A New Sampling Algorithm in Particle Filter for Geotechnical Analysis

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ABSTRACT: This paper discusses the applicability of the particle filter (PF) algorithms to geotechnical analysis through some numerical tests. Although several types of the PF algorithms have been proposed so far, this study focuses on three typical PF algorithms: sequential importance resampling (SIR), sequential importance sampling (SIS), and merging particle filter (MPF). First, a geotechnical parameter is identified using the three algorithms in both total stress and soil-water coupled analyses, and the effectiveness of each algorithm is investigated. The test results clarify that (1)SIS can be applied to non-Markov dynamics such as elasto-plastic problems, but degeneration problems are often encountered, and (2)MPF can avoid the degeneration problems, but it cannot be applied to non-Markov dynamics. To overcome the dilemma, an algorithm which can treat non-Markov dynamics and solve the degeneration problems is newly proposed. The proposed algorithm is applied to an element test, and the performance is demonstrated experimentally.

1. INTRODUCTION

Predictions based on field observations, so-called "type B" predictions (Lambe 1973), play a crucial role in safety assessments of structures during construction. This approach is the well-known "observational method (Peck 1969)," wherein remedial designs are introduced sequentially moving towards the most probable conditions based on sequentially observed data. When the observational methods are applied to practical problems, several types of filtering techniques, such as Kalman filter (KF, Kalman 1960), ensemble Kalman filter (EnKF, Evensen 1994), and the particle filter (PF, Gordon 1993; Kitagawa 1996), can be helpful (useful) tools. These methodologies have been successfully applied not only to earth science (e.g., Awaji et al. 2009) but also to geotechnical engineering (Murakami and Hasegawa, 1985; Murakami 1991).

In particular, the PF can be applied to nonlinear and non-Gaussian problems and has a high potential for application to geotechnical engineering. Shuku et al. (2012) discussed the applicability of the PF to parameter identification of Cam-clay model through numerical tests and model tests. Murakami et al. (2012) demonstrated the practicability of the PF by applying it to an actual settlement behavior observed in the Kobe Airport construction project.

Several types of PF algorithms, which have different sampling methods, have been proposed so far. The original PF (Gordon et al., 1993; Kitagawa 1996) adopts so-called sampling importance resampling (SIR), which resamples state variables every step of the simulation. The general algorithm of the PF is known as sequential importance sampling (SIS). This algorithm just calculates the likelihood of state variables, and it does not resample state variables during every step of the simulation. This algorithm is particularly effective in parameter identification for elasto-plastic geomaterials whose mechanical behaviour depends not only on the current stress state but also on the stress history (Shuku et al. 2012; Murakami et al. 2012).

The PF often encounters a problem called "degeneration", where all but one of the weights of a particle is very close to zero and a large computational effort is devoted to updating particles whose contribution to the estimation is almost zero. To solve the problem, a unique resampling algorithm called merging PF (MPF) was proposed by Nakano et al.(2007), and they demonstrated that the MPF can overcome the degeneration problem.

The important problem here is which algorithm is preferable for geotechnical analysis. Although several types of PF algorithms have been proposed, the performance of these algorithms for geotechnical analysis has not been discussed.

In this paper, PF algorithms, SIR, SIS, and MPF, are compared with each other to investigate their performance in geotechnical analysis. Firstly, the outline/concept of the PF and the algorithms are briefly shown. Secondly, the three algorithms are applied to numerical tests for deformation behaviour of a ground under monotonic loading in order to discuss the applicability. Then some technical issues of each algorithm are mentioned. Finally, an algorithm which can overcome the above issues is newly proposed, and its applicability is demonstrated by applying it to a numerical test.

2. PARTICLE FILTER

2.1 Outline of the PF

This study focuses on the PF and applies to geotechnical analysis. This is because the PF does not require assumptions of linearity and Gaussianity, but is applicable to general problems. An application example of the PF to geotechnical analysis includes the literature of Shuku et al. (2012) and Murakami et al. (2012).

2.2 Ensemble Approximation

The PF approximates PDFs via a set of realizations called an ensemble that has weights, and each realization is referred to as a "particle" or a "sample". For example, a filtered distribution at time *t*-1 $p(x_{t-1}|y_{1:t-1})$, where $y_{1:t-1}$ denotes $\{y_1, y_2, \cdots, y_{t-1}\}$, is approximated with ensemble $\{x_{t-1|t-1}^{(1)}, x_{t-1|t-1}^{(2)}, \cdots, x_{t-1|t-1}^{(N)}\}$ and weights $\{w_{t-1}^{(1)}, w_{t-1}^{(2)}, \cdots, w_{t-1}^{(N)}\}$ by the following equation:

$$p(x_{t-1}|y_{1:t-1}) \approx \frac{1}{N}\sum_{i=1}^{N} w_{t-1}^{(i)}\delta(x_{t-1} - x_{t-1|t-1}^{(i)}) \tag{1}$$

where *N* is the number of particles and δ is the Dirac delta function. $w_{t-1}^{(i)}$ is the weight attached to particles $x_{t-1|t-1}^{(i)}$ and should suffice $w_{t-1}^{(i)} \geq 0$ and $\sum_{i=1}^{N} w_{t-1}^{(i)} = 1$. Given the particle approximation, integral equations become sum equations.

