MOISTURE DIFFUSIVITY AND FINITE ELEMENT SIMULATION OF DRYING OF BANANA CV. KLUAI LEB MU NANG

Chayapat Phusampao

Abstract

A two-dimensional finite element model was developed to simulate moisture diffusion in Kluai Leb Mu Nang (Banana) during the drying process. The diffusivity and shrinkage of the banana used in this simulation were determined experimentally. The moisture diffusivity of the banana was determined by minimizing the sum of squares of the residuals between experimentally determined and numerically predicted data. The diffusivity values for bananas fall within the range of 4.15×10^{-6} to 6.22×10^{-6} m^2s^{-1}. Additionally, the shrinkage of the banana during drying was determined experimentally, and an equation based on physical concepts was fitted to the experimental data. A computer program in Compaq Visual FORTRAN version 6.6 was developed to simulate the finite element model, and it successfully predicted moisture diffusion during drying. The moisture content profiles of the banana were also predicted, providing accurate insights into the movement of moisture inside the banana during the drying process. The finite element model performed well in predicting moisture contents, with an RMSD of 0.925. This model can serve as a valuable tool to understand the dynamics of moisture movement without requiring extensive measurements. Furthermore, it can be used to obtain design data for dryers.

Keywords: Diffusivity, Finite element, Banana

Faculty of Education, Loei Rajabhat University, Muang District, Loei Province 42000

corresponding author e-mail: chayapatphu@gmail.com

Received: 16 August 2023; Revised: 7 October 2023; Accepted: 20 October 2023

DOI: https://doi.org/10.14456/lsej.2023.34

Introduction

Kluai Leb Mu Nang (Musa AA group) is typically dried using conventional dryers. The process of drying involves the controlled application of heat to eliminate a significant portion of the water typically present in food through evaporation. Drying fruits is a preservation method that effectively inhibits the growth of bacteria, yeasts, and mold by removing moisture from the fruits (Avhad et al., 2016; Seremet et al., 2016; Castro et al., 2018; Korese et al., 2021). A clear understanding and effective control of the drying process are crucial in establishing enhanced design guidelines. This necessitates providing an accurate description of the drying mechanism. Considerable theoretical and experimental work has been carried out in the past decades to describe the drying of biological materials. Several numerical methods are available for simulation studies. Among these methods, two have been mainly applied to model heat and mass transfer (Dobre et al., 2016; Shailesh & Ravi, 2017; Malekjani & Jafari, 2018; Winiczenko et al., 2018; Hou et al., 2020): the finite difference method and the finite element method. The finite element method has been extensively applied to modeling heat and mass transfer. This method operates on the assumption that any continuous quantity, such as moisture content, can be effectively approximated using a discrete model. The discrete model consists of a set of piecewise continuous functions defined across a finite number of sub-domains or elements. Elements are connected at nodal points along the boundaries, and their equations are obtained by minimizing a function of the physical problem. The finite element method has been extensively used to solve problems with irregular geometrical configurations and material properties that depend on temperature and moisture.

The diffusivity and volumetric shrinkage of bananas, along with the finite element modeling of their drying process, have not been previously reported. The finite element method is employed to solve problems with irregular geometrical configurations and material properties that depend on temperature and moisture. These elements are connected at nodal points along the boundaries, and their equations are obtained by minimizing a function of the physical problem. This study aims to simulate the drying of bananas using the finite element method, incorporating experimentally determined diffusivities.

Material and Method

The average initial moisture content of Kluai Leb Mu Nang (banana) was determined to be 250% (d.b.) based on the experiment. Prior to the experiment, the fruits were left at room temperature in the laboratory for approximately 20 hours to reach thermal equilibrium. The banana (Figure 1) was then dried in a laboratory dryer under controlled temperature conditions. Each experiment utilized approximately 500 g of banana. The drying process was conducted at an air velocity of 0.5,1.0 and 1.5 ms^{-1}. Temperature monitoring was done using a K-type thermocouple. The weights of the banana were recorded using an electronic balance with an accuracy of ±0.01 g at hourly intervals. The thin-layer drying tests were conducted at temperatures of 50, 60, and 70℃. Figure 2 depicts the schematic of the thin-layer drying process for bananas (Doymaz, 2010; Da Silva et al., 2014; Khawas et al., 2014).

