Analysis of Influencing Factors on Brazilian Test Results Based on A Complex-shaped Grain Model for Brittle Rock

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ABSTRACT: Brazilian test is a commonly used laboratory method to measure the tensile strength of brittle rock, but there are also many controversies. In this paper, influencing factors including the Poisson's ratio and the rock specimen thickness on Brazilian test results are investigated using PFC3D program based on a complex-shaped grain model which can capture all the characteristics of brittle rock in three-dimensional environment. The relationship between the Poisson's ratio and the tensile strength is observed through the comparison among three different models which presenting similar macro-properties except the Poisson's ratio and the tensile strength. The study of specimen thickness effect indicates that the Brazilian test results significantly overestimate the real tensile strength of the specimens with relatively larger thickness. Through investigating the stress-strain curves and crack developing processes of the Brazilian test specimens, conclusions are made that the Brazilian tensile strength will increase with the specimen thickness due to the great loading increment.

KEYWORDS: Brazilian test, Tensile strength, Poisson's ratio, Specimen thickness, Complex-shaped grain model.

1. INTRODUCTION

Tensile strength is one of the most important properties to characterize brittle rock in engineering research. It can be measured in the laboratory by performing direct-tension test or Brazilian test. Brazilian test is a simple indirect method to estimate the tensile strength of rock materials and is more widely used than direct-tension test in the laboratory because of the difficulties in making standardized rock specimen to meet the high requirements of direct-tension testing machine.

Traditional Brazilian test is achieved by applying diametric loading to disc specimen. And the tensile strength σ_t is calculated by the following formula:

$$\sigma_t = 2P / \pi Dt \qquad (1)$$

Where P is the maximum loading, D and t are the diameter and thickness of the rock disc specimen, respectively.

However, there have long been controversies and doubts about the validity of the Brazilian test, even after it was suggested as an effective method to determine the indirect tensile strength of rocks by the International Society for Rock Mechanics (ISRM, 1978). As early as 1964, Fairhurst had noted that in the Brazilian test, failure may occur away from the center of the disc and thus make the calculated tensile strength lower than the true value. Hudson et al. (1972) also illustrated that failure always initiated directly under the loading points if flat steel platens only were used to load the specimen. Wang et al. (1999) held that the primary crack initiation at the center region of the Brazilian disc during the test was a key problem, which must be solved properly. And some modified Brazilian tests were also put forward and studied by many researchers (Wang et al., 2004; Yu et al, 2009) in order to eliminate the stress concentration on the loading points.

In this research, our attentions will be paid to two of the influencing factors on the Brazilian test results. One factor is the Poisson's ratio. According to elasticity theory, Poisson's ratio will influence the three dimensional stress distribution in the Brazilian test specimen, but this effect on test results only received few attentions from researchers (Wijka, 1978; Chau and Wei, 2001; Yu et al. 2009). The other factor is the specimen thickness, which has previously been included in Eq. (1) to estimate the Brazilian tensile strength. The ISRM (1978) suggested that the thickness should be approximately equal to the specimen radius, while in the specifications for Brazilian test by the American Society for Testing and Materials (ASTM, 1988), the thickness-to-diameter ratio is between 0.5 and 1.0. However, as is well known, the Eq. (1) is derived from a two-dimensional elastic theory. It is impossible for the specimen to meet the two-dimensional condition in the laboratory due to the existence of specimen thickness. From this point of view, the smaller the specimen thickness is, the more accurate the Brazilian test results should be. In 1990, based on a series of statistical analyses, Newman and Bennett pointed out that the thickness-to-diameter ratio has a significant effect on the Brazilian tensile strength. Based on the finite element analyses and experimental results, Yu et al. (2006) concluded that Eq. (1) to estimate the Brazilian tensile strength of rock is inaccurate when the Brazilian disc has a significant thickness. Then in 2009, Yu et al. adopted a smaller thickness-to-diameter ratio of 0.2 in order to reduce the effect of three-dimensionally in their study.

