Probabilistic Approaches for Ultimate Resistance of Drilled Shafts in Sands Considering Spatial Variability

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ABSTRACT: In this paper, existing probabilistic approaches for determining the ultimate resistance of drilled shafts in sands considering the spatial variability of soil properties are evaluated and compared. The first approach is realized through random field modeling implemented with Monte Carlo simulation (MCS). The second approach is a simplified method based on the spatial averaging technique. The third approach is yet another simplified method based on the spatial correlation between spatial averages. The last two (simplified) approaches can be efficiently implemented without MCS so that much less computational effort is required. The comparison study shows that the three probabilistic approaches yield practically identical results (i.e., probability of failure in the designed drilled shafts). This study also highlights the importance of considering the spatial variability of soil properties and the model bias in the design of drilled shafts. Results indicate that: (1) neglecting spatial variability often leads to an overestimation of the probability of failure; (2) ignoring the model bias results in either overestimation or underestimation of the probability of failure, depending on the compression load applied to the drilled shafts.

KEYWORDS: Drilled shafts, Standard penetration test, Probability, Random field modeling, Spatial variability, Spatial average, Spatial correlation.

1. INTRODUCTION

The drilled shafts as a type of deep foundations are widely used in the geotechnical design. The ultimate resistance that a drilled shaft can carry consists of the shaft resistance and the toe bearing resistance. Many mechanical, empirical, and semi-empirical approaches are available to predict the shaft resistance and the toe bearing resistance of a drilled shaft (e.g., Meyerhof 1976; Reese and O'Neill 1988; Kulhawy 1991). Conventional geotechnical design of drilled shafts is realized through meeting the minimum required factor of safety, which is a deterministic approach and all input soil parameters are treated as constants. Due to the inherent variability of soil properties and the model uncertainty, a factor of safety greater than the minimum required value does not always guarantee safety. For this reason, it is rational to model the input soil parameters as random variables and perform the design by meeting the acceptable (target) failure probability requirement.

The effect of spatial variability of soil properties can have a significant influence on the reliability of deep foundations. The spatial variability is generally described by scale of fluctuation, which is the maximum distance within which the spatially random parameters are correlated (Vanmarcke 1977). In the traditional probabilistic analysis in which the spatial variability is ignored, the scale of fluctuation is by default set to be infinity or a relative large number, which is termed the spatial constant. The typical vertical scale of fluctuation for most soil parameters ranges between 0.1 m and 3 m, depending on the geological condition and composition in the field (Phoon and Kulhawy 1999). The study on probabilistic analysis of deep foundations considering spatial variability has been reported recently (e.g., Klammler et al. 2010; Zhang and Chen 2012; Luo and Juang 2012; Chen and Zhang 2013; Cao et al. 2013; Fan and Liang 2013a,b; Fan et al. 2014). Zhang and Chen (2012) examined the effect of spatial variability of standard penetration test (SPT) data on the bearing capacity of driven piles in sanThe importance of considering the spatial variability of soil properties in the probabilistic analysis of geotechnical problems has been recognized and is widely reported in literature (e.g., Fenton and Griffiths 2008; Griffiths and Fenton 2009; Huang et al. 2010; Luo et al. 2012a,b; Stuedlein et al. 2012; Zhang and Chen 2012; Chen and Zhang 2013). The most rigorous method to model the spatial variability of soil is the approach based on the random field theory (Vanmarcke 1977). In the random field modeling (RFM), the geotechnical model is first discretized into a series of elements with equal size. The correlation among the soil elements can be modeled by assuming a certain type of correlation structure, e.g., exponential correlation function, constant function, triangular function (Most and Knabe 2010). Then the spatial correlation can be dealt with some techniques such as the local averaging subdivision (Fenton and Griffiths 2008) or Cholesky decomposition (Fenton 1997). The Monte Carlo simulation (MCS) is generally implemented in random field modeling (RFM) and a sufficient number of MCS is required to reach converged results. Previous research shows that neglecting the spatial variability will lead to either overestimation or underestimation of the predicted probability of failure, depending on the limiting criteria (Griffiths and Fenton 2004).

In addition to the rigorous RFM approach, simplified approaches based on variance reduction technique (Vanmarcke 1983) can serve as alternatives in the probabilistic analysis of geotechnical problems when spatial variability is considered (e.g., Peschl and Schweiger 2003; Most and Knabe 2010; Luo 2012a,b). In these simplified approaches, in order to consider the spatial effect, the soil parameters with reduced variances are adopted in the probabilistic analysis. In this study, these simplified approaches are subdivided into two categories: (1) simplified approaches based on spatial averaging and (2) simplified approaches based on spatial correlation of spatial averages. It should be noted that the simplified approaches can be either realized through MCS or some efficient probabilistic methods such as first-order reliability method. Comparing with MCS, the simplified approaches implemented with efficient probabilistic methods consume much less computational effort, and thus, have greater potential as tools in the engineering practice.

In this paper, the aforementioned three probabilistic approaches for determining ultimate resistance of drilled shafts in sands under compression considering the spatial variability are presented and compared. The RFM approach is adopted to examine the spatial effect on the estimated failure probability at various levels of scale of fluctuation and variation in soil property and compression load. The results based on RFM are then used as a benchmark in the comparison study with the two simplified approaches. Comparison study shows that the three probabilistic approaches yield virtually the same results (i.e., identical probability of failure in the designed drilled shafts). However, the simplified approaches are preferred in the engineering practice, as they can be implemented with efficient probabilistic methods and require much less computational effort. This study also points to the importance of considering the spatial variability of soil property and the model bias in the design of drilled shafts. It is found that neglecting spatial variability leads to the overestimation of the probability of failure. Ignoring the model bias results in overestimation or underestimation of the probability of failure, depending on the load applied to the drilled shafts.