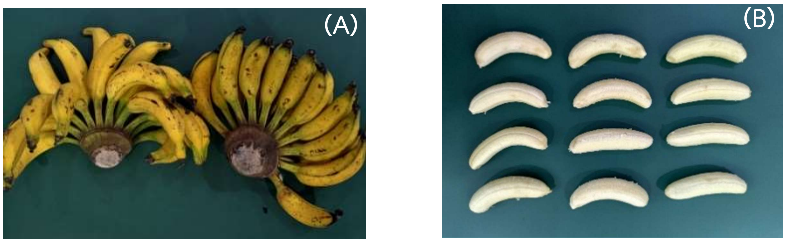

Figure 1 (A) Kluai Leb Mu Nang (banana), (B) peeled bananas

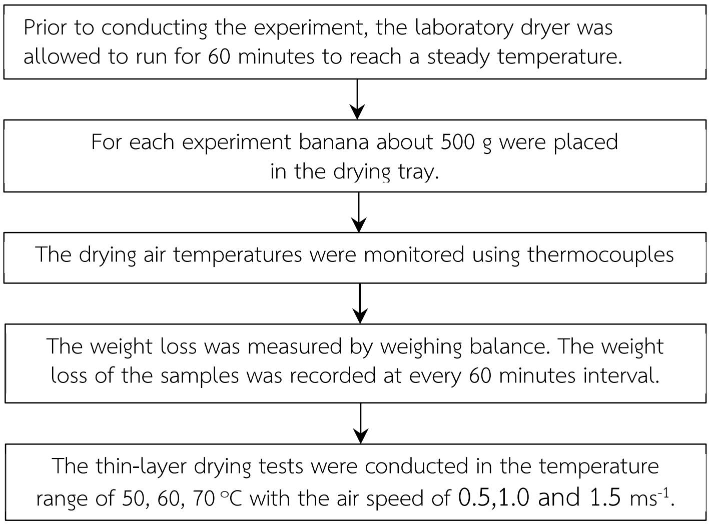

Figure 2 The schematic the thin-layer drying of Kluai Leb Mu Nang (banana).

The laboratory dryer is illustrated in Figure 3 through a schematic diagram. It comprises several components, including a blower, heaters, a drying chamber, and measurement instruments. The airflow rate is regulated by adjusting the fan speed control. The heating system involves an electric heater placed inside the duct. To control the temperature of the drying chamber, the heater power control is utilized. The homogeneity of air temperature and airflow was verified using thermocouples and a hot wire anemometer. The moisture content during the drying process was estimated based on the weight of the product samples and the calculated dried solid mass. To determine the precise dry solid mass of the product samples at the end of the experimental drying, the oven method was employed, subjecting the samples to a temperature of 103°C for 24 hours.

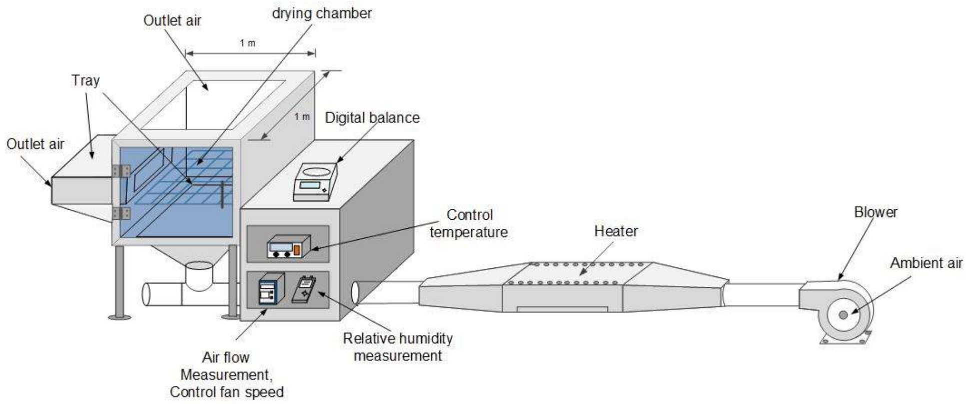

Figure 3 The schematic diagram shows the laboratory dryer.

