Stability Analysis of Jointed Rock Slope with Strength Reduction Method Based on a 3DEC Simulation

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ABSTRACT: To investigate the application of strength reduction method in the stability analysis of rock slope, a rock slope model with two sets of joints is established in the 3D discrete element code (3DEC) software. Different strength reduction algorithms are used to solve the slope safety factor and the slope failure form in critical condition. Results indicate that it is more reasonable to adopt the coordinated reduction method for the unjointed slope model. However, for the slope model with two groups of joints, the slope is stable when only the rock block parameters are reduced, resulting in a small amount of toppling deformation. The numerical results of only reducing the structural plane parameters are consistent with those of reducing all parameters. The safety factor of the rock slope model with two groups of joints is 1.30, and tensile shear failure occurs along the structural plane. Taking the convergence of the displacement of the monitoring points on the slope surface as the criterion of slope instability, the reduction factor when the slope is in a critical state can be obtained from the displacement curve. The reduction of structural plane parameters should be mainly considered when the discrete element strength reduction method is used to calculate the safety factor of rock slope.

KEYWORDS: Strength reduction method, Jointed rock slope, Safety factor, Numerical simulation, and Discrete element method.

1. INTRODUCTION

The stability of natural and artificial soil and rock slopes has always been a highly concerned problem in engineering (Mehmet 2019, Mao *et al.*, 2022, Mao *et al.*, 2023). Strength reduction method (SRM) in calculating safety factor is increasing used in the slope stability analysis, which has the three advantages. First, the nonlinear mechanical properties of rock and soil can be considered in numerical calculation, which can reflect the progressive failure process of rock and soil slope. Secondly, the sliding surface can be easily obtained from the calculation results once instability occurs. Moreover, the impact of complex causes on the slope can be considered.

The calculation of safety factor for evaluating a construction stability is one of the main approach in geotechnical engineering (Xiong, 2021). A nonlinear SRM technique was proposed that can represent the nonlinear behavior of a rock mass using the Hoek-Brown (HB) criterion in the FLAC (3D) program to analyze 3D slope stability (Shen, 2014). The convergence criterion cannot only calculate the safety factor, but also capture the shape of the failure surface. A coupling of reliability analysis methods with various mechanical approaches was developed for slope stability assessment. Both the cohesion and friction angle of the soil were assumed as uncertain input parameters, and modelled as correlated random variables. Besides, the limit equilibrium method and the shear strength reduction method based on finite elements were used to assess the stability of slopes of complex geometry (Barbara, 2018). Additionally, an adaptive finite element limit analysis based on the strength reduction technology was adopted to investigate the stability of rock slopes, and the safety factors obtained were validated via two real slope cases (Chen 2021, Sengani and Mulenga 2020). In the study, the rock mass disturbance D and slope angle β were in corporated into the determination of safety factor by implementing the disturbance weighting factor σ and the slope angle weighting factor f(β), respectively.

Li *et al.* (2021) introduced the discrete fracture network (DFN) in discrete element model to study the stability evaluation of rock slope. In this study, the whole process from field fracture acquisition to the DFN generation was applied to the 3D fracture rock slope stability analysis, which have a great significance for complex rock mass engineering assessment. It was also found that the effect of vertical side boundaries plays an important role in the stability of jointed rock slope in a ubiquitous joint model and the cohesive force is the main contribution to the resistant force of vertical side boundaries (Lu *et al.*, 2020; Wang *et al.*, 2022). In addition, a dynamic reduced method (DRM) combined with the strain-softening method was proposed to evaluate the possible slip surface of a highly heterogeneous rock slope (Chen *et al.*, 2018). The results illustrated that the DRM can provide a reliable prediction of the location of the slip surface for highly heterogeneous slopes with geological discontinuities in different length scales. As the cohesion c and internal friction angle φ play different roles in the progressive failure process of the slope, a double strength reduction method considered different reduction factors for these two parameters was proposed by Lu *et al.* (2021). The conclusions indicated that the distribution of plastic zone calculated by double strength reduction factor method is basically consistent with that of traditional method, and the impact degree of the two parameters to unstability is related to the inclination angle of the bottom sliding surface of the unstable block. The results indicated that it is necessary to adjust the deformation parameters of rock mass during the reduction of cohesion and internal friction angle, and the safety factors obtained by the proposed method are very close to those calculated by the limit equilibrium method (Yan, 2016).

The residual displacement increment criterion based on the displacement catastrophe criterion of characteristic points was proposed by Sun *et al.* (2021). The variational criterion was proposed and the slope stability can be determined by identifying the positive or negative of the variational value by Hua *et al.* (2022). These new criterions avoided the randomness of selecting characteristic points and the subjective error of judging the displacement mutation point artificially. Tu *et al.* (2016) introduced the energy mechanism of material failure in thermodynamic theory into slope engineering, and the energy equation of slope in the process of strength reduction was derived. The calculation results showed that the stability of slope using SRM is closely related to the change of energy. At the same time, four new instability criteria were proposed, which can improve the calculation accuracy and efficiency. Fang *et al.* (2020) built a new instability criterion based on the critical slope concept and the double strength reduction to evaluate the stability of slopes. The calculation results revealed that the reduction ratio of soil strength derived from the proposed method is more reasonable.

