Hypoplastic Model with Intergranular Strain: Dependence on Grain Properties and Initial State

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ABSTRACT: The Hypoplastic model is introduced and the procedure to determine the model parameters along with intergranular strain parameters is discussed in detail. Static trial tests and load/displacement controlled oedometer tests are performed to determine the hypoplastic model parameters. The dependence of the hypoplastic model parameters on initial density of granular materials is studied. Intergranular strain parameters are conventionally determined by resonant column tests. An innovative method based on static triaxial setup coupled with stress path test method is employed to determine the intergranular strain parameters. Parameters are determined for 5 different naturally existing sands. The dependence of the model parameters, on density, stress state and grain assembly properties of the sand is studied. The elastic strain range within which the incremental stiffness remains constant after strain reversal is studied in conjunction with the grain properties and the validity of the assumption that the governing parameter is a material independent constant is commented upon. The model parameters are verified against results of standard element tests.

KEYWORDS: Hypoplastic, Intergranular, Strain reversal, Incremental stiffness, Sand.

1. INTRODUCTION

The Hypoplastic model describes the mechanical behaviour of granular materials. The model assumes that granular materials are arranged in the form of simple granular skeleton and their state is defined by certain basic properties such as effective stress and void ratio. The Hypoplastic model is well suited to model the nonlinear and inelastic behaviour of dry granular soils. Typical soil characteristics like dilatancy, contractancy, different stiffnesses for loading and unloading as well as the dependency of stiffness on pressure and void ratio can be modelled. The first version of the hypoplastic model was formulated by Kolymbas (1991). The most widely used version was developed by von Wolffersdorff (1996). Since the behaviour of granular materials is non-linear even at small strain levels the model developed some problems under small scale cyclic loading under which an increased accumulation of strains was observed. In order to rectify the effect of ratcheting, Niemunis and Herle (1997) enhanced the model with the intergranular strain concept. The determination of Hypoplastic model parameters requires a series of triaxial and oedometer tests. The intergranular strain parameters are often assumed according to the work of Niemunis and Herle (1997) and have observed to provide satisfactory results in modelling various granular materials. However recently researchers have observed by means of dynamic and cyclic tests on Karlsruhe and Zbraslav sand that the intergranular strain parameters are density and stress state dependent (Plaxis Material Model Manual, 2007). The dependence of the intergranular strain parameters, on density and stress state of the sand is studied. The strain range within which the incremental stiffness remains constant after strain reversal is studied in conjunction with the grain properties and the validity of the assumption that the governing parameter is a material independent constant, is commented upon.

The paper aims to study the effect of grain assembly properties on the material parameters and to correlate the variation of the parameters to basic physical properties of the granular material. Five different kinds of sands were chosen for the determination of the material parameters. The basic physical properties of the sands were identified. A brief description on the method of determination of the hypoplastic material parameters is included. The dependence of the hypoplastic model parameters on initial density is studied. Intergranular strain parameters are comparatively difficult to evaluate and require dynamic tests or static tests with strain reversal (Niemunis and Herle, 1997) to be performed. The detailed explanation on the method of determination of the intergranular strain parameters is not available. The intergranular strain parameters are often assumed similar to the standard values as quoted by Niemunis and Herle (1997) due to lack of knowledge and difficulty level associated with the determination of these parameters. In this work, an innovative method involving the stress path controlled triaxial test is suggested and described, for the determination of the intergranular strain parameters.

2. EXTENDED HYPOPLASTIC MODEL

2.1 Hypoplastic Model

The hypoplastic model describes the stress state change of a simple granular skeleton. The granular framework is defined by the following properties:

- The state of the granular material is defined by the stress tensor and the void ratio.
- The grains are robust and are assumed to undergo no internal deformation or fracturing.
- The assembly of the grain skeleton is bound by an upper and lower void ratio depending on the existing mean effective pressure and hence macropores are accounted for.
- Deformations under similar boundary conditions are identical.
- It is assumed that granular materials are rate independent.

In the Hypoplastic model (von Wolffersdorff, 1996) the objective stress rate tensor $\dot{\sigma}$ is defined as a tensor valued function h of the effective Cauchy stress σ, the deformation rate D and the void ratio e as follows:

$$\dot{\sigma} = h(\dot{\sigma}, D, e) \tag{1}$$

The function (h) is first order homogenous with respect to strain rate and directionally homogenous with respect to stress. Hence the formulation is able to describe both elastic and plastic deformations. The Hypoplastic model without the intergranular strain parameter predominantly consists of 8 parameters. All of the parameters are closely related to the geometric and material constants of the grains and hence can be determined by standard index tests. The detailed description regarding the development and mathematical properties of the hypoplastic model can be found in Kolymbas (1988,1991), Wu, et. al. (1996), Gudehus, (1996), Bauer (1996).

2.2 Determination of Hypoplastic Material Parameters

The first set of parameters consist of the limiting void ratios e_{i0}, e_{c0} and e_{d0} corresponding to the upper bound, critical state and lower bound void ratio at zero pressure. These parameters are estimated from the minimum and maximum void ratio values of the granular material which are determined by standard index tests.

The critical friction angle φ_c determines the resistance of the granular material in monotonic shearing in critical state. It is evaluated from drained triaxial tests or by the determination of angle of repose. In this work the critical friction angle was determined by both the approaches..

