Geotechnical Engineering Journal of the SEAGS & AGSSEA Vol. 46 No.3 September 2015 ISSN 0046-5828

Modeling Undrained Shear Behavior of Reconstituted Clays considering the Effects of Initial Water Contents

X. Bian[1], L. L. Zeng[2], J. W. Ding[3] And Z. S. Hong[4]
[1]*Institute of Geotechnical Engineering, College of Civil and Transportation Engineering, Hohai University, Nanjing, China*
[2]*Institute of Geotechnical Engineering, College of Civil Engineering, Fuzhou University, Fuzhou, China*
[3,4]*Institute of Geotechnical Engineering, School of Transportation, Southeast University, Nanjing, China*
[1]*E-mail*: bianxia2010@gmail.com

ABSTRACT: This paper presents a new model for describing the undrained shear behavior of reconstituted clays due to the variation of initial water contents based on the concept of critical state soil mechanics. With the decrease of initial water contents, the reconstituted clays behave enhanced strength, stiffness and dilation, which are not involved in the Modified Cam Clay model. These features can be captured by introducing a new hardening parameter ('quasi-structure' strength) into the conventional critical state model. The 'quasi-structure' strength increases with the decrease of initial water contents. The available test data on the undrained shear behavior of reconstituted clays at different initial water contents are used to verify the proposed model, and the comparisons between computed and measured results show that the proposed model is able to predict the overall pattern of stress-strain curves, pore pressure variations and effective stress paths reasonably well, especially the ultimate undrained strength and pore pressure response at large strain.

KEYWORDS: Reconstituted clay, Initial water content, Constitutive modeling, Critical state.

1. INTRODUCTION

The mechanical behavior of reconstituted clays is often studied as the reference in understanding of the characteristics of natural clays (e.g. Leroueil et al., 1979; Burland, 1990; Leroueil, 1990; Chandler, 2000; Cotecchia & Chandler, 2000). A better understanding the mechanical behavior of reconstituted clays will eventually contribute to the knowledge of natural clays. Recently there are several studies related to the fundamental compression and stress-strain behavior of reconstituted clays (e.g. Cerato and Lutenegger, 2004; Hong et al., 2010; 2012; 2013). Hong et al. (2010) proposed that the virgin compression lines of reconstituted clays showed an inverse "S" shape from small stress level due to the effect of remolded yield stress. And the remolded yield stress possesses a decreasing tendency with initial water contents. Hong et al. (2010) suggested that the remolded yield stress was a main function that controlled the compressibility of reconstituted clays. In addition, Hong et al. (2013) showed that the undrained strength and stress-strain relationship of reconstituted clays are also varying with the initial water contents. Hence, the initial water content is one of the vital components controlling the mechanical behavior of reconstituted clays.

Gens (2010) suggested that "…the required development should be congruent with underlying themes of classical soil mechanics…This requires soil behavior, constitutive modeling, analysis and field observation to be brought together…" Following this concept, based on the new findings about the mechanical behavior of reconstituted clays, this paper presents a new constitutive model that incorporates the influence of initial water contents on stress-strain behavior of reconstituted clays into the conventional critical state model. The model has also been used to predict the behavior of reconstituted clays at different initial water contents in undrained shearing tests, allowing for a reasonably comprehensive evaluation.

2. MECHANICAL BEHAVIOR OF RECONSTITUTED CLAYS DUE TO INITIAL WATER CONTENTS

Understanding the characteristics of reconstituted clays with different initial water contents is vital and necessary to develop a rational constitutive model. The salient aspects have been presented as follows:

2.1 Typical virgin compression lines

As proposed by Hong et al. (2010), the initial water content is one of the main features to control the compression behavior of reconstituted clays as well as the intrinsic parameters defined by Burland (1990). Figure 1 shows the typical relationship between mean effective stress and the change in the specific volume of reconstituted clays with different initial water contents. The main characteristics of virgin compression lines of reconstituted clays with different initial water contents can be conducted from Hong et al. (2010): 1) there exists the remolded yield stress in virgin compression lines of reconstituted clays, which is similar to the consolidation yield stress of natural soft clays, resulting in an inverse "S" shape of VCL in a wide stress range from 0.5 kPa to 1600 kPa; 2) the VCL of a given reconstituted clay at a higher initial water content lies above that of a lower initial water content, indicating greater compressibility at higher initial water content when the stress level is larger than remolded yield stress.

