Seismic Analysis of Reinforced Soil Wall Considering Oblique Pull-out of Reinforcements: A Review

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ABSTRACT: Several methods are available for stability analysis of reinforced soil structures. However, most of these methods mainly concentrated on the horizontal pull-out of the reinforcement in spite of the evidences available that show the failure surface of reinforced soil structure will always intersect reinforcement layers diagonally due to the failure kinematics. It will cause oblique/transverse deformation to reinforcements across the failure surface. In the present paper, state-of-the-art review of earthquake stability analysis of reinforced soil-wall by employing the oblique/transverse pull of reinforcements is discussed. Formulations that are developed in various studies to determine the mobilization of diagonal pullout resistance of reinforcements, the amount of drag force triggered in the reinforcement sheets due to instability in the structure and the factor of safety against pull-out are presented. A comparative study is also carried out between existing models and methods that are used in determining the seismic stability of reinforced soil structure subjected to diagonal pullout of soil reinforcements. The comparative study shows the effect of various models and methods on the factor of safety against reinforced soil-wall stability and the influence of different parameters i.e., horizontal seismic acceleration, internal friction angle of soil, interface friction angle of soil and reinforcement, relative subgrade stiffness factor etc. Depending on the model used in analyses, the computed factor of safety may vary significantly.

KEYWORDS: Reinforced soil-wall, Oblique pull, Horizontal slice method, Earthquake, Soil reinforcement.

1. INTRODUCTION AND BACKGROUND

Reinforced soil structures are being vastly used all over the world, even in seismically active areas because of its versatility, cost effectiveness, structural flexibility, high load carrying capacity, long term durability and fast track construction. It is striking that even under severe earthquake shaking, reinforced soil structures performed quite well [e.g. 1994 earthquake in Northridge (Sandri, 1997; White and Holtz, 1997) and 2011 Tohoku earthquake (Koseki, 2012)] and no major earthquake induced failures were reported. This satisfactory performance under several earthquakes show the conservatism involve in static design procedures beside that a large factor of safety is used instead of adopting an appropriate seismic design. Thereby, in recent years, a great deal of attention has been paid to examine the behaviour of reinforced soil structures subjected to earthquake load, and various design procedures have been proposed. Theoretical approaches generally consist of limit equilibrium and the limit analysis method. Leshchinsky et al. (1995) came up with a new design approach for reinforced soil structures by using limit equilibrium method. The study was conducted without considering any earthquake load. Later, Ling et al. (1997) extended the study by considering the seismic force. The pseudo-static method was employed in this study, where the earthquake load is approximated as equivalent static force acting on the soil wedge which is likely to fail. Only the horizontal earthquake acceleration was considered in this study and impact of vertical earthquake acceleration was overlooked. The significance of vertical seismic acceleration on the safety of reinforced soil wall and slope was investigated by Ling and Leshchinsky (1998) and reported that for tieback analysis, upward and downward vertical seismic acceleration both are equally important. Bathurst and Cai (1995) presented a pseudo-static based Mononobe-Okabe (M-O) method to examine the earthquake stability of a geosynthetic reinforced segmental retaining wall and provided factor of safety against the internal and external stability as well as facing failure modes. The impact of the vertical earthquake force on a Tanata wall was investigated by Huang and Wang (2005) and they reported that the displacement and the stability of a Tanata wall is hardly influenced by any proportion of a vertical earthquake force compare to a horizontal earthquake force. The pullout resistance offered by soil reinforcements is a major factor contributing to the seismic stability of a reinforced soil wall. Garg (1998) proposed a theoretical formulation for the design of a gravity retaining wall with reinforced soil backfill and catalogued the design, construction and cost economics of an 11m high retaining wall in the Indian Himalaya. Keeping several constraints of the vertical slice method in mind, Shahgholi et al. (2001) proposed the Horizontal Slice Method (HSM), a new limit equilibrium method efficient in analysing stability of reinforced earth structures. In this method a failure wedge from the backfill soil is considered by identifying the failure plane. The failure plane makes an angle with the horizontal depending upon the soil parameters, seismicity and geometry of the reinforced earth structure. It is assumed that the failure soil mass is split into several horizontal slices and the equilibrium of each slice is taken into consideration in the stability analysis of the wall. Nouri et al. (2006) modified the existing HSM and upgraded it for a rigorous analysis. Unlike previous methods, here vertical and horizontal force equilibrium along with the moment equilibrium of each slice has been considered and that helps to achieve more accurate results. Analyses were carried out under the static load condition only. Nouri et al. (2008) employed the modified HSM in the analysis that estimate the critical reinforcement length, which is essential to keep the reinforced soil wall internally stable. Nimbalkar et al. (2006a,b) conducted a study to analyse the safety of a reinforced earth wall against internal failure by employing harmonic earthquake excitation at the base in the form of introducing new pseudo-dynamic method. The basic HSM is used in this study, in which vertical forces acting on a single slice and horizontal forces acting over the entire soil failure wedge are made balance in the analysis. The result reported in this study exhibits that the pseudo-static analyses underestimate the length of the reinforcement required to maintain the structural stability. Ahmad and Choudhury (2008, 2012) studied internal stability of reinforced waterfront soil-wall. Choudhury and Modi (2008) provided the displacements of reinforced soil-slopes considering seismic stability analysis using planar failure surface. Choudhury and Ahmad (2009) used pseudo-static method for external stability of reinforced waterfront soil-wall. Choudhury et al. (2007) used the pseudo-dynamic method to determine the reinforcement length required to maintain the earthquake induced sliding and overturning stability of a reinforced earth wall without taking into account influence of the amplification factor on the earthquake excitation. Basha and Babu (2010, 2011) proposed a probabilistic model for the efficient design of reinforced soil structures. Here impact of the soil friction angle, earthquake acceleration, geometry and properties of reinforcements on the factor of safety against internal failure of a reinforced earth wall are investigated. Pull-out capacity of the reinforcement is also studied. A logarithmic spiral failure surface and the pseudo-dynamic method with both vertical and horizontal sinusoidal excitations acted at the base level are considered in these analyses. Basha and Babu (2012) used the similar probabilistic model and pseudo-static approach to study the impact of the coefficient of variation (COV) of soil shear strength and reinforcement capacity, for a targeted value of the system reliability index on the reinforcement length and number of reinforcement layers under certain seismicity. Vahedifard et al. (2012) proposed a limit equilibrium solution to analyse stability of a reinforced soil retaining wall by considering failure surfaces of logarithmic spiral shape and the earthquake load was applied pseudo statically. Vahedifard et al. (2016) conducted a theoretical study for the optimization of profile facing elements of a reinforced earth retaining structure and reported that the concave profile of wall facing is the most efficient for practice. Zevgolis and Bourdeau (2017) presented an advance model for the reliability assessment of safety of a reinforced earth wall against internal failure subjected to static load. A limit equilibrium analysis is presented by Pain et al. (2017a) to examine the impact of frequency content of a harmonic excitation and dynamic soil properties on the internal stability of reinforced soil retaining structures by adopting log spiral failure mechanism with application of modified pseudo-dynamic method.

