# Probabilistic Stability Analyses of Reinforced Slope Subjected to Strip Loading

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**ABSTRACT:** The aim of the present study is to investigate the effect of uncertainty associated with soil friction angle ($\phi$) and soil unit weight ($\gamma$) on the stability of both unreinforced and reinforced cohesionless soil slopes subjected to strip loading. The magnitude of $CoV$ of $\phi$ and $\gamma$ are varied to account uncertainties. The location of the footing on the top of the slope is also changed. Stability of both unreinforced and reinforced slopes is presented in terms of factor of safety ($FoS$). Deterministic $FoS$ values are computed first by using a two-dimensional finite difference software FLAC. To perform probabilistic analyses, FLAC is combined with Monte Carlo simulations. The outcomes of the probabilistic analyses are presented in terms of probability of failure ($p_F$) and reliability index ($\beta$). The value of $\beta$ obtained from the present study is compared with the guidelines provided by USACE. It is found out that with the increase in the value of $CoV$, $p_F$ increases and $\beta$ decreases. As expected, the failure probability of slope is found to be maximum, when footing is placed on the edge of the unreinforced slope. With the inclusion of a single layer of geotextile in the slope for the same footing position, $p_F$ reduces drastically, and $\beta$ increases significantly. As footing position shifts from the slope edge, $p_F$ increases for a particular $CoV$ value of $\phi$ and $\gamma$. The effect of uncertainty related to $\phi$ is found to be more prominent with compared to the uncertainty related to $\gamma$. The influence of cross-correlation between $\phi$ and $\gamma$ is also studied. It is found that there is no significant change in the value of $p_F$ with the change in the value of cross correlation coefficient. Though the present study is related to a simple slope stability problem, but using the same methodology, probabilistic analyses of complex slopes can also be performed.

**KEYWORDS:** Probabilistic analyses; Reinforced slope ; Strip footing; FLAC; MCS

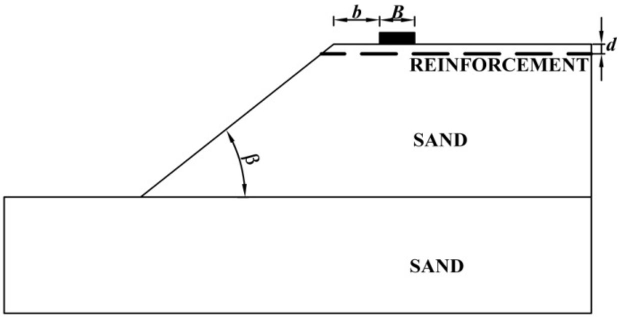
## 1. INTRODUCTION

As a consequence of the rapid urbanizations and non-availability of land in the hilly area, myriad structures are constructed now a days on or near the slope edge. When a structure is constructed on or near the edge of the slope, two types of failure are associated with it, namely, (a) bearing capacity failure of the foundation, and (b) slope failure. With the usage of different methods such as limit equilibrium, limit analysis, and finite element, various solutions were presented by Meyerhof (1957), Azzouz and Baligh (1983), Michalowski (1989), Narita and Yamaguchi (1990), Georgiadis *et al*. (2008), Shiau *et al*. (2011), Chakraborty and Kumar (2013), Leshchinsky (2015), Leshchinsky and Xie (2017), and Halder *et al*. (2017) for unreinforced soil slopes. With the rapid advancement in the field of reinforced-earth technology, researchers started to include various types of polymeric geosynthetic within the slope to enhance its stability as well as load carrying capacity of the foundation. By carrying out experimental as well as theoretical studies, Schneider and Holtz (1986), Leshchinsky and Boedeker (1989), Sawicki and Lesniewska (1989), Michalowski (1997), Zornberg *et al*. (1998), Mehdipour *et al*. (2013), Mehdipour *et al*. (2017) investigated stability aspect of reinforced soil slope. On the other hand, Huang and Tatsuoka (1994), Lee and Manjunath (2000), Yoo (2001), Blatz and Bathurst (2003), Latha *et al*. (2006), Latha and Rajagopal (2007), Alamshahi and Hataf (2009) studied the behaviour of footing placed on or near the slope edge by carrying out theoretical studies. It is to be mentioned here that all the previous works cited above do not consider the uncertainties associated with soil parameters, loading condition, reinforcement properties, and various assumptions and simplifications considered during the analysis. All of the previous works cited above consider soil parameters as deterministic variables. Liang *et al*. (1999), Duncan (2000), and Li *et al*. (2012) stated that two slopes might be susceptible to different levels of risks although they have the same factor of safety. Since the last few decades, several researchers (Dasaka and Babu, 2008; Babu and Singh, 2010; Luo *et al*., 2015; Liu *et al*., 2016) used probabilistic theory in solving various geotechnical engineering problems. By performing probabilistic analyses, one can directly include the effect of various types of uncertainties on the behaviour of the slope. Whitman (1984), Low *et al*. (1998), Liang *et al*. (1999), Duncan (2000), El-Ramly *et al*. (2002), Griffiths and Fenton (2004), Srivastava and Babu (2009), Wang *et al*. (2010), Griffiths *et al*. (2009, 2011), Cho (2010), Javankhosdel and Bathurst (2014, 2016), Luo and Bathurst (2017) introduced uncertainties associated with soil parameters in the unreinforced slope stability problems. On the contrary, the application of reliability on reinforced slope stability problems is found to be limited. The probabilistic analyses of two slopes were carried out by Kitch (1994). The initial layout of reinforcement was selected based on the deterministic design charts. Low and Tang (1997) carried out the stability analysis of a reinforcement embankment situated over soft clay and also proposed the procedures of calculating reliability index of the same by using spreadsheet technique. Luo *et al*. (2016) performed probabilistic analysis of geosynthetic reinforced slope by using strength reduction method incorporated in an open source finite element code. The influence of spatial variability of soil friction angle on the stability of both unreinforced and reinforced soil slopes is also investigated. Ferreira *et al*. (2016) included the uncertainties associated with soil unit weight, soil friction angle, soil-geosynthetic interface friction angle, and tensile strength of geosynthetic in the structural reliability calculation of both reinforced and unreinforced steep slopes according to EC-7. The effect of variability of each parameter on the slope stability was investigated by carrying out a sensitivity analysis.

