Time-Dependent Dynamic Characteristics of Model Pile in Saturated Sand during Soil Liquefaction

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ABSTRACT: In order to quantify the relation between soil stiffness and excess pore water pressure during liquefaction, the test data of a series of shaking table tests on model pile in saturated sand using a large biaxial laminar shear box conducted at the National Center for Research on Earthquake Engineering were analyzed. The pile tip was fixed at the bottom of the shear box to simulate the condition of a pile foundation embedded in a firm stratum. The pile head was mounted with steel disks to simulate the superstructure. In addition, strain gauges and mini-accelerometers were placed on the pile surface to obtain the response of the pile under shaking. Therefore, the model pile can be considered as a sensor to evaluate the changes of dynamic characteristics of soil-pile system during the shaking by using the time-frequency analysis and system identification technique. The results showed that the stiffness of the soil would increase with the dissipation of pore water pressure and the recovery of soil stiffness is directly related to the effective stress ratio of soil specimen.

KEYWORDS: Shaking table test, Soil liquefaction, Excess pore water pressure, Time-frequency analysis, System identification

1. INTRODUCTION

Soil liquefaction has been an important issue in the geotechnical earthquake engineering since the 1964 Niigata and Anchorage earthquakes. Recent earthquakes in New Zealand, and Japan still caused a great deal of liquefaction-induced damages on infrastructure. Liquefaction has been studied extensively over past 50 years. Extensive researches have been studied to understand the mechanism of liquefaction, liquefaction potential, hazard mapping and also to develop various countermeasures against liquefaction. However, soil liquefaction in nature is a quite complex process of phase transformation. If the variation of soil mechanical properties under actual earthquake accelerations could be estimated, the real seismic behavior of soil in field can be better understood and the weakening of soil due to the generation of excess pore water pressure could be appropriately considered in seismic design.

Many studies on dynamic characteristics of soil under earthquake loading were conducted in order to understand the dynamic behavior of saturated sand under earthquake shaking. Small soil specimens in the laboratory (e.g. Iwasaki et al., 1981; Chiang 1990; Pradhan et al., 1995), shaking table tests on sand specimens, under either 1 g or centrifugal condition (e.g. Tokimatsu et al., 2005; Ueng et al., 2006; Lee et al., 2012) have been used to investigate the relation between soil stiffness and excess pore water pressure, including post-liquefaction. Kostadinov and Towhata (2002) performed time-frequency analysis on the earthquake records of several liquefied sites using short-time Fourier transform (STFT) to investigate the change of the mean instantaneous frequency (MIF) of the ground seismic response. The MIF of each site was significantly lowered after liquefaction. Kramer et al. (2011) also conducted time-frequency analysis on earthquake records of liquefied sites using STFT, wavelet transform and Stockwell transform. Dramatic reduction in acceleration amplitude and predominant frequency were observed, showing that the liquefaction induced the quick softening of soil. The results of these studies indicated that the generation of excess pore water pressure led to the decrease of effective stress and resulted in reduction of modulus of soil

The foundation design code in Taiwan suggested to adopt the reduction factors for the mechanical parameters of liquefiable soil proposed in Japan Road Association (JRA, 1996) and in Architectural Institute of Japan (AIJ, 1998). However, these regulations are not based on solid theories and are inconsistent. Because existing related studies are quite limited, conservative considerations are usually adopted in engineering practice, causing uneconomical designs. Therefore, it is necessary to quantify the

relation of soil stiffness and excess pore water pressure during liquefaction for better understanding of soil behavior, including post-liquefaction.

Ueng et al. (2009) and Chen et al. (2012) have conducted a series of shaking table tests on model pile in saturated sand using a large biaxial laminar shear box developed at the National Center for Research on Earthquake Engineering to investigate the soil-structure interaction especially soil liquefaction and lateral spreading, in a liquefiable ground during earthquake. Strain gauges and accelerometers were placed on the pile surface to record the responses of the pile under shaking. The soil responses that near the pile and that far from the pile, including pore water pressure changes, accelerations, and settlements were also continuously measured during the shakings. These experimental data were utilized and analyzed to investigate the time-dependent characteristics of the model pile with the generation or dissipation of excess pore water pressure during the shaking via the time-frequency analyses and system identification technique.