The previous studies (e.g. Juran et al., 1988; Ochiai et al., 1996, Sobhi and Wu, 1996) that are carried out to probe the stability of reinforced earth structures assumed that the pullout of reinforcement from the backfill is purely axial and the transverse displacement of reinforcement along the failure surface is ignored. However, evidences from the field observations and experimental studies (Shewbridge and Sitar, 1989; Bergado et al., 2000) suggest that because of the failure mechanism (the failure wedge slide down along the failure surface), reinforcements experience a transverse or oblique pull along the failure surface and the pullout does not remain axial as shown in Figure 1 (Patra and Shahu, 2012). In case of axial pull, a uniform normal stress (i.e. soil overburden pressure) is considered over the reinforcement layers. However, in reality an oblique force acts on the reinforcements along the failure plane and mobilize additional normal stress. Since the mobilization of friction between soil and reinforcements interface is proportional to the normal stress, ignoring the oblique force may result conservatism in the design.

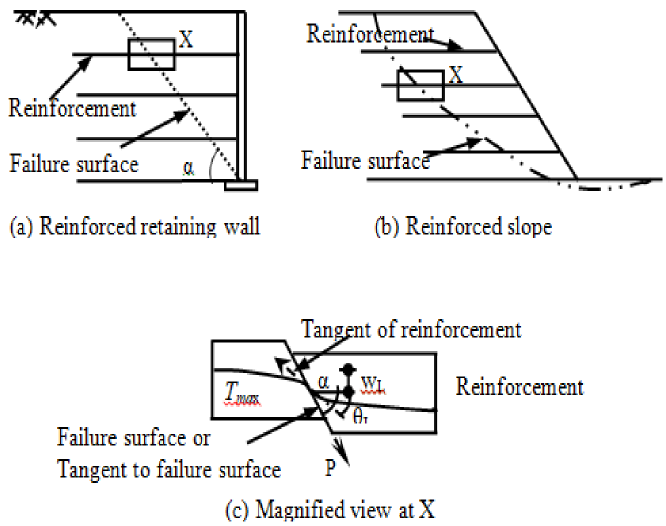

Figure 1 Kinematics of the reinforced structure failure (Patra and Shahu, 2012)

Madhav and Umashankar (2003) theoretically analysed the pull-out response of a rigid geosynthetics sheet subjected to small end displacement. The subgrade soil was represented by a number of Winkler's springs. Equilibrium equations proposed in the study do not take into account the ultimate distorted shape of the geosynthetic. Thereby, the study is only valid for minor end displacement. Later Shahu (2007) modified this study by incorporating the final deformed shape into the equilibrium equations. Furthermore, Patra and Shahu (2012) extended this modified work and presented a new model where the Pasternak model is used to illustrate the soil subgrade instead of the Winkler springs. Reddy et al. (2008a,b) employed the model of Madhav and Umashankar (2003) in analysing the stability of a vertical reinforced soil wall under static and earthquake load. The increase in friction at the interface between geosynthetic and soil caused by the transverse drag of geosynthetic sheet is considered in the calculation of the factor of safety and the result demonstrated that the factor of safety against pullout increased by 10% even at the very high level of seismic acceleration (k_h = 1). Gao et al. (2014) made few changes in the HSM proposed by Nouri et al. (2006) and employed it along with the Pasternak model (Patra and Shahu, 2012) to estimate the diagonal pullout resistance of a rigid sheet reinforcement and the factor of safety for a vertical reinforced soil wall based on the pseudo-static method. The results are compared with the previous results and observed that the factor of safety obtained in this investigation is higher than that in Reddy et al. (2008).

