

Overview and Interpretation of Stress-Relaxation of Soft Clay

L. Ye¹, Q. Y. Zhu², J. X. Liu³, P. P. Sun⁴ and Z. Y. Yin⁵

¹Department of Civil Engineering, Zhejiang University of Science and Technology, Hangzhou 310012, China

²State Key Laboratory for Geomechanics & Deep Underground Engineering, China University of Mining & Technology, Xuzhou, China

^{3,5}Department of Geotechnical Engineering, Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Tongji University, Shanghai 200092, China. LUNAM University, Ecole Centrale de Nantes, Nantes, France

⁴Department of Civil Engineering, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China

⁵E-mail: zhenyu.yin@gmail.com

ABSTRACT: Extensive laboratory tests and field observations show that soft clay exhibits significant stress relaxation characteristics. The evolution of pore pressure and stress of soft clays under both 1D and 3D stress relaxation tests was first studied based on published data from the view of the influence of strain rate and strain level prior to the relaxation phase. The relationship between parameters related to stress relaxation and liquid and plastic limits were discussed. Then, the stress relaxation behavior of soft clays under complex stress conditions was also analyzed. Furthermore, based on stress relaxation curves on double logarithmic plane, the stress relaxation coefficient was originally proposed, and an analytical solution of 1D stress relaxation was derived, which unified three time-dependent characteristics with their key parameters. Finally, the stress relaxation characteristics of soft clay were investigated from the aspect of stress-dilatancy of Hong Kong Marine clay under triaxial extension and stress relaxation conditions with some typical stress-dilatancy equations.

KEYWORDS: Soft clay, Stress relaxation, Rate-dependency, Stress-dilatancy

1. PREFACE

With the characteristics of high clay content, high water content and high void ratio etc., soft clay has a significant mechanical property of rheology (i.e. creep, stress relaxation and loading rate-dependency). Stress relaxation has close relationships with loading rate and creep, but it also has self-properties under different stress and strain condition. For example, stresses between structures are becoming weak with time. In reality, many structures likely subways, tunnels and underground-square are constructed underground and the structures or soil are likely to become unstable when lateral pressure lowers certain value caused by stress relaxation. So, to be able to provide safe and economy guides for constructing, the study of stress relaxation properties of soft clay becomes necessary.

In the past few years, many national scholars have done a series of test to study stress relaxation of soft soil such as 1D stress relaxation test and 3D drain and undrain stress relaxation test even stress relaxation test under complex stress. Augustesen summarized the previous study on stress relaxation then only simply pointed out their consensus but didn't do a deep study. So far, the study of soft soil stress relaxation is as follow and still has some shortcomings:

- (1) Existing studies are generally based on stress relaxation phase to study the relationship between stress and time, while stress reducing rate, the real starting time of stress relaxation and loading rate before the start of stress relaxation occurs are rarely mentioned.
- (2) Lacking of in-depth discussion of the relevance of the three rheological properties of soft clay and its key parameters (stress relaxation, creep, loading rate-dependency), and the applicability of the stress relaxation tests are limited.
- (3) Stress-dilatancy is an important mechanical properties of soil, is one of the foundations of soil constitutive relation. However, very few scholars conducted a depth research on stress-dilatancy characteristics and influencing factors of soft soil stress relaxation.

Based on the above issues, the authors believe it is necessary to do a review of existing test results and an in-depth analysis (Figure 1). Firstly, we summarize the study results of soft soil stress relaxation systematically by series tests under 1D to 3D and complex stress path condition. Based on stress relaxation curves

under double logarithmic coordinates, the stress relaxation coefficient was originally proposed, and an analytical solution of 1D stress relaxation was derived, which unified three time-dependent characteristics with their key parameters. Finally, the stress relaxation characteristics of soft clay were investigated from the aspect of stress-dilatancy of Hong Kong Marine clay under triaxial extension and stress relaxation conditions with some typical stress-dilatancy equations.

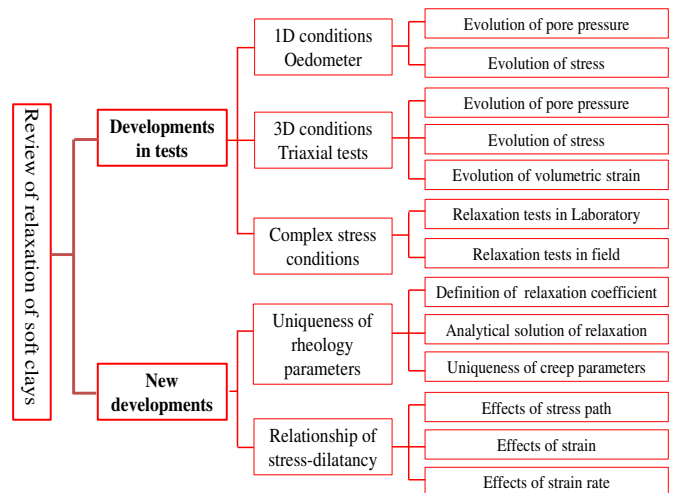


Figure 1 Schematic illustration of contents

2. REVIEW OF STRESS RELAXATION PROPERTIES

Stress relaxation is the phenomenon that stress continuously attenuates under a constant of soil deformation with time. Figure 2 is a schematic of a typical soil stress relaxation tests curve: point A to B is the stress load paths at a certain compression or shear rate. When the soil deforms to point A, stress relaxation tests was started (Figure 2a) and strain maintain a constant (Figure 2b), the soil stress decreases with time, (Figure 2c). In this section, the clay stress relaxation characteristics under different stress conditions are summarized: 1D compression, drained and undrained 3D compression and unconventional complex stress.

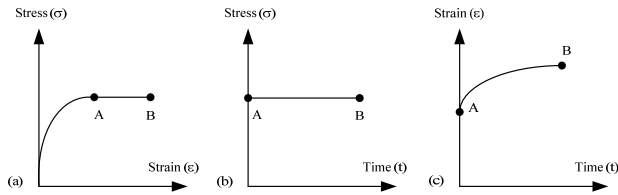


Figure 2 Relaxation test (A→B): (a) Stress-strain relationship; b) strain history; (c) stress history

2.1 One-dimensional stress relaxation test

One-dimensional stress relaxation test is under closed drainage condition in a one-dimensional oedometer, setting vertical deformation of the specimen maintaining constant to control displacement boundary, and measuring pore water pressure directly during the test, and then getting its relationship with time to study one-dimensional stress relaxation properties of soils. Based on the previous one-dimensional stress relaxation tests, we do this research mainly for a brief description of the following two aspects: (1) stress relaxation process of pore pressure variation; (2) the stress variation.

2.1.1 Pore pressure variation

Experimental results show that in the one-dimensional consolidation test, the effective stress reduced significantly after closing drainage conditions whether in primary or secondary consolidation phase. In the 1D stress relaxation tests, Yoshikuni et al.^[1] set vertical effective stress vary to 341 kPa with three different strain rates firstly (Figure 3a), and closed drainage conditions after the primary consolidation phase to observe the variation of pore water pressure with time (Figure 3b). Test results show that the pore water pressure increases gradually after the start of stress relaxation and the greater of the compression strain rates before the start of stress relaxation, the greater of the excess pore water pressure during the stress relaxation process.

It should be noted that, as a result of relaxation tests cannot be directly applied to construction engineering as soil creep and strain rate-dependency which are used commonly in practice. So compared with soil creep and strain rate-dependency, the current studies of soil stress relaxation are rare, and less in the one-dimensional test. If the relationship between stress relaxation and creep or strain rate-dependency is found (see the second part of this article), the stress relaxation tests can be promoted widely.