2. PROBABILISTIC ANALYSIS OF ULTIMATE RESISTANCE OF DRILLED SHAFTS

In the engineering practice, the ultimate resistance (Q_{ult}) of a drilled shaft can be estimated using in-situ test indices such as the standard penetration test (SPT) blow count N (e.g., Meyerhof 1976). Those empirical models generally correlate the toe bearing resistance and the shaft resistance with the SPT N value. The Q_{ult} is the sum of toe bearing resistance and shaft resistance. Figure 1(a) shows a schematic diagram of a drilled shaft with length of 30 m and diameter of 1 m under a compression load of F (note: this figure is not to scale). In this study, the following empirical model for the prediction of Q_{ult} of a drilled shaft is adopted (in *kN*, GEO 2006):

$$Q_{ult} = \sum_{i=1}^{n} \pi D L_i \overline{N}_i + 9.5\pi D^2 N_n / 4 \tag{1}$$

in which D is the diameter of the drilled shaft (m), L_i and $\overline{N}_i$ is the length (m) and the average N value in soil layer i, N_n is the N value for determining the toe bearing resistance. For an assumed idealized distribution of SPT N with depth as shown in Figure 1(b), N_n can be taken as the average N values between the shaft tip and $2D$ beneath the toe (O'Neill and Reese 1999). The two terms in Eq. (1) represent the shaft resistance and the toe bearing resistance, respectively.

In a deterministic analysis, Q_{ult} can be calculated with Eq. (1) if the profile of SPT N (for instance Figure 1(b), not to scale) is available, and thus Q_{ult} is a fixed value. The deterministic bearing capacity design is realized through meeting the required minimum factor of safety (FS), defined as the ratio of Q_{ult} over the load F.

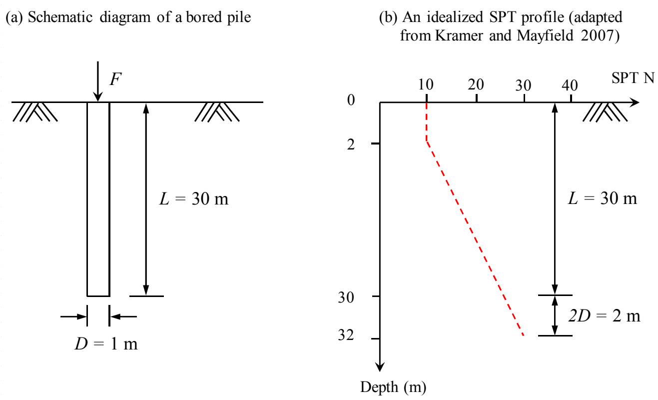


Figure 1 Design example of drilled shaft and the idealized profile of standard penetration test (not to scale)

However, it is well known that the field test indices generally involve large uncertainty mainly because of the inherent variability of the soil property. In addition, other sources of error such as the human error and the model uncertainty can also induce the uncertainty in the estimated Q_{ult}. Due to those uncertainties, the factor of safety greater than the required minimum FS (say 2.5) will not always guarantee safety. It is rational to model the soil parameters such as SPT N as random variables and consider the model uncertainty in the design. For this reason, the bearing capacity design of drilled shaft can be realized through meeting the acceptable (target) probability of failure (p_f) in the probabilistic analysis. In this study, p_f is defined as the probability that the load acting on the shaft exceeds the predicted ultimate resistance:

$$p_f = p\left(Q_{ult} < F\right) \tag{2}$$

The focus of this study is to investigate the effect of spatial variability and the model uncertainty on the estimated p_f in the bearing capacity design of drilled shafts.

3. PROBABILISTIC APPROACHES CONSIDERING SPATIAL VARIABILITY OF SOIL PROPERTY

In this paper, three probabilistic approaches are adopted: random field modeling approach, spatial averaging approach, and the spatial correlation of the spatial averages. These three approaches will be discussed in detail and compared in the probabilistic design of drilled shafts under compression loading. In all three approaches, the SPT N value normalized by the depth is assumed to follow a lognormal distribution, and thus the mean trend of N is shown in Figure 1(b) (adapted from the subsurface profile by Kramer and Mayfield 2007). The coefficient of variation (COV) of N can range from 0.1 to 0.7 (Zhang et al. 2009). Various levels of scale of fluctuation (θ) are assumed herein to investigate the effect of spatial variability on the probabilistic analysis. To consider the spatial variability, the exponentially decaying auto-correlation function (e.g., Jaksa et al. 1999) is adopted in this study:

$$\rho(\tau) = \exp\left(-\frac{2\tau}{\theta}\right) \tag{3}$$

where τ is the absolute distance between any two points in the random field and θ is the scale of fluctuation for $\ln N$.

3.1 Random field modeling of soil property

The random field theory (Vanmarcke 1977) is a rigorous approach to model the spatial variability of soil parameters. In this design example (Figure 1(a)), since the diameter of the shaft is relatively small (D = 1 m) compared to the horizontal scale of fluctuation, only the vertical spatial effect is considered in this study. Given the idealized profile of SPT N (adapted from the subsurface profile by Kramer and Mayfield 2007) as shown in Figure 1(b), the N value at the shallow depth (from 0 to 2 m) is relatively small (N = 10). The shaft resistance developed within this region has much less contribution to the total ultimate load. In this regard, the SPT N from 0 to 2 m is treated as constant for simplicity. The vertical spatial variability from depth of 2 m to 32 m is modeled using random field modeling (RFM). Considering that the mean trend of N increases with depth (Kramer and Mayfield 2007), the normalized N, defined as the ratio of N over depth, is constant with depth. In this study, the RFM of normalized N is realized using the Cholesky decomposition implemented with Monte Carlo simulation (MCS).

In RFM, the soil profile from 2 m to 32 m is discretized into intervals with equal thickness Δz, as illustrated in Figure 2(a). For simple demonstration purpose, only 10 elements are shown in Figure 2(a). First, the local averaging over the soil elements needs to be performed. The variance reduction factor (Γ^2) for the local averaging of N over a depth interval Δz can be obtained by setting τ = Δz and integrating the exponential function (Eq. 2), according to Vanmarcke (1983):

$$\Gamma^2 = \frac{1}{2}\left(\frac{\theta}{\Delta z}\right)^2\left[\frac{2\Delta z}{\theta} - 1 + \exp\left(-\frac{2\Delta z}{\theta}\right)\right] \tag{4}$$

In this study, the statistics of the equivalent normal distribution for the assumed lognormal distributed SPT *N* are first computed. Then the local averaging is realized by multiplying variance reduction factor (Γ^2) to the variance of the equivalent normal distribution. Thus the mean and the variance of the lognormal distribution with reduced variance can be obtained (Griffiths and Fenton 2004).

