

**THE PARENTAL TIME ALLOCATION
FOR THE THAI ECONOMY**


Col. Pittaya Gomolman

**A Dissertation Submitted in Partial
Fulfillment of the Requirements for the Degree of
Doctor of Philosophy (Economics)
School of Development Economics
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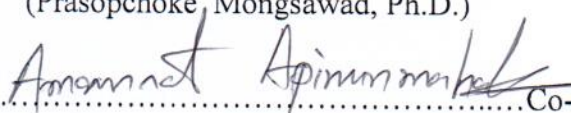
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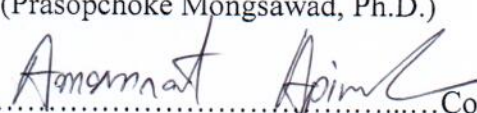
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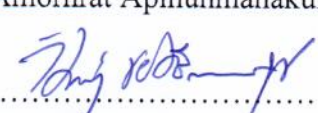
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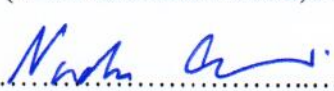
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ABSTRACT

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In the altruism hypothesis, parents care about the well-being of their children. They invest both their material resources and time into raising their children because they gain utility from such behaviors. Many economic literatures have focused on the material part of investment in children, such as the education expenditure and bequests. However, time investment is thought to be equally important to the development of children's skills and abilities. The parental time allocation for child care can affect basic cognitive and behavioral skills of a child, which in turn will have an impact, together with formal schooling, on his human capital and on his productivity. Since parent's time devoted to child care is taken away from leisure and/or paid employment, parents consider the benefits and costs of time spent on activities and make decisions to allocate their time between the activities in the best possible way that gives them maximum utility. These decisions are based on private returns and costs. From the macroeconomic perspective, a social planner would choose the amounts of parental time for child care so that it results in an optimal solution to achieve a social objective. This social optimal parental time allocation for child care may not coincide with the parents' decisions in a competitive economy. This draws our motivation to find out what the social parental allocation for child care would be in order to achieve social objectives, in particular, maximizing the total output and minimizing inequality. Our first objective in this study is to construct an

economic model with intergenerational transfer within the family that includes bequests, education and parental time allocation for child care, to mimic the earning and income inequality of Thailand. The second objective is to determine the parental time allocation for the Thai economy regarding different social goals such as minimizing earning inequality, minimizing wealth inequality and maximizing total output.

We constructed a five period overlapping generation model which differs from a general heterogeneous overlapping generation model. While recent studies focus on both unplanned and planned bequests (money and education) as intergenerational links, we add parental time allocation variables into the economic model. Parental time allocation variable is another channel, besides education, that help children develop their human capital which in turn increases efficient wage earning, hence better wealth in the future. We use the calibrated parameters from previous studies and compare the results with some statistics, in particular, education distribution and the earning and income distribution, generated by a previous study from the 2000 Household Socio-Economic Survey.

In general, the model generates the education distribution and the earning and income distribution reasonably close to that of the real data. Disabling the parental time allocation link in the model, while keeping all parameters with same values, we demonstrate that the model without parental time allocation is not as good as the benchmark model in terms of mimicking the education distribution and earning distribution. We conclude that the parental time allocation link is important and that the benchmark model can be used as a reference economy in any future analysis.

In order to find the solutions of parental time allocation for Thai economy, the benchmark model is modified slightly by taking the parental time allocation variable as exogenous. We alter different values of parental time allocation in the model to generate the results for three different scenarios, each of which corresponds to a social goal; minimize the earning GINI coefficients, minimize the wealth GINI coefficients and maximize the total output. For each scenario we explore further in order to see how the model generates the results in different environments by using different values of elasticity of earning with respect to parents' parental time allocation. To minimize wealth inequality the parental time allocation should be set at 12-15%. To

maximize the total output, the average parental time allocation should be set at 12-21% of available time. While increasing the parental time allocation will be beneficial in improving output and wealth equality, it comes at the cost of a wider earning gap.

Recommendations to improve the current situation regarding parental time are to improve the quantity and quality of parental time. To improve on the quantity of parental time, a government can introduce a policy to create an incentive for parents to give up working and leisure time in order to take care of their children. As far as the quality of parental time is concerned, a government can help improve the quality of parental time by creating a better environment for children or enhancing the parents' awareness on the benefits of increased parental time.

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CHAPTER 1

INTRODUCTION

1.1 Description of the Problem

In the altruism hypothesis, parents care about the well-being of their children. They invest both their material resources and time in raising their children because they gain utility from such behaviors. Many economic literatures have focused on the material part of investment in children, such as education expenditure and bequests. However, time investment is thought to be equally important to the development of children's skills and abilities. Casarico and Sommacal (2007) point out that time spent with children plays a large role in determining human capital achievements. In their concept, the intergenerational time transfer by a parent can affect basic cognitive and behavioral skills of a child, which in turn will have an impact, together with formal schooling, on his human capital and on his productivity. If parental time investment on children is one of the key factors in enhancing children's human capital, later in life and if, as it is recognized in the growth literature, human capital is a key engine for growth, the parental time allocation between paid work, leisure and childcare might have crucial implications for economic growth and development.

Similar to other resources, the amount of time invested in children is determined by the parents' budget constraints and their preferences. The parent's time devoted to childcare is taken away from leisure and/or paid employment, presenting an opportunity cost. Parents consider the benefits and costs of time spent on activities and make decisions accordingly and in such a way that maximizes utility. However, like other resources, their decisions on time allocation are based on private returns and costs. From the macroeconomic perspective, these parental time allocation decisions in aggregate affect an economy directly through the labor supply. Spending more time with children may enhance the children's human capital, thus increasing their

productivity and efficient wage earning in the future. However, it comes with a cost, for working parents must trade their working time with the time spent on children. A social planner would choose an amount of parental time for childcare so that it achieves a social objective. This social parental time may not coincide with the parents' decisions in a competitive economy. This draws our motivation to find out what the social parental time would be in order to achieve social objectives, in particular, to maximize the total output and to minimize the inequality.

Based on the total output and inequality of an economy, the study is focused on the general equilibrium overlapping the generation model with heterogeneous agents. Within this class of economic models, there are different saving motives which can be employed as a main assumption to explain an economy. Reviewing those motives is elaborated in chapter 2.

To the best of my knowledge, there are no papers exploring intergenerational transfer of time in the heterogeneity model. Therefore, the first part of this study introduces an altruistic parental time economic model where parents can allocate their time over leisure, working and childcare, expecting that giving more time on childcare will result in their children earning higher wages in the future. The same as giving proper education, this is a human capital development extension. The proposed model is calibrated to the Thai economy. Then the results of the model will be compared to the features of the Thai economy from a previous study. Having already established the parental time channel in the model, the second part of the study deals with the parental time as an exogenous variable in the model. The model has been tested through different values of parental time to find solutions for the Thai economy regarding different social goals such as minimizing earning and wealth inequality and maximizing total output.

This dissertation will contribute to the existing intergenerational transfer literature of Thailand. An OLG model with altruistic transfer motive via bequest, education and parental time is constructed to replicate the main features of the Thai economy, in particular, education distribution, earning distribution and income distribution. The model being studied will lead to solutions for desired parental time in the Thai economy. Hence, government interventions can be better planned to achieve social goals.

1.2 Research Objectives

The objectives of this study are twofold. The first objective is to construct an economic model with an intergenerational transfer within the family that includes bequest, education and parental time, to mimic the earning and income inequality of Thailand. The second objective is to determine the desired parental time for the Thai economy regarding different social goals such as minimizing earnings and wealth inequality and maximizing total output.

1.3 Scope of the Study

The areas of study are as follows:

1.3.1 The theoretical study by constructing an overlapping generation model with altruistic intergenerational transfer within the family including bequests, education and parental time.

1.3.2 The calibration and simulation to determine parental time for the Thai economy regarding different social goals such as minimizing earnings and wealth inequality and maximizing total output.

1.4 Research Questions

The study will seek to answer the following specific questions:

1.4.1 How well can an economic model with intergenerational transfer within the family including bequests, education and parental time, explain the earning and income inequality of Thailand?

1.4.2 What should parental time be for the Thai economy regarding different social goals, such as minimizing earning and wealth inequality and maximizing total output?

CHAPTER 2

LITERATURE REVIEW

Most theoretical studies of inequality and other macroeconomic variables are based on general equilibrium quantitative models with heterogeneous agents. The literature review employs general equilibrium quantitative models to mimic an economy and to explain inequality. The models are categorized as life-cycle and intergenerational transfer (involuntary transfer or accidental bequest and altruism) by motives of saving and transfers.

2.1 Life-Cycle

In a life-cycle related model, it is commonly assumed that an individual cares about himself. He only makes a saving to smooth his consumption over his life time. In this motive there is no altruistic monetary transfer linkage between individuals from different generations. Among academics studying inequality with a life-cycle hypothesis, Aiyagari (1994) modified a life-cycle model to involve uninsured idiosyncratic shocks. While the author deviates from a standard representative agent model by including a large number of heterogeneous agents, the aggregate variables are unchanged. With a combination of incomplete market and borrowing constraint, agents save in order to smooth their consumption in the face of uncertain individual labor incomes. The author compares income and wealth distribution generated from his model with those of the U.S. data. The model cannot generate the observed degrees of inequality. In particular, the Gini coefficients for net income and wealth are 0.12 and 0.32, respectively, compared with 0.4 and 0.8 in the U.S. data. Keeping the same features of heterogeneity, idiosyncratic shocks of income, incomplete market and borrowing constraints in the model, Huggett (1996) adds new features such as longevity uncertainty, income taxation and social security to improve the

match of U.S. wealth distribution. The main findings of his study are as follows. First, model economies with these features are able to replicate measure of both the aggregate wealth and the transfer wealth in the U.S. economy. Second, the model economies produce a number of features of the distribution of wealth in the U.S. In particular, the model can match the U.S. wealth Gini coefficient and the fraction of wealth held by the top 20 percent of U.S. households. However, there is still a gap in result findings from theory and observation. The model economies examined do not generate all the concentration of wealth in the upper tail of the distribution. In particular, the model economies generate only about half of the wealth held by the top 1 percent in the U.S.

In the same line of studies, Hendricks (2004) shows that life-cycle models have difficulties accounting for empirical relationship between lifetime earnings and household wealth. Quantitative life-cycle implies a far tighter relationship between earnings and wealth than is observed in U.S. data. As a result, life-cycle models understate wealth inequality among households with similar lifetime earnings. Incorporating several features thought to be important for understanding wealth inequality, in particular bequests and entrepreneurship, does not improve the model's ability to account for the data. These findings suggest that a standard life-cycle theory fails to account for an important source of wealth inequality.

The failure of the basic model to explain wealth inequality suggests that one needs to look at other mechanisms. Krusell and Smith (1998) argue that the macroeconomic model with heterogeneity features approximate aggregation. This approximation means, in equilibrium, all aggregate variables such as consumption, capital stocks, and relative prices can be almost perfectly described as a function of two simple statistics: the mean of the wealth distribution and the aggregate productivity shock. Therefore, consumers in the equilibrium face manageable prediction problems since the distribution of aggregate wealth is almost completely irrelevant for how the aggregates behave in the equilibrium. When the model is altered only by adding idiosyncratic, uninsurable risk, the resulting wealth distribution does not match the real data. The authors suggest that adding heterogeneity in preferences into the model may improve the result. Their main finding is that, their preference heterogeneity model can replicate some of the key features of the observed

data on the distribution of wealth. The wealth Gini coefficient is also quite close to that in the data.

The distributions of earning and wealth in the U.S. and in selected model economies are summarized in Table 2.1.

Table 2.1 Distributions of Earning and Wealth in the U.S. and in Selected Model Economies

	Gini	Bottom 40 percent	Top 5 percent	Top 1 percent
U.S. Economy				
Earnings	.63	3.2	31.2	14.8
Wealth	.78	1.7	54.0	29.6
Aiyagari (1994)				
Earnings	.10	32.5	7.5	6.8
Wealth	.38	14.9	13.1	3.2
Huggett (1996)				
Earnings	.42	9.8	22.6	13.6
Wealth	.74	.0	33.8	11.1
Krusell and Smith (1998)				
Earnings	-	-	-	-
Wealth	.82	-	55.0	24.0
Hendricks (2004)				
Earnings	-	-	-	-
Wealth	.47	0.7	39.4	14.0

The review of life-cycle literatures shows that most life-cycle model economies fail to generate enough savings to account for wealth inequality. This may be because households have neither the incentives nor the time to accumulate sufficiently large amounts of wealth. They neglected the parent-child link which may be the main incentive for wealth accumulation that accounted for the wealth inequality of the real data.

2.2 Intergenerational Transfer

The intergenerational transfer studies in economics involve the studies of causes, motives and behaviors of individuals regarding private transfers within a family, in particular by focusing on investigating the transfer motives and changes in transfer behaviors when the economy is affected by a government intervention program. These Intergenerational transfer motives can be sub-categorized into Involuntary transfers (accidental bequest) and Altruism.

2.2.1 Involuntary Transfers

The involuntary transfer hypothesis has been proposed as one of the causes of intergenerational transfers when a slow decumulation in retirement age which does not accord with life-cycle hypothesis was noticed in the U.S. Davies (1981) finds evidence that the slow dissave is not caused by intentional bequest motive but to a large extent by lifetime uncertainty. With imperfect private insurance markets, retired parents who have to cope with lifetime uncertainty may leave considerable accidental or precautionary bequests. These precautionary bequests represent the deferred consumption, had they lived longer. Presumably less well-off individuals are more likely to save more during retirement age and do not either want to trade with their children or make inter vivos transfers. The amount of such bequests should be proportional to life resources (with homothetic preferences), and decrease with age, pension coverage or private annuities, but be independent of the existence and income of children.

Among the studies on involuntary bequest's effects on the U.S. wealth inequality, Gokhale et al. (2000) constructs an overlapping generation model with uncertain lifespan. Each agent lives for at most 88 years with the first 22 years as a child, the second 22 years as a young adult who marries and has children, the third 22 years as a married, middle aged adult with no additional children, and the last 22 years as a married or widowed older adult facing lifespan uncertainty. Agents who die prior to reaching age 88 bequeath their wealth to their spouses. If their spouses are no longer living, they bequeath equal amounts to their children. Agents have life-cycle preferences with no intentional bequest motive. They leave bequests only because

their resources are not fully annuitized. In their model, inequality in inheritances can be influenced by many factors such as earnings inequality, transmission of earnings inequality across generations, number and spacing of children, heterogeneous rates of return, time preference, annuitization of retirement savings through social security, and progressivity of the income tax system. The authors calibrate the model to what appears to be the most realistic set of parameter values. The model generates a distribution of wealth that closely approximates to the degrees of inequality and skewedness in the actual U.S. data. In particular, the richest 1, 5 and 10 percent of their model hold 32.8, 49.4 and 58.8 percent of total wealth respectively, which is quite close to the corresponding 30.4, 51.0 and 62.5 percent figure in the Survey of Consumer Finances.

2.2.2 Altruism

The altruism hypothesis emerged as a clear cut alternative to self-interest, normally assumed to prevail in the market. A large number of literatures about altruism have been influenced by the work of Becker (1974, 1991). In Becker's model of altruism, parents care about the well-being of their children, using bequests and other gifts to obtain the desired distribution of resources within the family. When the pure altruism is operative the current generations are connected to future generations by a chain of intergenerational transfers. The model leads to Ricardian equivalence. Any marginal changes in public policy will result in full neutralization. In particular, a rise in social security benefits should lead to an equivalent increase in altruistic parent-to-child transfers (Barro, 1974). Many subsequent studies incorporate altruistic behavior in the models. For example, Kotlikoff and Summers (1981) use historical U.S. data to directly estimate the contribution of intergenerational transfers to aggregate capital accumulation, and Becker and Tomes (1979) study intergenerational mobility, human capital investment and inequality. Among the earlier partial equilibrium, quantitative studies, Davies (1982) analyzes the effects of various factors, including bequests, on economic inequality in a one-period model without uncertainty. In his set-up, one generation of parents cares about their children's future consumption, and there is a regression to the mean between parents' and children's

earnings. As a consequence, the income elasticity of bequests is high and inherited wealth is a major cause of wealth inequality.

Subsequent studies on income and wealth inequality include altruistic motive. For example, Laitner (2001) mixes life-cycle and dynastic behavior. In his model, all agents save for life-cycle purposes, but only some of them (a fraction λ of population) care about their own descendants. Such agents may choose to accumulate estates for bequests. Non altruistic families care solely about their own lives. In the model, there are perfect annuity markets. There is no earnings risk over the life-cycle. Hence, there are no precautionary savings. The author shows that the concentration in the upper tail of the wealth distribution generated from the model can match the real data by choosing the fraction of households that behaves as a dynasty.

Similar to Laitner (2001), Castaneda et al. (2003) also mix the main features of the life-cycle and of the dynastic hypothesis. However, without introducing a fraction of each type in an economy, they assume that all households in the model economies are altruistic and that they go through the life-cycle stages of working age and retirement. These households, with identical and standard preferences, receive an idiosyncratic random endowment of efficiency labor units. They do not have access to insurance markets and save in part to smooth their consumption. The households also save to supplement their retirement pensions and to endow their estates. The authors model explicitly some of the quantitative properties of the U.S. Social Security system. This feature gives the earnings-poor households few incentives to save. The calibration is done corresponding to the Lorenz curves of U.S. earnings and wealth as reported by the 1992 SCF. The authors also replicate the progressivity of the U.S. income and estate tax systems to account for the distortion of the labor and savings decisions, discouraging earnings-rich households both from working long hours and from accumulating large quantities of wealth. Furthermore the authors find that their model economy does a very good job of accounting for the U.S. distribution of earnings and wealth. In particular, they report the Gini index for their distribution of earning and wealth as 0.63 and 0.79 respectively, compared to those of 0.63 and 0.78 in the U.S. data. The share of earnings and wealth of the top 1 percent of households are 14.93 and 29.85 percent respectively in the model which almost exactly match the 14.76 and 29.55 percent in the U.S. data.

Nishiyama (2002) extends a standard heterogeneous agent overlapping generation model by adding two-way intergenerational altruism, lifetime uncertainty, a fertility shock, and borrowing constraints. In the model, households in the same dynasty play a Nash game in each period to determine their optimal consumption, working hours, inter vivos transfers, and savings. The model suggests that when deciding the level of bequests, a parent household considers the future utility of its child households, on average, about 20 percent less than the amount it considers for its own future utility. But, the parent household's motive for inter vivos transfers is much weaker than its motive for altruistic bequests. Although the model replicates the wealth distribution of the United States fairly well in term of the Gini coefficient of wealth distribution, which turns out to be 0.701 compared with 0.78 in the U.S. data, the share of wealth of the top 1 percent of households is 14.6 percent in the model which is lower than the 29.6 percent in the data. He concludes that the effects of bequests and inter vivos transfers on wealth distribution in the model are not very large.

2.2.3 Mixtures of Involuntary Transfer and Altruism

There are a large number of literatures that mix some features of involuntary transfer and altruistic transfer in a model. Among them, Heer (1999) develops an overlapping generation model in which heterogeneous agents face uncertain lifetime and leave both accidental and voluntary bequests to their children. Furthermore, agents face stochastic employment opportunities. The model is calibrated with regard to the characteristics of the US economy. The results indicate that bequests only account for a small proportion of observed wealth heterogeneity. The Gini coefficient only amounts to .45 compared with 0.79 of the U.S. data. The richest 5 percent hold about 20 percent of wealth compared with 50 percent of total wealth in the U.S. data. The author suggests that neglecting productivity heterogeneity within generations in the model may be one of the reasons why the endogenous wealth heterogeneity of his model is smaller than the one observed empirically. He considers that adding a channel for transfer human wealth may also improve the result.