The parameters granulate hardness h_s and exponent n describe the isotropic compression of loose sands under increasing effective mean pressure as follows:

$$e = e_0 . exp\left[-\left(\frac{3.p'}{h_s}\right)^n\right] \quad (2)$$

where, e = Void ratio

e_0 = Max void ratio

p' = Effective mean pressure

h_s = Granulate hardness

n = Exponent

n reflects the curvature of the compression curve whereas h_s governs the slope. They are determined from isotropic compression tests on loose samples of granular materials. Since placement of granular material under very loose condition in an isotropic compression test is difficult, h_s and n are obtained by one dimensional oedometer test.

The exponent α governs the peak friction angle and also indirectly influences the dilatancy of the material and is calibrated with the results of drained triaxial tests on dense samples.

The parameter exponent β which governs the stiffness of the granular materials is often calibrated from oedometer or triaxial test data on dense samples.

2.3 Intergranular Strain

The Hypoplastic model estimates deformations due to the rearrangement of granular skeleton under regular loading effectively but leads to accumulation of deformation on being subjected to excessive small amplitude stress cycles called as ratcheting. Hence the Hypoplastic model was extended and the intergranular strain parameters were introduced. The extended model took into account the behavior of the granular material under small strains due to change of direction of stress or strain path. The intergranular strain extension leads to the addition of the intergranular strain tensor (δ) which stores the most recent deformation history and also leads to an increase in the incremental stiffness ($E = dT/d\varepsilon$) of the material on change in direction of deformation (D).

The extended model states that if the state of stress and density of the material at point * (Figure 1) is similar even after being subjected to different deformation histories as indicated in Figure 1, a reversal in direction of deformation would lead to a variable increase in incremental stiffness depending on the nature of the reversal. A 180° reversal as indicated in Figure 1 (top left), would lead to an increase in stiffness according to $E_R = m_R E_0$ and a 90° reversal (middle left) would lead to an increase in stiffness as per $E_T = m_T E_0$ where m_R and m_T are intergranular strain parameters and E_0 is the asymptotic stiffness which the material inherits after long monotonic shearing in similar state. After being subjected to sufficient deformation (ε_{SOM}), the effect of change in direction of deformation is swept out of memory which is marked by the constancy of the incremental stiffness. The elastic range R, describes the strain range over which the stiffness of the material is strain independent. The size of the elastic range R is assumed to be constant irrespective of the state of stress and void ratio as indicated in Figure 1 (right) but whereas the magnitude of the elastic stiffness varies as per the stress state and void ratio. The intergranular strain model includes two other exponents β_χ and χ which govern the decay of $E_{R/T}$ after the change in deformation direction.

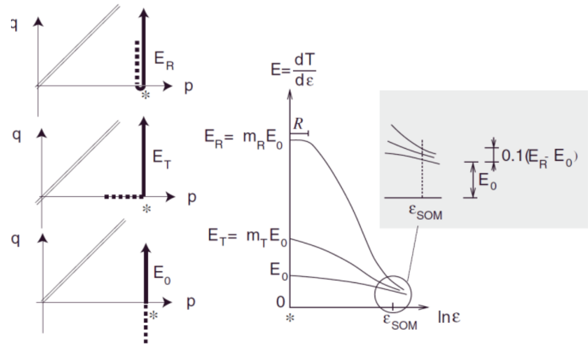


Figure 1 Incremental stiffness according to change in direction of deformation (Niemunis and Herle, 1997)

2.4 Determination of Intergranular Strain Parameters

The intergranular strain parameters can be determined from dynamic tests or static tests with strain reversal as stated earlier (Niemunis and Herle, 1997). The parameters as determined by Niemunis and Herle (1997) for Hochstetten sand has been seen to work quite reasonably well for finite element simulations and hence not much work has been done on the determination of these parameters. But some recent literature (PLAXIS 2D Reference Manual, 2014), has shown that the parameters are density and stress state dependent which calls for the need of identifying a simple and effective method of determination of the parameters. In this work stress path controlled static triaxial tests have been used to determine the intergranular strain parameters m_R, m_T and R which are explained in detail in the following sections. The experimental data was calibrated as per Niemunis and Herle (1997) to attain the exponential parameters.

3. GRANULAR MATERIAL

The hypoplastic parameters along with the intergranular strain parameters were determined for five different kinds of naturally existing sands. The sands were carefully chosen in order to incorporate sands of varying characteristics.

3.1 Sand Properties

The grain size distribution of the five sands are as depicted in Figure 2.The basic characterisation tests were performed on the sands and the results of the same have been tabulated in Table 1. It can be observed that all the four sands are of varying granulate properties and hence the resulting evaluated parameters would provide an insight on the inter relationship between grain assembly properties and material model parameters especially with respect to the intergranular strain parameters.