Moisture diffusivity of Kluai Leb Mu Nang (banana)

In this work, the moisture diffusivities of bananas were employed in the finite element model (Defraeye & Verboven, 2017; Chen et al., 2020). The equation for moisture diffusivity is expressed by.

$$\frac{M - M_e}{M_0 - M_e} = \sum_{n=1}^{\infty} \frac{4}{r^2 (\alpha_n)^2} \exp\left(- (\alpha_n)^2 \frac{D_{eff} t}{r^2} \right) \quad (1)$$

The effective moisture diffusivity D_{eff} (m^2s^{-1}) represents the conductive term of all moisture transfer mechanisms. Where M represents the moisture content (d.b.), n is the number of terms of the Fourier series, α_n is roots of the Bessel function of zero order, and t is drying time (s). Experimental data of bananas can be fitted to Eq. (1). The diffusivity can then be determined by minimizing the sum of squares of the differences between the predicted and experimental data.

Finite element modelling of Kluai Leb Mu Nang (banana)drying

Fick’s law of diffusion is used in modelling the moisture movement within banana during drying and the general equation which describes the moisture diffusion can be expressed as (Fadiji et al., 2018):

$$\frac{\partial M}{\partial t} = \nabla.(D\nabla M) \quad (2)$$

The following assumptions are made when solving Eq. (2) for banana using the finite element method: 1) Drying occurs at an isothermal condition. 2) Each component of the fruit is homogeneous. 3) The initial moisture content is uniform for each component of the fruit. 4) Moisture movement within the banana is two directional. In cartesian coordinates, with a constant diffusivity, the equation is expressed as follows:

$$\frac{\partial M}{\partial t} = D\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right) \quad (3)$$

with the initial conditions: at t = 0, $M = M_0$, t > 0 and boundary conditions:

$$-D\frac{\partial M}{\partial n} = h_m(M_s - M_e) \quad (4)$$

where h_m is the mass transfer coefficient (ms^{-1}), M_e is the equilibrium moisture content on a dry basis (%, d.b.), M_s is the surface moisture content on a dry basis (%, d.b.) and n is the magnitude of a normal vector to the surface. The finite element equations are obtained using Galerkin’s formulation of the weighted residual method and Eq. (3) using Galerkin’s method can be expressed as:

$$\int_{\Omega} [N]^T \left[D\left[\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right] - \frac{\partial M}{\partial t}\right] d\Omega = 0 \quad (5)$$

where [N] is a matrix of interpolating function and Ω is the fruit domain. A system of equations is developed by evaluating the weighted residual integral. Using

Green's theorem and simplifying the results, the final result is a system of first order differential equations:

$$[C]\left\{\frac{\partial M}{\partial t}\right\}+[K]\{M\}-\{F\}=0 \tag{6}$$

$$[C]=\int_{\Omega}[N]^{T}[N]\,d\Omega \tag{7}$$

$$[K]=\int_{\Omega}D[B]^{T}[B]d\Omega + h_m\int_{L}[N]^{T}[N] \tag{8}$$

$$\{F\}=h_m\int_{L}[N]^{T}M_e dL \tag{9}$$

$$M=[N]\{M\} \tag{10}$$

Where $[C]$ = global capacitance matrix

$[K]$ = global stiffness matrix

$\{F\}$ = load force vector

$\{M\}$ =vectors of unknown which can be defined as: $M=[N]\{M\}$

Eq. (10) in the finite difference form can be expressed as:

$$([C]+\Delta t[k])\{M\}_{t+\Delta t}=[C]\{M\}_t+\Delta t\{F\}_{t+\Delta t} \tag{11}$$

Where Δt is the time step. The final system of Eq. (11) has the following form

$$[A]\{M\}_{t+\Delta t}+[P]\{M\}_t-\{F_*\} \tag{12}$$

Where $[A]=([C]+\Delta t[K])$ (13)

$[P]=[C]$

$\{F_*\}=\Delta t\{M_e\}_{t+\Delta t}$ (14)

The banana is considered as a cylinder, and moisture moves from the interior of the fruit to the outward surface, exhibiting different diffusivities. Moisture diffusion is assumed to occur symmetrically throughout the peeled banana. A two-dimensional central axis symmetric finite element triangular grid is used to model the fruit and the only the domain in two dimensions is considered because of the geometric and transfer symmetries. Figure 4 shows finite element discretization for two dimensional sections of the banana consisting of 90 nodes and 138 triangular elements. The finite element method was implemented using Compaq Visual Fortran version 6.6. The flowchart for Fortran is shown in Figure 5.