In previous studies, experimental method or the finite element method was often used in the analyses of the Brazilian test. While in this paper, the three-dimensional discrete element analysis program PFC3D will be applied to study the influencing factors on Brazilian test results because of the superiorities of discrete element method (DEM) in dealing with the material failure naturally by simulating the failure evolution process from micro crack to macro fracture without any complex constitutive models (Zhang and Wong, 2012; Scholtes and Donze, 2013; Zhang et al., 2014). In PFC3D, a successful model with one set of micro-scale parameters that can capture all the characteristics of material is the foundation and prerequisite for all subsequent analyses. These characteristics or macro-scale properties of materials can be achieved by a series of laboratory tests (triaxial test, Brazilian test and so on), then the micro-scale parameters are chosen by means of a calibration process to reproduce the relevant material properties measured in these tests.

2. PFC MODELS

During the past 10 years, several researchers have attempted to model the macro-scale responses of brittle rocks using the PFC program. In 2004, a bonded-particle model (BPM) was proposed by Potyondy and Cundall in which the rock is represented by a dense packing of spherical particles that are bonded together at their contact points. Laboratory properties of Lac du Bonnet (LdB) granite obtained from a series of triaxial and Brazilian tests were then simulated by this model using PFC3D. Table 1 indicates the differences between the laboratory results of LdB granite after Martin CD (1993) and the simulation results of BPM after Potyondy and Cundall (2004).

It is worthwhile to note that the BPM successfully reproduces the elastic modulus, the Poisson's ratio and the unconfined-compressive strength of LdB granite, while huge differences exist in the values of the slope of the strength envelope, the friction angle and the cohesion. Besides, the tensile strength of the BPM is nearly three times of that of laboratory results. Potyondy and Cundall (2004) mentioned that these discrepancies may arise from the use of regular spherical grains in their model. Take into account this, Cho et al. (2007) proposed a two-dimensional clumped particle model (CPM), in which irregular-shaped particle assemblies (or clumps) acting as single particles are created to represent rock grains of certain sizes. The strength envelope of LdB granite is captured successfully by CPM in PFC2D, however, because of the limitation in two-dimensional analysis, the Poisson's ratio of Cho's model was not calibrated to match the laboratory result.

Table 1 Laboratory results and simulation results of LdB granite (Martin CD, 1993; Potyondy and Cundall, 2004)

Case	LdB granite	BPM
Elastic modulus	69±5.8 GPa	69.2 GPa
Poisson's ratio	0.26±0.04	0.26
Unconfined-compressive strength	200±22 MPa	198.8 MPa
Slope of strength envelope	13.0	3.28
Friction angle	59.0°	32.1°
Cohesion	30 MPa	55.1 MPa
Tensile strength	9.3±1.3 MPa	27.8* MPa

* This value is generated when the thickness is nearly the half of the diameter of Brazilian disc.

To overcome the limitation of bonded-particle model that if one matches the unconfined-compressive strength of a hard rock, then the tensile strength of the model will be much larger than that of the real rock, Potyondy (2010) developed a grain-based model (GBM) to respect the microstructure of brittle rock by overlaying a polygonal grain structure on the BPM and representing the interfaces via smooth-joint contacts. As provided in Table 2, the laboratory properties of Aspo diorite were all matched by GBM in PFC2D, with the exception of the Poisson's ratio, which is far less than that of Aspo diorite. And this discrepancy was also ignored due to the limitation in two-dimensional analysis of the GBM.

Table 2 Laboratory results and simulation results of Aspo diorite (Potyondy 2010)

Case	Aspo diorite	GBM
Elastic modulus	74 GPa	76 GPa
Poisson's ratio	0.27	0.12
Unconfined-compressive strength	193 MPa	194 MPa
Peak strength (7 MPa confinement)	231 MPa	228 MPa
Tensile strength	10.1 MPa	9.2* MPa

* This value is obtained by direct-tension test.

Table 3 lists the main advantages and disadvantages of these three previous models in reproducing the macro-properties of rocks. Although CPM and GBM overcome the disadvantages of BPM in strength envelope and tensile strength, both of them are prepared in a two-dimensional environment using the program PFC2D, in which the effects of Poisson's ratio on laboratory behaviors of rocks are ignored automatically. In addition, the direct-tension test is adopted in GBM to obtain the tensile strength of rock material, which avoiding the simulation of the Brazilian test. It is still uncertain that if these two models can reproduce the laboratory behaviors of rocks in a three-dimensional scale, thus in this paper, in order to study the Brazilian test results using PFC3D we first need to develop a new three-dimensional model that can reproduce all material properties of rocks.