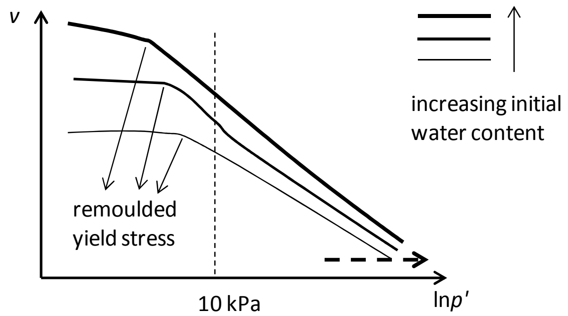


Figure 1 Typical virgin compression lines of reconstituted clays with different initial water contents

2.2 Typical undrained shear behavior

The initial water content also influences the stress-strain and pore pressure response of reconstituted clays (Hong et al., 2013). The schematic diagrams of stress-strain relationship for reconstituted clays with different initial water contents are plotted in Figure 2. It can be seen that the stress-strain curve of a given reconstituted clay at a higher initial water content lies below that at a lower initial water content, which indicates that with the increase of initial water contents, the strength and the initial shear modulus show a decreasing trend. However the pore pressure-strain curve shows that the pore pressure response of reconstituted clays during monotonic

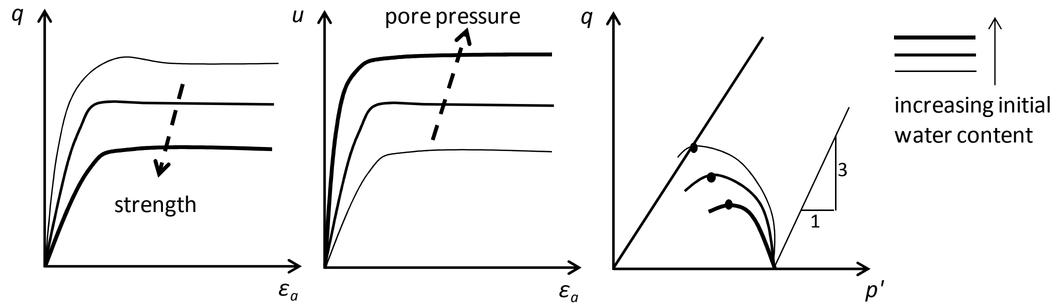


Figure 2 Conventional undrained triaxial compression test on reconstituted clays with different initial water contents

triaxial loading will increase with the increase of initial water content, which results in that the effective stress path at a higher initial water content below that of a lower initial water content.

2.3 Summary of mechanical responses due to initial water contents

The main effects of initial water contents to the mechanical behaviors of reconstituted clays can be summarized as follows:

1) stiffness and strength reducing with the increase of initial water contents;
2) compressibility and higher pore pressure response increasing with the increase of initial water contents. Hence, these aspects should be incorporated into constitutive model considering the influences of initial water contents to the shear yielding and the volumetric yielding of reconstituted clays.

3. THE PROPOSED MODEL CONSIDERING THE EFFECTS OF INITIAL WATER CONTENTS

In this section, a new constitutive model for reconstituted clays considering the effects of initial water contents is derived based on Modified Cam Clay model.

3.1 Reference yield surface

The Modified Cam Clay (MCC) model (Roscoe and Burland 1968) is developed for reconstituted clays and widely referenced to use in simulating mechanical response of natural clays by adding several important modifications representing the main features of natural clays (Alonso et al., 1990; Gajo and Wood, 2001; Liu and Carter, 2002; Wheeler et al., 2003; Baudet and Stallebrass, 2004; Desai et al., 2010;Liu et al., 2010;Uchida et al., 2012). Because it captures well the essential behavior of reconstituted soils (Desai et al., 1986; Muir Wood, 1990; Yu, 1998), the MCC model is chosen as the basis for this research. Hence, in this paper a constitutive model based on the main concept of critical state model framework is derived to simulate the effects of initial water contents to mechanical behaviors of reconstituted clays.

In this section, a brief introduction will be addressed to present the main features of MCC model. Figure 3 depicts the yield surface and compression behavior in the critical state model framework. The soil becomes elasto-plastic when the mean effective stress p' reaches the preconsolidation stress p'_0 and the volumetric deformation then dramatically changes. This volumetric yielding behavior is usually controlled by two parameters: κ is the slope of the swelling line, which defines the soil in the elastic region; λ is the slope of the normal compression line, which shows the soil behave in the elasto-plastic region. One of the major benefits of MCC model is its ability to capture volume changes more realistically for clays.

During the triaxial shearing stage, the deviator stress q will increase and the soil will behave elastically until the yield surface is reached. The yield surface for MCC model is described by an ellipse (Muir Wood, 1990), given by

$$f = q^2 / M^2 - p'p'_0 + p'^2 = 0 \tag{1}$$

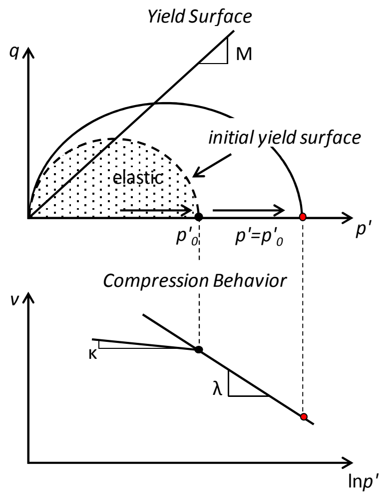


Figure 3 Yield surface and compression behavior of MCC model

This yield surface is shown in Figure 3. Note that the size of the yield surface is determined by the mean preconsolidation pressure and the change law of the size of yield surface (hardening law) is related to plastic volumetric strain. The flow rule of this model is assumed to be associated, and therefore, the plastic potential surface coincides with the yield surface.