Ocampo and Yuki (2006) investigate the quantitative importance of different savings motives on the distributions of wealth and consumption and aggregate capital

accumulation. In their heterogeneous overlapping generation model, agents differ in age, ability, earnings shocks, and inherited bequests. They also assume that there are uninsurable idiosyncratic risks associated with uncertain lifetime and earnings shocks. The model's parameter values are calibrated to match those observed in the U.S. data. The authors find that, in the baseline model, the top 1 percent and 5 percent hold wealth 24.17 percent and 49.78 percent respectively, compared with 29.55 percent and 53.5 percent in the U.S. data. To investigate the importance of each effect on inequality, the authors compare the baseline model with those of a model economy with complete annuity markets, an economy without earnings uncertainty and altruism. The numerical experiments have shown that different savings motives seem to differently affect savings behaviors of the heterogeneous population. The effect of completing annuity markets is dominant in the elderly population and results in a large increase in wealth and bequests inequalities through higher concentration of assets in the upper tail of the distribution. The authors explain that because poor people try to annuitize most of their wealth if such annuity securities are available. When the author takes out earnings uncertainty, savings by the young population, especially those in low income groups, decrease. The wealth inequality for the whole population improves because of the equalized lifetime earnings. Finally, the disappearance of altruism mainly affects savings behaviors of the old and rich population, and significantly reduces wealth and bequests inequalities by lowering the concentration of wealth in the upper tail of the distribution. In this model, altruism seems to be most important in explaining the distribution of wealth.

As suggested by Heer (1999) transferring human capital may be as important as transferring the physical capital in explaining inequality, De Nardi (2004) focuses on the transmission of both physical and human capital from parents to children. He shows that such intergenerational links can induce saving behavior that generates a distribution of wealth that is much more concentrated than that of labor earnings and that also makes the rich keep large amounts of assets in old age in order to leave bequests to descendants. The author adopts a computable, general equilibrium, incomplete-markets, life-cycle model in which parents and their children are linked by bequests, both voluntary and accidental, and by the transmission of earnings ability. The households save to self-insure against labor earnings shocks and life-span risk,

for retirement, and possibly to leave bequests to their children. The author finds that voluntary bequests can explain the emergence of large estates, and characterize the upper tail of the wealth distribution in the data. Accidental bequests alone, even if unequally distributed, do not generate more wealth concentration. The presence of a bequest motive also makes lifetime saving profiles more consistent with the data. The author explains that saving for precautionary and retirement purposes are the primary factors for wealth accumulation at the lower tail of the distribution, while saving to leave bequests significantly affects the shape of the upper tail. The bequest motive to save is thus stronger for the richest households who keep some assets for their children. The rich leave more wealth to their offspring, who, in turn, tend to do the same. This behavior generates some large estates that are transmitted across generations because of the voluntary bequests. A human capital link, through which children partially inherit the productivity of their parents, generates an even more concentrated wealth distribution. More productive parents accumulate larger estates and leave larger bequests to their children, who, in turn, are more productive than average in the workplace. Regarding the wealth inequality, the richest 1 percent and 5 percent in the model with bequest motive alone hold 14 percent and 37 percent respectively compared with 29 percent and 53 percent in the U.S. data. When the author includes bequest motive and productivity inheritance into the model, the figures increase to 18 percent and 42 percent.

With the similar assumptions made on the model in De Nardi (2004), Yang (2005) adds a borrowing constraint into his model. The households save to self-insure against labor earning shocks and life-span risk, for retirement, and possibly to leave bequests to their children. The members of successive generations are linked by bequests and by the children's inheritance of part of their parent's productivity. The households do not know the exact time and amount of inheritance, and neither are they allowed to borrow against its future. The existence of a borrowing constraint prevents households from smoothing consumption inter-temporally. Two households may have the same lifetime earnings, but one with positive earning shocks at a young age and a negative one at old age and vice versa. At retirement, these households will hold amounts of wealth that differ substantially. Inheritance adds another source of wealth heterogeneity among households with similar lifetime earnings. Some

earnings-poor households hold a large amount of wealth at retirement because they have inherited a large amount of assets. Some earnings-rich households receive no inheritance and thus own less wealth. The author also compares the benchmark economy with one without intergenerational links. This comparison indicates that heterogeneity of inheritance does not play a big role for the lower and middle income deciles, but does play an important role at generating wealth heterogeneity for the higher income deciles. The result confirms findings by De Nardi (2004), that a model without intergenerational links cannot generate a skewed wealth distribution comparable with the data. The Gini coefficient of wealth is only 0.64, compared with 0.72 in the benchmark economy and 0.78 in the data. The fraction of wealth held by the richest 1 percent is only 7 percent in the model, compared with 21 percent in the benchmark model and 29 percent in the data.

2.2.4 Educational Bequest

Early studies on income and wealth inequality seem to agree that intergeneration link, both physical wealth and human wealth, contributes to distribution and concentration in the upper tail of wealth distribution in the U.S. These works take earnings or wages as an exogenous random process and then proceeds to characterize the distributional implications of optimal consumption-savings and labor-leisure behavior. Huggett (2006) argues that this research agenda should integrate deeper foundations for the determinants of earnings and wages into these models by allowing earnings to be endogenous. He argues that when earnings are exogenous there is no channel for policy to affect consumption and welfare through earnings. This channel is arguably of first order importance. In fact, a dominant theme in the earnings distribution literature is that earnings profiles are determined by the optimal investment of time and resources into the accumulation of skills. If a government changes its policy, earning should be directly affected. In his model, each agent is endowed with some immutable learning ability and some initial human capital. Each period, an agent divides available time between market work and human capital production. Human capital production is increasing in learning ability, current human capital, and time allocated to human capital production. An agent maximizes the present value of earnings, where earnings in any period are the product of a rental

rate, human capital, and time allocated to market work. The assessment focuses on the dynamics of the cohort earnings distribution produced by the model from different initial joint distributions of human capital and learning ability across agents. The author establishes that the earnings distribution dynamics documented from the U.S. data can be replicated quite well by the model from the right initial distribution.

Suen (2014) generalizes the standard deterministic neoclassical growth model to include three important features, namely heterogeneity in time preference, human capital formation, and consumers direct preferences for wealth. The author states that the main purpose of introducing wealth preference in the model is to overcome difficulties in generating realistic wealth distribution found in the standard neoclassical growth model. The wealth distribution is degenerate and extremely unequal in the long run. The author shows that a non-degenerate wealth distribution can be obtained by assuming that consumers have direct preferences for wealth. In a model where there is no direct wealth preference, a consumer will choose to hold a constant positive level of financial wealth only when the equilibrium interest rate is identical to his discount rate. Once there is only one interest rate in the neoclassical model, it is not possible for consumers with different discount rates to maintain constant positive levels of wealth simultaneously. In the long-run equilibrium, the interest rate is equated to the lowest discount rate in the population. Thus, only the most patient consumers would have positive asset holdings. All other consumers with a discount rate greater than the equilibrium interest rate will deplete their wealth until it reaches zero. Introducing direct preferences for wealth changes this result by creating some additional benefits for holding financial assets. These additional benefits fundamentally change the consumers' saving behavior. In particular, consumers are now willing to maintain constant positive levels of wealth even if the interest rate is lower than their discount rates. These additional benefits not only prevent the consumers from depleting their wealth to zero, but also induce different types of consumers to hold different levels of wealth. Thus, the equilibrium wealth distribution is non-degenerate. Introducing human capital formation provides a channel via which time preference heterogeneity can lead to significant variations in earnings across consumers. It also helps create a strong positive correlation between earnings and capital income. Consumers with patience tend to have higher earnings

and financial wealth. This in turn generates a substantial degree of income inequality in the model. A calibrated version of the model with all three features is able to replicate the observed patterns of wealth and income inequality in the United States. In particular, the benchmark model result in wealth Gini coefficient of 0.816 compared with 0.816 in the U.S. data. For the income and wealth inequality, the richest 1 percent in the model holds 10.5 percent and 25.9 percent respectively, compared with 21.0 percent and 33.6 percent in the U.S. data. The richest 5 percent in the model hold 28.3 percent and 58.5 percent respectively compared with 36.9 percent and 60.3 percent in the U.S. data. The author illustrates the importance of including human capital in the model by showing the results from his model without human capital where the richest 1 percent in the model hold 18.6 percent and 54.4 percent respectively. The richest 5 percent in the model hold 26.6 percent and 70.5 percent.

Wisit Chaisrisawatsuk (2014) considers an alternative hypothesis that may account for inequality in income and wealth distribution via another persistence channel. The author argues that parents' concern for their children may be revealed during the parents' lifetime in the form of paying for a given education level, rather than after death as occurs in planned bequests. By paying for a child's education, parents can influence the likely level of earnings of a child. The author allows households to be subject to idiosyncratic risk. The wage income depends on an individual's age, education and idiosyncratic shock. The level of education has impact on both the level of earning and the probability of the level of earning. To establish the importance of this channel, the author compares equilibrium distribution with and without educational bequests. The model was set up to include three important factors: the planned money bequest, education as planned bequest, and the differences in idiosyncratic earning shocks by the level of education. The author compares the results generated by the model to the 2000 Household Socio-Economic Survey (HSES) produced by the National Statistical Office of Thailand. He finds that the model generates an education distribution closely similar to what is observed in the data. The model predicts that 71.04 percent of population will have an education lower than high school. The fraction of the population that chooses a high school education is 16.82 percent while only 12.14 percent of the population has a university education. In 2000, the Thai data indicated that 72.95 percent of the population does

not receive high school education while 16.38 percent does. However, 10.66 percent of the population receives a college degree. The Gini indexes generated by the model for earning and income are 0.6 and 0.51, compared with the actual data of 0.59 and 0.54 respectively. As for the top 1 percent to lowest 17.8 percent ratio, the model generates 18.28 for earning and 4.45 for income while the actual data are 19.66 and 5.32 for earning and income respectively. The author generates the percentage of shared data for the various quintiles and compares them with the actual data to see whether the model matches the shape of the earning and income distribution. Overall, the distribution of the model matches well with the data. He concludes that intentional bequest, education as planned bequest, and the differences in idiosyncratic earning shocks are the important factors in explaining earning and income inequality in Thailand.

2.2.5 Parental Time as Another Altruistic Bequest

According to Becker (1991), parents care about the well being of their children and influence their human capital and future earnings by devoting part of their time and wealth to their children. Much of the economic literature has focused on the material part of investment in children, such as education expenditure. However, time that parents spend with their children is equally important. Despite the importance of time transfers, this type of transfer has not received much attention in the theoretical literature. Generally, the amount of physical and time resources invested in children is determined by parents' budget constraints and their preferences, as perceived in the studied models.

Among a few studies on this agenda, Cardia and Ng (2003) use a two-period overlapping generation model with one-sided altruistic agents to analyze the role and importance of time transfers. In their model, agents raise children and work in period one. In period two, after retirement they make monetary transfers to their children, and/or help them raise their babies. Agents consume a market good and a home produced good. The home good consumed by the young is interpreted as childcare, and the home good consumed by the old is interpreted as old age care. Time and market goods are used in home production. The parents and grandparents both contribute their time to childcare. In their model, such parenting has two effects:

relieving the time constraint of the working generation by allowing them to devote more time to market work and relaxing the budget constraint by reducing the demand for purchased child inputs, such as day care and nannies. The authors calibrate the steady state of the model to match some basic stylized facts of the US economy. They compare the base case with economies in which time or money transfers are not operational. The finding is that although both time and money transfers positively affect capital accumulation, they differently affect the work effort. Monetary transfers directly translate into higher income which increases savings and capital accumulation. But as higher income discourages labor supply, this effect will partially offset and can even outweigh the inter-temporal substitution effect brought about by capital accumulation. In contrast, time transfers increase labor supply unambiguously since the only way the young can translate the time transfers into higher purchasing power is to increase work effort. The model's focus on time transfers and childcare makes this framework appropriate to study the macroeconomic effects of childcare policies. To this end, the study covers the steady state effects of three childcare policies. The authors find that subsidizing the time that the old spent parenting or on childcare expenses can raise the level of childcare without adverse general equilibrium effects on output and capital. In contrast, subsidizing the working young for them to spend more time on childcare will reduce the labor supply and thus the productive capacity of the economy.

Cardia and Michel (2004) use a standard two-period overlapping generation model to examine the behavior of an economy, where both intergenerational transfers of time and bequests are available. They assume that labor supply decisions are endogenous as the young can choose to work at home on a home-produced good or on the market place to produce a market good. The authors show that although both bequests and time transfers have both positive effects on capital accumulation, they act through different channels. Bequests increase savings and capital accumulation. This capital accumulation does not require the young to work more in the production of the market good. Time transfers, instead, increase capital accumulation by relaxing the young's time constraints and thus enabling them to work more. They also show that time transfers may take place when intergenerational altruism is insufficient to generate bequests. The critical level for operative time transfers depends on different

variables, rather than the ones needed for operative bequests. In particular, operative bequests depend on the capital intensity of the economy, while time transfers do not. The lower the capital intensity of the economy the higher is the critical value for positive bequests while the critical value for positive time transfers is not affected. This has an interesting implication with intuitive appeal. Although the degree of altruism may not be sufficient to generate bequests, there may still be important altruistic intergenerational transfers in the form of time transfers for less developed economies.

Casarico and Sommacal (2007) analyse the impact on growth of alternative childcare policies in a three period OLG model where human capital is formed both via childcare and formal schooling and where agents decide how many childcare services to purchase in the market, how much time to work and how much time to devote to childcare and leisure, both in working adulthood and old age. While the government can directly subsidize the purchase of childcare services by assuming that it cannot directly observe time devoted to child rearing, the only way to subsidize it is by implicitly or explicitly taxing the labor supply, which is perfectly observable. Through taxes, the government discourages labor supply but may provide incentives to increase childcare and/or leisure. It is clear that taxing labor supply is an imperfect instrument to achieve the government's goals. However, given the informational constraints, it affects individual's allocation of time. Explicit taxation means a proportional tax on wage earnings both during adult and old age, while implicit taxation refers to a benefit reduced for any additional unit of labor supplied. The authors show that the impact of different policies on growth depends on the parameter values. A reduction in tax rate or benefits of non-work for the adults increases the growth rate. On the contrary, such a reduction reduces the growth rate. This is caused by a fall in the childcare received which is not compensated by any increase in formal schooling.

Casarico and Sommacal (2012) develop a three-period OLG growth model where formal schooling and childcare enter the production function of human capital as complements. Childcare depends on the time that parents dedicate to child rearing and on the expenditure on goods and services which may impinge on the child's development (e.g books, toys, day-care center' services, pre-school programs, baby-

sitting). They compare two models to see how childcare affects child development. In the model where childcare has no effects on child development, labor income taxation affects human capital accumulation only through the decision to invest in formal schooling. In the second model, it also influences both directly and indirectly the growth rate through the change in the time where parents devote to childcare and the variation in the amount of childcare expenditure. The direct effect on human capital comes with the change in childcare, for a given level of formal schooling. The indirect effect passes through the complementarity between formal schooling and childcare in the process of skills' formation. The authors perform a numerical analysis of the model. When taxes are reduced, the net wage goes up, inducing people to work more and dedicate less time on childcare. This reduction in parental care may be compensated for by an increase in the amount of childcare expenditure. The authors find that the omission of childcare from the technology of skills' formation can quantitatively and significantly bias the results related to the effects of taxation on growth.

Hasimzade (2011) studied the effect of time allocation in a family on macro behavior of an economy. The author used an overlapping generations model to describe an economy where children's human capital is affected by parental childcare time and where parents' preference for spending time with children are determined endogenously, via transmission of preferences between generations, within and across the families. The model exhibits multiple steady-state equilibria. A positive externality in childcare time results in inefficiency of all competitive equilibria. Too little time is spent with children. As a result, a competitive economy underperforms, compared with the first best outcome, where parents' preference for childcare time is stronger, and the levels of output and human capital are higher than the one in the private equilibria.

Most literature agrees that the inclusion of altruistic human capital development in an economic model can improve the results in explaining inequality. Recent works focus on the models with education as a means to develop human capital when attempting to explain inequality. However, to our knowledge, there are no papers exploring intergenerational transfer of time in the heterogeneity model to explain inequality. Therefore we extend the existing economic models by adding

parental time as another intergenerational link to capture the parents' behaviors and impact of parental time investment on childcare.

CHAPTER 3

THE MODEL

3.1 General Description of the Overlapping Generation Model

This chapter explains the overlapping generation model that will be calibrated to the Thai economy. Individuals in this economy live for five periods with certainty.

3.1.1 Time Periods

3.1.1.1 The first period: Children

In the first period, individuals live with their parents. They do not make any economic decisions. Parents decide how much their children can consume, how much time they spend rearing their children and what level of education they should have, through altruistic motives.

3.1.1.2 The second period: Young adults

In the second period, individuals leave their parents and live on their own. They receive bequests left for them altruistically by their parents at the beginning of this period. They are endowed with a unit of time in which they decide how much time they spend on work and leisure. They also make decisions on their consumption and saving.

3.1.1.3 The third period: Parents

In the third period, parents raise a random number of children depending how many they choose to have. Parents are endowed with a unit of time in which they decide how much time they spend on working, leisure and rearing children. They also make the decisions about their own consumption, saving, their children's consumption, education level and bequests.

3.1.1.4 The fourth period: Old adults

In the fourth period, Old adults live without children since their children have left the family. Old adults still work or spend time not working and enjoy their leisure. They make decisions about their own consumption and saving.

3.1.1.5 The fifth period: Retirees

The last period of life is the retirement period. Individuals live without working, consume all what they have saved earlier and die at the end of this period.

A graphical model of the overlapping generation is illustrated in Figure 1

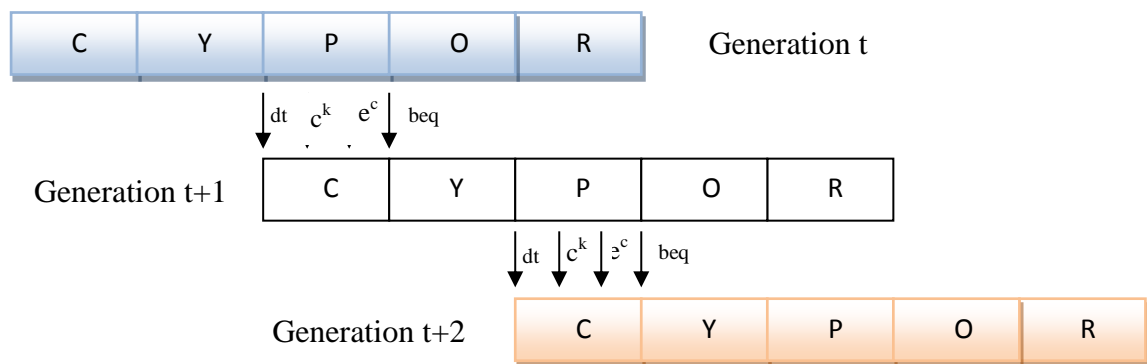


Figure 3.1 Graphical Model of the Overlapping Generation

Where	C	denotes	Children
	Y	denotes	Young adult
	P	denotes	Parent
	O	denotes	Old adult
	R	denotes	Retiree
	dt	is	parental time
	c^k	is	Consumption of children
	e^c	is	Education level of children
	beq	is	bequest

3.1.2 Random Events in the Model

There are two random events in the model that have significant effects on individuals. The first random event is an idiosyncratic shock which occurs at the beginning of each period. This shock differently affects individual's earning

depending on his own level of education and the amount of rearing time he gets from his parents in the first period. An agent with higher education will have a better chance of getting a better shock than the one with a lower education. The second random event is the number of children. At the beginning of parenthood, parents will randomly have a number of children, which may number from 1 to 5.