The next step is to build the correlation matrix using Eq. (3). The correlation matrix consists of the correlation coefficients for all the combinations of the elements in Figure 2(a). Figure 2(b) shows the illustrative example of spatial averages.

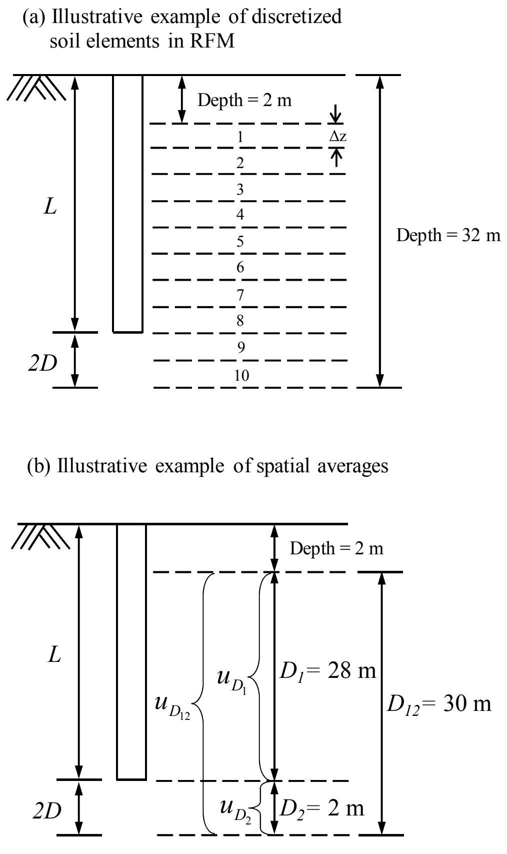


Figure 2 Illustrative example of discretized soil elements in random field modeling and spatial averages in simplified approach

For instance, to compute the correlation coefficient between Element 1 and Element 3, the absolute distance is set to be 2Δz. With Eq. (3) and a given scale of fluctuation *θ*, the correlation coefficient between Element 1 and Element 3, ρ_{13}, can be computed. This procedure can be repeated and all the elements in the correlation matrix (*ρ*) can be determined. In this study, the correlation matrix built with the correlation function is then decomposed by Cholesky decomposition method (Fenton 1997):

$$L \cdot L^T = \rho \tag{5}$$

With the lower triangle matrix *L*, the correlated standard normal random field can be obtained by linearly combining the independent variables as follows (Fenton 1997):

$$G_n(x_i) = \sum_{j=1}^{i} L_{ij} Z_j \tag{6}$$

where *M* is the number of elements in the random field; Z_j is a series of independent standard normal random variables. Then the lognormal random field of *N* can be generated through the following transformation (Fenton and Griffiths 2008):

$$N = \exp\{\mu_n + \sigma_n \cdot G_n(x_i)\} \tag{7}$$

where x_i is the spatial position at which *N* is modeled; μ_n and σ_n are the mean and the standard deviation of $\ln N$.

In the illustrative example shown in Figure 2(a), the spatially varied *N* values of Element 1 to 8 can be used to calculate the shaft resistance and the *N* values of Elements 9 and 10 can be used to calculate the toe bearing resistance. It is noted that Figure 1(a) is just a schematic diagram for illustration purposes and the actual element size in RFM (at an interval of 0.25 m) is much smaller than the specified *θ*. For the design example in Figure 1(a), the region with depth between 2 m and 32 m is subdivided into 120 elements with an equal interval of 0.25 m. With the aforementioned RFM procedure, the spatial variation of *N* is simulated at two levels of *θ* (0.5 m and 10 m) and the results are shown in Figure 3. It is observed that smaller *θ* yields more drastically varied *N*, indicating a more significant spatial effect. When the value of *θ* approaches zero, the correlation coefficient computed using Eq. (3) will be approximately zero (except the diagonal elements), indicating it is a stochastic simulation without any correlation among those elements. On the other hand, the larger *θ* results in more spatially uniformly distributed *N*. It should be noted that when the value of *θ* approaches infinity (or relative large comparing to the scale of the model), the correlation coefficients calculated using Eq. (3) will be approximately equal to unity, which means that this is the spatial constant case and all elements as in Figure 2(a) are perfectly correlated. The spatial constant case is equivalent to the traditional probabilistic analysis without considering the effect of spatial variability of soil property.

The aforementioned RFM procedure is for one realization of the random field. To investigate the inherent spatial variability, this procedure needs to be implemented with MCS. In each simulation, failure occurs if the estimated Q_{ult} is smaller than the compression loading (*F*). Given a sufficient number of MCS, the probability of failure can be calculated as the ratio of the number of failure over the total number of MCS (referring to Eq. 2).

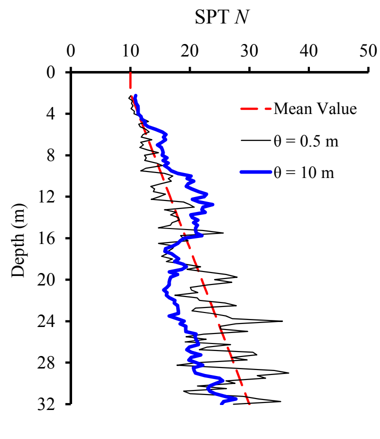


Figure 3 Simulated spatial variability of SPT *N* values based on the SPT profile in Figure 1 (assuming COV = 0.3)

3.2 Spatial averaging approach

The spatial variability of soil property over a certain region can also be approximately averaged as one random variable that represents a soil parameter (Vanmarcke 1977). This simplified approach is very useful and could be an alternative to RFM when a soil profile is simple and the characteristic length can be reasonably assessed (e.g., Peschl and Schweiger 2003; Most and Knabe 2010; Suchomel and Mašín 2010; Luo et al. 2012a, 2012b). In a random field where spatial variability exists, it is observed that the overall variation of soil property over a larger domain is generally smaller than that over a smaller domain. In other words, the variance of soil property over a large domain is reduced by a certain extent comparing with that of a small domain. This reduction in variance can be evaluated using the variance reduction factor. The variance reduction factor is a function of the scale of fluctuation θ and characteristic length L. Given the exponential correlation function (Eq. 3), the variance reduction function has the same form as Eq. (4) by replacing Δz with L:

$$\Gamma^2 = \frac{1}{2}\left(\frac{\theta}{L}\right)^2\left[\frac{2L}{\theta} - 1 + \exp\left(-\frac{2L}{\theta}\right)\right] \tag{8}$$