Then for the reconstituted clays with certain initial water content which can be represented by MCC model is chosen as reference yield surface. And the initial water content is defined as reference initial water content '$w_0^{\text{reference}}$'.

3.2 Strength change due to initial water contents

By defining the reference yield surface in section 3.1, it is clear that the yield surface of reconstituted clays with different initial water contents will locate differently with reference yield surface as shown in Figure 4. As depicted in Figure 2, with the decrease of initial water contents, the pore pressure decreases while the strength increases, which can be explained by plasticity theory. That sit with the decrease of initial water contents, the reconstituted clay exhibits enhanced strength and dilation. In the proposed model, these feathers are modeled by adding an additional parameter related to initial water contents to the reference yield surface defined by MCC model. Figure 4 illustrates the mechanism of strength enhancement and the mathematical representation of the yield surface expansion due to the decrease of initial water contents in the $q : p'$ space.

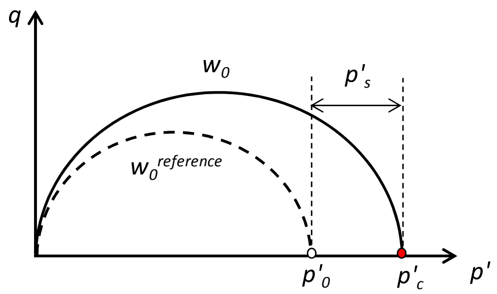


Figure 4 Schematic diagram of yield surface with different initial water contents

The additional hardening parameter ('quasi-structure' strength) p'_S is added to the yield function in such a way that the yield surface of lower initial water contents can expand to the right hand side of the reference one. As a result, enhanced strength will be obtained for reconstituted clays with lower initial water contents, while greater dilation is given by a greater normal angle to the tangent of the surface when the clay yields at a given mean effective stress, which will lead to the less pore pressure response in undrained tests.

It's assumed that the reference yield surface can be represented by MCC yield surface. Then adding the strength enlargement effect due to the decrease of initial water contents to the reference yield surface, Eq. 1 becomes

$$f = q^2 / M^2 - p'(p'_0 + p'_s) + p'^2 \tag{2}$$

where the 'quasi-structure' strength p'_S varies with the initial water content. When the 'quasi-structure' strength decreases towards 0, the yield function will recover to the conventional yield function of MCC model, which describes the mechanical behavior of reference initial water content.

After assuming that the change law of 'quasi-structure' strength p'_S simply relates to and degrades with the shear plastic strain, a simple equation as follow can be formed

$$dp'_s = -\chi p'_s d\varepsilon^p_d \tag{3}$$

where χ is the degradation constant which will vary with initial water contents, $d\varepsilon^p{}_d$ represents the plastic shear strain.

3.3 Yield function

By adding the 'quasi-structure' strength p'_S to the yield surface, the current stress state p'_0 will locate inside the initial yield surface p'_c. As a result, soil will behave elastic at the beginning of shearing according to MCC model. However, it has been widely accepted that irrecoverable plastic strains develop even when the stress state is inside the yield surface (Jardine, 1992; Liu and Carter, 2000; Mitchell and Soga, 2005). Hence, in order to provide a smooth transition from elastic to plastic behavior and more realistic pore pressure response under undrained test condition, a subloading ratio R, expressed as the ratio between current stress state and the size of yield surface (p'_0 / p'_c), is introduced, then the subloading surface becomes (Hashiguchi, 1989),

$$f = q^2 / M^2 - R \times p'(p'_0 + p'_s) + p'^2 \tag{4}$$

$$dR = u_1 (\frac{1}{R^m} - 1) \left\| d\varepsilon^p \right\| \tag{5}$$

where $d\varepsilon^p$ is the plastic strain vector, u_1 and m are the material constants, controlling the plastic deformation while the soil is elastic, which represents the development of plastic strain when the stress state is inside the yield surface. dR>0 represents the plastic state and dR<0 when the soil is elastic. The material constant u_1 and m will also vary with initial water contents.

By adopting the elastic behavior and associated flow rule from critical state theory, a constitutive model incorporating the effects of initial water contents can be derivate and the full details of the derivation of the proposed constitutive model is presented in Appendix A.