2. SHAKING TABLE TESTS

2.1 Model Pile and Sand Specimen

The model pile was made of an aluminum alloy pipe, with a length of 1600 mm, an outer diameter of 101.6 mm, a wall thickness of 3 mm and a flexural rigidity, $EI = 75 \text{ kN} \cdot \text{m}^2$. The pile was fixed vertically at the bottom of the shear box in both level ground and sloping ground cases. Hence, this physical models can be used to simulate the condition of a vertical pile embedded in level or sloping rock or within a firm soil stratum. The model pile was instrumented and placed inside the shear box before preparation of the sand specimen, attempting to simulate a pile foundation installed with minimum disturbance to the surrounding soil. In addition, various numbers of steel disks were attached to the top of the model pile to simulate various conditions of the superstructure. In the case of sloping ground test, the shear box was inclined 2° to the horizontal, simulating a mild infinite slope and the sloping direction of this test was defined as X direction, as shown in Figure 1.

Clean fine silica sand (Gs = 2.65, $e_{max} = 0.918$, $e_{min} = 0.631$, $D_{50} = 0.30$ mm) from Vietnam was used in this study for the sand specimen inside the laminar shear box. The sand specimen was prepared using the wet sedimentation method after placement of the model pile and instruments in the shear box. The sand was rained down into the shear box filled with water to a pre-calculated depth. The size of the sand specimen is 1.880 m × 1.880 m in plane and about 1.40 m in height before shaking tests. Details of biaxial

laminar shear box and the sand specimen preparation were described in Ueng et al. (2006)



Figure 1 Instrumentation on the model pile and within the sand specimen

2.2 Instrumentation

Two linear displacement transducers (LDTs) were set up to measure X- and Y-displacements of the pile top. They were mounted to the reference frames outside the shaking table. The strain gauges were placed on the pile surface to measure the bending strains of the model pile. There are 10 different depths with 15-cm spacing along the pile axis as shown in Figure 1. Vertical acceleration arrays along the pile were also installed on the model pile in X- and Y-directions for acceleration measurements. In addition, in order to observe the build-up and dissipation of the pore water pressures and accelerations in the sand specimen (the region near the pile and that far from the pile), mini-piezometers and mini-accelerometers were installed inside the box at different locations and depths. The responses of 15 layers of inner and outer frames at different depths of the laminar shear box were also recorded to evaluate the ground motion and liquefaction depth of the sand specimen using linear displacement transducers and accelerometers.

2.3 Test Plan

Shaking table tests were first conducted on the model pile without the sand specimen in order to evaluate the dynamic characteristics of the model pile itself. Sinusoidal and white noise accelerations with amplitudes from 0.03 to 0.05 g were applied in X and/or Y directions. The model pile in saturated level or sloping ground was then tested under one dimensional sinusoidal (1-8 Hz) and recorded accelerations at the Chi-Chi Earthquake and the Kobe Earthquake with amplitudes ranging from 0.03 to 0.15 g. Input motions were mainly imposed perpendicularly to the slope direction for the study of kinematic effect on the pile foundation (lateral spreading force) independently. White noise accelerations with amplitude of 0.03 g were also applied in both X- and Y-directions to evaluate the dynamic characteristics of the model pile within soil and the sand specimen. The height of the sand surface after each shaking test was measured to compute the settlement and density of the sand specimen.

2.4 Basic Results of Shaking Table Test

Shaking table tests on the model pile without sand specimen were conducted to evaluate the dynamic characteristics of the model pile itself. We consider the behavior of model pile without sand specimen under the shakings as a single-degree viscously damped system. Hence, the amplification curve was obtained from the Fourier spectral ratio of the measured acceleration of the pile top to that of the input motion. Table 1 and Table 2 list respectively the predominant frequencies of the model pile in level and sloping ground cases according to the test data. The average damping ratio of the model piles is about 1.4 % according to observations of the free vibration of the piles after the end of the input motions.