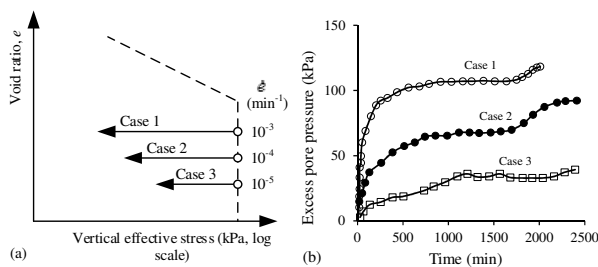


Figure 3 Excess pore-pressure generation during relaxation oedometer tests

2.1.2 Stress variation

In the one-dimensional undrained stress relaxation test, pore water pressure grew gradually. Accordingly, the effective stress of soil decreased (Figure 4a). Yin and Graham^[2] did the one-dimensional stress relaxation tests with reshape Erie soil, and found that vertical effective stress relaxation decreased rapidly at the beginning of the relaxation start, after 500min, the decreasing rate was slowing. Kim and Leroueil^[3] got the similar conclusions when tested Berthierville clay (Figure 4b). We can also get the stress relaxation law by one-dimensional compression test apparatus which could control vertical displacement and measure stress changes directly.

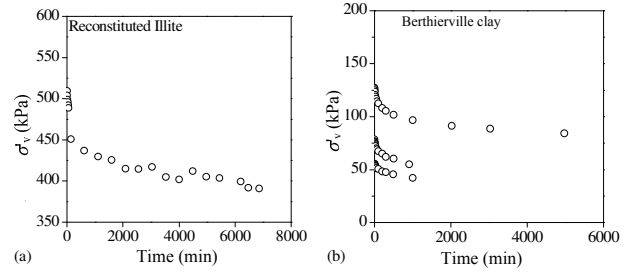


Figure 4 Effective stress versus time during relaxation oedometer tests

2.2 Triaxial stress relaxation tests

In triaxial stress relaxation test is also a test to study the soil properties under the condition of triaxial stress. It is needed to keep the confining pressure constant, first shear the specimen under a specific loading rate to predetermined initial strain and then control displacement boundary by fixing vertical displacement, at last the vertical stress and the pore water pressure under undrained tests could be measured directly or the volumetric strain under the condition of the drainage, eventually we can get their changing with the time.

With respect to the one-dimensional stress relaxation tests, soil triaxial stress relaxation tests can set different levels of lateral stress and perform under various stress paths (different K_0 , overconsolidated different degrees, different confining pressure, etc.) relaxation test. So triaxial stress relaxation tests is more in line with the actual needs of the project. Based on existing triaxial stress relaxation test results, the following issues are discussed which similar to research methods to of the one-dimensional stress relaxation. (1) stress variation (2) pore pressure variation under undrained conditions (3) strain variation under drainage condition.

2.2.1 Stress variation

The scholars have done a lot of efforts to study the stress variation during stress relaxation. (Table 1 [4-5, 7-8, 14-15, 24]). Lacerda & Houston did some tests on SFBM clay specimens with three different loading rates. Results showed that all of the stress relaxation curve shape are very similar, and the stress variation with time can be divided into two stages, stress decreased fast in first stage and slowly in second stage (Figure 5a), whereas in the plane of stress and the time (logarithmic coordinate system), the variation also could be divided into two stages, the stress with little attenuation at the first stage of, and then the stress varied linearly with logarithm of the time in second stage (Figure 5b). So far, the relationship between normalized stress q/q_0 and $\log(t)$ is mainly researched in the stress relaxation phase.

$$\frac{q}{q_0} = 1 - s \log\left(\frac{t}{t_0}\right), \quad t > t_0 \quad (1)$$

where, q is deviatoric stress, q_0 is the initial value of q at the beginning of stress relaxation, t_0 is initial equivalent time of stress relaxation whose value is the intercept on $\log t$ axis when extending the linear portion of relationship lines, s represent the stress decreasing rate with time, is the slop of the lines (Figure 5b).

' s ' and ' t_0 ' are two most important parameters in studying the stress relaxation. As for the relationship between ' s ', ' t_0 ' and soil properties, stress state, loading rate before stress relaxation and axial strain, different scholars have different understandings. For example, Sheahan thought that ' s ' is the inherent characteristics of the soil, and has nothing to do with the strain rate, strain value and OCR. While Murayama and Shibata^[24] thought that ' s ' is relevant to the strain value when relaxation starts. Oda et al.^[7] did series stress relaxation tests with four reshaped clay by numbers of loading rates,

and results show that ' s ' is related with loading rate before relaxation. In comparison, scholars study the characteristic ' t_0 ' less. A more general view is that the greater the loading rates before stress relaxation, the smaller the ' t_0 ', but lack of a quantitative description of the change principal and influencing factors^[14,15,25].

Table 1 Physical properties and test types for selected clays

Samples	type	w_L /%	w_P /%	I_P /%
Hongkong ^[15]	CU	60	32	28
SFBM ^[8]	CU	93	45	48
Kasaoka ^[7]	CU	62	37	25
Hayakita ^[7]	CU	63	33	30
Kucchun ^[7]	CU	81	40	41
Sheahan ^[14]	K_0 CU	45.3	22.0	23.3
Fujinomori ^[24]	CU	43.6	26.1	17.5
Le Flumet ^[5]	CD	38	24	14
Wanzhou ^[4]	CD	34.1	20.1	14

In this paper, the authors use the formula (1) fitted the stress relaxation test results of SFBM clay in Figure 5a, fitting results corresponding to ' s ' and ' t_0 ' are shown in Figure 5b, showed that slope ' s ' and ' t_0 ' both changes with loading rates and strain values before stress relaxation.

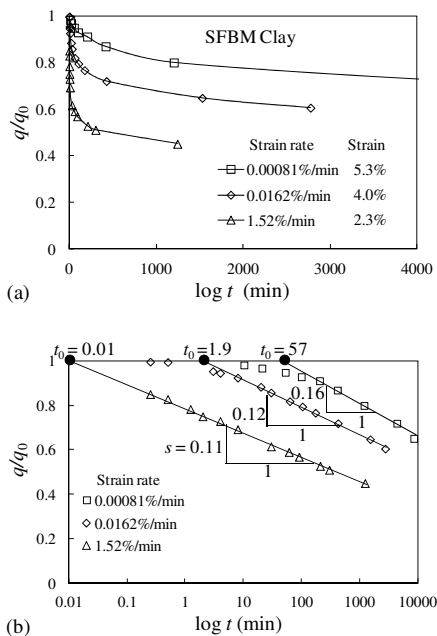


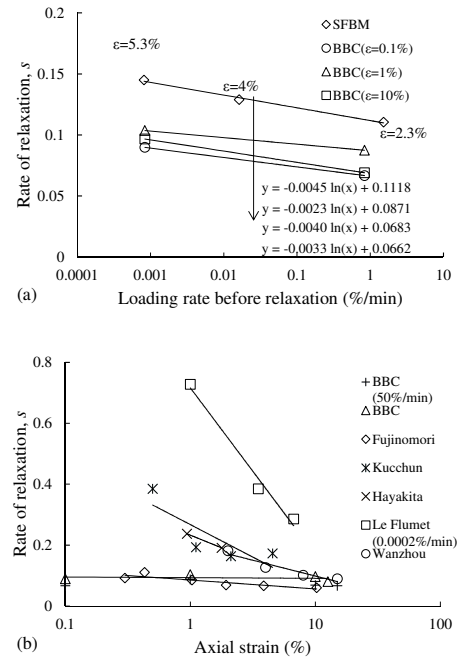
Figure 5 Undrained stress relaxation behavior of SFBM clay

In order to deepen the understanding of the stress relaxation characteristics, the author tried to analyze the factors influencing the stress relaxation rate s and stress relaxation initial time t_0 , based on the available literature experimental data.

(1) The factors affecting the relaxation rate ' s '

So far, undrained stress relaxation tests are few at the same strain value with different initial loading rates, we can only find that Sheahan et [14] did the stress relaxation study with BBC clay. Using equation (1) to analysis the relationship between ' s ' and other factors in stress relaxation tests of two initial loading rates at three different strains of the BBC clay, which shows that the stress relaxation rate s and loading rate before relaxation (in logarithm coordinate) is approximately linear relationship, and the slope of the linear between 0.005 to 0.013.