In this study, the characteristic length L, which specifies the averaging length for the one-dimensional case (Most and Knabe 2010), is set to be the vertical scale of the RFM region (30m, which is the depth between 2 m and 32 m). The reduced variance is the product of the variance reduction factor and the original variance. Similarly, when the variance reduction technique is used to simplify the effect of the spatial variability of a lognormal distributed N, as is assumed in this paper, only the variance of its equivalent normal distribution ($\ln N$) needs to be reduced through multiplying the variance reduction factor (Griffiths and Fenton 2004).

Using the spatial averaging technique, all the elements in the RFM region (for instance, Element 1 to Element 12 in Figure 2(a)) are combined as one random variable with the reduced variance in the subsequent probabilistic analysis. When there is no spatial variability (spatial constant case), the variance reduction factor is obtained by setting θ equal to infinity or a relatively large value. For the spatial constant case, the variance reduction factor is approximately equal to 1, indicating there is no reduction. In this spatial averaging approach, the continuously varying spatial random field of soil property is averaged over a certain domain, in which the geometric average is adopted to determine the shaft resistance; the geometric averaging of a lognormal random field can maintain the type of distribution and mean value of its equivalent normal distribution (Fenton and Griffiths 2008). In contrast, the conventional approach for ultimate resistant of drilled shafts employs the arithmetic average of N of a certain domain to compute the shaft resistance. For convenience of presentation and discussion later, this simplified approach using spatial averaging technique is denoted as S1 and is readily implemented with MCS in this paper.

3.3 Spatial correlation of spatial averages

The second simplified approach to investigating the effect of spatial variability is based on the spatial correlation of spatial averages (Vanmarcke 1977). Considering that the empirical model (Eq. 1) consists of two terms for the shaft resistance and toe bearing resistance, respectively, two spatial averages are classified and shown in Figure 2(b). The first spatial average (denoted as u_{D1}) covers the region from 2 m in depth to the bottom of the shaft (D_1 = 28 m), which is used to calculate the shaft resistance. The second spatial average (denoted as u_{D2}) covers the region from the bottom of the shaft to 2D below the tip (D_2 = 2 m), which is used to calculate the toe bearing resistance. In addition to u_{D1} and u_{D2}, a third spatial average (denoted as u_{D12}) that covers the region with a vertical distance of D_{12} (in Figure 2(b)) can also be determined.

The spatial averaging technique utilized in the simplified approach S1 is then used to determine the two spatial averages u_{D1} and u_{D2}. As such, the lognormally distributed N is first transformed into the equivalent normal distribution $\ln N$. Then the variance reduction factors for u_{D1} and u_{D2} (denoted as $\Gamma^2(D_1)$ and $\Gamma^2(D_2)$) can be computed with Eq. (4) by setting Δz equal to D_1 and D_2, respectively. Then the reduced variances for u_{D1} and u_{D2} are computed using the corresponding variance reduction factor and the variance of $\ln N$. It should be noted that u_{D1} and u_{D2} are in the same random field and thus the correlation between the two spatial averages needs to be considered in the probabilistic analysis. This paper adapts the correlation coefficient of two spatial averages proposed by Vanmarcke (1977), which takes the following form:

$$\rho_{u_{D_1},u_{D2}} = \frac{D_{12}^2\Gamma^2\left(D_{12}\right) - D_1^2\Gamma^2\left(D_1\right) - D_2^2\Gamma^2\left(D_2\right)}{2D_1D_2\Gamma\left(D_1\right)\Gamma\left(D_2\right)} \tag{9}$$

in which $\Gamma^2(D_1)$, $\Gamma^2(D_2)$ and $\Gamma^2(D_{12})$ are the variance reduction factors for the spatial averages u_{D1}, u_{D2} and u_{D12} respectively. These variance reduction factors are computed with Eq. (4) by setting Δz = D_1, D_2, and D_{12}, respectively.

This study adopts MCS to determine the statistical distribution of the estimated ultimate resistance (Q_{ult}) and the probability of failure at a certain level of compression load F. Comparing with the RFM approach and S1 based on spatial averaging technique, this second simplified method is an intermediate approach using the spatial correlation between spatial averages. In this paper, this simplified approach is denoted as S2 for convenience of presentation. The focus of this study is to compare the aforementioned three probabilistic approaches (RFM, S1 and S2) for determining the probabilistic ultimate resistance of a drilled shaft considering the effect of spatial variability.

4. PROBABILISTIC ANALYSIS OF DRILLED SHAFTS CONSIDERING SPATIAL VARIABILITY

In this section, the effect of spatial variability of SPT N on the probabilistic analysis is first investigated and demonstrated with RFM, which is a rigorous approach to deal with the spatial effect. The research results obtained from the RFM approach then serve as a benchmark in the subsequent comparison study for the two simplified approaches (S1 and S2). As will be shown in the subsequent sections of this paper, RFM, S1 and S2 yield comparable results (i.e., probability of failure) in the design example shown in Figure 1(a).