From the above analyses, complex shaped grains can make a better reproduction of the properties of rock materials. In the model to be developed in this study, the stamp logic proposed by Cho et al. (2007) are introduced into the three-dimensional environment to create complex shaped grains by stamping different spherical areas which correspond to the desired grain sizes so that the unbreakable grains can be formed by the particles within these spherical areas. A typical triaxial test specimen (63.4×31.7×31.7mm^3) of this complex-shaped grain model is shown in Figure 1, in which 159,594 particles are contained and 37,718 complex shaped grains represented by various colors are formed by clumping these particles.

Table 3 Comparison of some previous models for rock

	Advantages	Disadvantages
BPM	3-Dimension Poisson's ratio	Strength envelope Tensile strength Spherical particles
CPM	Strength envelope Tensile strength Irregular-shaped grains	2-Dimension Poisson's ratio
GBM	Strength envelope Direct tensile strength Polygonal grains	2-Dimension Poisson's ratio

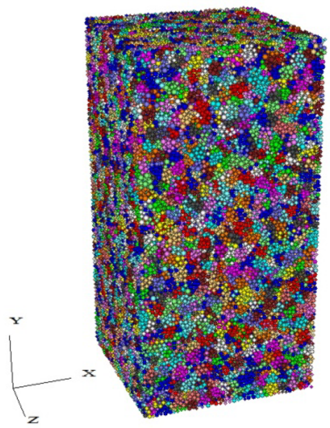

Figure 1 Triaxial test specimen of complex-shaped grain model (159,594 particles, 37,718 grains)

The calibration process described in Potyondy and Cundall (2004) is used to determine the micro-parameters of this complex-shaped grain model. After calibration, the micro-parameters listed in Table 4 are chosen to match the laboratory properties of LdB granite discussed in Table 1, in which $R_{\min}$ is the minimum particle radius, $R_{\max}/R_{\min}$ is the particle radius ratio, E_c is the contact modulus, $\bar{E}_c$ is the parallel bond modulus, k_n/k_s is the contact stiffness ratio (normal to shear), $\bar{k}_n/\bar{k}_s$ is the parallel bond stiffness ratio (normal to shear), $\bar{\lambda}$ is the parallel bond radius ratio, μ is the coefficient of friction, $\bar{\sigma}_n$ is the bond normal strength, $\bar{\sigma}_s$ is the bond shear strength, R_{stamp} is the radius of stamp and ρ is the density of particles.

About these micro-parameters, three points are worth mentioning: (1) the minimum particle radius is set to half of that of the BPM to increase the accuracy rating of measure results, (2) the bond shear strength is set to much higher value than that of bond normal strength to keep the material failing in a brittle fashion (Itasca Consulting Group Inc., 2005), (3) the non-uniform sizes of complex shaped grains are determined by specifying the radius of the stamp with a standard deviation.

Table 4 Micro-parameters used to represent the LdB granite

R_{min}	0.2975 mm	E_c	24 GPa
R_{max}/R_{min}	1.66	k_n/k_s	1.4
$\bar{\lambda}$	1.0	$\bar{E}_c$	24 GPa
μ	0.1	$\bar{k}_n/\bar{k}_s$	1.4
$\bar{\sigma}_n$	10±3 MPa	R_{stamp}	1.15±0.2 mm
$\bar{\sigma}_s/\bar{\sigma}_n$	32	ρ	2630 kg/m^3

The macro-scale properties of this complex-shaped grain model in this study are obtained by performing a series of triaxial tests and Brazilian tests. For triaxial tests, the specimen illustrated in Figure 1 is loaded by top and bottom walls, and the lateral walls are controlled by a servo-mechanism to apply constant confining stress to the specimen. The elastic constants are computed using the stress and strain increments occurring between the start of the test and the point at which one-half of the peak stress has been obtained (Potyondy and Cundall, 2004). For example, the Poisson's ratio is computed by

$$v = -\frac{\Delta\varepsilon_x}{\Delta\varepsilon_y} = -\frac{\frac{1}{2}\left(\Delta\varepsilon_x + \Delta\varepsilon_y\right)}{\Delta\varepsilon_y} = \frac{1}{2}\left(1 - \frac{\Delta\varepsilon_v}{\Delta\varepsilon_y}\right) \qquad (2)$$

where the average of the two lateral strains is used to approximate the lateral strain, and volumetric strain $\Delta\varepsilon_v = \Delta\varepsilon_x + \Delta\varepsilon_y + \Delta\varepsilon_z$.

