Simplified Model for Heat Transfer in Unsaturated Soils Considering a Nonisothermal Thermal Conductivity Function

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ABSTRACT: Heat transfer in unsaturated soils occurs primarily due to conduction, convection of pore fluids in both liquid and vapor forms, and latent heat transfer. Due to the complexity involved in simulating this coupled problem, this paper investigates a simplified model for heat transfer in unsaturated soils using a conduction analysis with a nonisothermal thermal conductivity function. Specifically, a relationship between the apparent thermal conductivity and degree of saturation that indirectly incorporates the effects of heat transfer due to convection and water phase change through temperature effects was defined based on experimental observations, and the governing equation for conductive heat transfer was reconsidered to account for the variation in nonisothermal thermal conductivity with respect to space and time. An axisymmetric analysis for horizontal heat transfer in a soil layer from a line heat source was performed using the simplified heat transfer model, and results were compared with a conventional isothermal conduction analysis. Further, a comparison of simulated soil temperatures from the simplified heat transfer model with measured temperatures from an experimental study on heat transfer in unsaturated silt shows a good match, indicating that the simplified model may be used for preliminary analyses of problems involving monotonic heating.

KEYWORDS: Apparent thermal conductivity, Heat transfer, Nonisothermal conditions, Unsaturated soil

1. INTRODUCTION

Heat transfer in unsaturated soils is a growing area of research because of many emerging geotechnical and geoenvironmental applications involving thermal effects. These applications include nuclear waste disposal, geothermal heat exchangers, energy piles, underground power cable systems, and thermal improvement of soils. Heat transfer in unsaturated soils may occur via several mechanisms, including conduction, convection associated with advective or thermally-induced water flow in liquid or gas phases, radiation (for near surface soil layers), and latent heat transfer due to water phase change (Philip and de Vries 1957). Coupled heat transfer and water flow analyses are complex and require several coupled constitutive relationships for the soil (Smits et al. 2011; Baser et al. 2018). To simplify this problem, this study investigates whether it is possible to simulate heat transfer in unsaturated soils by assuming that heat transfer can be simulated using a conduction analysis using an "apparent" thermal conductivity value that depends on temperature in a way that the effects of heat transfer due to convection and water phase change are considered. This simplification builds upon experimental studies by Campbell et al. (1994), Hiraiwa and Kasubuchi (2000), Smits et al. (2013), and Nikolaev et al. (2013), who observed that elevated temperatures can have a major effect on the relationship between the apparent thermal conductivity and degree of saturation. Specifically, a significant increase in apparent thermal conductivity with temperature is observed for soils having intermediate degrees of saturation. Although it is likely that these observations are due to the influences of other heat transfer mechanisms in unsaturated soils (i.e., convection, phase change) on the measurement of thermal conductivity using available techniques, it may be possible to take advantage of the measured nonisothermal apparent thermal conductivity relationships in developing a simplified model for heat transfer in unsaturated soils.

The relationship between thermal conductivity and the degree of saturation of soils under room temperature conditions has been widely investigated and several theoretical and semi-empirical models exist in the literature (de Vries 1963; Tarnawski and Leong 2012; Likos 2014; Lu and Dong 2015). Of these models, the thermal conductivity function (TCF) developed by Lu and Dong (2015) is particularly useful in heat transfer analyses in unsaturated soils as it is linked with the parameters of the soil-water retention curve (SWRC). Recently, Samarakoon et al. (2018) extended the TCF of Lu and Dong (2015) to account for the effects of temperature observed on the apparent thermal conductivity of soils having different degrees of saturation measured by Smits et al. (2013), Hiraiwa and Kasubuchi (2000), Nikolaev et al. (2013) and Campbell et al. (1994). Accordingly, this paper investigates the use of the nonisothermal apparent thermal conductivity function of Samarakoon et al. (2018) in a conduction analysis to define a simplified model for heat transfer in unsaturated soils. To do so, the governing equation for conduction was reconsidered to account for the fact that the apparent thermal conductivity will not be constant with respect to either space or time during heat transfer. Results from the simplified model for heat transfer in unsaturated soils are first compared with those from a conventional conduction analysis with a constant thermal conductivity. Next, results from the simplified model for heat transfer in unsaturated soils are compared with those from the experimental study by Baser et al. (2018) on coupled heat transfer and water flow in unsaturated silt.

2. BACKGROUND

Conductive heat transfer through soils is governed by Fourier's law, expressed in cylindrical coordinates as follows:

$$\dot{q} = -\lambda\left[\frac{\partial T}{\partial r} + \frac{1}{r}\frac{\partial T}{\partial \phi} + \frac{\partial T}{\partial z}\right] \tag{1}$$

where $\dot{q}$ is the heat flux (W/m^2), T is the temperature (°C), λ is the thermal conductivity (W/m°C) and r (m), ϕ (degrees), z (m) are the cylindrical coordinates. The use of cylindrical coordinates is suitable when evaluating heat transfer from a geothermal heat exchanger embedded in a vertical borehole, where the heat exchanger is treated as a line source.

