Validation of a New Simplified Hypothesis B Method for Calculating Consolidation Settlement of Clayey Soils Exhibiting Creep

J.-H. Yin[1,2] and W.-Q. Feng[1]
[1]*Department of Civil and Environmental Engineering,*
The Hong Kong Polytechnic University, Hong Kong, China
[2]*E-mail*: cejhyin@polyu.edu.hk

ABSTRACT: This paper introduces a new simplified Hypothesis B method for calculating consolidation settlement of clayey soils exhibiting creep. The general equations of the new simplified Hypothesis B method are presented and explained firstly. After this, four different cases are used to examine the validation of this new method. The four cases are: (i) a single layer of clay with laboratory test data, (ii) one layer of Hong Kong Marine Deposits (HKMD) with three different over-consolidation ratios (*OCR*s), (iii) one layer of HKMD with vertical drain, and (iv) two layers of HKMD and Alluvium. The fully coupled consolidation analyses of all four cases are done by using one commercial FE program using a soft soil model, one in-house developed FE program and a finite difference method using Yin and Graham's Elastic Visco-Plastic (EVP) model. The consolidated settlements of the same cases are also calculated using the new simplified Hypothesis B method and Hypothesis A method and are compared with values from numerical methods. The *relative errors* are calculated by using the FE results as reference. It is, from the above validation cases, found that the settlements calculated using the new simplified Hypothesis B method are closer to test data or the values from the fully coupled finite element (or finite difference) analyses with the least *relative errors*. Hypothesis A normally under-estimates the settlement a lot with the largest errors. The main conclusion is that the new simplified Hypothesis B method is very suitable for calculating consolidation settlement of clayey soils exhibiting creep and is easy to use by simple spreadsheet calculation.

KEYWORDS: Creep, Hypothesis A, Hypothesis B, Clay, Consolidation, Settlement, Vertical drain

1. INTRODUCTION

For clayey soils, the stress-strain behaviour is usually time-dependent, including consolidation process and viscous nature of soil skeleton (Bjerrum 1967; Graham et al., 1983; Leroueil et al., 1985). Under a constant loading, the viscous nature is usually named as creep. Following the consolidation study of soils, the creep is initially termed as "secondary" consolidation settlement to make a separation of the consolidation in history (Taylor 1948). Many researchers have taken efforts to consider the consolidation of clayey soils with creep in the calculation method based on Hypothesis A (Mesri and Godlewski, 1977; Choi, 1982; Mesri and Vardhanabhuti, 2006; Mesri, 2009) and Hypothesis B (Gibson and Lo, 1961; Barden, 1965; Bjerrum, 1967; Garlanger, 1972; Leroueil et al., 1985; Kelln et al., 2008, Nash and Ryde, 2001; Nash and Brown, 2013). Degago et al. (2011) made a critical investigation on the previous laboratory and field experiment results, and they found that Hypothesis B is correct to calculate the time-dependent behaviour of clays.

A simple calculation equation based on Hypothesis A is still widely used to calculate the total consolidation settlement by engineers due to its simplicity:

$$S_{totalA} = S_{primary} + S_{secondary} = \begin{cases} U_a S_f & for \quad t < t_{EOP,field} \\ U_a S_f + \dfrac{C_{\alpha e}}{1+e_o}\log(\dfrac{t}{t_{EOP,field}})H & for \quad t \geq t_{EOP,field} \end{cases} \tag{1}$$

where S_{totalA} is total settlement, $S_{primary}$ is "primary" consolidation settlement at a certain time, and $S_{primary} = U_a S_f$, in which U_a is the average degree of consolidation of clayey soils, S_f is the final consolidation settlement calculated by the soil stress-strain relationship (this will be described in details), $S_{secondary}$ is the "secondary" consolidation settlement, $S_{secondary} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t}{t_{EOP,field}})H$, in which $C_{\alpha e}$ is named as "secondary" consolidation coefficient, e_o is initial void ratio, $t_{EOP,field}$ is the time at "end-of-primary" (EOP) in field and H is the soil thickness. However, there are some limitations for this method: no "secondary" consolidation settlement occurs before the time of $t_{EOP,field}$. Also, the separation of "primary" and "secondary" consolidation is subjective due to the "end-of-primary" consolidation is infinite according to Tergazhi's theory. In Hypothesis A, the value of $t_{EOP,field}$ is taken as the time when $U_a = 98\%$. Comparatively, the approach based on Hypothesis B considers the creep in "primary consolidation" in the fully coupled consolidation analysis. Yin and Graham (1989, 1994) proposed and validated an elastic visco-plastic (EVP) constitutive model for time-dependent behaviour of clayey soils, which is an extension of Maxwell's model (linear elastic spring and linear viscous dash pot) (Yin, 2015). Afterwards, this constitutive model was adopted to analyse the couple consolidation analysis using a finite difference method to solve the equations of different thickness values of soil layers (Yin and Graham, 1996) and soft ground with vertical drains (Nash and Ryde, 2001) in 1-D straining. The computed results showed that the EVP constitutive model is suitable to analyse the time-dependent behaviour of clayey soils. One main limitation of this approach is that a computer program with numerical method is needed to solve the highly nonlinear partial different equations for the consolidation analysis, whereas the computer program is not always available for engineers or difficult for them to use. In this paper, a new simplified method based on Hypothesis B is proposed as a good approximation solution to easily and reasonably calculate the consolidation settlement of clayey soils exhibiting creep. This new simplified method is examined by comparing the calculation results and measured data or finite element (FE) simulation results with four cases including the test data of clayey soils, one single soil layer, one soil layer with vertical drain, and two-layered soil system. In addition, the simple method of Hypothesis A is also calculated and compared to illustrate the necessity of considering the creep during consolidation stage.

2. EQUATIONS OF THE NEW SIMPLIFIED HYPOTHESIS B METHOD

The total consolidation settlement, S_{totalB}, is the summation of consolidation settlement, $S_{primary}$, and creep settlement, S_{creep}. Based on the Hypothesis B and "equivalent time" concept, a new simplified method is proposed to calculate the consolidation settlement for 1-D straining condition and expressed as:

$$\begin{aligned} S_{totalB} &= S_{primary} + S_{creep} \\ &= U_a S_f + [\alpha S_{creep,f} + (1-\alpha) S_{secondary}] \\ &\quad (t \geq 1 \ for\ S_{creep,f}, t \geq t_{EOP,field}\ for\ S_{secondary}) \end{aligned} \tag{2}$$

where $S_{creep,f}$ is the creep settlement under the final effective vertical stress without excess pore water pressure coupling, the subscript "*creep*" indicates that this settlement is related to creep; $S_{secondary}$ is the same as that in Eq.(1), α is a parameter to illustrate the creep part of soil layer during the consolidation stage, it is in the range of 0 ~ 1. Referring to Figure 1, the vertical effective stress-strain state is valid for the soil nearby the drainage boundary, while the effective stress path inside the clayey soils away from the drainage boundary will be delayed due to the consolidation. For the soils nearby the drainage boundary, $S_{creep,f}$ in Eq. (2) is calculated at the final effective stress ignoring the coupling of the excess pore water pressure as the creep settlement. It is noted that when $\alpha = 0$, Eq. (2) is reduced to the Hypothesis A method, expressed as Eq. (1). When $\alpha = 1$, Eq. (2) can be returned to the simplified method proposed by Yin (2011). In the following part, how to determine the consolidation settlement and creep settlement will be explained in details.