All the previous pseudo-static studies do not take into account the actual dynamic nature of the earthquake loads and ignore the effects of time component, amplification, velocity of body waves travelling through the backfill and frequency content. Reddy et al. (2009) incorporated these effects except the amplification factor by adopting the pseudo-dynamic method in analyses and pointed out that the pseudo-static method gives conservative estimation.

The intent of the present study is to assess the effect of diagonal/oblique or transverse displacement of reinforcements on the stability of a reinforced earth wall subjected to earthquake shaking by reviewing different published literature. The assumptions made in developing different models have been pointed out, and finally a comparison is made between different studies to learn the effect of diagonal pull-out or displacement.

2. METHOD OF ANALYSIS

Schematic diagram of a reinforced earth retaining wall has been portrayed in Figure 2. The height of the wall is *H*. The backfill has *n* number of equally spaced reinforcement layers of length *L*. Therefore the distance between two consecutive reinforcement layers is *H/n* for the intermediate layers. The thickness of the top and the bottom layer is different from intermediate layers. γ is the unit weight of the retained soil. ϕ is the internal friction angle of the backfill soil and ϕ_r is the interface friction between the reinforcement layer and soil. Depth of the j^{th} layer reinforcement from the ground is h_j and the failure plane making an angle α with horizontal. Planar failure surfaces are considered in all the analyses since a substantial number of laboratory experiments, mostly shake table and few centrifuge tests, on the reinforced earth wall and slope models confirm that the commonly detected failure surface under the earthquake shaking is a logarithmic spiral which turns into a planar failure surface for near vertical slopes and vertical walls (Nouri et al., 2008). The values of the failure plane angle α changed with the changing value of earthquake excitation, soil internal friction angle ϕ and wall friction angle (Kramer, 1996).

2.1 Pseudo-static method

Reddy et al. (2008) and Gao et al. (2014) both used the conventional Mononobe-Okabe pseudo-static approach and the oblique drag of reinforcement for stability analysis of reinforced earth wall. They investigated the factor of safety for the entire wall against pullout failure of reinforcements. In these methods, inclination of the failure plane α (Figure 2) is calculated by using the Mononobe-Okabe pseudo-static method

$$\alpha = \phi - \psi + \tan^{-1}\left[\frac{-\tan(\phi-\psi) + C_{1E}}{C_{2E}}\right] \tag{1}$$

Where

$$C_{1E} = \sqrt{\tan(\phi-\psi)\left[\tan(\phi-\psi)+\cot(\phi-\psi)\right]\left[1+\tan(\delta+\psi)\cot(\phi-\psi)\right]} \tag{2}$$

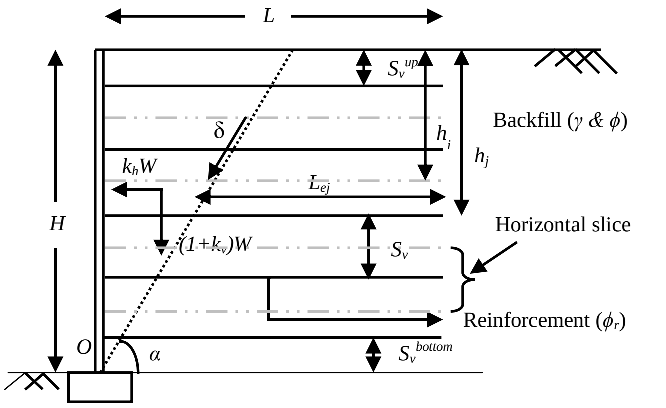

Figure 2 Schematic diagram of a reinforced earth wall with a planer failure surface and horizontal slices (Reddy et al., 2008)

$$C_{2E} = 1+\tan(\delta+\psi)\left[\tan(\phi-\psi)+\cot(\phi-\psi)\right] \tag{3}$$

and $\psi = \tan^{-1}\dfrac{k_h}{1-k_v}$ (4) (Kramer, 1996)

Where, k_h and k_v are the horizontal and vertical earthquake acceleration coefficient respectively.

This method is valid for only ϕ-$\psi \geq 0$; provided the backfill surface is plane and horizontal.
The following assumptions are made in both of these studies.

a) The internal friction angle, ϕ used in these analyses are factored where $\phi = \tan^{-1}[\tan(\phi_{peak})/F]$ and the value is equal to or less than $\phi_{residual}$. Here ϕ_{peak} and $\phi_{residual}$ are the peak and residual values of soil internal friction angle respectively. F, is a partial safety factor used to lower the value of soil strength parameters and can be obtained from a pertinent code of practice or proper engineering judgement.
b) Reinforcement sheets are rigid.
c) The backfill is isotropic and rigid plastic.
d) Shear resistance has completely mobilized at the interface between soil and reinforcement sheet irrespective of the relative displacement between them.