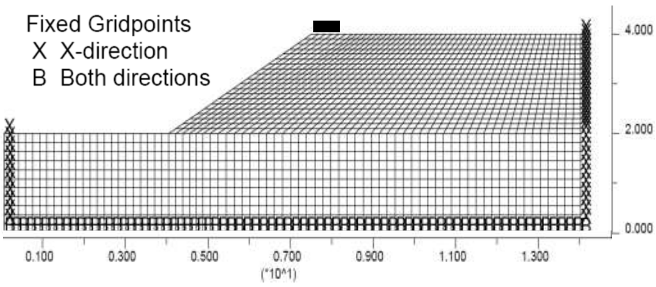
It is to be noted that except Luo *et al*. (2016), all the probabilistic stability analyses of reinforced soil slopes were based on the limit equilibrium method which has the disadvantage associated with the prior assumption of the critical failure surface. In addition to that, according to authors' knowledge, no such study is available where uncertainties related to soil parameters are considered in the slope stability analysis of both unreinforced and reinforced soil slopes subjected to strip loading. This study investigates the influence of uncertainties associated with soil friction angle ($\phi$) and soil unit weight ($\gamma$) on the stability analyses of both unreinforced and reinforced cohesionless soil slopes subjected to strip loading. The effect of cross correlation between $\phi$ and $\gamma$ on the probabilistic outcomes is also examined. A two-dimensional finite difference software Fast Lagrangian Analysis of Continua (FLAC) is used to carry out all deterministic analyses. Whereas, to perform probabilistic analyses, FLAC is used in combination with Monte Carlo simulations (MCS). Finally, outcomes of the probabilistic analyses are presented in terms of probability of failure ($p_F$) and reliability index ($\beta$).

## 2. PROBLEM STATEMENT

The objective of the present study is to incorporate the uncertainties associated with soil friction angle ($\phi$) and soil unit weight ($\gamma$) on the stability analyses of the both unreinforced and reinforced cohesionless soil slopes subjected to strip loading. To investigate the effect of reinforcement on the stability of the slope, a single layer of geotextile reinforcement is laid in the slope at various depths ($d$) measured from the bottom surface of the footing. A schematic diagram of the problem as well as a two-dimensional finite difference mesh which is used throughout this study is illustrated in Figures 1(a-b). The mesh consists of 1865 number of quadrilateral zones. After carrying out several numbers of trails, mesh size is finalized. The horizontal and vertical extents of the mesh are chosen in such a way so that stress should not reach to the boundary surfaces in both the directions. Both vertical and horizontal movement is restrained along the bottom boundary surface. Whereas, only vertical movement is kept permissible along the side boundary surfaces of the mesh. The strip loading is simulated on the top of the slope by applying uniform pressure over a zone of width $B$. The footing setback distance is denoted by $b$. The slope is assumed to be inclined at a constant angle of 30° with the horizontal axis. Both heights of slope and foundation soil are taken as $2B$. The soil is considered to be linearly elastic perfectly plastic material governed by Mohr-Coulomb failure criterion. Various properties of soil are taken from Yang *et al.* (2015) and enlisted in Table 1. In addition to that, soil is assumed to have a small cohesion value of 1 kPa to avoid numerical instabilities during analyses. Luo *et al.* (2016) reported that this amount of soil cohesion value have negligible effect on the numerical outputs. Geotextile is modeled by using cable elements in FLAC. It is a one-dimensional axial element which has resistance against tension or compression, but it cannot withstand bending moment. The material properties of geotextile element are also taken from Yang *et al.* (2015) and provided in Table 2.