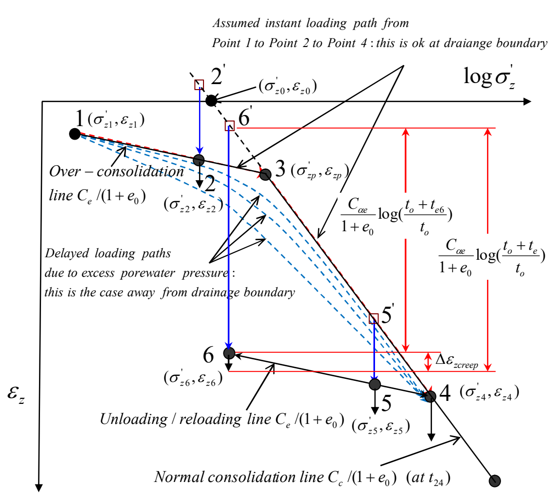


Figure 1 Relationship of strain and log (effective stress) with different consolidation states

2.1 Determination of Consolidation Settlement

The consolidation settlement is usually related to the average degree of consolidation of clayey soils, U_a, and the final consolidation settlement, S_f. For clayey soils, it can undergo the loading, unloading or reloading path. As shown in Figure 1, the loading state can be from point 1 $(\sigma'_{z1}, \varepsilon_{z1})$ to point 2, which is on the over-consolidation (*OC*) line, or from point 1 $(\sigma'_{z1}, \varepsilon_{z1})$ to point 4, which is on the normal consolidation (*NC*) line, the unloading state can be from point 4 to point 6, the reloading state is from point 6 to point 5.

The final consolidation settlement can be calculated based on the final stress state.

(i) Normal consolidation state (Point 1 to point 4)

$$S_f = \Delta\varepsilon_{z,1-4} H = \left[\frac{C_e}{1+e_o} log\left(\frac{\sigma'_{zp}}{\sigma'_{z1}}\right) + \frac{C_c}{1+e_o} log\left(\frac{\sigma'_{z4}}{\sigma'_{zp}}\right)\right] H \tag{3a}$$

(ii) Over consolidation state (point 1 to point 2, point 4 to point 6, point 5 to point 6)

$$S_f = \Delta\varepsilon_{z,1-2} H = \frac{C_e}{1+e_o} \log\left(\frac{\sigma'_{z2}}{\sigma'_{z1}}\right) H \ \ (point\ 1\ to\ point\ 2) \tag{3b}$$

$$S_f = \Delta\varepsilon_{z,4-6} H = \frac{C_e}{1+e_o} \log\left(\frac{\sigma'_{z6}}{\sigma'_{z4}}\right) H \leq 0 \ \ (point\ 4\ to\ point\ 6) \tag{3c}$$

$$S_f = \Delta\varepsilon_{z,6-5} H = \frac{C_e}{1+e_o} \log\left(\frac{\sigma'_{z5}}{\sigma'_{z6}}\right) H \geq 0 \ \ (point\ 6\ to\ point\ 5) \tag{3d}$$

The value of the average degree of consolidation, U_a, is related to the drainage condition and soil layer. In this paper, four cases including the test data of clayey soils, one single soil layer, one soil layer with vertical drain, and two-layered soil are considered. The details of U_a obtaining the will be different and presented in the examples.

2.2 Determination of Creep Settlement

In the new simplified method, creep settlement is calculated using the equation: $S_{creep} = \alpha S_{creep,f} + (1-\alpha) S_{secondary}$. Creep compression during the consolidation stage ($U_a \leq 98\%$) is $\alpha S_{creep,f}$, which indicates that creep compression occurs from the beginning in this new simplified method. Based on the "equivalent time" concept, the final creep settlement can be calculated based on the final effective stress state.

(i) Normal consolidation state (Point 1 to point 4)

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_0} \log\left(\frac{t_o + t_e}{t_o}\right) H \tag{4a}$$

In this case, the "equivalent time" $t_e = t - t_o$, where *t* is duration time of the current total vertical stress.

(ii) Over consolidation state (point 1 to point 2, point 4 to point 6, point 5 to point 6)

It is believed that the creep strain rate is just related to current stress-strain state and it is stress path independent for clayey soils. In other words, the creep strain rate from point 1 to point 2 is the same as the strain rate from point 2' (on the *NC* line) to point 2 by creep with the time duration of "equivalent time":

$$t_{e2} = t_0 \times 10^{(\varepsilon_{z2}-\varepsilon_{zp})\frac{(1+e_0)}{C_{\alpha e}}} \left(\frac{\sigma'_{z2}}{\sigma'_{zp}}\right)^{-\frac{C_c}{C_{\alpha e}}} - t_0 \tag{4b}$$

$$t_e = t - t_o + t_{e2} \tag{4c}$$

And the final creep settlement can be calculated:

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_0} \log\left(\frac{t_o + t_e}{t_o + t_{e2}}\right) H \tag{4c}$$

Similarly, the final creep settlement can be obtained using the following equations when clayey soils are on the state of point 6 or point 5:

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_0}\log\left(\frac{t_o + t_e}{t_o + t_{e6}}\right)H \tag{4d}$$

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_0}\log\left(\frac{t_o + t_e}{t_o + t_{e5}}\right)H \tag{4e}$$

The following sections present the application and verification of the above equations of the new simplified Hypothesis B method in the laboratory test, one layer of HKMD, one layer of HKMD with vertical drain, and two-layered system with HKMD and Alluvium. Hypothesis A method using Eq. (1) is also calculated to make a comparison. In this study, Yin and Feng (2017) defined the parameter, *relative error*, to obtain the accuracy of the new simplified Hypothesis B method at a certain time *t*. The *relative error* is defined as:

$$relative\ error = \left|(S_{cal} - S_{ref})/S_{ref}\right| \times 100\% \tag{5}$$

where S_{cal} is the calculated consolidation settlement from Hypothesis A method or the new simplified method, S_{ref} is the rigorous settlement at time *t*. For the laboratory test, the measured data is taken as the S_{ref}, and the FE simulations are taken as the S_{ref} in other cases.

3. VALIDATION OF THE NEW SIMPLIFIED HYPOTHESIS B METHOD FOR ONE SINGLE LAYER OF CLAY

3.1 Validation Using Measured Data by Berre and Iversen (1972)

The measured data on the consolidation behaviour of natural post-glacial marine clay exhibiting creep in the laboratory is presented by Berre and Iversen (1972). As a typical consolidation test with creep, this measured compression and excess porewater pressure data are used to evaluate the EVP constitutive model (Yin and Graham, 1996). In this part, this measured data Test 6 and Test H4 is adopted to compare with the calculation of the new simplified Hypothesis B method.