2.3 Prediction and Filtering Steps for the PF

We obtain the ensemble approximation for the forecast distribution $p(x_t|y_{1:t-1})$ at time *t* by the following calculation:

$$\begin{aligned} p(x_t|y_{1:t-1}) &= \int_{-\infty}^{\infty} p(x_t|x_{t-1})\,p(x_{t-1}|y_{1:t-1})\,dx_{t-1} \\ &\approx \sum_{i=1}^{N}\int_{-\infty}^{\infty} w_{t-1}^{(i)}\delta(x_{t-1} - x_{t-1|t-1}^{(i)})\,p(x_t|x_{t-1})\,dx_{t-1} \\ &= \sum_{i=1}^{N} w_{t-1}^{(i)}\delta(x_t - f_t(x_{t-1|t-1}^{(i)}, v_t^{(i)})) \\ &= \sum_{i=1}^{N} w_{t-1}^{(i)}\delta(x_t - x_{t|t-1}^{(i)}) \end{aligned} \tag{2}$$

where, $\{v_t^{(i)}\}_{i=1}^N$ is an independent and identically distributed (i.i.d.) sample set. The calculation means that each particle for the prediction ensemble, $x_{t|t-1}^{(i)}$, is obtained by the direct calculation of simulation models $f_t(x_{t-1|t-1}^{(i)}, v_t^{(i)})$.

Filtering

We obtain the ensemble approximation for filtered distribution $p(x_t|y_{1:t})$, and observation y_t by the following calculation:

$$\begin{aligned} p(x_t|y_{1:t}) &= \frac{p(y_t|x_t)\cdot p(x_t|y_{1:t-1})}{p(y_t|y_{1:t-1})} \\ &= \frac{p(y_t|x_t)\cdot p(x_t|y_{1:t-1})}{\int_{-\infty}^{\infty} p(y_t|x_t)\cdot p(x_t|y_{1:t-1})dx_t} \\ &= \frac{1}{\sum_j p(y_t|x_{t|t-1}^{(j)})w_{t-1}^{(j)}} \sum_{i=1}^N p(y_t|x_{t|t-1}^{(i)})w_{t-1}^{(i)}\delta(x_t - x_{t|t-1}^{(i)}) \\ &= \sum_{i=1}^N \tilde{w}_t^{(i)} w_{t-1}^{(i)}\delta(x_t - x_{t|t-1}^{(i)}) \\ &= \sum_{i=1}^N w_t^{(i)}\delta(x_t - x_{t|t-1}^{(i)}) \end{aligned} \tag{3}$$

where $\tilde{w}_t^{(i)}$ is defined as

$$\tilde{w}_t^{(i)} = \frac{p(y_t|x_{t|t-1}^{(i)})}{\sum_j p(y_t|x_{t|t-1}^{(j)})w_{t-1}^{(j)}} \tag{4}$$

If the observation system is linear, $p(y_t|x_{t|t-1}^{(i)})$ is given by the following equation:

$$p(y_t|x_{t|t-1}^{(i)}) = \frac{1}{(2\pi)^{m/2}|R_t|}\exp\left[-\frac{(y_t - H_t(x_{t|t-1}^{(i)}))^{\mathrm{T}} R_t^{-1}(y_t - H_t(x_{t|t-1}^{(i)}))}{2}\right] \tag{5}$$

where m is the number of state variables, H_t is the observation matrix, and R_t is the covariance matrix. Each weight $w_t^{(i)}$ is the product of $\tilde{w}_t^{(i)}$ and the previous time weight, namely,

$$w_t^{(i)} = \tilde{w}_t^{(i)} w_{t\text{-}1}^{(i)} \tag{6}$$

2.4 Sampling Methods

The central problem in PF is how to sample from $p(x_t|y_{1:t})$. The PF algorithm essentially consists of different ways of sampling. In this section, the differences among SIR, SIS, and MPF are briefly shown.

2.4.1 Sampling Importance Resampling (SIR)

The classic PF algorithm is known as the SIR (Gordon *et al.* 1993; Kitagawa 1996). The algorithm of SIR is summarized as follows:

1. *Initialization:*
   Generate an ensemble (set of particles) { $x_0^{(1)}, x_0^{(2)}, \cdots, x_0^{(N)}$ } from the initial distribution $p(x_0)$. Set $t = 1$.
2. *Prediction:*
   Each particle $x_{t-1}^{(i)}$ evolves according to numerical dynamic models such as FEM.
3. *Filtering:*
   After obtaining observation data y_t, calculate weight $w_t^{(i)}$, which expresses the "fitness" of the prior particles to the observation data, and assign a weight, $\tilde{w}_t^{(i)}$, to each $x_{t-1}^{(i)}$.
4. *Resampling:*
   Generate new particles { $x_t^{(1)}, x_t^{(2)}, \cdots, x_t^{(N)}$ } by resampling N times from the set of particles $x_{t-1}^{(i)}$, which is obtained in the filtering stage, where Pr $(x_t^{(i)} = x_{t-1}^{(i)}) = w_t^{(i)}$ and set weight $w_t^{(i)} = 1/N$. The set of determined particles $\{x_t^{(i)}\}$ results in an ensemble approximation of filtered distribution $p(x_t|y_{1:t})$.
   Set $t = t + 1$ and go back to Step 2.