The governing equation for heat transfer through a soil element is defined using Fick's law, as follows:

$$\rho C\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\lambda\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) \tag{2}$$

where ρ and C are the total density and specific heat capacity of the soil, respectively. Equation (2) indicates that heat transfer by conduction in soils depends primarily on the specific heat capacity and thermal conductivity of the soil. The degree of saturation is one factor that has a significant impact on thermal conductivity. Lu and Dong (2015) observed that at room temperature, distinct changes in thermal conductivity occur in the four water-retention regimes for unsaturated soils: the hydration for very dry soils with a degree of saturation close to 0, the pendular regime, the funicular regime, and the capillary for nearly-saturated soils with a degree of saturation close to 1.0.

As noted, there have been several studies that have investigated whether temperature will affect the relationship between thermal conductivity with degree of saturation. Smits et al. (2013) obtained continuous measurements of thermal conductivity for a uniform manufactured silica sand (30/40 sand) and a natural sand from the Great Sand Dunes National Park, CO. They used a Tempe cell to control the degrees of saturation in the specimens, and made continuous measurements of thermal conductivity during drying of the soil using a thermal needle from Decagon Devices of Pullman, WA. They repeated the drying experiments on different soils under temperatures of 30, 40, 50, 55, 60 and 70 °C by placing the Tempe cell in a temperature-controlled chamber. Campbell et al. (1994) measured the thermal conductivity of 10 soils including Royal soil at temperatures of 30, 50, 70 and 90 °C using an axially mounted heating probe and a microprocessor-controlled data logger. The thermal conductivity values of Ottawa sand and Richmond Hill fine sandy loam were measured by Nikolaev et al. (2013) for temperatures ranging from 2 to 92 °C in 10 °C increments. Their measurements were obtained using the guarded hot plate method, which is a steady-state technique involving a uniaxial heat flux through a specimen fixed between hot and cold plates. Hiraiwa and Kasubuchi measured the thermal conductivity of Ando soil and Red Yellow soil for temperatures ranging from 5 to 75 °C in 10 °C increments. A twin heat probe method with stainless-steel probes consisting of a heating wire and a thermocouple was used to obtain the measurements. Unlike Smits et al. (2013), Campbell et al. (1994), Hiraiwa and Kasubuchi (2000) and Nikolaev et al. (2013) used specimens prepared at different values of degree of saturation to obtain the relationship between thermal conductivity and degree of saturation at a given temperature, and didn't make continuous measurements of thermal conductivity during drying.

Although an increase in measured apparent thermal conductivity with temperature is observed for soils with intermediate degrees of saturation, it is uncertain whether these observations reflect an actual increase in thermal conductivity with temperature or are a result of increased heat transfer due to the contributions of other mechanisms of heat transfer in unsaturated soils. For example, Smits et al. (2013) attribute the enhancement in the measured apparent thermal conductivity to the additional heat transfer in the form of enhanced vapor diffusion and latent heat transfer (phase change). This is why the thermal conductivity measured in these tests is an "apparent" value. Accordingly, use of the nonisothermal apparent thermal conductivity relationships from these studies may indirectly incorporate the effects of these other heat transfer mechanisms.

3. SIMPLIFIED MODEL FOR HEAT TRANSFER IN UNSATURATED SOILS

3.1 Governing Equation for Conduction Considering a Nonisothermal Thermal Conductivity Value

A simplified model for heat transfer in unsaturated soils is defined in this study by reconsidering the governing equation for conduction accounting for the dependence of apparent thermal conductivity on temperature observed experimentally by Smits et al. (2013), Hiraiwa and Kasubuchi (2000), Nikolaev et al. (2013) and Campbell et al. (1994). This approach is simplified because it does not simulate the heat transfer due to convection of pore fluids or phase change, as considered by Baser et al. (2018), but instead assumes that the variation in apparent thermal conductivity with temperature indirectly accounts for these mechanisms of heat transfer. This simplified analysis cannot consider the impacts of drying during heating and a subsequent cooling process that were evaluated by Baser et al. (2018), but may be useful for preliminary evaluation of heat transfer problems.

The analysis was formulated for axisymmetric conditions as this is applicable to the analysis of geothermal heat exchangers installed in a vertical borehole or pile. Assuming heat transfer occurs only in the radial direction with no variation in temperature with depth, an axisymmetric model for horizontal heat transfer can be developed, as follows:

$$\rho C \frac{\partial T}{\partial t} r dr = \frac{\partial}{\partial r}\left(\lambda_a r \frac{\partial T}{\partial r} \right) dr \tag{3}$$

where λ_a is the apparent thermal conductivity that is a function of temperature and the initial degree of saturation. Using the product rule, Equation (3) can be reduced to:

$$\rho C \frac{\partial T}{\partial t} = \lambda_a \frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\frac{\partial \lambda_a}{\partial r} + \frac{1}{r}\lambda_a \frac{\partial T}{\partial r} \tag{4}$$

The chain rule is needed to consider the fact that λ_a is a function of temperature, as follows:

$$\rho C \frac{\partial T}{\partial t} = \lambda_a \frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\frac{\partial \lambda_a}{\partial T}\frac{\partial T}{\partial r} + \frac{\lambda_a}{r}\frac{\partial T}{\partial r} \tag{5}$$