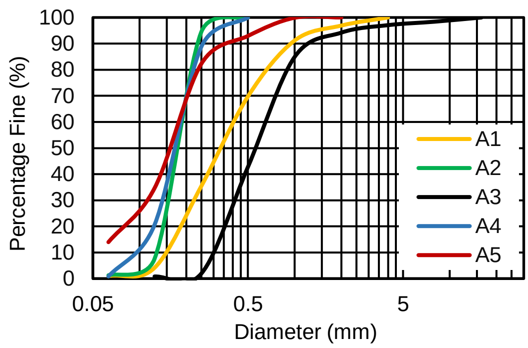


Figure 2 Grain size distribution of sands

Table 1 Physical properties of four sands

Sand	e_{max}	e_{min}	γ_{max} (g/cc)	γ_{min} (g/cc)	C_u
A1	0.799	0.385	1.914	1.473	2.32
A2	1.000	0.514	1.75	1.325	1.5
A3	0.715	0.456	1.809	1.546	2.1
A4	0.895	0.486	1.784	1.398	1.8
A5	1.116	0.672	1.585	1.253	2.4

3.2 Sample Preparation

Triaxial samples of sand of 50 mm diameter and a height of 125 mm were prepared using vacuum suction method. Standard oedometer samples of 70 mm diameter and 20 mm height were prepared. Dense and loose samples according to the maximum and minimum density of the sands as per Table 1 were prepared with sufficient quantity of water in case of saturated samples, in order to ensure complete saturation of the sample during testing.

4. HYPOPLASTIC PARAMETER EXPERIMENTS

The critical friction angle (φ_c) is determined based on the test method suggested by Cornforth (1973). A funnel filled with soil is slowly lifted releasing the soil on a flat surface. The angle made by the cone of the flat surface gives the value of the critical friction angle. In order to compare the results, critical friction angle was also determined using the results of shear test data.

In order to determine h_s and n, one dimensional compression oedometer test was performed on dry samples. The test was different from a standard oedometer test, the sample was only loaded at a constant deformation rate of 0.25 mm/min and the corresponding axial load developed was recorded. h_s and n were evaluated using the relations described below. The bulk modulus

$$K = -\dot{p}'/(\frac{\dot{e}}{1+e}) \quad (3)$$

where, $\dot{p}'$= rate of change of mean pressure
$\dot{e}$ = rate of change of void ratio and other terms as described earlier
Can be expressed with the help of equation 2 as

$$\mathrm{K} = \frac{1}{3}\frac{h_s}{n}\left(1+\frac{1}{e}\right)\left(\frac{3p'}{h_s}\right)^{1-n} \quad (4)$$

where, the terms are as described earlier.

When equation 3 is compared to conventional bulk modulus equation

$$K = \frac{p'(1+e)}{C_c} \quad (5)$$

where, C_c = Compression index
The following relation is generated

$$h_s = 3p'\left(\frac{ne}{C_c}\right)^{1/n} \quad (6)$$

Considering two load points on the oedometer compression curve p^1 and p^2 and corresponding compression indexes the exponent n can be evaluated from equation 6.

$$n = \frac{ln\left(\frac{e_1\ C_{c2}}{e_2\ C_{c1}}\right)}{\ln\left(\frac{p_2}{p_1}\right)} \quad (7)$$

The granulate hardness h_s is then evaluated according to the value of n. Further details can be found in Herle and Gudehus (1999). Typically the h_s and n are determined by compression tests on initially loose samples but in order to study the variation of the parameters with density, compression test with initial dense samples were also carried out.

The limiting void ratios e_{d0} which is the minimum void ratio can be reached by shearing the granular material under small amplitude cyclic loading under constant pressure. But for simplification the standard minimum void ratio index test according to DIN 18126 (1996) was performed and e_{d0} was assumed to be nearly equal to the obtained minimum void ratio. The e_{c0} has been found to be analogous to maximum void ratio (Herle, 1997). Hence maximum void ratio was found according to the standard index test as per DIN 18126 (1996). The e_{i0} is approximated to be around 1.2 times e_{max} (Herle and Gudehus, 1999).

Exponent α describes the effect of density on the peak friction angle of the granular material. The determination of α requires the determination of peak friction angle, dilatancy angle at a particular density by means of drained triaxial test. The values can then be used to determine the value of α using the following equation.

$$\alpha = \frac{\ln\left[6\frac{(2+K_p)^2+a^2K_p(K_p-a-\tan\vartheta_p)}{a(2+K_p)(5K_p-2)\sqrt{4+2(1+\tan\vartheta_p)^2}}\right]}{\ln\left(\frac{e-e_d}{e_c-e_d}\right)} \quad (8)$$

where, $K_p = \frac{1+\sin\varphi_p}{1-\sin\varphi_p}$ and $a = \frac{\sqrt{3}(3-\sin\varphi_c)}{2\sqrt{2}\sin\varphi_c}$
φ_p = Peak friction angle
ϑ_p = Dilatancy angle
φ_c = Critical friction angle
e = Void ratio
e_d = Min void ratio
e_c = Max void ratio

Exponent α can also be calibrated using results of drained triaxial test on dense samples (Bauer, 1996). Further details can be found in about the determination of α can be found in Herle (1998) and Karcher (2003).

Exponent β controls the increase of the pressure dependent stiffness with increasing density. The increase of the stiffness is predominant in dense specimens and hence the parameter is important for evaluation of stiffness in dense specimens. The factor is to be determined after the determination of all the other parameters of the hypoplastic model. The determination of β requires the stiffness modulus along with the void ratio of the material in dense and loose state, to be determined from oedometer test data. The exponent β can then be evaluated using the following equations.

$$\beta = \frac{\ln\left(\beta_0\frac{E_2}{E_1}\right)}{\ln\left(\frac{e_1}{e_2}\right)} \quad (9)$$

where, subscript 1 stands for loose and 2 for dense,
E = Stiffness modulus
$\beta_0 = \frac{3+a^2-a\sqrt{3}f_{d1}}{3+a^2-a\sqrt{3}f_{d2}}$
$f_d = \frac{e-e_d}{e_c-e_d}$ and the other terms are as described earlier.