2.4.2 Sequential Importance Sampling (SIS)

A general approach for filtering is known as SIS (Douset *et al.* 2001; Moral *et al.* 2006). The SIS algorithm can be viewed as a generalization of the SIR algorithm; it is based on using the importance sampling to estimate the expectations of functions of the state variables. The algorithm of SIS is summarized as follows:

1. *Initialization:*
   Generate an ensemble (set of particles) { $x_0^{(1)}, x_0^{(2)}, \cdots, x_0^{(N)}$ } from the initial distribution $p(x_0)$.
2. *Prediction:*
   Each particle $x_{t-1}^{(i)}$ evolves according to numerical dynamic models such as FEM.
3. *Filtering:*
   After obtaining observation data y_t, calculate weight $w_t^{(i)}$, which expresses the "fitness" of the prior particles to the observation data computed by Eq. (5), and assign a weight, $w_t^{(i)}$, to each $x_{t-1}^{(i)}$.
4. *Weight update:*
   The set of weighted particles $\{x_t^{(i)}\}$ results in an ensemble approximation of filtered distribution $p(x_t|y_{1:t})$.
   Set $t = t + 1$ and go back to Step 2.

2.4.3 Merging Particle Filter (MPF)

The PF often encounters a problem called "degeneration", where all but one of the weights of the particle is very close to zero and a large computational effort is devoted to updating particles whose contribution to the estimation is almost zero.

In order to solve the degeneration problem, Nakano et al. (2007) proposed the new algorithm called MPF and demonstrated its performance through numerical experiments using the Lorenz models. Since the MPF provides better estimations than the PF without a high computational cost, it can also be useful in geotechnical analysis. The algorithm of the MPF is summarized as follows:

1. *Initialization:*
   Generate an ensemble (set of particles) { $x_0^{(1)}, x_0^{(2)}, \cdots, x_0^{(N)}$ } from the initial distribution $p(x_0)$. Set $t = 1$.
2. *Prediction:*
   Each particle $x_{t-1}^{(i)}$ evolves according to numerical dynamic models such as FEM.
3. *Filtering:*
   After obtaining observation data y_t, calculate weight $w_t^{(i)}$, which expresses the "fitness" of the prior particles to the observation data, and assign a weight, $\tilde{w}_t^{(i)}$, to each $x_{t-1}^{(i)}$.

4. *Resampling:*
   Generate new particles $\{\hat{x}_t^{(1,1)},\cdots,\hat{x}_t^{(n,2)},\cdots,\hat{x}_t^{(1,N)}\cdots,\hat{x}_t^{(n,N)}\}$ by resampling $n \times N$ times from the set of particles $x_{t-1}^{(i)}$, which is obtained in the filtering stage, where Pr $(x_t^{(i)} = x_{t-1}^{(i)}) = w_t^{(i)}$ and set weight $w_t^{(i)} = 1/N$. Where, n is the number of particles to be merged.
5. *Merging and generating new samples:*
   Each particle in the new ensemble is generated as $x_{t|t}^{(i)} = \sum_{j=1}^{n} \alpha_j \hat{x}_{t|t}^{(j,i)}$. α_j is the merging weight. The set of determined particles $\{x_t^{(i)}\}$ results in an ensemble approximation of filtered distribution $p(x_t|y_{1:t})$.
   Set $t = t + 1$ and go back to Step 2.

Figure 1 shows the procedure of each algorithm.

3. NUMERICAL TESTS

This chapter presents application examples of the PF with SIR, SIS, and MPF algorithms to some numerical tests, and the effectiveness of each algorithm in geotechnical analysis is discussed.

3.1 Setup of Numerical Tests

In order to study the applicability of the PF algorithms, we applied them to a synthetic example of a total stress (non-coupled) and a soil-water coupled analysis. In this example, the deformation behavior of a ground under monotonic loading is simulated by FEM with a linear elastic model. The finite element mesh and the loading history are shown in Figures 2 and 3, respectively. In the coupled analysis, the left side and the right side of the mesh were assumed to be impermeable boundaries, whereas the top and bottom of the mesh were assumed to be permeable boundaries.

The placement of the observation points is also shown in Figure 2; the vertical displacements and the horizontal displacements are seen at S1-S3 and at L1-L3, respectively. The observed settlements and lateral displacements are shown in Figure 4.

Table 1 lists the parameters of the foundation ground, where E, ν, and k denote elastic modulus, Poisson's ratio, and coefficient of permeability, respectively. In this example, elastic modulus is set as the parameter to be identified.

The sets of particles for the parameters to be identified were generated with uniform random numbers in the range shown in Table 2. In MPF, the number of merged particles n was set to 3, and the weights α_j (j = 1, 2, ... , n) were set as follows (Nakano et al. 2007):

$$\alpha_1 = \frac{3}{4} \tag{7a}$$

$$\alpha_2 = \frac{\sqrt{13}+1}{8} \tag{7b}$$

$$\alpha_3 = \frac{\sqrt{13}-1}{8} \tag{7c}$$

which satisfy following equations.