4.1 Random field modeling (RFM) of spatial variability of SPT *N*

It is quite well known that the spatial variability of soil property has a significant influence on the probabilistic analysis of geotechnical engineering (Fenton and Griffiths 2008). It is advisable to compare the assumption of spatial constant case (no spatial variability) with those of various levels of spatial variability. In the RFM of the design example in Figure 1(a), the spatial constant case can be realized using a relatively large value of scale of fluctuation, say θ = 100 m. A series of sensitivity studies with the following ranges of parameters is performed:

COV = 0.1, 0.2, 0.3 and 0.6
θ = 0.5 m, 3 m, 10 m, 50 m and 100 m

The variability of soil property results from various sources of uncertainty may be grouped into two categories: aleatory uncertainty and epistemic uncertainty. The aleatory uncertainty includes spatial variability and random testing errors, while the epistemic uncertainty is related to measurement procedures and limited data availability (Whitman 1996). The sample statistics adopted in this study represent only the spatial variability of soils. The effect of other sources of uncertainty on the ultimate resistance of drilled shafts is not the focus of this paper, although further investigation of those uncertainties is warranted. Given that the COV and the θ is for the normalized N, the aforementioned RFM procedure based on the Cholesky decomposition is performed using each pair of COV and

θ. In this study, the level of failure probability of interest is 10^{-5}. The number of MCS samples should be at least 10 times of the reciprocal of the target failure probability (Ang and Tang 2007). Thus the number of MCS in this study is set to be 10^6 for each pair of COV and θ. Then the statistical distributions of the Q_{ult} are obtained and the mean values and COV can be determined using MCS. It is found that the spatial variability has very limited influence on the mean of Q_{ult}.

Figure 4 shows the variation of the ultimate resistance as a function of COV and θ. It is observed that at the same level of COV of normalized *N*, the COV of ultimate resistance increases with θ. The spatial variability has a significant influence on the variation of the resulting ultimate resistance: smaller θ leads to smaller COV of Q_{ult}. In Figure 4, it is also observed that at the same level of θ, smaller COV of normalized *N* results in smaller COV of Q_{ult}. Through comparison of the COV of Q_{ult} for spatial constant case (θ = 100 m) and the case with spatial variability (say, θ = 3 m), it is concluded that there is an effect of reduction in the variation of *N* when spatial variability exists, which leads to a smaller variation of Q_{ult}. This conclusion is consistent with the theory of variance reduction, which will be presented in the approaches S1 and S2 in section 4.2. With the statistical distributions of Q_{ult}, the probability of failure for this design example can also be estimated with Eq. (2) if a certain level of compression load *F* is applied. Figures 5(a) and 5(b) show the estimated probability of failure at various combinations of *F* and θ for COV = 0.3 and 0.6, respectively. It should be noted that the horizontal axis is set in the reverse order and *F* = 2000 *kN* approximately corresponds to a factor of safety of 1.0. At the same level of θ, the probability of failure increases as *F* approaches to 2000 *kN*. At the same level of *F*, the probability of failure for the spatial constant case (θ = 100 m) is much larger than the case with spatial variability (say θ = 3 m). For instance, in Figure 5(a), at *F* = 1800 *kN*, p_f is estimated to be 27.4% for θ = 100 m, comparing with 2.4% for θ = 3 m. The research results here support the previous research conclusions that neglecting the spatial variability of soil property leads to the overestimation of the probability of failure in the design of piles and drilled shafts (e.g., Luo and Juang 2012; Zhang and Chen 2012). Thus, the need to consider the spatial effect in the design of drilled shafts is highlighted in this paper.

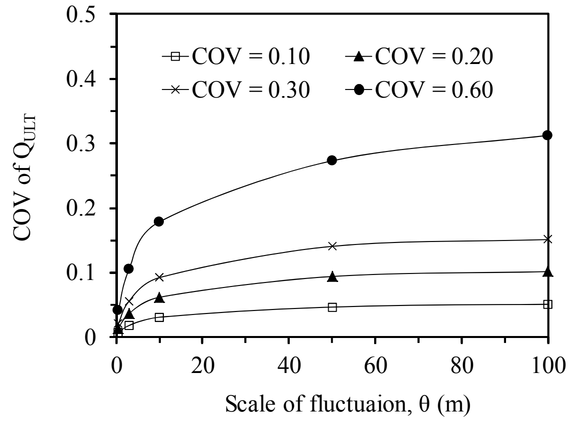
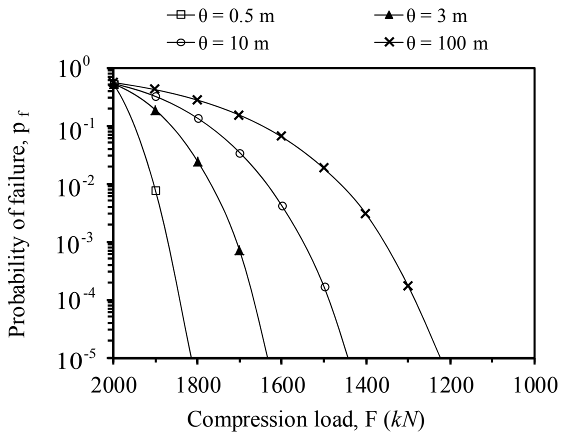
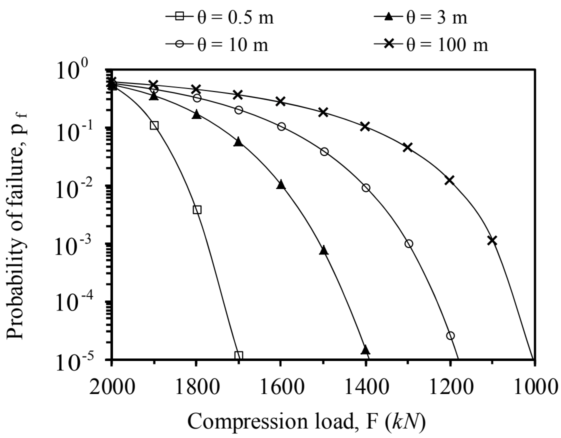


Figure 4 Coefficient of variation (COV) of ultimate resistance as a function of scale of fluctuation and COV



