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THESIS

THE LOSS OF QUANTUM COHERENCE INDUCED BY GAUSSIAN RANDOM POTENTIAL

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In this thesis, we study effects of the Gaussian random potential on the quantum coherence by using Feynman path integration method. It is found that the off-diagonal elements of the density matrix decrease as two characteristic decoherence time scale. The decrease of the off-diagonal elements was visualized. We have demonstrated that the decoherence process of a heavy mass and light mass particle can be controlled by a wide range of the coherence length.

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Student’s signature Thesis Advisor’s signature

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TABLE OF CONTENTS

Page

LIST OF FIGURES

THE LOSS OF QUANTUM COHERENCE INDUCED BY GAUSSIAN RANDOM POTENTIAL

INTRODUCTION

During the last 30 years it became clear that a transition role between quantum to classical is played by the natural environments interacting with a quantum system. Classical properties of interest system emerge as unavoidable where interaction switch on. By the meaning of transition, Born has interpreting the Shrodinger's wave function as the probability amplitude and no meaning in physical realm. Before observable, state of quantum system is in the superposition state weight by the probability amplitude corresponding for each state. Any observable or experiment acting on the system, quantum theory is implies that quantum state can be collapse to the single state and give the classical results corresponding with the observable. This content knows as problem of quantum measurement which still remains in the present day.

The process is widely known as "decoherence". This process proposed mechanism for the treating problem of quantum measurement. The coupling of the quantum system to an environment or subsystem generally is accepted that cause a rapid loss of coherence. The very notion of a quantum system consisting with other subsystems can be described adequately as a composite system (the interested system and thermal bath). The interacting with their environments comprising a large number of degree of freedom. The development of the decoherence program was reached in 1981–82 when Zurek published two papers on decoherence in Physical Review D. He pointed out that the importance view which is central to decoherence and the problem of the quantum-to-classical transition. Zurek developed the concept of environment-induced super-selection that it became a cornerstone of the decoherence program. He defined a precise framework for determining the pointer states and also showed how environment-induced super-selection effectively. Another important contribution to the development of the decoherence program was made in 1984 when he derived a general and simple expression from which typical decoherence time scales could be qualitatively evaluated.

The brief review about this view is follow by considering the standard double-slit experiment. Considering a wave packets, representing one a particle, have been separated and travel along two different paths. The initial state is linear superposition

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\Psi_1\rangle + \frac{1}{\sqrt{2}}|\Psi_2\rangle \tag{1}$$

The state of a particle passing through the slit 1 and 2 are defined as a Gaussian wave packet

$$|\Psi_1\rangle = \frac{1}{L^{1/2}\pi^{1/4}} exp\left[-\frac{\left(x+\frac{\Delta x}{2}\right)^2}{2L^2}\right]|\psi_1\rangle \;,\; |\Psi_2\rangle = \frac{1}{L^{1/2}\pi^{1/4}} exp\left[-\frac{\left(x-\frac{\Delta x}{2}\right)^2}{2L^2}\right]|\psi_2\rangle \tag{2}$$

The region where the wave packet overlap, an interference pattern will develop in the probability density $|\Psi(x)|^2$. The pattern on a screen becomes visible in the distribution of particles hitting , when the experiment is repeated many times, the fluctuating force due effect of environment assumed acts on the particle on its way towards the screen. Zurek illustrate some important feature of decoherence view about this content by consider a soluble model in one space dimension in which the system position coordinate $x(t)$ and the environment is modeled as a collection of harmonic oscillators coordinate $q(t)$. The action is

$$S[x,q] = \int_0^t dt\left[\frac{1}{2}M\dot{x}^2 + \sum_n \frac{1}{2}m_n\left(\dot{q}_n^2 - \omega_n^2 q_n^2\right) - \sum_n c_n x q_n\right] \tag{3}$$

The effect of environment present as the spectrum density defined as $I(\omega) = \sum_n \frac{c_n^2}{2m_n\omega_n}\delta(\omega-\omega_n)$. At time t = 0, the system and the environment are assumed to be uncorrelated $\hat{\rho}_{tot}(0) = \hat{\rho}_S(0)\otimes\hat{\rho}_B(0)$. To obtained the reduced density matrix, by illiminate the environmental degree of freedom, $\hat{\rho}_r(t) = Tr_B(\hat{\rho}_{tot})$ and applying

Feynman-Vernon influence functional method. In the high temperature limit, the master equation is obtained by

$$\frac{d\hat{\rho}_r(t)}{dt}=-\frac{i}{\hbar}\left[\frac{\hat{P}^2}{2m},\hat{\rho}_r(t)\right]-\gamma(x-x')\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)\hat{\rho}_r(t)-\frac{2m\gamma K_B T}{\hbar^2}(x-x')^2\,\hat{\rho}_r(t) \qquad (4)$$

The last term of the master equation turns out to cause decoherence of the system. To interpret it, he has been comparing the relative size of the terms, the last term contains a numerical factor of $1/\hbar^2 \approx 10^{6\varepsilon}$, while the other terms are either first or zeroth order in $1/\hbar$. Thus, for sufficiently large $|x-y|$, the last term will dominate. The simplified master equation has the solution

$$\rho(x,x',t)=\rho(x,x',0)exp\left(-\frac{2m\gamma K_B T(x-x')^2}{\hbar^2}t\right) \qquad (5)$$

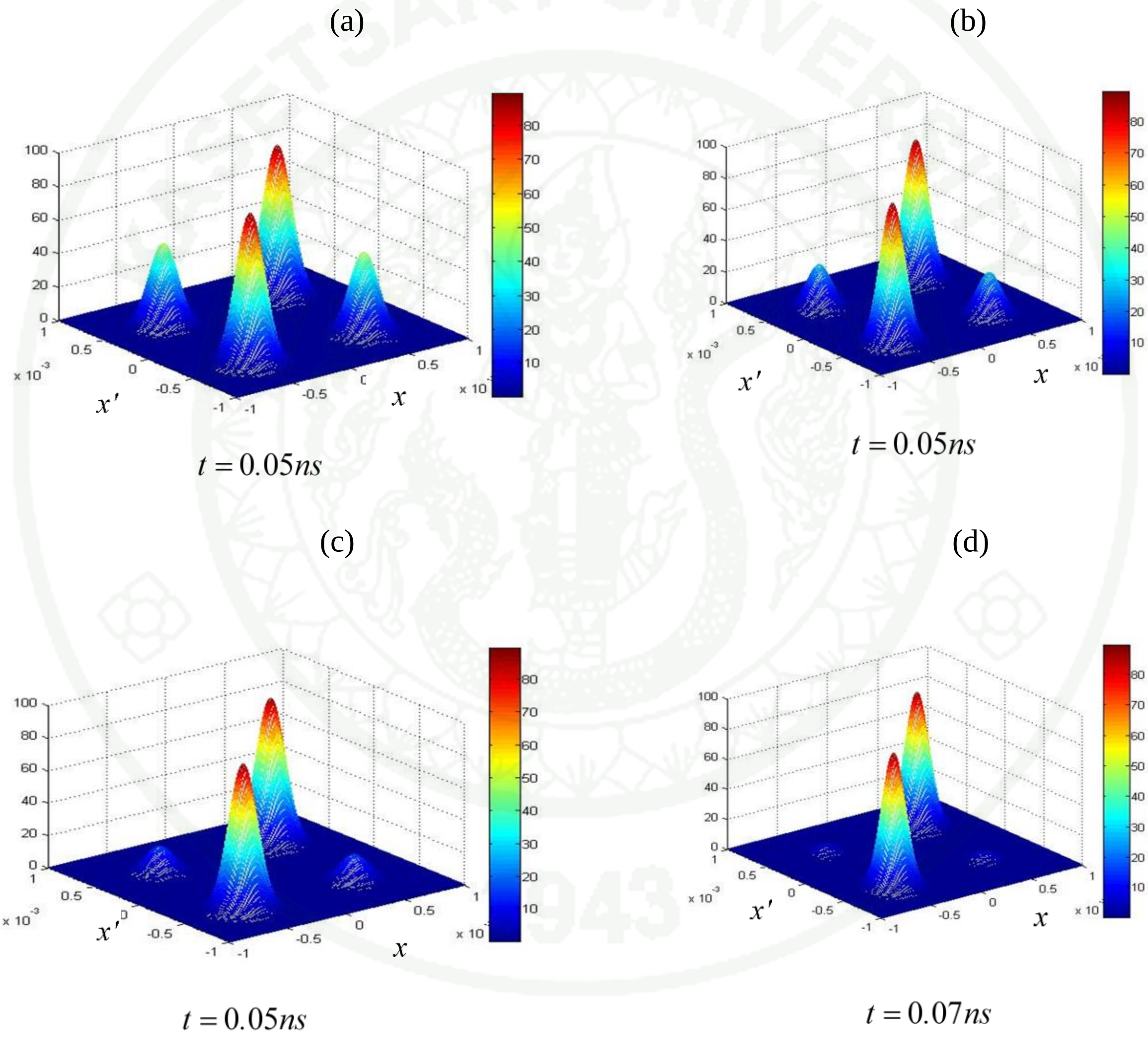
in the compact form

$$\rho(x,x',t)=\rho(x,x',0)exp\left(-\frac{t}{\tau_d}\right) \qquad (6)$$

where τ_d is defined as the decoherence time scale of system, $\tau_d=\tau_r\left[\frac{\lambda_{dB}}{(x-x')}\right]^2$ and $\lambda_{dB}=\frac{\hbar}{\sqrt{2mK_B T}}$ is the thermal de Broglie wave length, $\tau_r=\frac{1}{\gamma}$ is the relaxation time scale of system, γ is the relaxation rate it relate to the strange of interaction. By consider effect of environment on system, the term in exponent of Eq.(6) is decoherence factor which it destroys the off-diagonal element of density matrix. Recall that the interference pattern defined as $\hat{\rho}=|\Psi\rangle\langle\Psi|$. In the position representation

$$\rho(x,x')=|\Psi_1(x)|^2+|\Psi_2(x)|^2+2\,Re|\Psi_2(x)||\Psi_1(x)| \qquad (7)$$

The last term in Eq.(7) present as the interference term. The decoherence of system can be predict by expression Eq.(6-7), for visualize the matrix element as the macroscopic scale, the decoherence time scale τ_d is typically orders of magnitude smaller than the relaxation time scale τ_r. For instance, for a system at room temperature $T = 300K$ mass $m = m_e = 9.1 \times 10^{-31} Kg$ and the separation $\Delta x = 1mm$, thus if the relaxation time of system is choose in unit of time $\tau_r = 1s$ so the decoherence time scale is given as the order about $\tau_d \approx 10^{-11} s$. The decays off-diagonal term can be visualized in Figure 1.



(a) $t = 0.05ns$

(b) $t = 0.05ns$

(c) $t = 0.05ns$

(d) $t = 0.07ns$

Figure 1 Show as decays Off-diagonal density matrix. The peak near the diagonal correspond to the two possible location of particle. The peak away from the diagonal are due to quantum coherence. This plot we use scale $\Delta x = 1mm$, $L = 0.05mm$.