Table 1 Predominant frequencies of the model pile in a level ground

Mass on pile top	Aluminum pile	
	Freq., Hz	
No mass	21.0	
1 disk (37.84 kg)	5.74	
3 disks (114.1 kg)	3.17	
6 disks (226.1 kg)	2.19	

Table 2 Predominant frequencies of the model pile in a sloping ground

Mass on pile top	Aluminum pile Freq., Hz
No mass	22.9
6 disks (226.1 kg)	2.1

The dynamic characteristics of soil and soil-pile system were also evaluated by a series of shaking table tests on the model pile within the saturated sand specimen with small amplitude. The predominant frequencies of the soil and the soil-pile system for the model pile within the saturated sand of various relative densities in level ground and sloping ground were listed respectively in Table 3 and 4.

Table 3 Predominant frequencies of the soil and the aluminum pile in a level ground of different relative densities

Mass on pile top	e Soil density	Pile in soil	Soil
-	Dr, %	Freq., Hz	Freq., Hz
No mass	27.3	11.1	11.1
1 disk	38.1	11.6	11.6
1 disk	42.8	11.6	11.6
3 disks	42.9	8.79	12
3 disks	50.4	9.16	12.7
6 disks	70.2	5.0	13.4

Table 4 Predominant frequencies of the soil and the aluminum pile with 6 steel disks in a sloping ground of different relative densities

Soil density	Pile in soil	Soil
Dr, %	Freq., Hz	Freq., Hz
11.9	5.0	10.74
26.0	4.76	12.21
42.4	4.40	12.93
70.1	4.40	14.0

It can be seen that the predominant frequency of soil increases with the relative density, but that of the pile in soil may increase or decrease slightly with the relative density. The predominant frequency of pile in soil was affected by both the height and relative density of soil specimen except for the inertia effect. This might attribute to that the height of the soil specimen decreased with the increase of the relative density because of the tests in sequence. In addition, the predominant frequency of soil-pile system is significantly lower than that of the soil. Comparing the predominant frequencies of the model pile without and within sand specimen (e.g. Table 1 and Table 3, respectively), one can find that, except for the case without mass and with one steel disk of mass on the pile head, the predominant frequencies of the model pile in the sand specimen were higher than those without soil due to the confinement of the soil on the pile. Hence, the responses of the pile were mainly governed by the inertia force in the cases of model pile with 3 or 6 steel disks.

3. TIME-FREQUENCY ANALYSIS AND SYSTEM IDENTIFICATION

Fourier transform is a spectral analysis method widely used to obtain the frequency content of a vibration signal in the time domain. Based on the assumption of its theory, it is more appropriate for the signal is linear, periodic and stationary. However, for the case that the signal is nonlinear, non-periodic and non-stationary, spectral analysis method can not show the variation of the frequency content with time. In contrast, the time-frequency analysis can provide the characteristics of a signal in both time and frequency domains. It can thus capture the time-varying trends of the frequency content and avoid the ill effects of abnormal events or specific sources on the system identification.

Short-time Fourier transform (STFT) is one of the often used time-frequency analysis method (Mallat, 1999). The procedure is to divide the overall time history of the original signal into a shorter segment of a fixed duration with each segment partially overlapped with the next. Then Fast Fourier transform of the signal in each segment is conducted, giving the Fourier spectrum at the middle of duration of each segment. Finally, all the spectra at different moments are comprehended to obtain a Fourier amplitude-timefrequency distribution. Because STFT is not only easy to be programmed and executed but also able to effectively exhibit the time-frequency characteristics of the signal, it was adopted in this study.

In order to quantify the time-dependent predominant frequency of soil-pile system, a method of system identification technique, socalled short-time transfer function method (STTF), was proposed by this study to identify the time-varying predominant frequency of soil-pile system. The idea to extract fixed-duration sections continuously from the overall time histories of both the input and output signals with each section partially overlapped with the next, and the signal of each section is assumed to be stationary. The transfer function of each section is then calculated by the Fourier spectral ratio of the input to output signal. The predominant frequency at the middle of duration of each section is herein determined according to the first peak of the amplitude spectrum of the transfer function. Finally, all the predominant frequencies at different moments are identified to obtain the relation of time and the predominant frequency of the system.

4. CASE STUDIES- SHAKING TABLE TESTS

A series of shaking table tests on model pile within the saturated sand in both level and sloping ground at the NCREE to investigate the soil-structure interaction especially during soil liquefaction and lateral spreading were utilized for case study. In order to study the seismic behavior of the model pile during liquefaction, two shaking table tests with uniaxial earthquake excitation were adopted. One is the test on model pile in a level ground, and the other is the test in the sloping ground condition. The densely instrumentation on the pile and within the soil specimen were also continuously measured during the shakings. Thus, the methods mentioned in the previous section can be applied to analyze the test data.