3.1.3 Intergenerational Transfer and Saving Motives

A part of savings the agents make is from the life-cycle motive, to enable smooth consumption over their life time. An assumption is that agents in this model are altruistic towards their children. The intergenerational transfer occurs only in the third period when the parent agents are altruistic toward their children. Parents' utility increases when their children's utility and expected utility in future periods increases. The children's utility arises directly from the consumption and expected future incomes which in turn depend on education level, amount of time spent with parents and also any bequests given to the children.

3.2 The Household Problem

An individual usually lives life for a specific five periods. In each period denoted by $j \in (1, 2, \dots, 5)$ an agent has to face different economic environments which depend on the asset or wealth position, a , the education level, e , the time that his parents take care of him, dt , the idiosyncratic earning shock, z , the number of children in the household, n_c . There are three levels of education, primary school denoted by $e = 1$, high school denoted by $e = 2$ and university denoted by $e = 3$. There are 5 levels of parental time that parents can choose in order to take care of their children. Given that parents are endowed with a unit of time, dt_1 to dt_5 denote fractions of time: 0.03, 0.06, 0.12, 0.18 and 0.24 respectively, spent with a child. The household utility function is the constant relative risk aversion (CRRA) type:

$$U(c, lt) = \frac{[c^\gamma lt^{1-\gamma}]^{1-\mu}}{1-\mu} \quad (1)$$

where γ is the share of consumption in the utility.

μ is the curvature parameter.

It is the leisure time.

In the first period of life an agent is a child, and j is equal to one. He lives with his parents who make the decisions about his consumption, time to take care of him and the level of his education. An agent in this period does not make any economic decisions. As a result, an agent's utility function does not include his own utility during the childhood period.

In the second period, an individual lives alone after having been reared (parental time) and educated by his parents in the first period. Therefore, he enters this period (and the next) with a parental time level, an education level and an asset position which equals the value of the bequest left by parents in the previous period. An individual works to earn labor income which depends on the wage rate, w , and the effective labor supply, n . The effective labor supply depends on the age of the individual, the parental time level, the education level, the idiosyncratic earnings shock which he receives at the beginning of the period, and the number of each labor supplied. The effective labor supply is denoted by $n(j, dt, e, z, wt)$. Apart from labor income, an individual earns other sources of income from the principle and the interest of the bequest. The interest rate is denoted by $1+r$. Given these states and units of time that he has, the young adult must make decisions on his working time, wt , consumption level, c and saving, a' . In this period, the utility of the young adult individual depends on his consumption and leisure level. The optimization problem for the individual is:

$$v(j=2, a=beq, dt, e, z, n_c=0) = \max_{\{c, wt, a'\}} \{u(c, (1 - wt)) + \beta E[v(j + 1, a', dt, e, z', n_c|z)]\} \quad (2)$$

subject to

$$c + a' \leq wn(j, dt, e, wt, z) + (1 + r)a \quad (3)$$

Where β is the discount factor.

E is the expectation operator.

The Lagrange function in the second period is

$$\frac{[c^\gamma(1-wt)^{1-\gamma}]^{1-\mu}}{1-\mu} + \beta E[v(j+1, a', dt, e, z', n_c | z)] - \lambda(c + a' - wwt - (1+r)a) \quad (4)$$

First order conditions are:

$$c: [c^\gamma(1-wt)^{1-\gamma}]^{-\mu} \gamma c^{\gamma-1} (1-wt)^{1-\gamma} = \lambda \quad (5)$$

$$wt: [c^\gamma(1-wt)^{1-\gamma}]^{-\mu} (1-\gamma) c^\gamma (1-wt)^{-\gamma} = w\lambda \quad (6)$$

where λ is the Lagrange multiplier.

The solutions derived from the optimal condition are:

$$c(j=2) = \gamma[w - a' + (1+r)beq] \quad (7)$$

$$wt(j=2) = 1 - \left[\frac{(1-\gamma)c}{\gamma w} \right] \quad (8)$$

$$v(j=2) = \frac{[c^\gamma(1-wt)^{1-\gamma}]^{1-\mu}}{1-\mu} + \beta E v(j=3) \quad (9)$$

In the third period as a parent, the individual still works to earn labor income in the same manner as in the previous period. The individual receives a stochastic amount of children, n_c which can take the value between one and N_c . All parents in this model are altruistic towards their children. Given this state and the unit of time that he has, the parent must make decisions on his working time, wt , parental time level for rearing one of his children, dt_c , his consumption level, c , saving, a' , as well as bequest, consumption and education level of his children. In this period, the individual's utility function depends on his own consumption and leisure as well as the consumption of his children. As far as being forward looking is concerned, the parent also derives utility from the utility he expects himself and his children to obtain in the next period. The optimization problem for the individual is:

$v(j=3, a, dt, e, z, n_c) =$

$$\begin{aligned} & \max_{\{c, wt, a', c^k, dt_c, e_c, beq\}} \{u(c, (1-wt-n_c dt_c)) + b(n_c) n_c u(c^k) \\ & \quad + \beta b(n_c) n_c E[v^c(j=2, a=beq, dt_c, e_c, z', n_c=0) | z] \\ & \quad + \beta E[v(j+1, a', dt, e, z', n_c=0 | z)]\} \end{aligned} \quad (10)$$

subject to

$$c + n_c c^k + a' + n_c TC(e_c) + n_c beq \leq wn(j, dt, e, wt, z) + (1+r)a \quad (11)$$

where

v^c is value function of each child.

c^k is the consumption level of a child

dt_c is the parental time level that a parent choose to rear a child

e_c is the education level of a child

$TC(e_c)$ is the cost of education

n_c is the number of children

$b(n_c)$ is a discount factor with respect to the number of the children. Based on Knowles' work (Knowles 1999) the altruistic discount factor parameter is set to be equal to $b(n_c) = n_c^{-\theta}$, where θ equals to 0.55.

$\xi = 1 - \theta$

beq is bequest for a child

The Lagrange function in the third period is

$$\begin{aligned} & \frac{[c^\gamma(1 - wt - n_c dt_c)^{1-\gamma}]^{1-\mu}}{1 - \mu} + n_c^{\xi} \cdot \frac{(c^k)^{1-\mu}}{1 - \mu} + \beta n_c^{\xi} \cdot E v^c + \beta E v(j = 4) \\ & = 4) - \lambda(c + n_c c^k + a' + n_c TC(e_c) + n_c beq - wwt - (1 + r)a) \end{aligned} \quad (12)$$

First order conditions are:

$$c: [c^\gamma(1 - wt - n_c dt_c)^{1-\gamma}]^{-\mu} \gamma c^{\gamma-1} (1 - wt - n_c dt_c)^{1-\gamma} = \lambda \quad (13)$$

$$wt: [c^\gamma(1 - wt - n_c dt_c)^{1-\gamma}]^{-\mu} (1 - \gamma) c^\gamma (1 - wt - n_c dt_c)^{-\gamma} = w\lambda \quad (14)$$

$$c^k: n_c^{\xi} c^k^{-\mu} = n_c \lambda \quad (15)$$

where λ is the Lagrange multiplier.

The solutions derived from the optimal condition are:

$$c(j = 3) = \frac{[w - a' - n_c TC(e_c) - n_c beq + (1 + r)a - w n_c dt_c]}{1 + \frac{1-\gamma}{\gamma} + \left[\frac{1-\gamma}{\gamma w}\right]^{\varepsilon_1} \cdot \gamma^{-\frac{1}{\mu}} \cdot n_c^{\varepsilon_2}} \quad (16)$$

$$c^k = \left[\frac{1-\gamma}{\gamma w}\right]^{\varepsilon_1} \cdot \gamma^{-\frac{1}{\mu}} \cdot n_c^{\varepsilon_3} \cdot c(j = 3) \quad (17)$$

$$wt(j = 3) = 1 - n_c dt_c - \left[\frac{(1-\gamma)c}{\gamma w}\right] \quad (18)$$

$$v(j = 3) = \frac{[c^\gamma(1 - wt - n_c dt_c)^{1-\gamma}]^{1-\mu}}{1 - \mu} + n_c^{\xi} \cdot \frac{(c^k)^{1-\mu}}{1 - \mu} + \beta n_c^{\xi} \cdot E v^c + \beta E v(j = 4) \quad (19)$$

Where

$$\varepsilon_1 = (1 - \gamma) - \left[\frac{1-\gamma}{\mu}\right] \quad (20)$$

$$\varepsilon_3 = \frac{x_{i-1}}{\mu} \quad (21)$$

$$\varepsilon_2 = \varepsilon_3 + 1 \quad (22)$$

In the fourth period, an old adult lives alone since his children have already left the family. An old adult still works and his unit of time is divided between work and leisure. He makes decisions about his own consumption and saving for his own retirement in the next period. The optimization problem for an old adult in this period is:

$$v(j=4, a, dt, e, z, n_c=0) = \max_{\{c, wt, a'\}} \{u(c, (1 - wt)) + \beta E[v(j + 1, a', dt, e, z', n_c = 0|z)]\} \quad (23)$$

subject to

$$c + a' \leq wn(j, dt, e, wt, z) + (1 + r)a \quad (24)$$

The Lagrange function in the forth period is

$$\frac{[c^\gamma (1 - wt)^{1-\gamma}]^{1-\mu}}{1 - \mu} + \beta E[v(j + 1, a', dt, e, z', n_c|z)] - \lambda(c + a' - wwt - (1 + r)a) \quad (25)$$

First order conditions are:

$$c: [c^\gamma (1 - wt)^{1-\gamma}]^{-\mu} \gamma c^{\gamma-1} (1 - wt)^{1-\gamma} = \lambda \quad (26)$$

$$wt: [c^\gamma (1 - wt)^{1-\gamma}]^{-\mu} (1 - \gamma) c^\gamma (1 - wt)^{-\gamma} = w\lambda \quad (27)$$

where λ is the Lagrange multiplier.

The solutions derived from the optimal condition are:

$$c(j = 4) = \gamma[w - a' + (1 + r)a] \quad (28)$$

$$wt(j = 4) = 1 - \left[\frac{(1-\gamma)c}{\gamma w} \right] \quad (29)$$

$$v(j = 4) = \frac{[c^\gamma (1 - wt)^{1-\gamma}]^{1-\mu}}{1 - \mu} + \beta E v(j = 5) \quad (30)$$

The final period of an individual's life is as a retiree. An individual enters this period upon retirement. The only decision he will make is the amount of consumption. We assume that he has no planned bequest to give to his children in this period. All savings are consumed. Therefore, an individual entering this period will face the optimization problem:

$$v(j=5, a, dt, e, z=0, n_c=0) = \max_{\{c\}} \{u(c)\} \quad (31)$$

subject to

$$c = (1 + r)a \quad (32)$$

The solutions derived from the optimal condition are:

$$c(j = 5) = (1 + r)a \quad (33)$$

$$wt(j = 5) = 0 \quad (34)$$

$$v(j = 5) = \frac{c^{1-\mu}}{1-\mu} \quad (35)$$

The household problem can be summarized in general form as follows:

$$\begin{aligned} v(j, a, dt, e, z, n_c) = & \max_{\{c, wt, a', c^k, dt_c, e_{n_c}\}} \left\{ u \left(c, (1 - I_h)(1 - wt - I_{n_c} n_c dt_c) \right) + I_{n_c} b(n_c) n_c u(c^k) \right. \\ & + I_{n_c} \beta b(n_c) n_c E[v^c(j, a, dt_c, e_c, z', n_c) | z] \\ & + (1 - I_h)(1 - I_{En_c}) \beta E[v(j + 1, a', dt, e, z', n_c = 0 | z)] \\ & \left. + I_{En_c} \beta E[v(j + 1, a', dt, e, z', n_c | z)] \right\} \end{aligned} \quad (36)$$

subject to

$$c + I_{n_c} n_c c^k + (1 - I_h) a' + I_{n_c} n_c TC(e_c) + I_{n_c} n_c beq \leq (1 - I_h) wn(j, dt, e, wt, z) + (1 + r) \quad (37)$$

where the indicator variables are:

$$I_h = \begin{cases} 0, & \text{if } j < 5 \\ 1, & \text{if } j = 5 \end{cases} \quad (38)$$

$$I_{n_c} = \begin{cases} 0, & \text{if } j \neq 3 \\ 1, & \text{if } j = 3 \end{cases} \quad (39)$$

$$I_{En_c} = \begin{cases} 0, & \text{if } j \neq 2 \\ 1, & \text{if } j = 2 \end{cases} \quad (40)$$

3.3 The Problem of the Firms

There are two types of goods in the model. In one market, a representative firm produces commodity goods while the other produces education. Both markets are assumed to be competitive. The production function of both types of firms is of the standard Cobb-Douglas production function employing both capital and labor inputs.

$$Y = F^g(K_g, N_g) = K_g^\alpha N_g^{1-\alpha} \quad (41)$$

$$E = F^e(K_e, N_e) = K_e^\lambda N_e^{1-\lambda} \quad (42)$$

Where

Y represents the amount of commodity goods that firm produces

E represents the quantity of education

K_g is the aggregate capital used in commodity goods sector

K_e is the aggregate capital used in education sector

N_g is the aggregate labor employed in commodity goods sector

N_e is the aggregate labor employed in education sector

α is the capital share parameter in commodity goods sector

λ is the capital share parameter in education sector

p_g is the producer output price which is normalized to unity

p_e is the education price

r_g is the real interest rate in the commodity goods market

r_e is the real interest rate in the education market

w_g is the real wage rate in the commodity goods market

w_e is the real wage rate in the education market

the firm's profit maximization problems are as follows:

For commodity goods sector

$$\max_{\{K_g, N_g\}} p_g K_g^\alpha N_g^{1-\alpha} - r_g K_g - w_g N_g \quad (43)$$

The goods sector price is normalized to unity and the first order conditions are:

$$K_g: r_g = F_{K_g}^g(K_g, N_g) = \alpha K_g^{\alpha-1} N_g^{1-\alpha} \quad (44)$$

$$N_g: w_g = F_{N_g}^g(K_g, N_g) = (1 - \alpha) K_g^\alpha N_g^{-\alpha} \quad (45)$$

For education sector

$$\max_{\{K_e, N_e\}} p_e K_e^\lambda N_e^{1-\lambda} - r_e K_e - w_e N_e \quad (46)$$

First order conditions are:

$$K_e: r_e = p_e F_{K_e}^e(K_e, N_e) = p_e \lambda K_e^{\lambda-1} N_e^{1-\lambda} \quad (47)$$

$$N_e: w_e = p_e F_{N_e}^e(K_e, N_e) = p_e (1 - \lambda) K_e^\lambda N_e^{-\lambda} \quad (48)$$

The capital and labor are freely moved between the two sectors. This means prices of the production factors between sectors are the same. That is :

$$r = r_g = r_e \quad (49)$$

$$w = w_g = w_e \quad (50)$$

The amount of capital and labor used in each sector cannot exceed the total amount available in aggregate. This restriction can be expressed as:

$$K = K_g + K_e \quad (51)$$

$$N = N_g + N_e \quad (52)$$

3.4 Market Clearing Conditions

There are four competitive markets in this model, which have appropriate market clearing equations: Capital market, Labor market, Goods market and Education market, and must be specified. Because the model comprises heterogeneous individuals, the market clearing equations require aggregating over types of individuals. Therefore, additional notations are needed. Let $\varphi(j, a, dt, e, z, n_c)$ represents a fraction of agents of age j that holds the asset level a , and have parental time level dt , education level e , an idiosyncratic shock z , and a number of children n_c .

For the capital market, an aggregating stock of capital in the economy is simply adding the fraction of individuals with a specific state vector times their asset position a' , with the fraction of individuals with a specific state vector times the product of the number of children and bequest value.

$$K' = \int a' d\varphi(j, a, dt, e, z, n_c) + \int n_c beq d\varphi(j, a, dt, e, z, n_c) \quad (53)$$

For the labor market, the aggregate supply of labor is the sum of the product of the amount of effective labor supplied by a type of individual over all types of individuals.

$$N = \int n(j, dt, e, wt, z) d\varphi(j, a, dt, e, z, n_c) \quad (54)$$

In the goods market, the aggregate goods is the sum of all goods consumed by a type of individual and his children over all types of individuals plus the aggregate

stock of capital that individuals save at the end of the period minus the invested capital that is left from depreciation.

$$F^g(K_g, N_g) = \int (c + n_c c^k) d\varphi(j, a, dt, e, z, n_c) + K' - (1 - \delta)K \quad (55)$$

Note that the capital depreciation rate in this model will be set as one period in the model and is about 15-20 years. This means the capital is used up during this long period.

In the education market, the aggregate education demand is the sum of the product of the number of children born to a type of parents by the education level which that parent gives to the children over all the types of parents. For example, let us consider a fraction of parents who are characterized by an asset level of a , an idiosyncratic level of z , a number of children ($n_c = 2$), and a university education ($e_c = 3$). This type of individual generates a demand for six units of education. The education market clearing condition can be written as:

$$F^e(K_e, N_e) = \int n_c e_c(j = 3, a, dt, e, z, n_c) d\varphi(j = 3, a, dt, e, z, n_c) \quad (56)$$

From equation 51) and 53) the asset market equilibrium can be expressed as:

$$K' = K'_g + K'_e = \int a' d\varphi(j, a, dt, e, z, n_c) + \int n_c beq d\varphi(j, a, dt, e, z, n_c) \quad (57)$$

And from equation 52) and 54) the labor market equilibrium can be expressed as:

$$N = N_g + N_e = \int n(j, a, dt, e, wt, z) d\varphi(j, a, dt, e, z, n_c) \quad (58)$$

3.5 The Recursive Competitive Equilibrium

In the model, the individual state space is determined by the six-tuple $(j, a, dt, e, z, n_c) \in \mathcal{J} \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N}$ where $\mathcal{A} \subset \mathcal{R}_+$, $\mathcal{DT} \subset \mathcal{R}_+$, $\mathcal{E} \subset \mathcal{R}_+$, $\mathcal{Z} \subset \mathcal{R}_+$, and $\mathcal{N} \subset \mathcal{R}_+$. For any individuals, define the constraint set of an age j individual $\Omega_j(j, a, dt, e, z, n_c) \subset \mathbb{R}_+^5$ as all seven-tuples $(c, a', wt, c^k, dt_c, e_{n_c}, beq)$ such that the budget constraint (equation 37) is satisfied as well as the following non-negativity constraints hold:

$$\begin{aligned}
c &> 0 \\
c^k &> 0 \\
dt_c &> 0 \\
e_{n_c} &> 0 \\
a' &\geq 0 \\
wt &\geq 0
\end{aligned}$$

Let $v(j, a, dt, e, z, n_c)$ be the value of the objective function of an individual with state vector (j, a, dt, e, z, n_c) defined recursively as:

$$\begin{aligned}
v(j, a, dt, e, z, n_c) = & \max_{\{c, wt, a', c^k, dt_c, e_{n_c}\} \in \Omega_j} \left\{ u \left(c, (1 - I_h)(1 - wt - I_{n_c} n_c dt_c) \right) \right. \\
& + I_{n_c} b(n_c) n_c u(c^k) + I_{n_c} \beta b(n_c) n_c E[v^c(j, a, dt_c, e_c, z', n_c) | z] \\
& + (1 - I_h)(1 - I_{E n_c}) \beta E[v(j + 1, a', dt, e, z', n_c = 0 | z)] \\
& \left. + I_{E n_c} \beta E[v(j + 1, a', dt, e, z', n_c | z)] \right\}
\end{aligned}$$

where E is the expectation operator conditional on the current state of the individual.