Therefore, the final governing equation for heat transfer due to conduction with a temperature-dependent apparent thermal conductivity will be in the following form:

$$\rho C \frac{\partial T}{\partial t} = \lambda_a \frac{\partial^2 T}{\partial r^2} + \frac{\partial \lambda_a}{\partial T}\left(\frac{\partial T}{\partial r}\right)^2 + \frac{\lambda_a}{r}\frac{\partial T}{\partial r} \tag{6}$$

Although not investigated in this study, the governing equation for one-dimensional heat transfer with a nonisothermal apparent thermal conductivity, expressed in Cartesian coordinates, can be obtained as follows:

$$\rho C \frac{\partial T}{\partial t} = \lambda_a \frac{\partial^2 T}{\partial x^2} + \frac{\partial \lambda_a}{\partial T}\left(\frac{\partial T}{\partial x}\right)^2 \tag{7}$$

The finite difference formulation of Equation (6) for a forward time, central space finite difference scheme with a time step Δt can be obtained as follows.

$$\rho C \frac{\left(T_i^{t+1} - T_i^t\right)}{\Delta t} = \lambda_a(T_i^t)\frac{\left(T_{i+1}^t - 2T_i^t + T_{i-1}^t\right)}{\Delta r^2} + \frac{\partial \lambda_a\left(T_i^t\right)}{\partial T}\left(\frac{T_{i-1}^t - T_{i+1}^t}{2\Delta r}\right)^2 + \frac{\lambda_a\left(T_i^t\right)}{r_i}\left(\frac{T_{i-1}^t - T_{i+1}^t}{2\Delta r}\right) \tag{8}$$

In addition, the apparent thermal conductivity and specific heat capacity will vary with the initial degree of saturation (Baser et al. 2018). As this analysis neglects convection of water in liquid and gas phases, the apparent thermal conductivity and specific heat capacity corresponding to the initial degree of saturation (i.e., at the beginning of the heating process) are used in the simulation. The specific heat capacity is assumed to not depend on the temperature, but this needs to be confirmed through further experimental studies.

3.2 Nonisothermal Apparent Thermal Conductivity Function

The TCF by Lu and Dong (2015) at room temperature was used as the starting point in defining the relationship between apparent thermal conductivity and temperature at different degrees of saturation. The TCF of Lu and Dong (2015) is given as follows:

$$\frac{\lambda-\lambda_{dry}}{\lambda_{sat}-\lambda_{dry}}=1-\left[1+\left(\frac{S}{S_f}\right)^m\right]^{1/m-1} \tag{9}$$

where S is the degree of saturation, λ_{sat} is the thermal conductivity of saturated soil (which is treated as a fitting parameter as the model does not converge to λ_{sat} at S =1), λ_{dry} is the thermal conductivity of dry soil (i.e., at S=0), S_f is the degree of saturation at which the apparent thermal conductivity increases at its maximum rate, and m is the pore fluid connectivity network parameter. Lu and Dong (2015) found that the parameters m and S_f are linked with the parameters of the SWRC. This indicates that temperature effects on the SWRC may affect the TCF. Grant and Salehzadeh (1996) found that elevated temperatures cause a reduction in surface tension and water-solid contact angle, which lead to lower suctions and a shift in the SWRC. However, the shift in the SWRC is not significant for most soils, so it was assumed that this shift has a negligible effect on the TCF. Instead, it is assumed that effects of other heat transfer mechanisms in unsaturated soils (pore fluid convection and phase change) at different degrees of saturation cause the temperature effects on the measured apparent thermal conductivity.

The amount of thermally-induced vapor diffusion in unsaturated soils is expected to increase with decreasing degree of saturation due to the increased pathways for diffusion through air-filled voids. On the other hand, the occurrence of phase change in unsaturated soils is expected to increase with increasing degree of saturation due to the greater amount of water available in the pores, with a sharp onset at a given degree of saturation. To account for these two competing mechanisms, negligible phase change and vapor diffusion were assumed to occur at degrees of saturation less than S_f, with negligible changes in apparent thermal conductivity. A sharp jump in apparent thermal conductivity at S_f with a magnitude depending on temperature was assumed to account for the onset of phase change combined with enhanced vapor diffusion. A decay in apparent thermal conductivity with increasing degree of saturation above Sr is assumed to occur due to the decreasing amount of vapor diffusion. Following this logic, Equation (9) was modified by Samarakoon et al. (2018) to represent the observed effects of convective and latent heat transfer at different values of degree of saturation by adding a thermal adjustment term to the TCF of Lu and Dong (2015), as follows:

$$\frac{\lambda_a-\lambda_{dry}}{\lambda_{sat}-\lambda_{dry}}=1-\left[1+\left(\frac{S}{S_f}\right)^m\right]^{1/m-1} \quad \text{if } S < S_f \tag{10}$$

$$\frac{\lambda_a-\lambda_{dry}}{\lambda_{sat}-\lambda_{dry}}=1-\left[1+\left(\frac{S}{S_f}\right)^m\right]^{1/m-1}+a(1-S)\left(\frac{T-T_r}{T_r}\right)^b e^{-S} \quad \text{if } S > S_f \tag{11}$$

where *a* and *b* are fitting parameters. A comparison between the nonisothermal apparent thermal conductivity function at a random elevated temperature with the TCF of Lu and Dong (2015) at room temperature is shown in Figure 1. The thermal adjustment term (i.e., the convective term) is also shown for comparison. The apparent thermal conductivity equals the TCF by Lu and Dong (2015) at room temperature and when the degree of saturation is below S_f. At S_f, a sudden increase in apparent thermal conductivity occurs, followed by a nonlinearly increasing then decreasing trend as the degree of saturation increases.

