SWCC fitting equation, three fundamental assumptions may be identified from the three semi-empirical approaches. The first assumption is that soil is a porous medium consisting of a set of randomly, distributed and interconnected pores with radius r (Mualem 1986). The statistical distribution of r is characterised by f(r), which is known as pore size density function or PSD. By formulating different mathematical expressions for f(r)s, different statistical models and SWCC fitting equations can be derived. For example, Fredlund & Xing (1994) adopted the following expression for f(r) to formulate a SWCC using Eq. (10).

$$f(r) = \frac{mn(\psi/a)^{n-1}}{a[e + (\psi/a)^{n}]\{\log[e + (\psi/a)^{n}]\}^{m+1}}$$
(12)

where e is a mathematical constant equal to 2.71. The fitting coefficients a, n and m are used to best-fit the shape of a f(r) and hence a SWCC mathematically.

The second assumption is that any change of soil volume and hence f(r) due to a change of either suction or net stress or both is



Figure 14. Back-analysed *f*(*r*)s from laboratory measured SDSWCCs of CDT for tests SC0, SC40 and SC80

neglected implicitly. In other words, at a given  $\rho_d$ , of a soil, there is a unique f(r), which is best-fitted by a single set of empirical fitting coefficients.

The third assumption is that pore geometry of a soil can be analogous to a bunch of cylindrical capillary tubes with various tube diameters (Or & Tuller 2002). This suggests that the Young-Laplace equation is valid to describe the relationship between r and  $\psi$ .

To examine the importance of considering hydraulic hysteresis when using any of the three predictive methods, the laboratory measured drying and wetting SDSWCCs of CDT at 0kPa (i.e., SC0 in Figure 12(a)) are fitted. For the sake of illustration and discussion, the SWCC fitting equations,  $FX_F$  (i.e., Eqs (10) and (11)), are selected as an example. The fitted coefficients, *a*, *n* and *m*, are then used to back-analyse *f*(*r*) using Eq. (12). Figure 14 compares the back-analysed *f*(*r*)s from SC0-Dry (solid line) and SC0-Wet (double line). Obviously, the *f*(*r*) determined from the drying SDSWCC is distinctively different from that determined from the wetting one. It is clear that one should consider the effects of volume change and wetting-drying cycle when a predictive equation is used.

Similarly, to examine the relevancy of considering net normal stress when a fitting equation is used, the laboratory measured drying SDSWCCs at 40 and 80kPa of vertical stresses (i.e., SC40-Dry and SC80-Dry in Figures 12(b) and (c)) are fitted by  $FX_{F}$ . Figure 14 shows the comparison of back-analysed *f*(*r*)s among SC0-Dry (solid line), SC40-Dry (dash line) and SC80-Dry (dotted line)

and their corresponding fitting parameters. It is obvious that the f(r) of a laboratory CDT specimen is smaller at any r when it is subjected to a higher stress. This means that there is a reduction of soil pore volume under a higher applied stress.

Since the effects of net normal stress and hydraulic hysteresis are not considered explicitly in any of the three commonly used predictive equations, only drying  $k(\psi)$  at zero stress is predicted by Eqs (7) to (9) and compared with measured results



Figure 15. Comparisons between (a) fitted and laboratory drying SWCC of CDT at 0kPa; and (b) predicted and field drying SWCC of colluvium (Col) at 0.84m depth

## 7.3 Fitted SWCCs for predicting $k(\psi)$ s

Before predicting a  $k(\psi)$  using any of the three Eqs (7) to (9), a set of empirical coefficients is needed and they are determined by bestfitting a measured SWCC. The laboratory measured drying SDSWCC at zero stress of CDT (SC0) and the field measured drying SDSWCC of colluvium at 0.84m depth are fitted by BC<sub>F</sub>, VG<sub>F</sub> and FX<sub>F</sub>. The choice of one laboratory and one field measurements with and without considering net stress effects is to investigate the validity of the semi-empirical methods under different laboratory and field conditions.

Figure 15(a) shows the best-fitted drying SDSWCCs of CDT at zero stress and their comparisons with laboratory measured SC0 (solid squares). The empirical coefficients of each fitting equation are also shown. It is evident that all three fitted curves capture the measured soil-water characteristics of measured CDT well, in terms of AEV and desorption rate. The value of goodness-of-fit,  $R^2$ , of each fitting equation is greater than 0.94.