*(a)*




*(b)*

Figure 1 (a) Schematic diagram of the problem; (b) Typical Finite difference mesh used throughout the study

Table 1 Soil properties used in simulation (After, Yang *et al.*, 2015)

| **Property** | **Values** |
|---|---|
| Cohesion (kPa) | 1.0 |
| Friction Angle (°) | 36 |
| Dilation Angle (°) | 8 |
| Unit Weight (kN/m$^3$) | 1.6 |
| Bulk Modulus (kPa) | $4\times 10^4$ |
| Shear Modulus (kPa) | $2 \times 10^4$ |

Table 2 Geotextile properties used in simulation (After, Yang *et al.*, 2015)

| Type | Property | Value |
|---|---|---|
| Basic | Section Area (m$^2$/m) | $1.38 \times 10^{-3}$ |
| | Mass Density (g/ m$^2$/m) | 462 |
| Axial | Ultimate Strength (kN/m) | 10.80 |
| | Elastic modulus (kPa) | $7.14\times 10^5$ |
| Interface | Ultimate Strength (kN/m) | 23.80 |
| | Elastic modulus (kPa) | $2.38\times 10^3$ |

## 3. UNCERTAINTIES IN SOIL PARAMETERS

According to Phoon and Kulhawy (1999), various types of uncertainties in soil parameters are attributed to (i) the heterogeneity in in-situ soil mass due to wide range of constituents, (ii) measurement errors, (iii) errors in various assumptions and simplifications considered during the transformation of field or laboratory measurements into design properties. Similarly, such kinds of uncertainties also exist for reinforcement parameters and loading conditions. However, in the present study, only uncertainties in soil parameters are considered. The uncertainties in the measured soil properties are quantified by the mean ($\mu$) and coefficient of variation ($CoV$). Based on the prior works, a wide range of $CoV$ values of $\phi$ and $\gamma$ are considered for the present study (refer Table 3). In probabilistic analyses, each input variables follow a particular type of distribution. To avoid generation of negative values in MCS, $\phi$ and $\gamma$ are assumed to have lognormal distribution. Firstly, $\phi$ and $\gamma$ are considered to be uncorrelated random variables. However, in the real world, soil parameters are correlated with each other. So, cross correlation between $\phi$ and $\gamma$ is also considered afterward. Babu and Srivastava (2007), Wu (2013), Javankhosdel and Bathurst (2016) also consider positive correlations between $\phi$ and $\gamma$ in probabilistic analyses. In the above-mentioned literature, range of correlation coefficient between $\phi$ and $\gamma$ is in between 0.2-0.7.

Table 3 Typical coefficients of variations of $\phi$ and $\gamma$ used in MCS

| Parameter | $CoV$ (%) | Source |
|---|---|---|
| Soil friction angle ($\phi$°) | 1 - 20 | Lumb (1970), Harr (1984), Nguyen and Chowdhury (1984), Kulhawy (1992), Becker (1996), Cherubini (2000), Ferreira *et al.* (2016) |
| Soil unit weight ($\gamma$) | 3 - 20 | Harr (1984), Kulhawy (1992), Phoon and Kulhawy (1999), Cherubini (2000), Ferreira *et al.* (2016) |

## 4. MODELING METHODOLOGY

### 4.1 Slope Stability Analysis

Stability of the cohesionless soil slope in FLAC is expressed in terms of the factor of safety (*FoS*). Strength reduction method is used in FLAC for the calculation of FoS. In this technique, a series of simulations are run for trail factor of safety value ($F_{trial}$). The initial value of soil shear strength ($\phi$) is progressively reduced in each simulation by that $F_{trail}$ value until the slope reaches to the failure state. FLAC uses 'bracket and bisecting technique' where lower and upper brackets are set up for a particular $F_{trail}$ value. Lower bracket corresponds to that $F_{trail}$ value for which solution converges, and upper bracket corresponds to that $F_{trail}$ value for which solution does not converge. The average of these two bracket values are selected in the next trail and simulation is run again. If the solution converges then lower bracket value is substituted with the current $F_{trail}$ value and if the solution does not converge then upper bracket value is substituted with the current $F_{trial}$ value. Several simulations are carried out until the difference between upper and lower bracket values reach to a specified tolerance value. Similar kind of logic is also applicable for cable element which represents geotextile reinforcement.