Exponent β can also be calibrated using results of oedometer compression test on dense samples (Bauer, 1996). The exponents α and β are dependent on the value of h_s and n and need to be calibrated in tandem (Bauer, 1996).

5. STRESS PATH EXPERIMENTS FOR INTERGRANULAR STRAIN PARAMETERS

5.1 Experimental Setup

Simple static triaxial tests were performed in order to determine the intergranular strain parameters m_R, m_T and R. Stress path control method of testing was employed in order to subject the material to strain reversal after being subjected to different deformation histories. The tests were performed in the automated stress path module of the GDS (GDS Instruments, Hampshire, United Kingdom) Triaxial Testing System (GDSTTS). Since the speculated strain ranges are low and need to be measured accurately, on sample GDS Linear Variable Differential Transformer (LVDT) were installed. These LVDTs can measure small strains and are well suited for small strain stiffness measurements.

5.2 Stress Path Controlled Triaxial Test

GDS Triaxial Testing System equipped with automated stress path module is based on the classic Bishop and Wesley type stress path triaxial cell which directly controls the stress on the sample. The sand sample was cast at the required density and was placed in the triaxial chamber with the on sample LVDTs as described in the preceding section. The sample was installed with extension load cap, in order to ensure that the sample could even be subjected to extension/tension during loading. The sample was first made to undergo saturation ramp at a radial pressure of 410 kPa and back pressure of 400 kPa. Samples were ensured to have a B-check value of at least 0.98 in order to ensure satisfactory saturation. The samples were then subjected to isotropic consolidation at pressure 100 kPa or 200 kPa. The two different consolidation pressures were chosen in order to study the variation of the intergranular strain parameters with varying initial stress state conditions. The stress path controlled testing enabled the sample to be driven to desired p (mean stress) and q (deviator stress) stresses as depicted in Figure 3. The module ensures independent linear control of p and q on the sample; hence the samples were subjected to stress paths as depicted in Figure 3. Sand samples were subjected to three kinds of stress paths 1) 180° strain reversal 2) 90° strain reversal 3) no reversal. The points marked with * in Figure 1 have the same density and void ratio. The same was ensured in the experimental tests by casting similar samples (dense or loose) and subjecting them to the same preliminary consolidation stages and ensuring that the samples reached the same stress state point at start of the test. The samples were subjected to initial consolidation pressure of 100 kPa or 200 kPa pressure as described earlier and corresponding q and p for the different tests are marked in brackets in Figure 3. In order to study the effect of density on the intergranular strain parameters, the tests described in the preceding sections have been performed on both loose and dense samples of granular material.

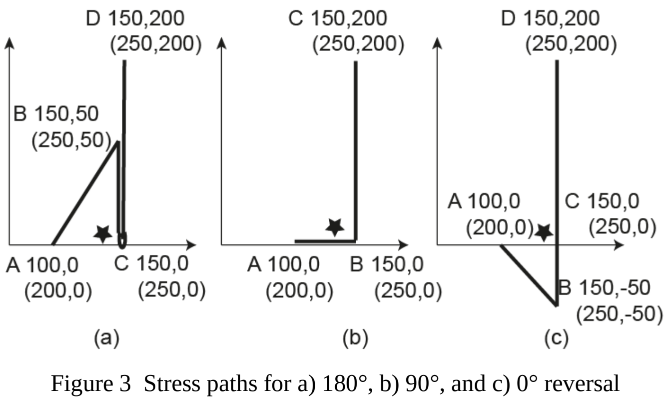


Figure 3 Stress paths for a) 180°, b) 90°, and c) 0° reversal

5.2.1 180° reversal

The sample is made to undergo the preliminary stages of saturation and consolidation as described in the preceding section. At the start of the stress path module the sample was subjected to 0 q stress and 100 kPa p stress (excluding back pressure) marked as point A in Figure 3(a). The q and p stresses were then linearly increased to reach 50 and 150 kPa (Point B Figure 3(a)), followed by reduction of q to 0 kPa at constant p of 150 kPa to reach point C (Figure 3(a)). This stage marked extension of the sample as q was reduced. As soon as q reached 0 kPa, the stress paths were programmed to make a 180° reversal. The q was increased to reach 200 kPa at constant p of 150 kPa to reach point D (Figure 3(a)). This stage marked the compression of the sample as q was increased hence the material underwent a 180° deformation reversal from extension to compression leading to the evaluation of the E_R which is ratio of incremental stress to strain after 180° strain reversal.

5.2.2 90° reversal

After consolidation, the material was subjected to further isotropic consolidation by increasing p to reach a value of 150 kPa as depicted in Figure 3(b) (Point B). The sample was then subjected to monotonic shearing by an increase in q to reach a value of 200 kPa at constant p of 150 kPa to reach point C (Figure 3(b)). This change from isotropic consolidation to compression marked a 90° reversal leading to the evaluation of E_T.