2.1.1 Method using Winkler spring model

Reddy et al. (2008) investigated stability of a reinforced earth wall subjected to earthquake excitation by considering the transverse displacement of reinforcements along the failure surface. The horizontal slice method developed by Shahgholi et al. (2001) and the model proposed by Madhav and Umashankar (2003) were used in this study. A rigorous solution needs to satisfy the three equilibrium conditions, horizontal and vertical force equilibrium along with the moment equilibrium equations for individual slices and the entire failure wedge, which is complex and needs tedious calculations to solve. Here, the formulation has been made more comprehensive by considering the vertical force equilibrium of each slice and horizontal force equilibrium of the entire failure wedge. Moment equilibrium equations have been overlooked as suggested by Shahgholi et al. (2001). In this case the downward movement of the failure wedge is considered as very small. For the i^{th}, slice, the algebraic sum of all vertical force components acting on it must be zero to reassure the equilibrium of the slice (Figure 3).

$$\sum F_{yi} = 0,$$

i.e.,

$$V_{i+1} - V_i - (1+k_v)W_i + S_i\sin\alpha + N_i\cos\alpha = 0 \tag{5}$$

Where, V_i and V_{i+1} are the vertical forces acting on the i^{th} slice, Wi is the weight of the i^{th} slice, N_i is the force acting perpendicularly on the slice failure plane, S_i is the tangential force acting along the slice failure plane and α is the failure plane angle with the horizontal.
For granular soils, the shear force S_i acting along the failure plane of the slice is a function of N_i, $\tan\phi$ and FS_{sr} and written as

$$S_i = \frac{N_i\tan\phi}{FS_{sr}} \tag{6}$$

Here, the value of FS_{sr} is considered as 1.0.

By substituting S_i of Equ. (6) into Equ. (5) and solving, expression for N_i is obtained

$$N_i = \frac{V_i - V_{i+1} + (1+k_v)W_i}{(\tan\phi/FS_{sr})\sin\alpha + \cos\alpha} \tag{7}$$

The vector sum of all horizontal force components acting on the failure wedge should be zero to satisfy equilibrium of the wedge. This condition is applied to get the expression for total tensile force $\sum t_j$, where t_j is the tensile force developed in j^{th} reinforcement due to wall instability and the value should be less than the ultimate strength of the reinforcement

$$\sum_{j=1}^{m} t_j = \sum_{i=1}^{n} N_i\sin\alpha - \sum_{i=1}^{n} S_i\cos\alpha + \sum_{i=1}^{n} W_i k_h \tag{8}$$

By considering the horizontal pull and ignoring the effect of transverse displacement, the total shear resistance mobilize in reinforcements can be determined by the following expression

$$\sum_{j=1}^{m} T_j = \sum_{j=1}^{m} 2\gamma h_j L_{ej}\tan\phi_r \tag{9}$$

Where, $h_j = (j-0.5)(H/n)$; the depth of the j^{th} reinforcement layer from the top and $L_{ej} = L-(H-h_j)\cot\alpha$; active length of the reinforcement shown in Figure 2. T_j =shear resistance mobilized in the j^{th} layer of reinforcement. m and n are the number of reinforcement layers and horizontal slices respectively.
The conventional factor of safety which take into account only the axial pullout of reinforcement is expressed as

$$FS_c = \sum_{j=1}^{m} T_j \bigg/ \sum_{j=1}^{m} t_j \tag{10}$$

But in reality, the failure plane intersect all reinforcement layers diagonally as the failure wedge slides downward (Figure 1). As a result of that an oblique or transverse displacement of reinforcements occurs along the failure surface, which in turn mobilizes an additional normal stress to the reinforcement layers. Since the mobilization of shear resistance is directly proportional to the applied normal stress, shear resistance mobilized at the reinforcement soil interface is considerably higher for the oblique deformation.

The procedure for calculating the pullout resistance of reinforcement, T_{Tj} of the j^{th} layer are similar to the conventional approach with an additional component for the increase in drag due to the transverse displacement and the expression is

$$T_{Tj} = 2\gamma h_j L_{ej} \tan\phi_r + P_j \tan\phi_r \tag{11}$$

Where, P_j is the additional transverse force on the j^{th} layer due to the oblique displacement of reinforcement.

$$P_j = \gamma h_j L_{ej} P_j^* \tag{12}$$

Where P_j^* is the normalized oblique force in the j^{th} layer. Besides overburden pressure exerting at the top and bottom, the inextensible reinforcement experience a transverse force P due to the transverse displacement w_L at the free end as shown in Figure 4, which induce an additional pressure at the soil below the reinforcement. The backfill is represented by a set of Winkler springs and the normalized tension $T_{j,k}^*$ and normalized displacement $W_{j,k}$ at the failure surface of the j^{th} layer are determined by adopting the model proposed by Madhav and Umashankar (2003).

$$P_j^* = \mu_j \frac{w_L}{L_{ej}} \frac{1}{z}\left[\frac{W_1+1}{2} + \sum_{k=2}^{z} W_{j,k}\right], \tag{13}$$

$$W_{j,k} = \frac{T_{j,k}^* z^2 (W_{j,k-1} + W_{j,k+1})}{\{2z^2 T_{j,k}^* + (\mu_j / 2\tan\phi_r)\}} \tag{14}$$

$$T_{j,k+1}^* = \frac{1}{2z}\left(\mu_j W_{j,k} \frac{w_L}{L_{ej}} + 2\right) + T_{j,k}^* \tag{15}$$

Where, $\mu_j = \mu\left(L_{ej}H\right) / \left(Lh_j\right)$ local subgrade stiffness factor, $\mu = k_s L / \gamma H$; a global relative subgrade stiffness factor, $w_L = \delta \sin\alpha$, δ = oblique displacement along the failure surface, k_s is the modulus of subgrade reaction and z is the number of elements the reinforcement is divided into.