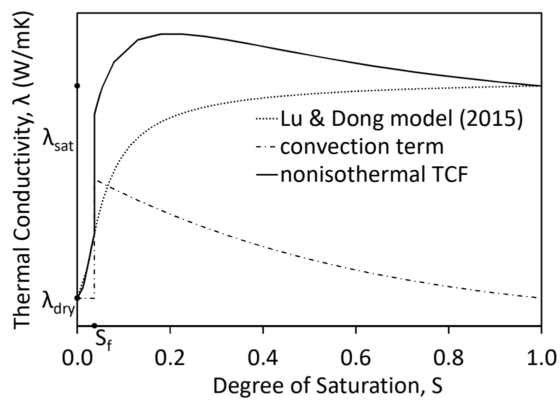

Figure 1 Comparison between the nonisothermal apparent thermal conductivity function and the TCF of Lu and Dong (2015)

Comparisons between the fitted nonisothermal apparent thermal conductivity function in Equations (10) and (11) and the measured data for different soils from Smits et al. (2013) are shown in Figures 2 and 3. The curves in Figure 2 for 30/40 sand reflect the impact of sand density on the apparent thermal conductivity, with the dense sand of dry density 1.77 Mg/m^3 having a consistently higher thermal conductivity than the loose sand having a density of 1.57 Mg/m^3. Similar comparisons between the fitted nonisothermal apparent thermal conductivity and soil data from Hiraiwa and Kasubuchi (2000), Nikolaev et al. (2013) and Campbell et al. (1994) for different soil types are shown in Figures 4, 5, and 6, respectively. Good agreement is observed for the soil types considered.

For the simplified heat transfer analysis in unsaturated soils, the derivative of λ_a with respect to T can be obtained from the derivative of Equation (11), as λ_a in Equation (10) for values of degree of saturation below S_f does not depend on the temperature. The derivative of Equation (11) is given as follows:

$$\frac{\partial\lambda_a}{\partial T}=(\lambda_{sat}-\lambda_{dry})a(1-S)e^{-S}b\frac{1}{T_r}\left(\frac{T-T_r}{T_r}\right)^{b-1} \tag{12}$$

Equations (10), (11), and (12) can be substituted into Equation (6) and solved for the temperature distribution with space and time using the finite difference method using the formation in Equation (8).

4. EVALUATION OF THE SIMPLIFIED MODEL FOR HEAT TRANSFER IN UNSATURATED SOILS

4.1 Example Problem

To evaluate the simplified model for heat transfer in unsaturated soils, axisymmetric, horizontal heat transfer in a soil layer away from a geothermal borehole heat exchanger represented as a line source was simulated. Specifically, a horizontal soil layer divided into elements of length Δr was considered with a heat source at one end having a constant temperature T_s and with the other end having a fixed ambient temperature T_r, as shown in Figure 7. This simple geometry permits an evaluation of both the transient and steady-state predictions from the simplified model for heat transfer in unsaturated soils. This geometry can also be used to simulate the heat transfer due to conduction using Equation (2) with a constant thermal conductivity value for comparison purposes.

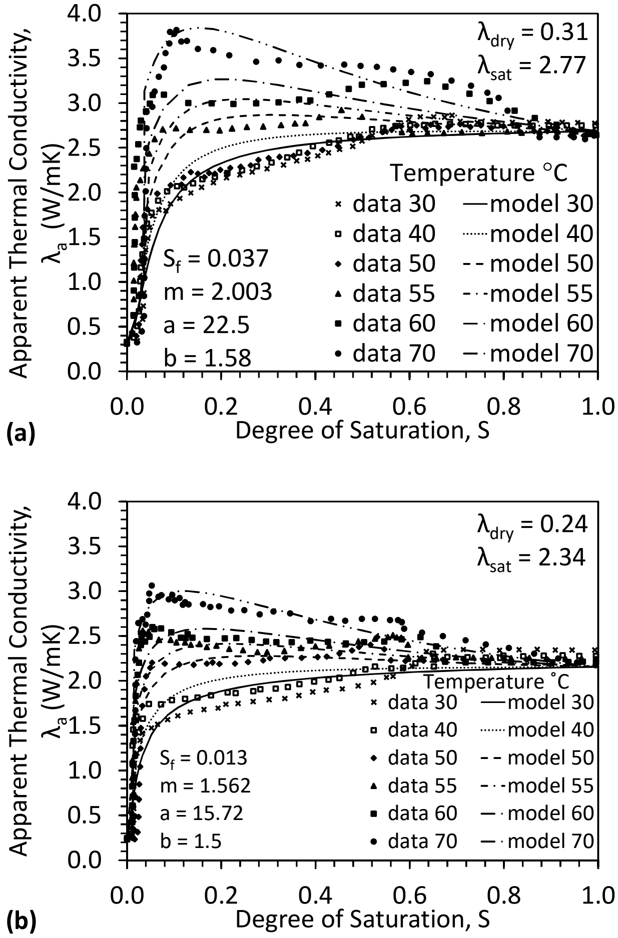

Figure 2 Nonisothermal apparent thermal conductivity function fitted to 30/40 sand data from Smits et al. (2013) at different densities: (a) Dense (tightly-packed); (b) Loose (loosely-packed)

