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Application of Deep Vertical Vibratory Compaction Using Resonance Amplification

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ABSTRACT: The mechanism which governs the compaction of granular soils is reviewed. Two different effects are identified: densification (volume change) and an increase in horizontal stress. The ground vibration velocity, which is directly related to shear strain, is a crucial parameter of soil densification. The implementation of deep vertical vibratory compaction (DVVC) is described. The compaction effect in granular soils depends primarily on shear strain amplitude and number of vibration cycles. Vibration amplification occurs at the resonance frequency of the vibrator-probe-soil system. Resonance can be achieved by changing the operating frequency of the vibrator. Resonance compaction (DVVCr), which utilizes the vibration amplification effect, is discussed. The shape and mass of the compaction probe are important parameters for enhancing the compaction effect. An important aspect of DVVCr is the monitoring and process control system (MPCS), which assists the machine operator in executing the compaction process. Resonance compaction has the advantage that the treatment process can be carried out at a significantly lower frequency than conventional vibratory driving, resulting in lower energy consumption.

KEYWORDS: Vibratory compaction, Granular soil, Resonance.

1. INTRODUCTION

Vibratory treatment of granular soils is widely accepted as an efficient method to solve geotechnical problems, such as total and differential settlements, improvement of soils subjected to cyclic loading (liquefaction), or homogenization of heterogeneous soil deposits. Correctly planned and competently executed, vibratory compaction can be technically and economically superior to alternative deep foundation solutions such as piling, grouting, static preloading, or other treatment methods. Vibratory compaction methods have been described in the literature (Broms, 1975; Mitchell, 1981; Massarsch, 1999; Schlosser, 1999; Kirsch and Bell, 2012). The European standard EN 14731:2005 defines ground treatment by deep vibratory compaction and covers depth vibrators and top-mounted vibrators. There is a need to distinguish between these two deep vibratory compaction methods. Therefore, Massarsch and Fellenius (2019) suggested the term "*Deep vertical vibratory compaction*" (DVVC) for methods using a vibrator mounted on top of a compaction probe. When horizontally oscillating depth vibrators are used, the term "*Deep horizontal vibratory compaction*" (DHVC) is more appropriate. The application of DHVC, also called "*vibroflotation*", is extensively described in the geotechnical literature. Compaction projects are usually designed and executed by specialist foundation contractors using proprietary equipment (Kirsch and Bell, 2012). This is usually not the case for DVVC, where projects are frequently carried out by construction companies using standard sheet piling equipment. General contractors have less experience with the efficient use of vibratory driving equipment. Therefore, standard vibrators and simple compaction probes (tube or beam) are frequently used, and DVVC is often carried out on a trial-and-error basis.

Before the start of a compaction project, the designer should analyze whether - or to what extent - compaction is needed. Unfortunately, compaction criteria are frequently chosen by the designer based on rules of thumb (often leaving the final decision to the foundation contractor). The most common compaction criteria are based on penetration tests applied to previous projects. The consequence of such a simple approach can be costly and result in inadequate compaction.

It is generally accepted that vibratory compaction causes the densification of granular soils (decrease in pore volume). However, a less well-known effect is that vibratory compaction permanently increases effective confining stress (causing pre-stressing). In general, only the densification effect is considered for design purposes. Nevertheless, the increase in confining stress can be an essential design consideration for geotechnical problems, such as settlement or liquefaction (Massarsch and Fellenius, 2020). This paper makes a distinction between soil densification (volume change) and stress changes (pre-stressing effect).

2. CAUSES OF VIBRATORY COMPACTION

Different hypotheses have been advanced to explain the compaction mechanism of granular soils. For instance, Kirsch and Kirsch (2016) proposed that: "*the stability of the structure of granular soils is destroyed by dynamic stresses when a critical acceleration of more than 0.5 g is reached. With increasing accelerations, the shear strength of the sand decreases until it reaches a minimum between 1.5 and 2 g. At this point, the soil is fluidized, and a further increase of acceleration causes dilation.*" Such concepts were developed to explain the vibratory driving of sheet piles in granular soils. Youd (1972) commented on using acceleration as the primary reason for compaction: "*Unfortunately, this approach has not led either to a clear understanding of the compaction process or to adequate methods for predicting compaction in situ*."