For Brazilian tests, the specimen is trimmed into a cylinder or disc (as shown in Figure 2) which is still in contact with lateral walls, and the lateral walls act as loading platens by moving toward one another at a certain velocity. It should be pointed out that in order to decrease the possible influence of specimen thickness on test results; the thickness t is chosen to be one tenth of the specimen diameter (31.7mm) in these Brazilian tests.

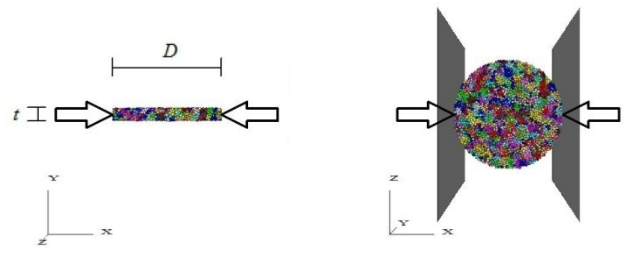

Figure 2 Schematic diagram of Brazilian test

Owing to the random distribution of particle radii and some micro-parameters, test specimens with different packing arrangements will produce different results. Mostly, researchers will carry out several repeated runs on specimens with different packing arrangements to obtain the average values of the test results (Potyondy and Cundall, 2004; Zhang and Wong, 2012, 2013). However, according to Zhang and Wong (2012), there is little difference between the results of specimens with different packing arrangements. Furthermore, in this present study, the goal is to solely study the influencing factors of Brazilian test results. Hence, in order to reduce the computation time, specimens with a certain kind of packing arrangement are adopted and no repeated runs are implemented on them in this study.

Table 5 compares the laboratory results of LdB granite and the simulation results of the present model in this study. It can be observed that all macro-properties of the present model agree well with that of the real LdB granite. In particular, the Poisson's ratio and the tensile strength are all captured by the complex-shaped grain model.

Table 5 Comparison of LdB granite and the complex-shaped grain model

Case	LdB granite	Complex-shaped grain model
Elastic modulus	69±5.8 GPa	67.4 GPa
Poisson's ratio	0.26±0.04	0.28
Unconfined-compressive strength	200±22 MPa	214 MPa
Slope of strength envelope	13.0	11.4
Friction angle	59.0°	57.0°
Cohesion	30 MPa	31.7 MPa
Tensile strength	9.3±1.3 MPa	10.9* MPa

* This value is generated when the thickness is one tenth of the diameter of Brazilian disc.

Figure 3 displays the strength envelope of LdB granite and the complex-shaped grain model. Despite the fact that the present model shows slightly higher unconfined-compressive strength and a smaller slope of the strength envelope, the agreement between these two envelopes is still excellent.

Compared with previous models, smaller particle size which provides more accurate simulation results, and irregular shaped grain which is more similar than a spherical particle to the grain microstructure of real rock, are adopted in the complex-shaped grain model developed in this study. From the results, the three-dimensional analyses and appropriate micro-parameters successfully reproduce the Poisson's ratio and the strength envelope of LdB granite. Moreover, by the simulation of the Brazilian test, the tensile strength of this model is very close to the laboratory result. From the point of view of the macro-properties reproduction, this complex-shaped grain model overcomes the main deficiencies of previous models we discussed in Table 3, and hence, providing us good conditions for further studying the influencing factors on Brazilian test results of brittle rock using PFC3D.

