Effect of Cyclic Strain History on Shear Modulus of Dry Sand using Resonant Column Tests

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ABSTRACT: A number of resonant column tests were performed on dry sand specimens to examine the effect of cyclic shear strain history, by including both increasing and decreasing strain paths, on the shear modulus (G) for different relative densities (D_r) and confining pressures (σ_3). The specimen was subjected to a series of cycles of increasing and decreasing shear strain paths approximately in a range of 0.001-0.1%. For a particular cycle, with a given strain amplitude, the shear modulus during the increasing strain path becomes always greater than that during the decreasing strain path. For a given cycle, irrespective of relative density of sand, the difference between the values of G associated with the increasing and decreasing strain paths becomes always the maximum corresponding to a certain shear strain level. The maximum reduction in the shear modulus, due to the cyclic variation of the shear strain, was noted to be around one fourth of the maximum shear modulus (G_0). This reduction in the shear modulus on account of the cyclic variation of shear strain increases generally with decreases in the values of both relative density and confining pressure. The study will be useful to examine the response of sand media subjected to earthquake excitation.

Keywords: Dynamic properties, earthquakes, resonant column tests, sands, shear modulus, vibrations.

1. INTRODUCTION

Resonant column tests are often recommended to find the variation in dynamic properties of soils with changes in (i) soil type, (ii) void ratio, (iii) degree of saturation and (iv) effective confining pressure (Hardin and Richart 1963; Hardin and Drnevich 1972a; Drnevich et al. 1978; Chung et al. 1984; Lo Presti et al. 1997; Kumar and Clayton 2007; Kumar and Madhusudhan 2011). Most of the available experimental data generally deal with the increasing strain path without completing the cyclic strain history (Sherif et al. 1977; Saxena and Reddy 1989; Avramidis and Saxena 1990; Hardin et al. 1994). A limited number of studies are, however, available which examine the possible changes in dynamic soil properties of soils due to cyclic strain history (Drnevich and Richart 1970; Shen et al. 1985; Alarcon-Guzman et al. 1989; Lo Presti et al. 1993; Li and Yang 1998; Li and Cai 1999). In most of these research investigations, the shear strain amplitude (γ) is gradually increased towards a certain pre-strain amplitude ($\gamma_{prestrain}$) and at $\gamma = \gamma_{prestrain}$, large numbers of strain cycles are induced. Subsequently, by starting again from the minimum (threshold) strain level, the shear strain is once again increased by following the increasing strain path towards the pre-strain amplitude of strain. It is generally noted from these studies that (i) hardly any change in the shear modulus occurs for shear pre-strain amplitudes lower than 0.01%, and (ii) on the other hand, an increase in the shear modulus often takes place with an increase in the number of cycles for $\gamma_{prestrain} > 0.01\%$. As compared to the other reported studies, Wichtmann and Triantafyllidis (2004) obtained shear modulus for sands covering both decreasing and increasing shear strain paths after prestraining. In contrast to most of the other reported results, these tests indicate that the shear modulus, for the subsequent strain cycles, reduces both during the decreasing strain as well as during increasing strain paths. The shear modulus for the decreasing strain path was found to become lower than that during the preceding increasing strain path. It is worth mentioning that in a typical earthquake excitation, initially the soil media is subjected to a few cycles of strains along the increasing strain path before reaching a particular peak amplitude of strain and then it is eventually subjected to a certain number of strain cycles along the decreasing strain path before ultimately attaining the zero strain. Therefore, it will be important to investigate the variation in the shear modulus along the increasing as well as decreasing strain paths. In the present study, similar to the investigation of Wichtmann and Triantafyllidis (2004), an attempt has been made to evaluate the extent of the variation in the magnitude of the shear modulus (G) of dry sand, by including both increasing and decreasing strain paths cycles, due to a repeated increase and decrease in the shear strain amplitude in a range of approximately 0.001-0.1%. It needs to be mentioned that Wichtman and Triantafyllidis (2004) studied the effect of cyclic strain history with an inclusion of large number of vibration cycles at certain pre-strain amplitude, covering both decreasing and increasing shear strain paths after pre-straining. On the other hand, in the present paper, tests have been performed without the application of a number of cycles at certain pre-strain amplitude. A series of resonant columns tests, in a torsional mode of vibrations, were performed by varying different values of relative density and confining pressure. The specimens have been subjected to repeated cycles of increasing and decreasing shear strain amplitudes in a range of approximately 0.001-0.1%. The effect of the cyclic variation of the strain on the shear modulus has been thoroughly quantified in all the cases.