In 1985, Joos and Zeh published a seminal paper that presented a detailed model for decoherence induced by the scattering of environmental particles. The article also included the first explicit numerical estimates of decoherence timescales for objects of various sizes and physical nature immersed into different types of environments. The idea of this view is the scattering of environmental particles as a one of the dominant sources of decoherence in the macroscopic domain and plays an important role in the emergence of classicality. They have considered an object (the interest system, A) that scatters a collection of environmental particles (the environment, B). Environmental scattering denoted as a collection of particles, such as photons or air molecules, scatters off the object of interest. By assumption that A and B are initially completely uncorrelated, the (pure-state) density matrix for the composite system at time $t=0$ is $\hat{\rho}_{tot}(0)=\hat{\rho}_A(0)\otimes\hat{\rho}_B(0)$. The initial state of the system with center-of-mass location x denoted by a position eigenstate $|x\rangle$ and $|\chi_i\rangle$ for the initial state of the incoming environmental particle. The object is assumed to be much more massive than the scattered environmental particle, such that the object is not disturbed by the scattering event. Thus the interaction yields the evolution of state. The effect of the scattering event can be expressed by the action of the scattering operator $\hat{S}$ (called as "S-matrix") on the initial state $|x\rangle|\chi_i\rangle\rightarrow\hat{S}|x\rangle|\chi_i\rangle$ the corresponding final reduced density matrix $\hat{\rho}_r(t)$ of the system is obtained by $\hat{\rho}_r(t)=Tr_B(\hat{\rho}_{tot})$ yields the time evolution of the reduced density matrix obeys master equation

$$\frac{d\rho(x,x',t)}{dt}=-F(x-x')\rho(x,x',t) \tag{8}$$

Where $F(x-x')$ is called decoherence factor , it plays the role of a localization rate and denoted the characteristic decoherence rate at which spatial coherences between two positions x and x' become locally.

The qualitative of this model is to give answer, "how fast a superposition become decohered as a consequence of environmental scattering?". They propose the two limiting case. (I) short-wavelength limit and (II) Long-wavelength limit.

For illustrating the decoherence dynamics of this view, Joos illustrate the decoherence dynamics of the reduced density matrix by choosing a particular initial state, namely, a pure-state Gaussian wave packet of width b centered at $x=0$,

$$\Psi\left(x,0\right)=\left(\frac{1}{2\pi b}\right)^{1/4} exp\left\{-\frac{x^2}{4b^2}\right\} \tag{9}$$

This yields the initial density matrix

$$\rho_0\left(x\ ,x'\ ,0\right)=\sqrt{\frac{1}{2\pi b}}\, exp\left\{-\frac{1}{4b^2}\left(x^2+x'^2\right)\right\} \tag{10}$$

Were the environmental interaction present, the ensemble width spreading is due to the fact that the scattering events lead to an decrease in the mean energy of a particle corresponding to uncertainty principle. That can be visualized the time evolution of the density matrix illustrate in Figure 2.

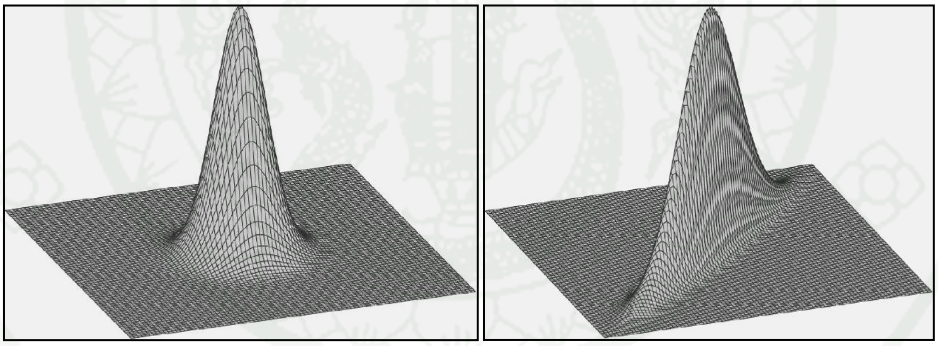


Figure 2 Time evolution of the Gaussian wave packet with the initial state given by Eq.(9), under the influence of a scattering environment. The width of the Gaussian in the off-diagonal direction $\Delta x=|x-x'|$ (representing spatial coherences) becomes progressively reduced, with the density matrix approaching a classical probability distribution of positions concentrated along the diagonal $(x=x')$

Both the papers by Zurek and by Joos and Zeh made it clear that decoherence constitutes an extremely fast and efficient process, especially on macroscopic scales. In 1991, Zurek published in Physics Today introduced decoherence view to a broader audience. The paper helped establish decoherence as a mainstay of physics, It is began to attract widespread attention from physicists, material scientists, and philosophers alike. Also, it has widely to study in various fields of physics such as very simple system to the broader of complicate condensed system. The study has been discussed in a several mechanisms. Examples as decoherence in quantum Brownian motion which considered in the hight and low temperature limit(Lombardo and Villar, 2005) that has been showed in a numerical estimation of decoherence rate. There is to analyze the temperature-dependent weak localization measurement and studied the effect of the zero-point fluctuation of environment, as a source of decoherence(Wang *et al*, 2000).

The problem of decoherence induced by the interactions in disordered system, has been of interest for a long time. Such as the subsequent theoretical debate focused on decoherence induced by Coulomb interaction in disordered electron system at the zero temperature (Mohanty and Webb, 1997) . In 1998, Golubev D. Zaikin considered the propagating electron amplitude in terms of the Feynman path integral. They have followed the analysis elaborated by the work of Chakravarty and Schmid (Chakravarty and Schmid,1986) which consider the propagation of an electron with the kinetic energy $\frac{1}{2}m\dot{x}^2$ in a potential of randomly distributed impurities $U_{imp}\left(x-x'\right)$ by addition to that the electron interacts with other electrons and two additional forces: (i) the damping force due to electron-electron collisions and (ii) the stochastic force due to the fluctuating electric field. After average overall random configuration, they have obtained that the electron decoherence time scale and point out that at the low temperature in mesoscopic conductors can be explained within the theory of weak localization provided the effect of intrinsic quantum fluctuations of the electric field, the results agree very well with the experimental data (Golubev and Zaikin, 1998).

In the last years, Lye has showed that the effect of the random potential on the long-rang correlations of density in Bose-Einstein condensates (BECs) and also exploring the role of interactions without loss of coherence control of dimensionality

(Lye *et al*, 2005). This problem had been considered as a quasi-1D simple harmonic oscillator moving in a weak random potential. They produced the random potential is by shining an off-resonant laser beam onto a diffusive plate and imaging the resulting speckle pattern on the BEC. Figure 3.

The total potential of system is combind from the harmonic potential and random potential(Modugno, 2006)

$$V_{tot} = \frac{1}{2} m\omega_x^2 x^2 + v_{random} \tag{11}$$

Representing in figure 4.

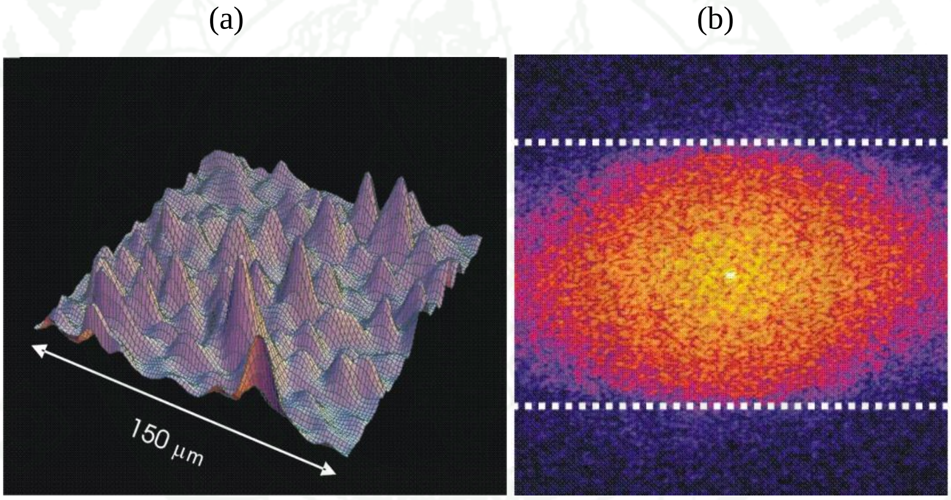


Figure 3 3D representation of the random potential for the BEC (a) and its Fourier transform (b). The dotted lines correspond to a length scale of about 10 μm in the axial direction.

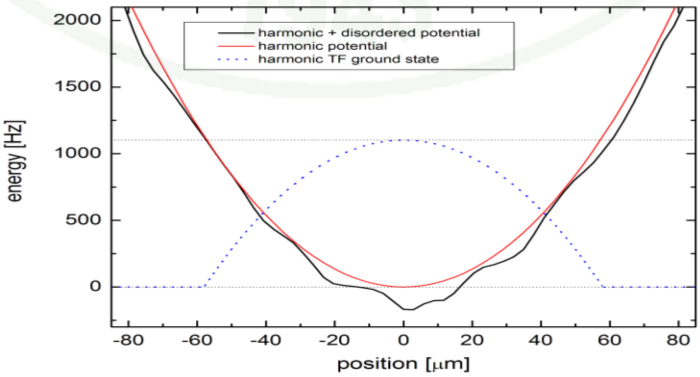
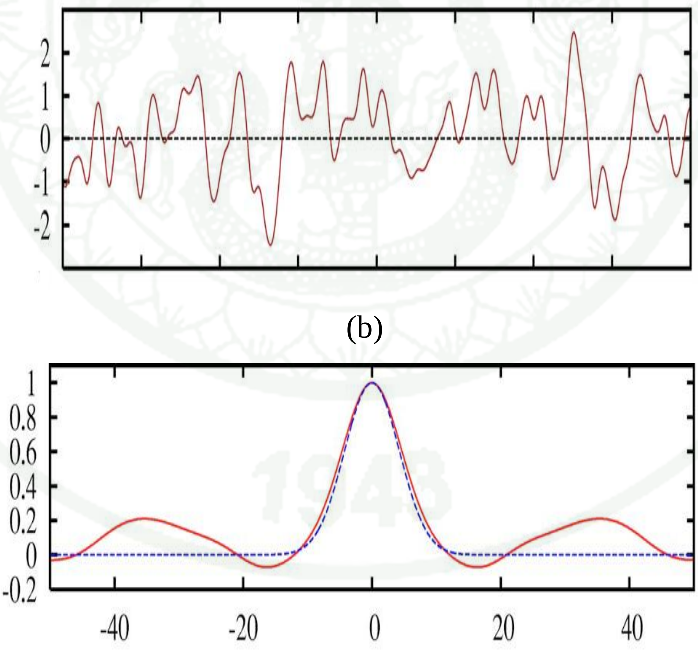


Figure 4 Typical shape of the combine potentials considered in Eq.(11)

This problem also very interesting in varies type of random potential because effects of disorder have also been observed for BEC in localization phenomena during the expansion in a one-dimensional (1D) waveguide and the process for fabrication of the microelectronics scale(Niimi, 2009)

In the similarly, Modugno has consider The effects of three different kinds of random potentials also show that the localization effects observed during the expansion in a 1D waveguide are mainly due to a classical trapping into single wells or between barriers of the random potential Figure 5. The qualitative behavior in this case is very similar to that of a periodic system. He noted that in observe localization phenomena in a 1D waveguide should have interference of multiple quantum reflections of matter waves. This could be achieved by reducing the correlation length of the random potential and size of defect(Modugno, 2006).