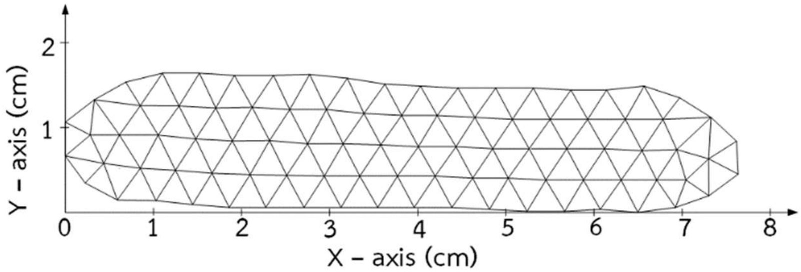

Figure 4 The mesh distribution and geometry were considered in the finite element model of Kluai Leb Mu Nang (banana).

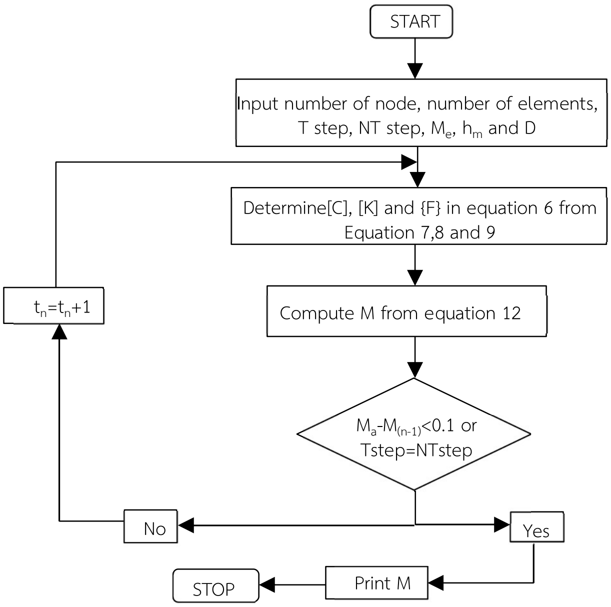

Figure 5 The schematic diagram illustrates the calculation of moisture content.

Shrinkage of Kluai Leb Mu Nang (banana)

To develop a mathematical model of volumetric shrinkage of bananas the following hypotheses are considered (Llave et al., 2016). The volume change due to the shrinkage of the product is equal to the volume of water evaporated. The pores in the product are occupied by water with density ρ_w. Let us now consider the mass of the product at a given time to be m and while the mass of dry matter is m_d and that of water is m_w. The volume of dry matter is V_d, the volume of water in the pores is V_w and the volume of the product is V. Now we can write

$$m = m_w + m_d ; V = V_w + V_d \tag{15}$$

Furthermore, it can be written as:

$$\rho = \frac{m}{V}, \rho_d = \frac{m_d}{V_d} \text{ and } M = \frac{m_w}{V_d} \tag{16}$$

where ρ is density of the product, ρ_d is density of dry matter and M is the moisture content of the product on a dry basis. The development of Eq. (15) and Eq. (16) gives the following expression:

$$V = V_0(A + \beta M) \tag{17}$$

Where

$$A = \frac{1}{1+aM_0}; \quad \beta = \frac{a}{1+aM_0} \text{ and } a = \frac{\rho_d}{\rho_w} \tag{18}$$

where M_0 is the initial moisture content, V_0 is initial volume and β is shrinkage banana If (M_0-M) is chosen to quantify the moisture removed at a time t, expression in Eq. (19) becomes:

$$V_r = 1 - \beta(M_0 + M) \tag{19}$$

$$\text{Where } V_r = \frac{V}{V_0} \tag{20}$$

Drying experiments were conducted to determine the shrinkage of bananas in a thin layer under controlled conditions. The dimensions of the bananas were measured during the drying process.