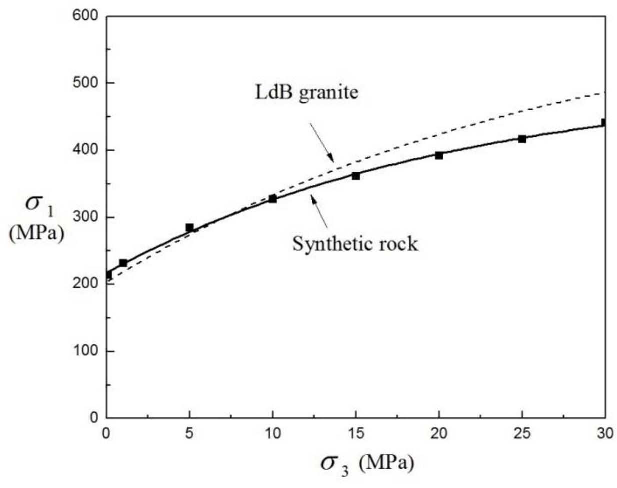

Figure 3 Strength envelopes of LdB granite and the complex-shaped grain model

3. RELATIONSHIP BETWEEN POISSON'S RATIO AND TENSILE STRENGTH

The Poisson's ratio and tensile strength are two separate macro responses of rock material. The relationship between them has often been overlooked, thus in this section, two additional rock models which followed the same generating procedure with the complex-shaped grain model are developed, while different micro-parameters are used in these three models in order to produce different macro responses.

In this section, the direct tensile strengths of these three models are also measured by performing direct-tension tests, during which the triaxial test specimen also need to be trimmed into a cylinder (see Figure 4), then the gray grains located at each end of the cylinder are griped and assigned to a constant velocity to simulate the equal and opposite tension to the direct-tension test specimen (shown in orange in Figure 4). Furthermore, in this study, both the thickness t and the diameter of the direct-tension test specimen are required to be equal to that of the Brazilian disc, thus to make sure that the specimens applied in these two kinds of tests are identical.

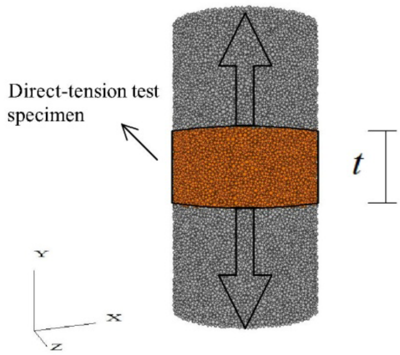

Figure 4 Schematic diagram of direct-tension test

Table 6 compares the macro-scale properties of these three models. There are no large differences in most of the results, except the Poisson's ratio and tensile strengths. This comparison illustrates that material models with similar macro responses but different Poisson's ratio may have different tensile properties, which verify the opinion that the CPM and GBM may not have reproduced all the behaviors of respective brittle rock exactly. Figure 5 shows the relationship between the Poisson's ratio and the tensile strengths.

Table 6 Comparison among three different rock models

Case	Comparison model #2	Complex-shaped grain model	Comparison model #3
Elastic modulus	70.2 GPa	67.4 GPa	68.7 GPa
Poisson's ratio	0.14	0.28	0.34
Unconfined-compressive strength	215 MPa	214 MPa	202 MPa
Slope of strength envelope	10.9	11.4	12.8
Friction angle	56.3°	57.0°	58.8°
Cohesion	32.6 MPa	31.7 MPa	28.2 MPa
Brazilian tensile strength	12.8* MPa	10.9* MPa	8.9* MPa
Direct tensile strength	13.3 MPa	8.0 MPa	7.8 MPa

* These values are generated when the thickness is one tenth of the diameter of Brazilian disc.

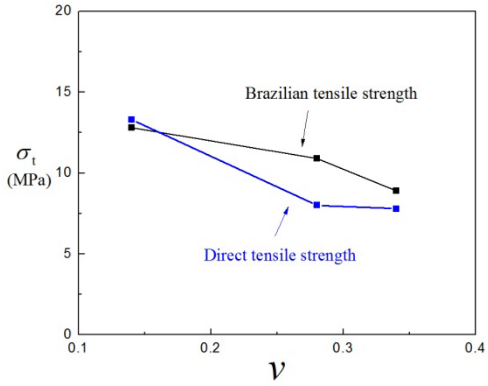

Figure 5 Relationship between the Poisson's ratio and the tensile strengths

More importantly, this comparison may reveal a relationship between the Poisson's ratio and tensile strength based on the test results of PFC3D models. It seems likely that a smaller Poisson's ratio corresponds to a larger tensile strength. Furthermore, both Brazilian tensile strength and direct tensile strength have such characteristic, which eliminates the possible effects of testing methods. However, the limited data in this study are far from sufficient to reach a conclusion. Considering that, for the time being, our study is limited in the field of numerical simulation, hence, no further studies are done to define this relationship in this paper.