Most of the existing research results specify a broken line structural plane as the potential sliding surface, which leads to the calculation results can not reflect the actual situation of the project. However, there are few researches on strength reduction using 3D discrete element method for multiple structural planes intersecting. In addition, further research is needed to consider the simultaneous reduction of rock and structural plane strength and the relationship between the reduction coefficients. Therefore, this study aims to simplify rock slope model and consider the reduction of rock mass strength and deformation parameters as well as structural plane strength parameters in the discrete element method.

2. NUMERICAL MODEL AND METHODOLOGY

2.1 Establishment of the Slope Model

2.1.1 Unjointed Rock Slope Model

The established numerical model of complete rock slope without joints is shown in Figure 1. Five monitoring points (A, B, C, D and E) are set on the longitudinal profile located at the centre of the model to monitor the horizontal and vertical top displacements. The discrete element model of unjointed slope is also established in Figure 2. The mechanical properties of geotechnical materials are described by Mohr Coulomb constitutive model and maximum tensile stress criterion. The upper boundary of the model is a free boundary, while the bottom and other boundaries are constrained. The mechanical parameters of rock mass and structural planeare listed in Table 1 and Table 2, respectively.

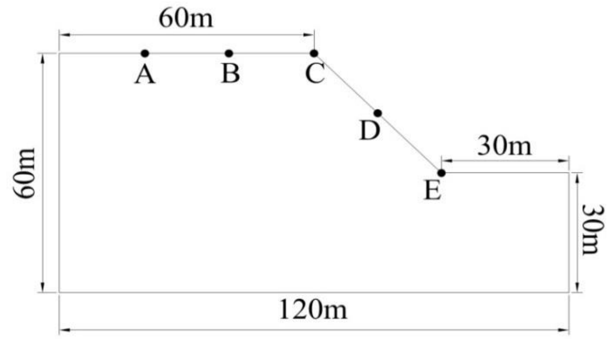


Figure 1 Unjointed slope model

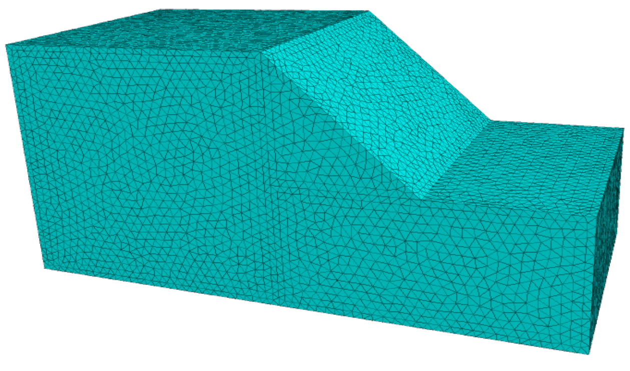


Figure 2 Discrete element model of unjointed slope

Table 1 Mechanical parameters of intact rock mass (Yuan, 2016)

Volumetric weight/ kN×m^{-3}	cohesion/ MPa	Internal friction angle/°	Elasticity modulus/ GPa	Poisson's ratio	Strength of extension/ MPa
2650	1.1	47.7	7.0	0.25	2.3

Table 2 Mechanical parameters of structural plane (Yan, 2016)

Normal stiffness/ GPa×m^{-1}	Shear stiffness/ GPa×m^{-1}	cohesion /MPa	Internal friction angle /°
6.0	3.0	0.15	24.0

2.1.2 Rock Slope Model with Two Sets of Joints

Based on unjointed slope model, a simplified model of rock slope with two groups of uniformly distributed joints is established in Figure 3 to study the influence of structural plane strength reduction. The model size and monitoring points of the slope model are consistent with those in Figure 1. The bedding joint inclination and spacing are 20° and 3 m, respectively. While reverse joint inclination and spacing are 80° and 6 m, respectively.

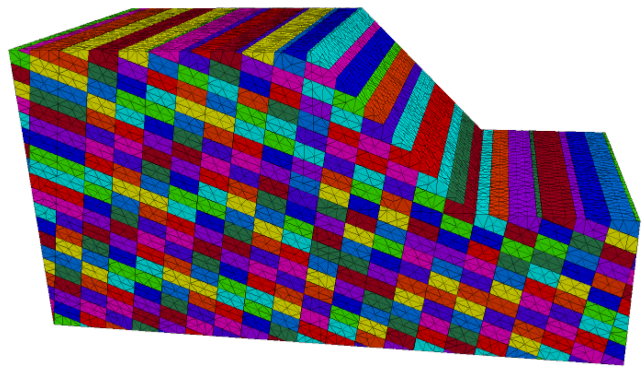


Figure 3 Discrete element model of slope with two sets of joints

2.2 Strength Reduction Method (SRM)

The SRM, which solves the slope safety factor is to make the slope reach the critical state by constantly adjusting the strength parameters of rock mass. Then, the ratio of the initial parameters of rock mass to the critical state parameters is taken as the slope safety factor. To study the influence of deformation parameters and tensile strength reduction on the safety factor of rock slope, two strength reduction algorithms are used to solve the safety factor of jointed rock slope. The first is that only the cohesion and internal friction angle of rock mass are reduced. The second is to reduce the deformation parameters and tensile strength of rock block while reducing cohesion and internal friction angle. The specific algorithm is as follows.