2. BASIC PROPERTIES OF SAND

Table 1 summarizes the basic gradation parameters, along with maximum and minimum unit weights of the chosen sand. Based on the SEM images as illustrated in Figure 1, sand particles are found to be generally angular in shape. As per the unified soil classification system, the sand is classified as a poorly graded sand (SP). The minimum and maximum unit weights of the sand were determined in accordance with the procedure described in ASTM D4254-method A and ASTM D4253-method 2A, respectively, and these values of the unit weights were noted to be 13.0 kN/m^3 and 15.8 kN/m^3.

Table 1 Properties of Chosen Sand

Specific gravity, G	2.64
Maximum dry unit weight, γ_{dmax} (kN/m^3)	15.84
Minimum dry unit weight, γ_{dmin} (kN/m^3)	12.98
D_{10} (mm)	0.23
D_{30} (mm)	0.40
D_{60} (mm)	0.60
Uniformity coefficient, C_u	2.61
Coefficient of curvature, C_c	1.16
Symbol as per unified classification nomenclature	*SP*

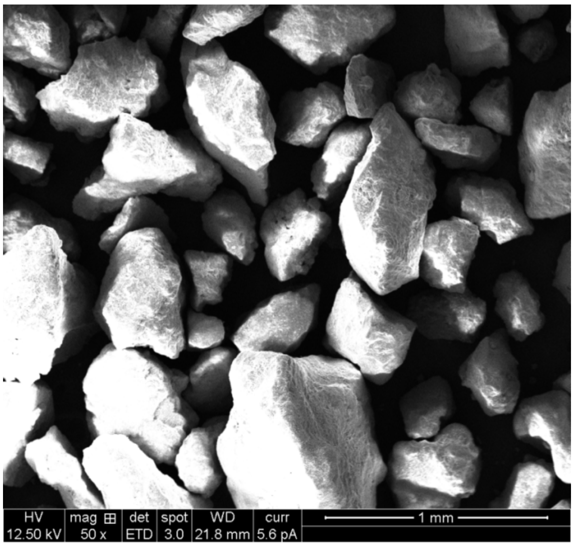

Figure 1 SEM image of sand sample

3. SPECIMEN PREPARATION TECHNIQUE

Cylindrical specimens of diameter 50 mm and height 100 mm were prepared at three different relative densities (D_r) close to 55%, 70% and 85%. In order to achieve a uniform relative density, specimens were prepared by using the pluviation technique. Necessary calibration exercise was done to relate the height of fall with the relative density of the chosen sand. The oven dried sand was first cooled in a desiccator for more than about 2 hours, and this sand was poured from a controlled height to the cylindrical split mould placed at the base of the resonant column apparatus. The dimensions of the specimen were accurately measured by using the Vernier calipers. To account for the thickness of the latex membrane, the necessary correction was also applied. This was done by measuring exactly the diameter of a 50 mm metallic mould both with and without the inclusion of the stretched membrane. The thickness of the membrane was accordingly taken as half the difference between the two diameter readings. The thickness of the membrane was found to be 0.29 mm.