$$\sum_{j=1}^{n} \alpha_j = 1 \tag{8}$$

$$\sum_{j=1}^{n} \alpha_j^2 = 1 \tag{9}$$

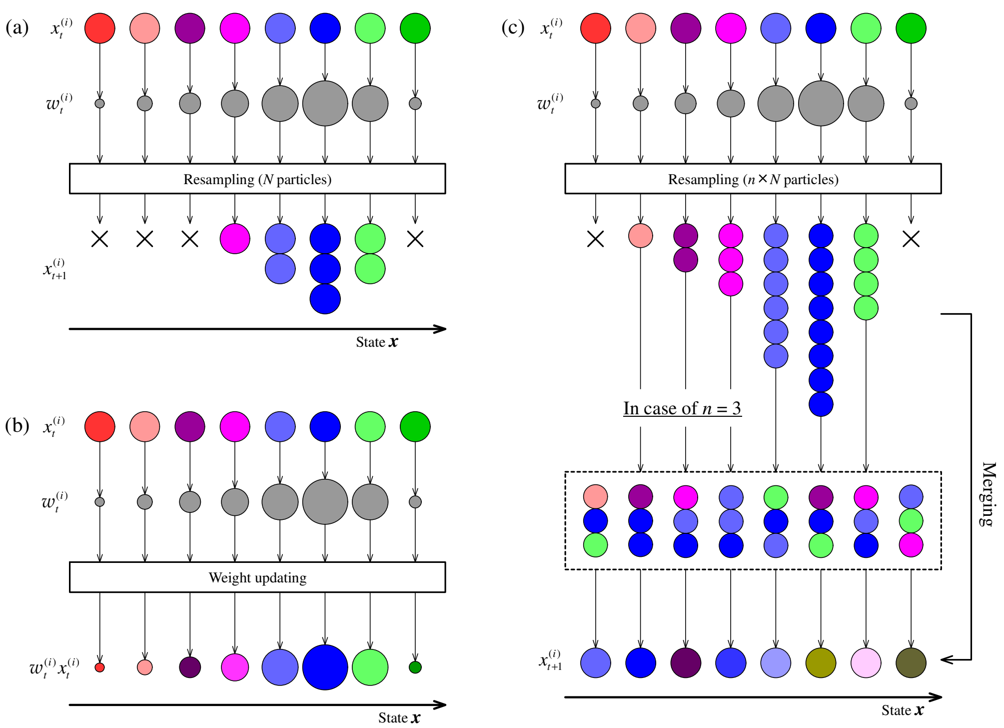


Figure 1 Algorithms of the (a) SIR, (b) SIS, and (c) MPF

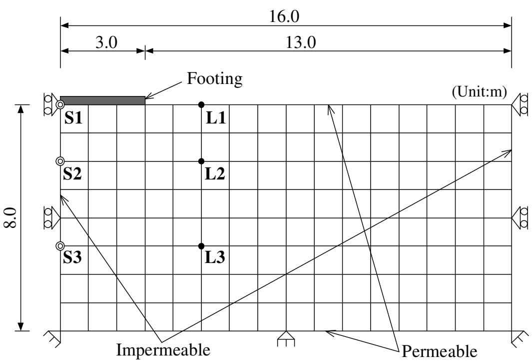


Figure 2 Finite element mesh

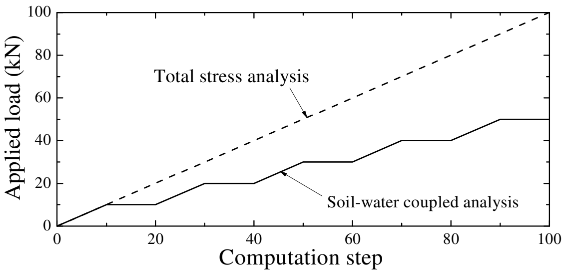
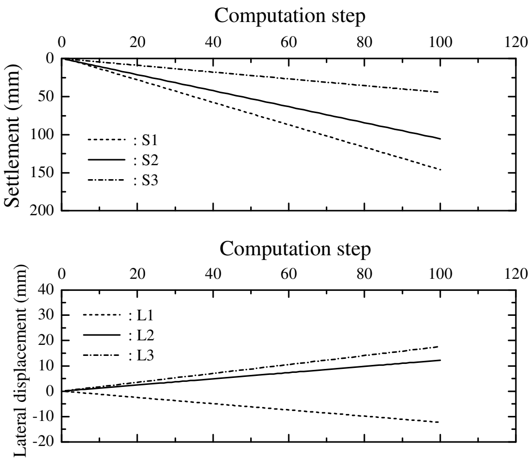
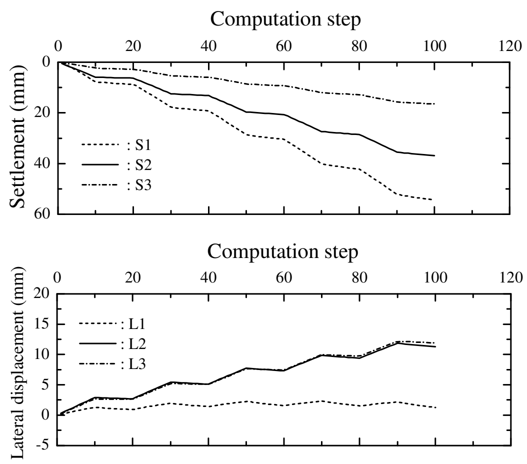


Figure 3 Loading histories



(a) Total stress analysis



(b) Soil-water coupled analysis

Figure 4 Observation data

Table 1 Geotechnical parameters used in the analysis

E (kN/m^2)	ν	k_v (m/day)
To be identified	0.25	1.0×10^{-4}

Table 2 Range of particle generation

Parameter	Range
E (kN/m^2)	1,000 ~ 15,000

The system noise and the observation noise were assumed to be zero, and the covariance matrix R_t in Eq.(5) is assumed to be a diagonal form which is expressed by the following equations for simplicity;

$$R_t = \sigma_i^2 \mathbf{I},\ \sigma_i^2 = 0.2 \tag{10a}$$

$$R_t = \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 \end{bmatrix} \tag{10b}$$

where σ^2 is the variance (mm^2), $\mathbf{I}$ is a unit tensor, and i is an observation number (S1 ~ S3, L1 ~ L3).