Extensive research in the field of earthquake engineering has provided a sound basis for an understanding of the compaction mechanism in granular soils. About fifty years ago, during the same period, three studies were published by eminent researchers: Brumund and Leonards (1972), Seed and Silver (1972), and Youd (1972). They concluded that shear strain is the critical parameter governing the densification process.

Based on a review of the effects of construction vibrations on granular soils by Massarsch (2002), the following general conclusions can explain the soil compaction process:

1. The magnitude of densification depends on the shear strain amplitude.
2. Compaction increases with the number of vibration cycles.
3. Ground vibrations generate cyclic horizontal vibrations, which cause a permanent increase in effective stress.

These factors will be discussed briefly in the following paragraphs.

2.1 Shear Strain Effect

Shear strain is a critical parameter for assessing the densification effect of granular soils. Vibrations, measured perpendicular to the direction of wave propagation, are caused by shear waves. The particle velocity can be measured by geophones placed on or below the ground surface (Massarsch, 2002). The shear strain amplitude can be determined from the following relationship, which is strictly valid only for plane waves but can be used to assess the effect of particle velocity on shear strain.

$$\gamma = \frac{v}{C_s} \tag{1}$$

where γ = shear strain, v = particle velocity measured perpendicular to wave propagation direction, and C_S = shear wave speed. C_s can be measured in the field or estimated from empirical relationships (Richart et al., 1970). Soil stiffness decreases with increasing strain, particularly in granular soils (silt, sand, or gravel). This strain-softening effect is illustrated by the results of a resonant column test (Massarsch 2000). The soil sample was silty sand with a plasticity index PI = 14. The bulk density at a water content of 32% was ρ = 1.870 kg/m^3. The following relationship exists between the secant shear wave speed, C_S, and the strain-dependent shear modulus, G_S.

$$C_s = \left(\frac{G_S}{\rho}\right)^{0.5} \tag{2}$$

where G_s = shear modulus and ρ = bulk density of the sample. The measured shear wave speed, C_S, and the derived secant shear modulus, G_S, are shown in **Figure 1**. The shear wave speed as well as the shear modulus, decrease with increasing shear strain. It is not generally appreciated that also the shear wave speed is strain dependent. For instance, at 0.1% shear strain, the shear wave speed decreases from about 205 m/s to 120 m/s.

The densification of granular soils (volume change) starts when a shear strain level of approximately 0.001% is exceeded. The particle velocity to achieve this shear strain level can be estimated from Eq. (1). Assuming medium-dense sand (C_s = 150 m/s), a particle velocity of about 20 mm/s causes $\gamma \sim$ 0.01%.

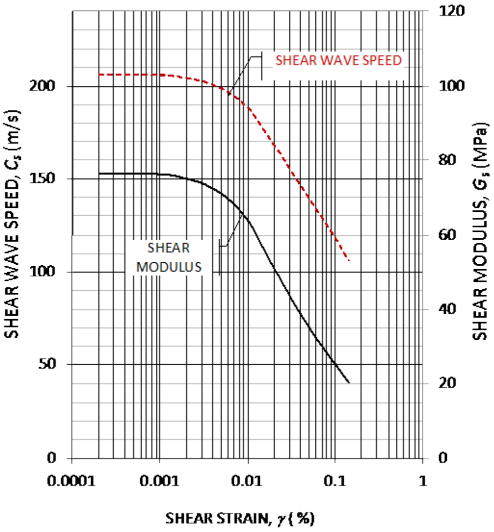


Figure 1 Effect of shear strain on shear wave speed (left axis) and secant shear modulus (right axis, determined by a resonant column test on silty sand

2.2 Number of Vibration Cycles

Another critical parameter that affects soil densification is the number of vibration cycles. Seed (1976) applied the "cumulative damage hypothesis" to convert random motions to equivalently damaging uniform cyclic motion. Green and Terry (2005) developed more sophisticated concepts for liquefaction analyses, considering the dissipated energy of random and uniform motions. However, for the assessment of vibratory densification with many uniform vibration cycles, the concept proposed by Seed (1976) appears to be sufficiently accurate. Based on the damage assessment of vibration cycles on granular soils, the following approximate relationship has been proposed by Massarsch (2000) and is represented graphically in **Figure 2**. For instance, 4.2 vibration cycles, v, at 50% of the maximum vibration velocity, v_{max}, have approximately the same distortion effect as one vibration cycle at v_{max}. For engineering purposes, it can be assumed that 100 vibration cycles, v, at approximately 30% of the peak vibration velocity, have an equivalent densification effect as one cycle at v_{max}. Consequently, increasing the number of vibration cycles will increase the densification effect. If vibratory treatment is performed at about 25 Hz and lasts 4 minutes in each layer, approximately 6000 cycles will be generated. Thus, the densification effect increases if the duration of vibratory treatment is extended.