4. APPARATUS AND TESTING PROCEDURE

Laboratory tests were performed by using a fixed–free configuration of the Resonant Column Apparatus (RCA) supplied by GDS Instruments, UK. Figure 2 presents a schematic diagram and the photographs of different components of the test apparatus. The RCA control box converts the digital sinusoidal waveform generated by the computer program into an analogue signal with the usage of a high-speed 16-bit data acquisition and the control card. The power amplifier amplifies the generated input signal and then transmits it back to the control box. The amplified input signal in turn is then supplied to the coils of the electromagnetic drive system. Torque is applied to the top of the specimen by means of this electromagnetic drive system which consists of a four-armed plate, attached to the top cap, having permanent magnets fixed to each arm and a support cylinder with four pairs of the drive coils. The application of the sinusoidal voltage to the coils results in the formation of a magnetic field which induces an oscillatory motion in the drive plate. The accelerometer attached to the drive plate measures the response of the top of the specimen. The charge amplifier amplifies the accelerometer response and transmits to the control box from where the analog voltage signal is converted to a digital signal and finally analyzed by using the computer program. A LVDT, with a sensitivity 0.0013 mm/V, is connected to the top of the specimen to measure the vertical deformation of the sample and its signal is transmitted to the computer through a serial data acquisition pad.

In order to find the resonant frequency of the sample associated with a given input voltage, the software allows running both broad and fine frequency sweep operations. In the broad/fine sweep, the frequency of the sample is varied at a specified frequency interval. During each increment of frequency, to ensure the minimum sample disturbance, the given sinusoidal signal is first steadily increased to a maximum amplitude. This is held for a specified number of cycles and, thereafter, it is gradually decreased in the same way. The accelerometer amplitude is then calculated by averaging the RMS value for each cycle in the middle 50% of the full data set. The peak amplitude corresponding to a particular frequency increment is then calculated from the known RMS value. Finally, a graph is plotted with a peak amplitude on the vertical axis and the corresponding frequency on the horizontal axis; the frequency corresponding to the maximum amplitude is taken as the resonant frequency of the sample with a given input voltage.

After preparing the specimen with a given relative density, the confining pressure of the required magnitude was then applied. The input excitation voltage to the coils was varied in order to induce different levels of strain in the specimen. For each chosen value of the input excitation voltage, the resonant frequency of the specimen was determined and the corresponding amplitude of the shear strain was measured. Starting from a minimum to maximum input voltage, this exercise was repeated in order to find corresponding values of the resonant frequencies and the strain levels. The minimum and maximum input voltage used in all the cases were 0.001V and 0.5V, respectively. After reaching the maximum input voltage, this procedure was continued again but by decreasing the strain to the lowest possible value with a reduction in the input voltage. Tests were performed for three different confining pressures, namely, 100 kPa, 300 kPa and 500 kPa. Even with the same relative density, for performing the test at a given confining pressure, a new specimen was prepared. The process of increasing and decreasing the strain, from lowest possible voltage to the highest voltage and then back again to the lowest voltage, is considered as one complete cycle. For all the cases, the variation of the shear modulus with strain was observed for a total of three and a half such cycles. The shear strains in different cases lie in a range 0.00063% - 0.1079%.

5. RESULTS AND DISCUSSION

Figures 3-5 show the variation of the shear modulus (G) with the cyclic changes in shear strain by including both increasing and decreasing strain paths for different cycles of excitation. For a particular cycle, both during the increasing and decreasing strain paths, the shear modulus decreases continuously with an increase in the shear strain from approximately 0.001% to 0.1%; for a given input excitation voltage, the exact strain level becomes a little different depending on the values of relative density, confining pressure and the strain cycle number. For a given cyclic variation of the strain, the magnitude of G, corresponding to a given strain level along the decreasing strain path, becomes always smaller than that during the increasing strain path. The curves of the decreasing strain paths for all the cycles were found to be very close to each other; at the same level of strain, the value of G for the subsequent cycle was found to be only a little lower than the previous cycle.