(a)

(b)

Figure 5 Typical shape of the Gaussian random potential (a) and the autocorrelation function the blue dash lines represent the expected average values over several realizations(b) in the length scale μm .

Now, the modern field of nano-electronics has brought about novel physical phenomena and created new challenges for their interpretation within the quantum theory. Among the most fundamental concepts are quantum coherence and interference effects. The interplay between coherence, disorder, interactions, and quantum decoherence has been studied in a variety of contexts and physical systems, such as transport of interacting electrons in quantum dots, quantum wires, and 2D structures, low temperature decoherence in metals and semiconductors and quantum information. An improved understanding of these and further phenomena is needed both from a fundamental point of view as well as for a variety of potential applications which need to understanding of decoherence in nanostructures.

OPJECTIVES

This thesis is devoted to the study deceherence in a simple system that the scale are large compared to dimensions of individual atoms, and such as, usually very sensitive to external effects. Therefore system under study considered as quantum systems interacting with their environments which we set their as disordered by statics Gaussian random potential. We explore decoherence caused by effects of the random potential fluctuations in small quantum scale and generate behaviore of decoherence time scale in various case comparing with the strange of random fluctuation in system. In literatures review, we introduce the concepts and machinery used in the theoretical treatment of decoherence for quantum systems and illustrate the formal corresponding example of the topic. In material and method, we present the method for treating problem. For this propose, we study the question about the influence of statical Gaussian random potential on decoherence. Also, we focus on the description of decoherence time scale for each correlation length of the random potential, comparing with the quantum wave length. In results and discussion, we illustrates the numerical results and visualize the matrix elements of the density operator of simple harmonic oscillator in its ground state.

LITERATURE REVIEWS

In this part of thesis, the theoretical tools for treating our model are discussed.

Brief Review of Path Integral

Consider evolution of particle mass M from a to b, let path $x(t)$ is the position of a particle at any time. For simplicity, x is a one-dimensional path. Then, if the particle has starting and terminal points $x(t_a) = x_a$ and $x(t_b) = x_b$

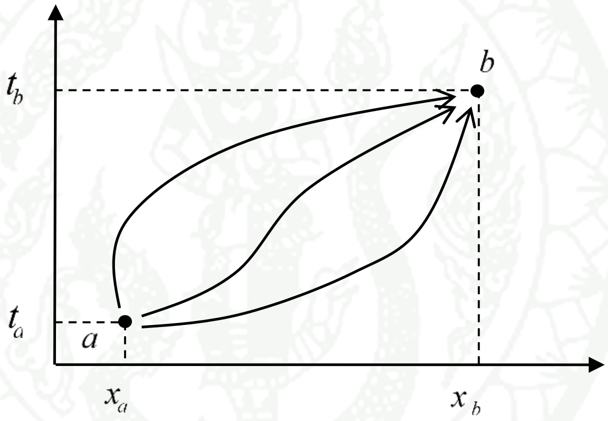


Figure 6 Diagram showing the possible paths from a to b

The action corresponding with lagrangian $L = T - V$ of the system is

$$S\left[x(t)\right] = \int_{t_a}^{t_b} L(\dot{x},x,t)dt \tag{12}$$

The action S is a functional of x, which means that S is a function of the whole path $x(t)$, and not just the function of the position $x(t)$ at certain time. Hamiltonian for a particle in a one-dimensional potential $V(x)$

$$\hat{H} = \frac{\hat{P}^2}{2m} + V(\hat{x}) \tag{13}$$

The state of a particle at final time obtained by solution of Schrodinger equation

$$\left|\psi\left(t_b\right)\right\rangle = e^{-\frac{i}{\hbar}\hat{H}\left(t_b - t_a\right)}\left|\psi\left(t_a\right)\right\rangle \tag{14}$$

In configuration space

$$\psi\left(x_b, t_b\right) = \int dx_a K\left(x_b, t_b; x_a, t_a\right)\psi\left(x_a, t_a\right) \tag{15}$$

where

$$K\left(x_b, t_b; x_a, t_a\right) = \left\langle x_b\right| e^{-\frac{i}{\hbar}\hat{H}\left(t_b - t_a\right)}\left|x_a\right\rangle \Theta\left(t_b - t_a\right) \tag{16}$$

is the amplitude for a particle to go from point at x_a time t_a to x_b at time t_b the total amplitude to go from a point *a* to another point *b* is contributed by every paths, including the classical path, $x_{cl}\left(t\right)$, and every paths gets the same weight, nevertheless, contributes at different phase $\Phi\left[x\left(t\right)\right]$which are given by the action along each path in units of $\hbar$. Feynman propose any phase is proportional to the action as

$$\Phi\left[x\left(t\right)\right] = \left(const\right)\exp\left\{\frac{i}{\hbar} S\left[x\left(t\right)\right]\right\} \tag{17}$$

The constant term is a normalizing factor corresponding with the weight probability from each path. The $K\left(x_b, x_a\right)$ called as the propagator, obtained by sum of all phases contribution from a to b

$$K\left(x_b, x_a\right) = \sum_{\substack{\text{Over all paths} \\ \text{from a to b}}} \Phi\left[x\left(t\right)\right] \tag{18}$$

The process to evaluate the propagator in Eq.(18) is called as path integration, however this work is very complicated. In 1948, Feynman proposed the way to perform the path integration by dividing the time intervals into small interval$\left(\varepsilon \to 0\right)$, This provides a set of times $t_{i+1} = t_{i+\varepsilon}$. At each time , t_i , one selects some special point x_i and contributes hole path, the resulting is

$$K\left(x_b,t_b;x_a,t_a\right) = \lim_{\varepsilon\to 0}\int\int\ldots\int exp\left\{\frac{i}{\hbar}S_{ba}\left[x\left(t\right)\right]\right\}\frac{dx_1}{A}\frac{dx_2}{A}\ldots\frac{dx_{N-1}}{A} \qquad (19)$$

Where A is the normalizing factor, Eq.(19), Feynman written the propagator as the short notation as

$$K\left(x_b,t_b;x_a,t_a\right) = \int_{x_a}^{x_b} D\left[x\left(t\right)\right]exp\left\{\frac{i}{\hbar}S\left[x\left(t\right)\right]\right\} \qquad (20)$$

It is called the Feynman's path integral. The symbol $\int_{x_a}^{x_b} D\left[x\left(t\right)\right]$ refers to an integration over all possible paths connecting the point a to b ,the probability for a particle to go from a to b is

$$P\left(x_a,x_b\right) = \left|K\left(x_b,x_a\right)\right|^2 \qquad (21)$$

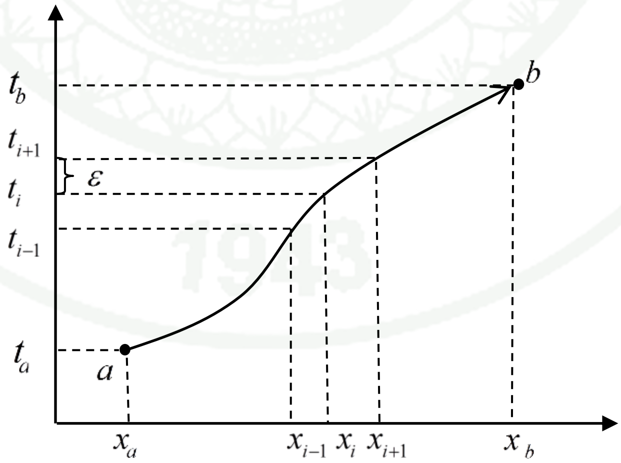


Figure 7 Diagram showing the sum over all paths. The specified times separated by very small intervals ε .

Feynman-Vernon Influence Functional

Considering a situation where a interest system acted on by external quantum sub-systems. If interaction between them is present by potential $V(x,q,t)$ For simplicity, we focus on one-dimensional coordinate, let x be the variable of the system A and q be the variable of the multi-dimensional coordinate others quantum sub-systems B, say the environment as seen in Figure 8

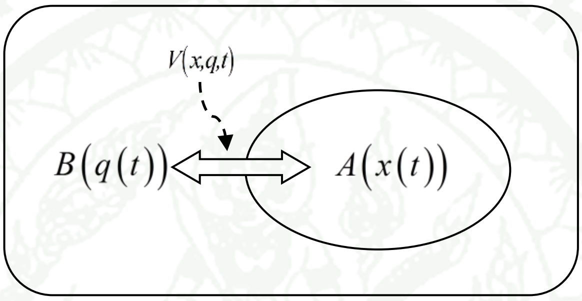


Figure 8 Quantum system A and B coupled by a potential $V(x,q,t)$

Following Feynman and Vernon in year 1963, by using the path integral approach, this formulation is call as "Feynman-Vernon influence functional". The propagator of the combined system is (Feynman-Vernon, 1963)

$$K(x_b,q_b;x_a,q_a)=\int_{x_a}^{x_b}\int_{q_a}^{q_b} D[x(t)]D[q(t)]exp\left\{\frac{i}{\hbar}(S[x]+S[q]+S_I[x,q])\right\} \qquad (22)$$

where $S_I[x,q]$ is the action of interaction between A and B, and $S[x]$, $S[q]$ are the actions of the system and environment, respectively.

The measurements process on the system A to give as the expectation value of a system observable, which is a function of the system variables only. Thus we must to eliminate the environmental variables but include all the quantum effects of the environment upon the system. This process is carry out the integral over all the paths $q(t)$.

The propagator can be rewritten in the form

$$K\left(x_b,t_b;x_a,t_a\right)=\int\limits_{x_a}^{x_b}D\left[x(t)\right]F\left[x\right]exp\left\{\frac{i}{\hbar}S\left[x(t)\right]\right\} \tag{23}$$

Where the functional

$$F\left[x\right]=\int\limits_{q_a}^{q_b}D\left[q(t)\right]exp\left\{\frac{i}{\hbar}\left(S\left[q(t)\right]+S_I\left[x(t),q(t)\right]\right)\right\} \tag{24}$$

is a functional of the whole path $x(t)$. This is also a total amplitude for the environmental system to go from point q_a to q_b at times t_a and t_b, respectively, under the influence of the interaction potential $V(x,q,t)$ with a specified path $x(t)$. If initially the system A is in state $\psi_n(x_a)$ and the environment B is in state $\phi_n(q_a)$ then the transition amplitude for A to go from n to m and B to go from n' to m' is

$$\begin{aligned}A_{mm',nn'}=\int\limits_{-\infty}^{+\infty}dx_b dq_b dx_a dq_a\int\limits_{x_a}^{x_b}\int\limits_{q_a}^{q_b}D\left[x(t)\right]D\left[q(t)\right]\psi_m^*\left(x_b\right)\phi_{m'}^*\left(q_b\right)\psi_n\left(x_a\right)\phi_{n'}^*\left(q_a\right)\\ \times exp\left\{\frac{i}{\hbar}S\left[x(t)\right]+S\left[q(t)\right]+S_I\left[x(t),q(t)\right]\right\}\end{aligned} \tag{25}$$

And the probability of the transition

$$\begin{aligned}P_{mm',nn'}=\int\limits_{-\infty}^{+\infty}dx_b dx'_b\, dx_a dx'_a\int\limits_{x_a}^{x_b}\int\limits_{x'_a}^{x'_b}D\left[x(t)\right]D\left[x'(t)\right]\psi_m^*\left(x_b\right)\psi_m\left(x'_b\right)\psi_n\left(x_a\right)\psi_n^*\left(x'_a\right)\\ \times F\left[x,x'\right]exp\left\{\frac{i}{\hbar}\left(S\left[x(t)\right]-S\left[x'(t)\right]\right)\right\}\end{aligned} \tag{26}$$