2.3 Horizontal Stress Change

An essential effect of vibratory compaction, which is not generally recognized, is the permanent increase in effective horizontal stress (lateral prestressing). This effect is independent of the compaction method used (DVVC, DHVC), as demonstrated by Massarsch and Fellenius (2020). Horizontal prestressing results in a pre-loading ("*overconsolidation*") effect, which is of importance for liquefaction and settlement (Seed and Silver, 1972; Massarsch, 1994).

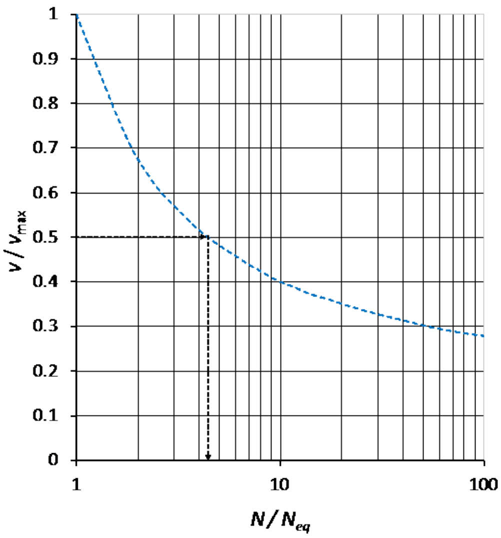


Figure 2 Influence of number of equivalent vibration cycles, N/N_{eq} on normalized vibration amplitude, v/vmax (Massarsch, 2000)

DVVC emits cylindrical shear waves generated along the shaft of the compaction probe (Massarsch, 2002). These shear waves oscillate perpendicular to the direction of wave propagation and attenuate similarly to surface waves (Massarsch and Westerberg, 1995b).

However, due to friction between the probe and the surrounding soil, vibrations are also emitted in the horizontal direction, generating horizontal compression waves. The horizontal stress increase, $\Delta\sigma_h$, can be estimated from the following relationship (Massarsch, 2002):

$$\Delta\sigma_h = v_h \, C_P \, \rho \tag{3}$$

where v_h = the vibration velocity in the direction of wave propagation, C_P = compression wave speed, and ρ = bulk density of the soil. Vibration measurements performed during vibratory compaction show that horizontal vibration amplitudes are approximately constant with depth (Massarsch, 2002). The permanent increase in horizontal effective stress is an important aspect of deep vibratory compaction and has been observed as a result of different vibratory compaction methods (Massarsch and Fellenius, 2020).

3. DVVC EQUIPMENT

Although the vibratory driving process appears to be relatively straightforward, this is not the case when applied to DVVC, as it requires understanding how vibration frequency affects probe penetration, compaction, and extraction. An example of a DVVC machine is shown in **Figure 3**. It comprises the following components: a) a vibrator, b) a compaction probe, and c) a monitoring and process control system (MPCS). These elements will be discussed in the following paragraphs.

During the past decades, powerful vibrators with variable frequency and variable amplitude have become available (Massarsch and Westerberg, 1995a; Massarsch et al., 2021a).

3.1 Vibrator

Massarsch and Westerberg (1995a) have described the practical application of modern vibrators with variable frequencies.

Figure 3 DVVC machine with a variable frequency vibrator and flexible double-Y probe

The vertical oscillation of the vibrator is generated by counter-rotating eccentric masses, the operating frequency of which can be varied during driving. The centrifugal force, F_v, depends on the eccentric moment, M_e, and on the circular frequency, ϖ, of the counter-rotating eccentric masses.

$$F_v = M_e \, \omega^2 \qquad (4)$$

where F_v = peak value of the centrifugal force, M_e = eccentric moment, ω = circular frequency ($2\,\pi\,f$). Thus, the magnitude of the centrifugal force of the vibrator is strongly affected by the vibration frequency.

