

Thaweedet Yantassanavanich 2007: Vanishing Sum and Roots of Unity.  
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Vichian Laohakosol, Ph.D. 26 pages.

A vanishing sum is an equation

$$u_1\alpha_1 + \cdots + u_k\alpha_k = 0,$$

where each  $\alpha_i$  is an  $n^{\text{th}}$  root of unity and  $u_i \in \mathbb{Q}$ .

In 1958, Schoenberg constructed a  $\mathbb{Z}$ -basis for the module whose elements satisfy a vanishing sum and proved identities about cyclotomic polynomials. Moreover he proved that if  $u \in \mathbb{Z}^n$  satisfies a vanishing sum, then it belongs to a  $\mathbb{Z}$ -module whose basis consists of all possible regular subpolygons inscribed in the regular  $n$ -gon.

In 2000, Lam and Leung proved that if  $u \in \mathbb{Z}^n$  satisfies a vanishing sum, then  $u$  can be represented as a linear  $\mathbb{Z}$ -combination of elements belonging to subgroups of the cyclic group of order  $n$ .

In this thesis, we construct a  $\mathbb{Z}$ -basis for the module of elements satisfying more general vanishing sums. Next, we define general cyclotomic polynomials and give their basic properties. Furthermore, we extend Schoenberg's results about the  $\mathbb{Z}$ -basis of a vanishing sum module as well as its geometric meaning to this general setting. In the last part, we compare between the main theorem of Lam and Leung with that of Schoenberg.

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