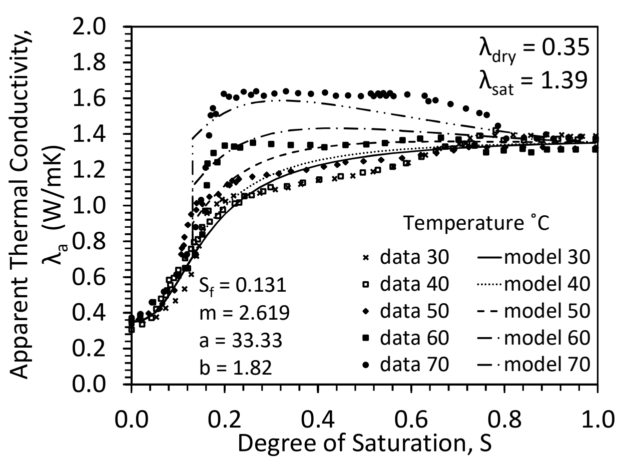

Figure 3 Nonisothermal apparent thermal conductivity function fitted to Great Sand Dunes sand data from Smits et al. (2013)

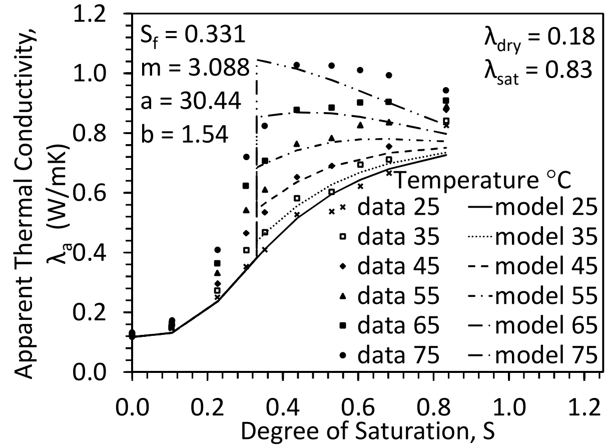

Figure 4 Nonisothermal apparent thermal conductivity function fitted to Ando soil data from Hiraiwa and Kasubuchi (2000)

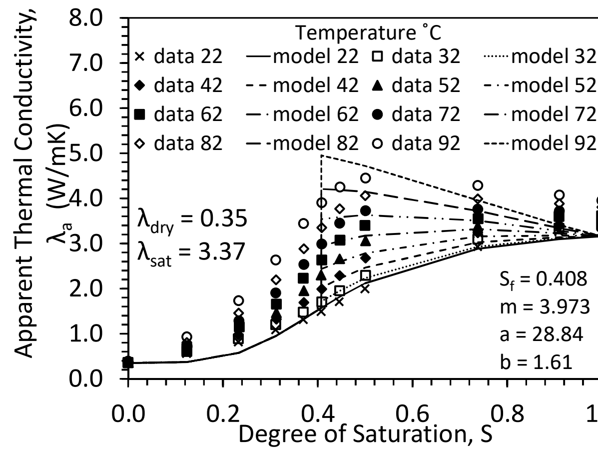

Figure 5 Nonisothermal apparent thermal conductivity function fitted to Ottawa sand data from Nikolaev et al. (2013)

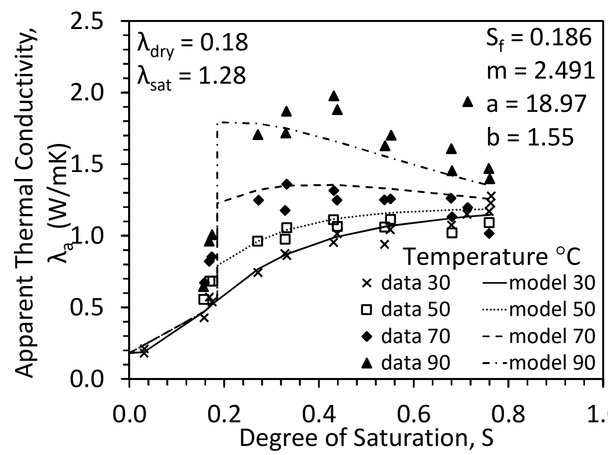

Figure 6 Nonisothermal apparent thermal conductivity function fitted to Royal soil data from Campbell et al. (1994)

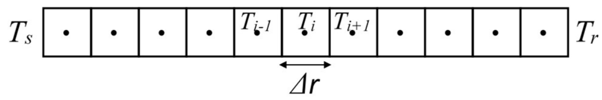

Figure 7 Discretization of a one-dimensional axisymmetric soil layer

For the comparison of the simplified model for heat transfer in unsaturated soils with a conventional conduction analysis, a horizontal soil layer having a length of 1m was divided into 20 elements of 0.05 m each. A time step of 0.1 seconds was used in the analysis. This domain geometry and the numerical formulation in Equation (8) were implemented and solved in Matlab.