The other important parameter governing the performance of the vibrator during compaction is the displacement amplitude, *s*, which can be calculated from the following relationship.

$$s = \frac{M_e}{m_t} \qquad (5)$$

where M_e = eccentric moment and m_t = total dynamic mass of the vibrating system. For details, reference is made to Massarsch and Fellenius (2005). The displacement amplitude is thus independent of the vibration frequency, f. To achieve effective compaction, it is important to generate a large vibration amplitude. Therefore, the total dynamic mass shall be kept to a minimum. The total dynamic mass, m_t, is the sum of all masses, which the vibrator must accelerate

$$m_t = m_v + m_{cl} + m_p \qquad (6)$$

where m_v = dynamic mass of the vibrator, m_{cl} = mass of the clamp, and m_p = mass of the compaction probe. Equipment manufacturers usually state the peak-to-peak (*double*) amplitude, S ($2s$). To achieve efficient compaction, the eccentric moment of the vibrator must be sufficient to move the compaction probe sufficiently during penetration, compaction, and extraction.

The displacement amplitude of the vibrating system (vibrator/probe) can be checked prior to the start of compaction by measuring the dynamic response of the suspended vibrator-clamp-probe system. The displacement amplitude of the vibrating probe should be at least 4 to 6 mm before the start of penetration. If this requirement is not met, either the eccentric moment of the vibrator must be increased or the mass of the probe must be reduced.

3.2 Compaction Probe

Different types of compaction probes have been described in the literature (Anderson, 1974; Broms, 1975; Wallays, 1982; Massarsch, 1991b; Cheng et al., 2013). The objective of DVVCr is to generate a large vibration amplitude in the surrounding soil. This is achieved most efficiently by a compaction probe with a low mass. Also, the cross-section and the shape of the probe are important for the transfer of vibration energy to the surrounding soil. The surface area of the compaction probe should be as large as possible. It was found that probes with a Y-shape or double-Y shape are the most effective transmitters of ground vibrations (Wallays, 1982). The probe should be provided with openings, thereby reducing the mass and enhancing the interaction between the probe with the surrounding soil, as shown in **Figure 4**.

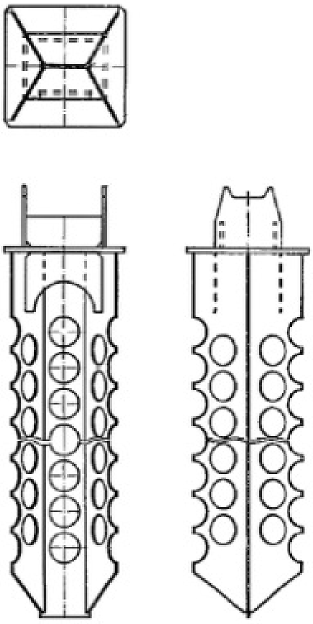


Figure 4 Double-Y probe with flexible shaft, cf. Figure 3

The penetration resistance along the probe shaft during penetration and vibratory compaction can be estimated based on cone penetration (CPT) results. Westerberg et al. (1995) have proposed a concept based on CPT. For practical purposes, it can be assumed that the resistance at the toe of the probe is equivalent to the cone resistance. While the toe resistance is not strongly affected by the vibration frequency in granular soils, the opposite applies to the shaft resistance. When the vibration frequency is at least 1.5 times higher than the system resonance frequency, the shaft resistance during penetration and extraction in granular soil is approximately 5 to 10% of the static sleeve resistance. However, when the vibrator is operated at the system resonance frequency, the probe and the surrounding soil start to oscillate "in phase," and the relative displacement between the probe and the soil becomes very small. At resonance, an almost "static friction" develops along the probe shaft, and the probe penetration speed slows down. At the same time, the displacement amplitude of the probe increases markedly, resulting in strong ground vibrations and compaction. Details of the probe-soil interaction during vibratory driving have been documented by field measurements (Massarsch et al., 2021b).

If the probe is extracted at the resonance frequency, the resisting force acting along the probe can be so high that they result in the collapse of the compaction machine, **Figure 5**.