Table 1 lists the initial effective stress, initial strain, and time duration at Increment 5 and the basic parameters used in the simplified methods. The pre-consolidation pressure, σ'_{zp}, should be firstly determined, and there should be a relationship with the pre-consolidation pressure and the initial effective stress-strain state:

$$\sigma'_{zp} = 10^{[(\varepsilon_{z1}-\varepsilon_{z0})+(\frac{C_c}{V}\log\sigma'_{z0}-\frac{C_e}{V}\log\sigma'_{z1})]\frac{V}{C_c-C_e}} \tag{6}$$

Substituting the values in Table 1 into Eq. (6), the pre-consolidation pressure is 112.88 kPa for Test 6 soils, and 111.05 kPa for Test H4. Comparing the values of σ'_{zp} and those of $\sigma'_{z,f}$ in two tests, the final effective stress-strain states are at point 4. Therefore, Eq. (3a) is used to calculate the final consolidation strain and their values are listed in Table 1. The *coefficient of volume compressibility*, m_v, *coefficient of consolidation,* c_v, can be obtained from Eq. (7):

$$m_v = \frac{\Delta\varepsilon_f}{\Delta\sigma'_z} \qquad c_v = \frac{k_v}{m_v\gamma_w} \tag{7}$$

where γ_w is the water unit weight, taken as 9.81 kN/m^3. Using the values of m_v and c_v, the average degree of consolidation, U_v, can be calculated by substituting value of c_v into following equations:

$$U_v = \sqrt{\frac{4T_v}{\pi}} \qquad for\, U_v \le 0.6$$
$$U_v = 1 - 10^{-(\frac{T_v + 0.085}{0.933})} \qquad for\, U_v > 0.6 \tag{8}$$

where $T_v = \frac{c_v t}{d^2}$ and d is the length of the longest drainage path.

Table 1 Parameters and main values calculated in the simplified Hypothesis B method at Increment 5

Parameters	C_e=0.0236; C_c=0.9313; $C_{\alpha e}$=0.0413; V=2.56; t_0=40 mins; σ'_{z0}=79.2kPa; k_v=3.0×10^{-6} *m/min*
Test 6	$\varepsilon_{z,1}$=5.51%; $\sigma'_{z,1}$=90.3kPa; σ'_{zp}=112.88 kPa; $\sigma'_{z,f}$=140.5 kPa; $\Delta\varepsilon_f$=3.55%; m_v=0.0744 kPa^{-1}; c_v=4.11×10^{-6} *m²/min*
Test H4	$\varepsilon_{z,1}$=5.25%; $\sigma'_{z,1}$=89.2kPa; σ'_{zp}=111.05 kPa; $\sigma'_{z,f}$= 134.7 kPa; $\Delta\varepsilon_f$=3.14%; m_v=0.0690 kPa^{-1}; c_v=4.43×10^{-6} *m²/min*

Lastly, the creep compression can be calculated using Eq. (4a). Figure 2 shows a comparison of curves from measured data in tests, an EVP model using finite difference (FD) method, Hypothesis A method, the new simplified Hypothesis B method for Test 6 and Test H4 at Increment 5.

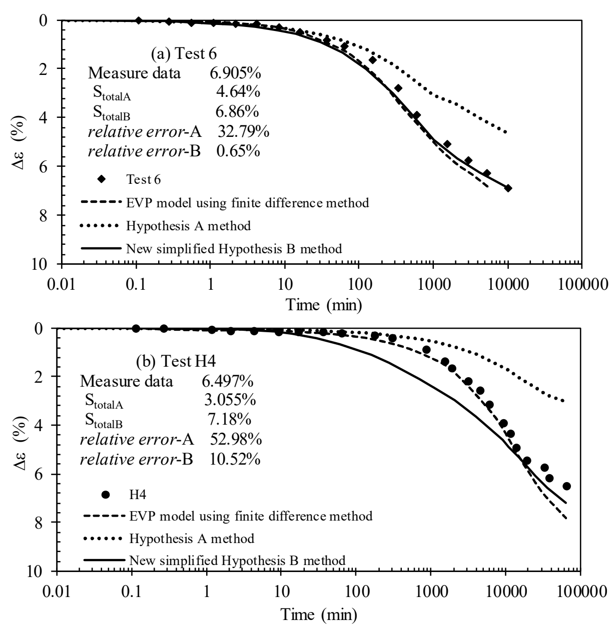


Figure 2 Comparison of strain-log(time) curves from two tests, EVP model using finite difference method, Hypothesis A method, and new simplified Hypothesis B method. (a) Test 6, (b) Test H4 (Berre and Iversen, 1972)

From Figure 2, it is found that Hypothesis A method largely underestimates the average strain for Test 6 and Test H4, and there is a good agreement between the test data and the calculation result of the new simplified Hypothesis B method by using $\alpha = 0.8$. The end duration of Test 6 and Test H4 are 5694 mins and 61450 mins, respectively. And the *relative errors* are calculated at the end duration of each test and its values for Hypothesis A method and the new simplified method are calculated and shown in Figure 2. Therefore, we can make sure that this new simplified method is more suitable than Hypothesis A method to predict the consolidation settlement of soils exhibiting creep for one layer of clayey soil.

3.2 Validation with Finite Element Simulation of Hong Kong Marine Deposits

Hong Kong International Airport (HKIA) is planning to construct a third runway. For the environmental and political concerns, all marine deposits, including HKMD, cannot be removed and must be kept or improved in situ. Therefore, the consolidation settlement, especially the post-construction settlement will be a bigger concern to safe operation of the third runway. In fact, the current construction of artificial islands on the seabed of Hong Kong as part of Hong Kong-Zhuhai-Macau Link project (29.6 km in length) will also face this problem of possible large settlements in the future. In this section, "Upper Marine Clay" layer with 4 m thickness is selected as one soil layer for consolidation analysis with values of soil parameters reported by Koutsoftas et al. (1987), Handfelt et al. (1987) and Zhu et al. (2001). In order to better interpolate the creep compression, different *OCR* values (*OCR*=1, 1.5, 2) are adopted in this simplified method calculation and finite element (FE) simulation.

Two FE programs are used for the fully coupled consolidation analysis: one is the software *Consol* developed by Zhu and Yin (1999a, 2000), and the other one is *Plaxis* software (2D 2015 version). In the analysis, the 1D EVP model (Yin and Graham, 1989; 1994) is implemented in software *Consol* and a soft soil creep (SSC) model in *Plaxis* software (2D 2015 version) are adopted in the FE analysis. The 1-D EVP model was applied by Zhu and Yin (1999a, 2000) and Zhu et al. (2001) for consolidation analysis. The description of the soft soil creep model is referred to Vermeer and Neher (1999) and Plaxis user's manual (2015). This soft soil model has been widely used in consolidation simulations by Degago et al. (2011) and Nash and Brown (2013). In the simulation, the top is drain and the bottom is set as impermeable. Values of all parameters used in FE consolidation simulation are listed in Table 2. The initial *OCR* value is input easily in a menu in *Plaxis*; while this *OCR* value is calculated by giving the pre-consolidation pressure with depth in *Consol* software (Zhu and Yin, 2000). In all FE simulations, a uniform surcharge of 20 kPa is regarded as a sudden loading and is kept for 18250 days (50 years).