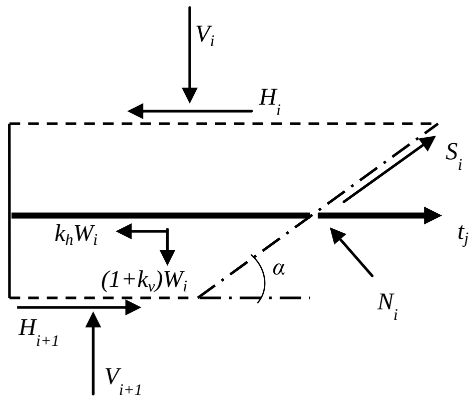

Figure 3 Forces acting on a typical horizontal i[th] slice (Reddy et al., 2008)

By taking into account the oblique pullout effect, the total shear resistance mobilized in the reinforcement layers is

$$\sum_{j=1}^{m} T_{Tj} = \sum_{j=1}^{m} 2\gamma h_j L_{ej} \tan\phi_r + \sum_{j=1}^{m} P_j \tan\phi_r \tag{16}$$

The Factor of safety FS_T, considering the oblique pull-out is obtained as the ratio of Equ. (16) to Equ. (18)

$$FS_T = \sum_{j=1}^{m} T_{Tj} \bigg/ \sum_{j=1}^{m} t_j \tag{17}$$

2.1.2 *Method using Pasternak model*

Gao et al. (2014) used the oblique pull-out model developed by Patra and Shahu (2012) and HSM proposed by Nouri et al. (2006) to study the impact of earthquake forces on the stability of a reinforced earth wall. The subgrade soil is depicted by the linear-elastic Pasternak model in the calculation. In this study, large end deformation of sheet reinforcement is considered and the effect of the deformation is incorporated into equilibrium equations.

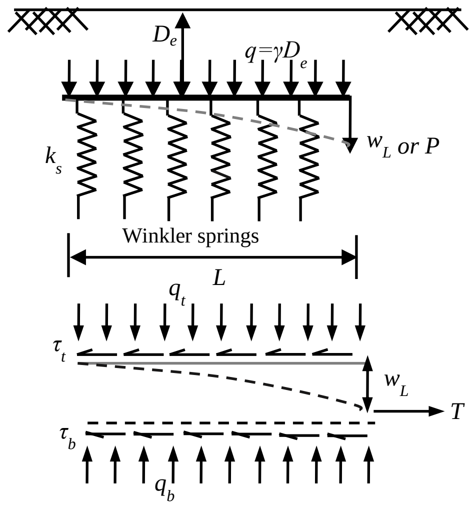

Figure 4 Schematic diagram of the proposed model by Madhav and Umashankar (2003), used in Reddy et al. (2008)

The shear resistance mobilized in each reinforcement, T_j, subjected to oblique pull-out is obtained as

$$T_{Tj} = 2\gamma h_j L_{ej} \tan\phi_r T_{Tj}^{*} \tag{18}$$

Where $L_{ej} = L-(H-h_j)\cot\alpha$ from the Figure 2.

$$T_{Tj}^{*} = \frac{1}{2z\cos\alpha}\sum_{k=1}^{z}\left\{\begin{bmatrix}\mu_j W_{j,k} W_{j,L} + 2 \\ -z^2 G_j^{*} W_{j,L}\left(W_{j,k+1} - 2W_{j,k} + W_{j,k-1}\right)\end{bmatrix}\cos\theta_{j,ck}\right\} \tag{19}$$

In which, z is the number of small segments into which j^{th} layer reinforcement is divided. μ_j and G_j^{*} are the local subgrade stiffness factor and local subgrade shear stiffness factor respectively. The expression for μ_j is the same as given in Reddy et al. (2008) and $G_j^{*} = G^{*}(HL/h_jL_{ej})$ where, $G^{*} = (GH_s/\gamma HL)$

G = shear modulus of the backfill soil, H_s = thickness of the shear layer assumed in the Pasternak model, L = total length of the reinforcement, L_{ej} = active length of the reinforcement (Figure 2). $W_{j,L}$ and $W_{j,k}$ are the normalized displacement of reinforcement at failure surface, and at node k of the j^{th} reinforcement layer respectively.

To improve the calculation efficiency, Gao et al. (2014) modified the HSM *(5N-1)* formulation proposed by Nouri et al. (2006) to a new formulation, consist of *4N* equations and employed it to determine of the tensile force generate in reinforcements to preserve the seismic stability of a reinforced earth wall. Here, along with the vertical and horizontal force equilibrium of each slice, the moment equilibrium for the whole failure wedge is considered in the formulation. An iterative procedure is adopted in order to estimate the value of t_j and $\sum t_j$. The factor of safety, FS_T is also presented in this study and the expression is same as the equation (17). At equilibrium, forces acting on an arbitrary i^{th} slice is shown in Figure 5.