To study the impact of strain during stress relaxation to ' s ', the authors summarized the stress relaxation properties at different strains while the same loading rates with different clay (Figure 6b), the results show that ' s ' of BBC clay under two different loading rates and Fujinomori clay is hardly affected by vertical strain, while it decreases with the increasing of vertical strain for Kucchun clay, Hayakita clay and Le Flumet clay, and the decreasing rate of Le Flumets is the most rapid which maybe to do with its undrained condition. Above all, stress relaxation rate ' s ' under undrained condition is hardly affected by vertical strain when stress relaxation occurs.

Figure 6 (a)Influence of the strain rate prior stress relaxation to relaxation rate s ; (b) Influence of strain lever to relaxation rate s

(2) The factors affecting the initial equivalent time ' t_0 ' of stress relaxation

With same methods, we use equation(1) to analyze the relationship between ' t_0 ' and other factors, and know that ' t_0 ' is linearly relevant to loading rate (in double logarithm coordinates) as vertical strain remain constant (Figure 7a), and fitting results indicate that slop of fitting lines vary between 1.034 to 1.147. But fitting slop value is 1.119 for SFBM clay, and ' t_0 ' of SFBM clay has nothing to do with the strain. Besides the undrained stress relaxation tests with Le Flumet clay in which ' t_0 ' decreases with strain in double logarithmic scale (slope is 1.133), ' t_0 ' of others are hardly relevant to stress relaxation strain.

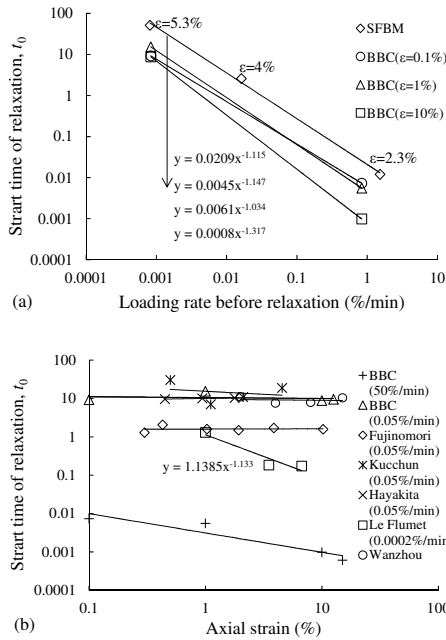


Figure 7 (a) Influence of strain rate prior stress relaxation to t_0 ;
(b) Influence of strain lever to t_0

(3) Relation between s , t_0 and liquid limit and plastic limit

Based on the above study, we remain the loading rates and strain constant, then discuss the relationship between ' s ' and ' t_0 '. The authors selected Hayakita, Kucun, BBC and Fujinomori four clays, and they distributed in CL and OH areas in Casagrande plastic figure. Firstly, setting the loading rate 0.05 %/min and vertical strain 1%, we get a ' t_0 ' and ' s ' accordingly in stress relaxation test, then drew out their relationship with liquid limit and plastic index in Figure 8 which gives a linear fitting equations and regression coefficient R^2 . The results showed that the strain relaxation parameters ' s ' has a certain linear relationship with soil liquid limit and plasticity index while hardly between t_0 and soil liquid limit and plasticity index as shown in Figure 9.

2.2.2 Pore pressure variation under undrained condition

In undrained triaxial stress relaxation tests, the studies of Lacerda and Houston [8], Akai [16], Murayama & Shibata [9] have shown that the pore water pressure hardly changed in the whole process of stress relaxation. While the studies of Silvestri et al [10], Zhu et al [15] and Sheahan [14] had shew that a small excess pore water pressure occurs during the tests, for example, excess pore pressure was -0.5%~4.2% of confining pressure in stress relaxation of Hong Kong clay. In addition, the research results of Oda & Mitachi [7] and Sheahan [14] showed that when the loading rate before stress relaxation exceeds 50% /h, it will produce a large excess pore water pressure during the stress relaxation process. Actually, there are many factors like strain rate, strain and stress state before the start of the stress relaxation (compression or elongation test) affecting the excess pore water pressure in the stress relaxation tests. However, there is no certain conclusion to explain the mechanism of the changes of pore water pressure during stress relaxation tests.

2.2.3 Volumetric variation under drained condition

As the soil samples are saturated, pore water can enter or exit samples freely in tests, and the volumetric change of pore equals to the volumetric change of soil samples. The results showed that [4-5], there is almost no change of the volumetric of the soil in the relaxation process, which is consistent with the conclusion "the pore water pressure remains almost fixed value in undrained stress relaxation tests". Similarly, the strain rate, strain and stress state

before the start of stress relaxation in drained conditions will also affect the volumetric variation. However, the actual affecting laws are still unclearly.

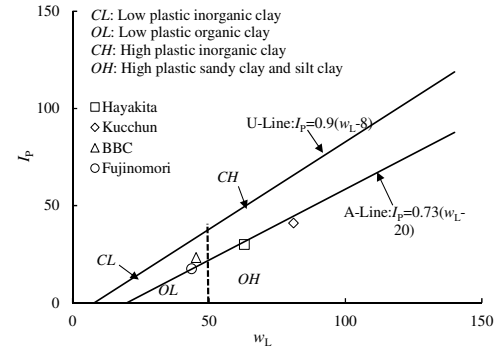


Figure 8 Classification of selected soils in plasticity chart

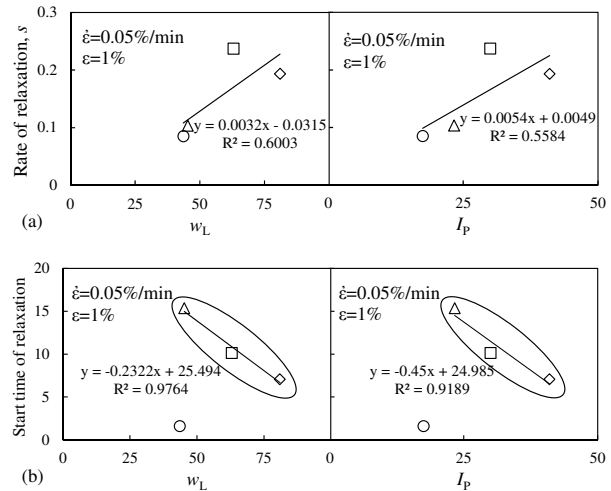


Figure 9 Relationship between s , t and w_L , I_P

2.3 Unconventional stress relaxation tests

In actual project practice, soil suffered a more complicated stress state than that in 1D or 3D tests. Therefore, it is necessary to do some unconventional stress relaxation tests with some unideal units, like unconventional laboratory tests, field tests etc.

2.3.1 Laboratory lateral pressure test

Comparing with triaxial tests, there are fewer examples using unconventional apparatus to do unconventional soil stress relaxation. Laboratory lateral pressure tests one of the most typical tests. In lateral pressure tests, we insert the cylindrical lateral pressure measuring device into soil vertically, and inflate the lateral film by increasing the pore pressure in vertical vessel, and then the pressure of film will be transferred to the lateral soil to produce strain of lateral soil. We can obtain the parameters of underground foundation soil by measuring the soil pressures and deformations. In order to control the boundary conditions and soil uniformity more effectively, Hicher's team developed an indoor lateral pressure test apparatus [19, 26-27] (Figure 10). The instrument can imitate the lateral pressure conditions in triaxial pressure chamber. It has the functions of monitoring the development of pore water in the hole wall and the development of lateral pressure under the condition fixing the lateral displacement. Yin and Hicher [19] did a series stress relaxation test use the instruments shown in Figure 10, when the displacement of tunnel wall is 3.5% of hole initial radius δ_{ra} , the stress relaxation tests were started (Figure 11), all the stress relaxation tests process last about 2×10^5 s. As can be seen from the figure, the lateral pressure has a linear relationship with time in logarithmic axial,

which is consistent with the results of triaxial stress relaxation, in addition, the pore water pressure in hole wall is gradually reduced from 62 kPa to 57 kPa during stress relaxation stage, and then stabilized gradually.