Table 2 Values of parameters used in FE simulation

FE type	Value
Plaxis	κ^*=0.02172; λ^*=0.174; μ^*=0.0076; k_v=1.9×10^{-4} m/day; c'=0.1 kPa; ϕ'=30°
Consol	κ/V=0.01086; λ/V=0.174; ψ/V=0.0076; t_0=1day; k_v=1.9×10^{-4}m/day; σ'_{z0}*=1 kPa;

*: σ'_{z0} is the value of the effective vertical stress when the vertical strain of the reference time line is zero $(\varepsilon_{z0}=0)$. Further details can be referred in Zhu and Yin (2000).

For the calculation of the new simplified method, it should divide the soil layer into several sub-layers with thickness 0.5m to obtain an accurate consolidation settlement because the stress-strain relationship of soils is nonlinear, as show in Figure 1. The initial strain of each sub-layer is assumed to be zero, while the initial vertical effective stress, $\sigma'_{z0,i}$, and pre-consolidation stress, $\sigma'_{zp,i}$, are determined based on the depth, soil weight, and *OCR* values.

$$\begin{aligned}
\sigma'_{z0,i} &= (\gamma_{clay} - \gamma_w) z_i \\
\sigma'_{zf,i} &= \sigma'_{z0,i} + \Delta\sigma'_z \\
\sigma'_{zp,i} &= OCR \times \sigma'_{z0,i}
\end{aligned} \tag{9}$$

where z_i is the depth of each mid-location of sub-layer *i*, γ_{clay} is the clayey soil weight, $\gamma_{clay} = 15kN/m^3 \cdot \Delta\sigma'_z$ is the stress increment, taken as 20kPa in this study, $\sigma'_{zf,i}$ is the final effective stress state for different depths of sub-layer soils. For *OCR*=1, the initial effective stress is the same as the pre-consolidation effective stress. Eq. (3a) is used to calculate the consolidation settlement, and the total settlement of soil layer is summation of each sub-layer of soils:

$$S_f = \sum_{i=1}^{n} S_{f,i} \tag{10}$$

Based on this value of total settlement, the *coefficient of volume compressibility*, m_v, *coefficient of consolidation,* c_v, can be calculated and listed in Table 3. Eq. (8) is used to calculated the average degree of consolidation, U_v, with time. Afterwards, the creep compression is also calculated by using Eq. (4a).

To make a clarification, Hypothesis A method is also used to calculate the settlement with time in both "primary consolidation" and the "secondary consolidation" period using Eq. (1). There is no difference of "primary consolidation" in Hypothesis A method and consolidation settlement in new simplified method. Thus, the calculated result difference of Hypothesis A method and the new simplified method is only due to the calculation of creep compression and "secondary consolidation".

Table 3 Parameters and main values calculated in the simplified Hypothesis B method for HKMD

Type	Value
HKMD	C_e=0.0913; C_c=1.4624; $C_{\alpha e}$=0.0639; *V*=3.65; t_0=1 day; k_v=1.9×10^{-4} *m/day*
OCR=1	ε_{zp}=0; s_f=0.918 m; m_v=0.0115 *kPa*$^{-1}$; c_v=0.00169 *m*^{2}*/day*
OCR=1.5	ε_{zp}=0.0044; s_f=0.653 m; m_v=0.0082 *kPa*$^{-1}$; c_v=0.00237 *m*^{2}*/day*
OCR=2	ε_{zp}=0.0075; s_f=0.465 m; m_v=0.0058 *kPa*$^{-1}$; c_v=0.00333 *m*^{2}*/day*

As shown in Figure 3, it is found that the simulation results from *Consol* are the same as those from *Plaxis* modelling. These FE modellings are regarded as the credible rigorous results to evaluate the accuracy of the new simplified method and Hypothesis A method. The time duration of 50 years is used to calculate the *relative error* by adopting Eq. (5). Again, the calculation results from Hypothesis A method obviously underestimate the total settlement in the whole stage, and the *relative errors* are in the range of 16.34% ~ 31.60%. Comparatively, there is a good agreement between the results from the new simplified Hypothesis B method and two FE simulations, and the values of *relative error* are within 5%. Detailed information of the *relative errors* at the time *t=50 years* for three *OCR* values are also shown in Figure 3.

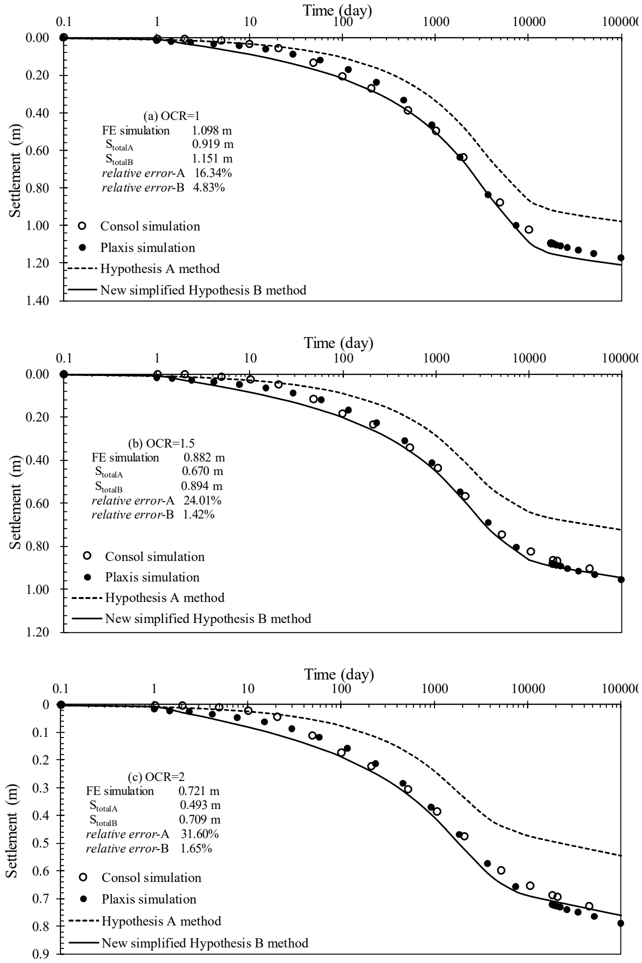


Figure 3 Comparison of settlement-log(time) curves from Hypothesis A method, two FE models, and the new simplified Hypothesis B method for 4 m thick layer: (a) *OCR*=1; (b) *OCR*=1.5; and (c) *OCR*=2.