Equilibrium moisture content of Kluai Leb Mu Nang (banana)

The researcher fitted five models to the isotherm data of bananas. The modified and Oswin fitted the best and it was followed by the modified Chung-pfost, and the Day and Nelson models. The agreement between the best fitted models and experimental data was excellent. For simplicity and consistency effect of temperature, Oswin model was selected for use in this simulation. The equilibrium moisture content model used is:

$$M_e = \frac{53.426517 - 0.295723T}{\left(\frac{1}{a_w} - 1\right)^{\frac{1}{2.473582}}} \quad (21)$$

Where M_e is equilibrium moisture contents of banana (%, d.b.), T is temperature (°C) and a_w is water activity (decimal). The water activity is equal to the relative humidity (%) divided by 100.

Results

Diffusivities of Kluai Leb Mu Nang (banana)

The determination of diffusivities for banana involved the minimization of the sum of squares between experimental and predicted values derived from thin-layer drying data. The experimental investigations were carried out on banana, subject to controlled conditions of air temperature. The obtained moisture diffusivities for the banana are presented in Table 1. The mean value of the moisture diffusivity for banana in this study falls within the range of 4.15×10^{-6} to 6.22×10^{-6} m^2 s^{-1}. It was observed that the diffusivity of the banana is strongly influenced by temperature and can be expressed as a function of temperature using the Arrhenius-type equation, given by:

$$D = 6.8 \times 10^{-6} e^{\left(-2249.42/T_{ab}\right)} \quad (22)$$

Figure 6 portrays the correlation between the moisture diffusivity of bananas and the reciprocal of the absolute drying air temperature. The graphical representation provides compelling evidence that the diffusivity data of Bananas exhibits a strong conformity to the Arrhenius-type equation, signifying a robust fitting.

Table 1 The moisture diffusivities of Kluai Leb Mu Nang (banana).

Air velocity (ms^{-1})	Diffusivity (m^2s^{-1}) at temperature			Mean Diffusivity (m^2s^{-1})
	50 °C	60 °C	70 °C	
0.5	2.05×10^{-6}	4.02×10^{-6}	6.37×10^{-6}	4.15×10^{-6}
1	2.48×10^{-6}	5.43×10^{-6}	8.45×10^{-6}	5.45×10^{-6}
1.5	3.81×10^{-6}	6.17×10^{-6}	8.68×10^{-6}	6.22×10^{-6}

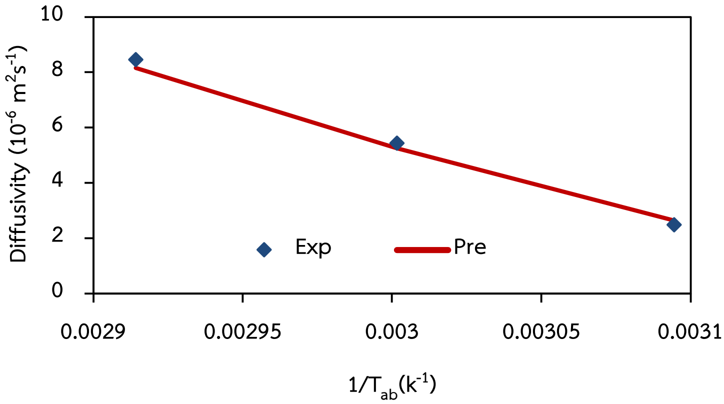

Figure 6 Moisture diffusivity of banana as a function of the reciprocal of temperature at an air velocity of 1 ms^{-1} (Exp: diffusivity measured from the experiments; Pre: diffusivity predicted from the Arrhenius-type equation).