4. EFFECT OF SPECIMEN THICKNESS ON TENSILE STRENGTH

In order to examine the effect of specimen thickness on the Brazilian tensile strength, 10 Brazilian discs with different thickness are prepared for Brazilian tests. All of these discs are trimmed from the triaxial test specimen of the present complex-shaped grain model, and have the same diameter (31.7mm) but different thickness-to-diameter ratio t/D which varies from 0.1 to 1.0. Meanwhile, 10 direct-tension test specimens which are in one-to-one correspondence and are identical in shape and size with those Brazilian discs are also prepared. Direct-tension tests are performed on them to compare with the Brazilian test results.

Completely different results obtained by these two testing methods are shown in Figure 6. It can be seen that for all specimens, direct tensile strengths keep unchanged at 8MPa, while the Brazilian tensile strengths present a growing trend (10.9MPa ~ 26.8MPa) with the increase of the specimen thickness. Although 10.9MPa is very close to the direct tensile strength and the Brazilian tensile strength may well be much closer to the direct tensile strength when the specimen thickness continues to decrease, it is the direct tensile strength which more truly reflects the tensile properties of brittle rock.

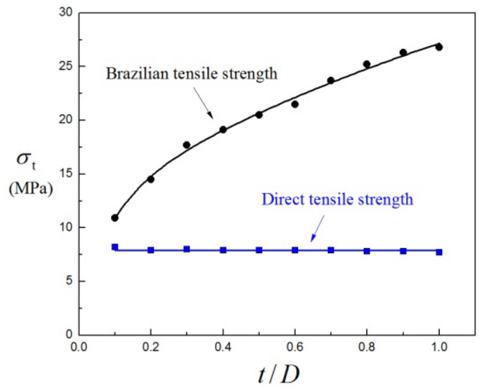

Figure 6 Effect of specimen thickness on the tensile strength.

The diverse trends of the test results indicate that the Brazilian test may well not be reasonable, especially in a larger thickness condition, and furthermore, verify that there does exist a thickness effect on the Brazilian tensile strength. According to Formula (1), the increase of specimen thickness may cause a decrease of Brazilian tensile strength, but why a growing trend of the results is observed in these Brazilian tests, the superiority of PFC3D in micro-structure simulating will provide us a new view to explore this question.

Figures 7 and 8 show the stress-strain curves and the crack developing processes of the Brazilian test specimens with different thickness-to-diameter ratio (0.1 and 0.5, respectively). During Brazilian test, tension-induced parallel-bond failure will occur when the parallel-bond normal strength between two bonded particles has been exceeded, then red lines lying between the two previous bonded particles are used to represent the tension-induced cracks. In Figure 7 ($t/D=0.1$), it was evident that in the early stage of loading, relatively few cracks randomly formed inside of the specimen, and with the increase of loading, more cracks formed and evenly distributed along the loading direction (Figure 7b). As the loading continues to grow, macro fracture initiated from the crack interconnection and traveled through the center of the specimen in a direction parallel to the loading (Figure 7c). Near peak loading, one or more macro fracture cut across the specimen, and more and more cracks occurred throughout the specimen. The parallel-bond distribution in the specimen at this time is also shown in Figure 7e, in which each black line represents the parallel-bond lying between two bonded particles, and obvious white space induced from the parallel-bond failure, namely the macro fractures, could be clearly observed.

In Figure 8 ($t/D=0.5$), similar crack developing processes can be found in the early stage of loading. Cracks are first formed inside of the specimen and then evenly distributed along the loading direction (Figures 8a and 8b). After that, macro fractures occurred from the center of the specimen and propagated along the loading direction (Figure 8c). However, due to the increased thickness, under same strain condition, significantly more cracks were observed in Figure 8d than the case of $t/D=0.1$ in Figure 7d, but at the same time less parallel-bond failures occurred in Figures 8e and 8f, which respectively show the parallel-bond distribution in the center section and one end section of the specimen, both of which have same thickness with the specimen in Figure 7e. As the loading reached maximum, more obvious macro fractures induced from parallel-bond failures (as seen in Figures 8h and 8i) cut across the specimen, meanwhile cracks had already spread almost all over the specimen.