Corresponding to three different values of σ_3, namely, 100 kPa, 300 kPa and 500 kPa, Table 2 presents the difference (G_{diff}) between the values of the shear modulus associated with the increasing and decreasing strain paths for a given cycle, at a given strain level, in terms of the percentage with respect to the maximum shear modulus (G_0); note that the modulus G_0, for given values of D_r and σ_3, herein corresponds to the increasing strain path for the first cycle with the minimum strain which is roughly 0.001%. The magnitude of G_{diff} becomes always the maximum corresponding a certain strain level which usually varies between 0.005% and 0.01%. The maximum value of G_{diff} was always found to become the highest for the first cycle, and for the subsequent cycles, this difference was generally

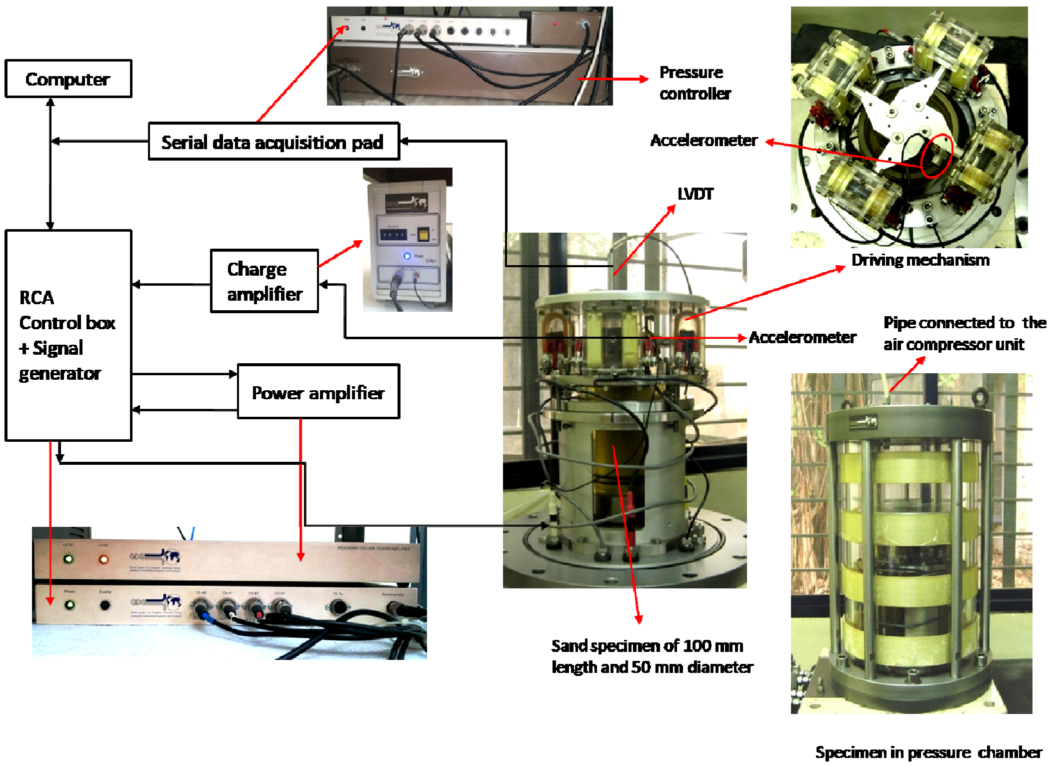

Figure 2 A schematic diagram along with the photographs of the different components of the Resonant Column Apparatus