Six cases, which have different particle numbers (N = 16, 32, 64, 128, 256, 512), were analyzed in this example. In all cases, all observation data (S1~S3, L1~L3) were available for identification.

3.2 Results and Discussions

Figures 5 and 6 show the time evolution of the identified parameters for N=16, 64, and 256 in the total stress analysis and the coupled analysis, respectively. In the PF, identified parameters are expressed in the form of probability density functions (PDFs) approximated by discrete samples called particles. Therefore, the weighted mean value is defined as the identified parameters in this paper:

$$\bar{\phi}_t = \sum_{i=1}^{N} w_t^{(i)} \phi_t^{(i)} \tag{11}$$

where $\bar{\phi}_t$ and $\phi_t^{(i)}$ indicate the identified parameter at time step t and the parameter of particle number (i) at time step t, respectively.

In the total stress analysis, all the algorithms with N=16 do not converge to the true value. However, the identified parameters with N=64 and 256 converge to the true value, although the identification starts with an incorrect $\bar{\phi}_0$ purposefully. These results verify the effectiveness of the PF algorithms for parameter identification in geotechnical analysis. In particular, SIS and MPF algorithms can stably identify the geotechnical parameter, and the MPF needs fewer particles than SIR for accurate parameter identification. We could also see the above trends in soil-water coupled analysis (Figure 5).

The values of the identified parameters at the end of the computation (computation step 100) for the total stress and the soil-water coupled analyses are summarized in Tables 3 and 4. We can see the effects of the number of particles N from these tables, and it is clear that better identifications can be produced as more samples are used. Even when the number of samples was increased to 32, the SIR provided a worse identification than the SIS and MPF. When the number of samples was increased to 64, the identified parameters of SIR became as good as those by the SIS and MPF.

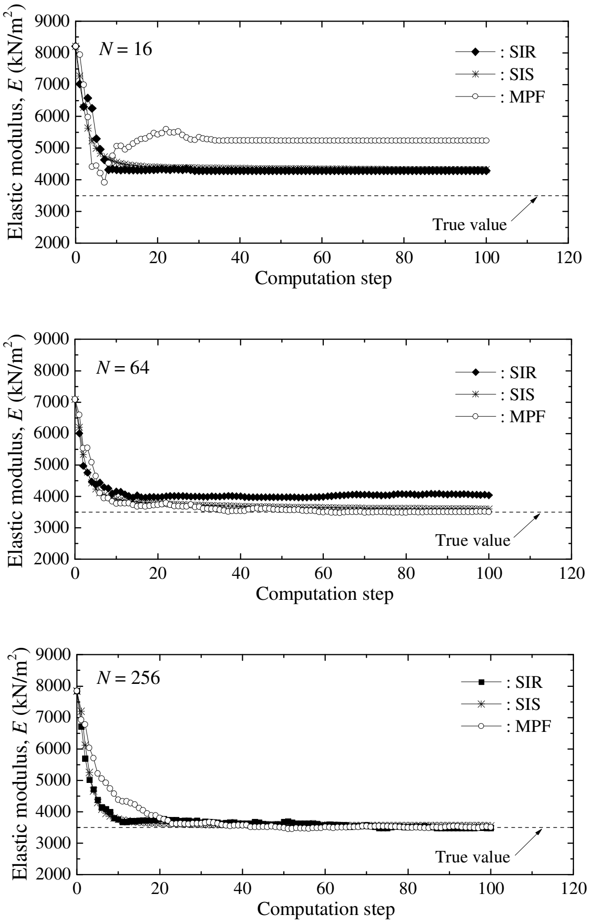


Figure 5 Identified parameters in the total stress analysis

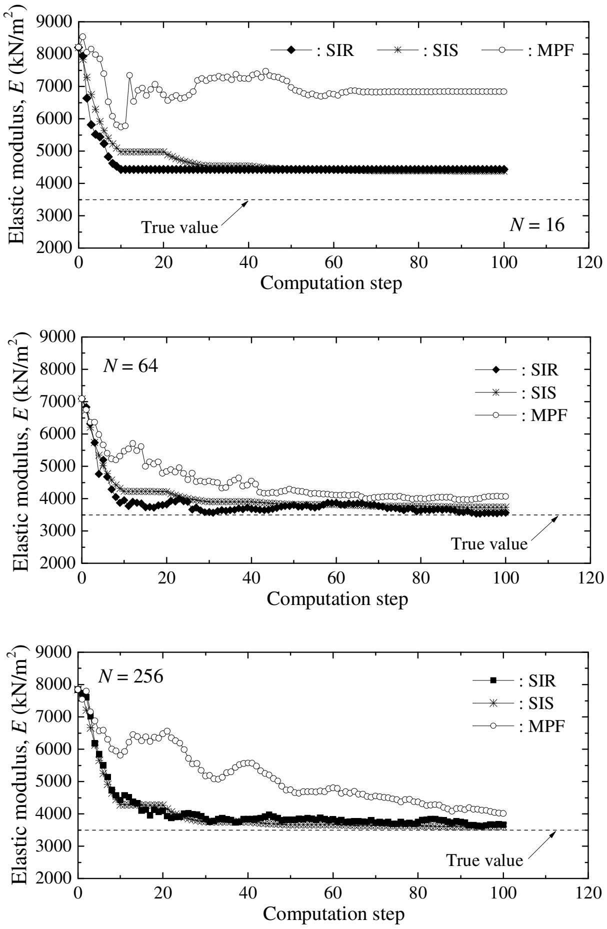


Figure 6 Identified parameters in the coupled analysis

Table 3 Identified parameters at end of the computation (Total stress analysis)