Comparing the phenomena in specimens with different thickness, the discrepancies may arise from the diversities of crack propagation path in three-dimensional environment. As is known to all, macro fractures caused by crack propagation need to appear not only in the direction of loading, but also in the direction of thickness to make the specimen broken. For a specimen with a smaller thickness, the crack propagation in the direction of the thickness is more easily to form macro fracture that travel across the specimen. However, for a specimen with a larger thickness, greater force is needed to promote the development of macro fracture in the direction of thickness, the loading required for its broken is far more than that of a thinner specimen. Exactly because of the greater force that leads to overdeveloped fractures in the direction of loading (Figures 8h and 8i) and more cracks occurring throughout the specimen (Figure 8g) when the specimen is broken. When the loading increment in Brazilian test significantly exceeds the specimen thickness increment, according to Eq. (1), Brazilian tensile strength undoubtedly will increase with the specimen thickness.

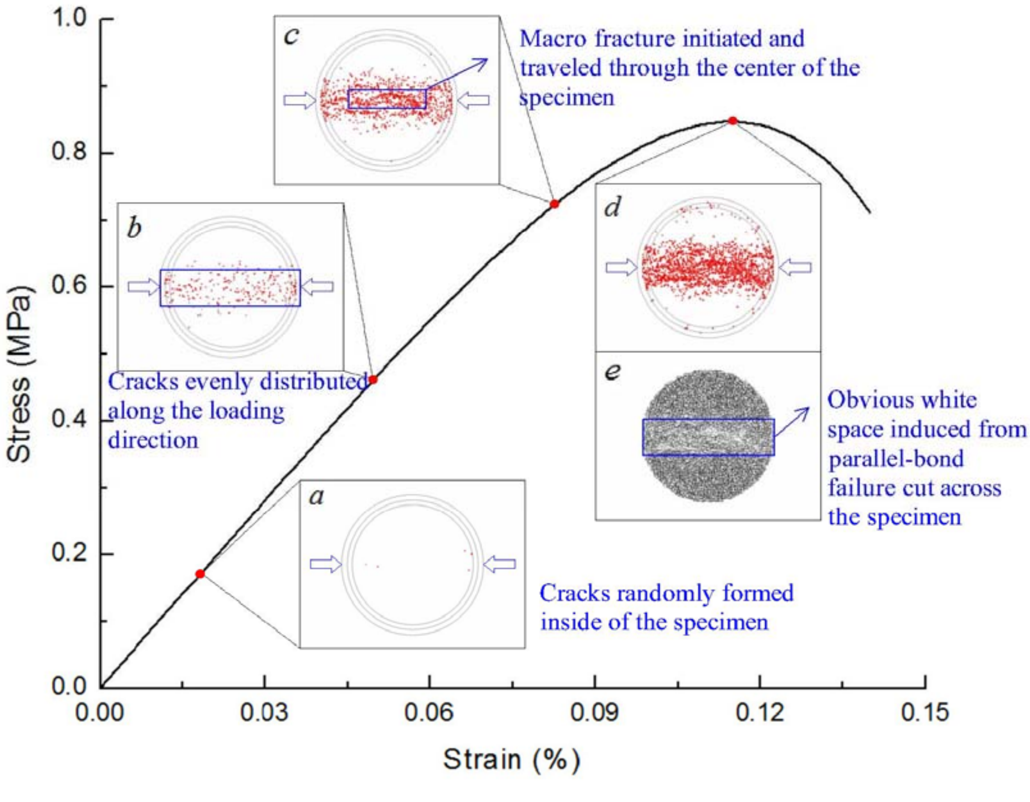

Figure 7 The stress-strain curve and crack developing processes of specimen with $t/D=0.1$