3.5.1 Definition of Equilibrium

A steady state competition equilibrium consists of

Value functions : $J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$;

Decision rules : $J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$,

$a' : J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$, $wt : J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$,

$c^k : J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$, $dt_c : J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$,

$e_{n_c} : J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$ and $beq : J \times \mathcal{A} \times \mathcal{DT} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{N} \rightarrow \mathbb{R}_+$;

Aggregate production factors $\{K_g, N_g, K_e, N_e\}$;

Prices $\{w, r, p_e\}$ and Invariant distribution $\varphi(j, a, dt, e, z, n_c)$ such that

3.5.1.1 given the price $\{w, r, p_e\}$, the value function v and decision rules $a', c, c^k, wt, dt_c, e_{n_c}, beq$ solve the consumer problem;

3.5.1.2 given the price $\{w, r, p_e\}$, the aggregate production factors $\{K_g, N_g, K_e, N_e\}$ solve the firms' profit maximization problems by satisfying equations(41) through (52);

3.5.1.3 the goods market clears;

$$F^g(K_g, N_g) = \int (c + n_c c^k) d\varphi(j, a, dt, e, z, n_c) + K' - (1 - \delta)K$$

3.5.1.4 the education market clears;

$$F^e(K_e, N_e) = \int n_c e_c d\varphi(j = 3, a, dt, e, z, n_c)$$

3.5.1.5 the asset market clears;

$$K' = \int a' d\varphi(j, a, dt, e, z, n_c) + \int n_c beq d\varphi(j, a, dt, e, z, n_c)$$

3.5.1.6 the labor market clears ;

$$N = \int n(j, dt, e, wt, z) d\varphi(j, a, dt, e, z, n_c)$$

3.5.1.7 letting T be an operator which maps the set of distributions into itself, aggregation requires

$$\varphi'(j + 1, a', dt, e, z', n_c) = T(\varphi)$$

and T be consistent with individual decisions.

3.5.2 Transition of State Space

The T operator for this model depends on the transition matrix for the education specific earnings, Π_z^e , the probability matrix that specifies the number of children that may arrive in the third period which we specify as Π_{n_c} , and the decision rules for $a'(j, a, dt, e, z, n_c)$, $c(j, a, dt, e, z, n_c)$, $c^k(j, a, dt, e, z, n_c)$, $wt(j, a, dt, e, z, n_c)$, $dt_c(j, a, dt, e, z, n_c)$, $e_{n_c}(j, a, dt, e, z, n_c)$ and $beq(j, a, dt, e, z, n_c)$. The transition matrix (T_φ) shows how to find the distribution of agents across the state space for the next period (φ') given this period's distribution (φ). The transition matrix tells us how an agent moves from the current state space and age to the next. To elaborate, children move to young adults, young adults move to parents, parents move to old adults, and old adults move to retirees. Thus, the current agent distribution matrix which is used to operate on the transition matrix is composed of agents with children, young adults, parents and old adults. And the result of the next period agent distribution matrix is composed of agents with young adults, parents, old adults and retirees. The children cohort is not included as they become young adults. It can be formally written as:

$$\varphi'_{noc}(j + 1, a', z', n'_c) = T(\varphi) = T_\varphi^T \varphi_{nor} \quad (59)$$

Where T_φ is the transition matrix, the superscript T stands for the transpose of matrix.

φ'_{noc} is the next period distribution of agent for every generation except the children generation.

φ_{noc} is today's distribution without retirement generation.

To construct the transition matrix, it requires the education dependent probability matrix Π^e and the decision rules. The agent's decision rules allow us to know the position in the next period, and the probability matrix shows the likelihood of that position. The probability matrix Π^e is a matrix containing the probability of an agent with education, e , moving from a specific place in the state space today to a place in the state space next period. Therefore, the education dependent probability matrix Π^e for any age and education level is:

$$\Pi^e = \begin{bmatrix} \Pi_{12}^e & \cdot & \cdot & \cdot \\ \cdot & \Pi_{23}^e & \cdot & \cdot \\ \cdot & \cdot & \Pi_{34}^e & \cdot \\ \cdot & \cdot & \cdot & \Pi_{45}^e \end{bmatrix} \quad (60)$$

where Π_{jk}^e , ($j = 1, \dots, 4$ and $k = 2, \dots, 5$) represents the transition matrix for an individual with education level e currently of age j who will move to age k . As long as j does not equal 2, $\Pi_{12}^e = \Pi_{34}^e = \Pi_z^e$ where

$$\Pi_z^e = \begin{bmatrix} \pi_{11}^e & \pi_{12}^e \\ \pi_{21}^e & \pi_{22}^e \end{bmatrix} \quad (61)$$

Where $\pi_{zz'}^e$ is the probability of moving from state z with education e to state z' with the same education level e . With p denoting primary education, h denoting high school education and u denoting university education, the probability of moving from state z with education e to state z' with the same education level can be specifically defined as:

$$\pi_{zz'}^e = \begin{bmatrix} \pi_{zz'}^p & 0 & 0 \\ 0 & \pi_{zz'}^h & 0 \\ 0 & 0 & \pi_{zz'}^u \end{bmatrix} \quad (62)$$

The transition matrix of an individual with education level e currently of age 2 (young adult) moving to age 3 (parent) Π_{23}^e , will be different from $\Pi_{12}^e = \Pi_{34}^e = \Pi_z^e$. While the transition matrix will only depend on the education level specific earning probabilities in other periods, in the period of moving from young ($j = 2$) to parent ($j = 3$), the probability matrix of an agent will depend on both the education level specific earning probabilities and the unconditional matrix associated with the number

of children. Defining the unconditional probability of having n_c children as π_{n_c} , the probability of moving from state z with education e today to state z' with education e and one children is $\pi_{zz'}^e * \pi_1$. And such probabilities with different numbers of children are computed similarly such as a probability of $\pi_{zz'}^e * \pi_2$ for two children. Hence, the transition matrix for an individual with state ($j = 2, z, e$) is written as:

$$\Pi_{23}^e = \Pi_z^e \otimes \Pi_{n_c} = \begin{bmatrix} \pi_1 \pi_{11}^e & \cdot & \cdot & \cdot & \pi_5 \pi_{11}^e & \pi_1 \pi_{12}^e & \cdot & \cdot & \cdot & \pi_5 \pi_{12}^e \\ \pi_1 \pi_{21}^e & \cdot & \cdot & \cdot & \pi_5 \pi_{21}^e & \pi_1 \pi_{22}^e & \cdot & \cdot & \cdot & \pi_5 \pi_{22}^e \end{bmatrix} \quad (63)$$

As the agent approaches a retirement period, he does not work, resulting in Π_{45}^e equal zero. Hence, idiosyncratic shocks are irrelevant. Finally, the off diagonal elements are equal to zero because an agent cannot move across two age periods, stay in their current age, or move to a younger age. As the result, the education dependent probability matrix Π^e for any age and education level is:

$$\Pi^e = \begin{bmatrix} \Pi_{12}^e & \cdot & \cdot & \cdot \\ \cdot & \Pi_{23}^e & \cdot & \cdot \\ \cdot & \cdot & \Pi_{34}^e & \cdot \\ \cdot & \cdot & \cdot & \Pi_{45}^e \end{bmatrix} = \begin{bmatrix} \Pi_z^e & \cdot & \cdot & \cdot \\ \cdot & \Pi_z^e \otimes \Pi_{n_c} & \cdot & \cdot \\ \cdot & \cdot & \Pi_z^e & \cdot \\ \cdot & \cdot & \cdot & 0 \end{bmatrix}$$

3.5.3 Population Growth

As an exogenous constant rate of population growth is allowed in the model, the fraction of a new entry of children cohort, without adjustment for growth, will increase every period. Therefore failure to properly account for population growth can adversely affect the calculation of the invariant distribution matrix. The aim of accounting for population growth is to have identical distribution matrix between the models with and without growth. If the probabilities of moving from state z to state z' of models with and without population growth are identical, then agents in both cases will be distributed in the same proportion across states. Hence, the ratio of agents in generation j and state z to the total population of generation j with and without population growth models will be the same, that is:

$$\frac{x_{jz}}{pop_j^o} = \frac{y_{jz}}{pop_j^g} \quad (64)$$

where x_{jz} is the number of people in generation j in state z without population growth model.

y_{jz} is the number of people in generation j in state z of model with population growth.

pop_j^0 is the total population of generation j of the model without population growth.

pop_j^g is the total population of generation j of the model with population growth.

The concept of accounting for population growth in the model is firstly to find the invariant distribution matrix when population growth does not occur. Then we adjust this distribution for population growth.

The transition matrix indicates the next period distribution of agents for young adults, parents, old adults and retirement generation. In order to complete the distribution of agents, the distribution of children in the next period is required. This distribution can be calculated based on the parent's decision on the education level, parental time and the amount of bequest. Therefore, the fraction of children in each period can be stated as:

$$\begin{aligned} \varphi'_c(\text{beq} = a', j = 1, z', dt_c = dt, e_c = e) \\ = \left(\frac{1}{1+g}\right) \int_{dt_c=dt, e_c=e, \text{beq}=a'} n_c d\varphi'(j = 3, a', z', dt, e, n'_c) \end{aligned} \quad (65)$$

This equation indicates that the fraction of children with parental time level dt , education e and bequest a' in the next period is the sum of the number of children whose parents made a decision on a specific parental time level, education level and bequest equal to dt , e and a' respectively. Since the calculation of children's fraction is summing up the number of children it is affected by the population growth which can be adjusted for by $1/(1+g)$, where g is population growth rate.

The household distribution matrix with zero population growth rate for next period can be written as:

$$\varphi'_{g=0} = \begin{bmatrix} \varphi'_c \\ \varphi'_{noc} \end{bmatrix} \quad (66)$$

The last step is to account for the population growth in the distribution matrix with zero population growth, equation 66). In order to use equation 66), the exact number of population in each generation is needed so that the total population can be

normalized to unity. The sum of all generations must be equal to one accordingly. Since population growth is constant and exogenous, the population of each period is computed by multiplying population growth with the population of the next period. For example, the old adult population is the result of population growth timing the retirement population. The population of children, young adults, parents and retirees can be defined as pop_j for $j = 1, 2, \dots, 5$. The population in each generation can be written as:

$$\begin{aligned} pop_4 &= (1 + g)pop_5 \\ pop_3 &= (1 + g)pop_4 = (1 + g)^2 pop_5 \\ pop_2 &= (1 + g)pop_3 = (1 + g)^3 pop_5 \\ pop_1 &= (1 + g)pop_2 = (1 + g)^4 pop_5 \end{aligned}$$

The total population normalized to unity can be written as:

$$1 = pop_5 + (1 + g)pop_5 + (1 + g)^2 pop_5 + (1 + g)^3 pop_5 + (1 + g)^4 pop_5 \quad (67)$$

Finally adjusting the distribution matrix for the population growth is relatively straightforward.

The total population for each generation with no population growth would be 0.2. Population growth can be taken into account by multiplying the distribution of generation j by $pop_j / 0.2$. Therefore, the aggregate household distribution matrix can be written as:

$$\varphi' = \begin{bmatrix} \varphi'_{g=0}(j = 1, a', z', e)(pop_1/0.2) \\ \varphi'_{g=0}(j = 2, a', z', e)(pop_2/0.2) \\ \varphi'_{g=0}(j = 3, a', z', e, n_c)(pop_3/0.2) \\ \varphi'_{g=0}(j = 4, a', z', e)(pop_4/0.2) \\ \varphi'_{g=0}(j = 5, a', z', e)(pop_5/0.2) \end{bmatrix} \quad (68)$$

3.6 Calibration

In this section, functional forms for the utility function, production function, parameters for the functions, and random variables are discussed.

3.6.1 Model Period

In this model economy, an individual is assumed to live for five 15-year periods.

3.6.2 Preferences

In characterizing the individual decision, a form of the utility function must be specified. As generally used in many studies on the overlapping generation model, the constant relative risk aversion (CRRA) type utility function, $U(c, lt) = [c^\gamma lt^{1-\gamma}]^{1-\mu} / (1 - \mu)$ is chosen with γ the share of consumption in the utility function and μ the curvature parameter. For the share of consumption, we use the value 0.268 as in Wisit Chaisrisawatsuk (2014) who used the earning and consumption data from 1990, 1992, 1994, 1996, 1998 and 2000 Thailand Household Socio-Economic and the first order condition for labor supply decision $\gamma/(1 - \gamma) = c/w(wt)$ to solve for γ . Aiyagari (1994) has used three different values for the curvature parameter: 1, 3 and 5. All three values of curvature parameter are examined and yield similar results. Hence the results with the curvature parameter equal to 3 are reported. The preference structure of an individual also involves a discount factor parameter and an altruistic discount factor for the number of children. In the previous studies, the annual discount factor values are 0.91, as in Huggett (1994), 0.924 as in Castaneda et al. (2003), 0.934 as in Nishiyama (2000), to 0.96, as in and Aiyagari (1994), Castaneda et al (1998) and Yang (2005). In Wisit Chaisrisawatsuk (2014)'s study, the discount factor for a period of 15 years is set to 0.24. The discount factor in this research is set to be 0.24. Based on Knowles' work (Knowles 1999) the altruistic discount factor parameter is set to be equal to $b(n_c) = n_c^{-\theta}$, where θ equals to 0.55. This parameter implies that today's altruistic discount factors with 1-5 children in the model are 1.37, 1, 0.83, 0.73 and 0.66 respectively. Therefore the next period's altruistic discount factors are 1.37β , β , 0.83β , 0.73β and 0.66β respectively.

3.6.3 Technology

The standard Cobb-Douglas production function is used as a functional form. Specifically, the aggregate commodity production function is $(K_q, N_q) = K_q^\alpha N_q^{1-\alpha}$ and the aggregate education production function is $Edu(K_e, N_e) = K_e^\lambda N_e^{1-\lambda}$. The

parameter α is the capital share parameter in the commodity sector, and λ is capital share parameter in the education sector. Pranee Tinakorn and Chalongphob Sussangkarn (1998) report capital share data in agriculture, industry, manufacturing and service sector in Thailand from 1980 to 1995. Based on their findings, this research uses the average capital share of agriculture, industry and manufacturing sectors for the commodity sector. This value is 0.60. For the capital share in the education sector we average the values of the estimates capital share in the service sector from the 1980 to 1995 in Pranee Tinakorn and Chalongphob Sussangkarn (1998), which is 0.61910.

3.6.4 Random Variables

The values for two random events in the model: idiosyncratic shocks and the number of children are based on the work of Wisit Chaisrisawatsuk (2014). The probability transition matrix is:

$$\Pi_z^e = \begin{bmatrix} \pi_{11}^e & \pi_{12}^e \\ \pi_{21}^e & \pi_{22}^e \end{bmatrix} \quad (69)$$

Where π_{ij}^e is a probability of being in state i for this period entering to state j for the next period of an agent with education level, e , and for $i = 1, 2$ and $j = 1, 2$. Let $e=1$ represent being in the bad state and $e=2$ represent being in the good state. The probability transition matrix is shown below.

$\Pi_z^{e=1}$	$\begin{bmatrix} 0.62 & 0.38 \\ 0.53 & 0.47 \end{bmatrix}$
$\Pi_z^{e=2}$	$\begin{bmatrix} 0.38 & 0.62 \\ 0.21 & 0.79 \end{bmatrix}$
$\Pi_z^{e=3}$	$\begin{bmatrix} 0.27 & 0.73 \\ 0.08 & 0.92 \end{bmatrix}$

To interpret the probability transition matrix, consider the agent with primary education in the bad state, he will stay in the bad state with a probability of 0.62. In contrast, an individual who starts in a bad state and has an education level of high school will stay in the bad state only 38 percent of the time. Intuitively an agent with

primary education, starting in a bad state, will have a higher probability of staying in this state than an agent with a high school education.

As for the number of children, the unconditional probabilities of numbers of children are as shown.

$$\Pi_{n_c} = [\pi_{n_c}^1 \quad \pi_{n_c}^2 \quad \pi_{n_c}^3 \quad \pi_{n_c}^4 \quad \pi_{n_c}^5] = [0.3924 \quad 0.3454 \quad 0.1405 \quad 0.0717 \quad 0.05]$$

where $\pi_{n_c}^k$ is the probability of having k children for $k = 1, 2, \dots, 5$.

3.6.5 Parental Time in Human Capital Development

In Casarico (2007), parental time spent interacting with children is shown to be a crucial element in the development of the children's human capital. The children human capital, h_c , is produced using inputs: the childcare x received during the first period of life and the time investment in formal schooling e undertaken in the same period:

$$h_c = q(e \cdot \bar{h})^\omega x^{1-\omega} \quad (70)$$

where h represents human capital of a parent while \bar{h} denotes the average level of human capital of parent generation in the economy, representing an intergenerational externality. $q > 0$ is a parameter. The parameter ω in the human capital production function is the elasticity of earnings with respect to schooling with $0 < \omega < 1$. Assuming that there is no human capital depreciation during an individual's lifetime, the human capital is acquired through all periods. Childcare production function used in this research is in line with that of Casarico (2007).

$$x = (dt \cdot h)^{\sigma_1} Care^{\sigma_2} \quad (71)$$

where dt is time transfer from parents to children; $Care$ is childcare services purchased on the market (e.g. baby-sitting, day-care centers); $\sigma_k > 0$ with $k = 1, 2$ and $\sigma_1 + \sigma_2 = 1$. The production function postulated implies that the inputs to produce childcare are complements.

Solving equations 70 and 71 together yields:

$$h_c = q(e \cdot \bar{h})^\omega ((dt \cdot h)^{\sigma_1} Care^{\sigma_2})^{1-\omega} \quad (72)$$

In Croix and Doepke (2003), the elasticity of earnings with respect to schooling, ω , in actual data goes from 0.4 to 0.8. This research, in line with Casarico

(2007), uses 0.6 as a value of ω . For the values of the parameters of the childcare production function, σ_1 and σ_2 are set to be equal 0.5.