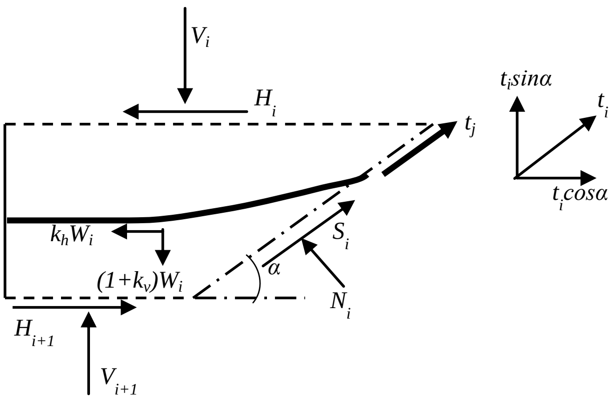

Figure 5 Forces acting on the i^{th} slice (Gao et al., 2014)

2.1 Pseudo-dynamic method

Unlike pseudo-static method, pseudo-dynamic method can partially capture the dynamic nature of the earthquake load like, the time component, amplification factor, the velocity of body waves and frequency of an earthquake is considered in the analysis. Steedman and Zeng (1990) first introduced the concept of pseudo-dynamic method and it is further modified by Choudhury and Nimbalkar (2005, 2006), by incorporating the primary wave velocity and amplification factor. In the pseudo-dynamic analysis, it is assumed that the horizontal and vertical harmonic excitations with the magnitude of acceleration a_h ($=k_hg$) and a_v ($=k_vg$) respectively act at the bottom of the wall and backfill. Here g is the gravitational acceleration. Primary and shear waves begin to propagate through the backfill at the same time and there is no phase difference between these propagating waves. Therefore, the magnitude of the horizontal and vertical acceleration at any depth d from the surface level of the backfill and at any time t, without considering any amplification (thus amplification factor f_s =1), can be demonstrated as,

$$a_h(d,t) = a_h \sin\left[\omega\left(t-\frac{H-d}{V_s}\right)\right] \tag{20}$$

$$a_v(d,t) = a_v \sin\left[\omega\left(t-\frac{H-d}{V_p}\right)\right] \tag{21}$$

Where ω is the angular frequency of earthquake excitation, V_s and V_p are the shear and primary wave velocity respectively. Though the pseudo-dynamic method has recently been modified by Pain et al. (2015, 2016, 2017b) and Rajesh and Choudhury (2017a, b, c), it is yet to be used for the seismic stability analysis of reinforced soil-wall extensively.

Reddy et al. (2009) extend the study of Reddy et al. (2008b) by using the pseudo-dynamic instead of pseudo-static method, in seismic stability analysis of a reinforced soil wall. The effect of oblique pull out of reinforcements on the shear resistance mobilization is also considered in this study.

Leave aside the equation of horizontal and vertical seismic inertia forces, all other formulations and mechanisms are same for both the studies (Reddy et al., 2008b and Reddy et al., 2009). For the pseudo- dynamic approach, the horizontal and vertical inertia forces acting on the i^{th} slice, of weight W_i, are calculated by the following equations

$$q_{hi} = W_i k_h \sin\left[2\pi\left(\frac{t}{T}-\xi+\frac{h_i}{TV_s}\right)\right] \tag{22}$$

$$q_{vi} = W_i k_v \sin\left[2\pi\left(\frac{t}{T}-\frac{\xi}{1.87}+\frac{h_i}{TV_p}\right)\right] \tag{23}$$

Where, the period of lateral shaking, $T=2\pi/\omega$, $\xi = H/TV_s$ and V_p/V_s = 1.87 for most of the geological materials (Das, 1993)

The tensile force, t_j, induced in the reinforcement is calculated by taking the vertical and horizontal force equilibrium of each slice and the total tensile force is obtained as

$$\sum_{j=1}^{m} t_j = \sum_{i=1}^{n} N_i \sin\alpha - \sum_{i=1}^{n} S_i \cos\alpha + \sum_{i=1}^{n} W_i q_{hi} \tag{24}$$

Where $N_i = \dfrac{V_i - V_{i+1} + (1+q_{vi})W_i}{(\tan\phi/FS_{sr})\sin\alpha + \cos\alpha}$ and the value S_i is calculated by following equation (6).

Calculations for the mobilization of shear resistance in reinforcements is same as Reddy et al. (2008) and the factor of safety for conventional method FS_C and by considering oblique pull FS_T is calculated by adopting the Equation (10) and (17) respectively.

3. COMPARATIVE STUDY

A comparative study has been presented among the three methods by considering different soil parameters and seismicity. Soil parameters have been taken same as it is considered in the study by Reddy et al. (2008 & 2009) and Gao et al. (2014). The factor of safety is the parameter that represent the stability of a reinforced soil wall under static or seismic conditions. Therefore, here graphs have been plotted (Figure 6 and Figure 7) to show the variation of the factor of safety with changing seismicity and relative subgrade stiffness factors. The impact of the subgrade shear stiffness factor G^* is also presented in both the comparisons.

The horizontal seismic acceleration coefficient k_h has been taken within the range of 0 to 0.3 as suggested by Ling et al. (1997), Michalowski (1998) and Ausilio et al. (2000) and the value of μ and G^* are according to Reddy et al. (2008) and Gao et al. (2014). Gao et al. (2014) reported that for k_h = 0.2, the factor of safety obtained for G^* = 0 and G^* = 50 are 57% and 18% higher than the values obtained in Reddy et al. (2008) which is noticeable in Figure 6. It can be observed that with the increase of the global shear stiffness factor G^*, mobilization of shear resistance at the interface between soil and the reinforcement decreases drastically and becomes almost steady after a certain value of G^*.