①The reduction coefficient is determined and the cohesion and internal friction angle after reduction are calculated according to Equation 1 (Jiang, 2015).

$$\left.\begin{aligned} c_i &= c_0 / F_i \\ \varphi_i &= \varphi_0 / F_i \end{aligned}\right\} \qquad (1)$$

where, c_i、 ϕ_i are the cohesion and internal friction angle after reduction; c_0、 ϕ_0 are the initial cohesion and internal friction angle; F_i is the reduction factor.

②The constants α , β , and λ are calculated according to the following Equation 2 (Yan, 2016).

$$\left.\begin{aligned} \alpha &= \frac{1-2\upsilon_0}{\sin\varphi_0} \\ \beta &= \frac{2c_0\cdot\cos\varphi_0}{(1+\sin\varphi_0)\cdot f_{t0}} \\ \lambda &= \frac{2c_0\cdot\cos\varphi_0}{(1-\sin\varphi_0)\cdot E_0} \end{aligned}\right\} \qquad (2)$$

where, α, β, and λ are the introduced constants; f_{t0} and E_0 are initial tensile strength and elastic modulus respectively; the rest of the signs are the same as before.

③ The tensile strength and deformation parameters after reduction are calculated according to the cohesion, internal friction angle and corresponding constants after reduction by Equation 3 (Yuan, 2016).

$$\left.\begin{aligned} \upsilon_i &= \frac{1}{2}\left(1-\alpha\cdot\sin\varphi_i\right) \\ f_{ti} &= \frac{2c_i\cdot\cos\varphi_i}{\left(1+\sin\varphi_i\right)\cdot\beta} \\ E_i &= \frac{2c_i\cdot\cos\varphi_i}{\left(1-\sin\varphi_i\right)\cdot\lambda} \end{aligned}\right\} \qquad (3)$$

where, f_{ti}、ϑ_i, and E_i are tensile strength, poisson's ratio and elastic modulus after reduction, respectively; the rest of the signs are the same as before.

④ The slope stability is calculated based on the reduced parameters. If the slope does not reach the critical state, adjust the reduction coefficient, and repeat steps ① to ③ until the critical state is reached.

In order to study the influence of structural plane parameter reduction on slope stability and failure form, three cases of strength reduction methods are used to calculate the safety factor of slope model with two groups of joints. (1) Only strength parameters of structural plane are reduced; (2) Only rock mass strength, deformation parameters and tensile strength are reduced; (3) All the above parameters are reduced at the same time. The same coefficient is used to reduce the strength of rock mass and the two groups of structural plane parameters during calculation.

3. RESULTS AND ANALYSIS

3.1 Results of Unjointed Slope Model

3.1.1 Results of Only Rock Mass Strength Parameters are Reduced (The First Case)

The calculated safety factor of the slope model is 1.49 when only the cohesion, internal friction angle are reduced. The displacement contour of rock slope in critical condition is presented in Figure 4. At this point, the slope produces a circular sliding surface, and the maximum displacement is at the toe of the slope, about 0.9 cm. The distribution of plastic zone in the model is shown in Figure 5, in which the slope near the sliding surface is mainly subject to shear failure and a few areas are subject to tension shear composite failure. Figure 6(a) shows the horizontal displacement curve of each monitoring point. It can be seen that the displacements of the top and trailing edge of the slope are relatively small, while the displacement of the slope surface and toe is large. The displacement growth rate is large at the initial stage. The displacement gradually tends to be stable after reaching the critical state. The vertical displacement curve of each monitoring point is shown in Figure 6(b). The change trend of the curve is consistent with that of the horizontal displacement. The displacement at the top of the slope is large, and the displacement at the foot of the slope is upward.