4. DEVELOPMENT OF INPUT PARAMETERS

Eight parameters are required to define the proposed model completely in this study: M, λ, κ, υ', p'_S, χ, R, u_1 or m. The value of M, λ, κ, υ' are standard parameters used in the MCC model (Wood, 1990). Additional hardening parameters p'_S and its degradation constant χ are introduced herein to describe the effects of initial water contents on the mechanical behavior of reconstituted clays. The subloading ratio R and evolution parameter u_1 or m represent the plastic strain effects inside the yield surface.

Hong et al. (2013) performed a series of undrained triaxial tests on reconstituted clays with various initial water contents. Each sample was normally consolidated samples and applied four confined consolidation pressure (e.g. 25 kPa; 50 kPa; 100 kPa; 200 kPa) before strain controlled shearing. These experimental data will be used to develop the input parameters. Table 1 summarizes the soil parameters for the proposed model.

4.1 Critical state input parameters

λ is the slope of isotropic compression line in $v : \ln p'$ space. There are four different isotropic consolidation pressures for each reconstituted sample. Then λ can be derived from the linearity of these four points in $v : \ln p'$ space.

κ is the slope of swelling line. However the swelling test did not be conducted, then the slop of swelling κ is calculated based on the empirical relationship between λ and κ, which is κ =(1/3~1/4) λ (Wood, Mackenzie, and Chan, 1992). In this study, $\kappa = 0.25\lambda$ is used.

Assuming a typical value of Poisson's ratio for clay material equals to 0.3. The values of λ, κ and υ' at reference initial water content are used for all other samples in the proposed model during calculation.

M is the critical state stress ratio, which can be determined by conventional triaxial test. Table 1 shows that with the increase of initial water contents, the value of M decreases. In this study, the experimentally measured M at different initial water contents is used for each sample.

4.2 Model-specific input parameters

a. The 'quasi-structure' strength p'_S and degradation constant χ

Figure 5 illustrates the mechanism of how to determine the 'quasi-structure' strength based on initial water contents. As shown in Figure 5, it is assumed that the current stress state p'_0 of lower initial water contents lie on the swelling line of reference initial water content. Then the initial yield stress p'_c for lower initial water contents will be larger than that of reference initial water content p'_0, where the difference between p'_c and p'_0 can be defined as the 'quasi-structure' strength p'_S. In this study, initial water content w_0=156.4% for Wenzhou clay and initial water content w_0=113.9% for Kemen clay is chosen to be the reference initial water content, then the 'quasi-structure' strength used in this study is summarized in Table 2. It can be seen that the 'quasi-structure' strength p'_S decreases with the increase of initial water contents and becomes 0 when the initial water content equals to reference initial water content

Table 1 Summary of input parameters in this study

Soil	Initial water content (%)	Critical state input parameters				Model-specific input parameters			
		M	λ	κ	v'	χ	R_0	m	u_1
Wenzhou clay	156.4 #	0.95	0.238	0.06	0.30	-	-	-	-
	135.5	1.04				200	0.74	0.5	200
	118.5	1.09				500	0.68	0.5	500
	100.1	1.16				2000	0.57	0.5	2000
Kemen clay	113.9 #	1.05	0.170	0.043	0.30	-	-	-	-
	98.0	1.11				500	0.76	0.5	500
	83.4	1.17				1000	0.68	0.5	1000
	70.4	1.24				2000	0.57	0.5	2000
	63.0	1.29				3000	0.53	0.5	3000

Note: # represents the reference initial water content

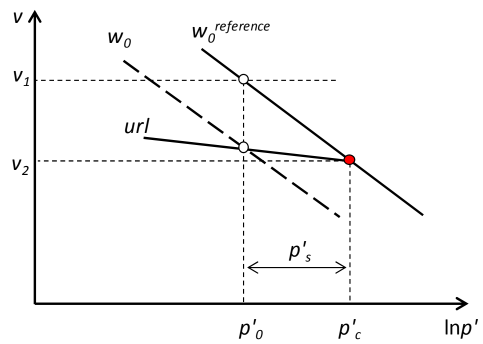


Figure 5 Determination of ‘quasi-structure’ strength

The degradation constant χ is determined by fitting the experimental data, which varies with initial water contents as well.

Table 2 ‘quasi-structure’ strength of reconstituted clays at different initial water contents

Confine pressure p'_0 (kPa)		25	50	100	200
Sample	Initial water content (%)	quasi structure strength p'_s (kPa)			
Wenzhou clay	156.4 #	0	0	0	0
	135.5	9	18	34	66
	118.5	12	24	47	90
	100.1	20	39	75	143
Kemen clay	113.9 #	0	0	0	0
	98.0	8	16	31	61
	83.4	12	24	48	93
	70.4	20	38	74	139
	63.0	24	47	88	165

b. Initial subloading ratio R_0 and evolution parameter m or u_1

R_0 represents the relative distance between initial stress state and initial yield surface, which can be expressed as ($=p'_0$ / p'_c). As shown in Table 1, with the increase of initial water contents, R_0 also increases and the initial subloading surface will gradually close to initial yield surface. The evolution parameter m or u_1 is also determined by fitting the experimental data, which changes with initial water content.