Constant temperature boundary conditions were imposed at both ends of the domain. A heat source with a constant temperature value of T_s =70 °C is located at one of the domain. At the other end of the domain, the temperature was maintained at T_r=30 °C. The initial temperature is equal to 30 °C. The initial degree of saturation was 0.1 (i.e., relatively dry conditions) throughout the domain and was constant with time. Regarding the soil in the example problem, the properties of the loosely-packed 30/40 sand reported by Smits et al. (2013) and shown in Figure 2(b) were considered. In the simplified model for heat transfer in unsaturated soils, the apparent thermal conductivity will vary with temperature according to Equation (11) (i.e., nonisothermal properties), while in the conventional conduction analysis the thermal conductivity is constant and equal to 1.70 W/mK (i.e., isothermal properties). In both analyses, a specific heat capacity of 830 J/kgK was used for the 30/40 sand having a dry density of 1.57 Mg/m^3.

4.2 Comparison of the Simplified Model for Heat Transfer in Unsaturated Soils with a Conventional Conduction Analysis

The temperature in the soil domain was predicted as a function of space and time using both the simplified model for heat transfer in unsaturated soils (results in figures labelled as nonisothermal) and the conventional conduction analysis with a constant thermal conductivity (results in figures labelled as isothermal). In both analyses, heat transfer occurs from the constant temperature heat source at one end to the constant temperature heat sink at the other end. However, both models predict different steady state temperature distributions.

Time series of temperature at the midpoint of the horizontal sand column are shown in Figure 8. The temperature is observed to increase at a higher rate and reach a higher value in the simplified model for heat transfer in unsaturated soils, indicating that it indirectly incorporates the effects of the other mechanisms of heat transfer in unsaturated soils. Thermal equilibrium is reached after approximately 130 hrs for the nonisothermal case whereas it is reached after approximately 140 hrs for the isothermal case. The results in Figure 9 show the steady state distribution in temperature in the soil domain after 15 days. Although the temperature at the ends of the soil domain are the same for both models because of the imposed boundary conditions, the soil in the middle of the domain has a higher temperature for the nonisothermal analysis.

The trend in apparent thermal conductivity with time in Figure 10 follows a similar trend to the increase in temperature at this location shown in Figure 8. The thermal conductivity in the isothermal analysis is constant. The spatial variation of the apparent thermal conductivity at steady state conditions in both models is shown in Figure 11. The apparent thermal conductivity is highest at the location of the heat source, and decreases with distance until it approaches the isothermal value at the other end of the domain. This follows the pattern of the spatial distribution of temperature due to the temperature dependence of the apparent thermal conductivity function in Equation (11). The apparent thermal conductivity considered in the simplified model for heat transfer only considers the increased effects of convection of pore fluids and phase change in unsaturated soils at elevated temperatures, and does not represent the actual thermal conductivity of the soil.

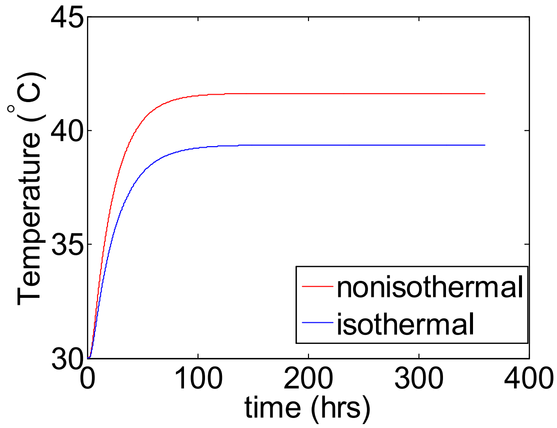

Figure 8 Time series of temperature at the midpoint of the horizontal 30/40 sand (loosely packed) layer (initial degree of saturation = 0.1, source temperature = 70 °C)

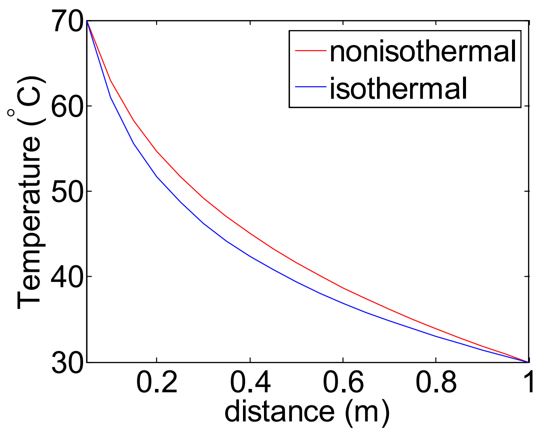

Figure 9 Steady state spatial distribution of temperature after 15 days for the 30/40 sand (loosely packed) layer (initial degree of saturation = 0.1, source temperature = 70 °C)