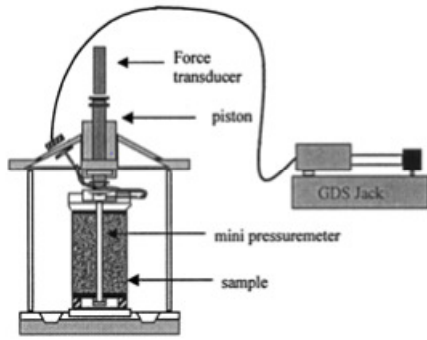


Figure 10 Modified pressio-triax apparatus

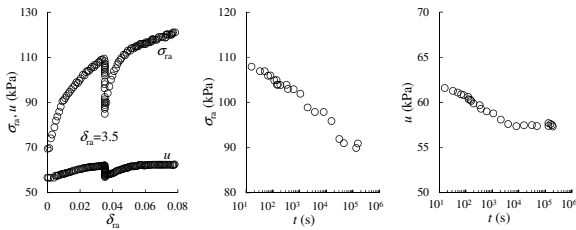


Figure 11 Experimental results of pressure meter stress relaxation test

2.3.2 Field tests

Because the field tests need a strict tests environment and large human and material supplements, the cases of field tests are rather less. To explore the stress relaxation properties of soil in site, Sun Jun excavated a hole of 11m length, 2m high and 3m span beside a railway tunnel as a hole for experimental[28], the test loading system by screw jacks and measuring ring composition, the carrier board area 30cm × 30cm (Figure 12a). Test loading system was composed of screw jacks and the load rings, the area of the carrier plate was 30cm × 30cm (Figure 12a). During the experiment, carrier plate was pressed into the ground, when the deformation reaches y_0 , stop pulling the jack, and from this point, they recorded the rings' readings. Figure 12b show the changes of elastic resistance versus time after the start of relaxation, the results show that elastic resistance decreased in the relaxation process gradually, but did not relax reach to zero.

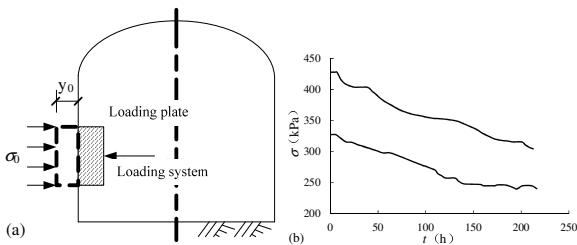


Figure 12 Elastic resistance stress relaxation curve for loess clay

3. THE CORRELATION BETWEEN STRESS RELAXATION AND CREEP AND RATE-DEPENDENCY

Stress relaxation, creep, rate-dependency are reactions of the rheological properties of soil under different stress-strain state. Despite their tests laws can be expressed through the appropriate rheological formulas, the correlation of strain relaxation formulas with the creep and rate-dependency formulas are still not be studied. In addition, it can be found in the literature, the consistency between creep formula and rate-dependency formula has been fully validated [29, 30]. Based on this, an analytical solution of 1D stress relaxation was derived, while we didn't do derivation with triaxial tests as the great complexity of stress-dilatancy.

3.1 Stress relaxation factors

Based on the results of stress relaxation tests (Figure 13) SFBM clay [8], RHKMD [15], Le Flumet clay [5], Saint-Herblain clay [19], reshaped Erie soil [2], Berthierville clay [3]). Stress relaxation coefficient R_α was originally defined in double logarithmic coordinates in which time $\ln(t)$ is linearly with stress $\ln(p)$ after a period of time t_α .

$$R_\alpha = -\frac{\Delta \ln q}{\Delta \ln t} \quad (2)$$

R_α , the slope of the $\ln(q)-\ln(t)$ figure, represents the decreasing rates of stress relaxation with the time. t_α , like t_0 , is the equivalent time of stress relaxation in double logarithmic axis.

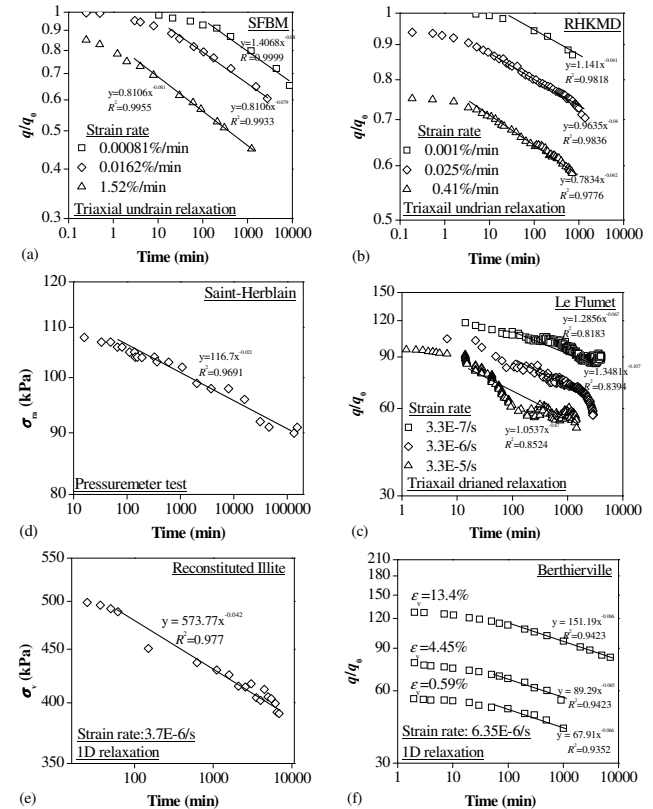


Figure 13 Effective stress versus time during relaxation oedometer tests

By fitting the stress relaxation test results of SFBM clay in Figure 5a with equation (2), we find that unlike t_0 and s , R_α is nearly the same with three different loading rates, while t_α decreases with the loading rates, but bigger than counterpart t_0 . Using a similar method, all the R_α and t_α of different clays in Table 1 were measured. And, the relationship between them and loading rate and strain levels before stress relaxation were studied.

Figure 14a shows that loading rate before the stress relaxation has almost no effect on the R_α . Comparison of Figure 6b and Figure 14b can be seen, vertical strain has a similar effect on the relaxation rate R_α and s . And, the relationship between t_α and loading rates and vertical strain is also similar to that of t_0 . t_α of SFBM clay and BBC clay vary from 0.913 ~ 1.123. We can also find that vertical strain has little effect on t_α from Figure 14b. Additionally, the relationships between R_α , t_α and liquid and plastic limit are drew out in Figure 15, meanwhile the linear fit equation and regression coefficient R2 are showed which indicates R_α , t_α are affected by liquid and plastic limit the same as t_0 and s , but not so obvious.

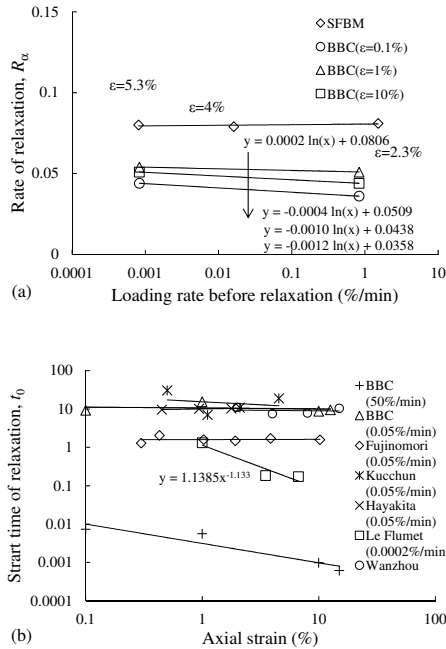


Figure 14 (a) Influence of strain rate prior stress relaxation to t ;
(b) Influence of strain lever to t .