4. GENERALIZATION OF NEW SIMPLIFIED HYPOTHESIS B METHOD

4.1 One Single Layer of HKMD with Vertical Drain

Vertical drains have been widely installed in soft soil ground to accelerate the consolidation of soft soils to provide the horizontal/radial drainage path (Olson and Roy, 1977; Hansbo, 1981; Bergado et al., 2002; Conte et al., 2009; Walker and Indraratna, 2009; Lei et al., 2015). The insertion of vertical drains usually results in some disturbances of soft soils surrounding the drains, termed as "smear zone", which has a large influence on the horizontal consolidation (Hawlader et al., 2002; Zhu and Yin, 2004a). Barron (1948) presented an approximate solution to calculate the average degree of horizontal consolidation considering the effect of smear and well resistance based on equal-strain consolidation assumption, expressed as:

$$U_r = 1 - \exp(-8T_r/\mu) \tag{11}$$

where T_r is the time factor for radial consolidation, and μ is a factor to consider the smear zone in radial drainage. The formulas of T_r and μ are expressed as:

$$T_r = \frac{c_r t}{4r_e^2} \tag{12}$$

$$\mu = \frac{n^2}{n^2 - S^2}\ln(n/S) - 3/4 + \frac{S^2}{4n^2} + \frac{k_r}{k_s}\left(\frac{n^2 - S^2}{n^2}\right)\ln S$$

where n is the space ratio, $n = r_e/r_w$, S is the smear zone ratio, $S = r_s/r_w$; The overall average degree of consolidation of soil layers is usually calculated by using Carrillo's formula (1942):

$$U_a = 1 - (1 - U_v)(1 - U_r) \tag{13}$$

The average degree of vertical consolidation, U_v, is calculated by using Eq. (8) This solution is widely used in the practice design for vertical drains, and it is named as Carrillo-Barron method.

Zhu and Yin (2001a, 2001b) presented a rigorous solution for the soils considering vertical and horizontal drainages under a ramp load and introduced the design charts for geotechnical engineers to use, afterward, this solution is extended to consider the smear effect with practical solution charts (Zhu and Yin, 2004a). It is termed as Zhu-Yin method. The main equations are presented as follows:

$$U_a = \begin{cases} \dfrac{T}{T_c} - \sum_{m=1}^{\infty}\sum_{n=1,3,5\ldots}\dfrac{32C_m\omega}{n^2\pi^2T_c\xi}\left[1 - \exp\left(-\dfrac{\xi T}{\omega}\right)\right] & T \le T_c \\ 1 - \sum_{m=1}^{\infty}\sum_{n=1,3,5\ldots}\dfrac{32C_m\omega}{n^2\pi^2T_c\xi}\left[1 - \exp\left(-\dfrac{\xi T_c}{\omega}\right)\right]\exp\left(-\dfrac{\xi}{\omega}(T - T_c)\right) & T > T_c \end{cases} \tag{14}$$

where T is a normalized time factor, $T = \left(\frac{c_r\mu_1^2}{r_w^2} + \frac{c_v\pi^2}{4H^2}\right)t$, T_c is the normalized construction time factor, $T_c = \left(\frac{c_r\mu_1^2}{r_w^2} + \frac{c_v\pi^2}{4H^2}\right)t_c$, L is a parameter related to the vertical and horizontal consolidation, $L = \frac{c_v r_w^2}{c_r H^2}$, η is the consolidation coefficient of smear zone and undisturbed zone, $\eta = \sqrt{\frac{c_r}{c_{rs}}}$, ω, ξ and C_m are three shorten terms, expressed as:

$$\omega = \mu_1^2 + \frac{\pi^2 L}{4}$$

$$\xi = \mu_m^2 + \frac{n^2\pi^2 L}{4}$$

$$C_m = \frac{\left[V_1^m \eta\mu_m\right]^2}{(N^2 - 1)\eta^2\mu_m^2\left\{N^2\left[W_0^m N\mu_m\right]^2 - \left[V_1^m\eta\mu_m\right]^2 + \left(1 - \dfrac{c_{rs}}{c_r}\right)S^2\left[V_1^m S\eta\mu_m\right]^2\right\}} \tag{15}$$

Further details can be referred in Zhu and Yin (2001a, 2001b, 2004a).

For HKMD, Prefabricated Vertical Drain is one efficient method in the ground improvement to accelerate the consolidation. In fact, Foott et al. (1987) reported a test fill at the site of HKIA using vertical drains to accelerate consolidation. Zhu *et al.* (2001) described that installed vertical drain characteristics with equivalent radius, r_w, is 27.45*mm*, the smear zone is five times of equivalent radius, $r_s = 137mm$, and equivalent radius of unit ground in the test site, r_e, is 787.5*mm*. Geological profile of HKMD with a thickness of 4*m* is studied here as one single soil layer with vertical drain examples. the vertical drain with smear zone (*S*=1, *S*=3), with different spacing (*n*=58.33, 116.67), are also considered in this study to illustrate their effect.

As shown in Figure 4, it is found that the new simplified Hypothesis B methods using Carrillo-Barron method and Zhu-Yin method agree well with FE modelling for different *OCR* values. Due to Carrillo-Barron method plays the same role as Zhu-Yin method in calculating the total average degree of consolidation of soil layer with vertical drain (Zhu and Yin, 2004b), the curves of the new simplified method are overlapped. In this section, three time durations are considered to illustrate the accuracy of calculation method within and after consolidation (U_a=50%, 98%, and *t*=36500 days). The *relative error* values are in the range of 0.9% ~ 11.3%. Comparatively, obvious underestimation of Hypothesis A method, with *relative errors* from 9.9% ~ 22.4%, are observed. It also indicates that creep settlement is essential to be calculated within the consolidation stage. Similarly finding is observed in Figure 5 and Figure 6, which compare with different spacing (*n*=29.17, 58.33, 116.67) and different smear zones (*S*=1, 3, 5), detailed obtained *relative error* values are also shown in Figures 4~6.