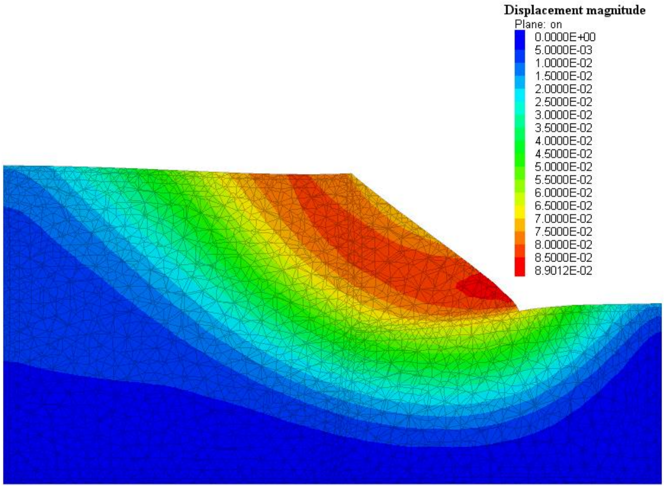


Figure 4 Contour of slope displacement

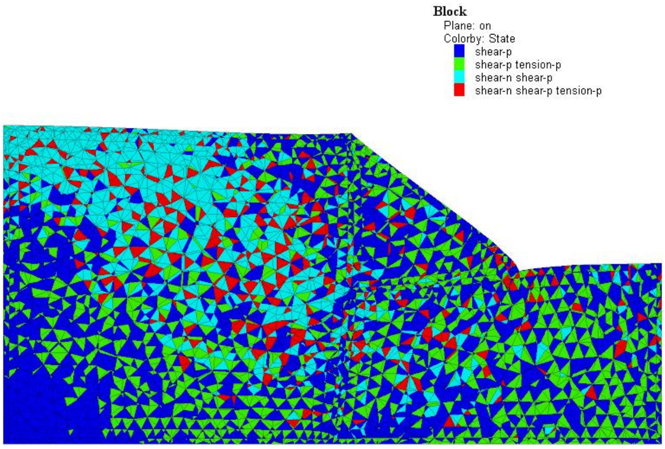
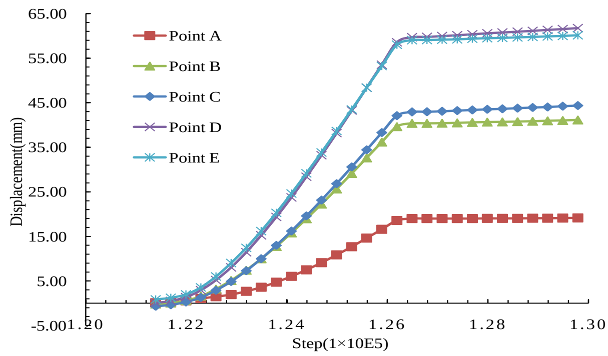
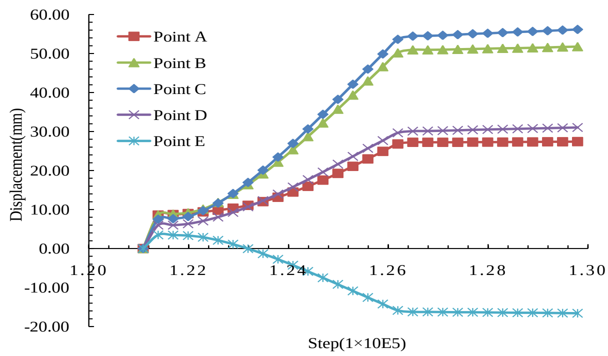


Figure 5 Distribution of plastic zone in slope model



(a) Horizontal displacement curve of monitoring points



(b) Vertical displacement curve of monitoring points

Figure 6 Displacement curve of monitoring points

3.1.2 Results of Reduced Rock Mass Strength, Deformation Parameters and Tensile Strength (The Second Case)

The safety factor of the slope model is 1.21 when the cohesion, internal friction angle, poisson's ratio and elastic modulus are reduced. The displacement contour of rock slope in critical condition is shown in Figure 7. The slip form of the slope is consistent with Figure 4, but the maximum displacement is about 7.5 cm. The slope deformation in critical state is larger due to the reduction of rock mass deformation parameters under the same stress condition. The distribution of the plastic zone in the slope model is shown in Figure 8. The slope near the sliding surface is dominated by tensile and shear failures, and tensile failure is dominant on the sliding surface after the reduction of tensile strength. The calculated results are consistent with the practical situation.