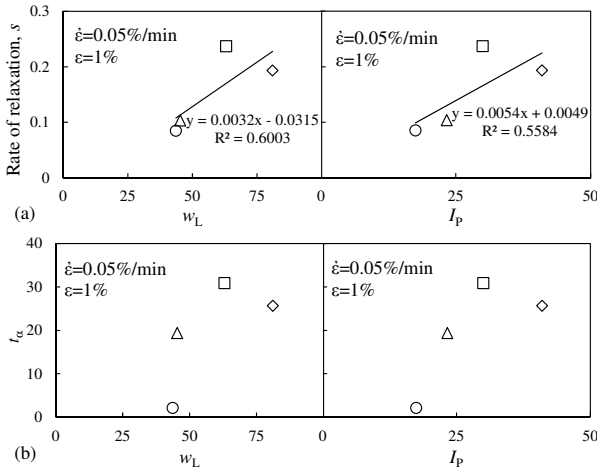


Figure 15 Relationship between R_α , t_α and w_L , I

3.2 Analytical solution of 1D stress relaxation

Based on the loading rate-dependency of Clay, YIN ZHENYU et al. [31] proposed a one-dimensional elastic viscoplastic model. The expression of the model is as following.

$$\dot{\epsilon}_v = \frac{\kappa}{1+e_0} \frac{\dot{\sigma}_v'}{\sigma_v'} + \dot{\epsilon}_v^r \frac{\lambda - \kappa}{\lambda} \left(\frac{\sigma_v'}{\sigma_p^{r'} \exp\left(\frac{1+e_0}{\lambda - \kappa} \epsilon^{vp}\right)} \right)^\beta \quad (3)$$

In above equation, β the rate-dependency coefficient, is the slop in $\log(\sigma_p^0) - \log(d\epsilon/dt)$ plane. We consider $\dot{\epsilon}_v = 0$ in 1D stress relaxation tests, and assume that the reference pre-consolidation pressure stress changes from $\sigma_p^{r'}$ to σ_p^r , vertical stress is σ_v' . We took σ_p^r as the initial value of reference pre-consolidation pressure stress during the whole tests, thus the relationship between stress and plastic volumetric strain in stress relaxation process is as following.

$$\frac{\kappa}{1+e_0} \frac{\dot{\sigma}_v'}{\sigma_v'} + \dot{\epsilon}_v^r \frac{\lambda - \kappa}{\lambda} \left(\frac{\sigma_v'}{\sigma_p^r \exp\left(\frac{1+e_0}{\lambda - \kappa} \epsilon^{vp}\right)} \right)^\beta = 0 \quad (4)$$

Volumetric strain remains always a constant in the process of tests, so plastic volumetric strain is equal to elastic volumetric strain except for a minus symbol.

$$\epsilon^{vp} = - \frac{\kappa}{1+e_0} \frac{\dot{\sigma}_v'}{\sigma_v'} \quad (5)$$

From formula (5) and (4), we get the following equation

$$\frac{\kappa}{1+e_0} \frac{\dot{\sigma}_v'}{\sigma_v'} + \dot{\epsilon}_v^r \frac{\lambda - \kappa}{\lambda} \left(\frac{\sigma_v'}{\sigma_p^r \exp\left(-\frac{\kappa}{\lambda - \kappa} \int \frac{\dot{\sigma}_v'}{\sigma_v'} dt\right)} \right)^\beta = 0 \quad (6)$$

Because $\int \frac{\dot{\sigma}_v'}{\sigma_v'} dt = \ln \sigma_v' - \ln \sigma_{vi}'$, formula (6) can be changed to formula (7).

$$\frac{\kappa}{1+e_0} \frac{\dot{\sigma}_v'}{\sigma_v'} + \dot{\epsilon}_v^r \frac{\lambda - \kappa}{\lambda} \left(\frac{\sigma_v'}{\sigma_p^r \left(\frac{\sigma_v'}{\sigma_{vi}'}\right)^{\frac{\kappa}{\lambda - \kappa}}} \right)^\beta = 0 \quad (7)$$

First-order differential of the vertical stress is derived by formula (7)

$$\dot{\sigma}_v' = - \dot{\epsilon}_v^r \frac{(1+e_0)(\lambda - \kappa)}{\lambda \cdot \kappa} \left(\frac{1}{\sigma_p^r \cdot \sigma_{vi}^{\frac{\kappa}{\lambda - \kappa}}} \right)^\beta \sigma_v'^{\frac{\lambda\beta}{\lambda - \kappa} + 1} \quad (8)$$

All the parameters in formula (8) could be regarded as constants except for vertical stress σ'_v for certain samples. To get the solution of the one-dimensional differential equation conveniently, equation (8) is changed a more general form

$$(\sigma'_v)' = A(\sigma'_v)^m \quad (9)$$

Where,

$$A = -\dot{\epsilon}_v^r \frac{(1+e_0)(\lambda-\kappa)}{\lambda \cdot \kappa} \left(\frac{1}{\sigma_p^r \cdot \sigma_{vi}^{\frac{\kappa}{\lambda-\kappa}}} \right)^\beta, m = \frac{\lambda\beta}{\lambda-\kappa} + 1$$

Solving the first-order differential equation (9), and the solution is

$$\frac{(\sigma'_v)^{1-m}}{1-m} = At + C \quad (10)$$

C is the constant of the equation which can be gotten by initial stress conditions when the stress relaxation starts, that is to say $t=0$, $\sigma'_v = \sigma'_{vi}$, thus

$$C = \frac{\sigma_{vi}^{1-m}}{1-m} \quad (11)$$

Putting the expression of C into equation (10)

$$\sigma'_v = \left(A(1-m)t + \sigma_{vi}^{1-m} \right)^{\frac{1}{1-m}} \quad (12)$$

Substituting constant A and m into equation (12), the complete expression of the vertical stress in the process of stress relaxation is obtained.

$$\sigma'_v = \left(-\dot{\epsilon}_v^r \frac{(1+e_0)(\lambda-\kappa)}{\lambda \cdot \kappa} \left(\frac{1}{\sigma_p^r \cdot \sigma_{vi}^{\frac{\kappa}{\lambda-\kappa}}} \right)^\beta \left(\left(-\frac{\lambda\beta}{\lambda-\kappa} \right) t + \sigma_{vi}^{-\frac{\lambda\beta}{\lambda-\kappa}} \right) \right)^{\frac{\lambda-\kappa}{\lambda\beta}} \quad (13)$$

3.3 The unity of the rheological parameters

In the process of stress relaxation when $t > t_0$, σ_{vi}^{1-m} in equation (12) is infinitely small compared to $A(1-m)t$. Therefore the equation (12) can be transferred in to logarithmic form

$$\ln \sigma'_v = \frac{1}{1-m} [\ln A(1-m) + \ln t] \quad (14)$$

After the simplify, we get the differential value of $\ln(\sigma'_v)$ to $\ln(t)$

$$\frac{\Delta \ln \sigma'_v}{\Delta \ln t} = \frac{1}{1-m} = -\frac{\lambda-\kappa}{\lambda \cdot \beta} \quad (15)$$

The equation (15) and equation (2) showed that stress relaxation coefficient R_α can be expressed by three material parameters λ, κ and β .

$$R_\alpha = \frac{\lambda-\kappa}{\lambda\beta} \quad (16)$$

As for certain samples, parameters λ and κ are constants, the stress relaxation R_α has one relationship with rate-dependency β .