(a) COV = 0.3



(b) COV = 0.6

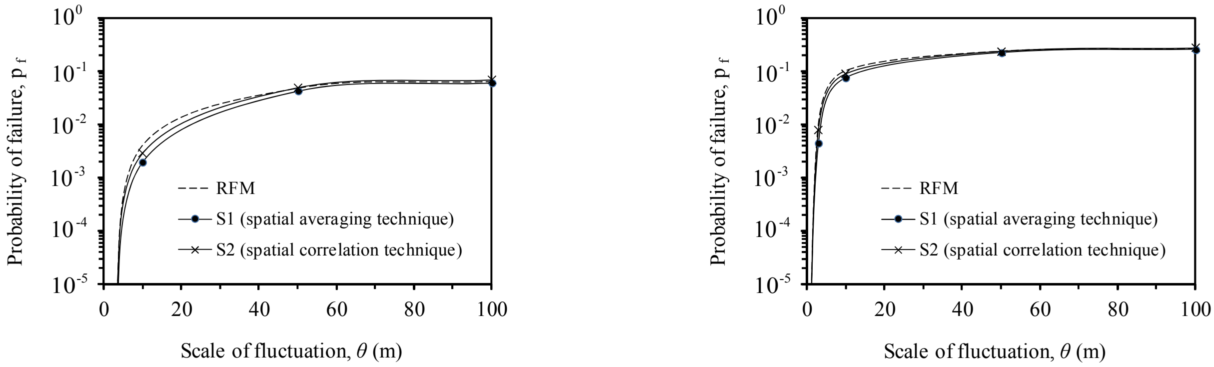
Figure 5 Estimated probability of failure at various levels of compression load using RFM

4.2 Comparison between RFM and simplified approaches

The focus of this paper is to compare the estimated probability of failure among the three approaches, RFM, S1 and S2. With the aforementioned procedures for RFM, S1 (simplified approach based on spatial averaging) and S2 (simplified approach based on spatial correlation between spatial averages), a series of MCS are performed to investigate the effect of spatial variability of SPT *N* on the probability of failure for the design example in Figure 1(a) at a compression load of 1600 *kN*. For each combination of COV and θ, the number of MCS is 10^6 in all three probabilistic approaches. Following the procedures presented in Section 3, the comparisons of estimated probability of failure at various levels of scale of fluctuation are shown in Figure 6(a) and 6(b) for COV = 0.3 and 0.6, respectively. It is found that the three approaches (RFM, S1 and S2) yield virtually identical probability of failure at various combinations of COV and θ. The simplified approaches (S1 and S2) can serve as alternatives to the RFM approach in this case study.

To further demonstrate the potential of the simplified approaches in the probabilistic analysis of drilled shafts considering spatial variability, another series of comparison studies is performed at various levels of compression load (F). The same procedures are repeated and the comparisons of the estimated failure probability among the three approaches are shown in Figure 7 for COV = 0.3. For the four levels of scale of fluctuation (θ = 0.5 m, 3 m, 10 m and 100 m) shown in Figure 7, the RFM approach and the simplified approaches (S1 and S2) results in almost identical probability of failure, regardless of the magnitude of compression load F. The same procedures of RFM, S1 and S2 are further repeated at a higher level of variation of normalized N, COV = 0.6, and the results are shown in Figure 8 for θ = 0.5 m, 3 m, 10 m and 100 m. It is also observed in Figure 8 that the three probabilistic approaches are almost equivalent at the higher variation of normalized N. The research results presented in Figures 7 and 8 indicate the simplified approaches (S1 and S2) can yield consistent probability of failure as those obtained from the RFM approach, and thus S1 and S2 can serve as alternatives to RFM.

In this comparison study, all three approaches are conducted using MCS. It should be noted that RFM necessarily involves the random field theory and MCS, and thus it may not be practical in the engineering practice due to the large computational effort required. Considering that the simplified approaches (S1 and S2) can be easily realized with analytical methods, such as point estimate method (PEM), first-order second moment method (FOSM), and first-order reliability method (FORM), which require much less computational effort comparing to MCS and can be implemented easily in a spreadsheet (e.g., Luo and Juang 2012), the simplified approaches may be preferred in the probabilistic analysis of drilled shafts considering the spatial variability of soil property. For the two simplified approaches (S1 and S2), S2 is relatively complicated since three variance reduction factors are needed and the spatial correlation (as in Eq. 6) has to be considered. S1 is relatively easier because the entire random field is treated as one spatial average and only one variance reduction factor needs to be computed. This study advocates S1 as it requires the least amount of computational effort in the analysis of the ultimate resistance of drilled shafts.



(a) COV = 0.3 (b) COV = 0.6

Figure 6 Comparison of estimated probability of failure at various levels of scale of fluctuation among RFM, S1 and S2 at F = 1600 *kN*