### 4.2 Monte Carlo Simulation

There are several methods such as first-order reliability method (FORM), second-order reliability method (SORM), point estimate method (PEM), etc. which are used for calculation of reliability or safety index ($\beta$) as well as the probability of failure ($p_F$). However, above mentioned analytical methods are complex in nature and good knowledge of probability and statistics is also required. Whereas, Monte Carlo simulation (MCS) is a probabilistic method which is comparatively easier to compute the probability of failure and a little background knowledge in probability and statistics is required. The advancement of computer makes MCS more computationally efficient. The basic steps of MCS method are provided in Halder and Mahadevan (1999) and Ferreira *et al*. (2016). In the present study, depending upon $\mu$ and $CoV$ value, $N$ number of uncorrelated log-normal random variables $\phi$ and $\gamma$ are first generated in MATLAB by using inbuilt 'logrand' command and then stored in ASCII files. After that, in each simulation FLAC takes one $\phi$ and $\gamma$ value from ASCII file and calculates *FoS*. Such procedures are carried out for $N$ number of times. Here, $N$ denotes the total number of simulations. Figures 2(a-b) showed that there are no significant changes in the mean of *FoS* and standard deviation of *FoS* after 1500 number of simulations. Thus the final value of $N$ is fixed as 1500. After calculating *FoS* at the end of each simulation, it is checked whether calculated *Fo*S value is less than one. If $FoS < 1$, it denotes failure. On the other hand, if $FoS \geq 1$, it means the slope is safe. By this way, total number of failures can be calculated and counted. Now, the total number of failure ($N_f$) is divided by the total number of simulations ($N$) to obtain the $p_F$. So, the expression is $p_F = \dfrac{N_f}{N}$. All of the above procedures are executed by writing a program in the FISH language which is embedded in FLAC. Following the methodology proposed by Nguyen and Chowdhury (1985), and Ching and Phoon (2012), depending upon correlation coefficient between $\phi$ and $\gamma$ the correlated lognormal variables $\phi$ and $\gamma$ are generated in MATLAB. For the sake of completeness, the procedure is provided here very briefly.

(i) Independent standard normal variables ($Z_1$ and $Z_2$) are generated by using MATLAB command 'normrnd'. .

(ii) Correlated standard normal variables ($Y_1$ and $Y_2$) are generated by using $Y = LZ$. Here, $L$ denotes the lower triangular matrix obtained from the Cholesky decomposition of $R$ matrix, where $R = LL'$. $R$ is the matrix which constitutes of correlation coefficients between variables. If there are two random variables, $R$ is obtained by the following expressions:

$$R = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix} \quad (1)$$

$\rho_{12}$is the correlation coefficient between two random variables.

(iii) Mean and standard deviation values of normal variables ($\mu$ and $\sigma$) are then transferred to the mean and standard deviation values of lognormal variables ($\mu_{ln}$ and $\sigma_{ln}$) by following equations.

$$\mu_{\ln} = \ln(\mu) - 0.5\sigma_{\ln}^2 \quad (2)$$

$$\sigma_{\ln}^2 = \ln(1 + CoV^2) \quad (3)$$

*a*

(iv) Now, correlated lognormal variables are generated by using Eqn. (4).

$$X_i = \exp\left(\mu_{\ln} + \sigma_{\ln} Y_i\right) \quad (4)$$

Appendix -A provides a flowchart of probabilistic analysis which is carried out in the present study.

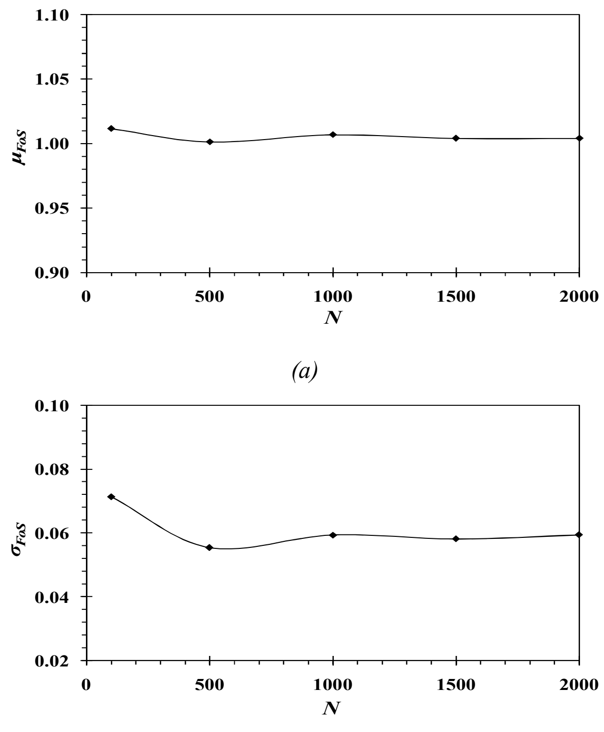



Figure 2 (a) Variation of mean of *FoS* with $N$; (b) Variation of standard deviation of *FoS* with $N$

### 4.3 Reliability index

In the present study, limit state function is given by the following equation:

$$FoS < 1 \quad (5)$$

It also represents the limit state boundary which demarcates the safe zone and failure zone. By using the term reliability index ($\beta$), probabilistic assessment of slope failure is carried out. $\beta$ value can be obtained from the $p_F$ value by using the following expression :

$$\beta = \Phi^{-1}(1 - p_F) \tag{6}$$

It is found that actual cumulative distribution of *FoS* matches very well with the log-normal fit. So, *FoS* is considered as lognormally distributed. By invoking 'loginv' command in MATLAB, $\beta$ is calculated.