Figure 5 The collapse of a compaction rig because of extracting the probe at the resonance frequency

When the probe is pulled at system resonance, the densified soil can become looser (decompaction), which can be observed by the development of an inverse dome on the ground surface. Thus, probe extraction must be carried out at a high frequency (at least 1.5 times the system resonance frequency).

3.3 Monitoring and Process Control

To execute DVVCr efficiently, the entire compaction process must be monitored. Powerful computerized systems have become available, which can be used on construction sites (Massarsch and Wersäll, 2019). The MPC developed for DVVCr is shown in **Figure 6**. Such a system is not only a passive documentation system but can also be used by the machine operator and project management (on-site or in the office) to follow and actively control the compaction process.

The MPCS makes it possible to optimize the compaction process in real time. The following parameters are measured directly: position of pile/sheet pile (GPS coordinates); date and time (hh:mm:ss); depth of sheet pile (m); vibrator frequency (Hz); acceleration of vibrator (cm/s^2); hydraulic pressure of power supply (MPa) and ground vibration velocity (mm/s); eccentric moment (kgm - optional); and static force (kN - optional). From these measurements, the following parameters can be derived: frequency of vibrator (rpm); movement amplitude at pile head (mm); probe penetration velocity (cm/min); centrifugal force (kN); penetration depth (m) and vibration cycles per depth interval (cycles/cm).

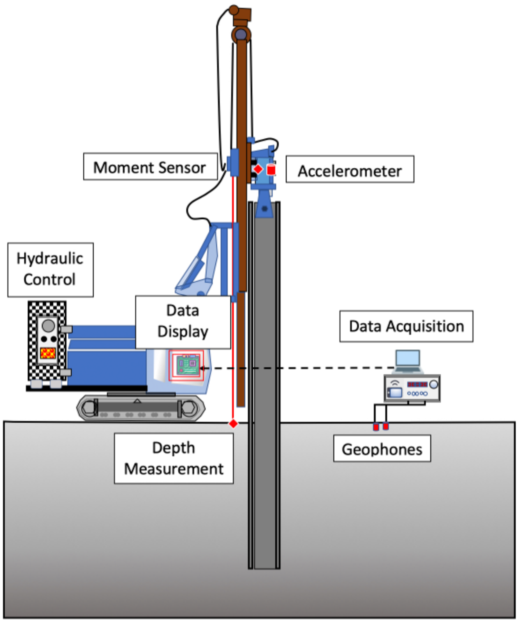


Figure 6 Principle of Monitoring and Process Control System (MPCS), showing sensors mounted on the rig and ground surface (Massarsch and Wersäll, 2019)

The vibrator operator can view all relevant information (measured and derived parameters) on a screen. In addition, important performance parameters and visual guidance (arrows, bar diagrams, or dials) can be indicated to assist the machine operator during the entire compaction process.

4. RESONANCE COMPACTION

Deep vertical vibratory compaction using resonance amplification has been successfully applied to a variety of projects (Massarsch and Vanneste, 1988; Massarsch, 1991a; Massarsch, 1991b; Choa et al., 2001; Massarsch and Fellenius, 2005, Liu and Cheng, 2012, Cheng and Liu, 2013; Massarsch et al., 2017; Massarsch and Fellenius, 2017 and Massarsch et al., 2021b).

To achieve optimal vibratory compaction, a suitable treatment process must be chosen (Massarsch and Fellenius, 2005). During vibratory driving at a high frequency, much of the vibration energy is consumed as heat along the shaft of the probe, and soil densification will be low. When the vibration frequency is gradually lowered, probe penetration speed decreases, and ground vibrations increase. At resonance, the compaction probe and the soil oscillate "in phase." Thus, the probe acts as an antenna for transferring vibration energy from the vibrator to the surrounding soil.

The frequency at system resonance is affected by several parameters, such as the dynamic properties of the compaction system (mass of probe, eccentric moment of vibrator) and the dynamic properties of the soil (shear modulus), which change with strain levels and frequency. Thus, estimating the system resonance frequency is a complex task (Guangyin et al., 2012). However, it is relatively simple to measure the dynamic ground response and system resonance on site by a triaxial geophone **Figure 7**.