Comparisons between field measured and best-fitted drying SWCCs of colluvium at 0.84m depth are depicted in Figure 15(b). The coefficients used to best-fit each field measured SWCC are also illustrated. Except VG<sub>F</sub>, the fitted curves (see dash and solid lines) appear to have comparable AEV and desorption rate with the field measurements and they have  $R^2$  values greater than 0.93. In contrast, a substantially smaller AEV and lesser desorption rate are obtained from the SWCC fitted by VG<sub>F</sub> (see dotted line), resulting in a relatively smaller  $R^2$  value of 0.83. Obviously, for suctions larger than 10kPa, there are distinctively different desorption rates "extrapolated" by each SWCC fitting equation. This is because each

fitting equation adopted different empirical methods to determine  $\theta_r$ . As revealed from the field test results shown in Figures 11 and 13,  $\theta_r$  of CDT could not be determined because the test duration required for the drying phase of the IP test to reach  $\theta_r$  was longer than 24 days, which were carried out in the field already (Ng *et al.* 2011). This illustrates an inherent potential problem of any predictive method that relies on an accurate determination of  $\theta_r$ . Although it is theoretically sound to measure  $\theta_r$  by the IPM, the measurement may be impractical since it may take a considerable long duration to achieve it. This is particularly so for clayey soils, which have relatively low  $k(\psi)$  at high suctions. This then defeats the purpose of using semi-empirical equations, if an accurate measurement of  $\theta_r$  is required for predicting  $k(\psi)$  properly.

### 7.4 Comparisons between predicted and measured $k(\psi)$ s

Figure 16(a) compares laboratory measured and predicted drying  $k(\psi)$ s of CDT at 0kPa. It can be observed that both ML<sub>P</sub> (dash line) and VG<sub>P</sub> (dotted line) predict a constant  $k(\psi)$  at  $k_s$  when suction is less than the AEV<sub>0</sub> (i.e., 2.7kPa, refer to Figure 12(a)). For suction beyond the AEV<sub>0</sub>, the  $k(\psi)$ s predicted by ML<sub>P</sub> and VG<sub>P</sub> are always larger than the measured values by more than one order of magnitude. Moreover, it is observed that the discrepancy between the measured and the two predicted  $k(\psi)$ s increases with increasing suctions. The maximum over-prediction can be found at suction of 80kPa and it is about two orders of magnitude. Between these two predictive equations, the ML<sub>P</sub> seems to be more promising.

In contrast, when using the predictive equation  $FL_P$  (solid line), the measured  $k(\psi)$  of CDT is always under-estimated for suction ranging from AEV<sub>0</sub> to 80kPa. The maximum under-prediction is found to be up to two orders of magnitude at suction of 80kPa. Also it can be deduced that the discrepancy between the measured and the predicted  $k(\psi)$ s increases with increasing suctions.



Figure 16. Comparisons between (a) fitted and laboratory drying  $k(\Psi)$  of CDT at 0kPa; and (b) predicted and field drying  $k(\Psi)$  of colluvium (Col) at 0.84m depth

Figure 16(b) shows the comparisons between the field measurements and predicted drying  $k(\psi)$ s of colluvium at 0.84m depth. The shaded region denotes the range of  $k_s$  measured in the mid-levels of Hong Kong (i.e.,  $2 \times 10^{-6} - 9 \times 10^{-4}$ m/s) (GCO 1982). For simplicity, an average  $k_s$  of  $4 \times 10^{-5}$ m/s is selected (solid circle)

for predicting  $k(\psi)$  to compare measured values. It is obvious that the predicted shape of  $k(\psi)$ s using both ML<sub>P</sub> (dash line) and VG<sub>P</sub> (dotted line) is inconsistent with that of the measured  $k(\psi)$ . This is likely caused by the inherent potential problem of the two predictive methods that rely on an accurate determination of  $\theta_r$  (refer to Figure 15(b)). On the other hand, as shown in Figure 16(b), predictions by using FL<sub>P</sub> (solid line) appear to show comparable shape with measured  $k(\psi)$  one, although the measured  $k(\psi)$  of is slightly underpredicted by less than one order of magnitude at any suction.