### 4.4 Validation of Numerical Model

The present numerical results are validated with the results provided by Luo *et al*. (2016). Luo *et al*. (2016) computed *FoS* of unreinforced slopes for various values of $\phi$. The details of model geometry and soil properties are enlisted in Luo *et al.* (2016). Figure 3 shows the comparison between the values of FoS obtained from Luo *et al.* (2016) and the present study. *FoS* values obtained from the present study are found to be matched quite well with those values of *FoS* obtained from Luo *et al*. (2016).

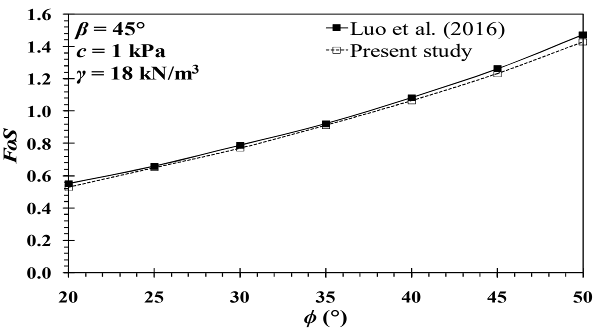



Figure 3 Comparison between the value of *FoS* with $\phi$ obtained from Luo *et al*. (2016) and the present study

## 5. RESULTS AND DISCUSSIONS

Results obtained for the deterministic and probabilistic analysis of both unreinforced and reinforced cohesionless soil slopes subjected to the strip loading are presented and discussed in detail in the following subsections.

### 5.1 Deterministic Analyses

At first, a series of deterministic numerical simulations are carried out by changing footing position (*b/B* = 0, 1, and 2) on the top of the unreinforced soil slope. A uniform pressure is applied on the nodes which represent footing position on top of the slope. The applied pressure is kept on increasing until *FoS* value reaches to unity. It is to be mentioned that same procedure is followed for all footing setback distances. Figure 4 indicates the variation of *FoS* with the applied pressure for various *b/B*. For a particular *b/B* value, *FoS* reduces with an increase in the value of applied pressure.

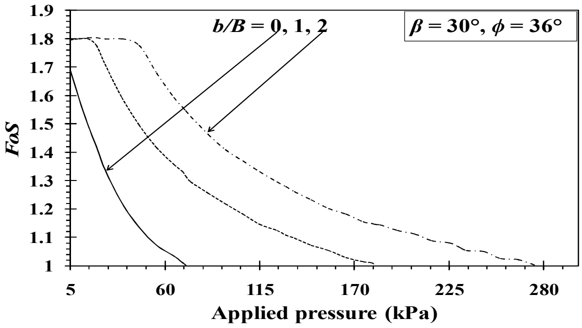



Figure 4 Variation of *FoS* with Applied pressure for *b/B* = 0, 1, and 2

As expected, with the increase in *b/B* value, the critical value of applied pressure which is required to bring the slope on the verge of failure (i.e., *FoS* = 1), increases.

For example, with the change in *b/B* value from 0 to 2, the critical value of applied pressure varies from 72 kPa to 275 kPa. From the results, it can be stated that the stability of the slope is found to be most vulnerable when footing is placed on its edge (i.e., *b/B* = 0). So, the critical value of the applied pressure for *b/B* = 0 case (i.e.,72 kPa) is to be considered as the reference load in the subsequent analyses meant for reinforced slope.

A series of numerical simulations are then carried out to investigate the effectiveness of the geotextile-reinforcement in improving the stability of the cohesionless soil slope subjected to strip loading. Footing setback distances (*b/B* = 0, 1, and 2) and position of reinforcement (*d/B*) are varied in these analyses. It is to be noted that in all simulations a uniform pressure of 72 kPa is applied to simulate the strip loading on top of the slope. The improvement in the stability of the slope is expressed by a dimensionless factor named improvement factor ($I_F$). It is the ratio between the value of *FoS* obtained for reinforced slope subjected to a uniform pressure of 72 kPa to the unity value of *FoS* obtained for unreinforced slope subjected to a uniform pressure of 72 kPa at the *b/B* = 0 position. The variation of *d/B* and $I_F$ for various *b/B* values are shown in Figure 5. It indicates that $I_F$ increases up to a certain increase in *d/B* value after that it decreases. The shear strain increment plots shown in Figure 6 various reinforcement positions in the slope with *b/B* = 0 case also endorse the above-stated fact. When reinforcement is placed at a smaller depth (*d/B* = 0.20), it deforms first and then redistributes the load more in the downward direction. Shear strain extends to the toe of the slope. On the other hand, when reinforcement is placed at a higher depth (*d/B* = 0.40), slope fails above the reinforcement layer. Here, in this case, shear strain also propagates towards the slope face above the reinforcement layer.