Shrinkage of Kluai Leb Mu Nang (banana)

Figure 7 shows the comparison between the predicted and experimental data of banana shrinkage, which varies as a function of moisture removal. As part of this analysis, the following equation was developed to describe the volume shrinkage of the banana:

$$\frac{V}{V_0} = -0.00568(M_0 - M) + 0.9968\,,\ R^2 = 0.94. \tag{23}$$

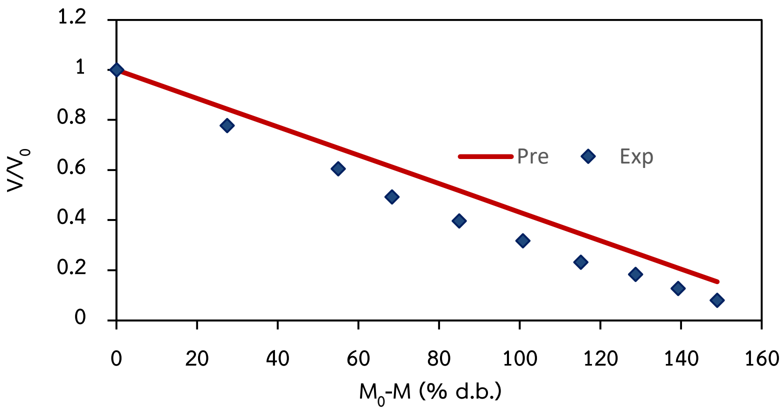

Figure 7 The predicted (Pre) and experimental (Exp) values of the shrinkage of bananas (V/V_0) during drying, as a function the change of moisture content, (M_0-M).

Finite element simulated drying

A computer program was developed using Compaq Visual FORTRAN version 6.6 to predict changes in moisture content during the drying process. The program utilized a finite element model, and data from bananas dried in a laboratory dryer were employed to validate the model. Figure 8 shows the comparison between predicted moisture contents generated by the finite element model and experimental data. The model incorporated experimentally determined values for banana diffusivity and shrinkage. Notably, the finite element model exhibited remarkable accuracy in predicting the moisture contents (RMSD = 0.925). This achievement can be attributed to the accurate experimental determination of banana diffusivity, as well as considering its shrinkage during the drying process. Additionally, the two-dimensional finite element model provides enhanced flexibility in accurately describing the banana's geometry.

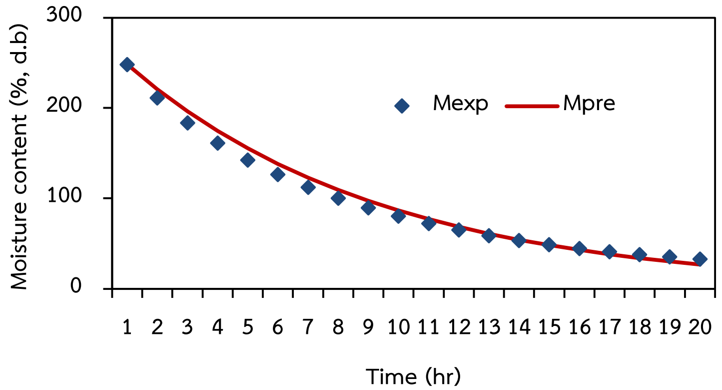

Figure 8 The predicted moisture contents of bananas using the finite element model (M_{pre}) were compared with the experimental moisture contents (M_{exp}) of bananas at T = 60°C and air velocity of 1 ms^{-1}).

Discussion

The finite element model accurately predicts the moisture content of bananas. This accuracy is attributed to the consideration of banana diffusivities. Additionally, another reason for its success could be the 2-D nature of the finite element model, providing greater flexibility in describing the geometry of the components. Some researchers (Nilnont et al., 2012) have demonstrated that moisture diffusivity also follows the Arrhenius-type dependency on temperature, with a pre-exponential factor that linearly depends on the initial moisture content. While using diffusivity as a function of temperature in the finite element simulation predicts the drying of banana well, incorporating diffusivity as a function of both temperature and initial moisture content would enhance the flexibility of the finite element model. This enhancement would allow for more accurate predictions of banana drying across different initial moisture content levels.

Conclusions

A two-dimensional finite element model for drying Kluai Leb Mu Nang (banana) was developed, and a computer program was created using Compaq Visual FORTRAN version 6.6 to simulate the model. The diffusivity and shrinkage of the bananas used in the drying simulation were determined experimentally. These properties were found to be functions of temperature and moisture removal, respectively. The mean values of the diffusivity of banana ranged from 4.15×10^{-6} to 6.22×10^{-6} m^2s^{-1}, and the shrinkage increased with higher moisture removal. In addition, a shrinkage model for bananas was developed as part of this work.