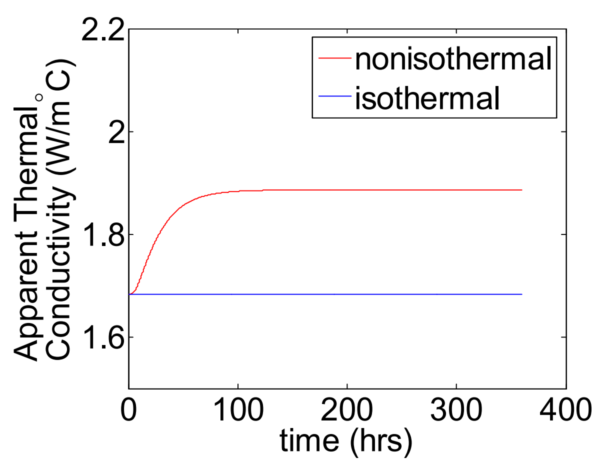

Figure 10 Trends in apparent thermal conductivity with time at the midpoint of the horizontal 30/40 sand (loosely packed) layer (initial degree of saturation = 0.1, source temperature = 70 °C)

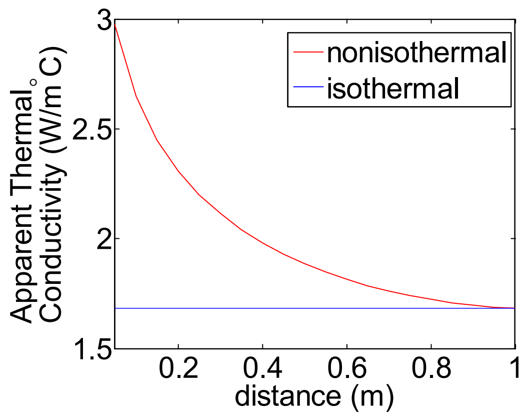

Figure 11 Spatial variation of apparent thermal conductivity after 15 days for 30/40 sand (loosely packed) layer (initial degree of saturation = 0.1, source temperature = 70 °C)

Further, it must be noted that the simplified model for heat transfer in unsaturated soils does not consider thermally induced changes in degree of saturation of the soil. In reality, the unsaturated soil near the heat source will experience a decrease in degree of saturation, which will lead to a decrease in the actual thermal conductivity (Baser et al. 2018). This decrease in actual thermal conductivity cannot be predicted using this model, which means that subsequent cooling processes will be affected by the decrease in actual thermal conductivity. This indicates that the simplified model for heat transfer in unsaturated soils is only suitable for simulation of monotonic heating.

5. COMPARISON WITH EXPERIMENTAL DATA

Experimental results from Baser et al. (2018) for heat transfer in a tank-scale heat injection experiment using unsaturated, compacted Bonny silt were used for comparison with the simulated results from the simplified model for heat transfer in unsaturated soils. Bonny silt is classified as an inorganic silt (ML) according to the Unified Soil Classification System and the soil specimen used in the experiment has a dry unit weight of 14 kN/m^3, a porosity of 0.46, and an initial degree of saturation of 0.42. For an initial degree of saturation of 0.42, the specific heat capacity of Bonny silt is equal to 1840 J/kgK. The values of λ_{sat}, λ_{dry}, S_f and m were obtained from Table 1 of Lu and Dong (2015), and are 1.28, 0.37, 0.145, and 2.62, respectively. Values of the nonisothermal TCF fitting parameters *a* and *b* were estimated to be 19 and 1.55, respectively, based on the range of these parameters obtained for the soil types in Figures 2 through 6. The TCF at room temperature based on Lu and Dong (2015) and the nonisothermal TCF at 60 °C based on Samarakoon et al. (2018) are shown in Figure 12. The relationships in this figure indicate that the greatest increase in apparent thermal conductivity with temperature occurs around a degree of saturation of 0.35, close to the initial degree of saturation of 0.42 in the experiment of Baser et al. (2018).

To simulate the tank-scale experiment of Baser et al. (2018) shown in Figure 13, horizontal heat transfer from the centre of an axisymmetric domain with a length of 0.275 m was considered. Heat transfer was imposed by applying a constant boundary temperature of 60 °C at the heat source and 23.5 °C at the far end. The initial temperature was also taken as the ambient temperature of 23.5 °C and the initial degree of saturation was considered to be 0.42 consistent with the experimental set up. The model discretization is also shown in Figure 13, with the left-hand side of the model corresponding to the centre of the tank and the right-hand side of the model corresponding to the edge of the tank.

A comparison between the spatial distribution of temperature from the experiment after 5 hrs of heating and the temperatures obtained from the simplified model is shown in Figure 14. Good agreement can be observed between the experimental results and the numerical simulation using the nonisothermal apparent thermal conductivity function. The over-estimate of temperature closer to the heat exchanger in the simplified model for heat transfer in unsaturated soils can be attributed to the fact that the model does not consider thermally induced drying of the soil near the heat exchanger. This thermally induced drying will lead to a decrease in degree of saturation and a corresponding decrease in thermal conductivity. Nonetheless, the simplified model captures the general trend in the data, indicating that it may be useful for preliminary analysis of monotonic heating.