Figure 7 Tri-axial geophone for measurement of resonance frequency at 4 m distance from compaction probe

The effect of vibrator frequency on the vertical ground vibration velocity is illustrated in **Figure 8**.

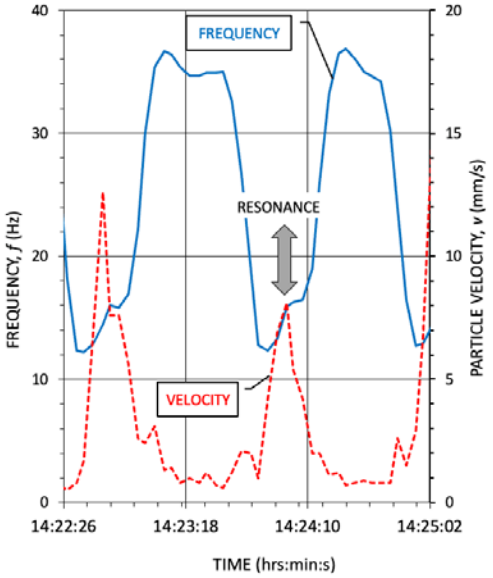


Figure 8 Effect of vibrator operating frequency on vertical ground vibration velocity (RMS) (Massarsch et al., 2021a)

At a driving frequency of 38 Hz, ground vibrations are low (less than 3 mm/s RMS). When the frequency is reduced to 12 Hz, ground vibrations are amplified by a factor of 5 to 10. Also note the reduction of hydraulic pressure when the frequency is reduced, which implies that the energy consumption is significantly reduced.

The MPCS can be programmed to optimize the different stages of resonance compaction. Measured and derived parameters can be displayed to the vibrator operator in real time and assist in optimizing the compaction process (Massarsch and Wersäll, 2019).

A typical example of the treatment process (penetration and extraction) is illustrated in **Figure 9**. During the initial insertion, the probe is vibrated at a high frequency (>35 Hz) to ensure fast penetration. At about 75% of the maximum treatment depth, the vibrator frequency is gradually lowered until system resonance is achieved. Thereafter, the vibrator frequency is increased to its maximum, and the probe is withdrawn by typically 2 m. The probe is subsequently re-inserted (at low frequency) and withdrawn (at high frequency) in several steps. Massarsch et al. (2017b) and Massarsch and Fellenius (2019) have given examples of the practical application of the resonance compaction process.

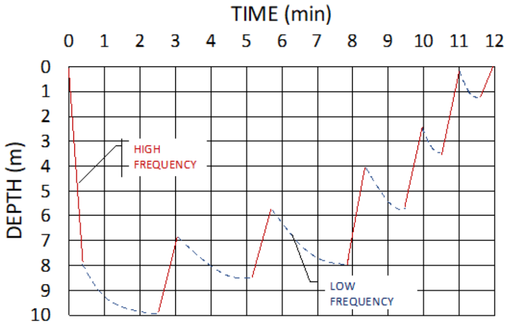
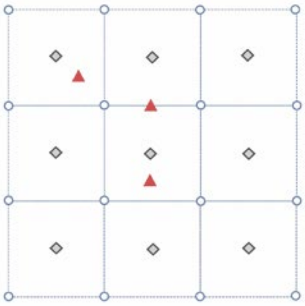


Figure 9 Example of probe penetration and extraction process during resonance compaction

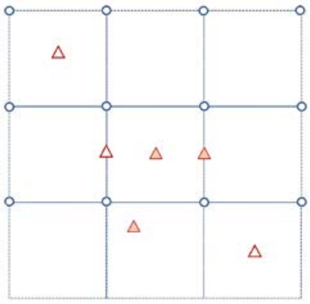
Thanks to the shape of the double-Y probe (**Figure 4**), the zone of influence around the vibrating probe has an almost rectangular shape. Therefore, the grid of compaction points can be chosen in a rectangular pattern, which results in significant savings compared to a conventional, triangular pattern.

The distance between compaction points depends on several factors, such as a) the size of the compaction probe, b) the eccentric moment of the vibrator, and c) the required degree of compaction. Based on experience, it is recommended to perform DVVCr in two passes. This has several advantages: the first treatment pass achieves a more homogeneous soil volume, and treatment can be carried out swiftly. During the second pass, the already compacted soil columns will confine the penetrating probe, assuring probe verticality. The increased horizontal stresses achieved by the first pass enhance the compaction effect significantly.