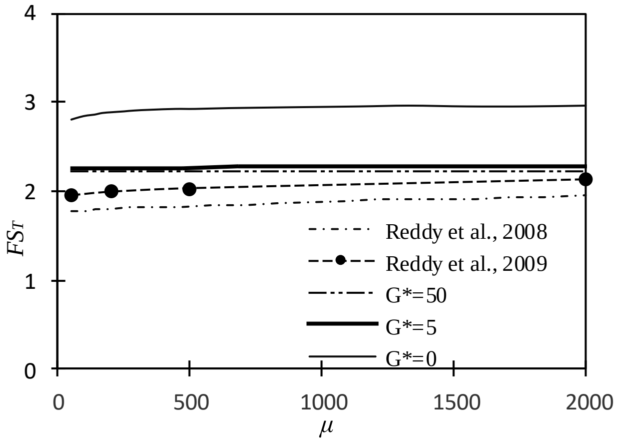

Figure 6 Variation of Factor of safety with μ and G^* (k_h =0.2, k_v =0.5 k_h, *L/H = 0.5, n =5,* ϕ_r/ϕ *=2/3,* ξ*=0.3{for pseudo-dynamic}*).

Figure 7 presents the change of safety factor to horizontal earthquake acceleration k_h for different models discussed in this study and the effect of global subgrade shear stiffness. The Pasternak model used in the formulation gives higher shear resistance mobilization in oblique pull-out of the reinforcement compare to formulations use Winkler spring model. However, the difference gets narrowed as the horizontal earthquake acceleration increases and the reason being, as the horizontal earthquake acceleration rises, the failure plane angle α drops causing a decrease in the tolerable final deformation of reinforcements and the shear resistance mobilized in the reinforcements decrease.

At low seismicity, when the value of seismic acceleration coefficient is small, as the normalized displacement along the failure surface increases, the factor of safety against pull-out increases sharply and significantly. Because, when normalized displacement values get larger due to kinematics of failure, an additional bond resistance gets mobilized along the active length of the reinforcements, leads to higher pull-out resistance. However, as the seismic acceleration increases, the influence of normalized displacement gets dissipated. When the seismic acceleration coefficient value is high, the failure plane makes a lesser angle with the horizontal resulting in a decrease in the effective/active length of the reinforcement beyond the failure surface into the backfill and the bond resistance decline. Variation of FS_T with normalized displacement for different k_h value and a comparison in results between pseudo-static and pseudo-dynamic methods has been shown in Figure 8. At static condition (k_h= 0), when normalized displacement W increases from 0.001 to 0.01, the value of FS_T increased by 23.5%. However, the increase in FS_T is only 9% at k_h=1, for the same increment of W (Reddy et al., 2008b). Instead of a constant seismic acceleration coefficient throughout the depth, the pseudo-dynamic method considers phase change of seismic acceleration in the backfill. As a result, for any seismic acceleration and normalized displacement, pseudo-dynamic method gives higher FS_T and more economical design compare to the pseudo-static method.

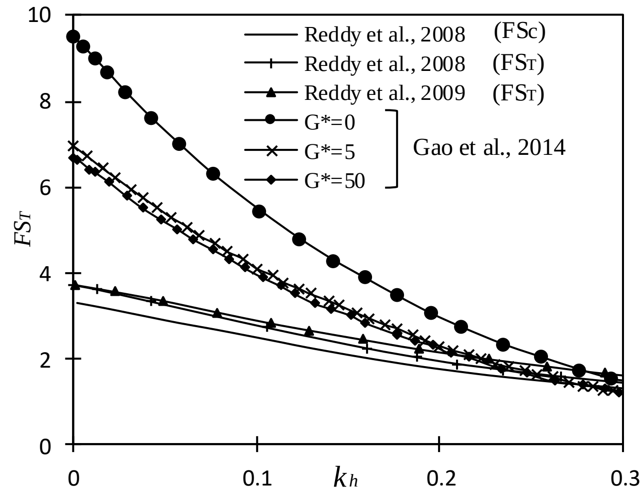

Figure 7 Variation of safety factor with horizontal seismic acceleration coefficient and the effect of G (k_v =0.5k_h, *L/H = 0.5, n =5,* ϕ_r/ϕ *=2/3,* ξ*=0.3 {for pseudo-dynamic}*).

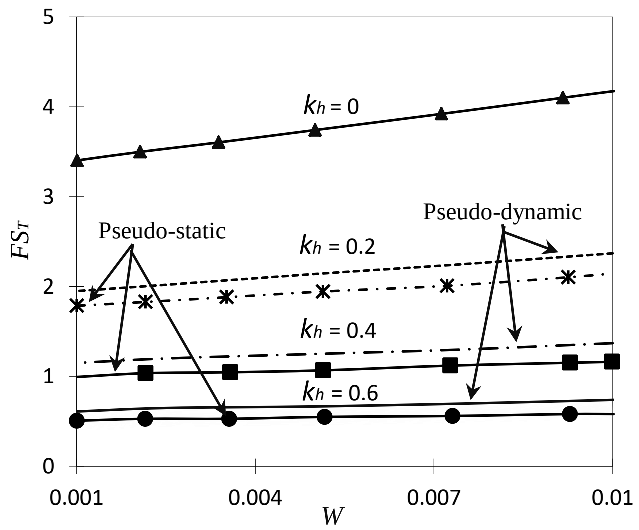

Figure 8 Variation of safety factor, FS_T with normalized displacement for different k_h values (k_v =0.5k_h, *n =5, L/H =0.5,* ϕ*=30,* ϕ_r/ϕ *=2/3,* μ *=2000,* ξ*=0.3 {for pseudo-dynamic}*).