4.1 Case I-Level Ground

A shaking table test on model pile in a level ground under onedimensional recorded acceleration in Chi-Chi earthquake with an amplitude of 0.15 g in Y direction was conducted to study the effect of pore water pressure on the pile behavior in liquefiable soil with a relative density of 40.8 %. The pile tip was vertically fixed at the bottom of the shear box with a superstructure of 37.84 kg, and the model pile was densely instrumented with mini-accelerometer and strain gauges. Therefore, the model pile can be considered as a sensor to evaluate the changes of dynamic characteristics of soil-pile system during the shaking.

Figure 2 shows the measured acceleration time histories of the pile top and input motion. In addition, the time histories of excess pore water pressure ratios (r_u) at different depths of the free-field piezometer array are also shown in Figure 3. It can be observed based on the measured excess pore water pressures that the sand at a shallower depth liquefied at about 18 seconds, and afterwards the excess pore water pressures were nearly dissipated at around 70 seconds. The depth of liquefaction was determined based on the measured pore water pressures in the sand specimen and accelerations on the frames. In this test, the liquefied depth of the sand specimen reached about a depth of 79 cm.



Figure 2 The measured acceleration time histories of the pile head and input motion



Figure 3 Time histories of excess pore water pressure ratios at different depths (free field, location D in Figure 1)

4.1.1 Time – Frequency Analysis

Figure 4 shows the result of time-frequency distribution of the measured acceleration on the pile head by STFT method under onedimensional earthquake shaking. It can also be observed that the time-frequency distribution during the shaking can be divided into four stages: (i) before the shallower depth of soil liquefied prior to 15 seconds, the frequency content of the soil-pile system is mainly ranged from 10-14 Hz in accordance with dynamic characteristics of the ground under small excitation. (Table 1) (ii) In the period of initial liquefaction during 15 - 40 sec, the main response of soil-pile system exhibits the frequency in 4 - 6 Hz. (iii) the main frequency of soil-pile system seems to increase with time from 4 Hz to 12 Hz during the period of 40 - 70 sec. (iv) after 70 sec, the main frequency of soil-pile system is kept in a range from 12-14 Hz. Based on the observation above, one can find that the main response of frequency has a remarkable drop after the initial liquefaction occurs, and afterward the main response recovered with time due to the dissipation of excess pore water pressure. This result provided good evidence that stiffness of soil is strongly affected by the changes of excess pore water pressure, and also in accordance with the previous studies. However, for a trade-off of the selection of time and frequency resolution, the result could only present an outline in frequency response of soil-pile system during the shaking.



Figure 4 Time-frequency distribution of the measured acceleration on the pile head by STFT method under 1D earthquake shaking (level ground)

4.1.2 System Identification

The results of time-frequency analysis of STFT just provide a sketch of the seismic behvaior of model pile in a saturated soil during the shaking table test. Therefore, the time-dependent predominant frequency of model pile in a saturated sand during earthquake shaking was processed by short-time transfer function method (STTF), as shown in Figure 5. It can be seen that the predominant frequency of the soil-pile system substantially decreases before liquefaction (from 15 - 20 sec), and later the predominant frequency increase with time at the period of 45 - 70 sec. After 70 sec, the predominant frequency remains constant. Comparing the analysis results of short-time Fourier transform and short-time transfer function in the case of earthquake shaking (Figures 4 & 5), it was found that the results are almost the same. Based on the results of the time-frequency analysis and system identification of the shaking table test, it can be seen that the generation and dissipation of excess pore water pressure has great effects on the stiffness of soil and it would result in the variations of dynamic characteristics of soil-pile system. In addition, the short-time transfer function method can be used to identify the dynamic characteristics of the time-dependent

system, and can obtain the reasonable results for the quantification study.