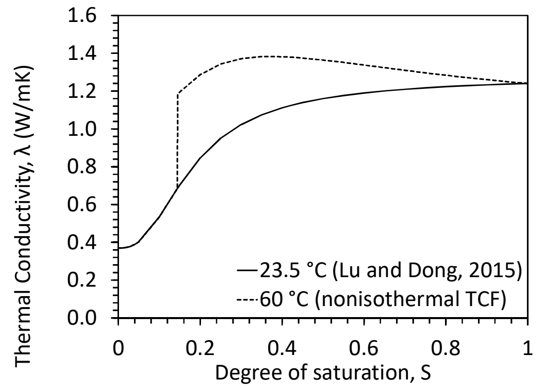

Figure 12 TCFs for Bonny silt at room temperature (Lu and Dong 2015) and at 60 °C (nonisothermal)

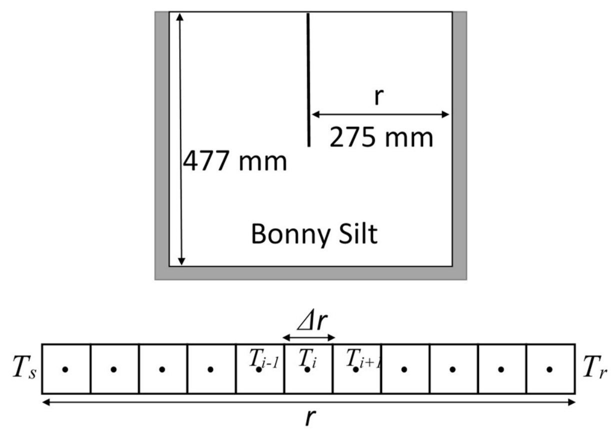

Figure 13 Schematic diagram of the test set up of Baser et al. (2018) and the model discretization

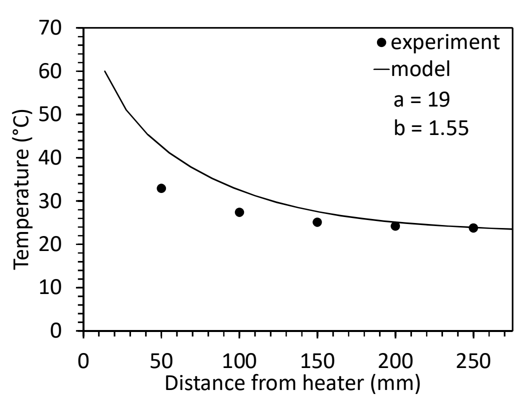

Figure 14 Comparison of spatial distribution of temperature after 5 hrs with experimental data for Bonny silt from Baser et al. (2018)

6. CONCLUSION

This paper presents a simplified model for heat transfer in unsaturated soils that involves a conduction analysis combined with a nonisothermal apparent thermal conductivity function. The apparent thermal conductivity function is assumed to indirectly incorporate the effects of mechanisms of heat transfer in unsaturated soils other than conduction as well (i.e., thermally-induced convection of pore water in liquid and vapor forms along with latent heat transfer due to water phase change). The nonisothermal apparent thermal conductivity function used in this study also accounts for the changes in magnitude of the effects of these other mechanisms of heat transfer as the degree of saturation in the soil changes. To account for the changes in apparent thermal conductivity with temperature during heat transfer process, the governing equation for conductive heat transfer was reconsidered to account for the spatial and temporal variation of apparent thermal conductivity.

The simplified model for heat transfer in unsaturated soils was first evaluated by comparing the predicted temperatures with those from a conventional conduction analysis using a constant thermal conductivity value for an example problem involving axisymmetic, horizontal heat transfer through a soil layer from a heat source to a heat sink. The temperature at the midpoint of the soil layer was observed to increase at a higher rate and reach a higher equilibrium temperature in the simplified model using a nonisothermal apparent thermal conductivity function than when conduction with a constant thermal conductivity was used in the heat transfer analysis. This confirms that the simplified model for heat transfer in unsaturated soils indirectly reflects the roles of the other mechanisms of heat transfer in unsaturated soils that lead to enhanced heat transfer. As expected, the trends in the apparent thermal conductivity at different locations followed similar trends to the change in temperature in the soil.

The simplified model for heat transfer in unsaturated soil was also compared with experimental data from a heat transfer problem in unsaturated silt. A reasonable agreement was obtained between the experimental and numerically simulated results, although the simplified model led to an overprediction of temperature closer to the heat source because it is not capable of considering the reduction in the actual thermal conductivity associated with thermally induced drying. This indicates that the simplified model may only be useful in preliminary analyses of problems involving monotonic heating of unsaturated soils, but not in problems involving cyclic heating and cooling of unsaturated soils where it is critical to consider the changes in degree of saturation and the corresponding effects on the actual thermal conductivity. Comparison with more experimental studies of heat transfer in different types of unsaturated soils over a wider range of degrees of saturation will help better understand the applicability of this simplified model to different heat transfer problems in unsaturated soils.

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