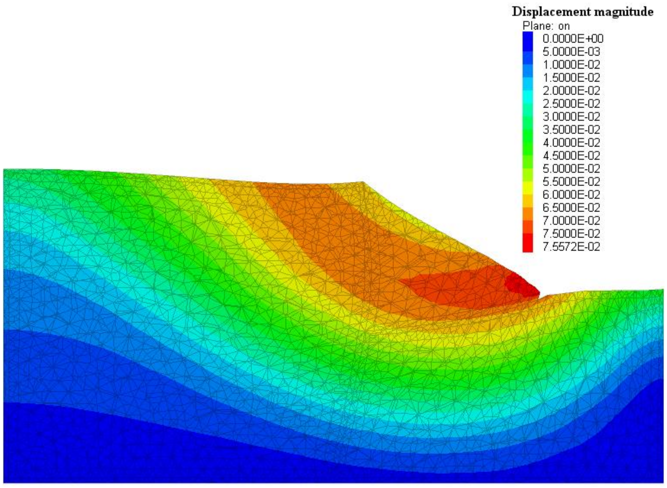


Figure 7 Contour of slope displacement

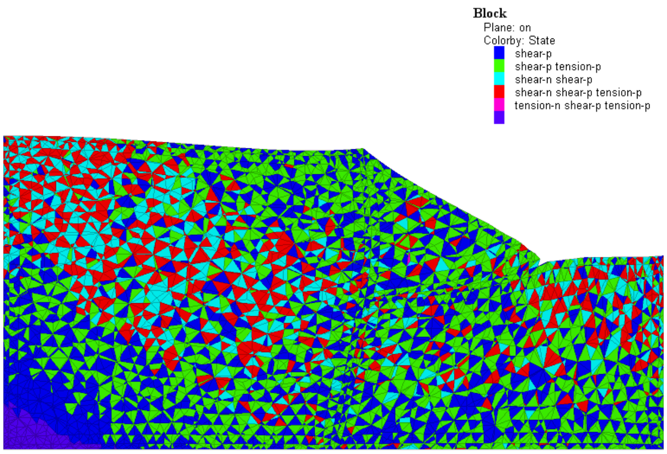
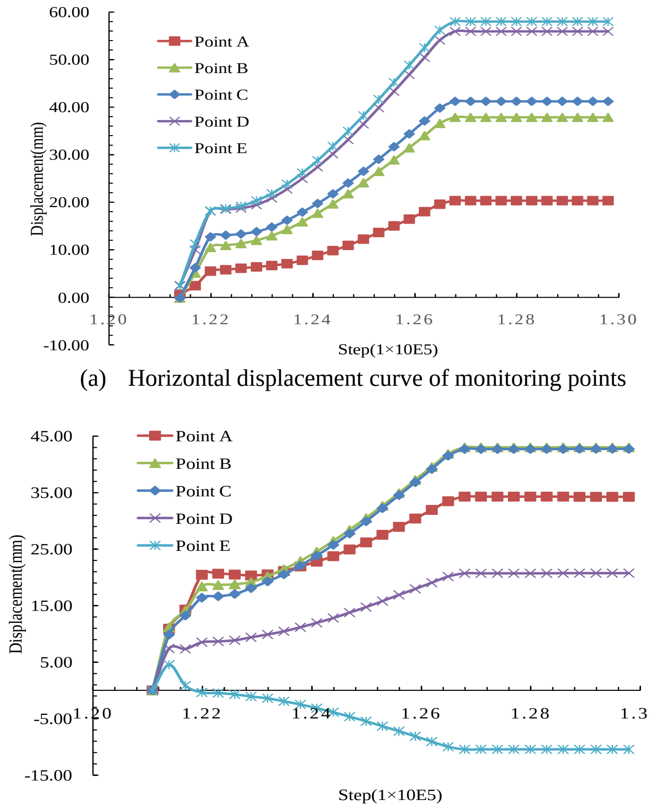


Figure 8 Distribution of plastic zone in slope model

The horizontal and vertical displacement curves of each monitoring point are shown in Figure 9(a) and Figure 9(b), respectively. The trend of development curve is basically consistent with that in Figure 6, and the increase of displacement shows obvious stage. The reduction of deformation parameters and tensile strength leads to tensile failure in some areas of the model, so the displacement curves of each monitoring point fluctuate to a certain extent.



(a) Horizontal displacement curve of monitoring points

(b) Vertical displacement curve of monitoring points

Figure 9 Displacement curve of monitoring points

3.2 Results of Slope Model with Two Sets of Joints

3.2.1 Results of Only Reduced Structural Plane Parameters (The First Case)

The calculated safety factor of the slope model is 1.30 when only the structural plane parameters are reduced. The displacement contour of rock slope in critical condition is shown in Figure 10. At this time, the displacement at the top of the slope reaches the maximum value of 0.11 m. The upper rock mass slides downward along the bedding joints under the action of gravity and pushes the lower rock mass to finally form an integral bedding sliding, further resulting in an obvious displacement interface in the slope. It can be seen from the displacement contour that the rock mass at the rear edge of the slope produces tension cracks along the structural plane with an angle of 80° when the slope rock mass slips. This phenomenon can be observed more clearly through the horizontal displacement curve of each monitoring point in Figure 11(a), where the horizontal displacement curve of each point presents an obvious stage accelerated development under the critical state. This is because the slope block accelerated sliding along the longitudinal structure under the thrust of the upper rock mass and its own gravity. The displacement difference between the block and the upper rock mass results in the formation of tensile fractures with the increase of displacement. Therefore, the thrust force of the block decreases and the slip acceleration decreases. The horizontal displacement at point C is the largest, and the displacement difference between two points A and B at the back edge of the slope is about 2 cm, resulting in obvious tensile cracks generated. The displacement at point D on the slope is less than that at the top of the slope, while point E at the foot of the slope is stable and no obvious damage occurs. The vertical displacement curve of each monitoring point is shown in Figure 11(b). The curve shape is close to the horizontal displacement and has the same stage characteristics. The displacement at the top of the slope is the largest, the trailing edge and the slope surface are small and the slope foot is stable. The vertical displacement of the monitoring points on the slope top and the slope surface fluctuates slightly, which is due to the unloading rebound of the lower rock mass caused by the sliding of a small number of blocks on the slope surface.