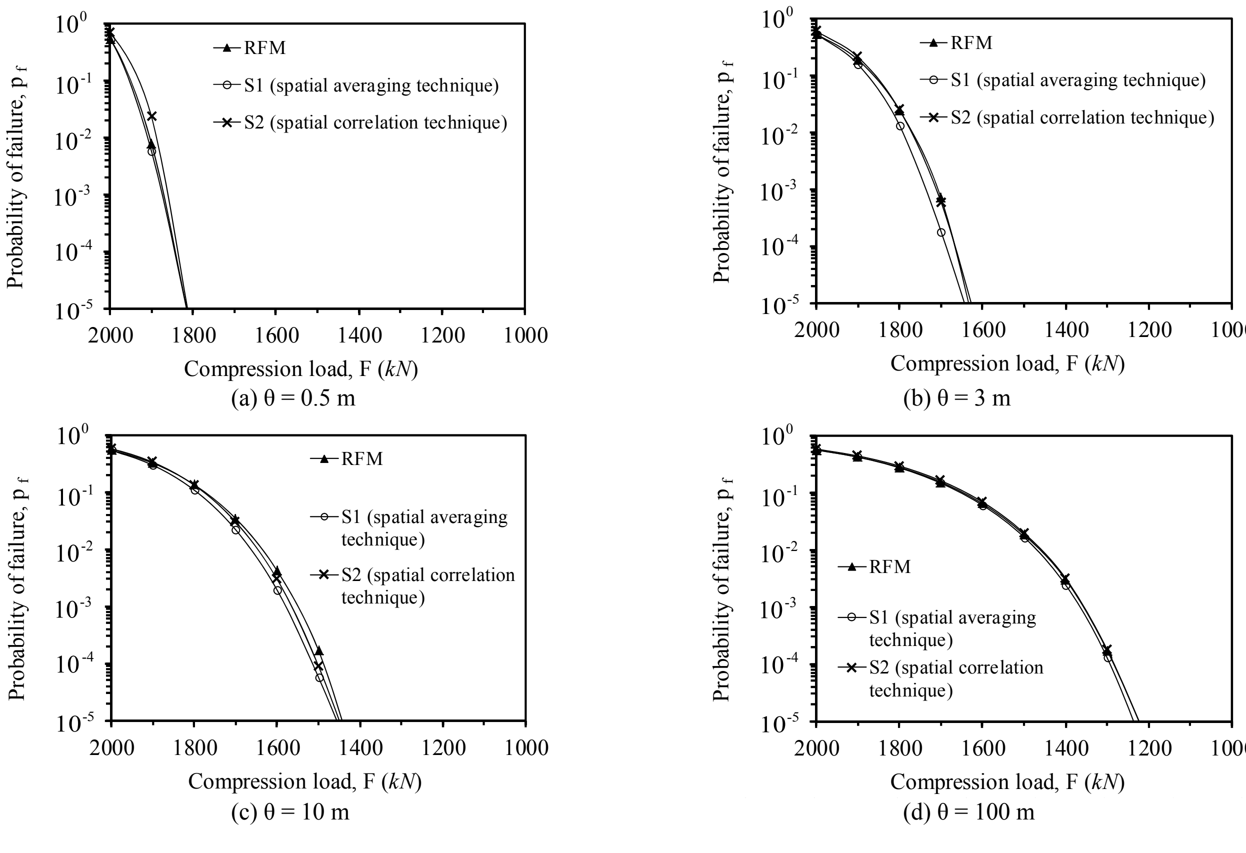


Figure 7 Comparison of estimated probability of failure among the three approaches RFM, S1 and S2 for COV = 0.3

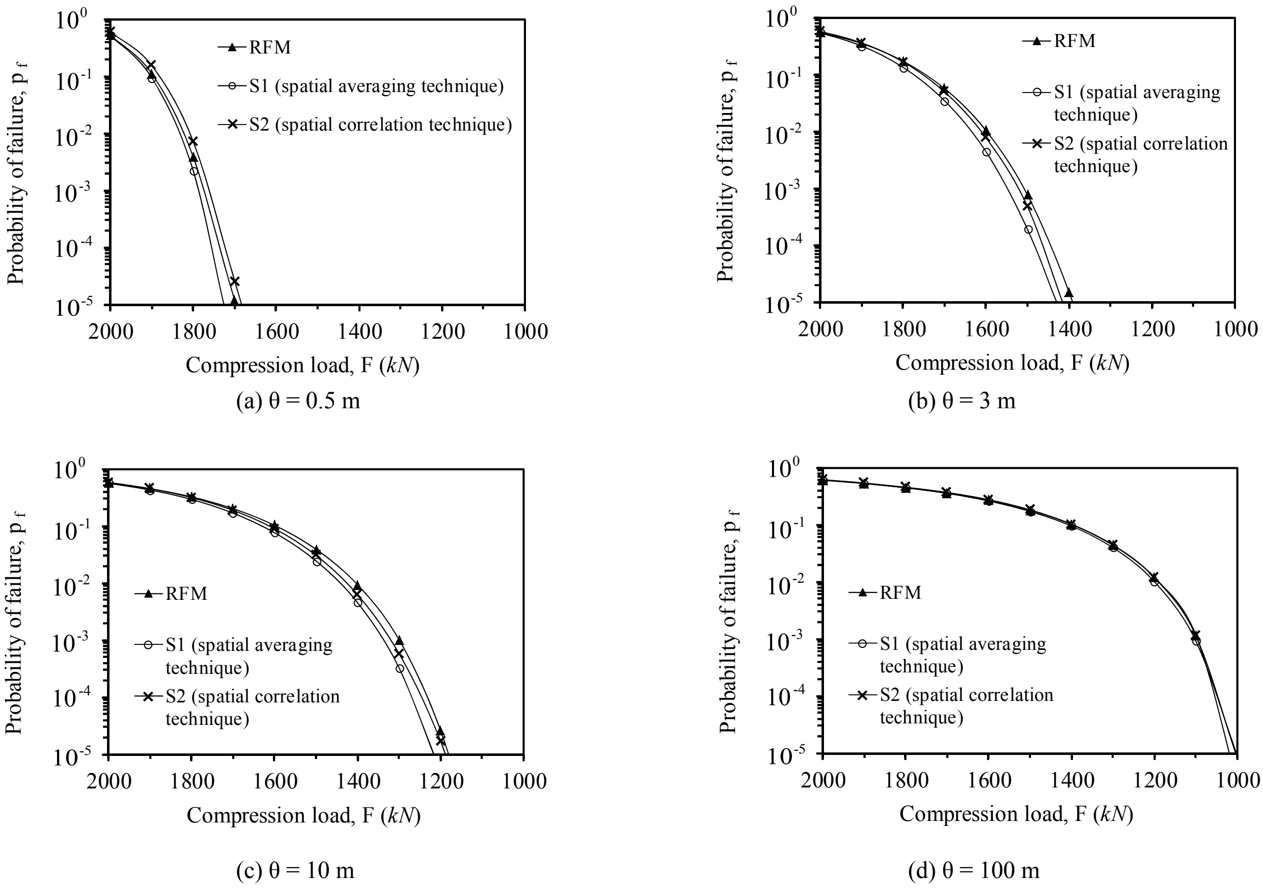


Figure 8 Comparison of estimated probability of failure among the three approaches RFM, S1 and S2 for COV of = 0.6

4.3 Effect of model bias on the probability of failure

The probabilistic analysis of the ultimate resistance of drilled shafts presented so far is performed assuming no model bias in the empirical model (Eq. 1). When the model bias is to be considered, Eq. (1) can be adapted by multiplying a bias factor (*BF*), defined as the ratio of the measured over the predicted ultimate resistance, express as follows:

$$Q_{ult} = BF \cdot \left(\sum_{i=1}^{n} \pi D L_i \overline{N}_i + 9.5 \pi D^2 N_n / 4 \right) \tag{10}$$

In the previous probabilistic analysis, the *BF* is assumed to be a constant of 1.0. When the model bias exists, the mean of *BF* may not necessarily be unity and should be modeled as a random variable. Zhang et al. (2009) calibrated the model bias of Eq. (1) using a database in Hong Kong. The calibration results by Zhang et al. (2009) are adopted herein. The mean value of *BF* (μ_{BF}) is set to be 1.41. Two levels of COV of *BF* (COV_{BF}) are selected to be 0.2 and 0.3. In this paper, the *BF* is assumed to follow the lognormal distribution (Zhang et al. 2009).