In order to estimate a earning profile, a parental time factor which is a ratio between children human capital assumed that they are given the same level of education by the same parents and the idiosyncratic shock is required. From equation 72), a parental time factor is then expressed as:

$$\frac{h_c(e, \bar{h}, h, Care, dt2)}{h_c(e, \bar{h}, h, Care, dt1)} = \frac{q(e \cdot \bar{h})^\omega ((dt2 \cdot h)^{\sigma_1} Care^{\sigma_2})^{1-\omega}}{q(e \cdot \bar{h})^\omega ((dt1 \cdot h)^{\sigma_1} Care^{\sigma_2})^{1-\omega}} = \left[\frac{dt2}{dt1} \right]^{\sigma_1(1-\omega)} \quad (73)$$

For the amount of time parents spend on their children, Cardia and Ng (2003) report estimation for the time devoted to child care by parents a value of 10 hours per week or 9 percent of total time (Total time is assumed to be 16 hours a day. That is the available time after the sleeping time (assumed to be 8 hours a day) is excluded) For grandparents, Cardia and Ng (2003), using the HRS (Health and Retirement Study), find for the US grandparents spend on average almost 9 hours per week or 8 percent of total time looking after grandchildren. Hill and Stafford (1985) report that young households spend between 381 and 813 minutes per week (5.7 percent and 12 percent of total time) on child care. Hotz and Miller (1988) estimate that the amount of time required to care for a newborn is about 660 hours per year, or 12.69 hours per week (11.3 percent of total time). Leibowitz (1974b) suggests 144.51 minutes per day of an average couple in the survey are spent on physical care of the child, while 131.6 minutes are spent on educational care. These two types of child are add up to 4.6 hours per day with each spending about 2.3 hours or 14.4 percent of total time. Most of the studies report the time parents spend on their children ranges from 8 to 15 percent of the total time. Nonetheless, the model in this research will cover a wider range of time parents spend on their children from 3 - 24 percent of the total time. Within this range, the parental time parameter is set to 5 levels as shown in the following table:

Table 3.1 Parental Time Parameter

Parental time parameter	Percentage of the total available time of 16 hours	Parental time in total
dt1	3	30 minutes
dt2	6	1 hour
dt3	12	2 hours
dt4	18	3 hours
dt5	24	4 hours

Given a parental time parameter, the parental time factors can be calculated using equation 73. This parental time factor is a relative term that compares labor supply efficiency between a group of individuals to the group with the lowest parental time with the same age, education level, and earning shock. The magnitude of the factor depends not only on the differences between parental time but also the value of elasticity of earning with respect to parental time $\sigma_1(1 - \omega)$. The elasticity of earning with respect to parental time $\sigma_1(1 - \omega)$ has two components. One is σ_1 , a parameter of the childcare production function. It indicates the productivity of parental time towards child care. The other component is $(1 - \omega)$, the productivity of child care towards earning profile. Having taken the value of ω and σ_1 equal to be 0.6 and 0.5 respectively, it is found that the rate of substitution between σ_1 and $(1 - \omega)$ is around 1.11. That is if σ_1 is to be reduced by 1 percent, then $(1 - \omega)$ has to be increased by around 1.11 percent to keep the same value of parental time factor.

3.6.6 Effective Labor Supply and Earning Profile

The effective labor supply $n(j, dt, e, z, wt)$ of an individual is a product of an individual's earning profile and working time (wt). The earning profiles used in the research are calculated based on age (j), human capital (h) which is a function of education level (e) and parental time (dt) and earning shock (z). While in previous research, the earning profile is computed based on ages, education levels and earning shock using Household Socio-Economic data. This research is further extended to include parental time effects in the existing earning profile.

The earning profile can be calculated by the following steps. Firstly the working households are classified by their ages: 16-30 (young); 31-45 (parent); 46-60 (old), and by education level (primary, high school and university). Therefore based on only age and education level, there are 9 groups. For each group, the arithmetic mean of earning is calculated as a reference point. For each individual, if his earning exceeds the reference mean in his group, the individual is in good state of earning shock, otherwise he is considered of being in bad state. By introducing the earning shock, each household group is separated into 2 subgroups (bad and good states). Therefore households can be classified into 18 groups based on age, education level and earning shock. In each age-education level-earning shock, average earning is calculated. The labor supply efficiency is computed by dividing the average earning of each group by average earning of the young primary education group with the bad state group. Since in the research we classify parental time into 5 levels as in Table 2, for each age-education level-earning shock group, we divide it into 5 groups of parental time level by multiplying the labor supply efficiency by corresponding parental time factors. Eventually the earning profile is classified into 90 groups based on age, education level, parental time and earning shocks. The earning profile is elaborated in detailed in Appendix A:

The following table summarizes the parameter values used in the research model.

Table 3.2 Parameter Values

	Function	Parameter
Utility function	$U(c, lt) = [c^\gamma lt^{1-\gamma}]^{1-\mu} / (1 - \mu)$	$\gamma = 0.26 \quad \mu = 3$
Discount factor	$\beta, b(n_c) = n_c^{-\theta}$	$\beta = 0.24 \quad \theta = 0.55$
Production	$Q(K_q, N_q) = K_q^\alpha N_q^{1-\alpha}$	$\alpha = 0.6$
function	$Edu(K_e, N_e) = K_e^\lambda N_e^{1-\lambda}$	$\lambda = 0.619$

3.7 Computational Procedure

In this section, the computation of the overlapping generation model is explained in detail.

3.7.1 The procedure starts with conjecture values for aggregate capital demand and aggregate labor demand, as well as corresponding relative price of education.

3.7.2 Given the values of aggregate capital and labor demand, the first order conditions from the various firms' problem are used to compute sets of capital and labor allocation in goods market and education market.

3.7.3 With every set of capital and labor allocation and the relative price of education, the researcher calculates the factor prices (interest rate and wages) in both sectors then selects the allocation that makes the factor prices equal in both markets.

3.7.4 With the common interest rate, wage and education price, the household problem can be solved and the decision rules can be identified.

3.7.5 Using the decision rules and the law of motion, the researcher computes the invariant distribution of individuals.

3.7.6 With the invariant distribution of individuals, the education demand and supply can be calculated. Hence, the market clearing condition can be checked. If the education market is not clear, the conjecture relative education price must be updated with a repeated procedure until the market is cleared.

3.7.7 Likewise, the invariant distribution of individuals can be used to find labor supply. After checking the labor market for clearing condition, the conjectured aggregate labor demand can be altered until the labor market is cleared.

3.7.8 Finally the capital market is checked for clearing with the possible adjusting of aggregate capital demand.

As the utility function used in this research involves altruistic motive, a parent's value function includes a child's value function. It can be seen from this type of utility function that a parent must be concerned about his children. Later on, these children must be concerned about their offspring. So based on the altruistic logic, a parent cares about his descendants. It is simply stated that the value function of a parent becomes an infinite object. As a result, the methods to solve this individual

problem involve value function iteration and backward induction. Considering the value function for an individual of $j = 3$, a parent's value function is:

$$\begin{aligned} v(j = 3, a, dt, e, z, n_c) = & \max_{\{c, wt, a', c^k, dt_c, e_c, beq\}} \{u(c, (1 - wt - n_c dt_c)) \\ & + b(n_c) n_c u(c^k) \\ & + \beta b(n_c) n_c E[v^c(j = 2, a = beq, dt_c, e_c, z', n_c = 0)|z] \\ & + \beta E[v(j + 1, a', dt, e, z', n_c = 0|z)]\} \end{aligned}$$

From the above equation, the term $(E[v^c(j = 2, a = beq, dt_c, e_c, z', n_c = 0)|z])$ is a child's future value function or simply a young adult's value function. The term $(E[v(j + 1, a', dt, e, z', n_c = 0|z)])$ is his value function in the next period or simply an old adult's value function.

Solving this problem starts from conjecturing a value function of a child which is the same as young adult's value function. Using the backward induction method, the optimal decision in each point (dt, e, a, z) on the state-space grid for the retirement, the old adult, and their value function can be solved by assuming that the retirement saving is zero. Using the optimal old adult value function and the conjectured value function, leads to solutions for the parent's optimal decisions at each point (a, dt, e, z, n_c) , on the state-space grid. This is done by comparing the parents' utility for each feasible child education level e_c and parental time level. Using e_c , dt and the research algorithm leads to the optimal decision for parental and child consumption, working hours and investment made per child (parental time, education and bequest). Substituting the parent value function into the young adult value function results in the optimal decision and value function of young adult generation on each state-space grid (dt, e, a, z) . The method of updating the conjectured child value function with the young adult one is repeated until the value function converges.

CHAPTER 4

FINDINGS

4.1 Benchmark Model

While many literature reviews state that both unplanned and planned bequests (money and education) have contributed to explaining inequality, a few literatures argue that parental time can have a role in developing human capital. In this research, the parental time variable is included into an economic model to explain inequality in the Thai economy. This economic model extends from a general heterogeneous overlapping generation model in many ways. It includes an altruistic motive of parents towards children that appears in different forms: voluntary bequest, education and parental time. Parental time variable performs like education in helping children develop their human capital, which in turn increases efficient wage earning and more wealth in the future. The model is calibrated and the generated results are compared with the statistics from the 2000 Household Socio-Economic Survey produced by the National Statistical Office of Thailand. (The results are taken from Wisit Chaisrisawatsuk (2014)) In particular, the results are compared with the Thai data in terms of education distribution, concentration and skewness statistics and the distribution of earnings and income.

Table 4.1 compares the education distribution between the results generated by the model and the Thai data. It can be seen that the model reasonably generates education distribution close to that of the real data. In particular, the model predicts that 70.02 percent, 15.86 percent and 14.12 percent of the population have an education as high as primary school, high school and university respectively, compared with the real data that reports 72.95 percent, 16.38 percent and 10.66 percent, respectively. For both primary school and high school, the model slightly under predicts the real values by around 2.93 0.52 and percent each.

However, it over predicts the fraction of population graduating from university by about 3.46 percent.)

Table 4.1 The Education Distribution for Benchmark Model

	Benchmark Model	2000 Data
Primary School	70.02	72.95
High School	15.86	16.38
University	14.12	10.66

Table 4.2 compares the concentration and skewness statistics between the results generated by the model and those observed in the Thai data. Although there are a few statistics which the model cannot predict well, like the education distribution, mostly the model is able to predict reasonably well. The concentration statistics in table 4.2 comprise the Gini coefficients and the ratio of the top 1 percent to the lowest 17.8 percent of the population. The Gini coefficients generated by the model for earning and income are 0.56 and 0.53 respectively. The values are under predicted, compared with the real values of 0.59 and 0.54 respectively. For the skewness ratio of the top 1 percent to the lowest 17.8 percent of the population, the model generates 18.58 for earning and 4.83 for income. Those values are under predicted, compared with the values of the real data of 19.66 and 5.32 respectively.

The skewness statistics in table 4.2 consist of the location of mean, skewness coefficient and the ratio of the mean to median. For the location of mean, the model generates 67.80 for earning and 67.49 for income. Both are under predicted when compared with the real values of 68.5 and 70.0 respectively. For ratio of mean to median, the model generates 1.83 for earning compared with the real value of 1.72 while it generates the ratio of 1.64 for income, slightly under predicting the real values of 1.67. For the skewness coefficient, it can be seen that the model under predicts the skewness coefficient of 12.77 for earning and 10.25 for income compared with the real values of 13.61 and 14.5 respectively. In general, it is conclusive from most of the concentration and skewness statistics, that the model generates a slightly less concentrated and skewed result than that of the real data. Although a few

statistics from the model departs from those of the real data, most of them suggest that the model can capture the key features that explain the earning and income distribution in Thailand.

Table 4.2 Concentration and Skewness Statistics for Benchmark Model

	Earning		Income	
	Benchmark Model	2000 Data	Benchmark Model	2000 Data
Gini	0.56	0.59	0.53	0.54
Top 1				
percent/lowest				
17.8 percent	18.58	19.66	4.83	5.32
skewness				
Location of				
Mean	67.80	68.5	67.49	70.0
Skewness				
Coefficient	12.77	13.61	10.25	14.5
Mean/Median	1.83	1.72	1.64	1.67

In evaluating the model, the percentage shares of earning and income distribution by quintiles are generated and compare with those from the real data in table 4.3. It can be seen that the percentage share of earning and income distribution by quintiles generated by the model are close to those of the real data. Overall, the results comparing earning distribution and income distribution show that the earning distribution from the model better matches the real data than the income distribution does. Considering the 1st quintile for earning and income distribution, the model predicts 0.97 and 2.34 compared with the real data of 0.89 and 2.5 respectively. In the 2nd quintile, the model predicts 5.48 and 4.19 for earning and income distribution, compared with the real data of 5.59 and 6.77 respectively. In the 3rd quintile, our model predicts 11.27 and 12.46 for earning and income distribution, compared with the real data of 11.71 and 12.09 respectively. In the 4th quintile, the model predicts 24.61 and 24.74 for earning and income distribution, compared with the real data of

21.89 and 20.81 respectively. Lastly for the top quintile, the model predicts 57.66 for earning and 56.26 for income compared with the real data of 60.48 and 57.86 respectively. In general the model predicts less inequality than the real data. It also predicts less earning and income concentration than the real data as suggested by the concentration statistics examined earlier.

Table 4.3 Percentage Share of Earning and Income by Quintiles for Benchmark Model

Quintiles	Earning		Income	
	Benchmark Model	2000 Data	Benchmark Model	2000 Data
1 st	0.97	0.86	2.34	2.5
2 nd	5.48	5.59	4.19	6.77
3 rd	11.27	11.71	12.46	12.09
4 th	24.61	21.89	24.74	20.81
5 th	57.66	60.48	56.26	57.86

It is stated earlier that the objective of inclusion of the parental time effect on human capital was to improve the explanatory ability of the previous model with the planned monetary and education bequest on the inequality of Thailand. The results generated from the benchmark model, with parental time to improve children's human capital, can match the shape of education distribution and the shape of earning and income distribution of the real data. In order to confirm that the model with parental time can actually improve the explanatory ability of the previous model, the results generated from both models with and without parental time are compared. Table 4.4 compares the education distribution generated among the models with and without parental time and the Thai data. In general, the two models with and without parental time generate the education distribution close to the shape of the real distribution. For primary school distributions, while both models under predict the real data, the benchmark model predicts a slightly closer distribution to the real data than the model without parental time. For high school distribution, while the benchmark model slightly under predicts the real data, the model without parental time over predicts the

real data. For university education distribution, both models over predict the real values. However, the model without parental time predicts slightly better.

Table 4.4 Comparison of the Education Distribution Among Models

	Model Without Parental Time	Benchmark Model	2000 Data
Primary School	67.29	70.02	72.95
High School	21.15	15.86	16.38
University	11.56	14.12	10.66

Table 4.5 displays the comparison of concentration and skewness statistics between the model without parental time and the benchmark model. For the concentration statistics for both earning and income which are the Gini coefficient and the ratio of the top 1 percent to the lowest 17.8 percent, the benchmark model predicts closer to the real data than the model without parental time. As for the skewness statistics for both earning and income, the result from comparing both models is inconclusive. While the benchmark model predicts closer to the real data than the model without parental time in terms of the location of means of earning, the model without parental time is superior in terms of the location of means of income. For the skewness coefficients of earning the benchmark model predicts closer to the real data than the model without parental time. However, for the skewness coefficients of income, the model without parental time predicts closer than the benchmark model. For the ratio of mean to median, the model without parental time predicts closer to the real data than the benchmark model in terms of earning. However, the benchmark model is superior in terms of income.

Table 4.5 Comparison of the Concentration and Skewness Statistics Among Models

	Earning			Income		
	Without Parental Time	Benchmark Model	2000 Data	Without Parental Time	Benchmark Model	2000 Data
Gini	0.51	0.56	0.59	0.51	0.53	0.54
Top 1 percent/lowest 17.8 percent skewness	16.76	18.58	19.66	4.22	4.83	5.32
Location of Mean	66.10	67.80	68.5	69.36	67.49	70.0
Skewness Coefficient	19.73	12.77	13.61	13.86	10.25	14.5
Mean/Median	1.71	1.83	1.72	1.56	1.64	1.67

Table 4.6 Percentage Share of Earning and Income by Quintiles Among Models

Quintiles	Earning			Income		
	Without Parental Time	Benchmark Model	2000 Data	Without Parental Time	Benchmark Model	2000 Data
1 st	1.51	0.97	0.86	2.51	2.34	2.5
2 nd	6.61	5.48	5.59	4.42	4.19	6.77
3 rd	13.46	11.27	11.71	13.63	12.46	12.09
4 th	28.78	24.61	21.89	23.37	24.74	20.81
5 th	49.64	57.66	60.48	56.07	56.26	57.86

Performance comparison of the two models is conducted to explain inequality by examining the percentage share of earning and income distribution by quintiles generated by both models in table 4.6. It is evident that the overall percentage share of

earning distribution by quintiles generated by the benchmark model is better matched to the real data than the model without parental time. Interestingly, both models generate similar shapes for income distribution to represent the distribution of real data.

It is noted that the benchmark model comprises three intergenerational links: bequest, education and parental time while the model without parental time comprises only two links: bequest and education. To complete the comparison, the sum of bequest values of the two models will be compared. Bequests from the two models are about the same, which means utility from giving education and parental time in the benchmark model should be about the same as utility from giving only education in the other model too, given the same opportunity cost. Although it is evident that adding parental time in the model can alter earning distribution, it hardly changes the income distribution of the economy since income comprises wage earning and earning from saving. Result comparisons in table 4.4-4.6 lead to a conclusion that parental time is another channel which parents can use to improve the future earning of their children. The parental time in the model acts like education but at a finer scale that parents can choose from. This is why in the benchmark model parents choose more primary school for children than high school. Choosing primary school at a lower cost while allocating more time must be more optimized for the parents.

4.2 Parental Time for Thai Economy

In the previous sections, we introduce a heterogeneous economic model with altruistic intergenerational transfers from parents to children that include bequest, education and parental time. After comparing the results generated by the benchmark model with the Thai economic data, the benchmark model is able to mimic the important features such as the education distribution, earning and income distributions of the Thai economic data. We then disable the parental time link in the benchmark model, keeping all parameters with the same values, and run a statistical program for the result. We find that the model without parental time is not as good as the benchmark model in terms of mimicking the education distribution and earning distribution. Therefore, we conclude that the parental time link is important and that

the benchmark model can be used as a reference point for the economy in any future analysis.

In this section, the benchmark model is analyzed to find solutions of parental time for the Thai economy. The main assumption for this analysis is that there is a government to implement a policy relating to time allocation for parenting. In addition, the benchmark model is slightly modified by taking the parental time variable as exogenous. In the process of finding desired parental time, the values of parental time are changed from 3 percent to 33 percent with an increment of 3 percent.

There are three different scenarios to be analyzed for optimality; each of which corresponds to a social goal. In the first scenario the social goal is to minimize earning inequality (minimize Earning GINI coefficient). In the second scenario, the government is to minimize wealth inequality (minimizing Wealth GINI coefficient). And in the final scenario the government aims to maximize the total output of the economy. In the model, earning is defined as wage payment received from work while wealth is the saving of each individual, not including earnings. And the total output comprises total goods consumption, education consumption and aggregate capital supply. Since the model is a closed economy, there is no net export value. For each scenario, desired parental time is sought to achieve the social goal. These findings serve as a very good source of information for relevant policy makers to devise related policies for the Thai economy.

4.2.1 Parental Time to Minimize Earning Inequality (Minimize Earning GINI Coefficient)

4.2.1.1 Finding Desired Parental Time to Minimize Earning Inequality

In this section, we assume that the elasticity of earning with respect to parental time $\sigma_1(1 - \omega)$ takes the value of 0.2. In Figure 2, the parental time are plotted against corresponding percentage change in earning GINI coefficients. The x-axis represents the values of parental time, presented in terms of ratios of the parental time over total available time, while the y-axis represents the percentage changes in earning GINI coefficients from the earning GINI coefficients of the benchmark model.