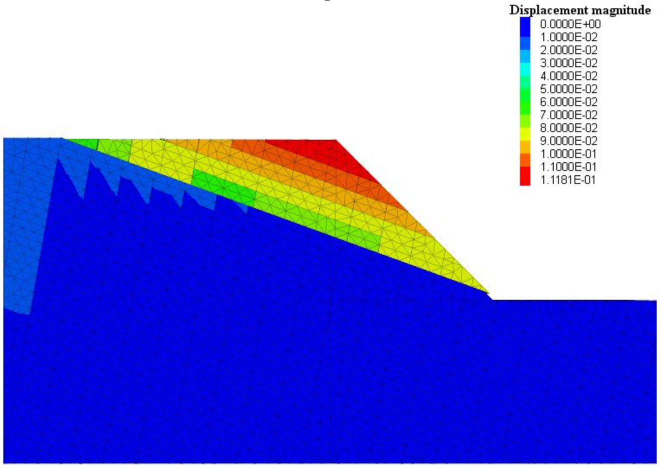
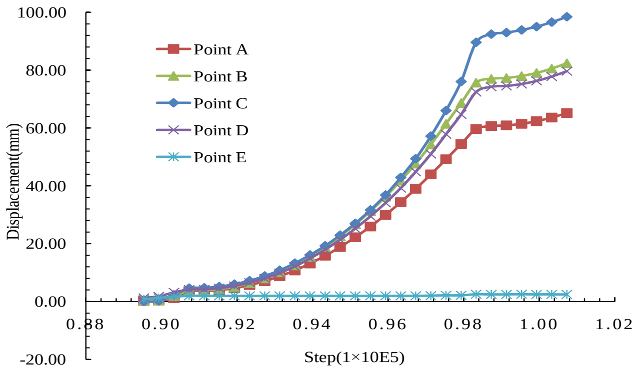
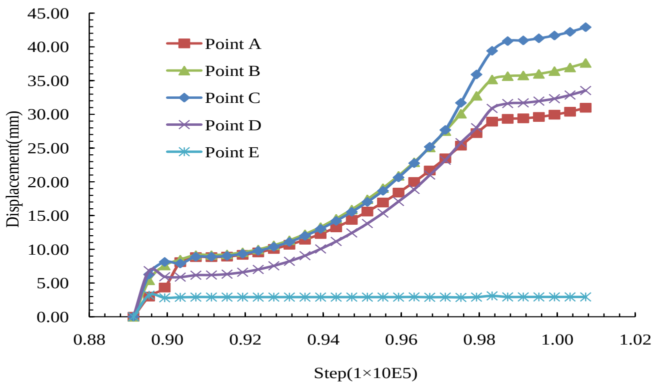


Figure 10 Contour of slope displacement



(a) Horizontal displacement curve of monitoring points



(b) Vertical displacement curve of monitoring points

Figure 11 Displacement curve of monitoring points

3.2.2 Results of Only Reduced Rock Blocks Parameters (The Second Case)

The results show that the slope is in a stable state. The safety factor of the slope obtained by only reducing the rock block strength and deformation parameters has little correlation with the reduction factor. The stability of the slope is mainly affected by the parameters of the structural plane, but is less affected by the parameters of the rock block. The slope displacement contour is shown in Figure 12, which the reduction factor is 4.0. It can be seen from the figure that there is no overall slip in the slope after the rock block parameters are reduced, but vertical tension cracks are generated along the anti tilting structural plane. The upper rock mass of the slope has a displacement pointing out of the slope and appears toppling deformation. The rock mass presents the deformation characteristics of soft rock. The horizontal displacement curve of each monitoring point in the slope is shown in Figure 13(a). It can be seen that the horizontal displacement of each monitoring point increases gradually, and the slope tends to be stable after a small amount of displacement. The deformation at the top of the slope is the largest, and the deformation of the slope back edge is slightly small. The rock mass on the slope is basically stable without obvious damage. The vertical displacement of each monitoring point is shown in Figure 13(b). It can be seen that the vertical displacement of the monitoring point at the top of the slope is large, while the vertical displacement of the slope surface is small. The top of the slope has a large settlement due to gravity and reduction of rock deformation parameters.