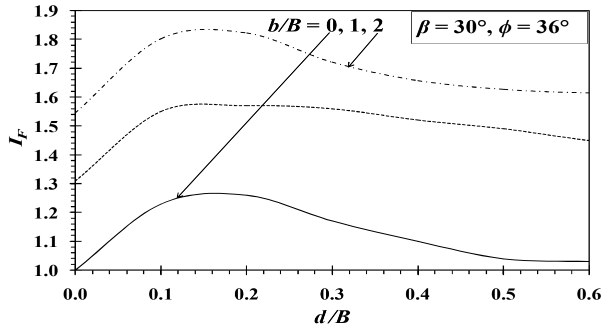
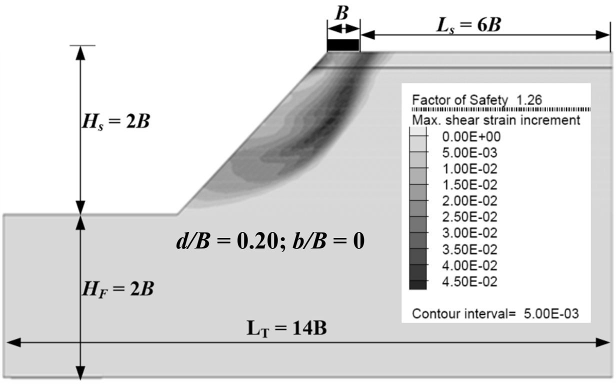


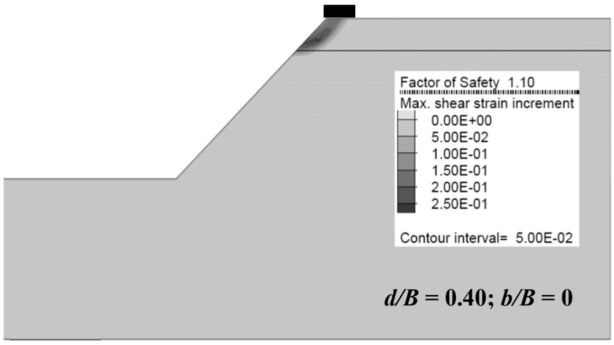

Figure 5 Variation of $I_F$ with *d/B* for *b/B* = 0, 1, and 2

### 5.2 Probabilistic Analyses

By varying the *CoV* of $\phi$, the effect of uncertainties related to soil friction angle on the unreinforced and reinforced slope stability is considered. It is to be mentioned here that all the probabilistic analyses of reinforced slopes are carried out by placing the reinforcement at the critical depth where maximum reinforcing efficiency is obtained from deterministic analyses. Figure 7a shows the variation of *CoV* of $\phi$ with the $p_F$ of unreinforced and reinforced slope for various *b/B* values. Here, *CoV* of $\gamma$ is kept constant as 5% for all the analyses. With the increase in the value of *CoV* of $\phi$ for a particular *b/B* value, $p_F$ increases. For an unreinforced slope with *b/B* = 0, it is found out that when *CoV* of $\phi$ changes from 5% to 20%, $p_F$ changes from 0.49 to 0.53. It is also observed that with the increase in *b/B* value, the chance of slope failure reduces significantly. When reinforcement is used, the probability of slope failure reduces drastically. For a combination of *b/B* =0, *CoV* of $\phi$ = 15%, and *CoV* of $\gamma$ = 5%, $p_F$ of unreinforced slope is found to be 0.52. On the other hand, for the same combination of *b/B*, *CoV* of $\phi$, and *CoV* of $\gamma$, $p_F$ of the geotextile-reinforced slope is computed as 0.10.

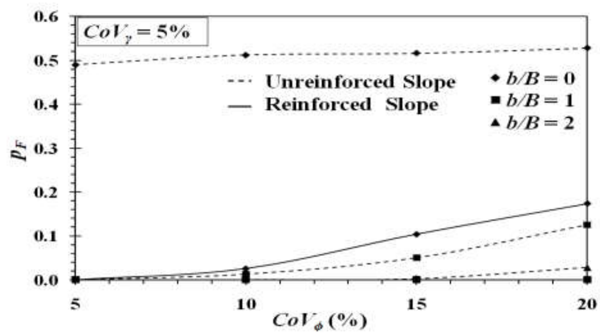



(a)

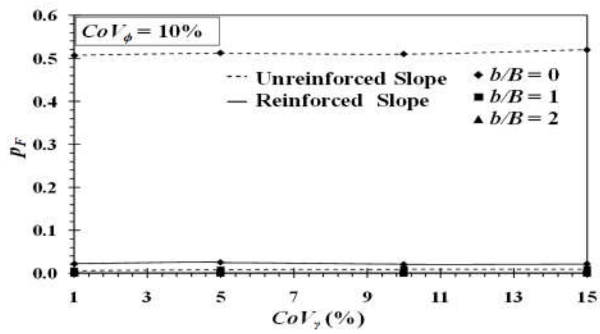



(b)

Figure 6 (a) Shear strain increment plot when reinforcement is placed at smaller depth ($d/B$ =0.20); (b) Shear strain increment plot when reinforcement is placed at higher depth ($d/B$ =0.40)