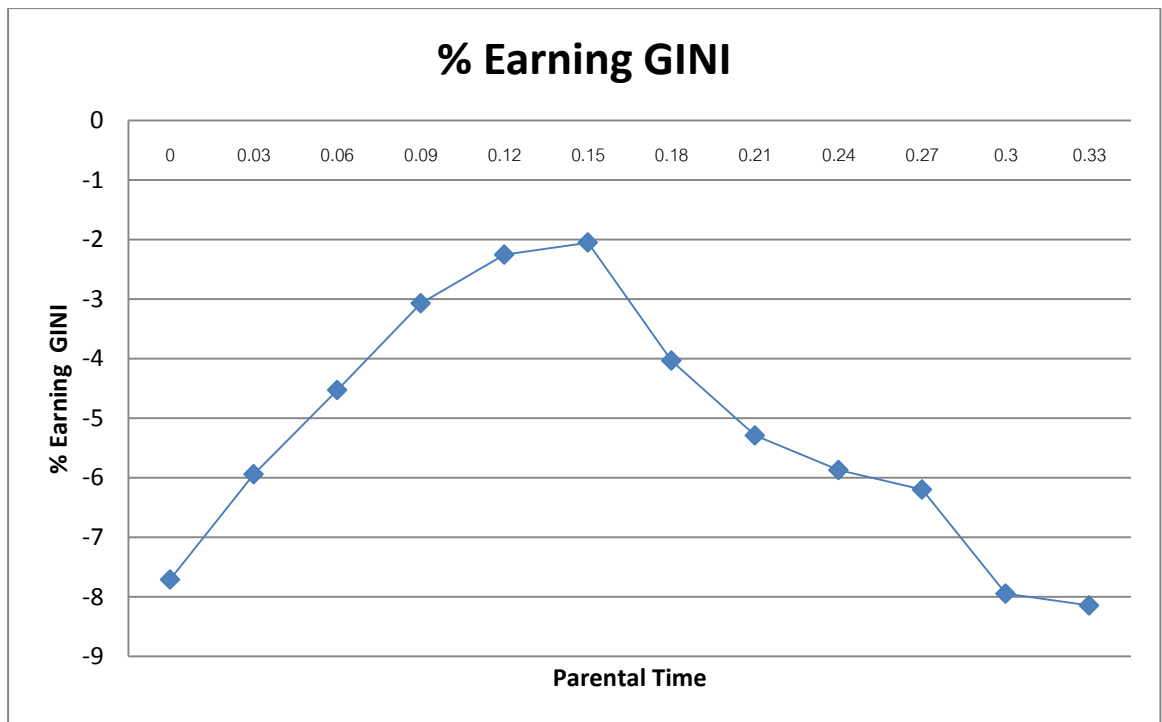


Figure 4.1 Percent Change in Earning GINI Coefficient VS Ratio of Parental Time Over Available Time

Figure 4.1 shows that enforcing the parental time with children by the government generally improves earning equality from the benchmark model as the percentage changes of earning GINI coefficients are negative for the entire range of parental time. When the parental time is zero the earning GINI is lower than that of the benchmark model by about 7.7 percent. After enforcing the parental time by an average of 3 percent or 30 minutes per day for each child raises earning GINI coefficient by 1.9 percent. When the parental time is at a low level, the earning GINI coefficient grows at an average rate of 0.5 percent for every percent of increase in parental time. As the parental time has been increased up to 9 percent, the percentage change in earning GINI coefficient reaches about -3 percent. Then an increase in parental time by one percent will raise GINI by only about 0.07 percent. The earning GINI coefficient reaches the maximum level at -2 percent where the parental time is at 15 percent. Further increases in parental time beyond 15 percent results in the decrease of earning GINI coefficients. The percentage change in earning GINI

coefficient is at the minimum of -8.1 when the parental time is beyond 33 percent or about 5 hours and 20 minutes.

How does an increase in parental time increase earning inequality at the early stage and then make earning more equal as the parental time keeps increasing? Earning of an individual is a product of his efficient wage earning and the amount of his working time. An increase in parental time allocation certainly results in an increase in efficient wage earning for the entire population. However, the amounts of working time are dependent on individual preference. For young and old adults, an increase in efficient wage earning has an ambiguous effect on working time. However, for parents, enforcing more parental time means a lesser amount of working time. In order to trace how an increase in parental time affects working times and average earnings of each group, the ratios of total working time over available time against parental time are plotted in figures 4.2-4.4 as follows.

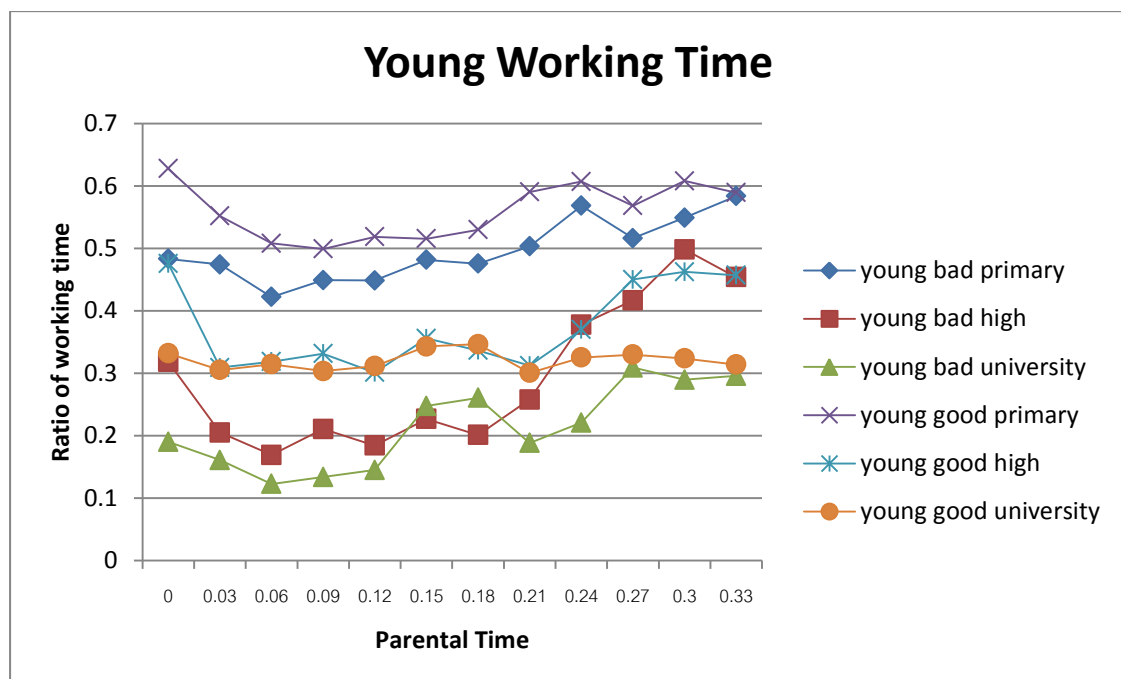


Figure 4.2 Working Time for Young Adult VS Parental Time

Figure 4.2 displays the plots of working hours against parental time for each group in the young adult generation. In general, those who have primary school

education work harder than high school finishers and university graduates. Within each education level, the ones with a good status work slightly harder than the others. Considering the amount of working time of each group when parental time changes, all young adults reduce their working time when their efficient wage earning capacity increases. The lower earning groups, like primary school finishers, reduce working time more than the higher earning groups, like university graduates, when the parental time is set at 3 percent to 6 percent. The working hours become relatively constant when the parental time is between 6 - 15 percent. For the parental time of more than 18 percent, the working hours increase for most groups. Again, the lower earning groups increase their working hours more than the higher earning groups.

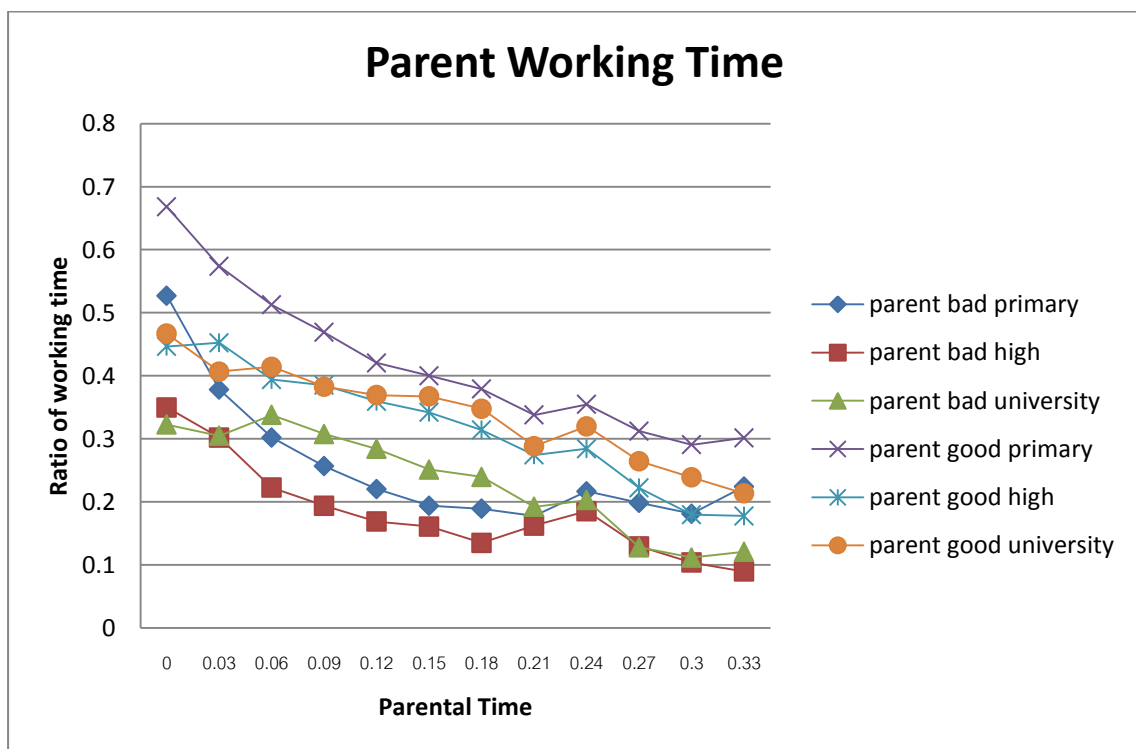


Figure 4.3 Working Time for A Parent VS Parental Time

For parent generation, figure 4.3 displays the plots of a parent's working time against parental time. All parents are forced to work less when the government enforces more working time. For example a parent who finishes primary school and having a good status has to reduce working time from 57 percent of

available time when he must give 3 percent parental time to 30 percent of available time, when he must give 30 percent parental time. Considering the differences in rate of reduction of working time between groups, it is obvious that between the parental time of 0 percent and 18 percent, the lower earning parents reduce working time at a higher rate than the high earning parents. However, the lower earning parents reduce working time at a lower rate than the high earning parents for parental time of more than 18 percent.

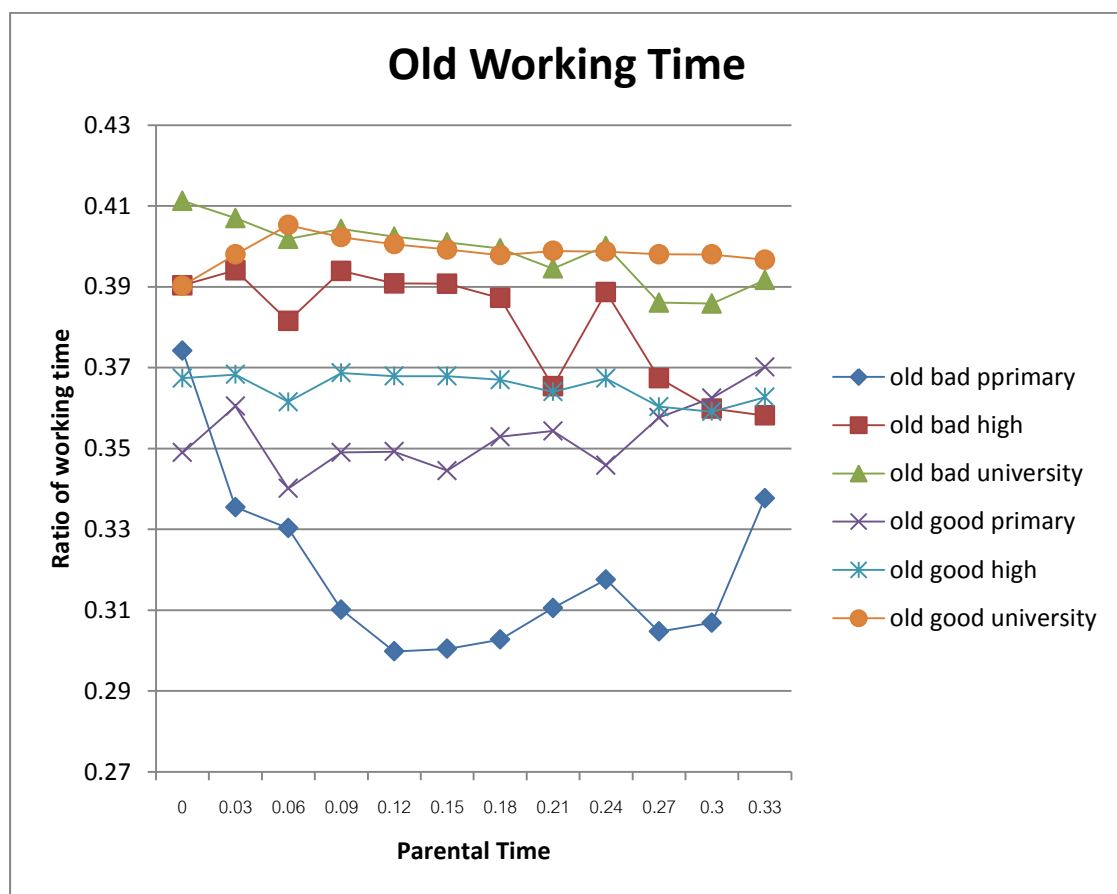


Figure 4.4 Working Time for an Old Adult VS Parental Time

For the older adult generation, figure 4.4 displays the plots of old adults' working time against parental time. With the exception of old primary school finishers who have a bad status, all groups of older adults are least affected by the increase in efficient wages. Their working times have not changed much. The high

earning old adults slightly reduce their working time. The lower earning adults reduce their working time much more when the parental time is between 0 - 12 percent. As the result they will gradually increase their working time as parental time is raised.

Figure 4.2-4.4 help to give a clearer picture of earning gaps between the high earners and the low earners. The changes in earning GINI coefficients are due not only to the fact that the parental time causes a bigger gap in efficient wages among those who have different education levels and states, but also to how different groups adjust their working time given higher efficient wages. Although an increase in efficient wages makes individuals adjust their working hours, the low earners and high earners adjust working hours differently. When the parental time is low, an increase in parental time causes the low earning groups to reduce their working time comparatively more than the high earning groups. Hence earning gaps grow at this stage. When parental time is high the increase in efficient wages again affects the lower earners more than the higher ones in terms of working hours. However, the effect is in the opposite direction. By raising the already high parental time, low earners in all generations will work harder, narrowing earning gaps. When the parental time is low, enforcing more parental time causes stage earning inequality to grow. However, it becomes less after the parental time reaches 15 percent.

4.2.1.2 The parental time to minimize earning inequality with different values of the elasticity of earning with respect to parental time

This section explores sensibility of earning inequality due to the change of elasticity of earning with respect to parental time. The value of elasticity of earning with respect to parental time calculated as $\sigma_1(1 - \omega)$ takes the values of 0.1, 0.15, 0.2, 0.25 and 0.3 respectively. Figure 4.5 shows the plots of percentage change in earning GINI coefficients against the ratios of parental time with different values of elasticity of earning with respect to parents' parental time. In general, enforcing parents to allocate time for children mostly improves earning equality from that of the benchmark model for all the ranges of parental time. All graphs representing different values of the elasticity of earning with respect to parental time take the same inverted bowl shape. In the early stage of increasing parental time from 0 - 6 percent the percentage changes in earning GINI coefficients increase (at almost constant rates with $\sigma_1(1 - \omega)$ equal to 0.1, 0.15 and 0.2 and at an increasing rate with ω equal to

0.25 and 0.3). Further increases in parental time will cause an increase in percentage change in earning GINI coefficients but at decreasing rates until they reach the maximum. Thereafter, the percentage change in earning GINI coefficients will decrease.

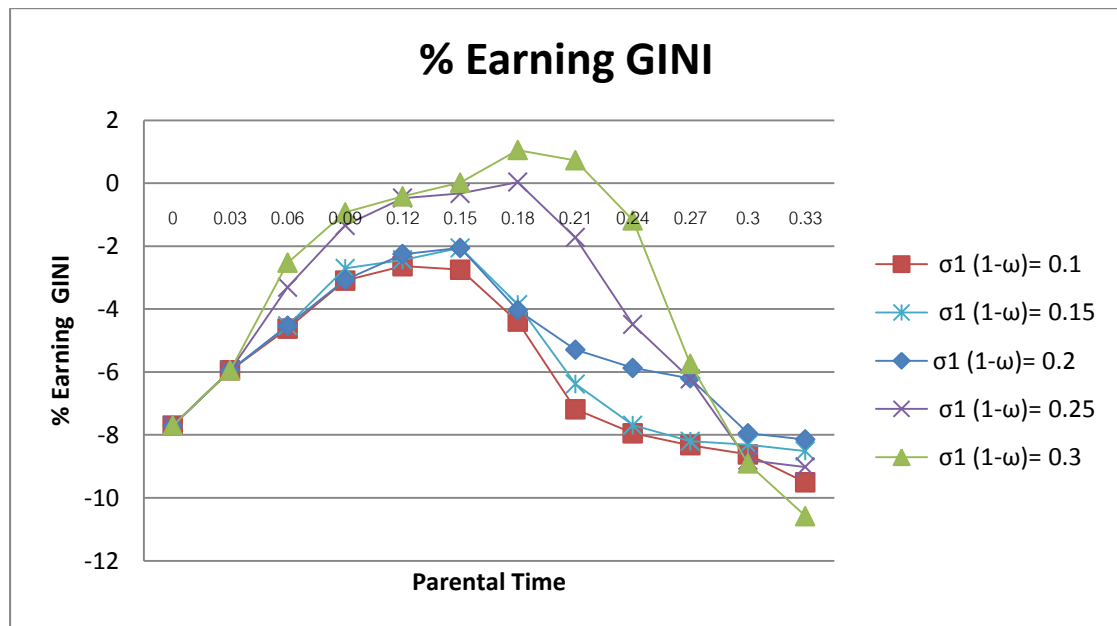


Figure 4.5 Percentage Change in Earning GINI Coefficients against the Ratios of Parental Time with Different Values of Elasticity of Earning with Respect to Parental Time

Comparing the graph lines among different values of the elasticity of earning with respect to parental time, overall the increase in value of the elasticity of earning with respect to parental time will shift most of the graph line upwards. The higher value of the elasticity of earning with respect to parental time means the bigger gap of earning wages is by the same amount which parental time can produce. These bigger gaps of efficient wages contribute to bigger earning gaps across the population. With the elasticity of earning with respect to parental time is at the lowest level of 0.1. The percentage change in earning GINI coefficient rises 1.9 percent for the first 3 percent increase in parental time, 1.4 percent for the second 3 percent increase in parental time and 1.6 percent for the third 3 percent increase in parental time. The

maximum percentage change in earning GINI coefficients is -2.6 percent when the parental time is at 12 percent or about 2 hours. Further increases of parental time from that point will cause the percentage change in earning GINI coefficients to decrease. The percentage change in earning GINI coefficient will be at the minimum point when the parental time is 33 percent or about 5 hours and 15 minutes. With the elasticity of earning with respect to parental time is at the highest level of 0.3. The percentage change in earning GINI coefficients rises 1.9 percent for the first 3 percent increase in parental time, 3.7 percent for the second 3 percent increase in parental time, and 1.7 percent for the third 3 percent increase in parental time. Further increases in parental time will cause the increase in percentage change in earning GINI coefficient but at decreasing rates around 0.25 percent for every percent increase in parental time until they reach maximum points. The maximum percentage change in earning GINI coefficients is about 1 percent compared with that of the benchmark model when the parental time is at 18 percent or about 3 hours. Further increase of parental time from that point will cause the percentage change in earning GINI coefficients to decrease. Table 10 sums up some critical values of percentage change in earning GINI coefficient for each value of the elasticity of earning with respect to parental time. The minimum earning GINI coefficients are at the 33 percent of parental time.