5. COMPARISONS BETWEEN EXPERIMENTAL DATA AND COMPUTED RESPONSES

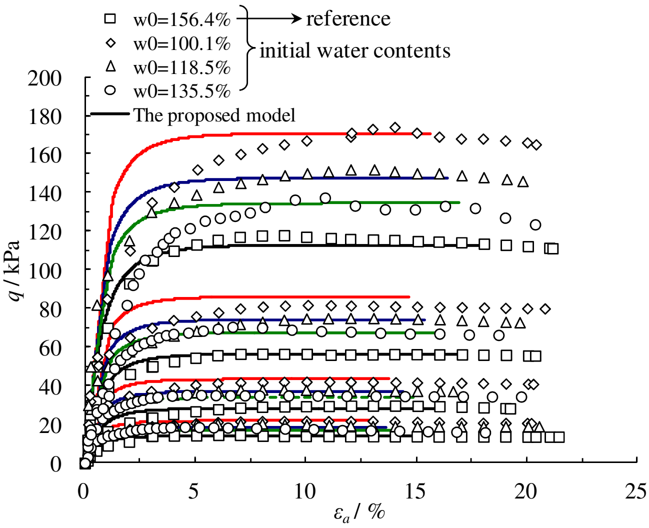
The comparisons between the measured values of stress–strain curves, pore pressure variations and effective stress paths during shearing stage in triaxial tests and the values predicted by the proposed model are plotted in Figures 6, 7 and 8, respectively. In these Figures, dot points represent the experimental data, while the normal lines represent the computed results of the proposed model.

Since initial water content w_0=156.4% for Wenzhou clay and initial water content w_0=113.9% for Kemen clay is chosen to be the reference initial water content. Then the MCC model is used to simulate the mechanical behavior of reconstituted clay at reference initial water content. It is clear that the effective stress path and stress-strain-pore pressure relationship at reference initial water content can be represented precisely by MCC model.

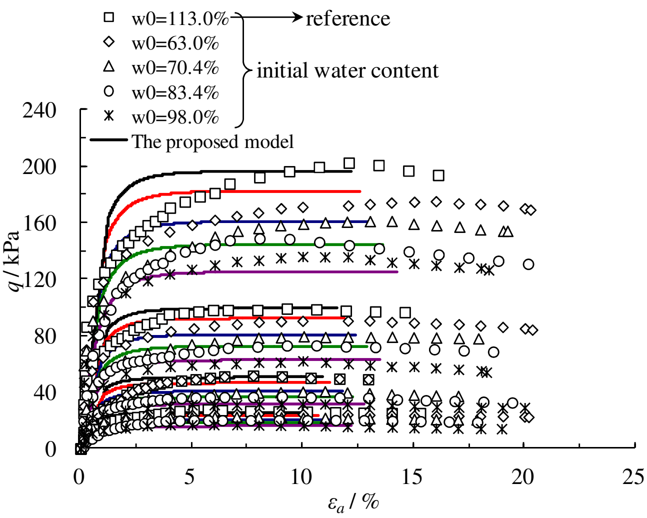
Figures 6, 7 and 8 show a significant correlation between predicted values by the proposed model and the experimental data. The stress-strain curves in Figure 6 predicted by the proposed model are able to capture the overall trend at different initial water contents for both Wenzhou clays and Kemen clays. More importantly, the proposed model predicts the ultimate undrained strength at large strain precisely for all reconstituted clays at different initial water contents.

The ultimate values of the pore pressure response in Figure 7 are predicted relatively close by the proposed model for both Wenzhou clays and Kemen clays. However, it seems that the proposed model over-estimates the increase rate of pore pressure at the beginning stage of shearing. It should be pointed out that the pore pressure transducer is embedded in the bottom of specimens during triaxial test. During the shearing stage, the measured pore pressure tends to be captured by transducer in the bottom with delayed effect. This phenomenon may cause the difference between computed and experimental data curves.

It is evident in Figure 8 that due to the increase of undrained strength and decrease of pore pressure response, the computed effective stress path at lower initial water contents lie on the right of that at higher initial water content and become more nearly vertical. It is noteworthy that the computed effective stress paths of all reconstituted clays can simulate the experiment effective stress paths reasonably well. These comparison results imply that it is sufficient to simulate the strength enhancement due to initial water contents using the ‘quasi-structure’ strength technique.