To establish the optimal resonance compaction procedure, compaction trials should be carried out at the start of the project. The objectives are to establish the optimal spacing between compaction points and the duration of compaction required to achieve the specifications. A typical trial compaction pattern is shown in **Figure 10**.



a) first compaction pass



b) second compaction pass

Figure 10 Example of trial compaction in two passes with recommended locations of penetration test points

The probe penetration velocity, v_P, which can be measured by the MPCS, reflects the soil strength. In loose granular soils, the penetration velocity will be high but decreases with increasing compaction (Massarsch et al., 2021). As the vibrator frequency, f (Hz), is known, it is possible to convert v_P, (cm/min) into an equivalent number of penetration cycles, c_P per depth interval (cycles/cm):

$$c_P = \frac{f}{v_P} \tag{7}$$

If for instance, the probe penetration velocity at 40 Hz is 500 cm/min (typical for medium-dense sand), the number of vibration cycles will be c_P = 4,8 cycles/cm (or 96 cycles/20 cm). This value can be related to, for instance, the CPT penetration resistance, q_c.

Trial compaction performed at different compaction grid spacings can be used to determine the probe penetration velocity. This information can be programmed into the MPCS and used to guide the machine operator during the execution of the compaction work.

An important practical consideration is that, at resonance, due to the lower operating frequency, the energy consumption of the hydraulic system decreases significantly. A lower operating frequency reduces the number of oscillations of the vibrator bearings, which increases the life length of the vibrator.

5. CONCLUSIONS

The primary factors which cause soil densification (volume change) in granular soils are the shear strain amplitude (related to the ground vibration velocity) and the number of vibration cycles (duration of compaction). However, vibratory compaction also results in a permanent increase in the horizontal effective stress.

Deep vertical vibratory compaction (DVVC) uses a vibrator mounted on top of a specially designed compaction probe. The main components of the DVVC system are a) a vibrator with variable frequency; b) a probe with a low mass; c) a probe with a double Y-shape; d) openings in the wall of the probe, which achieve efficient interaction with the surrounding soil.

When DVVC is carried out at the resonance frequency of the vibrator-probe-soil system (DVVCr), ground vibrations are amplified, and the compaction efficiency is enhanced. The vibrator-probe-soil system oscillates in phase with the surrounding soil, and the compaction probe acts as an antenna, transferring the vibration energy from the vibrator to the soil. Ground vibrations are amplified while the required compaction energy decreases.

For the execution of DVVCr a measuring and process control system (MPCS) is used, which can be programmed to optimize the compaction process.

The system resonance frequency can be readily measured in the field using a triaxial geophone installed at approximately 4 m from the probe.

The resonance frequency is strongly influenced by the mass of the vibrating system and the stiffness of the soil. As the shear wave speed increases because of compaction, also the resonance frequency rises.

Probe penetration velocity is high in the uncompacted soil but decreases due to densification. The probe penetration velocity can be used to monitor the compaction effect in real time.

6. LIST OF NOTATIONS

C_P	compression wave speed
c_P	number of penetration cycles per depth interval
C_S	shear wave speed
DHVC	Deep horizontal vibratory compaction
DVVC	Deep vertical vibratory compaction
DVVCr	Deep vertical vibratory compaction using resonance
f	frequency
F_v	peak value of the centrifugal force
g	gravitational acceleration
G	shear modulus
m_{cl}	mass of the clamp
m_t	total dynamic mass of the vibrating system
M_e	eccentric moment
m_p	mass of the compaction probe
MPCS	Measuring and process control system
m_v	dynamic mass of the vibrator
N	number of vibration cycles
N_{eq}	equivalent number of vibration cycles
q_c	cone penetration resistance
RMS	*root mean square*
s	displacement amplitude
S	peak-to-peak amplitude (2s)
V	particle velocity measured perpendicular to wave propagation direction
v	particle velocity
v_h	vibration velocity in the horizontal direction
v_{max}	maximum particle velocity
$\Delta\sigma_h$	horizontal stress,
ρ	bulk density of soil
γ	shear strain
ω	circular frequency (2 π f)

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