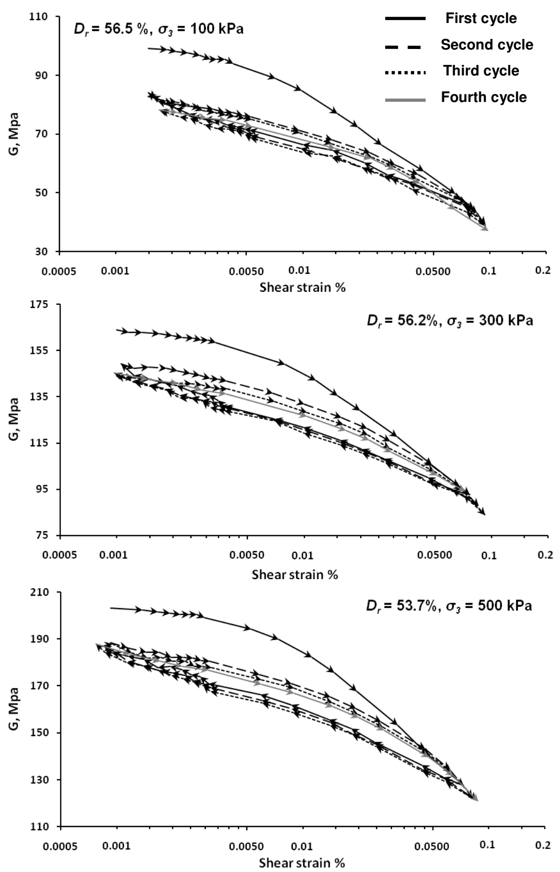

Figure 3 The variation of *G* with an increase and decrease in shear strain for different values of σ_3 at D_r = 53.7% -56.5%

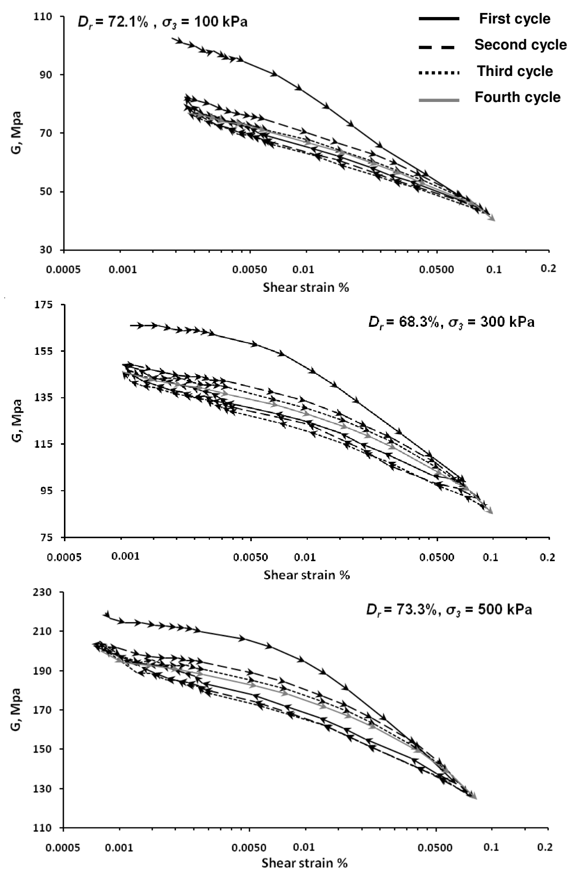

Figure 4 The variation of *G* with an increase and decrease in shear strain for different values of σ_3 at D_r = 68.3% -73.3%

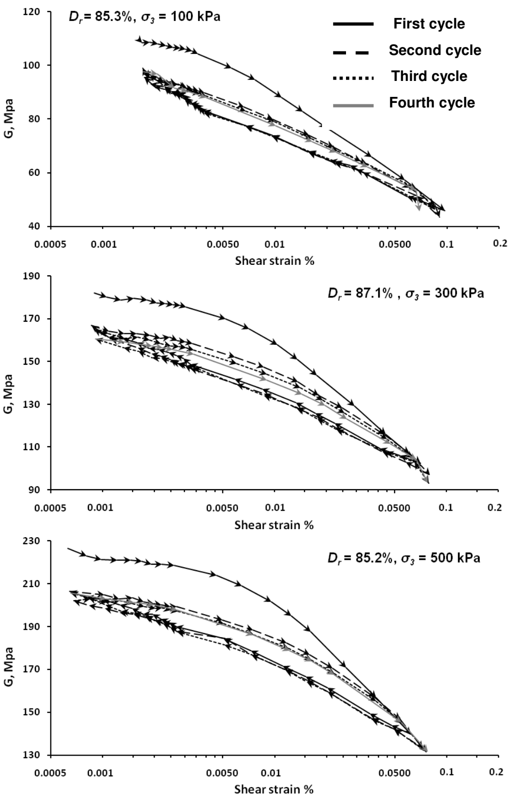

Figure 5 The variation of G with an increase and decrease in shear strain for different values of σ_3 at D_r = 85.2% - 87.1%

noted to decrease. The value of G_{diff} also reduces generally with an increase in the confining pressure. For the first cycle, the maximum value of G_{diff} was around 21.5 %. It should be mentioned that the difference G_{diff} was normalized with respect to G_0 rather than using the value of G corresponding to the appropriate strain level; the magnitude of G varies (reduces) with an increase in the strain level and the trend of the results on this basis was not found to be always consistent especially at high strain level.