Where

$$\begin{aligned}F\left[x,x'\right]=\int\limits_{-\infty}^{+\infty}dq_b dq'_b\, dq_a dq'_a\int\limits_{q_a}^{q_b}\int\limits_{q'_a}^{q'_b}D\left[q(t)\right]D\left[q'(t)\right]\phi_{m'}^*\left(q_b\right)\phi_{m'}\left(q'_b\right)\phi_{n'}\left(q_a\right)\phi_{n'}^*\left(q'_a\right)\\ \times exp\left\{\frac{i}{\hbar}\left(S\left[q(t)\right]-S\left[q'(t)\right]+S_I\left[x(t),q(t)\right]-S_I\left[x'(t),q'(t)\right]\right)\right\}\end{aligned} \tag{27}$$

The functional *F* depends on the environmental system B, the initial and final states of B, and the coupling between A and B. It includes all the effects of the environment upon the system when the environment has a transition from state *n’* to *m’*. If we don’t care about the environmental system because we not measure it, so all the final states of the environmental system are possible to sum over all the final state of B the functional becomes

$$\begin{aligned} F\left[x,x'\right] = \int_{-\infty}^{+\infty} \delta\left(q_b - q'_b\right) dq_b dq'_b\, dq_a dq'_a \int_{q_a}^{q_b}\int_{q'_a}^{q'_b} D\left[q\left(t\right)\right] D\left[q'\left(t\right)\right] \phi_{n'}\left(q_a\right)\phi^{*}_{n'}\left(q'_a\right) \\ \times exp\left\{\frac{i}{\hbar}\left(S\left[q\left(t\right)\right] - S\left[q'\left(t\right)\right] + S_I\left[x\left(t\right),q\left(t\right)\right] - S_I\left[x'\left(t\right),q'\left(t\right)\right]\right)\right\} \end{aligned} \qquad (28)$$

The effects of external systems on a system can be represented by this functional. We call as the *“influence functional”* , it is the form of double path integral. In practice, the problem in real systems depend on the interaction between system and environment and the way to perform double path integral are very complicate.

Reduced Density Matrix

In 1932, John von Neumann has propose to use the density matrix for describe the state of a quantum system which interacting with other quantum systems. It is defined as a combination of pure state weighted by a corresponding probability. If we express $\left|\Psi\right\rangle$ as a superposition of basis states $\left|\Psi\right\rangle = \sum_i \left|\Psi_i\right\rangle$ the corresponding density operator written in this basis $\left\{\left|\Psi_i\right\rangle\right\}$ written as

$$\hat{\rho} = \sum_{ij} p_{ij} \left|\Psi_i\right\rangle\left\langle\Psi_j\right| \qquad (29)$$

Where $p_{ij} = c_i c_j^*$ is the weight probability which are real value. The terms $i \neq j$ on the right-hand side of this equation represent the quantum coherence state between the different components $\left|\Psi_i\right\rangle$. Accordingly, they are usually referred to as interference terms, or off-diagonal terms. If, the state is pure with $i = j = 1$, $p = 1$ then the density matrix is reduced to the form of pure state density matrix, $\hat{\rho} = |\Psi\rangle\langle\Psi|$

The operation always acts on some operator of an orthonormal basis $\{|\psi_i\rangle\}$, to perform the operation define as

$$Tr\left(\hat{O}\right) = \sum_i \langle\psi_i|\hat{O}|\psi_i\rangle \tag{30}$$

Called as trace operation, let us choose the eigenstates $|a_i\rangle$ of $\hat{A}$, with corresponding discrete eigenvalues as the orthonormal basis for evaluating the trace of $\hat{O} = \hat{\rho}\hat{A}$, yields

$$\begin{aligned} Tr\left(\hat{\rho}\hat{A}\right) &= \sum_i \langle a_i|\left(|\Psi\rangle\langle\Psi|\right)\hat{A}|a_i\rangle \\ &= \sum_i a_i \left|\langle a_i|\Psi\rangle\right|^2 \end{aligned} \tag{31}$$

The term $\left|\langle a_i|\Psi\rangle\right|^2$ is simple form of the Born probability of the outcome a_i in a measurement represented by $\hat{A}$. This is the definition of the expectation value $\langle\hat{A}\rangle$ of the observable $\hat{A}$. The connection between the mathematical procedure of the trace of the operator $\hat{\rho}\hat{A}$ and the expectation value of a measurement is known as the trace rule

$$\langle\hat{A}\rangle = Tr\left(\hat{\rho}\hat{A}\right) \tag{32}$$

another properties of the density operator such as positivity$\left(\hat{\rho} = \hat{\rho}^+\right)$, normalization ,$\langle\Psi|\hat{\rho}|\Psi\rangle \geq 0 \left(\forall|\Psi\rangle \in \mathrm{H}\right)$ and self – adjointedness, $Tr\left(\hat{\rho}\right) = 1$.

The reduced density matrix obtained by tracing out partial degrees of freedom of a coupled system denoted by $\hat{\rho}_r$. If a quantum system A that is correlated with another system B. The reduced density matrix given by $\hat{\rho}_r = Tr_B\left(\hat{\rho}_{tot}\right)$.

The integral over all environmental coordinate is

$$\hat{\rho}_r(t) = \int_{-\infty}^{\infty} dq_f dq'_f \, \delta\left(q_f - q'_f\right)' \hat{\rho}_{tot}(t) \qquad (33)$$

Here, this expression means that the trace is to be performed using an orthonormal basis of the Hilbert space of B only. Accordingly, the operation Tr_B is also referred to as the partial trace over B and may be interpreted as an “averaging” over the degrees of freedom of the unobserved system B.

Consider the situation where we make measurement only on the system A, the final states of the other subsystem (environment B) must be summed over as before. The coordinate representation of the total density matrix for the system A and the environment B denoted as $\rho(x,q;x',q')$, and the expectation value of an operator $\hat{O}$ which acts only on the system variable written as

$$\begin{aligned} \left\langle \hat{O} \right\rangle &= Tr\left(\hat{O}\hat{\rho}\right) = \int\int \left[\int dq \int \rho\left(x,q;x',q'\right) \delta\left(q-q'\right) dq' \right] O\left(x,x'\right) dxdx' \\ \left\langle \hat{O} \right\rangle &= \int\int \rho_r\left(x,x'\right) O\left(x,x'\right) \delta\left(x-x'\right) dxdx' \end{aligned} \qquad (34)$$

where $\hat{\rho}_r = Tr_B\left(\hat{\rho}_{tot}\right)$ is the reduced density matrix for the system A. In order to get the time evolution for any physical quantity, we shall derive the propagation of $\hat{\rho}_r$ in the functional representation. Since state of composite system

$$\psi\left(x_b,q_b\right) = \int\int dx_a dq_a K\left(x_b,q_b;x_a,q_a\right) \psi\left(x_a,t_a\right) \qquad (35)$$

Then the time evolution operator of system is defined as

$$\begin{aligned} J\left(x_b,q_b;x'_b,q'_b \middle| x_a,q_a,x'_a,q'_a\right) &= \int_{x_a}^{x_b} \int_{q_a}^{q_b} D\left[x(t)\right] D\left[q(t)\right] \int_{x'_a}^{x'_b} \int_{q'_a}^{q'_b} D\left[x'(t)\right] D\left[q'(t)\right] \\ &\times exp\left\{ \frac{i}{\hbar} \left(S\left[x(t)\right] - S\left[x'(t)\right] + S\left[q(t)\right] - S\left[q'(t)\right] + S_I\left[x(t),q(t)\right] - S_I\left[x'(t),q'(t)\right] \right) \right\} \end{aligned} \qquad (36)$$

The density matrix for total system can be written as

$$\rho\left(x_b,q_b;x'_b,q'_b\right)=\int dx_i dx_i{}' dq_i dq_i{}' J\left(x_b,q_b;x'_b,q'_b\middle|x_a,q_a,x'_a,q'_a\right)\rho\left(x_a,q_a;x'_a,q'_a\right) \qquad (37)$$

Here and in the following we have replace $(x,q,0)$ and (x,q,t) with (x_a,q_a) and (x_b,q_b) respectively. The reduced density matrix is obtained by

$$\begin{aligned}\rho_r\left(x_b,x'_b\right)&=\int_{-\infty}^{\infty} dq_f dq_f{}' \rho\left(x_b,q_b;x'_b,q'_b\right)\delta\left(q_b-q'_b\right)\\ &=\int_{-\infty}^{\infty} dx_a dx_a{}' J_r\left(x_b,x'_b\middle|x_a,x'_a\right)\rho_r\left(x_i,x'_i\right)\end{aligned} \qquad (38)$$

In general the expression of J_r is very complicated since it depends on the initial state (at t = 0) of the system. But for simplicity we can assume that initially the system and the environment are uncorrelated

$$\rho_0\left(x_a,q_a;x'_a,q'_a\right)=\rho_s\left(x_a,x'_a\right)\rho_E\left(q_a,q'_a\right) \qquad (39)$$

Then J_r does not depend on the initial state system and can be written as

$$\begin{aligned}J_r\left(x_b,x'_b;x_a,x'_a\right)&=\int_{x_a}^{x_b}\int_{x'_a}^{x'_b} D\left[x(t)\right]D\left[x'(t)\right]F\left[x,x'\right]exp\left\{\frac{i}{\hbar}\left(S\left[x\right]-S\left[x'\right]\right)\right\}\\ &=\int_{x_a}^{x_b}\int_{x'_a}^{x'_b} D\left[x(t)\right]D\left[x'(t)\right]exp\left\{\frac{i}{\hbar}\left(A_{eff}\left[x,x'\right]\right)\right\}\end{aligned} \qquad (40)$$

where $A_{eff}\left[x,x'\right]=S\left[x\right]-S\left[x'\right]+\delta A\left[x,x'\right]$ is the effective action for composite system and $\delta A\left[x,x'\right]$ is called the influence action (or influence phase). The influence functional can be written in the double path integral form

$$F\left[x,x'\right]=\int_{-\infty}^{+\infty}\delta\left(q_b-q'_b\right)dq_b dq'_b\, dq_a dq'_a\, K_{Ex}\left(q_b,q_a\right)K^*_{Ex}\left(q'_b,q'_a\right)\rho_{Ex}\left(q_a,q'_a\right) \qquad (41)$$

Where

$$K_{Ex}\left(q_b,q_a\right)=\left\langle q_b\left|U_{Ex}\left(x,t\right)\right|q_a\right\rangle \text{ and } K^*_{Ex}\left(q_b,q'_a\right)=\left[\left\langle q_b\left|U_{Ex}\left(x',t\right)\right|q'_a\right\rangle\right]^* \qquad (42)$$

And $\rho_{Ex}\left(q_a,q'_a\right)$ is the initial density matrix of external system. which the form is different by depend on the initial condition such as where in thermal equilibrium at a temperature $\beta^{-1}=1/K_BT$. The density matrix $\rho_{Ex}\left(q_a,q'_a\right)$ satisfies the Boltzmann distribution .