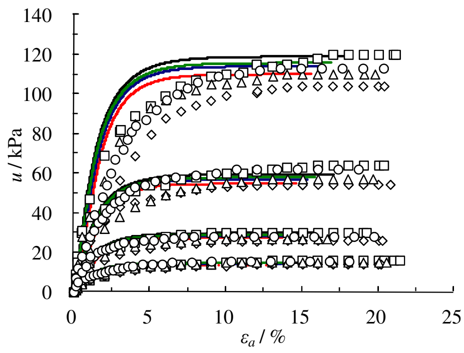
(a) Wenzhou clays



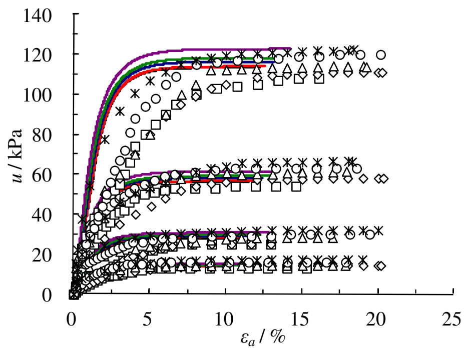
(b) Kemen clays

Figure 6 Stress-strain curves during shearing at different initial water contents

The evaluation parameter m or u_l used as the best fit curves in Figures 6, 7 and 8 will decrease with the increase of initial water contents. This illustrates that the proportion of plastic strain accumulating within elastic region will increase with the increase of initial water contents, resulting in full plastic at reference initial water content. The degradation constant χ of reconstituted clays shows a decreasing tendency with the increase of initial water contents. This implies that the 'quasi-structure' strength effect of reconstituted clays degrades more rapidly at lower initial water contents under shearing.

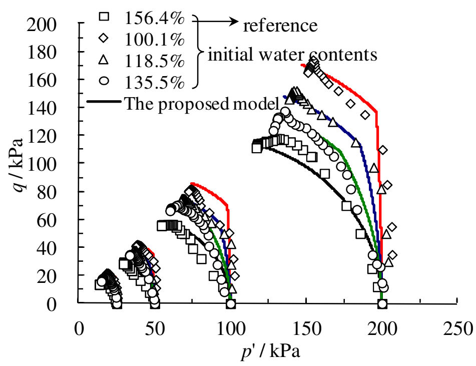


(a) Wenzhou clays

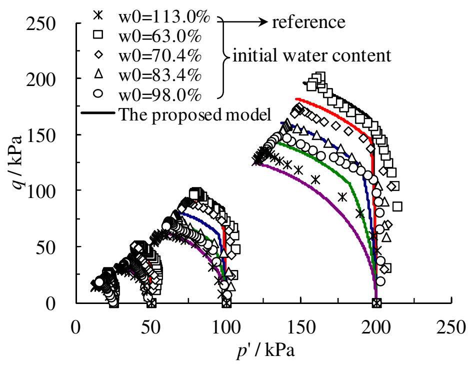


(b) Kemen clays

Figure 7 Pore pressure-strain curves during shearing at different initial water contents



(a) Wenzhou clays



(b) Kemen clays

Figure 8 Effective stress paths during shearing at different initial water contents

6. CONCLUSIONS

In this paper, a new constitutive model for reconstituted clays considering the effects of initial water content is proposed, and the model incorporates the enhanced strength, stiffness and dilation due to decrease of initial water contents. The 'quasi-structure' strength is introduced and added to the yield function of MCC model to simulate those features.

By comparing with undrained triaxial test results, it is found that the proposed model can capture the overall pattern of stress-strain curves, pore pressure-strain curves and effective stress paths of reconstituted clays at various initial water contents reasonably well. In addition, the ultimate undrained strength and pore pressure response at different initial water content can be predicted precisely by the proposed model.

7. ACKNOWLEDGEMENTS

This study is supported by the National Natural Science Foundation of China (Grant No. 41372309; No. 41172240). In addition, a grant from the Scientific Research Foundation of Graduate School of Southeast University in support of this study is also gratefully acknowledged.

8. REFERENCES

Alonso, E., Gens, A., and Josa, A. (1990). "A constitutive model for partially saturated soils." Géotechnique, 40(3), 405-430.

Baudet, B., and Stallebrass, S. (2004). "A constitutive model for structured clays." Géotechnique, 54(4), 269-278.

Burland, J. (1990). "On the compressibility and shear strength of natural clays." Géotechnique, 40(3), 329-378.

Chandler, R. (2000). "Clay sediments in depositional basins: the geotechnical cycle." The Quarterly Journal of Engineering Geology and Hydrology, 33(1), 7-39.

Cerato, A., and Lutenegger, A. (2004). "Determining Intrinsic Compressibility of Fine-Grained Soils." Journal of geotechnical and geoenvironmental, 130(8), 872-877.