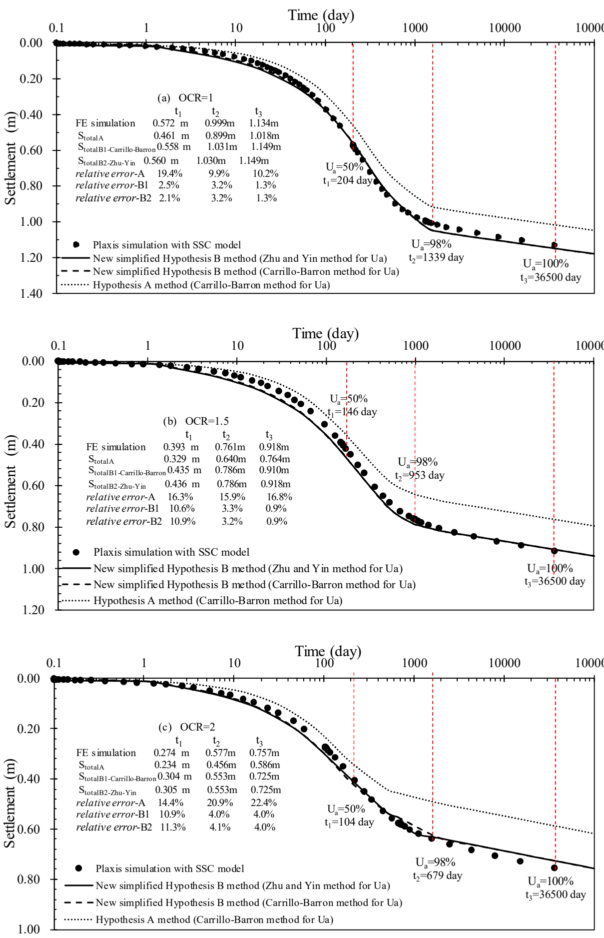


Figure 4 Comparison of settlement-log(time) curves from FE simulations, the new simplified Hypothesis B method, and Hypothesis A method for one soil layer with vertical drain (*S*=5, *n*=29.17): (a) *OCR*=1; (b) *OCR*=1.5; (c) *OCR*=2

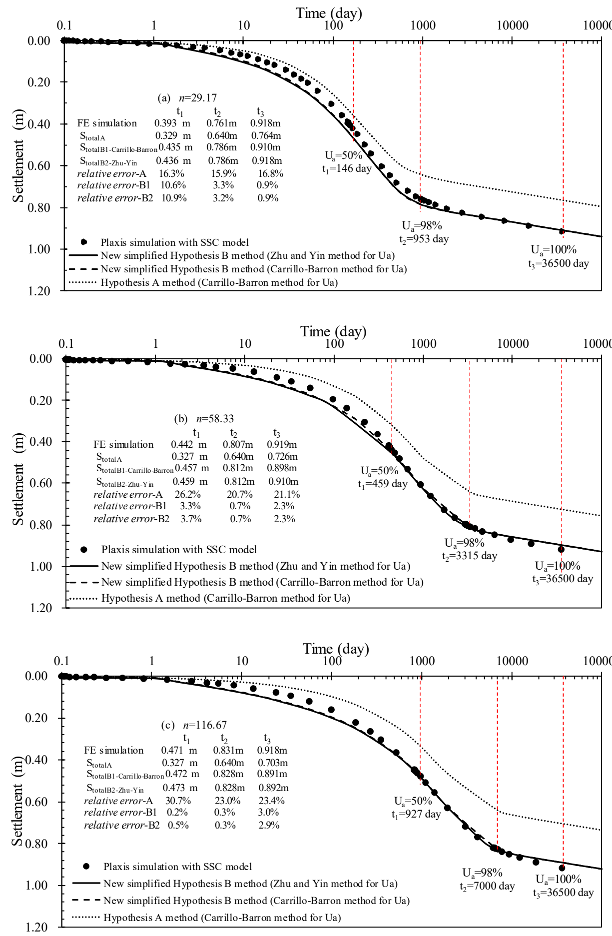
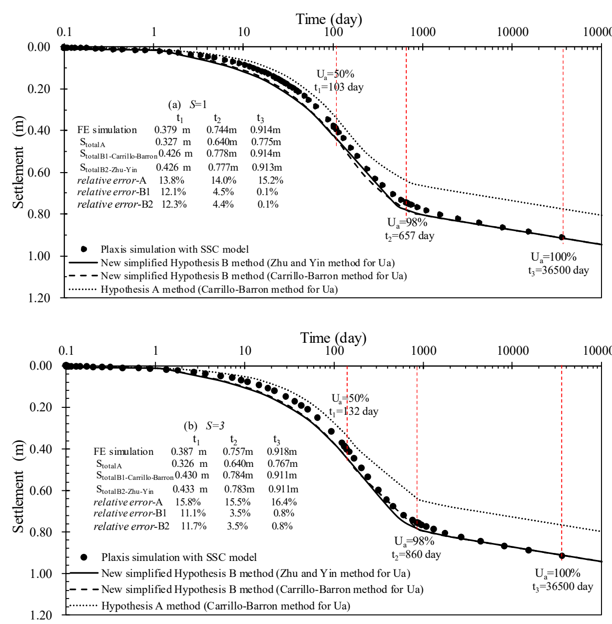


Figure 5 Comparison of settlement-log(time) curves from FE simulations, the new simplified Hypothesis B method, and Hypothesis A method for one soil layer with vertical drain (*S*=5, *OCR*=1.5): (a) *n*=29.17; (b) *n*=58.33; (c) *n*=116.67



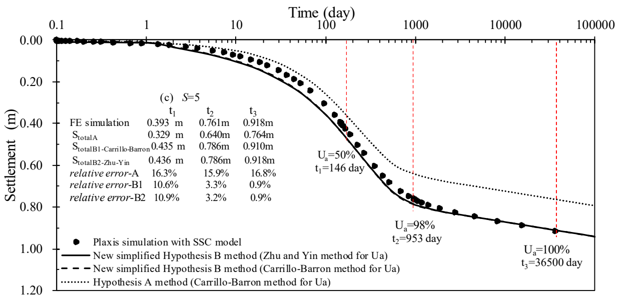


Figure 6 Comparison of settlement-log(time) curves from FE simulations, the new simplified Hypothesis B method, and Hypothesis A method for one soil layer with vertical drain (*n*=29.17, *OCR*=1.5): (a) *S*=1; (b) *S*=3; (c) *S*=5

4.2 Multi-layered HKMD and Alluvium

In reality, there are more than one soil layer in the field geological profile, and the multi-layered soils are extensively studied by previous researchers (Schiffman and Stein, 1970; Xie, 1994; Xie et al., 1999; Xie et al., 2002). A simplified procedure is proposed to convert multiple soil layers into one single soil layer using the equivalent thickness (US Department of the Navy, 1982). Afterward, an analytical approach and solution charts for double soil layers under the ramp load are presented for different depths, and different consolidation behaviours between the double soil layers and a simplified one single soil layer are demonstrated (Zhu and Yin 1999b; 2005). In this part, US Navy method and Zhu and Yin's method are adopted here to calculate the average degree of consolidation, U_a, for multi-layered soils, and the simplified Hypothesis B method is generalized as:

$$\begin{aligned} S_{totalB} &= \sum_{i=1}^{n} S_{primary''i} + \sum_{i=1}^{n} S_{creepi} = U_a \sum_{i=1}^{n} S_{fi} + \sum_{i=1}^{n} [\alpha S_{creep,fi} + (1-\alpha) S_{''secondary''i}] \\ &= U_a \sum_{i=1}^{n} \varepsilon_{fi} H_i + \sum_{i}^{n} \{[\alpha \varepsilon_{creep,fi} + (1-\alpha)\varepsilon_{''secondary''i}] H_i\} \quad for\ t \geq 1 day (t \geq t_{EOP,field}\ for\ S_{''secondary''i}) \end{aligned} \tag{16}$$

where $\sum_{i=1}^{n} S_{''primary''i}$ is the "primary" consolidation settlement of *n* soil layers, U_a and $\sum_{i=1}^{n} S_{fi}$ are the average degree of consolidation and the total "primary" consolidation settlement of *n* soil layers, $\sum_{i=1}^{n} S_{creepi}$ is the total creep settlement of *n* soil layers, $\sum_{i=1}^{n} S_{creep,fi}$ $\sum_{i=1}^{n} S_{''secondary''i}$ are the total final creep settlement and the total "secondary" consolidation settlement of *n* soil layers. If *n*=1, Eq. (16) can return to the same format of Eq. (2). Therefore, Eq. (16) is an extension formula to calculate the consolidation settlement of multi-layered soils exhibiting creep.