The path integral on the right hand side of (41) can be evaluated which yields the general result present as the influence phase of system.

$$F\left[x,x'\right]=exp\left\{\frac{i}{\hbar}\delta A\left[x,x'\right]\right\} \qquad (43)$$

All effect of external system included in the influence action $\delta A\left[x,x'\right]$. The explicit form of $\delta A\left[x,x'\right]$ is very complicate to perform evaluated. In practice, it is often to assume that interaction is weak that can be apply to use the perturbation method.

We noted for the general properties of the influence functional when $x\left(t\right)=x'\left(t\right)$ then $F=1$. And if the interaction between the system and the environment is zero then $F=1$ and the influence phase $\delta A\left[x,x'\right]$ is zero. In this case, the reduced evolution operator $J_r\left(t\right)$ is just the evolution operator for the Liouville-Neumann equation. In others case evolution of reduced density matrix is known as the the master equation and satisfy the non-Markovian dynamics.

MATTERIAL AND METHOD

Model for Disordered System

Our system is modeled as a quantum particle moving in disordered environment which set as the random scattering potential distributed in a volume of material. The interaction between a particle and random fluctuation potential caused the imperfection of the system. There are assumed that the random potential present as scatters in the system. The scatters being determined by its own field $v\left(x-R_n\right)$ located at site R_n. The random scattering potential felt by a particle is sum of individual random scattering potentials

$$V\left(x\right)=\sum_{n=1}^{N}v\left(x-R_n\right) \tag{44}$$

The Fourier's transform of random scattering potential is

$$V\left(k\right)=\sum_{n=1}^{N}v\left(k\right)exp\left(-ik\cdot R_n\right) \tag{45}$$

We have introduced the Fourier's transform of the potential of individual scatters

$$v\left(k\right)=\int_{\Omega}dx\exp\left(-ix\cdot k\right)v\left(x\right) \tag{46}$$

Ω denoting the volume of system. The individual scatters which is called the random scattering potential supposed as a Gaussian form

$$v\left(x-R_n\right)=v_0\,exp\left[-\frac{\left|x-R_n\right|^2}{l^2}\right] \tag{47}$$

Where v_0 is the strength of the individual scattering potential. The particle is allowed to interacting with N identical fixed scattering centers

These scattering centers are assumed to be distributed with the probability,

$$P\left(R_n\right)=\frac{1}{(length)} \qquad (48)$$

The mean potential energy E_0 .

$$E_0=\left\langle V\left(x,t\right)\right\rangle_{\{R_n\}}=\prod_{n=1}^{N}\int dR_n P\left(R_n\right)V\left(x,t\right)=\int dRv\left(x-R\right) \qquad (49)$$

And the correlation function, $w\left(x-x'\right)$ are given by

$$w\left(x-x'\right)=\left\langle V\left(x,t\right)V\left(x',t'\right)\right\rangle_{\{R_n\}}=\int_R dRv\left(x-R\right)v\left(x'-R\right) \qquad (50)$$

We suppose correlation function as a Gaussian form

$$w\left(x-x'\right)=\xi_L\, exp\left[-\frac{\left|x-x'\right|^2}{L^2}\right] \qquad (51)$$

Where ξ_L is the correlation strength having the dimension of energy square $\xi_L=\lambda\sqrt{\pi}Lv_0^2$ and λ is density of scatters per unit length of system, $L=\sqrt{2}l$ is the correlation length of the random potential fluctuation.

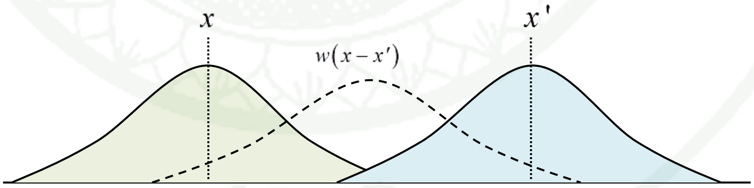


Figure 9 The correlation between pairs scattering potential

Time Evolution of System in Random Potential

Hamiltonian of a quantum particle moving in 1D and being scattered by the static random scattering potential can be written as

$$\hat{H}_{tot}=\hat{H}_0+V\left(\hat{x}\right) \qquad (52)$$

Where $\hat{H}_0$ is the Hamiltonian of unperturbed particle and $V\left(\hat{x}\right)$ is the potential due to the presence of random scatters, with the environment of random scatters assumed to have no internal dynamics.

The total density operator obeys the Liouville-Neumann equation

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar}\left[\hat{H}_{tot}, \hat{\rho}(t)\right] \tag{53}$$

Then the time evolution of density operator

$$\hat{\rho}(t;R_n) = \hat{U}(t)\hat{\rho}_i(0)\hat{U}^{\dagger}(t) \tag{54}$$

Where the evolution operator is

$$\hat{U}(t) = exp\left(-\frac{i}{\hbar}\hat{H}_{tot}(x,R_n)t\right) \tag{55}$$

and $\hat{\rho}_i(0)$ is the statistical operator at some initial time. By using path integral approach, rewritten Eq.(54) as

$$\rho(x,x',t;R_n) = \langle x'|\hat{U}(t)\hat{\rho}_i(0)\hat{U}^{\dagger}(t)|x\rangle \tag{56}$$

Inserting the completeness relation $\int_{-\infty}^{+\infty} dx|x\rangle\langle x| = 1$ and $\int_{-\infty}^{+\infty} dx'|x'\rangle\langle x'| = 1$

$$\rho(x,x',t;R_n) = \int_{-\infty}^{+\infty} dx_a \int_{-\infty}^{+\infty} dx_a{}'\langle x|\hat{U}(t)|x'_a\rangle\langle x'_a|\hat{\rho}_i(0)|x_a\rangle\langle x_a|\hat{U}^{\dagger}(t)|x'\rangle \tag{57}$$

This expression clearly seen that can be written as the path integral form

$$\rho\left(x,x',t;R_n\right)=\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx_a\int_{x_a}^{x}D\left[x\right]\int_{x'_a}^{x'}D\left[x'\right]exp\left\{\frac{i}{\hbar}\left(S_0\left[x\right]-S_0\left[x'\right]\right)\right\}$$
$$\times exp\left\{-\frac{i}{\hbar}\left[\int_0^t dtV\left(x\left(t\right)\right)-\int_0^t dt'V\left(x'\left(t'\right)\right)\right]\right\}\rho_i\left(x_a,x'_a,0\right) \qquad (58)$$

To introduced the randomness of system, we need to averaged over all random potential configuration where the total density matrix after tracing out of random potential call as the reduced density matrix , $\rho_r\left(x,x',t\right)=\left\langle\rho\left(x,x',t;R_n\right)\right\rangle_{\{R\}}$. Then the tracing out of random potential becomes

$$\left\langle\rho\left(x,x',t;R_n\right)\right\rangle_{\{R\}}=\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx_a\int_{x_a}^{x}D\left[x\right]\int_{x'_a}^{x'}D\left[x'\right]exp\left\{\frac{i}{\hbar}\left(S_0\left[x\right]-S_0\left[x'\right]\right)\right\}\rho_S\left(x_a,x'_a,0\right)$$
$$\times\prod_{n=1}^{N}\left[\int_0^R dR_nP\left(R_n\right)exp\left\{-\frac{i}{\hbar}\left[\int_0^t dtV\left(x\left(t\right)\right)-\int_0^t dt'V\left(x'\left(t'\right)\right)\right]\right\}\right] \qquad (59)$$

The last term in of Eq.(59) present as the influence functional, defined by

$$F\left[x,x'\right]=\prod_{n=1}^{N}\left[\int_0^R dR_nP\left(R_n\right)exp\left\{-\frac{i}{\hbar}\left[\int_0^t dtV\left(x\left(t\right)\right)-\int_0^t dt'V\left(x'\left(t'\right)\right)\right]\right\}\right]$$
$$=\left\langle exp\left\{-\frac{i}{\hbar}\left[\int_0^t dtV\left(x\left(t\right)\right)-\int_0^t dt'V\left(x'\left(t'\right)\right)\right]\right\}\right\rangle_{\{R\}} \qquad (60)$$

Now, the reduced density matrix can be written in the simple form as

$$\rho_r\left(x,x',t\right)=\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx'_aJ\left(x,x',t\middle|x_a,x'_a,0\right)\rho_S\left(x_a,x'_a,0\right) \qquad (61)$$

Here $J\left(x,x',t\middle|x_a,x'_a,0\right)$ is the propagator of density matrix , controlling the time evolution of reduced density matrix and can be written in path integral form as

$$J\left(x,x',t\middle|x_a,x'_a,0\right)=\int_{x_a}^{x}D\left[x\right]\int_{x'_a}^{x'}D\left[x'\right]F\left[x,x'\right]exp\left\{\frac{i}{\hbar}\left(S_0\left[x\right]-S_0\left[x'\right]\right)\right\} \qquad (62)$$

Following the perturbation theory and limiting the weakness of the scattering potential, we expand Eq.(60) in the form of Dyson series, suppose the time integral of exponential along a path is small compare to $\hbar$. Then the part of the exponential can be expanded as series

$$exp\left[-\frac{i}{\hbar}\left\{\int_0^t dtV\left(x,t\right)-\int_0^t dt'V\left(x',t'\right)\right\}\right]=1-\frac{i}{\hbar}\left\{\int_0^t dtV\left(x,t\right)-\int_0^t dt'V\left(x',t'\right)\right\}$$
$$+\frac{1}{2}\left(-\frac{i}{\hbar}\right)^2\left\{\int_0^t dtV\left(x,t\right)-\int_0^t dt'V\left(x',t'\right)\right\}^2+... \qquad (63)$$

Using this expansion and interchanging to symmetry time variable $t'\leftrightarrow s_1$, $t\leftrightarrow s_2$, we have obtained the series

$$F\left[x,x'\right]=F^{(0)}\left[x,x'\right]+F^{(1)}\left[x,x'\right]+F^{(2)}\left[x,x'\right]+...F^{(n-1)}\left[x,x'\right] \qquad (64)$$

Where

$$F^{(0)}\left[x,x'\right]=\left\langle 1\right\rangle_{\{R\}}=\prod_n\int_0^R dR_nP\left(R_n\right)\left(1\right)$$

$$F^{(1)}\left[x,x'\right]=-\frac{i}{\hbar}\left\langle\int_0^t ds_1V\left(x\left(s_1\right)\right)-\int_0^t ds_2V\left(x'\left(s_2\right)\right)\right\rangle_{\{R\}}$$

$$F^{(2)}\left[x,x'\right]=\frac{1}{2}\left(-\frac{i}{\hbar}\right)^2\left\langle\int_0^t\int_0^t ds_1ds_2V\left(x,s_1\right)V\left(x,s_2\right)-2\int_0^t\int_0^t ds_1ds_2V\left(x,s_1\right)V\left(x',s_2\right)+\int_0^t\int_0^t ds_1ds_2V\left(x',s_1\right)V\left(x',s_2\right)\right\rangle_{\{R\}}$$

And the n-th order

$$F^{(n)}\left[x,x'\right]=\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\left\langle\left(\int_0^t ds_1V\left(x\left(s_1\right)\right)-\int_0^t ds_2V\left(x\left(s_2\right)\right)\right)^n\right\rangle_{\{R\}} \qquad (65)$$

Finally, we have obtained

$$F\left[x,x'\right]=\left\langle\sum_{i=1}^N\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\left(\int_0^t ds_1V\left(x',s_1\right)-\int_0^t ds_2V\left(x,s_2\right)\right)^n\right\rangle_{\{R\}} \qquad (66)$$