In addition, many scholars [29-33] have studied the relationship between the rate-dependency β and secondary consolidation coefficient $C_{\alpha e}$ ($C_{\alpha e} = \Delta e / \Delta \ln t$) and summarized the equation.

$$\beta = \frac{\lambda-\kappa}{C_{\alpha e}} \quad (17)$$

Substituting equation (17) to (16), we can unify the parameters of rheological properties by the following formula.

$$R_\alpha = \frac{\lambda-\kappa}{\lambda\beta} = \frac{C_{\alpha e}}{\lambda} \quad (18)$$

That is to say equation (18) can indicate all the three parameters relating to rheological properties have relationships. Namely, we can get three rheological parameters by only one experiment (secondary consolidation tests, loading rate test or stress relaxation tests).

Reshaped Erie clay and Berthierville clay soil are selected as research soil in this paper to verify the unity of the three rheological parameters. Further, their stress relaxation coefficients are 0.042 and 0.0657 respectively (Figure 13e and f), and then simulating their stress relaxation tests with these coefficients shown in Figure 16. The results show that the stress relaxation solution of this paper can fit stress relaxation properties ideally.

Using the known R_α from equation (18) to calculate the stress relaxation coefficients β of reshaped Erie clay and Berthierville clay, $\beta = 17.9$ and $\beta = 14.2$ respectively. Comparing the experimental values in $\log(\sigma'_{p0}) - \log(d\epsilon/dt)$ lines with the deriving values from R_α on Figure 17, we find that they agree well.

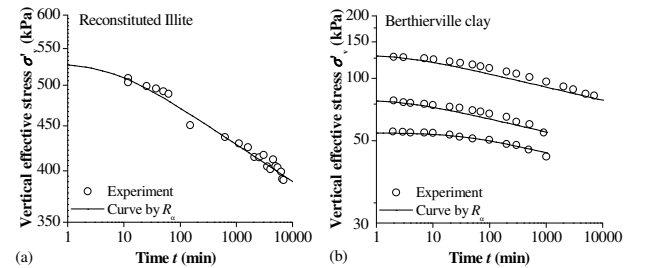


Figure 16 Comparison of between experimental and theoretical results for stress relaxation

Additionally, Yin and gramham[2] have discovered that the secondary consolidate coefficients $C_{\alpha e} / ((1+e_0)) = 0.004$ of reshaped Erie clay and $C_{\alpha e} = 0.027$ of Berthierville clay in experiments are very similar to the values calculated by R in equation (18) $C_{\alpha e} = 0.0042$ of reshaped Erie clay and $C_{\alpha e} = 0.032$ of Berthierville clay. Accordingly, we can use the stress relaxation coefficients R_α to predict the secondary consolidate coefficients $C_{\alpha e}$.

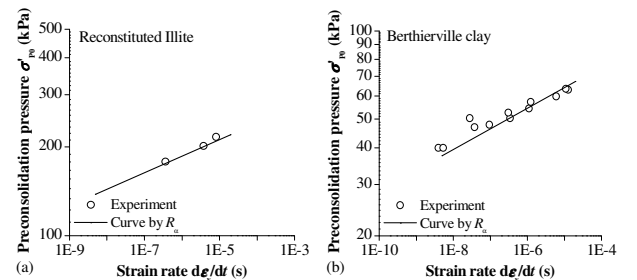


Figure 17 Comparison between theoretical curves using β derived from R_α and measurements on the strain rate-dependency of preconsolidation pressure

4. DISCUSSIONS OF STRESS-DILATANCY

The third author did a series of CRS undrained triaxial compression and extension stress relaxation tests with typical Hong Kong Marine clay (HKMD). The basic physical indicators of HKMD are $G_s = 2.66$, $w_p = 28\%$, $w_L = 60\%$, $I_p = 32$. The discussions of stress-dilatancy relationship in the process of stress relaxation will be illustrated in triaxial compression and tension tests two aspects in the following contents.

4.1 Triaxial compression tests

4.1.1 The stress relaxation relationship

The initial loading rate and strain value when stress relaxation occurs are both different of two contrast tests in the triaxial compression tests. We know that stress relaxation occurs after the soil arriving critical state and the strain value has hardly effects on stress relaxation, so the effects caused by strain value could be ignored, and then how the loading rate affect stress-dilatancy will be illustrate in detail below. The initial loading rate of two trials tests were $0.025\% / \text{min}$ and $0.41\% / \text{min}$, and different initial loading rate also caused a different variation of the deviatoric stress in the process of the stress relaxation (Figure 18). The deviatoric stress was $0.75q_0$ when the stress relaxation starts at the loading rate $0.41\% / \text{min}$ (q_0 is the maximum shear stress values in the compression process).

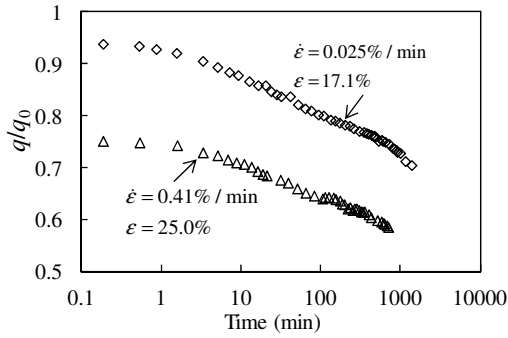


Figure 18 Stress relaxation for triaxial compression test

4.1.2 The stress-dilatancy features in stress relaxation

(1) Plastic strain incremental

According to the classical elastic-plastic theory, the total volumetric strain incremental and total deviatoric strain incremental consist of two parts: the elastic strain part and the plastic strain part.

$$d\epsilon_v = d\epsilon_v^e + d\epsilon_v^p \quad (19)$$

$$d\epsilon_d = d\epsilon_d^e + d\epsilon_d^p \quad (20)$$

As volumetric strain is always remain a constant in undrained triaxial tests, the sum of plastic volumetric strain incremental and elastic volumetric strain incremental is zero, thus, $d\epsilon_v^p + d\epsilon_v^e = 0$. We can calculate the plastic volumetric strain incremental by mean effective stress.

$$d\epsilon_v^p = -d\epsilon_v^e = -\frac{dp'}{K} \quad (21)$$

In above equation, volumetric modulus $K = (1 + e_0)p' / \kappa$; mean effective stress $p' = p_0 + q/3 - u$. Then we can get the variation relationship of plastic volumetric strain incremental versus time in stress relaxation phase in Figure 19a. Furthermore, volumetric strain

is zero in undrained tests, thus $d\epsilon_a + 2d\epsilon_r = 0$ so the deviatoric strain incremental in undrained triaxial tension tests is obtained.

$$d\epsilon_d = \frac{2}{3}(d\epsilon_a - d\epsilon_r) = d\epsilon_a \quad (22)$$

In the process of stress relaxation, the deviatoric strain incremental is

$$d\epsilon_d = \frac{2}{3}(d\epsilon_a - d\epsilon_r) = d\epsilon_a = 0 \quad (23)$$

The elastic deviatoric incremental in equation (20) can be calculated by deviatoric incremental dq

$$d\epsilon_d^e = \frac{dq}{3G} \quad (24)$$

In above equation, shear modulus G can be calculated by volumetric modulus K and Poisson's ratio.

$$G = \frac{3K(1 - 2\nu)}{2(1 + \nu)} \quad (25)$$

The Poisson's ratio of soft clay is about $\nu = 0.2$. Combining equation (20), (22) and (25), the plastic deviatoric strain incremental in undrained triaxial tension can be expressed below.

$$d\epsilon_d^p = d\epsilon_a - \frac{dq}{3G} \quad (26)$$

The plastic deviatoric strain incremental in undrained stress relaxation is expressed below by combining equation (20), (23) and (24).

$$d\epsilon_d^p = -\frac{dq}{3G} \quad (27)$$

Accordingly, the variation law of the plastic deviatoric strain incremental versus time in the phase of stress relaxation of triaxial compressing tests can be obtained in Figure 19b, then, we can also find the variation laws of plastic strain incremental ratio versus time in Figure 19c. The figure indicates that plastic volumetric strain incremental, the plastic strain incremental and plastic deviatoric strain incremental ratio under high loading rates are all greater than the case of low rates.