Table 4.7 Percentage Change in Earning GINI Coefficient for Each Value of the Elasticity of Earning with Respect to Parental Time

$\sigma_1(1 - \omega)$	Percent Parental Time When GINI at Maximum	Maximum percent Change in Earning GINI Coefficient	Minimum percent Change in Earning GINI Coefficient (At 33 percent Parental Time)
0.1	12 (2 hours)	-2.6	-9.5 percent
0.15	15 (2 hours 24 Minutes)	-2.05	-8.5 percent
0.2	15 (2 hours 24 Minutes)	-2.05	-8.15 percent
0.25	18 (3 hours)	0.04	-9.0 percent
0.3	18 (3 hours)	1.05	-10.6 percent

4.2.2 Parental Time to Minimize Wealth Inequality (Minimize Wealth GINI Coefficient)

4.2.2.1 Finding the desired parental time to minimize wealth inequality

With the elasticity of earning with respect to parental time $\sigma_1(1 - \omega)$ is 0.2 Figure 4.6 shows the plots of parental time against corresponding percentage change in wealth GINI coefficient. As in the previous sections, the x-axis represents the ratios of the parental time over total available time while the y-axis represents the percentage changes of the wealth GINI coefficient from the wealth GINI coefficient of the benchmark model.

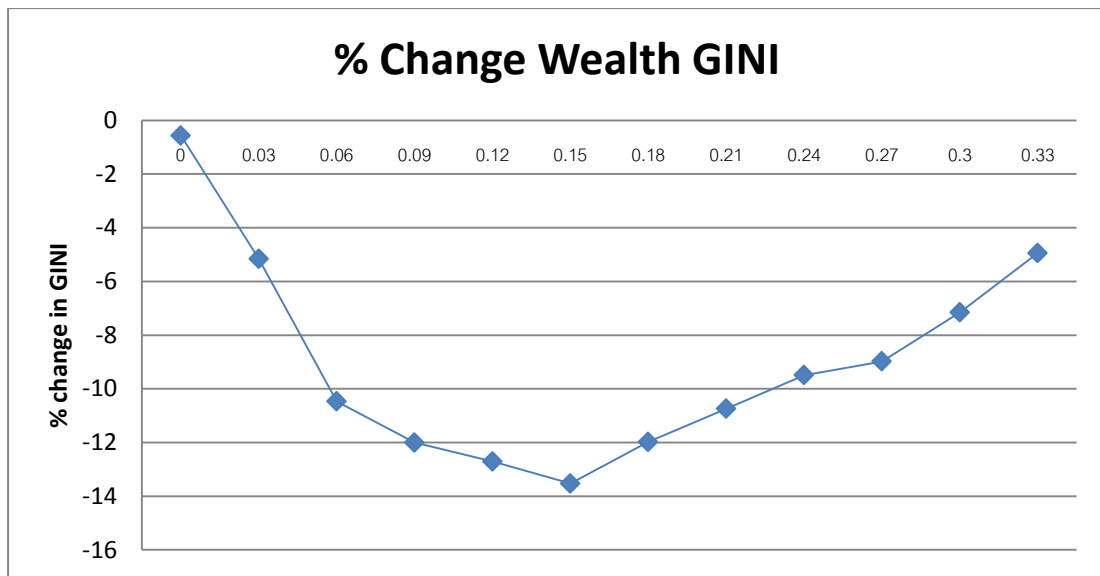


Figure 4.6 Percent Change in Wealth GINI Coefficient VS Ratio of Parental Time Over Available Time

Figure 4.6 shows that the shape of percentage change in wealth GINI coefficient plotted against parental time is a bowl shape. At the start, enforcing the parental time by an average 3 percent or 30 minutes per day for each child actually decreases the wealth GINI coefficient to -5.2 percent. A further 3 percent increase in parental time will decrease the wealth GINI coefficient another 5.3 percent. Then the wealth GINI coefficient decreases at a slower rate of 0.34 percent for every percent of increase in parental time. As the parental time has been increased up to 15 percent the percentage change in wealth GINI coefficient reaches the minimum value of about -13.5 percent. Then from the minimum point an increase in parental time by one percent will raise the wealth GINI coefficient by about 0.37 percent. The percentage change in wealth GINI coefficient reaches -4.94 percent as the parental time is at 33 percent.

The plots of percentage change in wealth GINI coefficient in figure 4.6 takes a bowl shape and looks similar to a mirror reflecting a shape of plots of percentage changes in the earning GINI coefficient in figure 2. How does an increase in parental time decrease wealth inequality at the early stage and then make wealth inequality grow again as the parental time keeps increasing? As defined earlier,

wealth is the savings that individuals make in each generation. In order to understand why the plots of wealth GINI coefficients take such a shape we must trace back to see how an increase in parental time affects the average savings of each group. The plots of savings against parental time are presented in figure 4.7 and 4.8 for the young adult and parent generation.

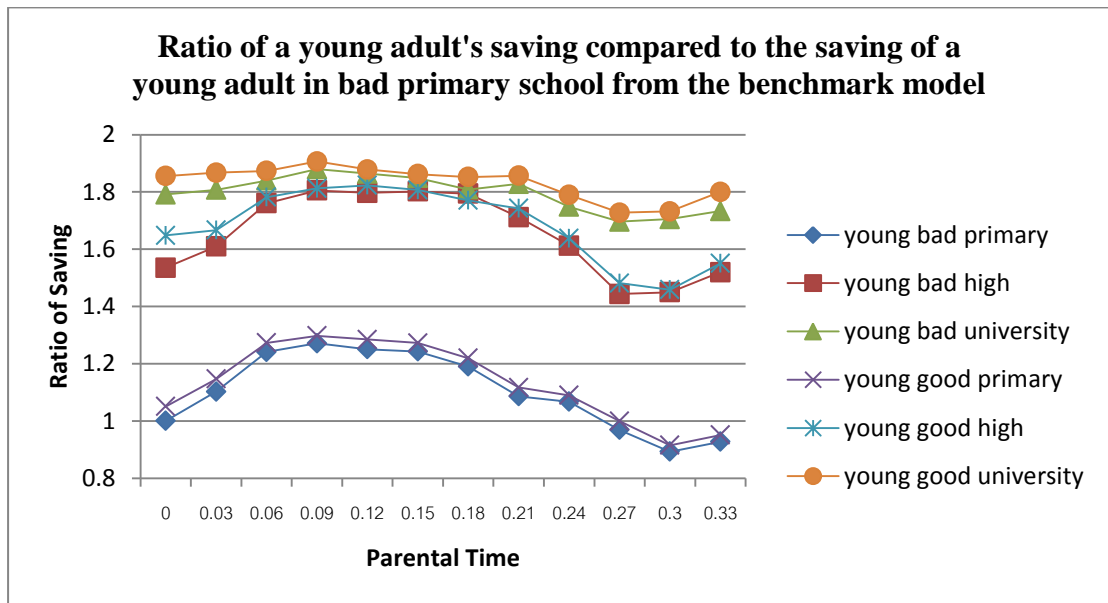


Figure 4.7 Plot of Young Adult Wealth VS Ratio of Parental Time Over Available Time

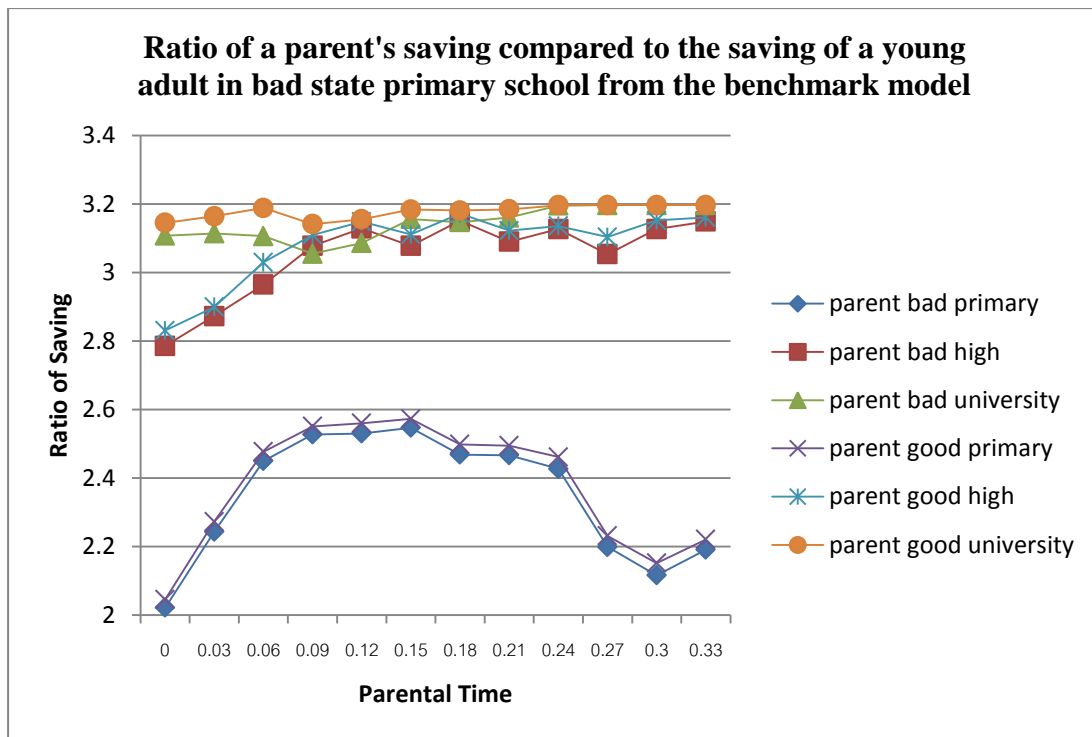


Figure 4.8 Parent Wealth VS Ratio of Parental Time Over Available Time

In Figures 4.7 and 4.8, the x-axis represents the ratios of the parental time over total available time while the y-axis represents the ratios of an average saving of an individual corresponding to the parental time over the savings of an individual from the benchmark model. In general, Figures 4.7 and 4.8 show a similar pattern in which individuals with primary education who are the majority of the population tend to save less than those with high school and university education. Their saving behavior is also more sensitive toward any changes in earning. When the parental time is small, increasing it and the earnings causes individuals with primary education to increase their savings at a higher rate than others. On the other hand, high earners can afford not to increase their savings as much for smooth and even consumption as their wages are higher in future periods. When the parental time is between 0 percent and 6 percent the wealth GINI coefficient decreases. When the parental time reaches about 6-9 percent, further increase in parental time does not affect the saving of low earners as much as it does at the beginning. The wealth GINI coefficient still decreases but at a lower rate. When the parental time is between 9-15

percent all individuals keep saving about the same amount. And during this time the gap between the richest and the poorest in terms of saving is closest. Increasing parental time of greater than 15 percent will result in low earners to gradually decreasing their saving, making the wealth GINI coefficient increase. So at the early stage, enforcing more parental time causes wealth inequality to become less. However, it later increases after the parental time reaches 15 percent.

4.2.2.2 The parental time to minimize wealth inequality with different values of the elasticity of earning with respect to parental time

As in the previous sections, to explore the sensibility of wealth inequality due to the change in elasticity of earnings with respect to parental time, the elasticity of earnings with respect to parental time is changed to 0.1, 0.15, 0.2, 0.25 and 0.3. Figure 4.9 shows the plots of percentage change in wealth GINI coefficient against the ratios of parental time with different value of elasticity of earning with respect to parental time. In general, all graphs representing different values of the elasticity of earnings with respect to parental time take the same shape. In the early stage of the increase of parental time from 0 percent to about 9 percent the percentage changes in wealth GINI coefficient decrease. Further increases in parental time will cause the percentage change in wealth GINI coefficient to reach a minimum. Thereafter, the percentage change in wealth GINI coefficient will increase.

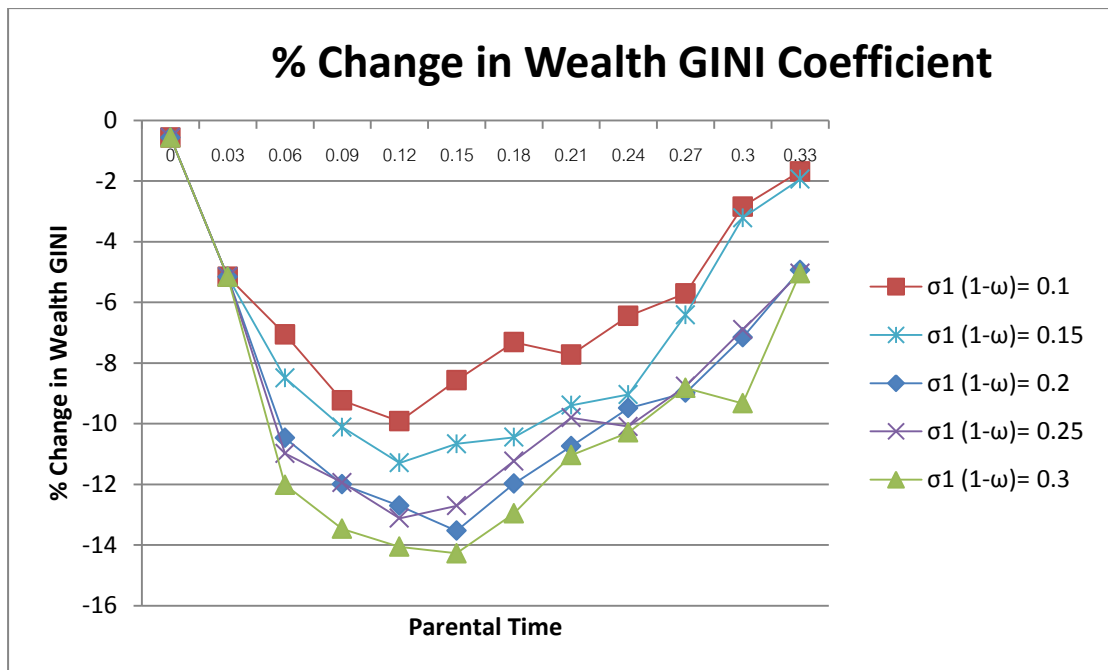


Figure 4.9 Percentage Change in Wealth GINI Coefficient against the Ratios of Parental Time with Different Values of Elasticity of Earnings with Respect to Parental Time

Comparing the graph lines among different values of the elasticity of earnings with respect to parental time reveals that the increase in value of this elasticity will pull almost the entire graph line downwards. The higher value of the elasticity means the bigger gap of wealth inequality, then the gap that is parental time can narrow. When the elasticity of earnings with respect to parental time is at the lowest level of 0.1, the percentage change in wealth GINI coefficient drops to -5.2 percent for the first 3 percent increase in parental time. Then the wealth GINI coefficient drops 1.9 percent for the second 3 percent increase in parental time and another 2.2 percent for the third 3 percent increase in parental time. The minimum percentage change in wealth GINI coefficient is -9.9 percent when the parental time is at 12 percent or about 2 hours. Further increases of parental time from that point will cause the percentage change in the wealth GINI coefficient to increase. When the elasticity of earnings with respect to parental time is at the highest level of 0.3, the percentage change in wealth GINI coefficient drops to -5.2 percent for the first 3

percent increase in parental time. Then the wealth GINI coefficient drops another 6.9 percent for the second 3 percent increase in parental time and 1.4 percent for the third 3 percent increase in parental time. Further increases in parental time will cause the decrease in percentage change in wealth GINI coefficient, but at decreasing rates around 0.25 percent for every one percent increase in parental time until they reach the minimum. The minimum percentage change in wealth GINI coefficient is -14.27 percent when the parental time is at 15 percent or about 2 hours and 30 minutes. Further increases of parental time from that point will cause the percentage change in wealth GINI coefficient to increase. Table 4.8 sums up the minimum values of percentage change in the wealth GINI coefficient for each value of the elasticity of earnings with respect to parental time.

Table 4.8 Percentage Change in Wealth GINI Coefficient for Each Value of the Elasticity of Earning with Respect to Parental Time

$\sigma_1(1 - \omega)$	Percent Parental time When Wealth GINI at Minimum	Minimum percent Change in Wealth GINI Coefficient
0.1	12 (2 hours)	-9.9
0.15	12 (2 hours)	-11.29
0.2	15 (2 hours 30 minutes)	-13.53
0.25	12 (2 hours)	-13.12
0.3	15 (2 hours 30 minutes)	-14.27

4.2.3 Parental Time to Maximize Total Output

4.2.3.1 Finding the desired parental time to maximize total output

As in the previous section, the assumption is that the elasticity of earning with respect to parental time, $\sigma_1(1 - \omega)$, takes the value of 0.2. In Figure 4.10, the parental time are plotted against corresponding percentage change in total outputs. The total output for the economy is taken as the sum of goods consumption, education consumption and saving. On the x-axis, the values of parental time is presented in term of ratios of the parental time over total available time while on the

y-axis, the percentages of total outputs are calculated as percentage change in total output from the total output of the benchmark model.