To illustrate the effect of model bias, the design example in Figure 1(a) is used herein and the COV of normalized *N* and θ are assumed to be 0.3 and 3 m, respectively. Following the aforementioned procedure of the RFM approach, MCS is performed to determine the probability of failure at various compression loads *F*, for two combinations of μ_{BF} and COV_{BF}: (1) μ_{BF} = 1.41, COV_{BF} = 0.2 and (2) μ_{BF} = 1.41, COV_{BF} = 0.3. The estimated probability of failure as a function of *F* is shown in Figure 9. For comparison purpose, the case with no model bias, which is the curve with the RFM approach in Figure 7(b), is re-plotted in Figure 9.

The effect of mode bias on the probabilistic analysis is significant. If the curve of no model bias and the curve with COV of 0.2 are compared, a critical load of approximately 1800 *kN* is observed: when the load on the drilled shaft is relatively small (smaller than 1800*kN*), the probability of failure for the curve with model bias is larger than that without model bias. This trend reverses when the load exceeds 1800 *kN* and the failure probability for the case with model bias is smaller than that without considering model bias. It is concluded that if the model bias is neglected, the probability of failure is overestimated at relatively large load and the probability of failure is significantly underestimated when the load is relative small. The smaller value of *F* indicates a larger factor of safety. This indicates even though a conservative deterministic design is adopted (using a large factor of safety), the actual probability of failure can be higher than expected. In other words, a "conservative" deterministic design can be un-conservative because of the existence of model bias. Similar conclusions are drawn if the curve of no bias and the curve with COV of 0.3 are compared. Therefore, the model bias should be included in the probabilistic analysis of the ultimate resistance of drilled shaft.

Finally, a comparison among the three probabilistic approaches (RFM, S1 and S2) with the consideration of model bias is conducted. In this comparison study, the model bias follows a lognormal distribution with μ_{BF} = 1.41, COV_{BF} = 0.2. The COV and θ are assumed to be 0.3 and 3 m, respectively. Figure 10 shows the probability of failure with various levels of load based on MCS. The three approaches still yield identical results when the model bias is considered in the probabilistic analysis. However, it is advisable to adopt the simplified approaches (S1 and S2) in the probabilistic analysis of drilled shafts considering the spatial variability of soil properties. The simplified approaches can be implemented with efficient probabilistic technique in lieu of MCS, which significantly saves the computational effort (e.g., Luo et al. 2012a, 2012b).

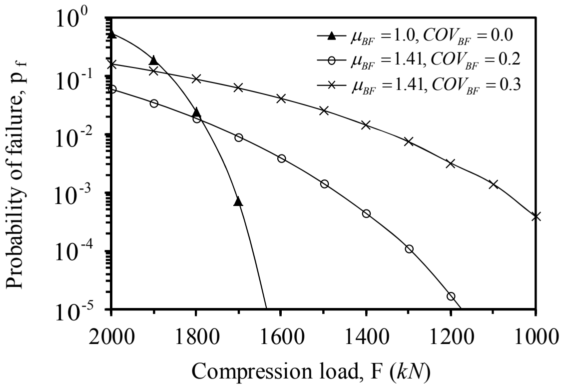


Figure 9 Effect of model bias on probability of failure using RFM

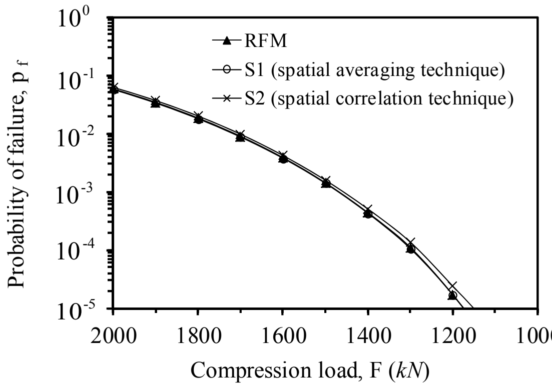


Figure 10 Comparison of estimated probability of failure among the three approaches RFM, S1 and S2 with the consideration of model bias

5. CONCLUSIONS

In this paper, a series of probabilistic analysis of the ultimate resistance of drilled shafts considering the effect of spatial variability of soil property is conducted. The procedures of three probabilistic approaches for modeling spatial variability are illustrated: a traditional random field modeling (RFM), a simplified approach based on spatial averaging technique (S1), and a simplified approach based on spatial correlation between spatial averages (S2). In this study, the effect of spatial variability of SPT *N* on the estimated probability of failure is investigated using the RFM approach. It is concluded that the negligence of spatial variability leads to the overestimation of the probability of failure in the design of drilled shafts. The results based on RFM are then used as a benchmark in the subsequent comparison study.

The focus of this paper is to compare the three probabilistic approaches (RFM, S1 and S2) for the design of drilled shafts. The results show that the simplified approaches (S1 and S2) can yield consistent probability of failure with those obtained from the RFM approach, and thus S1 and S2 can serve as alternatives to RFM. In lieu of Monte Carlo simulation, it should be noted that the simplified approaches (S1 and S2) can also be easily realized using reliability methods, which requires much less computational effort and thus can be a practical tool in the probabilistic analysis of drilled shafts considering spatial variability. Comparing with S2, S1 requires less computational effort and may be preferred in the engineering practice. This study also shows the model bias should be included in the probabilistic analysis of the ultimate resistance of drilled shafts. Neglecting the model bias will result in an overestimation or underestimation of the probability of failure, depending on the load applied to the drilled shafts.

It should be noted that the results presented in this paper are limited to single drilled shaft under compression load considering one-dimensional spatial variability with the assumed soil parameters. Further studies to consider the effects of three-dimensional soil variability in more realistic scenarios involving drilled shaft group subjected to more complex loading conditions are warranted.

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