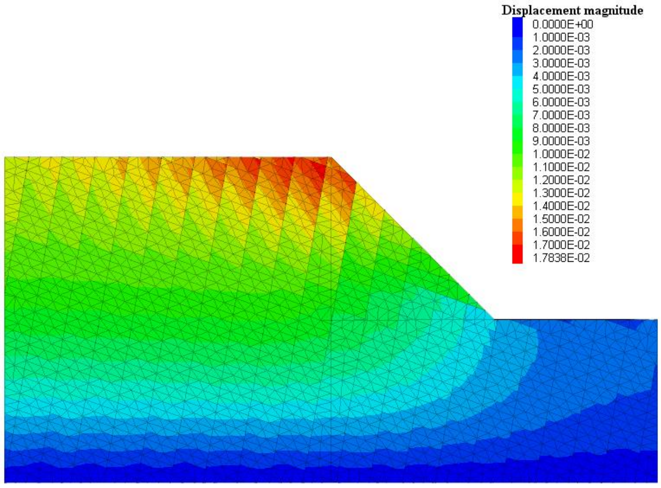
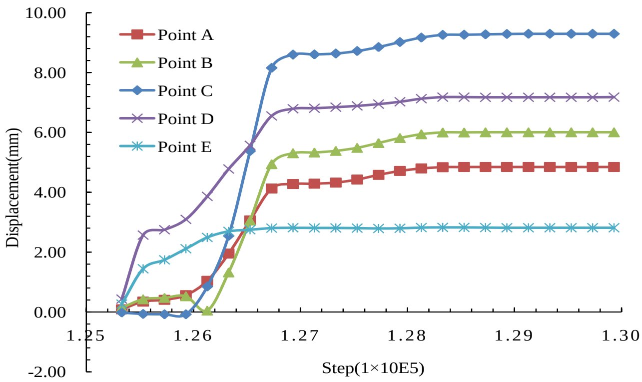
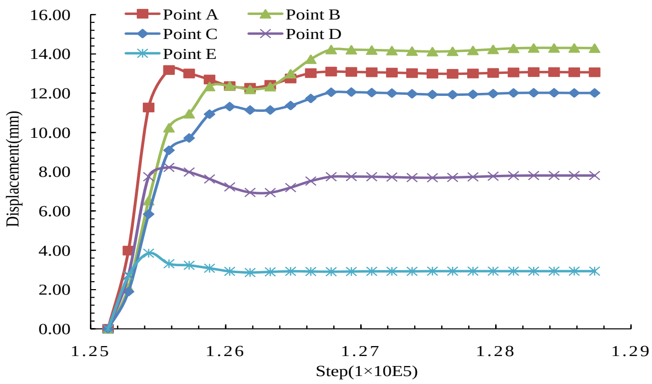


Figure 12 Contour of slope displacement



(a) Horizontal displacement curve of monitoring points



(b) Vertical displacement curve of monitoring points

Figure 13 Displacement curve of monitoring points

3.2.3 Results of Reduced Rock Blocks and Structural Plane Parameters (The Third Case)

The safety factor of the slope model is 1.30 when rock blocks and structural plane parameters are both reduced at the same time. The displacement contour of the slope in the critical state is shown in

Figure 14. It can be seen that the failure form and displacement amount are similar to the condition that only the structural plane parameters are reduced. The horizontal and vertical displacement curves of each monitoring point are shown in Figure 15. The change trend of each displacement curve is basically consistent with Figure 11. However, the displacement is slightly small because the stiffness of rock mass decreases after the rock block parameters are reduced, resulting in a part of displacement being replaced by deformation.

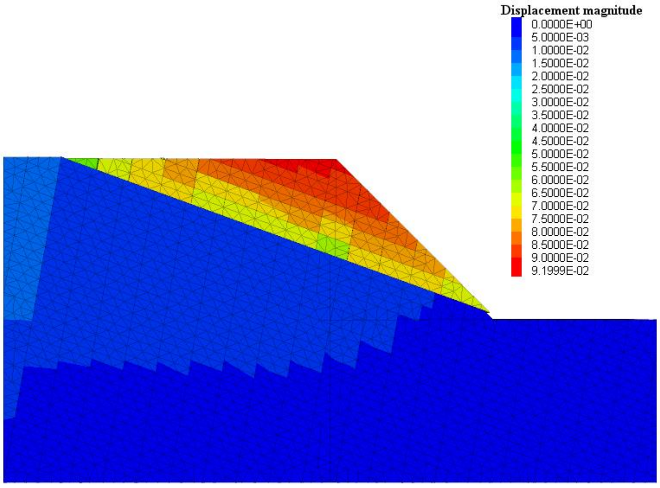
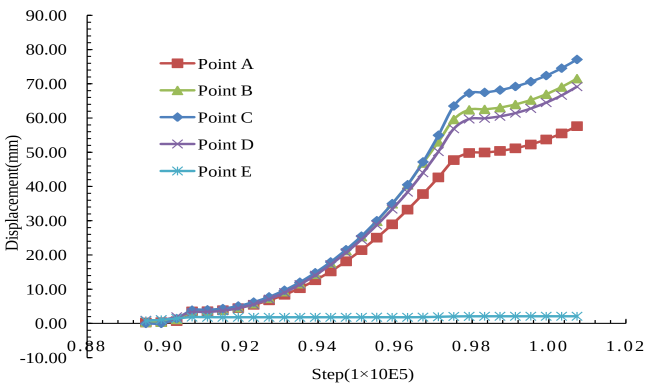
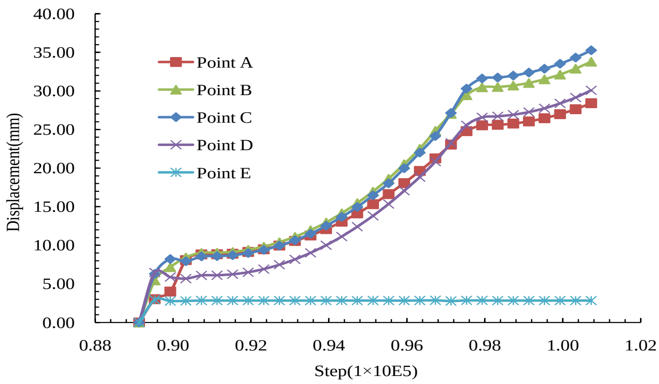