4. CONCLUDING REMARKS

Reinforcements in reinforced earth wall experience non-axial pull-out since the unstable failure wedge moves downward and intersect the reinforcement layers diagonally. The vertical component of the diagonal pull produce additional normal stress at the underneath of the reinforcement layer, resulting extra shear stresses mobilization and as a consequence development of large pullout force. In this state-of-the-art review, a comparative study and formulations of the seismic shear resistance mobilization subjected to oblique pullout (or displacement) of reinforcement is presented and the study shows that,

- There is significant increase of shear resistance mobilization occur along the reinforcement when oblique pullout of reinforcement is considered instead of purely axial pull. However, the value of conventional safety factor, FS_C, and FS_T, decline when the horizontal earthquake acceleration value increases. Because, the gradient of the failure plane α drops (Equation 1) that leads to shortening of the active reinforcement length and subsequently reduction in shear resistance mobilization.
- The difference between the values of FS_C and FS_T widen as the angle of soil internal friction angle, the quantity of reinforcement layers, soil reinforcement interface friction angle and length of the reinforcement increase.
- The factor of safety FS_T, declines with the increase of G^* value and reaches to a steady state after a certain value of G^*. Also, with the increase of the k_h value, the changing rate of the FS_T becomes smaller and approaches steady state.
- The safety factor FS_T obtained by using the Pasternak model gives a higher value than the Winkler spring model and the difference decreases with the increase in k_h value.
- The pseudo-static method gives conservative result compare to the pseudo-dynamic method because of the non-identical approaches are used in considering the seismic inertia forces. The difference between FS_T and FS_C decrease with the increase of earthquake excitation.
- For pseudo-dynamic approach, as the velocity of shear wave increases or the period of lateral seismic shaking increases, the value of ξ decreases and as a result of that factor of safety FS_C and FS_T get reduced.

Only the factor of safety against pull-out failure of reinforcements has been discussed in this paper. Factor of safety against tensile failure of reinforcements and the external stability (i.e. overturning and sliding) of the wall were not considered in this paper. The backfill soil is assumed to be dry cohesionless in the study. Planar failure surface has been considered in the paper instead of curved failure surface. However, the present method can be useful for practical design of reinforced soil-wall under seismic condition considering internal stability.

5. NOTATIONS

a_h, a_v	amplitude of horizontal and vertical seismic acceleration (m/s^2)
b, H	width and height of the reinforced wall (m)
FS_C	factor of safety against pull-out considering axial pull only
FS_T	factor of safety against pull-out considering transverse pull of reinforcement
FS_{sr}	the ratio of available shear resistance to the required shear resistance along the failure plane
g	acceleration due to gravity (m/s^2)
G	shear modulus of the backfill soil (N/m^2);
G^*	global subgrade shear stiffness factor (dimensionless)
G_j^*	local subgrade shear stiffness factor (dimensionless)
H_s	thickness of the shear layer assumed in the Pasternak model (m)
h_j	depth of j^{th} layer reinforcement from the ground (m)
h_i	depth of i^{th} horizontal slice (m)
k_h, k_v	horizontal and vertical seismic acceleration coefficient (dimensionless)
K_s	modulus of subgrade reaction (kN/m^3)
L	length of reinforcement (m)
L_{ej}	active length of j^{th} layer reinforcement (m)
m	number of reinforcement layers
n	number of horizontal slices
N_i	force perpendicular to the failure plane of i^{th} slice (kN)
P_j	additional transverse force on j^{th} layer due to oblique displacement of reinforcement (kN)
P_j^*	normalized oblique force in j^{th} layer of the reinforcement (dimensionless)
q_{hi}, q_{vi}	horizontal and vertical seismic inertia force acting on i^{th} slice (N/m)
S_i	tangential force acting along the failure plane (kN)
T_j	tensile force mobilized at the j^{th} layer of reinforcement due bond resistance (kN)
T_{Tj}	Tensile force mobilized at the j^{th} layer of reinforcement due to bond resistance and transverse force P_j (kN);
t_j	tensile force developed in the j^{th} layer of reinforcement (kN)
T	period of lateral shaking (s)
V_p, V_s	primary and shear wave velocity (m/s);
V_i	vertical inter slice force at the i^{th} slice (kN)
w	transverse displacement of reinforcement at any point (m)
w_L	transverse displacement of reinforcement at failure plane (m)
W_i	weight of the i^{th} slice (kN)
W	normalized transverse displacement of reinforcement (δ/L) (dimensionless)
$W_{j,L}$	normalized displacement of reinforcement at failure surface of the j^{th} reinforcement layer (dimensionless)
$W_{j,k}$	normalized displacement of reinforcement at node k of the j^{th} reinforcement layer (dimensionless)
z	number of elements the reinforcement is divided into
α	failure plane angle with the horizontal (deg)
δ	oblique displacement along the failure surface (m)
ϕ	internal friction angle of soil (deg)
ϕ_r	interface friction angle between soil and reinforcement layer (deg)
γ	unit weight of backfill soil (kN/m^3)
μ	global subgrade stiffness factor (dimensionless)
μ_j	local subgrade stiffness factor (dimensionless)
ω	natural angular frequency of base shaking (rad/s)
ξ	ratio of height of the wall to wavelength of the vertically propagating seismic wave (dimensionless)

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