(2) Several typical equation of stress-dilatancy

Reynolds discussed the physical performance of stress-dilatancy equations firstly, and then, Rowe^[34] and Roscoe et al.^[35] introduced two different dilatancy equations, on the base of which scholars have established many constitutive equations.

In Rowe's stress-dilatancy equation, it is assumed that the ratio of input energy and output energy is constant K . In the extension tests, the input energy is $2\sigma_r d\epsilon_r^p$, and output energy is $\sigma_a d\epsilon_a^p$, thus Compression condition:

$$\frac{\sigma_a}{\sigma_r} = K \left(1 - \frac{d\epsilon_v^p}{d\epsilon_a^p} \right) \quad (28)$$

Extension condition:

$$\frac{\sigma_r}{\sigma_a} = K \left(1 - \frac{d\epsilon_v^p}{d\epsilon_a^p} \right) \quad (29)$$

If we use the variables in critical state soil mechanics, the above equations could be expressed below.

Compression condition:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{9(M - \eta)}{3M - 2M\eta + 9} \quad (30)$$

Extension condition:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{-9(M + \eta)}{9 + 2M\eta - 3M} \quad (31)$$

η is the ratio value of deviatoric stress and mean effective stress (q/p'). Additionally, Roscoe et al [35] proposed a dilatancy equation based on the energy dissipation of triaxial stress, which shows that the plastic incremental equals to the energy dissipated in friction process, namely

Compression condition:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = M - \frac{q}{p} \quad (32)$$

Extension condition:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = -M - \frac{q}{p} \quad (33)$$

These two equations can inflect the flow rules in the Original Cambridge Model proposed by Schofield and Wroth [36]. With the development of the Modified Cambridge Model, Roscoe and Burland [37] proposed another dilatancy equation which is widely used in practice. It has the same form in triaxial compression and triaxial extension.

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{M^2 - \eta^2}{2\eta} \quad (34)$$

Based on this, the inclined yield surface and plastic potential surface theories have been accepted widely in the anisotropic constitutive model [38, 39].

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{M^2 - \eta^2}{2(\eta - \alpha)} \quad (35)$$

In above equation, α is the slope of yield surface or plastic potential surface on the p' - q plane, $\alpha_{K0} = (\eta_{K0}^2 + 3\eta_{K0} - M^2)/3$ can be used to estimate the initial value of K_0 consolidation of clay [39], if $\alpha=0$ equation (35) can be changed to the form equation (34). α also changes with stress ratio and plastic strain, which is called anisotropy. The value of M can be calculated by following equations.

Compression condition:

$$M = \frac{6 \sin \phi}{3 - \sin \phi} \quad (36)$$

Extension condition:

$$M = \frac{6 \sin \phi}{3 + \sin \phi} \quad (37)$$

4.1.3 Impacts by compression rate to the stress-dilatancy features in stress relaxation tests

Based on Figure 19c and Figure 18, we can obtain the relationship between deviatoric strain incremental ratios and stress ratio of these two stress relaxation tests in Figure 20. It shows that stress ratio η hardly changes during the whole phase of stress relaxation, about $\eta=0.5$. Although the plastic strain incremental ratio d changes between -2 to 4 virus relaxation time, there is nearly no difference of the stress-dilatancy relationship between these two compression rates, which may be associated with the critical state of samples when stress relaxation occurs. If we plot the four compression stress-dilatancy relationship in Figure 20, it can be found all the formulas are not able to describe the stress-dilatancy characteristics in compressive stress relaxation phase.

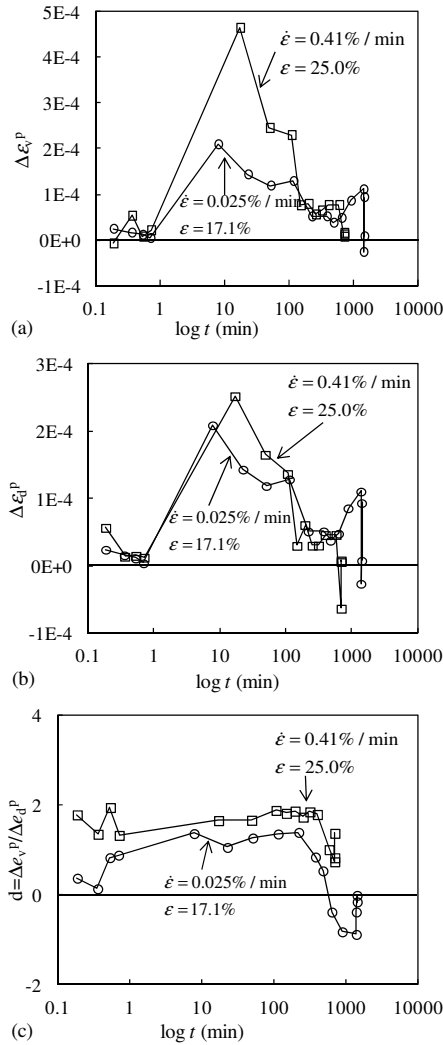


Figure 19 Triaxial compression relaxation stage: (a) plastic volume strain increment versus time; (b) plastic deviatoric strain increment versus time; (c) plastic strain increment ratio versus time

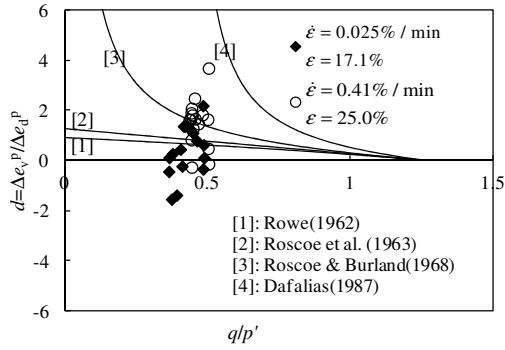


Figure 20 Influence of compression rate on stress-dilatancy behavior of the stage of stress relaxation for HKMD

4.2 Triaxial extension tests

Isotropic consolidating the samples with mean effective stress $p'_0 = 400$ kPa firstly, then, starting stress relaxation tests after shearing the samples to a certain strain with particular extension rate, and then repeating the triaxial extension tests continually after stress relaxation reaches a certain stage. Specific test programs and the relationship between the deviatoric stress and vertical strain are shown in Figure 21. The stage 1, 2 and 3 of Figure 21 will be discussed mainly.

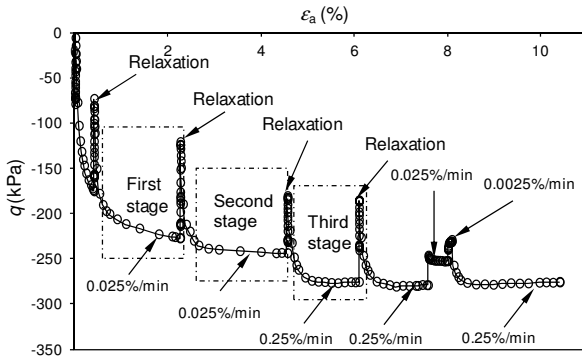


Figure 21 Relationship of axial strain versus deviatoric stress for step-changed constant strain rate undrained triaxial extension test on HKMD

4.2.1 Stress-dilatancy properties in stress relaxation phase

We set the first stage of the test in Figure 22 as an example in this section. Normalized deviatoric stress versus time in logarithmic axis is plotted in Figure 23. Tests results show that the stress relaxation curves of HKMD are consistency with others of ordinary clay, and have the following parameters $s=0.06$, $t_0=0.3$ min, $R_\alpha=0.06$, $t_\alpha=0.36$ min.