Figure 14 Contour of slope displacement



(a) Horizontal displacement curve of monitoring points



(b) Vertical displacement curve of monitoring points

Figure 15 Displacement curve of monitoring points

4. DISCUSSIONS

4.1 Unjointed Slope Model

The difference of safety factor for unjointed rock slope between strength reduction the first case and the second case are 0.28. The development process and failure form of slope deformation and instability under critical state are basically the same, which are circular are sliding. The safety factor calculated by the second case of strength reduction is smaller and the slope deformation is larger in the critical state. The plastic zone distribution of the first case and the second case of SRM is obviously different. The slip surface plastic zone obtained by the latter is mainly dominated by tension-shear mixed failure, which is more obvious near the back edge of the slope. Therefore, the results calculated by the second case of SRM are more in line with the practical situation.

4.2 Slope Model with Two Sets of Joints

The SRM of three different cases are used to calculate the safety factor of rock slope with two groups of joint, and the result of the third case is more reasonable. The strength and distribution characteristics of structural plane control the stability of rock slopes with well-developed structural planes. The safety factor and failure mode of the rock slope are mainly affected by the strength of the structural plane, and the influence of rock block strength and deformation parameters can be ignored. Shear failure occurs on the sliding surface after the slope structural plane parameters are reduced, and the rock mass at the back edge of the slope crest produces tensile cracks along the vertical structural plane. The obtained failure form is in good agreement with the actual situation (Yuan, 2016). The convergence of the displacement of the monitoring point is taken as the instability criterion of the slope. The relationship between the displacement of the monitoring points and the reduction coefficient in the same time step in the first case and the third case are shown in Figures 16 and 17. It can be seen that the displacement of point C and point D in the model gradually increases in the same time step as the reduction coefficient increases from 1.1 to 1.5, and the slope of the curve gradually increases. The displacement of monitoring points is smaller when the reduction coefficient is less than 1.3 and gradually converges, while it increases rapidly when it is greater than 1.3. Therefore, 1.3 is determined as the reduction coefficient of slope in critical state. The relationship between the displacement of monitoring point and the reduction coefficient obtained in the second case for slope stability is basically a straight line, so the graph is not listed.

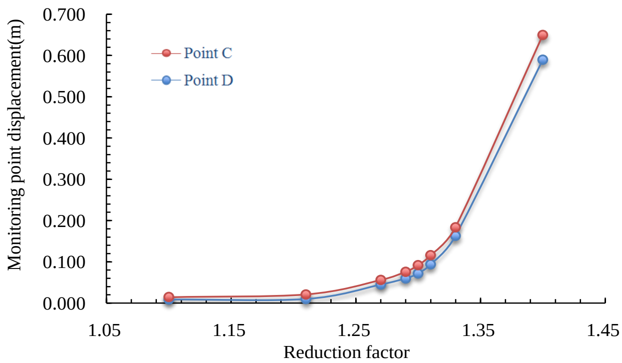


Figure 16 Relationship between displacement of monitoring points and reduction factor (the first case)

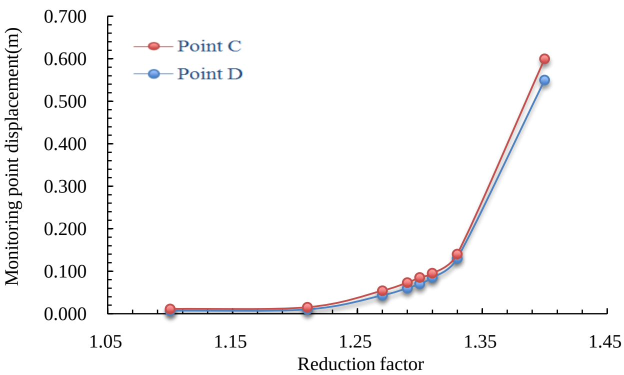


Figure 17 Relationship between displacement of monitoring points and reduction factor (the third case)

5. CONCLUSIONS

This paper uses the3D discrete element code (3DEC) to investigate the application of strength reduction method in the stability analysis of rock slope. Different strength reduction algorithms are used to solve the slope safety factor and the slope failure form in critical condition. Based on the simulation results, the following conclusions can be drawn:

(1) The unjointed slope produces circular arc sliding failure, while the slope with two groups of joints produces tension-slip failure along the structural plane.

(2) The safety factor of the rock slope obtained by the coordinated reduction method with simultaneous reduction of strength, deformation parameters and tensile strength is smaller, and the calculation results are more reasonable.

(3) The stability and failure form of rock slope are mainly controlled by the strength parameters of the structural plane. The safety factors obtained by only reducing the structural plane parameters and simultaneously reducing the structural plane and rock block parameters are consistent, and the failure form is basically the same under the critical state.

(4) There is no sliding failure of the slope when only the rock parameters are reduced. It produces a small amount of toppling deformation, showing the plastic deformation characteristics of soft rock.

6. ACKNOWLEDGMENTS

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