Table 2 The variation of G_{diff} with Shear Strain for Different Cycles

σ_3 (kPa)	Shear strain (%)	The ratio of G_{diff} to G_0 in %								
		D_r = 53.7% - 56.5%			D_r = 68.3% - 73.3%			D_r = 85.2% - 87.1%		
		Cycle 1	Cycle 2	Cycle 3	Cycle 1	Cycle 2	Cycle 3	Cycle 1	Cycle 2	Cycle 3
100	0.0025	21.48	3.51	4.54	20.83	4.41	2.60	15.22	3.36	3.13
	0.0050	21.48	5.78	5.99	21.63	6.01	3.60	18.13	6.49	5.37
	0.0075	19.62	6.20	6.40	20.43	6.61	4.61	17.02	6.04	4.93
	0.0100	18.59	5.78	6.20	18.02	6.61	4.41	15.45	5.82	5.37
	0.0250	9.71	5.99	4.54	8.01	6.41	4.61	9.40	4.93	3.81
300	0.0025	14.40	6.40	3.91	14.97	5.22	4.00	12.70	5.63	4.02
	0.0050	16.00	7.11	5.69	16.71	6.27	5.92	14.79	6.75	4.50
	0.0075	15.64	6.75	5.33	15.67	6.27	5.74	14.31	7.07	5.14
	0.0100	14.40	6.75	5.87	14.10	5.92	5.92	13.18	7.07	5.79
	0.0250	8.89	6.04	4.98	8.88	7.14	5.74	8.20	5.63	5.14
500	0.0025	12.57	4.42	3.42	11.20	5.53	4.30	12.33	4.37	2.70
	0.0050	13.43	5.27	4.99	12.74	6.60	5.52	12.72	4.75	4.50
	0.0075	12.86	5.27	5.13	12.74	7.06	5.68	12.72	5.27	4.37
	0.0100	11.97	5.27	5.13	11.97	7.06	5.52	12.08	5.52	4.63
	0.0250	7.84	5.42	4.85	7.21	6.75	5.52	6.55	5.40	4.50

For different relative densities and confining pressures, Table 3 presents the values of G_{diff} for the difference between the increasing strain path of the first cycle and the decreasing strain path of the last cycle corresponding to different strain levels. Note that the maximum reduction in the value of shear modulus occurs corresponding to a strain level of roughly 0.005%. The reduction in the shear modulus increases generally with decreases in the values of confining pressure and relative density.

Table 3 The Variation of Maximum G_{diff} with Shear Strain for Different Values of Relative Density and Confining Pressure

Shear strain (%)	The ratio of G_{diff} to G_0 in %								
	D_r = 53.7% - 56.5%			D_r = 68.3% - 73.3%			D_r = 85.2% - 87.1%		
	100 kPa	300 kPa	500 kPa	100 kPa	300 kPa	500 kPa	100 kPa	300 kPa	500 kPa
0.0025	22.72	16.18	13.97	23.83	17.58	13.51	15.22	14.79	12.46
0.0050	23.54	17.24	14.97	24.43	18.80	14.73	18.81	15.11	14.13
0.0075	22.30	16.18	14.40	23.03	17.93	14.43	17.24	15.11	13.49
0.010	21.06	15.82	13.40	20.43	16.54	13.51	15.90	14.47	12.72
0.025	10.74	9.78	8.70	10.41	10.79	8.59	9.63	9.49	7.45

The variation of the shear modulus with shear strain for different values of relative density and confining pressure, corresponding to the first cycle of increasing strain path, has been provided in Figure 6. The magnitudes of G were found to increase continuously with an increase in the values of both confining pressure and relative density.