For US Navy method, it is usually to convert soil layer 2 into an equivalent thickness of soil layer 1, using:

$$\begin{aligned} H_2' &= H_2 (c_{v1}/c_{v2})^{1/2} \\ T &= \frac{c_{v1} t}{(H_1 + H_2')^2} \end{aligned} \tag{17}$$

where H_2 is the height of the soil layer 2, H'_2 is the equivalent thickness of soil layer 2 as if it is made up of soil layer 1, c_{v1} and c_{v2} are the coefficients of consolidation for layer 1 and layer 2, respectively. *T* is the overall time factor of the whole deposit. After the conversion, the two-layered soil can be treated as one soil layer, following the procedures in *3.2*.

For Zhu and Yin's method, two independent parameters, *p* and *q*, are defined:

$$\begin{aligned} p &= \frac{\sqrt{k_2 m_{v2}} - \sqrt{k_1 m_{v1}}}{\sqrt{k_2 m_{v2}} + \sqrt{k_1 m_{v1}}} \\ q &= \frac{H_1 \sqrt{c_{v2}} - H_2 \sqrt{c_{v1}}}{H_1 \sqrt{c_{v2}} + H_2 \sqrt{c_{v1}}} \end{aligned} \tag{18}$$

The average degree of consolidation of two-layered soil system, U_a, can be determined as:

$$U_a = \begin{cases} \dfrac{T_c}{T} - \sum_{n=1}^{\infty} \dfrac{c_n}{\lambda_n^4 T_c} [1 - \exp(-\lambda_n^2 T)] & T \leq T_c \\ 1 - \sum_{n=1}^{\infty} \dfrac{c_n}{\lambda_n^4 T_c} [1 - \exp(-\lambda_n^2 T_c)] \times \exp[-\lambda_n^2 (T - T_c)] & T \geq T_c \end{cases} \tag{19}$$

where *T* and T_c are the time factor and construction time factor, $T = \dfrac{c_{v1} c_{v2} t}{\left(H_1 \sqrt{c_{v2}} + H_2 \sqrt{c_{v1}}\right)^2}$, $T_c = \dfrac{c_{v1} c_{v2} t_c}{\left(H_1 \sqrt{c_{v2}} + H_2 \sqrt{c_{v1}}\right)^2}$ λ_n is the root of the equation $\sin\theta + p\sin(q\theta) = 0$ for both top and bottom drained condition (*condition1*) and the equation $\cos\theta - p\cos(q\theta) = 0$ for one side drained condition (*condition2*). Values of c_n are determined by the following equation:

$$c_n = \begin{cases} \dfrac{2[m_{v1} H_1 \xi \sin(\lambda_n \xi) + m_{v2} H_2 \omega \sin(\lambda_n \omega)]^2}{\omega^2 \xi^2 (m_{v1} H_1 + m_{v2} H_2)[m_{v1} H_1 \xi \sin^2(\lambda_n \xi) + m_{v2} H_2 \omega \sin^2(\lambda_n \omega)]} & condition1 \\ \dfrac{2[m_{v1} H_1 \xi \cos(\lambda_n \xi)]^2}{\omega^2 (m_{v1} H_1 + m_{v2} H_2)[m_{v1} H_1 \xi \cos^2(\lambda_n \xi) + m_{v2} H_2 \omega \sin^2(\lambda_n \omega)]} & condition2 \end{cases} \tag{20}$$

Details of the derivation could be found in Zhu and Yin (1999b, 2005), Feng and Yin (2017).

With one single layer of HKMD, the "Upper Alluvium" is also considered in this part to consist the two layer soils. The thickness of alluvium is also 4m with three *OCR* values (*OCR*=1, 1.5, 2). FE simulations are the same as the one single soil layer, and not repeated here. Following the idea of Pyrah (1996), two cases are considered in two-layered soil and the values of parameters are listed in Table 4.

Table 4 Parameter values in the simplified Hypothesis B method for HKMD and Alluvium

Type	Value
HKMD	C_e=0.0913; C_c=1.4624; $C_{\alpha e}$=0.0639; *e*=2.65; t_0=1day; k_v=1.9×10^{-4} m/day (*Case I*); k_v=1.9×10^{-3} m/day (*Case II*)
Alluvium	C_e=0.05; C_c=0.2993; $C_{\alpha e}$=0.016; *e*=1; t_0=1day; k_v=5.18×10^{-4} m/day (*Case I*); k_v=5.18×10^{-5} m/day (*Case II*)

It is found that α is related to *OCR* and can be taken as $\alpha = 0.4 + 0.2\,OCR$. For *OCR*=1, 1.5, and 2, we have $\alpha = 0.6, 0.7, 0.8$. All the procedures are the same as one single layer of HKMD, except that the "equivalent time" for *OCR*=2. After adding the 20 kPa, the stress state of HKMD is normal consolidation (*NC*) state, while that of Alluvium is over-consolidation (*OC*) state. Eqs. (3b) and (4c) are used to calculate the final consolidation settlement and creep compression. The value of t_{e2} are shown in Figure 7, details can be referred to Feng and Yin (2017). And the main calculated parameter values for HKMD and Alluvium with different OCR values are listed in Table 5.

Table 5 Main values calculated in the simplified Hypothesis B method for HKMD and Alluvium

Type	Value
OCR=1	s_{f1}=0.918m; m_{v1}=0.0115 kPa^{-1};
	s_{f2}=0.114m; m_{v2}=0.00142 kPa^{-1};
	c_{v1}=0.00169 m^2/day, c_{v2}=0.03709 m^2/day (*case I*);
	c_{v1}=0.0169 m^2/day, c_{v2}=0.003709 m^2/day (*case II*);
OCR=1.5	s_{f1}=0.653m; m_{v1}=0.0082 kPa^{-1};
	s_{f2}=0.032m; m_{v2}=0.00032 kPa^{-1};
	c_{v1}=0.00237 m^2/day, c_{v2}=0.167 m^2/day (*case I*);
	c_{v1}=0.0237 m^2/day, c_{v2}=0.0167 m^2/day (*case II*);
OCR=2	s_{f1}=0.465m; m_{v1}=0.0058 kPa^{-1};
	s_{f2}=0.019m; m_{v2}=0.00024 kPa^{-1};
	c_{v1}=0.00333 m^2/day, c_{v2}=0.222 m^2/day (*case I*);
	c_{v1}=0.0333 m^2/day, c_{v2}=0.0222 m^2/day (*case II*);