N	SIR	SIS	MPF
16	4291.10	4314.99	5230.40
32	4768.00	3704.21	3669.90
64	4037.71	3601.64	3505.57
128	3716.80	3613.42	3590.84
256	3486.28	3551.81	3502.12
512	3602.41	3553.62	3577.26

Table 4 Identified parameters at end of the computation (Soil-water coupled analysis)

N	SIR	SIS	MPF
16	4432.10	4378.81	10877.23
32	4488.40	3871.51	4210.67
64	3565.75	3735.58	4058.79
128	3672.75	3661.60	3966.24
256	3657.71	3605.85	4004.60
512	3881.91	3631.51	4115.70

The MPF requires fewer samples than the SIR and thus would be a more efficient algorithm. In the results of the coupled analysis, identified parameters by the MPF do not converge to the true values, and parameter identification by the MPF seems to be difficult. A reason of the difficulty includes the dependency of pore water pressure. The relationship among displacements, pore pressure, and elastic modulus in geotechnical analysis is shown in Figure 6. In total stress analysis, deformation behaviour of the grounds is governed only by elastic modulus. On the other hand, deformation behaviour of geomaterials in soil-water coupled fields depends not only on geotechnical parameters but also on pore water pressure. Therefore, in the case of soil-water coupled problems, resampling algorithms cannot be effective, and the SIS is the most preferable algorithm of the three algorithms.

Figures 7 and 8 show the PDFs of the parameters identified at the end of the computation. In these figures, the vertical axes represent the frequency, while the horizontal axes represent the parameter values. In both the total stress analysis and the coupled analysis, SIR and SIS clearly encounter degeneration phenomena, and the PDFs are represented by fewer particles. However, the PDF obtained by the MPF is broad even at the end of computation, and it is concluded that the MPF can solve the degeneration problems in geotechnical analysis.

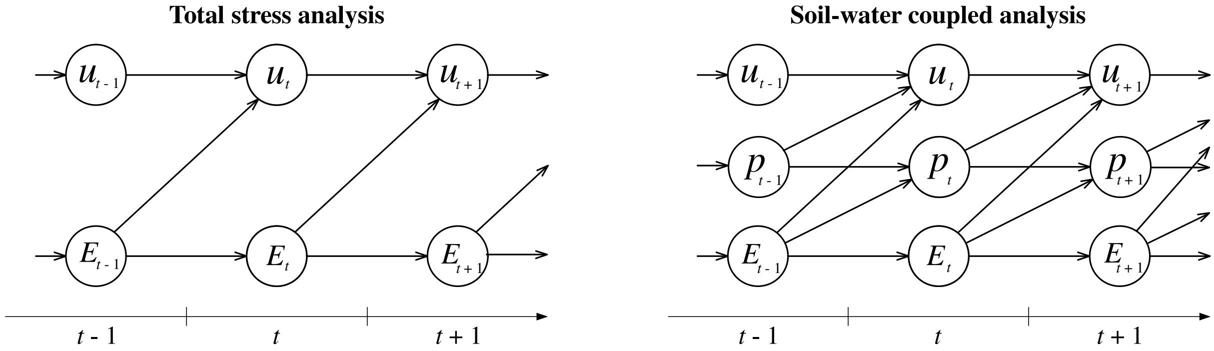


Figure 6 The relationship among displacements (u_t), pore water pressure (p_t), and elastic modulus (E_t) in geotechnical analysis

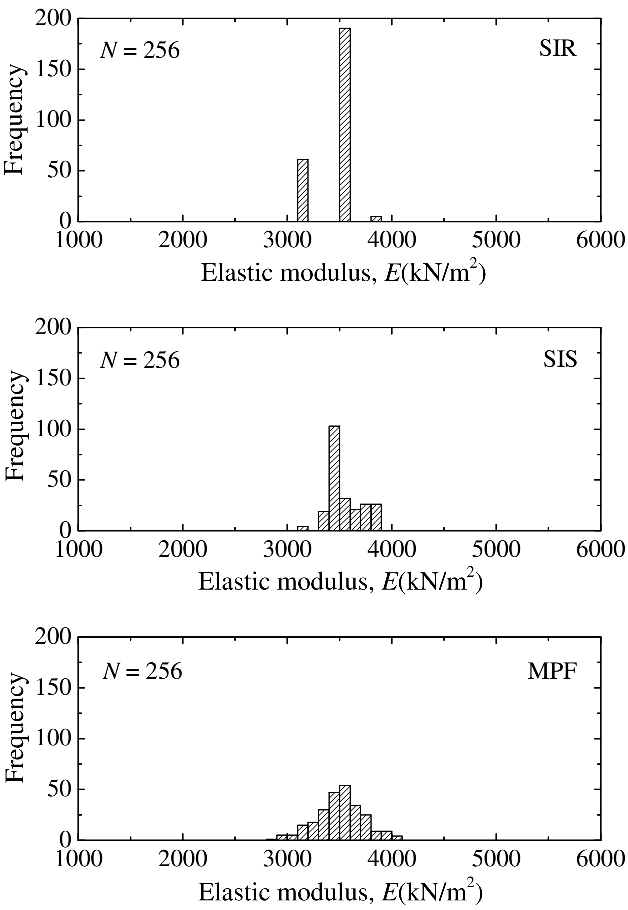


Figure 7 PDFs of the identified parameters (total stress analysis)