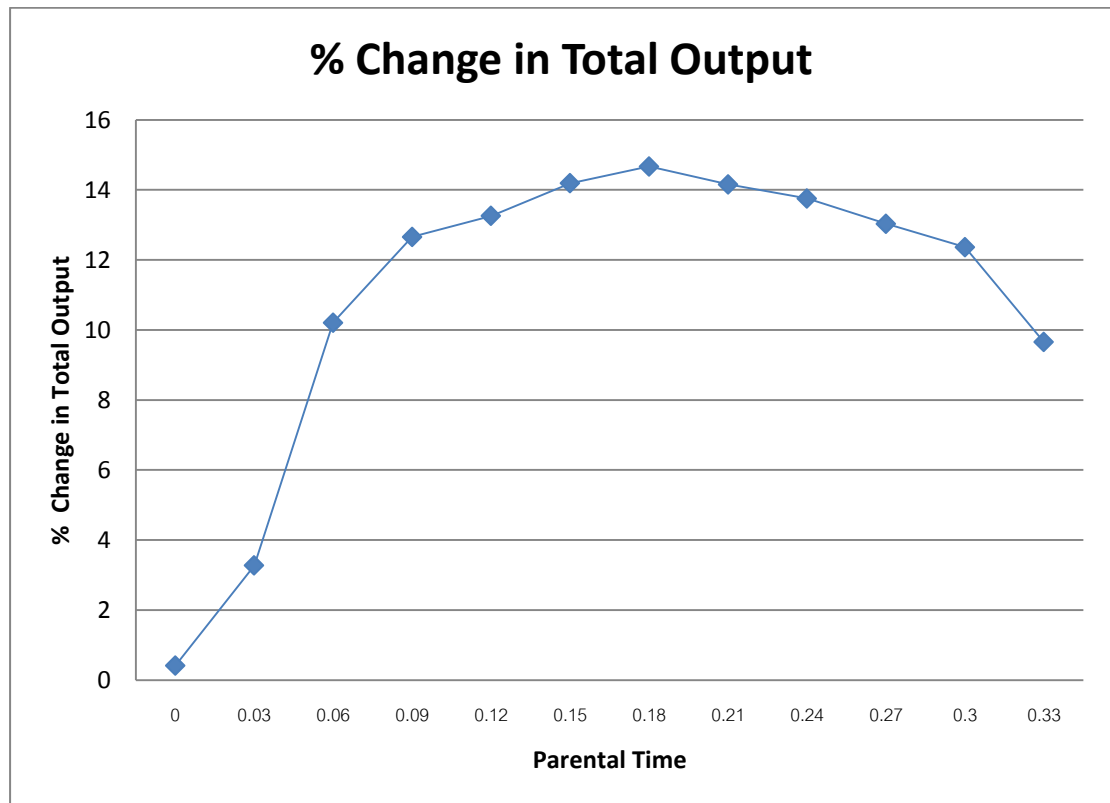


Figure 4.10 Percent Change in Total Output VS Ratio of Parental Time Over Available Time

From Figure 4.10, when the government enforces the parental time from start by an average of 3 percent or 30 minutes per day for each child, the percentage change in total output rises by around 2.8 percent. As the parental time increases for another 3 percent from 3 - 6 percent, the percentage change in total output rises sharply by another 7 percent. The rate of increase in the percentage change in total output from a percentage increase in parental time at this stage is at the highest. After the parental time has reached 6 percent, further increases in parental time cause the percentage change in total output to rise at a decreasing rate. It is noted that the percentage change in total output reaches the maximum value of 14.67 percent when the parental time is at 18 percent or about 3 hours per day. Further

increases in parental time beyond 18 percent leads to the percentage change in total output to decrease although it is still positive when compared with the total output of the benchmark model.

As discussed in the previous chapter, the parental time is one of the factors to build up human capital. Increasing parental time results in increases in human capital for the entire population. These increases in human capital are reflected through higher efficient wage earnings for each level of education. With higher efficient wage earnings, individuals can earn more from working for the same amount of working time. As for the economy as a whole, an increase in parental time means an increase in the aggregate labor productivity. Given the same amount of aggregate capital, the total output of the economy can be more productive. However, the increase in parental time also comes with costs. Parents will have to reallocate the time left available from taking care of children to leisure and work. In general, this means that parents will have less working time which in turn has a negative effect on the labor supply. Although enforcing more parental time from parents increases efficient wage earning, its cost is fewer working hours for parents. In the previous section, Figure 4 shows that parents work less when the parental time is increased. So for the labor supply in aggregate, as the parental time increases labor supply increases at the beginning. For further increases in parental time, the cost of losing parent labor supply grows such that it slows down the increase in percentage change in labor supply. Finally at the high level of parental time the percentage change in labor supply decreases. Section 4.2.2.1 the low earning groups are more sensitive to change in their savings when the parental time is changed. When the parental time is at the low level, an increase in parental time increases total savings as low earning groups increase their savings. When the parental time is high, enforcing more parental time will make the low earning groups decrease savings. As the result, the aggregate saving decreases. To sum up, when the parental time is at a low level, the benefits of enforcing more parental time exceed the cost of reducing working time. Hence the total output along with wages and savings increases. Once the level of parental time is already high, any further increase causes the costs such as reduction of parents' working time and total savings to exceed the benefit of lifting efficient wage earnings. Hence, the total output is maximized at the parental time of 18 percent.

4.2.3.2 The parental time to maximize total output with different values of the elasticity of earning with respect to parental time

As in the previous section, the elasticity of earning with respect to parental time is altered for total output sensibility analysis. The altered values are 0.1, 0.15, 0.2, 0.25 and 0.3. Figure 4.11 shows the plots of percentage change in total output against the ratios of parental time with different value of elasticity of earnings with respect to parental time. In general all graphs representing different values of the elasticity of earning with respect to parental time take the same shape. In the early stage of increasing parental time from 0 percent to 6 percent the percentage change in total output increases with the increasing rates. Further increases in parental time will cause the increase in percentage change in total output, but at decreasing rates until they reach their maximum. Thereafter, the percentage change in total output will decrease.

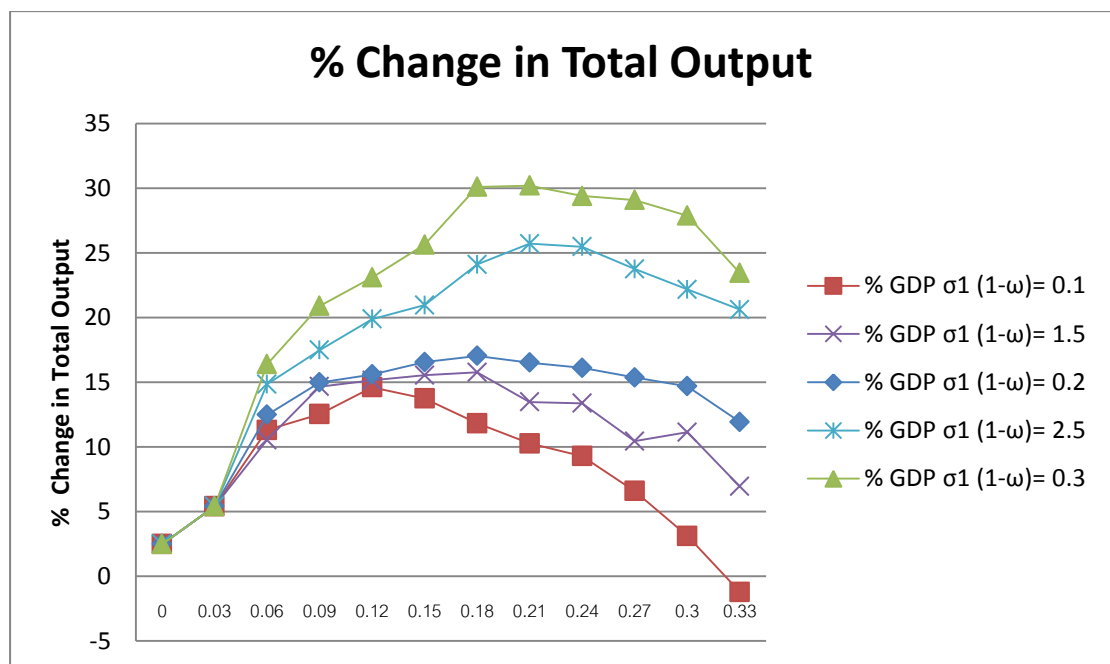


Figure 4.11 Plot of Percent Change in Total Output VS Ratio of Parental Time Over Available Time for Different Values of Elasticity of Earning with Respect to Parental Time

By comparing the graph lines among different values of the elasticity of earning with respect to parental time, it is obvious that increasing the value of the elasticity of earning with respect to parental time will lift the entire graph line upwards. The higher value of the elasticity of earning with respect to parental time means higher productivity for the parental time variable on efficient wages (human capital) given the same education level. With the elasticity of earnings with respect to parental time at the lowest level of 0.1, the percentage change in total output rises from 2.8 percent for the first 3 percent increase in parental time, to 5.8 percent for the second 3 percent increase in parental time. The maximum percentage change in total output is 12.29 percent when the parental time is at 12 percent or about 2 hours. Further increases of parental time from that point will cause the percentage change in total output to decrease. The percentage change in total output will be negative if the parental time is more than 30 percent, or about 5 hours. With the elasticity of earnings with respect to parental time at the highest level of 0.3, the percentage change in total output rises from 2.8 percent for the first 3 percent increase in parental time to 10.8 percent for the second 3 percent increase in parental time. The maximum percentage change in total output is 27.55 percent, when the parental time is at 21 percent or about 2 hours and 20 minutes. Further increases of parental time from that point will cause the percentage change in total output to decrease. Table 4.9 sums up the maximum percentage change in total output for each value of the elasticity of earnings with respect to parental time.

Table 4.9 Maximum Percentage Change in Total Output for Each Value of the Elasticity of Earning with Respect to Parental Time

$\sigma_1(1 - \omega)$	Percent Parental time	Maximum percent Change in Total Output
0.1	12 (2 hours)	12.29
0.15	18 (3 hours)	13.41
0.2	18 (3 hours)	14.66
0.25	21 (3 hours 20 minutes)	23.16
0.3	21 (3 hours 20 minutes)	27.55

4.2.4 Discussion on Desired Parental Time For Thai Economy

The summary of solutions for all three scenarios discussed previously is reported in Table 4.10

Table 4.10 Summary of Solutions for Parental Time for Thai Economy

$\sigma_1(1 - \omega)$	Percent Parental time When Earning GINI at Maximum	Percent Parental time for Minimum Earning GINI Coefficient	Percent Parental time for Minimum Wealth GINI Coefficient	Percent Parental time for Maximum Total Output
0.1	12 (2 hours)	33 (5 hours 20 minutes)	12 (2 hours)	12 (2 hours)
0.15	15 (2 hours 24 Minutes)	33 (5 hours 20 minutes)	12 (2 hours)	18 (3 hours)
0.2	15 (2 hours 24 Minutes)	33 (5 hours 20 minutes)	15 (2 hours 30 minutes)	18 (3 hours)
0.25	18 (3 hours)	33 (5 hours 20 minutes)	12 (2 hours)	21 (3 hours 20 minutes)
0.3	18 (3 hours)	33 (5 hours 20 minutes)	15 (2 hours 30 minutes)	21 (3 hours 20 minutes)

Setting parental time allocation according to one specific social goal is a straight forward step. However, choosing the best solution seems to be a difficult task, given the unclear social goal as shown in Table 4.10. For example, with the elasticity of earnings with respect to parental time equal to 0.2, setting the solution for parental time to 18 percent to achieve the maximum total output will come with a cost of widening earning inequality. On the other hand, minimizing earning inequality by either keeping the parental time allocation at a minimum or setting it to the maximum will not improve the situation in total output or on wealth inequality. However, when the social goal is unclear, it seems reasonable to set the target on maximizing the total output which automatically improves wealth equality. Thus setting the cost of earning inequality at around the widest among the entire range of parental time. Although enforcing more parental time may widen the human capital gap compared with enforcing less parental time, the earning equality has been improved from the benchmark model.

In section 4.1, the benchmark model produces the results from the utility maximization problem with the parental time as a choice variable. The table in Appendix B displays the percentage shares of children by parental time levels, education levels and numbers of children. Which can be summarized that 40.6 percent of the children are given 3 percent of parental time, 24.5 percent of the children are given 6 percent of parental time, 14.1 percent of the children are given 12 percent of parental time, 12.2 percent of the children are given 18 percent of parental time and 8.6 percent of the children are given 24 percent of parental time. Therefore, on average, each child in the benchmark model gets parental time for about 8.6 percent or 1 hour and 20 minutes.

Having known the current situation regarding parental time and having set a social target on one of the desired parental times, what policies should be implicated to move the parental time from the current situation to the specific target? There are a few directions to improve the current human capital based on parental time: to improve the quantity and quality of parental time. Firstly, to improve the quantity of parental time, a government can introduce an incentive that parents value high enough to give up working time and leisure time to take care of children such as tax-related incentives. With less working time parents can allocate more time to

taking care of children, thus increasing parental time. A subsidy policy can be implemented for the parent generation as well. This subsidy should work as an earning insurance so that parents with high earnings can work less and take care of their children more.

Secondly as for improving the quality of parental time, the economic model in this paper does not include a variable that directly represents the quality of parental time. Although this paper follows the human capital production function as in Casarico (2007) in which human capital of parents represents the quality of parental time as shown in equation 72, modification of the function is done and its relative terms are used to find the parental time factors.

$$h_c = q(e \cdot \bar{h})^\omega ((dt \cdot h)^{\sigma_1} Care^{\sigma_2})^{1-\omega} \quad (72)$$

$$\frac{h_c(e, \bar{h}, h, Care, dt2)}{h_c(e, \bar{h}, h, Care, dt1)} = \frac{q(e \cdot \bar{h})^\omega ((dt2 \cdot h)^{\sigma_1} Care^{\sigma_2})^{1-\omega}}{q(e \cdot \bar{h})^\omega ((dt1 \cdot h)^{\sigma_1} Care^{\sigma_2})^{1-\omega}} = \left[\frac{dt2}{dt1} \right]^{\sigma_1(1-\omega)} \quad (73)$$

In equation 73, an assumption is that the children are given the same level of education by the same parents, and the heterogeneity of the parents' human capital is neglected. Including the heterogeneity of the parents' human capital in the model should theoretically allow the quality of parental time to affect human capital. In this concept, a parent with a higher level of human capital should provide a higher quality time for the same parental time compared with a parent with lower level of human capital. However, in reality, forcing a higher human capital parent to spend time with his child does not necessarily mean that his child gets high quality time. The high human capital parent can be with a child but do nothing to develop human capital. So the parent's human capital may not be an accurate representation of the quality of parental time.

As far as the model is concerned, the elasticity of earnings with respect to parental time, $\sigma_1(1 - \omega)$, can be viewed as the quality component of parental time. When the parental time is relatively less productive, the elasticity of earnings with respect to parental time may be at the lowest level of 0.1 in this study. The quality of parental time is not so good. For a parent with very high value of elasticity of earnings with respect to parental time, 0.3 in this study, his quality of parental time is high.

Given the same parental time, a parent with higher elasticity of earnings with respect to parental time or higher quality of parental time, should raise children with higher earning capability. Taking an example from Figure 4.11, in order to raise the total output to around 15 percent compared to the benchmark model, the parents with higher quality parental time ($\sigma_1(1 - \omega)=3$) require only 5 percent of the total time with their children while the parents with lower quality parental time ($\sigma_1(1 - \omega) = 1.5$) require 11 percent of their total time. This implies that keeping the quantity of parental time fixed, the total output of the economy can be improved by improving the quality of such parental time. In this sense a policy recommendation for the government would be to improve the quality of parental time for those with a very low level of elasticity to the standard level of society. For example, the government can build a better environment for children to be raised in or enhance parents' awareness on the benefit of parental time.

CHAPTER 5

CONCLUSION

The first objective of this study is to construct an economic model with intergenerational transfer within the family that includes bequests, education and parental time, in order to mimic the earning and income inequality of Thailand. The second objective is to determine parental time for the model economy to achieve three social goals; minimizing earning and wealth inequality, and maximizing total output.

A five period overlapping generation model is constructed and differs from a general heterogeneous overlapping generation model in many ways. While recent studies focus on both unplanned and planned bequests (money and education) as intergenerational links, a parental time variable is added into the economic model. Parental time variable is another channel, besides education, that helps children develop their human capital which in turn increases efficient wage earning and wealth in the future. The calibrated parameters from previous studies are used and the results are compared with some statistics, in particular, education distribution and the earning and income distribution, generated by a previous study from the 2000 Household Socio-Economic Survey.

In general, the model generates the education distribution and the earning and income distribution reasonably close to those of the real data. Then the parental time link in the model is disabled, keeping all parameters with the same values. The finding is that the model without parental time is not as good as the benchmark model in terms of mimicking the education distribution and earning distribution. Therefore, it is concluded that the parental time link is important and that the benchmark model can be used as a reference economy in any future analysis.

In order to find the desired solutions of parental time for the Thai economy, the benchmark model is slightly modified by taking the parental time variable as exogenous. Altering different values of parental time in the model leads to a generating of the results for three different scenarios, each of which corresponds to the social goals: minimizing earning GINI coefficients and wealth GINI coefficients and maximizing the total output. For each scenario, the model generates various results in different environments using different values of elasticity of earnings with respect to parents' parental time. To minimize wealth inequality the parental time should be set at 12-15 percent, and to maximize the total output, the average parental time should be set at 12-21 percent of available time. While increasing the parental time will be beneficial in improving output and wealth equality, it comes with the cost of wider earning gaps.

Suggestions are presented here to improve the current situation on parental time: to improve on the quantity and quality of parental time. To improve on the quantity of parental time, a government can introduce a policy to create an incentive for parents to give up working and leisure time in order to take care of their children. As far as the quality of parental time is concerned, a government can help improve the quality of parental time by creating a better environment for children or enhancing parents' awareness on the benefits of increased parental time. A recommendation for future research is that a variable representing the quality of parental time should be included in any model for future study.

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APPENDIX

APPENDIX A

The Earning Profile

Generation	Parental time	Education	Bad State	Good State
Young Adults	dt1	Primary School	1	1.945
	dt2		1.15	2.237
	dt3		1.32	2.567
	dt4		1.43	2.781
	dt5		1.52	2.956
	dt1	High School	1.21	2.66
	dt2		1.392	3.059
	dt3		1.597	3.511
	dt4		1.73	3.804
	dt5		1.839	4.043
	dt1	University	1.817	4.76
	dt2		2.09	5.474
	dt3		2.398	6.283
	dt4		2.598	6.807
	dt5		2.762	7.235
Parents	dt1	Primary School	1.04	4.74
	dt2		1.197	5.451
	dt3		1.374	6.2
	dt4		1.489	6.778
	dt5		1.582	7.205
	dt1	High School	3.415	8.22
	dt2		3.927	9.453
	dt3		4.508	10.85
	dt4		4.883	11.755

Generation	Parental time	Education	Bad State	Good State
	dt5		5.191	12.494
	dt1	University	5.92	12.86
	dt2		6.808	14.789
	dt3		7.814	16.975
	dt4		8.466	18.39
	dt5		8.998	19.547
Old Adults	dt1	Primary School	1.096	6.86
	dt2		1.26	7.889
	dt3		1.447	9.055
	dt4		1.567	9.89
	dt5		1.666	10.427
	dt1	High School	2.5	9.74
	dt2		2.875	11.201
	dt3		3.3	12.857
	dt4		3.575	13.928
	dt5		3.8	14.805
	dt1	University	7.6	14.38
	dt2		8.74	16.537
	dt3		10.032	18.982
	dt4		10.868	20.563
	dt5		11.552	21.858

APPENDIX B

Results from the Benchmark Model

Percentage Share of Children by Number of Children, Education and Parental time in Benchmark Model

No.of children	Primary School						High School						University					
	dt1	dt2	dt3	dt4	dt5	Sum	dt1	dt2	dt3	dt4	dt5	Sum	dt1	dt2	dt3	dt4	dt5	Sum
	0.03	0.06	0.12	0.18	0.24		0.03	0.06	0.12	0.18	0.24		0.03	0.06	0.12	0.18	0.24	
1	1.227	0.998	1.214	0.817	0.768	5.023	0.055	0.556	1.286	0.000	0.433	2.330	0.824	3.521	1.980	4.509	1.034	11.868
2	6.424	5.531	1.647	4.301	2.989	20.893	2.586	3.439	4.378	0.289	0.000	10.693	2.011	0.241	0.000	0.000	0.000	2.252
3	7.730	6.635	1.265	1.439	1.238	18.308	1.909	0.430	0.000	0.000	0.000	2.339	0.000	0.000	0.000	0.000	0.000	0.000
4	7.836	2.411	1.045	0.237	2.157	13.686	0.362	0.000	0.000	0.000	0.000	0.362	0.000	0.000	0.000	0.000	0.000	0.000
5	9.491	0.754	1.303	0.560	0.000	12.108	0.138	0.000	0.000	0.000	0.000	0.138	0.000	0.000	0.000	0.000	0.000	0.000
Sum						70.019						15.862						14.120

BIOGRAPHY

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