Figure 5 The time-dependent predominant frequency of model pile within saturated sand during earthquake shaking (level ground)

4.1.3 Effect of Pore Water Pressure Changes

In order to investigate the relation of pore water pressure and predominant frequency of soil-pile system, the representative parameter should be firstly integrated from all the responses of the pore water pressures in the sand specimen to present the state of the specimen. Because of the stiffness of soil related to the vertical effective stress and vertical effective stress also related to the excess pore water pressure, the average pore pressure ratio (r_u, ave) is used to represent a average state of sand specimen in this study. The idea is to calculate the weighted excess pore water ratios of vertical piezometer array in the free field within the specimen, and the weighting is the tributary length of each piezometer. The tributary length of each piezometer is a half of the distance between the closest upper and lower piezometers. If a piezometer is close to the soil boundary, its tributary length is the distance from the soil boundary to the midpoint between it and the next piezometer. The tributary length can be calculated by the following diagram, as shown in Figure 6. Each piezometer of the array has the excess pore water ratio time history $r_{u,i}(t)$ and the tributary length of each piezometer is d_i . The average of excess pore water ratio time history of sand specimen can be calculated by

$$r_{u,ave}(t) = \sum_{i=1}^{n} r_{u,i}(t) \times d_i / \sum_{i=1}^{n} d_i$$
(1)
$$I_1 + I_2 + I$$



piezometer

Figure 6 The diagram of the tributary lengths at different locations

Figure 7 shows the average pore water pressure ratio time history of sand specimen. On the basis of concept of effective stress in saturated sand, the average effective stress ratio time history can be obtained by

$$ESR_{ave}(t) = 1 - r_{u,ave}(t) \tag{2}$$

and the average effective stress ratio time history of sand specimen is shown in Figure 8.



Figure 7 Average pore water pressure ratio time history of sand specimen



Figure 8 Average effective stress ratio time history of sand specimen

Figure 9 is the comparison of the predominant frequency time history of soil-pile system and the average effective stress ratio time history of sand specimen. It can be seen that the trend of the predominant frequency is similar to that of the average effective stress ratio. Furthermore, comparing this result at the period of 20-40 second with the predominant frequency of the model pile without and within the soil specimen (Table 1 & 3), one can find that predominant frequency of the model pile within the soil is only slightly larger than that of model pile without soil specimen while the soil specimen is partially liquefaction. This inferred that the stiffness of the soil almost vanished during the period of initial liquefaction.



Figure 9 The predominant frequency vs. average effective stress ratio time history (level ground)

4.2 Case II-Sloping Ground

In the case of the shaking table test on model pile in a sloping ground under one-dimensional recorded acceleration in Chi-Chi earthquake with an amplitude of 0.10 g in Y direction was also analysed in both time-frequency analysis and system identification techniques. The superstructure of 226.1 kg was fixed on the pile head and the relative density of sand specimen is about 27 %.

Figure 10 shows the measured acceleration time histories of the pile head and input motion. Moreover, Figure 11 presents the time histories of excess pore water pressure ratios (r_u) at different depths of the free-field piezometer array. It can be seen that the sand at a shallower depth liquefied at about 20 seconds, and afterwards the excess pore water pressures were totally dissipated at around 45 seconds. The liquefied depth of the sand specimen was about 45 cm in this test.



Figure 10 The measured acceleration time histories of the pile head and input motion (sloping ground)



Figure 11 Time histories of excess pore water pressure ratios at different depths (sloping ground)

Applying the time-frequency analysis of short-time Fourier transform and short-time transfer function techniques, the calculated time-frequency distribution and the identified predominant frequency of model pile within saturated sand during earthquake shaking were compared, as shown in Figure 12. It can be seen that the predominant frequency of the soil-pile system gradually decreases before liquefaction (from 10 - 20 sec), and minimum occurs at the onset of initial liquefaction (about 20 sec) whereas the predominant frequency increase with time at the period of 30 - 50 sec. After 50 sec, the predominant frequency is almost kept constant. The results also show that the variation of stiffness of soil due to the excess pore water pressure changes resulting in the changes of dynamic characteristics of soil-pile system. The trends of these results are almost the same, but the technique of short-time transfer

function can obtain more precise value of predominant frequency of the soil-pile system.



Figure 12 The comparison of the analysis results of STFT and STTF in the case of sloping ground test

Based on the average effective stress ratio approach mentioned in the previous section, the predominant frequency time history of soil-pile system and the average effective stress ratio time history of sand specimen are shown in Figure 13. It can be seen that the trend of the predominant frequency is also similar to that of the average effective stress ratio. The result also indicated that the predominant frequency of soil-pile system would increase with the dissipation of pore water pressure after liquefaction. In other word, the stiffness of the soil would increase with the dissipation of pore water pressure, and the recoverable increment of soil stiffness is directly related to the decrease of excess pore water pressure.