5.2.3 0° reversal

The sample was made to undergo extension after consolidation by reducing q to reach -50 kPa and by increasing p to 150 kPa to reach point B (Figure 3(c)). The sample was then subjected to monotonic shearing by increasing q to reach 200 kPa at constant p of 150 kPa. The incremental stiffness E_0 was evaluated after the stress path crossed point C (Figure 3(c)).

6. RESULTS

6.1 Hypoplastic model parameters

The hypoplastic model parameters of the five sands are as tabulated in Table 2. It can be observed that the material parameters of the sands vary with their physical properties. The friction angle marginally increases with increasing C_u of soils which was also observed by Herle and Gudehus (1999). It can be observed that h_s and n are not only dependent on the physical properties but also on the initial density of the granular material. Figure 4 describes the determination of h_s and n from oedometer compression test results from two sets of data points (A-B and D-E) as described in previous sections for both initially dense and loose samples. It can be observed that the nature of the curves for dense and loose sample vary and hence the parameters h_s and n vary as per the initial density of the sand. It can also be observed by comparing values in Table 2 and 1 that n is closely related to the granulometric property of the sands and increases with reducing C_u (Schultze and Moussa, 1961). h_s on the other hand is found to increase with increasing C_u. The minimum void ratio e_{d0} decreases with increasing C_u due to filling of voids with smaller particles (Youd, 1972). Sand A5 is an anomaly and negates the above state general trends. The probable reason can be due to the presence of excessive finer particles (more than 15%), which do not allow the sand to reach a lower void ratio owing to the difficulty in compaction. α governs the dependence of peak riction angle on density and hence is found to be related to the granulometric property of sands (Mogami and Yoshikoshi, 1968; Koerner, 1970; Holubec and Appolonia, 1973). α is found to marginally increase with increasing φ_c which is indirectly related to C_u as already discussed. The parameter values obtained lie in the range as suggested by Herle and Gudehus (1999) barring sand A5 consisting of higher fraction of fines.

6.2 Intergranular strain tests

The evaluation of the intergranular strain parameters revealed that the parameters vary with different granular materials. The parameters are stress state sensitive and vary according to the density of the granular material. The following section describes the variation of the intergranular strain parameters with stress state and initial density of the granular material.

Table 2 Hypoplastic model parameters

	φ_c (°)	h_s (MPa)	n	α	β	e_{d0}	e_{c0}	e_{i0}
A1	33.7	552_L	0.36_L	0.04	2.76	0.38	0.8	0.92
		853_D	0.48_D					
A2	32.3	980_L	0.35_L	0.03	2.51	0.51	1.0	1.15
		2277_D	0.43_D					
A3	33	410_L	0.47_L	0.04	1.52	0.47	0.72	0.82
		436_D	0.60_D					
A4	32	351_L	0.57_L	0.03	2.32	0.49	0.89	1.03
		542_D	0.74_D					
A5	32.9	37_L	0.4_L	0.007	6.03	0.67	1.12	1.28
		109_D	0.68_D					

L= Loose , D= Dense

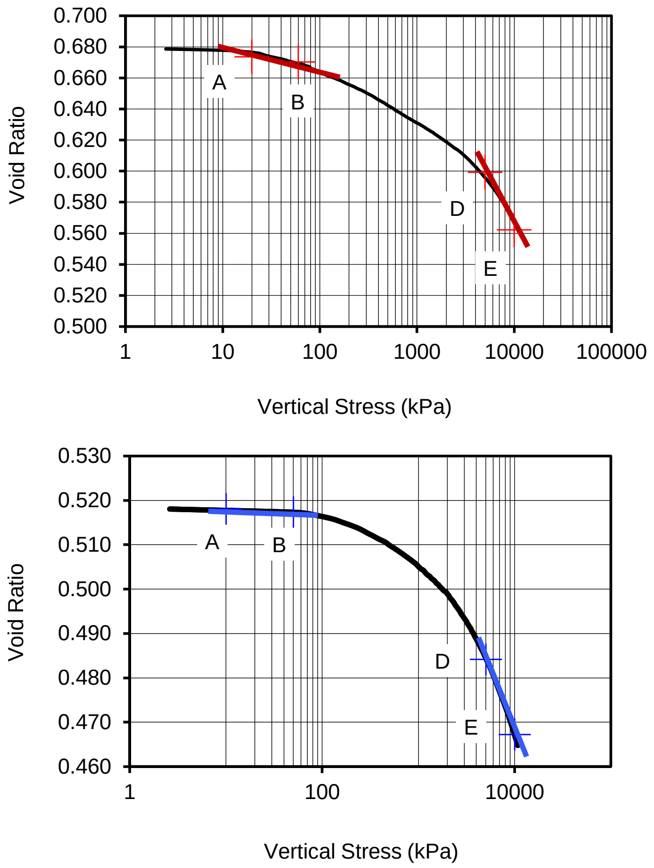


Figure 4 Determination of h_s and n from oedometer test results for loose sample (top) and dense sample (bottom)

6.2.1 Variation of stiffness with reversal in deformation path

The actual applied mean and deviatoric stress on the granular material in the stress path controlled triaxial test for an initial consolidation pressure of 100 kPa is as described in Figure 5. The stiffness was evaluated after the point of reversal (indicated with * in Figure 3) as ratio of axial stress to axial strain. The variation of the stiffness after the deformation path reversal over strain is as described in Figure 6. It was observed that the incremental stiffness was highest after a 180° reversal followed by stiffness for 90° reversal. Eventually the incremental stiffness values decreased with increasing deformation and asymptotically reached a constant value which was equivalent to the stiffness under no deformation path reversal as observed in Figure 6. The strain (ε_{SOM}) beyond which the effect of deformation path reversal is negligible or the strain beyond which the stiffness reaches a constant value are as tabulated in Table 3. Variation of stiffness for 180° and 90° reversal is compared in Figure 7 and it was observed that for all the granular material m_R was found to be 1.5 - 2.5 times m_T.