Where the random average is defined by

$$\langle O\rangle_{\{R\}}=\int\frac{dR_1}{\Delta X}\int\frac{dR_2}{\Delta X}\dots\int\frac{dR_N}{\Delta X}O \qquad (67)$$

Then the right hand side of Eq.(66) becomes

$$\prod_{i=1}^{N}\left[\int\frac{dR_1}{\Delta X}\int\frac{dR_2}{\Delta X}\dots\int\frac{dR_N}{\Delta X}\sum_{i=1}^{N}\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\left(\int_0^t ds_1 v\left(x-R_i\right)-\int_0^t ds_2 v\left(x'-R_i\right)\right)^n\right] \qquad (68)$$

For completely random distribution Eq.(68) can be written as

$$\left[\int\frac{dR}{\Delta X}\sum_{i=1}^{N}\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\left(\int_0^t ds_1 v\left(x-R\right)-\int_0^t ds_2 v\left(x'-R\right)\right)^n\right]^N \qquad (69)$$

Here N denoted the number of impurities situated in region of one dimensional line density ΔX . Following Fridberg and Luttinger,1975 in limit of $N\to\infty,\ \Delta X\to 0$ such that $\frac{N}{\Delta X}=\lambda$ is finite. Rewritten Eq.(69) in the alternating form as

$$\lim_{N\to\alpha}\left[1+\lambda\int\frac{dR}{N}\left\{\sum_{i=1}^{N}\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\left(\int_0^t ds_1 v\left(x-R\right)-\int_0^t ds_2 v\left(x'-R\right)\right)^n-1\right\}\right]^N \qquad (70)$$

By using the identity $\lim_{N\to\infty}\left(1+\frac{x}{N}\right)^N=e^x$,the above expression becomes

$$exp\left[\lambda\int dR\left\{\sum_{i=1}^{N}\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\left(\int_0^t ds_1 v\left(x-R\right)-\int_0^t ds_2 v\left(x'-R\right)\right)^n-1\right\}\right] \qquad (71)$$

Here define series of influenc functional as

$$F\left[x,x'\right]=exp\left[\delta W_n\left[x(t),x'(t')\right]\right] \qquad (72)$$

Were, the correlation functions of random potential, $\delta W_n\left[x(t),x'(t')\right]$ present as

$$\begin{aligned}\delta W_n\left[x(t),x'(t')\right] &= \lambda\int dR\left\{\sum_{i=1}^{N}\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\left(\int_0^t ds_1 v(x-R)-\int_0^t ds_2 v(x'-R)\right)^n-1\right\}\\ &= \lambda\int dR\left\{exp\left[-\frac{i}{\hbar}\left(\int_0^t ds_1 v(x-R)-\int_0^t ds_2 v(x'-R)\right)\right]-1\right\}\end{aligned} \tag{73}$$

Thus, we can expand the term in exponential above relation analogue to series of influence functional and only keep up to second order so that the influence functional can be written as

$$F\left[x,x'\right]=exp\left\{-\frac{1}{2\hbar^2}\int_0^t\int_0^t dtdt'\,\delta W\left[x(t),x'(t')\right]\right\} \tag{74}$$

Where the correlation functional

$$\delta W\left[x(t),x'(t')\right]=w\big(x(s_1)-x(s_2)\big)-2w\big(x(s_1)-x'(s_2)\big)+w\big(x'(s_1)-x'(s_2)\big) \tag{75}$$

and the correlation functions

$$\left.\begin{aligned} w\big(x(s_1)-x(s_2)\big) &= \lambda\int_R dRv\big(x(s_1)-R\big)v\big(x(s_2)-R\big)\\ w\big(x(s_1)-x(s_2)\big) &= \lambda\int_R dRv\big(x(s_1)-R\big)v\big(x'(s_2)-R\big)\\ w\big(x'(s_1)-x'(s_2)\big) &= \lambda\int_R dRv\big(x'(s_1)-R\big)v\big(x'(s_2)-R\big)\end{aligned}\right\} \tag{76}$$

By assumption that the length under consideration is mesoscopic, so that the electron experiences a random potential, we need only discuss the average properties of physical quantities, which can be done in terms of the statistically averaged density matrix. Assuming the random scattering potential to be Gaussian distributed with zero mean we have $\langle....\rangle_R$ now denotes the statistical average with respect to the ensemble of random scattering potential, Furthermore, the above Gaussian description neglects multiple scattering off the same random scattering. The form of correlation can be given by

$$\left.\begin{aligned} w\left(x(t_1)-x(t_2)\right) &= \xi_L\, exp\left[-\frac{\left|x(t_1)-x(t_2)\right|^2}{L^2}\right] \\ w\left(x(t_1)-x'(t_2)\right) &= \xi_L\, exp\left[-\frac{\left|x(t_1)-x'(t_2)\right|^2}{L^2}\right] \\ w\left(x'(t_1)-x'(t_2)\right) &= \xi_L\, exp\left[-\frac{\left|x'(t_1)-x'(t_2)\right|^2}{L^2}\right] \end{aligned}\right\} \tag{77}$$

We need to evaluate the influence functional. In this process, following Eq.(77). It is conveniently to represent these correlation functions in Fourier's decomposition

$$w(x-x') = \int \frac{dk}{2\pi} v(k)\, exp\left\{ik(x-x')\right\} \tag{78}$$

Where $v(k) = 2\sqrt{\pi}L\, exp\left(-\frac{1}{4}k^2L^2\right)$ is the Fourier transform of $w(x-x')$. So that, the influence functional can be written as

$$F[x,x'] = exp\left\{-\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\int\frac{dk}{2\pi}v(k)\,\delta W_k\left[x(t),x'(t')\right]\right\} \tag{79}$$

Here

$$\begin{aligned} \delta W_k\left[x(t),x'(t')\right] = exp\left\{ik\left(x(t)-x(t')\right)\right\} + exp\left\{ik\left(x'(t)-x'(t')\right)\right\} \\ -2\,exp\left\{ik\left(x(t)-x'(t')\right)\right\} \end{aligned} \tag{80}$$

Now, the time evolution operator $J\left(x,x',t\middle|x_a,x'_a,0\right)$ after average over all random configurations becomes

$$\int_{x_a}^{x} D[x]\int_{x'_a}^{x'} D[x']exp\left\{\frac{i}{\hbar}\left(S_0[x]-S_0[x']\right)-\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\int\frac{dk}{2\pi}v(k)\delta W_k\left[x(t),x'(t')\right]\right\} \tag{81}$$

This results implies that a quantum particle scattering on the random potential and the effect of interaction induces a coupled trajectories are contained in the last term of exponential. Behavior of the reduced evolution operator $J\left(x,x',t\middle|x_a,x'_a,0\right)$ is contributing the non-local influence functional connect to different time histories and makes the state of the system depend on it states at previous times.

Now, the average density matrix can be written as

$$\begin{aligned}\rho_r(x,x',t)=&\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx_a\int_{x_a}^{x}D[x]\int_{x'_a}^{x'}D[x']exp\left\{\frac{i}{\hbar}\left(S_0[x]-S_0[x']\right)\right\}\\ &\times exp\left\{-\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\,\delta W_k\left[x(t),x'(t')\right]\right\}\rho_S(x_a,x'_a,0)\end{aligned} \tag{82}$$

In order to evaluate the reduced density matrix, we may proceed to find average $\rho_r(t)$ in the form of

$$\rho_r(x,x',t)=\rho_0(x,x',t)\left\langle exp\left\{-\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\int\frac{dk}{2\pi}v(k)\delta W_k\left[x(t),x'(t')\right]\right\}\right\rangle_S \tag{83}$$

Where $\rho_0(x,x',t)=\langle x|e^{-\frac{i}{\hbar}\hat{H}_0 t}\hat{\rho}_i(0)e^{\frac{i}{\hbar}\hat{H}_0 t}|x'\rangle$. The functional average $\langle F\rangle_S$ defined as

$$\langle F\rangle_S=\frac{\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx'_a\rho_0(x_a,x'_a)\int_{x_a}^{x}D[x]\int_{x'_a}^{x'}D[x']F[x,x']exp\left\{\frac{i}{\hbar}\left(S_0[x]-S_0[x']\right)\right\}}{\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx'_a\rho_0(x_a,x'_a)\int_{x_a}^{x}D[x]\int_{x'_a}^{x'}D[x']exp\left\{\frac{i}{\hbar}\left(S_0[x]-S_0[x']\right)\right\}} \tag{84}$$

By applying the cumulant expansion

$$\left\langle exp\left(A\left[x,x'\right]\right)\right\rangle_S = exp\left\{\left\langle A\left[x,x'\right]\right\rangle_S + \frac{1}{2!}\left(\left\langle A\left[x,x'\right]^2\right\rangle_S - \left\langle A\left[x,x'\right]\right\rangle^2_S\right) + \ldots\right\} \tag{85}$$

Limiting of the weakness of scattering potential, we can approximate this functional average as first order $\left\langle exp\left(A\left[x,x'\right]\right)\right\rangle_S \approx exp\left\{\left\langle A\left[x,x'\right]\right\rangle_S\right\}$. Then the reduced density matrix can be written as

$$\rho_r\left(x,x',t\right) = \rho_0\left(x,x',t\right) exp\left\{-\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\int_{-\infty}^{\infty}\frac{dk}{2\pi}v\left(k\right)\left\langle \delta W_k\left[x\left(t\right),x'\left(t'\right)\right]\right\rangle_S\right\} \tag{86}$$

The average on the right-hand side of Eq.(86) can be expand in the first two cumulants

$$\left\langle exp\left(ik\left(x-x'\right)\right)\right\rangle_S = exp\left\{ik\left\langle x-x'\right\rangle_S - \frac{k^2}{2}\left(\left\langle\left(x-x'\right)^2\right\rangle_S - \left\langle x-x'\right\rangle^2_S\right)\right\} \tag{87}$$

To obtain $\rho_r\left(x,x',t\right)$ we have to find the average $\left\langle \delta W_k\left[x\left(t\right),x'\left(t'\right)\right]\right\rangle_S$ which is the form $\left\langle exp\left[ik\left(x\left(t\right)-x'\left(t'\right)\right)\right]\right\rangle_S$, $\left\langle exp\left[ik\left(x'\left(t\right)-x'\left(t'\right)\right)\right]\right\rangle_S$ and $\left\langle exp\left[ik\left(x\left(t\right)-x\left(t'\right)\right)\right]\right\rangle_S$. These can be expressed in term of following average $\left\langle x\left(\tau\right)x\left(\sigma\right)\right\rangle_S$ and $\left\langle x^2\left(\tau\right)\right\rangle_S$. Such average are obtain from a characteristic functional that unperturbed system is linearly coupled to the external force $J\left(\tau\right)$ and $J'\left(\tau\right)$ with the action, $S_{ext}\left[J,J'\right] = \int_0^t d\tau J\left(\tau\right)x\left(\tau\right) - \int_0^t d\tau J'\left(\tau\right)x'\left(\tau\right)$. Written the form of generating functional average as

$$G_0\left[J,J'\right] = \left\langle exp\left\{\frac{i}{\hbar}\left[\int_0^t d\tau J\left(\tau\right)x\left(\tau\right) - \int_0^t d\tau J'\left(\tau\right)x'\left(\tau\right)\right]\right\}\right\rangle_S \tag{88}$$

From Feynman and Hibbs, 1965 , Sayakanit, 1974 and Hu, 1993, the average $\left\langle x(\tau)x(\sigma)\right\rangle_S$ and $\left\langle x(\tau)x'(\sigma)\right\rangle_S$ can be obtained in the form

$$\begin{aligned}\left\langle x(\tau)x(\sigma)\right\rangle_S &= \frac{(i\hbar)^2}{G_0[J,J']}\frac{\delta}{\delta J(\tau)}\left\{\frac{\delta}{\delta J(\sigma)}G_0[J,J']\right\}_{J=J'=0}\\ \left\langle x(\tau)x'(\sigma)\right\rangle_S &= \frac{-(i\hbar)^2}{G_0[J,J']}\frac{\delta}{\delta J(\tau)}\left\{\frac{\delta}{\delta J(\sigma)}G_0[J,J']\right\}_{J=J'=0}\end{aligned} \tag{89}$$

For the propose to evaluation the density matrix , we should apply these expression to our problem.