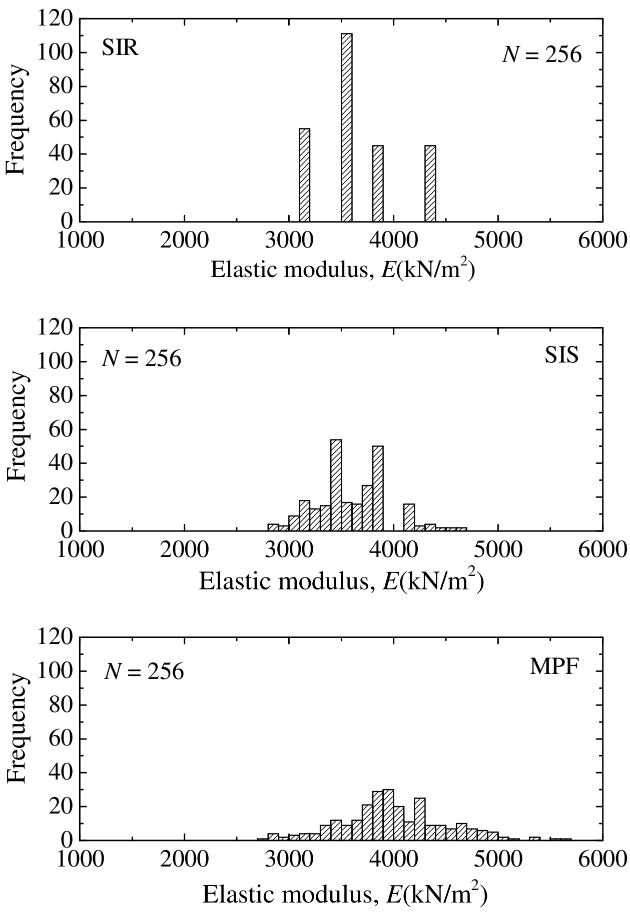


Figure 8 PDFs of the identified parameters (coupled analysis)

4. MPF ALGORITHM FOR NON-MARKOV DYNAMICS

Although the MPF avoids the degeneration problem, it cannot be applied to non-Markov dynamics such as elasto-plastic problems where the deformation behaviour is governed not only by the current stress state but also by the history. On the other hand, SIS algorithm can be applied to non-Markov dynamics (Shuku et al. 2012; Murakami et al. 2012), but it often encounters the degeneration problem (Doucet et al. 2000).

To overcome the dilemma, a simple algorithm is newly proposed herein. The concept is extremely simple, and it is combination of the SIS and the MPF. The conceptual illustration of the proposed algorithm is shown in Figure 9. In the proposed method, after resampling by the MPF at step1, the computation starts from step0 again and the resampled particles are used as initial ensemble members. After that, if the computation is advanced to step2, the particles are resampled by the MPF and the computation starts from step 0 again. These procedures are continued until the end of the computation. Cleary, this algorithm can be applied to non-Markov dynamics and can solve the degeneracy problem.

To investigate the applicability of the proposed method, simulation of one-dimensional consolidation is conducted. The schematic illustration of the numerical test is shown in Figure 10. In this analysis, in order to produce a non-Markov process, the following equation is used in elastic constitutive model for simplicity;

$$E_{t+1} = E_0 \cdot \exp(\varepsilon_t^{v}) \tag{12}$$

where, E_0 is the initial values of elastic modulus, ε_t^{v} is the volumetric strain at time t. Clearly, a non-Markov process can be produced by using the Eq.(12) in elastic constitutive models.

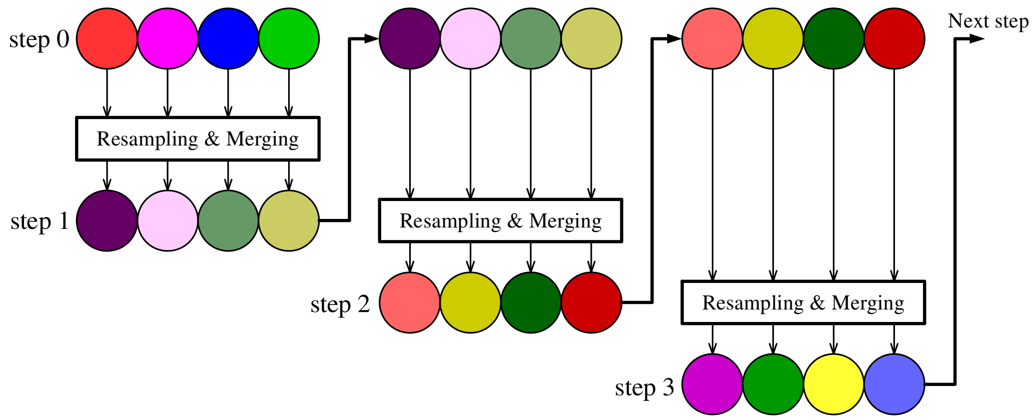


Figure 9 Conceptual illustration of the proposed algorithm

In this analysis, we attempt to identify the elastic modulus expressed by the Eq.(12), and the geotechnical parameters of the element are shown in Table 1. The sets of particles for the parameters to be identified were generated with uniform random numbers in the range shown in Table 2. The settlement observed on the top of the element is used for identification in this analysis.