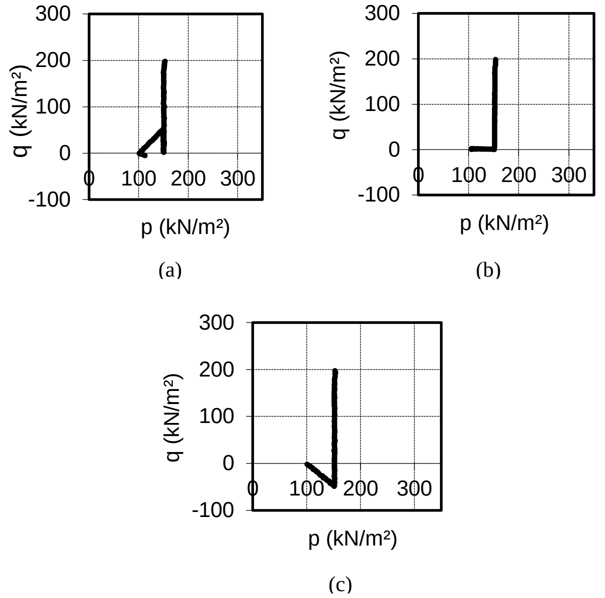


Figure 5 Applied q-p stress paths for a) 180°, b) 90°, and c) 0° reversal in triaxial test

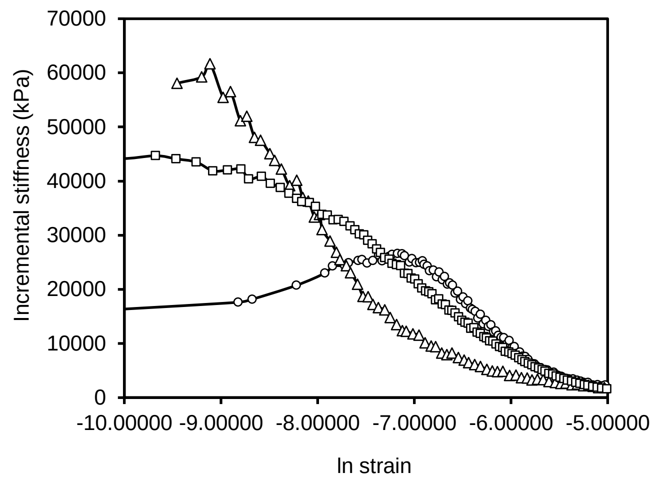


Figure 6 Variation of stiffness with reversal for A5 sand

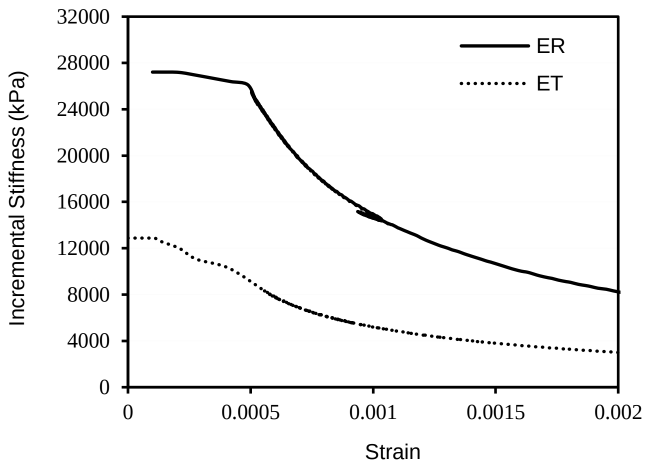


Figure 7 Stiffness variation with strain for 180° and 90° reversal for A3 sand (loose state)

6.2.2 Variation of stiffness with nature of sand

The evaluation of m_R requires the analysis of the variation in stiffness of the granular material after 180° reversal in deformation path. Figure 8 describes the variation of the stiffness after 180° reversal for the five sands. The stiffness is higher for pure sands with higher coefficient of uniformity. Sands A3 and A5 are found to be outliers and this can be probably be justified by the presence of more amount of fines (15%) in A5 soil and more percentage fraction of gravel in A3 (Figure 2). Hence it can be suggested that stiffness parameters not only depend on the index properties of the sand but also on the nature of particles present in the granular material (Biryaltseva et. al., 2016). The presence of gravel and fines leads to substantial changes in the stiffness parameters of the sand.