The finite element model successfully predicts moisture contents during drying. It's based on the finite element method, which approximates continuous quantities like moisture content using a discrete model composed of piecewise continuous functions over finite sub-domains or elements. This model provides valuable insights into moisture movement dynamics without the need for extensive measurements and design data, enhancing our understanding of transport processes within Kluai Leb Mu Nang (banana).

Acknowledgements

The author would like to thank Program of Physics, Department of Science, Faculty of Science and Technology. Thanks also go to Professor Dr. Serm Janjai, Department of Physics, Faculty of Science, Silapakorn University.

References

Avhad MR, Marchetti JM. Mathematical modelling of the drying kinetics of Hass avocado seeds. Industrial Crops and Products 2016;(91):76-87.

Castro AM, Mayorga EY, Moreno FL. Mathematical modelling of convective drying of fruits: A review. Journal of Food Engineering 2018;(223):152-167.

Chen C, Venkitasamy C, Zhang W, Khir R, Upadhyaya S, Pan Z. Effective moisture diffusivity and drying simulation of walnuts under hot air. International Journal of Heat and Mass Transfer 2020;(150):119283.

Da Silva WP, E Silva CMDPS, Gama FJA, Gomes JP. Mathematical models to describe thin-layer drying and to determine drying rate of whole bananas. Journal of the Saudi Society of Agricultural Sciences 2014;(13):67-74.

Defraeye T, Verboven P. Convective drying of fruit: Role and impact of moisture transport properties in modelling. Journal of Food Engineering 2017;(193):95-107.

Dobre T, Pârvulescu OC, Guzun AS, Stroescu M, Jipa I. Heat and mass transfer in fixed bed drying of non-deformable porous particles. International Journal of Heat and Mass Transfer 2016;(103):478-485.

Doymaz İ. Evaluation of Mathematical Models for Prediction of Thin-Layer Drying of Banana Slices. International Journal of Food Properties 2010;13(3):486-497.

Fadiji T, Coetzee CJ, Berry TM, Ambaw A, Opara UL. The efficacy of finite element analysis (FEA) as a design tool for food packaging: A review. Biosystems Engineering 2018;(174):20-40.

Hou L, Zhou X, Wang S. Numerical analysis of heat and mass transfer in kiwifruit slices during combined radio frequency and vacuum drying. International Journal of Heat and Mass Transfer 2020;(154):119704.

Khawas P, Das AJ, Dash KK, Deka SC. Thin-layer drying characteristics of Kachkal banana peel (Musa ABB) of Assam, India. International Food Research Journal 2014;21(3):1011-1018.

Korese JK, Achaglinkame Matthew A, Chikpah SK. Effect of hot air temperature on drying kinetics of palmyra (Borassus aethiopum Mart.) seed-sprout fleshy scale slices and quality attributes of its flour. Journal of Agriculture and Food Research 2021;(6):100249.

Llave Y, Takemori K, Fukuoka M, Takemori T, Tomita H, Sakai N. Mathematical modeling of shrinkage deformation in eggplant undergoing simultaneous heat and mass transfer during convection oven roasting. Journal of Food Engineering 2016;(178):124-136.

Malekjani N, Jafari SM. Simulation of food drying processes by Computational Fluid Dynamics (CFD); recent advances and approaches. Trends in Food Science & Technology 2018;(78):206-223.

Nilnont W, Thepa S, Janjai S, Kasayapanand N, Thamrongmas C, Bala BK. Finite element simulation for coffee (Coffea arabica) drying. food and bioproducts processing 2012;(90):341-350.

Winiczenko R, Górnicki K, Kaleta A, Martynenko A, Mankowska MJ, Trajer J. Multi-objective optimization of convective drying of apple cubes. Computers and Electronics in Agriculture 2018;(145):341-348.

Seremet L, Botez E, Nistor OV, Andronoiu DG, Mocanu GD. Effect of different drying methods on moisture ratio and rehydration of pumpkin slices. Food Chemistry 2016;(195):104-109.

Shailesh GA, Ravi NM. Mathematical model for heat and mass transfer during convective drying of pumpkin. Food and Bioproducts Processing 2017;(101):68-73.