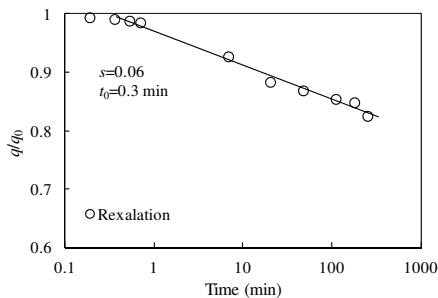


Figure 22 Stress relaxation results for the first stage

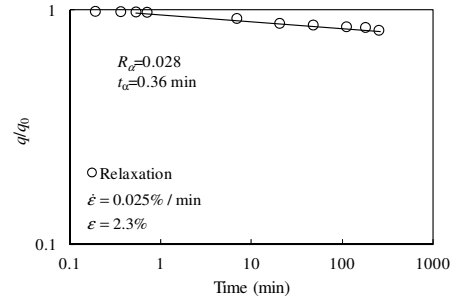


Figure 23 Normalized deviatoric stress versus time in logarithmic axis

Adopting the methods describing the relationship of plastic deviatoric ratio and stress ratio, we can get the relationship of plastic deviatoric ratio and stress ratio in triaxial extension phase and stress relaxation phase in Figure 24. As is shown in Figure 24, the stress ratio η at the beginning of the phase is greater in stress extension phase which follows a stress relaxation phase (absolute value of η greater than 0.6). The value of plastic strain incremental ratio d (less than -0.2) is small relatively when stress ratio is great, but it is between -0.8~-2.2 in stress relaxation phase which is greater obviously than that in loading phase.

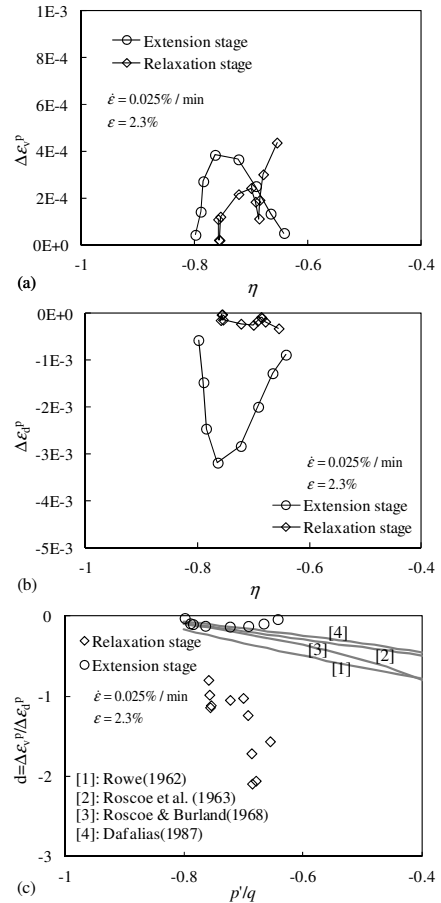


Figure 24 Triaxial extension and stress relaxation stage: (a) plastic volume strain increment versus stress ratio; (b) plastic deviatoric strain increment versus stress ratio; (c) stress-dilatancy relationship

If we plot the four extension stress-dilatancy relationship in Figure 24c, it can be found all the formulas are able to describe the stress-dilatancy characteristics in undrained extension process with high stress ratio, while they couldn't describe the stress-dilatancy properties of clay in stress relaxation phase at the same time.

4.2.2 The impact of strain value to stress-dilatancy properties

In order to study the impact of strain value when stress relaxation occurs to stress-dilatancy properties, we analyze the results of the stress relaxation tests of second CRS loading phase in Figure 22, then, the relationship between plastic strain incremental ratio d and stress ratio η of this phase is plotted in Figure 25, meanwhile, the results of first CRS loading stage and stress relaxation stage are also plotted in the Figure as well as four stress-dilatancy equations. The results showed that the four stress-dilatancy equations can describe stress-dilatancy properties of these two undrained extension stages effectively. The relationship of plastic strain incremental ratio d and stress ratio η in stress relaxation process is nearly the same for two different stress relaxation strain values 2.3% and 4.6%, namely, with the decrease of $|\eta|$, $|d|$ becomes greater, and the slopes of two lines are nearly equal.

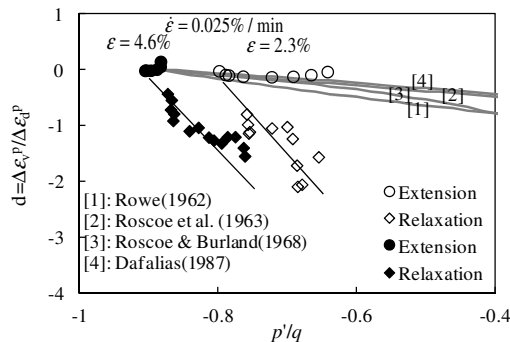


Figure 25 Stress-dilatancy behavior at the same extent rate and different relax strain conditions for HKMD

4.2.3 The impacts of loading rates to stress-dilatancy properties

Above conclusions indicate that the strain value at the start of stress relaxation has little impact on stress-dilatancy properties. Therefore, we can ignore the impact of strain value when analyzing the stress-dilatancy properties under different loading rates. So, for contrast, the CRS loading and stress relaxation tests results are discussed in second (0.025%/min) and the third (0.25%/min) stages in Figure 22. The results of stress relaxation tests and CRS loading of the second and third stages as well as four stress-dilatancy equations are all plotted in Figure 26. The results showed that the four stress-dilatancy equations can describe stress-dilatancy properties of these two undrained extension stages effectively. The relationship of plastic strain incremental ratio d and stress ratio in stress relaxation process is affected slightly by loading rate. In the stress relaxation phase, the decrease of stress ratio η under high loading rate is faster than that under low loading rate, while the relationship of strain incremental ratio d and stress ratio η is hardly affected by loading rates.

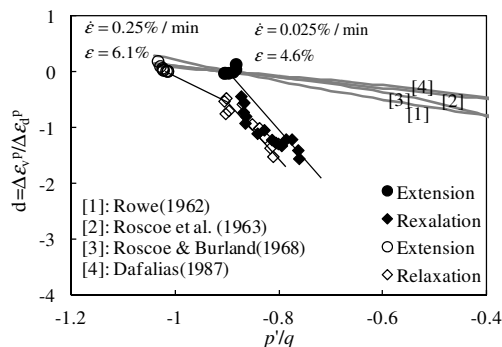


Figure 26 Influence of extent rate on stress-dilatancy behavior of the stage of stress relaxation for HKMD

5. CONCLUSION

In this paper, the authors summarized systematically the research results about stress relaxation properties done by many scholars, and then discussed the correlation between rheological properties of clay, and proposed some new discussion on soil stress relaxation. The conclusions are as follow.

- (1) Based on logarithmic and double logarithmic axial, the affecting factors for stress relaxation coefficient s and R_α and stress relaxation equivalent time t_0 and t_α are studied. The stress relaxation rate s and loading rate before relaxation (in logarithm coordinate) is approximately linear relationship, and the slope of the linear between 0.005 to 0.013 and R_α has nothing to do with vertical value. ' t_0 ' is linearly relevant to loading rate as well as t_α (in double logarithm coordinates) as vertical strain remain constant, and t_0 vary between 1.034 to 1.147, t_α between 0.913 to 1.123, they both have nothing to do with the vertical value.
- (2) The strain relaxation parameters ' s ' has a certain linear relationship with soil liquid limit and plasticity index while hardly between t_0 and soil liquid limit and plasticity index, and the impact of liquid limit and plastic index to R_α and t_α is also not so obvious.
- (3) Based on the rate-dependency rheological equations, an analytical solution of stress relaxation and the coefficient were derived. The study found that a certain equivalence between the stress relaxation coefficient, rate-dependency coefficient and secondary consolidation coefficient. This conclusion will provide a theoretical basis for practical application of stress relaxation tests.
- (4) Four stress-dilatancy equations can describe the stress-dilatancy properties in undrained extension test under high stress ratio effectively, whereas, they couldn't be applied in the stress relaxation phase of triaxial compression tests and triaxial extension tests.
- (5) Loading rate before stress relaxation only affects the stress-dilatancy properties at the early stage of stress relaxation. The stress-dilatancy properties have nothing to do with the strain value at start of stress relaxation.

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