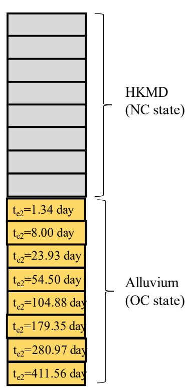


Figure 7 Values of "equivalent time" t_{e2} in sub-layers of Alluvium with *OCR*=2

The calculation results of Hypothesis A method, the new simplified Hypothesis B method using US Navy method for U_a, Zhu and Yin's method for U_a are shown in Figure 8. Three values of time (U_a=50%, 98%, and t=100000 days) are taken to illustrate the consolidation characteristics of two-layered soil layer for two cases. Hypothesis A method obviously underestimates the consolidation settlement, comparing with FE simulation, for all the cases with different *OCR* values. For Case I, US Navy method predicts almost the same value of U_a as Zhu and Yin's method, and this results that calculated curves from new simplified Hypothesis B method are overlapped, which is very close to the FE simulations. For Case II, there is a large difference between the results from the simplified method using US Navy method and that adopting Zhu and Yin's method. The difference is mainly due to the two-layered soil simplification into one single soil layer, which does not consider the flow continuity of interface between two layers (Zhu and Yin, 2005). Thus, it is recommended to use Zhu and Yin's method to obtain the average degree of consolidation, U_a, for two-layered system.

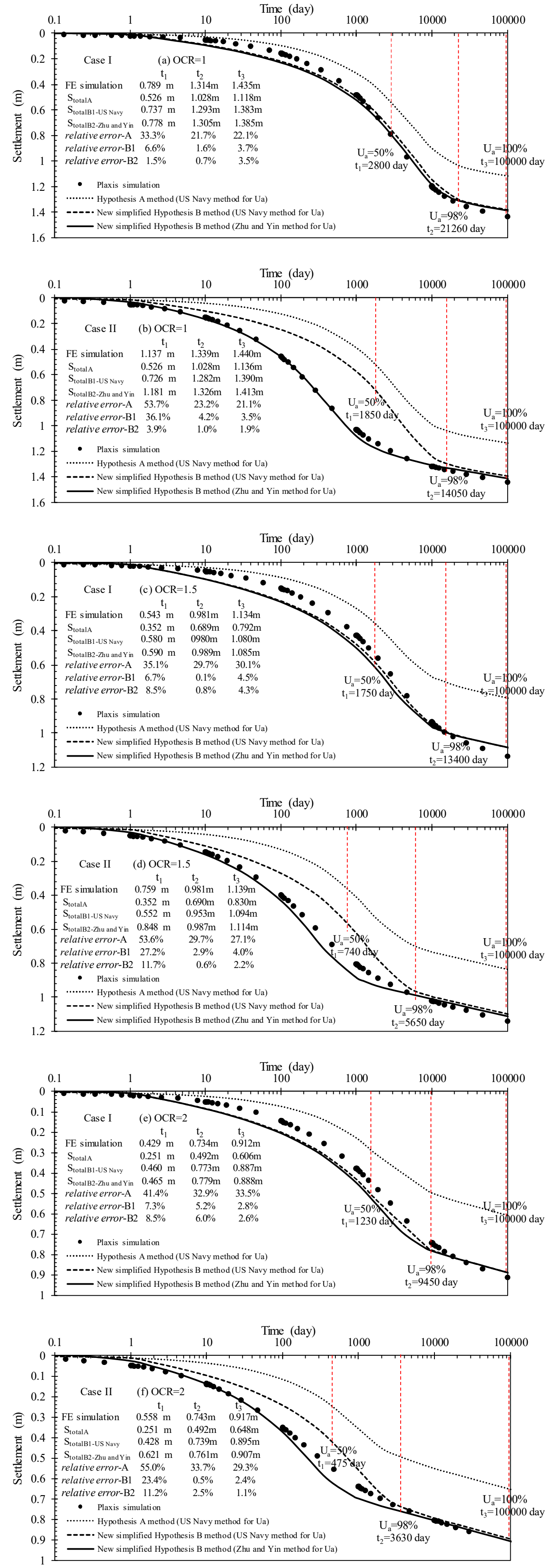


Figure 8 Comparison of settlement-log(time) curves of two-layered soils ((4m HKMD and 4m Alluvium)) for Case I and Case II with *OCR* =1, 1.5, 2 obtained from FE simulations and using the extended new simplified Hypothesis B method

5. CONCLUSIONS

In this paper, a new simplified method based on Hypothesis B is presented, and validated in four different cases. The parameter, *relative error*, is defined to evaluate the accuracy of the new calculation method based on the measured data from tests or simulation results from finite element analyses. Main conclusions are drawn as followings:

(a) The new simplified method is easy to use with hand-calculation and this method can correctly capture the creep compression within and after the primary consolidation under both normal consolidation state and over-consolidation state.

(b) Comparing with the measured data of Berre and Iversen (1972) and FE simulations, Hypothesis A method overestimates settlements with larger relative errors, which means that the creep settlement is not properly considered by this method.

(c) For one-layered soil, it is verified that the new simplified Hypothesis B method generally predicts consolidation settlements very close to the measured data and results from FE analyses with *relative errors* less than 10.5%.

(d) For one layer of soil with vertical drain, it is found that Carrillo-Barron method plays the same role as Zhu-Yin's method to obtain the average degree of consolidation, U_a, in the simply calculation cases. By adopting these two approaches for U_a, calculation, results of the new simplified Hypothesis B method agree well with the FE simulations.

(e) The new simplified Hypothesis B method is generalized to calculate the consolidation settlement of multi-layered soils exhibiting creep. It is recommended to use Zhu and Yin's method rather than US Navy method for calculating the average degree of consolidation, U_a, of two-layered soils.

It is also recognized that there are still some challenging works which need to be done for this simplified method, such as thick-layered soils, two-layered soils with vertical drains, and the generalized multi-layered soil system considering construction time, *etc*. In addition, the application of this new simple method for the real project is still limit. Further work will be conducted in this area too.

6. ACKNOWLEDGEMENT

The work in this paper is supported by a research grant (project No. 51278442) from National Natural Science Foundation of China (NSFC), a general research fund (GRF) (project No. PolyU PolyU 152796/16E) from Research Grants Council (RGC) of Hong Kong Special Administrative Region Government of China, grants (4-BCAW, G-YN97, B-Q43L, B-Q55N) from The Hong Kong Polytechnic University, China. We also acknowledge the supports by a National State Key Project "973" grant (Grant No.: 2014CB047000) (sub-project No. 2014CB047001) from Ministry of Science and Technology of the People's Republic of China, a CRF project (Grant No.: PolyU12/CRF/13E) from Research Grants Council (RGC) of Hong Kong Special Administrative Region Government of China, and Research Institute for Sustainable Urban Development of The Hong Kong P9olytechnic University.

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