Figure 11 shows the time evolution of identified parameters with *N*=256. In spite of resampling, the proposed method can identify elastic modulus with high accuracy even in non-Markov dynamics. It can be found that the SIR and the MPF cannot identify geotechnical parameters in non-Markov processes. The PDFs of the identified parameters at computation step 35 are shown in Figure 12. We show the results of SIS and MPF herein. The PDF obtained by the proposed algorithm consists of many types of samples, and the proposed method does not encounter the degeneration problem. These results verify the effectiveness of the proposed algorithm.

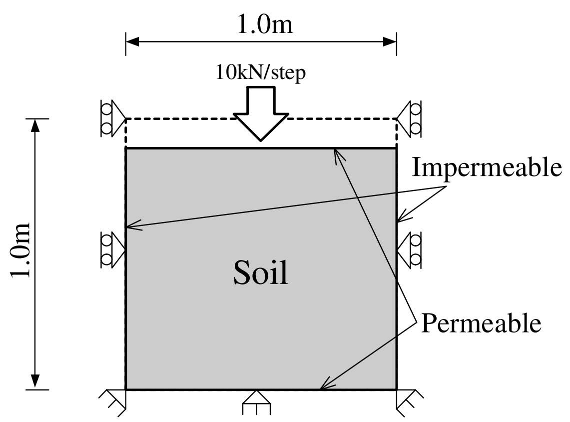


Figure 10 Setup of simulation of one-dimensional consolidation

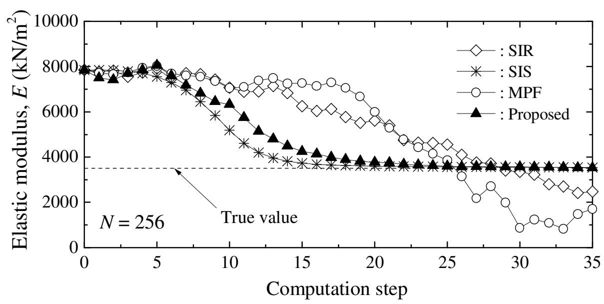


Figure 11 Identified parameters in the element test.

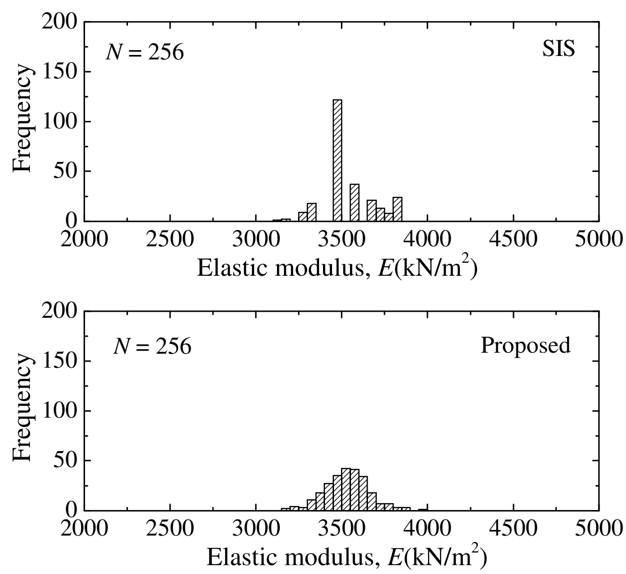


Figure 12 PDFs identified by the SIS and the proposed method

5. CONCLUSION

This paper has discussed the applicability of the three PF algorithms, SIR, SIS, and MPF, to geotechnical analysis thorough total stress and soil-water coupled analyses. In addition, the algorithm which can be applied to non-Markov dynamics and can solve the degeneration problem has been newly proposed. In addition, the proposed algorithm has been applied to a numerical test to investigate its effectiveness. The following remarks can be noted:

(1) The numerical tests have shown that the geotechnical parameters identified by the SIR, SIS, and MPF have converged into their true values, and the usefulness of the PF algorithms for geotechnical analysis was presented.

(2) Deformation behaviour of geomaterials in soil-water coupled fields depends not only on geotechnical parameters but also on pore water pressure. Therefore, resampling algorithms cannot produce accurate identification and SIS is the most preferable algorithm in case of soil-water coupled problems.

(3) PF algorithm which can solve degeneration problems and treat non-Markov dynamics was newly proposed by focusing on the dilemma about SIS and MPF.

(4) The simulation results of one-dimensional consolidation have shown that the proposed algorithm can be applied to non-Markov dynamics and solve degeneration problems.

Since this study has treated just elastic models for simplicity, the obtained results cannot be extended to elasto-plastic problems. Further research which focuses on elasto-plastic models such as Cam-clay model and the discussion are necessary.

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