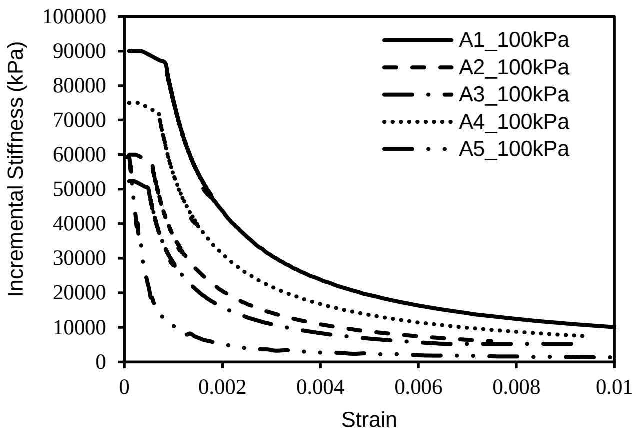


Figure 8 Stiffness variation with strain for 180° reversal (dense state)

6.2.3 Variation of stiffness with stress state

The dependence of the incremental stiffness on the stress state can be observed in Figure 9. It can be observed that the stiffness values increase as the material is subjected to higher consolidation pressure and eventually a higher p and q stresses. This observation shows that the intergranular strain parameters would be stress state sensitive and are not unique for a particular granular material.

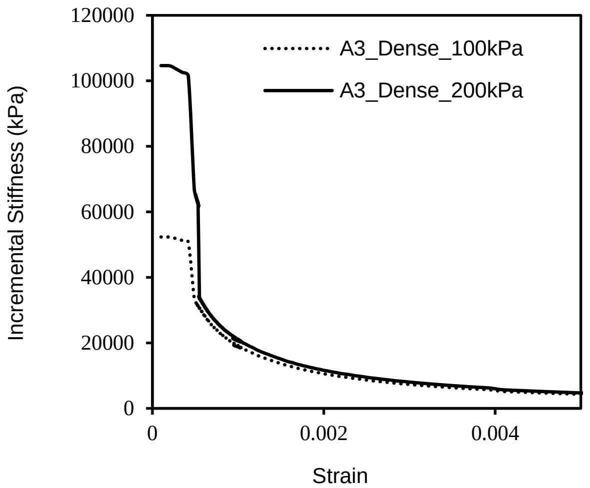


Figure 9 Stiffness variation stress state for A3 sand (dense state)

6.2.4 Variation of stiffness with density

The incremental stiffness varied as per the density of the sand as can be observed in Figure 10. Sand in denser state showed higher stiffness values than in loose state suggesting that the corresponding intergranular strain parameters would be density dependent.

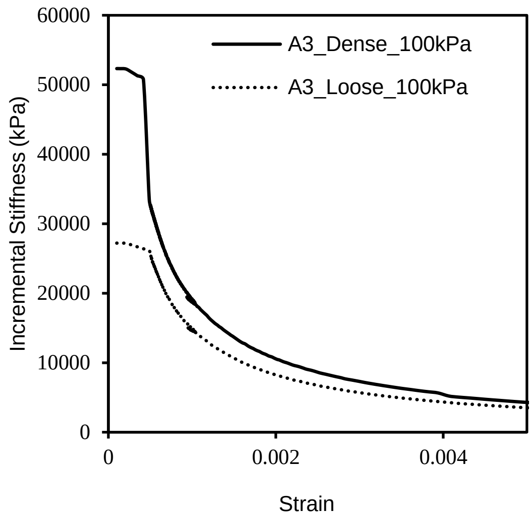


Figure 10 Stiffness variation density for A3 sand

6.2.5 Elastic strain range

The elastic strain range R is defined as the strain until which the stiffness of the material remains constant after deformation reversal. The values of R for all the five sands are as tabulated in Table 3. It is interesting to observe that R is neither sensitive to the nature of the granular material nor to the stress state and density. It can be observed that as suggested by Niemunis and Herle (1997), the value of R can be closely be approximated to 0.0001.

6.2.6 β_χ and χ

β_χ and χ which define the decay of $m_{R/T}$ were calibrated using the experimental results and are as tabulated in Table 3 for varying initial density.

6.3 Intergranular strain parameters

Table 3 tabulates all the intergranular strain parameters for two different initial density conditions for an initial consolidation pressure of 100 kPa. The intergranular stiffness coefficients are found to be highly sensitive to density, whereas as the other parameters can be nearly be assumed to be density independent.

Table 3 Intergranular strain parameters

Sand	m_R	m_T	R	β_χ	χ	ε_{SOM}
A1	6.6 $_L$	4.25 $_L$	0.0001	0.7	1	0.0003
	3.26 $_D$	2.93 $_D$		0.6	1	0.00024
A2	8.68 $_L$	4.44 $_L$	0.00011	0.5	2	0.0005
	5.48 $_D$	3.65 $_D$		0.6	2	0.0004
A3	7.53 $_L$	5.35 $_L$	0.0001	1	1	0.00011
	2.91 $_D$	2.25 $_D$		1	1	0.00013
A4	8.40 $_L$	4.95 $_L$	0.00009	1	1	0.00015
	5.25 $_D$	2.25 $_D$		1	1	0.0002
A5	-	-	0.00012	0.3	6.0	0.00028
	3.69 $_D$	2.78 $_D$				