Cotecchia, F., and Chandler, R. (2000). "A general framework for the mechanicalal behaviour of clays." Géotechnique, 50(4), 431-447.

Gajo, A., and Wood, D. (2001). "A new approach to anisotropic, bounding surface plasticity: general formulation and simulations of natural and reconstituted clay behaviour." International Journal for Numerical and Analytical Methods in Geomechanicals, 25(3), 207-241.

Gens, A. (2010) "Soil–environment interactions in geotechnical engineering." Géotechnique 60(1), 3-74.

Desai, C. S., Somasundaram, S., and Frantziskonis, G. (1986). "A hierarchical approach for constitutive modelling of geologic materials." International Journal for Numerical and Analytical Methods in Geomechanicals, 10(3), 225-257.

Desai, C. S., Sane, S., & Jenson, J. (2010). Constitutive modeling including creep-and rate-dependent behavior and testing of glacial tills for prediction of motion of glaciers. International Journal of Geomechanics, 11(6), 465-476.

Hashiguchi, K. (1989). "Subloading surface model in unconventional plasticity." International Journal of Solids and Structures, 25(8), 917-945.

Hong, Z., Yin, J., and Cui, Y. (2010). "Compression behaviour of reconstituted soils at high initial water contents." Géotechnique, 60(9), 691-700.

Hong, Z., Zeng, L., Cui, Y., and Cai, Y. (2012). "Compression behaviour of natural and reconstituted clays." Géotechnique, 62(4), 291-301.

Hong, Z., Bian, X., Cui, Y., Gao, Y., and Zeng, L. (2013). "Effect of initial water content on undrained shear behaviour of reconstituted clays." Géotechnique, 63(6), 441–450.

Jardine, R. (1992). "Some observations on the kinematic nature of soil stiffness." Soils and Foundations, 32(2), 111-124.

Leroueil, S., Tavenas, F., Rochelle, P. L., and Roy, M. (1979). "Behavior of destructured natural clays." Journal of geotechnical engineering, 105(6), 759-778.

Leroueil, S., Tavenas, F., and Locat, J. (1985). Discussion on "Correlations between index test and the properties of remoulded clays." Géotechnique, 35(3), 223-226.

Leroueil, S. (1990). "The general and congruent effects of structure in natural soils and weak rocks." Géotechnique, 40(3), 467-488.

Liu, M. D. & Carter, J. P. (1999). "Virgin compression of structured soils." Géotechnique, 49(4), 43-57.

Liu, M. D., & Carter, J. P. (2000). "Modelling the destructuring of soils during virgin compression." Géotechnique, 50(4), 479-483.

Liu, M., and Carter, J. (2002). "A structured Cam Clay model." Canadian Geotechnical Journal, 39(6), 1313-1332.

Liu, M. D., Carter, J. P., & Airey, D. W. (2010). Sydney soil model. I: Theoretical formulation. International Journal of Geomechanics, 11(3), 211-224.

Mitchell, J., and Soga, K. (2005). Fundamentals of soil behavior. John Wiley & Sons, Hoboken, N. J.

Roscoe, K., and Burland, J. (1968). On the Generalized Stress-strain Behaviour of Wet Clay. Engineering, Engineering Plasticity, Cambridge University Press, 535-609.

Uchida, S., Soga, K., and Yamamoto, K. (2012). "Critical state soil constitutive model for methane hydrate soil." Journal of geophysical research, 117(B03209).

Wheeler, S., Naatanen, A., Karstunen, M., and Lojander, M. (2003). "An anisotropic elastoplastic model for soft clays." Canadian Geotechnical Journal, 40(2), 403-418.

Wood, D. (1990). Soil behaviour and critical state soil mechanicals. Cambridge University Press.

Wood, D., Mackenzie, N., and Chan, A. (1992). Selection of parameters for numerical predictions. In Predictive Soil Mechanicals: Proceedings of the Wroth Memorial Symposium, Oxford, UK. Thomas Telford, London. pp. 496–512.

Wroth, C. P. & Wood, D. M. (1978). The correlation of index properties with some basic engineering properties of soils. Canadian Geotechnical Journal. 15(2), 137–145.

Yu, H. (1998). "CASM: A unified state parameter model for clay and sand." International Journal for Numerical and Analytical Methods in Geomechanicals, 22(8), 621-653.

Zeng, L., Hong, Z., and Cui, Y. (2014). "Modelling Virgin Compression of Reconstituted Clays with Different Initial States." Géotechnique, submitted.

APPENDIX A: DERIVATION OF THE CONSTITUTIVE LAW

The yield surface considering the influences of initial water contents is adopted in this study. The subloading surface is described in $q : p'$ space is

$$f = q^2 / M^2 - R \times p'(p'_0 + p'_s) + p'^2 \tag{A1}$$

where p' and q are effective mean stress and deviator stress respectively, p'_0 is current effective stress, p'_S represents the 'quasi-structure' strength accounting the effects of initial water contents, and R represents the subloading ratio.