As the predictability of  $k(\psi)$  by Eqs (7) to (9) relies on the availability of  $\theta_r$  (refer to Figure 15(b)), it is not surprising to observe large discrepancies between the measured and predicted  $k(\psi)$ s shown in Figure 16(b). This highlights an important limitation and hence the predictability of semi-empirical predictive methods.

### 8. SUMMARY AND CONCLUSION

Direct measurements of field and laboratory stress-dependent  $k(\psi)$ s and SDSWCCs of a decomposed silty clay (CDT) have been conducted using the Instantaneous Profile (IP) Method. An *in-situ* permeability test using the IP method was carried out on a natural hillslope, whereas a new 1D stress-controllable soil column was developed to control stress and boundary drainage conditions independently for laboratory tests. Effects of net normal stress and drying-wetting cycles on both  $k(\psi)$  and SDSWCC of CDT were investigated. With the measured field and laboratory  $k(\psi)$ s, the predictability of some common semi-empirical equations were evaluated. Some key conclusions may be drawn as follows:

Stress-dependent  $k(\psi)$  and SDSWCC of CDT

- (a) Owing to a 5% increases of dry density and changes of PSD due to an increase of net normal stresses from 0kPa to 80kPa, the measured permeability function of CDT decreases by up to one order of magnitude, when expressing in terms of matric suction (i.e.,  $k(\psi)$ ), and by up to two orders of magnitude, when expressing in terms of VWC (i.e.,  $k(\theta_w)$ ).
- (b) At any of the three stress levels (i.e., 0kPa, 40kPa and 80kPa), both measured  $k(\psi)$  and SDSWCCs of CDT are found to be hysteretic. At a given soil suction, owing to a greater VWC along a drying SDSWCC, drying  $k(\psi)$  is generally larger than the wetting one by about a half order of magnitude.
- (c) When net normal stress increases from 0kPa to 80kPa, the size of a hysteresis loop for each pair of drying and wetting  $k(\psi)$  and SDSWCC appears to reduce substantially.
- (d) When permeability of CDT is expressed in terms of VWC,  $k(\theta_w)$ s, average drying permeability at any stress level is found to be higher than the average wetting permeability at low VWC consistently. However, when VWC increased to higher values, the average drying permeability becomes lower.
- (e) To further investigate and quantify the effect of net normal stress on redistribution of PSD of a soil, it would be useful to carry out microstructural analysis such as mercury intrusion porosimetry (MIP).

## Observed differences between field and laboratory measurements of $k(\psi)$

- (f) At a given soil suction, field measured wetting  $k(\psi)$  is found to be higher than the drying one by about one and a half orders of magnitude. This opposes to the laboratory measurements that drying  $k(\psi)$  is often higher.
- (g) For suctions less than 10kPa, field measured  $k(\psi)$ s of CDT are always higher than the laboratory result by up to two orders of magnitude. The higher *in-situ* measured  $k(\psi)$ s may be because of preferential subsurface flow along geological features like cracks, fissures and rootlets, which were not taken into account in the laboratory test.

Verifications of common predictive methods

- (h) By revealing the assumptions made in each predictive equation, it is clear that one should consider the effects of volume change and wetting-drying cycle when a predictive equation is used.
- (i) By comparing with laboratory measured drying  $k(\psi)$  using the 1D soil column at zero net normal stress, estimates by using the predictive methods proposed by Mualem (1976) and Fredlund *et al.* (1994) appear to capture the measurements reasonably well, although the former and the latter over- and under-estimated measured values by about one order of magnitude, respectively.
- (j) If any predictive method replies on an accurate determination of residual water content, the predictability of these methods should be verified with care and use with caution.

## 9. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support by the Geotechnical Engineering Office (GEO), Civil Engineering and Development Department of the Government of the Hong Kong Special Administrative Region (SAR) (Dr H. W. Sun and Ir H. N. Wong) for funding the field test presented in this paper. The research funds OAP06/07.EG01 and HKUST/9/CRF/09 provided by Arup and Arup (Dr J. W. Pappin) and the Research Grants Council of the Government of the Hong Kong SAR, respectively, are acknowledged.

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