Harmonic Oscillator in Random Scattering Potential

In order apply to physical system, we focus on the most simplest and very important know as the harmonic potential. We assume that a particle is confine as a harmonic oscillator, Hamiltonian of a quantum particle can be written as

$$\hat{H}_0 = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega_0^2\hat{x}^2 \tag{90}$$

The initial density matrix of the harmonic oscillator is in its ground state energy. In general, state of system can be solve exactly(Feynman and Hibbs, 1965)

$$\Psi_n(x) = \frac{1}{\sqrt{n!}}\left[\frac{1}{\sqrt{2m\omega}}\left(m\omega x - \frac{\partial}{\partial x}\right)\right]^n\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} exp\left(-\frac{m\omega}{2\hbar}x^2\right) \tag{91}$$

Corresponding to the energy , $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$. For the ground state, state of system describe as a Gaussian form

$$\Psi_0(x) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{\frac{1}{4}} exp\left(-\frac{m\omega_0}{2\hbar}x^2\right) \tag{92}$$

Since the ground state is pure, we obtained the pure state density matrix

$$\rho_0\left(x\ ,x'\ ,0\right)=\sqrt{\frac{m\omega_0}{\pi\hbar}}\,exp\left\{-\frac{m\omega_0}{2\hbar}\left(x^2+x'^2\right)\right\} \tag{93}$$

Which can be represent in Figure 11 , where system is setting in nanoscale $\left(10^{-9}m\right)$, m is an electron mass $9.1\times10^{-31}kg$, ω_0 is correspond to frequency at ground state energy E_0 given as $E_0=1eV$.

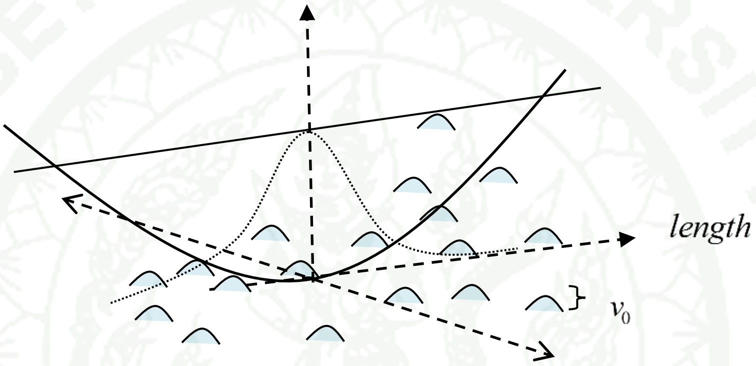


Figure 10 Showing the ground state harmonic oscillator by present of random scattering potential

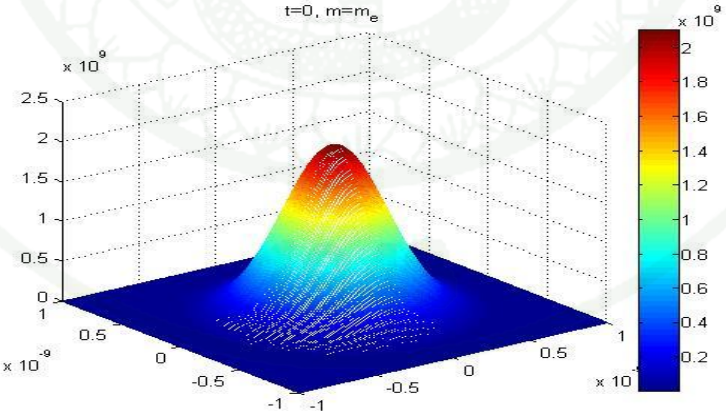


Figure 11 Density matrix of a ground state harmonic oscillator.

In process evaluation of the reduced density matrix for our problem. Recall that the general form of reduced density matrix Eq.(86) , Thus for conveniently, rewritten reduced density matrix in compact form

$$\rho_r\left(x,x',t\right)=\rho_0\left(x,x',t\right)exp\left\{-\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\Phi\left(t,t'\right)\right\} \tag{94}$$

Where

$$\begin{aligned}\Phi\left(t,t'\right)=&\int\frac{dk}{2\pi}v\left(k\right)exp\left(-\frac{k^2}{2}\left\langle\left(x\left(t\right)-x\left(t'\right)\right)^2\right\rangle_S\right)+\int\frac{dk}{2\pi}v\left(k\right)exp\left(-\frac{k^2}{2}\left\langle\left(x'\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S\right)\\&-2\int\frac{dk}{2\pi}v\left(k\right)exp\left(-\frac{k^2}{2}\left\langle\left(x\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S\right)\end{aligned} \tag{95}$$

Substitute $v\left(k\right)$ from Eq.(95) , thus integral of right hand side of Eq.(95) is a Gaussian form which can be perform exactly, thus rewritten

$$\Phi\left(t,t'\right)=\int\frac{dk}{2\pi}exp\left(-k^2A\right)-2\int\frac{dk}{2\pi}exp\left(-k^2B\right)+\int\frac{dk}{2\pi}exp\left(-k^2C\right) \tag{96}$$

Where defined parameters are

$$A=\frac{L^2}{4}+\frac{1}{2}\left\langle\left(x\left(t\right)-x\left(t'\right)\right)^2\right\rangle_S,B=\frac{L^2}{4}+\frac{1}{2}\left\langle\left(x'\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S,C=\frac{L^2}{4}+\frac{1}{2}\left\langle\left(x'\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S \tag{97}$$

By using relation $\int_{-\infty}^{+\infty}\exp\left(-ax^2+bx\right)dx=\sqrt{\frac{\pi}{a}}\exp\left(-\frac{b^2}{4a}\right)$, performing Gaussian integral We have obtained

$$\Phi\left(t,t'\right)=\left[\left(1+\frac{2\left\langle\left(x\left(t\right)-x\left(t'\right)\right)^2\right\rangle_S}{L^2}\right)^{-\frac{1}{2}}-2\left(1+\frac{\left\langle\left(x'\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S}{L^2}\right)^{-\frac{1}{2}}+\left(1+\frac{2\left\langle\left(x\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S}{L^2}\right)^{-\frac{1}{2}}\right] \tag{98}$$

Any terms in the bracket can be evaluated by using relation

$$\left\langle\left[x(s_1)-x(s_2)\right]^2\right\rangle_s=\left\langle x^2(s_1)\right\rangle_s+\left\langle x^2(s_2)\right\rangle_s-2\left\langle x(s_1)x(s_2)\right\rangle_s$$

$$\left\langle\left[x'(s_1)-x'(s_2)\right]^2\right\rangle_s=\left\langle x'^2(s_1)\right\rangle_s+\left\langle x'^2(s_2)\right\rangle_s-2\left\langle x'(s_1)x'(s_2)\right\rangle_s$$

$$\left\langle\left[x(s_1)-x'(s_2)\right]^2\right\rangle_s=\left\langle x^2(s_1)\right\rangle_s+\left\langle x'^2(s_2)\right\rangle_s-\left\langle x(s_1)x'(s_2)\right\rangle_s-\left\langle x'(s_1)x(s_2)\right\rangle_s \qquad (99)$$

The term in right hand side is obtained by Eq.(89). To perform evaluated these relation, the generating functional corresponding with our problem is given by

$$\begin{aligned}G_0[J,J']=&\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx'_a\rho_0(x_a,x'_a)\int_{x_a}^{x}D[x]\int_{x'_a}^{x'}D[x']exp\left\{\frac{i}{\hbar}\left(S_0[x]-S_0[x']\right)\right\}\\&\times exp\left\{\frac{i}{\hbar}\left[\int_0^t d\tau J(\tau)x(\tau)-\int_0^t d\tau J'(\tau)x'(\tau)\right]\right\}\end{aligned} \qquad (100)$$

Where $S_0[x]$ and $S_0[x']$ is the action of isolated particle with difference path

$$S_0[x]=\int_0^t dt\left[\frac{1}{2}m\dot{x}^2-\frac{1}{2}m\omega_0^2x^2\right] \text{ and } S_0[x']=\int_0^t dt'\left[\frac{1}{2}m\dot{x}'^2-\frac{1}{2}m\omega_0^2x'^2\right] \qquad (101)$$

Written the generating functional in the compact form

$$G_0[J,J']=\int_{-\infty}^{+\infty}dx_a\int_{-\infty}^{+\infty}dx'_a\rho_0(x_a,x'_a)K\left(x_b,t|x_a,0;J\right)K^*\left(x'_b,t|x'_a,0;J'\right) \qquad (102)$$

The propagator $K\left(x_b,t|x_a,0;J\right)$ is similarly to the force harmonic oscillator which can be solve exactly as

$$K\left(x_b,t\middle|x_a,0\right)=\sqrt{\frac{m\omega_0}{2\pi i\hbar\,sin\left(\omega_0 t\right)}}\,exp\left\{\frac{i}{\hbar}\left[\frac{m\omega_0}{2\,sin\,\omega_0 t}\left(\left(x_b^2+x_a^2\right)cos\left(\omega_0 t\right)-2x_b x_a\right)\right]\right.$$
$$+\frac{x_a}{sin\left(\omega_0 t\right)}\int_0^t dt' J\left(t'\right)sin\left(\omega_0\left(t-t'\right)\right)+\frac{x_b}{sin\left(\omega_0 t\right)}\int_0^t dt' J\left(t'\right)sin\left(\omega_0 t'\right)$$
$$\left.-\frac{1}{m\omega_0\,sin\left(\omega_0 t\right)}\int_0^t dt'\int_0^t ds J\left(t'\right)J\left(s\right)sin\left(\omega_0 s\right)sin\left(\omega_0\left(t-t'\right)\right)\right\} \qquad (103)$$

For the process evaluation $G_0\left[J,J'\right]$, it is conveniently rewritten the propagator as a simple form

$$K\left(x_b,t\middle|x_a,0\right)=P_0\,exp\left\{ax_b^2+bx_a^2-2bx_b x_a+c_1 x_a+c_2 x_b+d\right\} \qquad (104)$$

And the initial density matrix of ground state harmonic oscillator

$$\rho_0\left(x_a,x'_a,0\right)=\sqrt{\frac{m\omega_0}{\pi\hbar}}\,exp\left\{-A\left(x_b^2+x_a^2\right)\right\} \qquad (105)$$