Figure 13 The predominant frequency vs. average effective stress ratio time history (sloping ground)

5. CONCLUSION

Based on the test data of shaking table test at NCREE, the timedependent behavior of model pile in liquefiable soil during liquefaction process have been investigated by using time-frequency analysis and system identification technique. It can be seen that the time-dependent dynamic characteristics of soil-pile system can be reasonably identified by both time-frequency analysis and system identification technique. The average effective stress ratio of the specimen proposed in this study can probably represents the condition of soil specimen. The trend of the predominant frequency is similar to that of the average effective stress ratio. In addition, the stiffness of the soil almost vanished during the period of initial liquefaction. It was also found the stiffness of the soil would increase with the dissipation of pore water pressure, and the recoverable increment of soil stiffness is directly related to the decrease of excess pore water pressure. Further analyses of the test data will be performed to obtain more information on the relation between stiffness of the soil and excess pore water pressure and set up a model to assess the seismic behavior of liquefied soil for more reasonable seismic design.

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7. REFERENCES

- Architectural Institute of Japan (AIJ). (1988) Recommendations for Design of Building Foundations, Architectural Institute of Japan. (in Japanese)
- Chen, C. H., Ueng, T. S., and Chen, C. H. (2012) "Shaking table tests on model pile in saturated sloping ground", The 15th World Conference on Earthquake Engineering, Paper No.1250, Lisbon, Portugal, September 24-28.
- Chiang, K. L. (2005) The behavior of saturated sand subject to undrained monotonic loading after the cyclic loading. MS thesis, National Taiwan University, Taipei, Taiwan. (in Chinese).
- Iwasaki, T., Tokida, K., and Kimata, T. (1981) Study on the feasibility of liquefaction evaluation methods for sandy ground on seismic design, Report No. 1729, Public Works Research Institute, Ministry of Construction, Tsukuba, Japan. (in Japanese).
- Japan Road Association (JRA). (1996) Design Specifications for Highway Bridges. Japan Road Association: Tokyo. (in Japanese).
- Kostadinova, M. V., and Towhata, I. (2002) "Assessment of liquefaction-inducing peak ground velocity and frequency of horizontal ground shaking at onset of phenomenon", Soil Dynamics and Earthquake Engineering, 22, Issues 4, pp309-322.
- Kramer, S. L., Hartvigsen, A. J., Sideras, S. S., and Ozener, P. T. (2011) "Site response modeling in liquefiable soil deposits", 4th IASPEI / IAEE International Symposium: Effects of Surface Geology on Seismic Motion, Santa Barbara, CA.
- Lee, C. J., Wang, C. R., Wei, Y. C., and Hung, W. Y. (2012) "Evolution of the shear wave velocity during shaking modeled in centrifuge shaking table tests", Bulletin of Earthquake Engineering, 10, Issues 2, pp401-420.
- Pradhan, T., Kiku, H., and Sato, K. (1995) Effect of fines content on behavior of sand during the process to liquefaction, Earthquake Geotechnical Engineering, Ishihara (ed.), Balkema, Rotterdam, pp823-828.
- Mallat S. (1999) A Wavelet Tour of Signal Processing, 2nd edition, Academic Press.
- Tokimatsu, K., Suzuki, H. and Sato, M. (2005) "Effect of inertial and kinematic interaction on seismic behavior of pile with embedded foundation", Soil Dynamics and Earthquake Engineering, 25, Issues 7-10, pp753-762.
- Ueng, T. S., Wang, M. H., Chen, M. H., Chen, C. H., and Peng, L. H. (2006) "A large biaxial shear box for shaking table tests on saturated sand. Geotechnical Testing Journal, 29, Issues 1, pp1-8.
- Ueng, T. S. and Chen, C. H. (2009) "Performance of model piles in liquefiable soil during shaking tests", IS-Tokyo, International Conference on Performance-Based Design in Earthquake Geotechnical Engineering— from case history to practice pp523-529, Tsukuba, Japan, June 15-18.