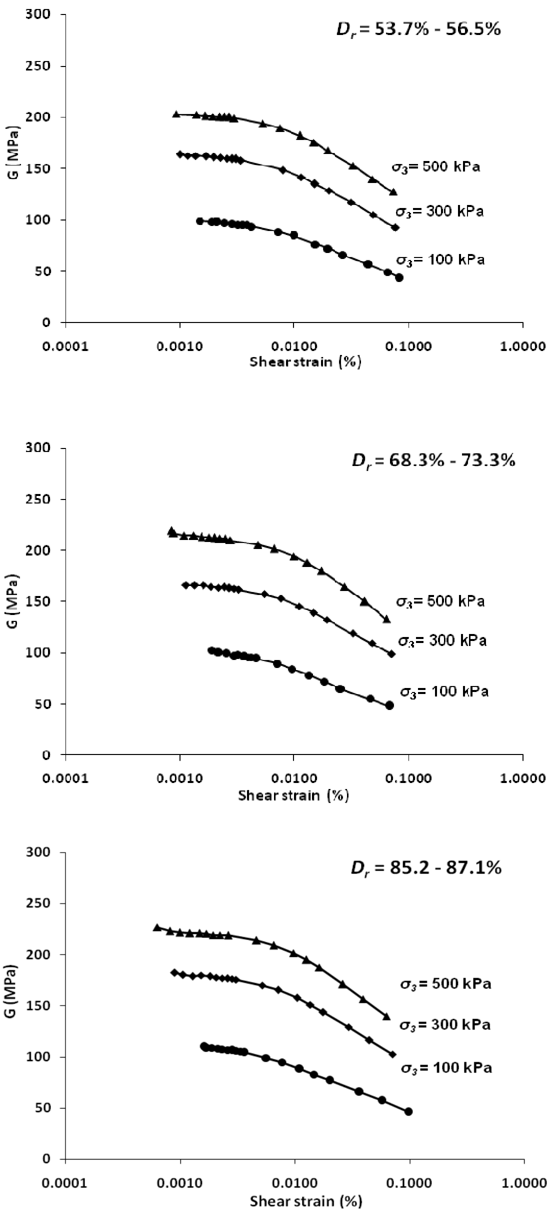

Figure 6 The variation of G with shear strain for different combinations of relative density and confining pressure

6. COMPARISON OF MAXIMUM SHEAR MODULUS (G_0)

The obtained values of G_0, that is, the magnitude of the shear modulus corresponding to minimum strain level for the first increasing strain path, were compared with the following empirical correlation given by Hardin and Drnevich (1972b):

$$G_0 = 102132.65\frac{(2.973-e)^2}{(1+e)}\sigma_3^{1/2} \quad (1)$$

In Eq. (1), the values of the confining pressure, σ_3 and G_0 are expressed in N/m^2 and e is the void ratio of sand. The comparison of the results between the two studies has been illustrated in Figure 7. The present experimental values of G_0 are found to be marginally greater than that predicted with the correlation of Hardin and Drnevich (1972b).