(a)

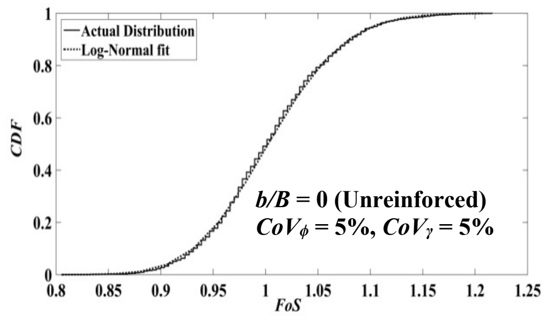



(b)

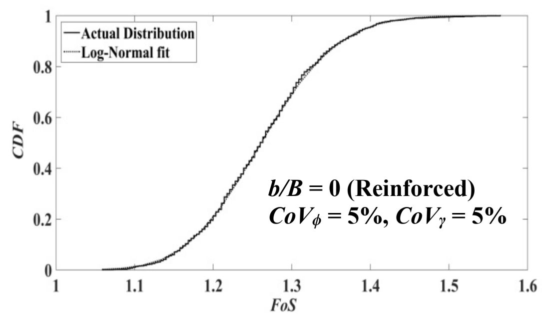
Figure 7 (a) Variation of $p_F$ with $CoV_\phi$ of unreinforced and reinforced slopes with $b/B$ = 0, 1, and 2; (b) Variation of $p_F$ with $CoV_\gamma$ of unreinforced and reinforced slopes with $b/B$ = 0, 1, and 2

As the footing shifts from the edge of the reinforced slope, $p_F$ decreases and becomes insignificant. The influence of uncertainties associated with $\gamma$ on the stability of reinforced and unreinforced slopes are investigated by varying *CoV* of $\gamma$. For these cases, *CoV* of $\phi$ is kept constant at a value of 10%. Figure 7b clearly indicates that $p_F$ increases with an increase in *CoV* of $\gamma$ for a particular value of $b/B$. However, the effect is not as prominent as it is observed when *CoV* of $\phi$ is varied. Similar results are also reported in previous literature. With the increase in $b/B$ value, $p_F$ value obtained from unreinforced as well as reinforced slopes are found to be very less, and the magnitude is in the order of $10^{-4}$.

An attempt is also made in this study to present the probabilistic outcomes of the slope stability analyses in terms of reliability index ($\beta$). It is a well-known fact that there is no exact cumulative distribution function (CDF) for output variables, and it is difficult to measure also. So, *FoS* is assumed to follow the lognormal distribution as both the input random variables are considered as lognormally distributed. This is also supported by the Figures 8(a-b) where actual CDF of *FoS* obtained from the present study is matched very well with the lognormal distribution of *FoS* values based on $\mu$ and $\sigma$ values of *FoS*.



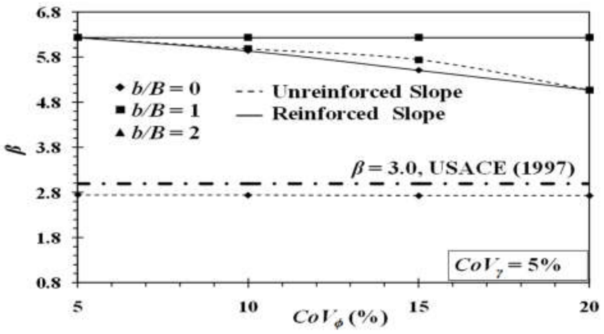

(a)



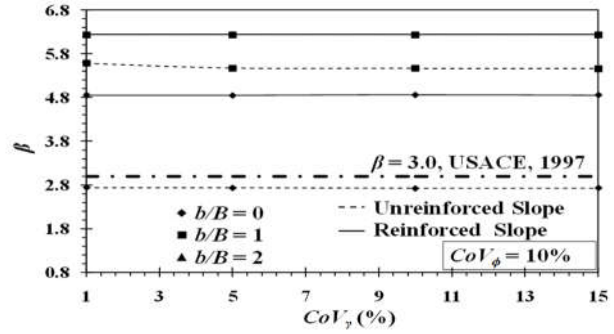

(b)

Figure 8 (a) Plot of Actual Distribution of *FoS* and Log-Normal fit of *FoS* for (a) unreinforced slope and (b) reinforced slope

Figures 9(a-b) show the obtained values of reliability index for various cases. It is found out that the relation between $p_F$ and $\beta$ is reciprocal. As $p_F$ increases, $\beta$ decreases and vice versa. It is to be mentioned here that USACE (1997) provided some guidelines regarding choice of reliability index for geotechnical and infrastructure project. For good and average performance of the systems, values of $\beta$ should be 4.0 and 3.0, respectively. When footing is placed on the edge of the unreinforced slope, $\beta$ value is found to be less than 3.0, thus making the overall system vulnerable. With the inclusion of a single layer of geotextile, $\beta$ increases significantly and found to be higher than $\beta$ = 3.0, which is required for the average performance of the system. It is found that with the increase in $b/B$; for unreinforced slope, $\beta$ increases. For reinforced slope with $b/B$ = 1 and 2, obtained values of $\beta$ are found to be greater than 3.0. It means overall performance of the footing-reinforced slope system improves with respect to the unreinforced case.