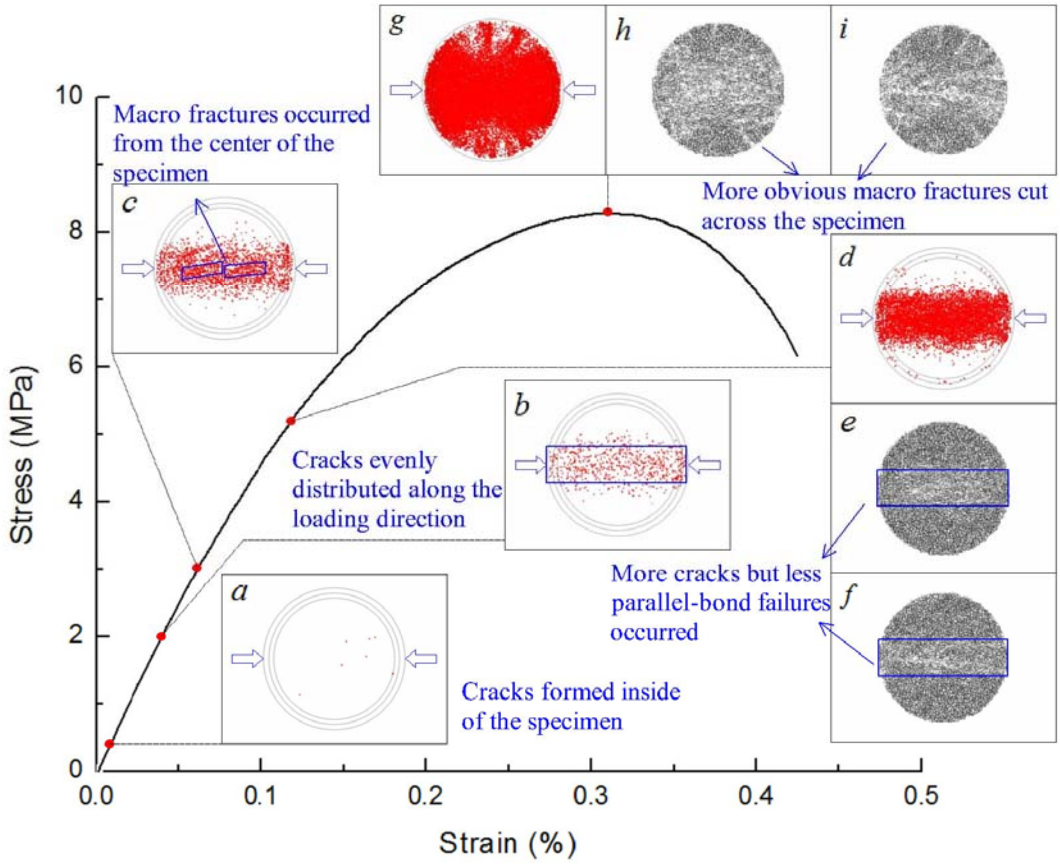

Figure 8 The stress-strain curve and crack developing processes of specimen with $t/D = 0.5$

5. CONCLUSIONS

In this study, the influencing factors on Brazilian test results, including the Poisson's ratio and specimen thickness, have been investigated using discrete element analysis program PFC3D. The success of the proposed complex-shaped grain model in reproducing many macro-properties of brittle rock, such as the Poisson's ratio, tensile strength and strength envelope, not only overcomes the main deficiencies of previous models but also provides us a solid foundation for the Brazilian test study.

A comparison among the test results of three different complex-shaped grain models successfully captures a relationship between the Poisson's ratio and tensile strength which seems likely that a material with a smaller Poisson's ratio may have a larger tensile strength. However, due to the limitation of numerical simulation, no further studies are done to define the relationship in this paper.

Ten Brazilian tests and ten direct tensile tests are performed on cylinder specimens with different thickness in PFC3D. Brazilian tensile strengths present a growing trend while the direct tensile strengths keep unchanged with the increase of specimen thickness. The reason of this phenomenon is explored by comparing the stress-strain curves and crack developing processes of the Brazilian test specimens with different thickness, which indicate that the increase of specimen thickness will cause great increase in the loading required for specimen broken, and when the loading increment significantly exceeds the thickness increment, Brazilian test results will increase with the specimen thickness, hence lead to overestimating the real tensile strengths of the specimens.

6. ACKNOWLEDGEMENTS

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