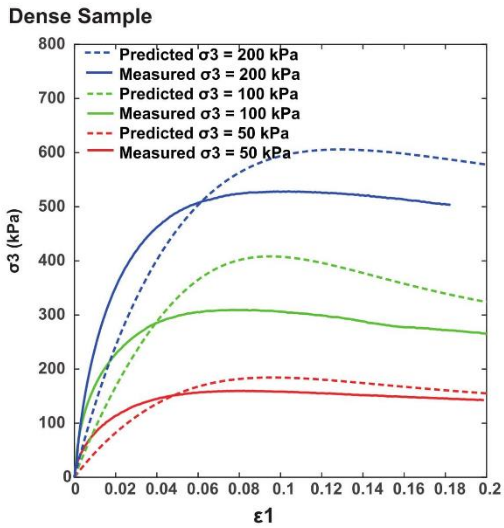
L= Loose , D= Dense

7. CALIBRATION AND VERIFICATION OF PARAMETERS

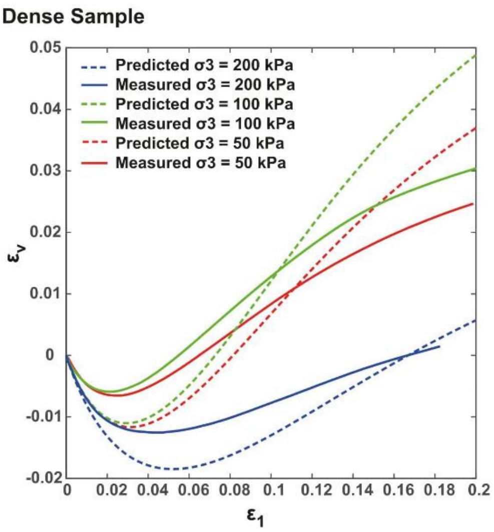
The hypoplastic parameters determined and discussed in the previous sections are verified on the basis of the results on some basic elementary tests. It has been discussed in the earlier sections that the parameters are sensitive with respect to initial stress and density state. Hence the calibration of the parameters is of utmost importance so as to deliver optimised material behaviour prediction under a wide range of states. An optimising routine was written using which, certain hypoplastic model parameters could be optimised in order to best predict the material behaviour under both loose and dense conditions. The parameters that are associated with the physical behaviour of the sands and are material constants such as friction angle, minimum and maximum void ratios where held constant where the other parameters which were found to be density and stress state dependent could be varied in order to achieve a best prediction of the experimental results. A set of three triaxial tests (dense specimen) and one oedometer test (loose specimen) were performed on sand A5 and the results of the same were used to verify the hypoplastic model parameters. Sand A5 for calibration, firstly, its determined material properties were out of the range as suggested by Herle and Gudehus (1999).

The parameters were optimised till best predictions were obtained with minimum error. Figure 11 depicts the measured and predicted results for the triaxial and oedometer tests. The hypoplastic model is best suited for prediction of a single element deformation under homogenous conditions. Hence the material behaviour prediction of the model in the initial part of the triaxial test is important as after the peak the material undergoes non uniform deformation and hence does not abide by rules of single element deformation. The oedometer test closely resembles an element test and hence should be given more priority with respect to the calibration of the model parameters. The calibrated model parameters of the A5 sand are as tabulated in Table 4. It can be observed that the parameters representing the physical properties of the sand such as friction angle, void ratios and intergranular strain parameters have been kept constant as obtained from experimental tests.

(a)



(b)



(c)

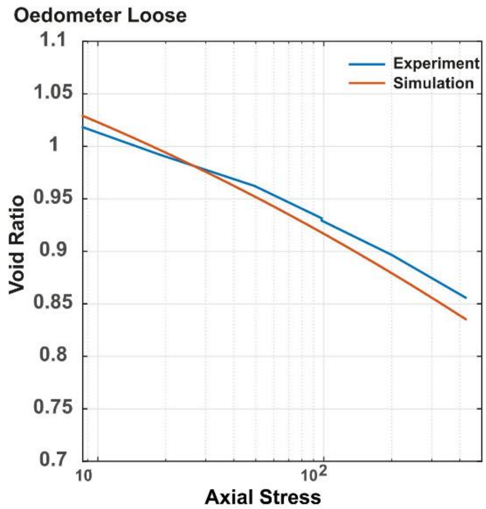


Figure 11 (a) Measured value for triaxial test, (b) Predicted value for triaxial test, (c) Oedometer test results

Table 4 Calibrated hypoplastic model parameters

Parameter	Value
φ_c (°)	32.97
h_s (MPa)	195
n	0.168
α	0.25
β	1.03
e_{d0}	0.678
e_{c0}	1.116
e_{i0}	1.283
m_R	2.78
m_T	3.69
R	0.00012
$\beta\chi$	0.3
χ	6

8. CONCLUSION

The hypoplastic parameters are closely associated with the granulometric properties of the sands. The hypoplastic parameters along with the intergranular strain parameters are not only material dependent but also vary as per the stress state and initial density. The h_s and n are dependent on the initial density. An innovative method using the stress path controlled simple triaxial test set up has been developed in order to determine the intergranular strain parameters. The intergranular strain parameters can be easily be determined by simple stress path controlled triaxial tests and hence not be considered according to existing literature. The intergranular stiffness is dependent on both the stress state and initial density. The m_R and m_T in dense initial state was found to be to the order of 1.8 times the m_R and m_T at loose state. The m_R and m_T were found to double with doubling of confining pressure from 100 to 200 kPa, indicating that the parameters are highly sensitive to the initial stress state. The elastic strain range in which the incremental stiffness after deformation reversal is constant was found to be nearly constant and hence the assumption that R is material independent constant is justified. The hypoplastic material parameters are stress state and density dependent and hence need to be calibrated according to the expected test or on field conditions in order to deliver best material behaviour predictions.

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