(a)




(b)

Figure 9 (a) Variation of $\beta$ with $CoV_\phi$ of unreinforced and reinforced slopes with $b/B$ = 0, 1, and 2; (b) Variation of $\beta$ with $CoV_\gamma$ of unreinforced and reinforced slopes with $b/B$ = 0, 1, and 2

The influence of cross-correlation coefficient on the failure probability of the slopes is discussed with the help of Figure 10. It is observed that there is no significant change in the value of $p_F$ with an increase in the value of $\rho$. For example, for an unreinforced slope with $b/B$ = 0, the value of $p_F$ changes from 0.51 to 0.52, with an increase in the value of $\rho$ from zero to 0.75.

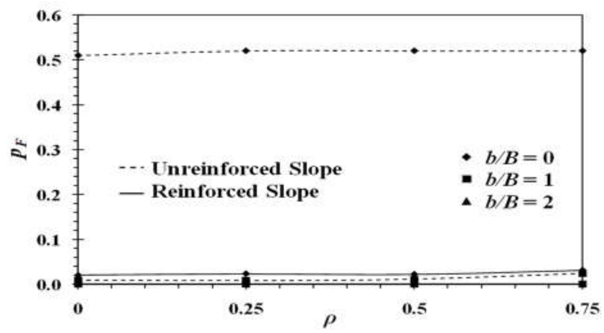



Figure 10 Variation of $p_F$ with $\rho$ for unreinforced and reinforced slopes with $b/B$ = 0, 1, and 2

## 6. REMARKS

Present study primarily deals with the consideration of uncertainties related to soil friction angle and soil unit weight in the stability analyses of a simple dry slope, made of sand and also subjected to strip loading. The influence of fill material other than sand, slope inclination, position of ground water table and rainfall on the stability assessment of slope are kept beyond the scope of the present study. The effectiveness of using multiple numbers of reinforcement and various types of reinforcement are also not examined. However, by using the same methodology as provided in APPENDIX-A the effect of above mentioned parameters on the probabilistic assessment of slope can be investigated in future.

## 7. CONCLUSIONS

The stability of unreinforced and reinforced cohesionless soil slopes subjected to strip loading is investigated by performing deterministic as well as probabilistic analyses. The influences of footing setback distance and depth of geotextile embedment on the *FoS* values of slopes are also examined. The effect of uncertainties related to $\phi$ and $\gamma$ are considered by varying *CoV*. Cross correlation is assumed to exist between two random variables, $\phi$ and $\gamma$. Major outcomes from the present study are detailed below:

(i) With the increase in the value of $b/B$ for unreinforced slope, the required pressure to bring the slope on the verge of failure increases. For example, the magnitude of applied pressure corresponding to *FoS* =1, is found to be 72 kPa and 275 kPa, respectively, for $b/B$ = 0 and 2. However, with the inclusion of a single layer of reinforcement, *FoS* value is found to be increased significantly for the same applied pressure. Under the application of 72 kPa, *FoS* of the reinforced slope with $b/B$ = 0, is increased by an amount of 1.26 times with respect to the unreinforced slope with $b/B$ = 0. The maximum increment in the value of *FoS* is observed when reinforcement is placed at a particular depth. For example, $d_{cr}$ is found to be 0.20$B$ for a reinforced slope with $b/B$ = 0.

(ii) The probability of slope failure increases with an increase in the *CoV* value of $\phi$ and $\gamma$ for a particular $b/B$ value. However, the effect is more pronounced when slope is unreinforced and $b/B$ = 0. With the inclusion of geotextile layer, $p_F$ value reduces drastically for a particular value of *CoV*. It is also observed that with respect to $CoV_\gamma$, $CoV_\phi$ is found to have more impact on the $p_F$ value.

(iii) For an unreinforced slope with $b/B$ = 0, $\beta$ value is found to be less than 3.0, thus making the overall system vulnerable. With the inclusion of a single layer of geotextile, $\beta$ increases significantly and found to be higher than $\beta$ = 3.0, which is required for the average performance of the system as suggested by USACE (1997). It is observed that, $\beta$ increases with the increase in the value of $b/B$ for slope. For reinforced slope with $b/B$ = 1.0, and 2.0, obtained values of $\beta$ are found to be greater than 3.0.

(iv) The influence of cross-correlation coefficients on the failure probability of the slopes is also investigated. It is found that the impact of cross-correlation between $\phi$ and $\gamma$ on the failure probability of slope is very minimal.

(v) It should be mentioned here that though the present study is carried out for a simple slope stability problem, but using the same methodology, probabilistic analyses of complex slopes can also be performed.

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**APPENDIX- A**