Assuming that current stress point lies on the subloading surface then:

$$f(\sigma', p'_0, p'_s, R) = 0 \tag{A2}$$

The yield function of the proposed model contains 5 variables: effective stress σ′ (p' and q)、p'_0、p'_S and R. Then the consistency equation becomes

$$df = \frac{\partial f}{\partial \sigma'} d\sigma' + \frac{\partial f}{\partial R} dR + \frac{\partial f}{\partial p'_0} dp'_0 + \frac{\partial f}{\partial p'_s} dp'_s = 0 \tag{A3}$$

$$\frac{\partial f}{\partial \sigma'} = \frac{\partial f}{\partial p'} \frac{\partial p'}{\partial \sigma'} + \frac{\partial f}{\partial q} \frac{\partial q}{\partial \sigma'} \tag{A4}$$

In critical state models, the stress-dependent elastic soil bulk modulus K' is given by (Roscoe and Burland, 1968)

$$K' = \frac{v}{\kappa} p' \tag{A5}$$

where v is the specific volume (=1+e), κ is the slope of swelling line and p' is the mean effective stress. Whilst the shear modulus G' can be derived from bulk modulus given by

$$G' = \frac{3(1-2\upsilon')K'}{2(1+\upsilon')} \tag{A6}$$

where υ' is the effective poison ratio.

Then the elastic stiffness matrix D^e can be written as

$$D^e = \begin{bmatrix} K' & 0 \\ 0 & 3G' \end{bmatrix} \tag{A7}$$

It is assumed that the deformation of clay can be decomposed into elastic and plastic components given by

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \tag{A8}$$

And the elastic constitutive law:

$$d\sigma = D^e d\varepsilon^e \tag{A9}$$

The associated flow rule can be described as:

$$d\varepsilon^p = \Lambda \frac{\partial g}{\partial \sigma'} = \Lambda \frac{\partial f}{\partial \sigma'} \tag{A10}$$

And conventional hardening rule used by many critical state models is adopted in this study to describe the change law of the size of yield surface at reference initial water content:

$$dp'_0 = \frac{1+e}{\lambda - \kappa} p'_0 d\varepsilon^p_v \tag{A11}$$

The change law of 'quasi-structure' strength p'_S is

$$dp'_s = -\chi p'_s d\varepsilon^p_d \tag{A12}$$

Adopting the definition of dR from Hashiguchi (1989),

$$dR = u_1 (\frac{1}{R^m} - 1) \left\| d\varepsilon^p \right\| \tag{A13}$$

By substitute Equations A8and A9 into Equation A10 gives:

$$d\sigma = D^e \left[d\varepsilon - \Lambda \frac{\partial f}{\partial \sigma'} \right] \tag{A14}$$

Substituting Eqs. A3, A11, A12, A13 and A14, the plastic multiplier Λ can be expressed as:

$$\Lambda = \frac{\left(\frac{\partial f}{\partial \sigma'}\right)^T D^e d\varepsilon}{\left(\frac{\partial f}{\partial \sigma'}\right)^T D^e \left(\frac{\partial f}{\partial \sigma'}\right) - \left(\frac{\partial f}{\partial R}\right) u_1 \left(\frac{1}{R^m} - 1\right) \left\| \frac{\partial f}{\partial \sigma'} \right\| - \zeta^{p'_0} + \zeta^{p'_s}}$$

Where

$$\zeta^{p'_0} = \left(\frac{\partial f}{\partial p'_0}\right) \frac{1+e}{\lambda - \kappa} p'_0 \left(\frac{\partial \varepsilon^p_v}{\partial \varepsilon^p}\right)^T \left(\frac{\partial f}{\partial \sigma'}\right) \tag{A16}$$

$$\zeta^{p'_s} = \left(\frac{\partial f}{\partial p'_s}\right) \chi p'_s \left(\frac{\partial \varepsilon^p_d}{\partial \varepsilon^p}\right)^T \left(\frac{\partial f}{\partial \sigma'}\right) \tag{A17}$$

Then the plastic stiffness matrix can be written as,

$$d\sigma_{ij} = \left(D^e - \frac{\left(\frac{\partial f}{\partial \sigma'}\right)^T D^e d\varepsilon \left(\frac{\partial f}{\partial \sigma'}\right)}{\left(\frac{\partial f}{\partial \sigma'}\right)^T D^e \left(\frac{\partial f}{\partial \sigma'}\right) - \left(\frac{\partial f}{\partial R}\right) u_1 \left(\frac{1}{R^m} - 1\right) \left\| \frac{\partial f}{\partial \sigma'} \right\| - \zeta^{p'_0} + \zeta^{p'_s}} \right) d\varepsilon$$

(A18)