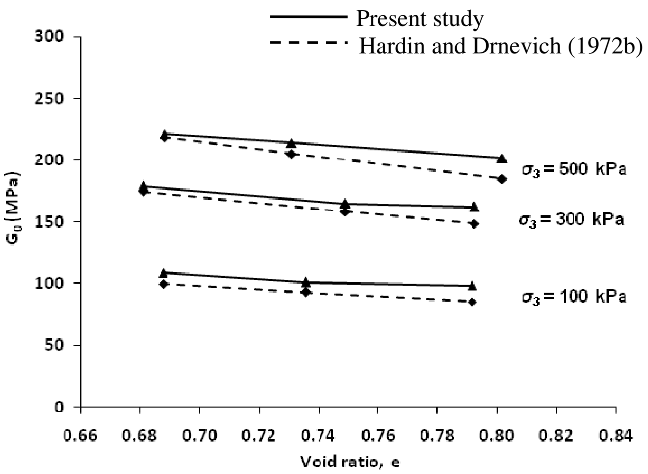

Figure 7 A comparison of G_0 at different combinations of e and σ_3

7. REMARKS

The literature on tests' results, where the effect of pre-staining has been included, generally indicates either no change or a very marginal increase in the shear modulus (Drnevich and Richart 1970; Shen et al. 1985; Alarcon-Guzman et al. 1989; Lo Presti et al. 1993; Li and Yang 1998; Li and Cai 1999). On the other hand, in the present study, without exerting any pre-staining, a decrease in the shear modulus has been noted with an inclusion of the decreasing shear strain path. This observation is in line with the data reported earlier by Wichtmann and Triantafyllidis (2004). It indirectly reveals the significance of including a decreasing shear strain path while performing resonant column tests to find the effects of earthquake excitation.

The observed variation of the shear modulus of dry sand, during decreasing and increasing strain paths, has to be obviously associated with the corresponding variation of the void ratio of the sand specimen. A decrease in the shear modulus needs to be on account of an increase in the void ratio of the specimen. The present testing has been conducted on dry sand specimens. For dry specimens, it was not possible to directly measure the changes in the void ratios during the cyclic variation of small levels of strain. This would perhaps form the scope of future research work in order to justify the observed trend of results. In a resonant column apparatus, it is not possible to measure directly the diameter of the specimen immediately after applying the excitation. In order to measure the diameter, an electronic measuring device needs to be attached to the periphery of the specimen. This will induce a dynamic loading on the cylindrical specimen in the event of any excitation and, therefore, it will become a difficult task to apply the theory on which the working of the equipment is based.

The LVDT is placed on the top of drive mechanism for which the necessary calibration exercise is always carried out by using Aluminum calibration bars. Based on the LVDT readings before and after performing the tests, the changes in the heights and corresponding changes in relative densities of the specimens in the different cases are shown in Table 4. It can be noted that the changes (increase) in the relative densities of the samples are found to vary only between 0.07%-0.18%; this difference is found to be practically insignificant.

Table 4 Changes in Heights and Relative Densities of the Specimens after Performing the Tests for Different Combinations of Relative Densities and Effective Confining Pressures

σ_3 (kPa)	D_r (%)	Initial length, L(mm)	Change in LVDT reading (mm)	Final length, L(mm)	Final D_r (%)	Increase in Dr (%)
100	56.51	99.00	-0.035	98.965	56.69	0.18
	72.07	100.00	-0.014	99.986	72.14	0.07
	85.34	100.16	-0.015	100.145	85.41	0.07
300	56.22	99.00	-0.021	98.979	56.33	0.11
	68.26	100.10	-0.016	100.084	68.35	0.09
	87.14	100.42	-0.011	100.409	87.19	0.05
500	53.69	99.80	-0.028	99.772	53.83	0.14
	73.32	99.70	-0.018	99.682	73.4	0.08
	85.18	100.00	-0.015	99.985	85.25	0.07

8. CONCLUSIONS

The variation of the shear modulus G with the cyclic changes in the shear strain was examined by gradually increasing the number of cycles. Both increasing and decreasing strain paths were included in the investigation. The results were obtained for different relative densities and confining pressures. For a given strain, the shear modulus along the increasing strain path becomes always higher than that during the decreasing stain path. For the first strain cycle, as compared to the increasing strain path, the shear modulus reduces significantly along the decreasing strain path. However, for subsequent strain cycles, this reduction in the shear modulus becomes generally continuously smaller. The maximum reduction in G takes place corresponding to the shear strain roughly equal 0.005%, and this maximum reduction in G with respect to G_0 was found to be of the order of around 24%. The reduction in the shear modulus due to the cyclic variation of strain increases generally with decreases in the values of the confining pressure and relative density. In a typical earthquake excitation, the soil strata is subjected to a few cycles of strains along the increasing strain path before reaching a peak amplitude of strain and then eventually it is exerted to a number of strain cycles along the decreasing strain path. Therefore, the results obtained will be useful in predicting the response of sand media during a typical earthquake excitation.

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