Where defined parameters

$$P_0=\sqrt{\frac{m\omega_0}{2\pi i\hbar\,sin\left(\omega_0 t\right)}},\quad A=\frac{m\omega_0}{2\hbar},\quad a=\frac{im\omega_0\,cos\left(\omega_0 t\right)}{2\hbar\,sin\,\omega_0 t},b=\frac{im\omega_0}{2\hbar\,sin\,\omega_0 t}$$
$$c_1=\frac{i}{\hbar\,sin\left(\omega_0 t\right)}\int_0^t dt' J\left(t'\right)sin\left(\omega_0\left(t-t'\right)\right),\quad c_2=\frac{i}{\hbar\,sin\left(\omega_0 t\right)}\int_0^t dt' J\left(t'\right)sin\left(\omega_0 t'\right)$$
$$d=-\frac{1}{m\omega_0\,sin\left(\omega_0 t\right)}\int_0^t dt'\int_0^t ds J\left(t'\right)J\left(s\right)sin\left(\omega_0 s\right)sin\left(\omega_0\left(t-t'\right)\right)\Bigg\} \qquad (106)$$

Substituted Eq.(104-107) in Eq.(102) and taken some algebra, we have obtained

$$G_0\left[J,J'\right]=\sqrt{\frac{m\omega_0}{\pi\hbar}}\,exp\left\{-\frac{m\omega_0}{2\hbar}\left[x_b^2+x'^2_b\right]+x_b\frac{i}{\hbar}\int_0^t dt' J\left(t'\right)e^{-i\omega_0\left(t-t'\right)}+x'_b\frac{i}{\hbar}\int_0^t dt' J'\left(t'\right)e^{i\omega_0\left(t-t'\right)}\right.$$
$$\left.-\frac{i}{\hbar m\omega_0}\int_0^t dt'\int_0^{t'} ds\left[J\left(s\right)J\left(t'\right)sin\left(\omega_0\left(t-t'\right)\right)e^{-i\omega_0\left(t-s\right)}-J'\left(s\right)J'\left(t'\right)sin\left(\omega_0\left(t-t'\right)\right)e^{i\omega_0\left(t-s\right)}\right]\right\} \qquad (107)$$

Following the process for evaluation, Eq.(89) and Eq.(99) thus, finally we get

$$
\begin{aligned}
\left\langle \left[x(s_1)-x(s_2)\right]^2 \right\rangle_s &= 2x^2\left[\cos^2(\omega_0 s_1)+\cos^2(\omega_0 s_2)-1\right]-2x^2\cos\left[\omega_0(s_1+s_2)\right] \\
&+\frac{2\hbar}{m\omega_0}\left[\sin^2(\omega_0 s_1)+\sin^2(\omega_0 s_2)\right]+2ix^2\sin\left[\omega_0(s_1+s_2)\right]-ix^2\left[\sin(2\omega_0 s_1)+\sin(2\omega_0 s_2)\right] \\
&+\frac{2i\hbar}{m\omega_0}\left[\sin(2\omega_0 s_1)+\sin(2\omega_0 s_2)\right]+Q_1(t)
\end{aligned}
\tag{108}
$$

$$
\begin{aligned}
\left\langle \left[x'(s_1)-x'(s_2)\right]^2 \right\rangle_s &= 2x'^2\left[\cos^2(\omega_0 s_1)+\cos^2(\omega_0 s_2)-1\right]-2x'^2\cos\left[\omega_0(s_1+s_2)\right] \\
&+\frac{2\hbar}{m\omega_0}\left[\sin^2(\omega_0 s_1)+\sin^2(\omega_0 s_2)\right]-2ix'^2\sin\left[\omega_0(s_1+s_2)\right]+ix^2\left[\sin(2\omega_0 s_1)+\sin(2\omega_0 s_2)\right] \\
&-\frac{2i\hbar}{m\omega_0}\left[\sin(2\omega_0 s_1)+\sin(2\omega_0 s_2)\right]+Q_2(t)
\end{aligned}
\tag{109}
$$

$$
\begin{aligned}
\left\langle \left[x(s_1)-x'(s_2)\right]^2 \right\rangle_s &= x^2\left[2\cos^2(\omega_0 s_1)-1\right]+x'^2\left[2\cos^2(\omega_0 s_2)-1\right] \\
&+\frac{2\hbar}{m\omega_0}\left[\sin^2(\omega_0 s_1)+\sin^2(\omega_0 s_2)\right]-2xx'\cos\left[\omega_0(s_2-s_1)\right]-ix^2\sin(2\omega_0 s_1) \\
&+ix'^2\sin(2\omega_0 s_2)+\frac{2i\hbar}{m\omega_0}\sin\left[\omega_0(s_2-s_1)\right]
\end{aligned}
\tag{110}
$$

Where

$$
\begin{aligned}
Q_1 &= -\frac{\hbar}{m\omega_0}\left[\sin(\omega_0 s)\sin(\omega_0 t')\right]-i\frac{\hbar}{m\omega_0}G(s_1-s_2) \\
Q_2 &= -\frac{\hbar}{m\omega_0}\left[\sin(\omega_0 s)\sin(\omega_0 t')\right]+i\frac{\hbar}{m\omega_0}G(s_1-s_2)
\end{aligned}
\tag{111}
$$

and

$$
G(s_1-s_2)=\left[\sin(\omega_0 s)\sin(\omega_0 t')\right]\Theta(t'-s)+\left[\cos(\omega_0 s)\sin(\omega_0 t')\right]\Theta(s-t') \tag{112}
$$

Also, we observe that

$$
\left\langle x(s_1)-x(s_2)\right\rangle_s=\left\langle x(s_1)-x'(s_2)\right\rangle_s=\left\langle x'(s_1)-x'(s_2)\right\rangle_s=0 \tag{113}
$$

Now, we get the average density matrix defined by Eq.(94). It is shows that dissipation is present via the correlation function defined by Eq.(95). Clearly, it is produced by an exponential factor, $exp\left\{-\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\Phi\left(t,t'\right)\right\}$, Where $\Phi\left(t,t'\right)$ can be written in the compact form

$$\Phi\left(t,t'\right)=\left[\left(1+\frac{2a\left(t\right)}{L^2}\right)^{-\frac{1}{2}}-2\left(1+\frac{2b\left(t\right)}{L^2}\right)^{-\frac{1}{2}}+\left(1+\frac{2c\left(t\right)}{L^2}\right)^{-\frac{1}{2}}\right] \qquad (114)$$

And the parameter $a(t),b(t),c(t)$ determined by relation, Eq.(109-111)

$$a\left(t\right)=\left\langle\left(x\left(t\right)-x\left(t'\right)\right)^2\right\rangle_S,b\left(t\right)=\left\langle\left(x\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S,c\left(t\right)=\left\langle\left(x'\left(t\right)-x'\left(t'\right)\right)^2\right\rangle_S \qquad (115)$$

Physically the above obtained result, By considering the time evoluation of density matrix after averaged overall random scattering potential, we should regard that the spreading of the wave packet, scattering on impurities, and the interference of scattering wave lead to decoherence. We expect the initial Gaussian wave packet to evolve over time if decoherence due to random scattering potential is taken into account. We anticipate that the interaction with the potential will make it increasingly hard to observe spatial coherences over a given distance $\left|x-x'\right|$. That is, we expect the coherence length $\beta=\left|x-x'\right|$ to decrease over time and squeezing the Gaussian wave packet Eq.(93) in the direction perpendicular to the diagonal, $x=x'$. We expect that this situation can be actually occurred. So, we need to confirm these intuitions by studying the time evolution of some of the relevant quantities obtained by using the explicit expressions Eq.(114) and Eq. (115) in next section.

RESULT AND DISCUSSION

Decoherence Time Scale

In previos section, we obtained the general form of influence phase as double integral in time $\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\Phi\left(t,t'\right)$, We propose that it destroys the phase coherence of system. For demonstrates, the function $\Phi\left(t,t'\right)$ is depended on time variables defined by Eq.(108-110) which can not be integrated exactly , however for nontrivial case we can focus on asymptotic behaviors by considering condition that the coherence would be destroyed rapidly, shorter than time scale of system $\left(2\pi/\omega_0\right)$, $\tau_d \prec\prec t$, where τ_d is decoherence time scale of system. By expanding influence phase as a power series

$$\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\Phi\left(t,t'\right) \cong \Lambda^{(0)} + \Lambda^{(1)}t + \Lambda^{(2)}t^2 + \ldots \qquad (116)$$

By using the shot time condition , we observe that the integral is double integral in time, then the prefactor occurred by

The first term ; $$\Lambda^{(0)} = \frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\,\Phi\left(t,t'\right)\Big|_{s_1=s_2=0} = 0 \qquad (117)$$

The second term ; $$\Lambda^{(1)} = \frac{\xi_L}{2\hbar^2}\frac{\delta}{\delta t}\left(\int_0^t\int_0^t dtdt'\,\Phi\left(t,t'\right)\Big|_{s_1=s_2=0}\right) = 0 \qquad (118)$$

The third term ; $$\Lambda^{(2)} = \frac{\xi_L}{2\hbar^2}\frac{\delta^2}{\delta t\delta t'}\left(\int_0^t\int_0^t dtdt'\,\Phi\left(t,t'\right)\Big|_{s_1=s_2=0}\right) = \frac{\xi_L}{\hbar^2}\left[1-\left(1+2\left(\frac{x-x'}{L}\right)^2\right)^{-1/2}\right] \qquad (119)$$

Keep up only $\Lambda^{(2)}$, thus we have obtained

$$\frac{\xi_L}{2\hbar^2}\int_0^t\int_0^t dtdt'\Phi\left(t,t'\right)=\frac{\xi_L}{\hbar^2}\left[1-\left(1+2\left(\frac{x-x'}{L}\right)^2\right)^{-1/2}\right]t^2 \qquad (120)$$

Now, the time evolution of density matrix can be written in the form

$$\rho_r\left(x,x',t\right)=\rho_0\left(x,x',t\right)exp\left\{-\frac{t^2}{t_d^2}\right\} \qquad (121)$$

We observe that the density matrix defined by Eq.(121) is multiplied by decaying factor $exp\left\{-\frac{t^2}{t_d^2}\right\}$ it decreases exponentially and characterized by the characteristic time for decoherence to occur will be defined as the decoherence time scale(τ_d^{-2})

$$\tau_d^{-2}=\frac{\xi_L}{\hbar^2}\left[1-\left(1+2\left(\frac{x-x'}{L}\right)^2\right)^{-1/2}\right] \qquad (122)$$

This is the decoherence time scale for a harmonic oscillator due to the Gaussian random potential disorders, it depends on the strength of correlation between scattering potential(ξ_L) and the correlation length$\left(L\right)$.

In order to analyze decoherence time scale, we plot the function given by Eq.(122) versus the coherence length of system $\beta=\left|x-x'\right|$. The curve in Figure 12 appears to be concave up from $\beta=0$ to $\beta=L/2$ (the critical points). For $\beta\succ L/2$, the curve is concave down.The off-diagonal element,where $\beta\succ L/2$,disappears faster than the region $0 \prec \beta \prec L/2$. The time evolution of the off-diagonal initial density matrix, with the Gaussian wave packet Eq.(93), separated by a distance of $\beta = |x - x'|$ decreases exponentially over no single characteristic time scale. We shall be interested in two asymptotic characteristics of decoherence.

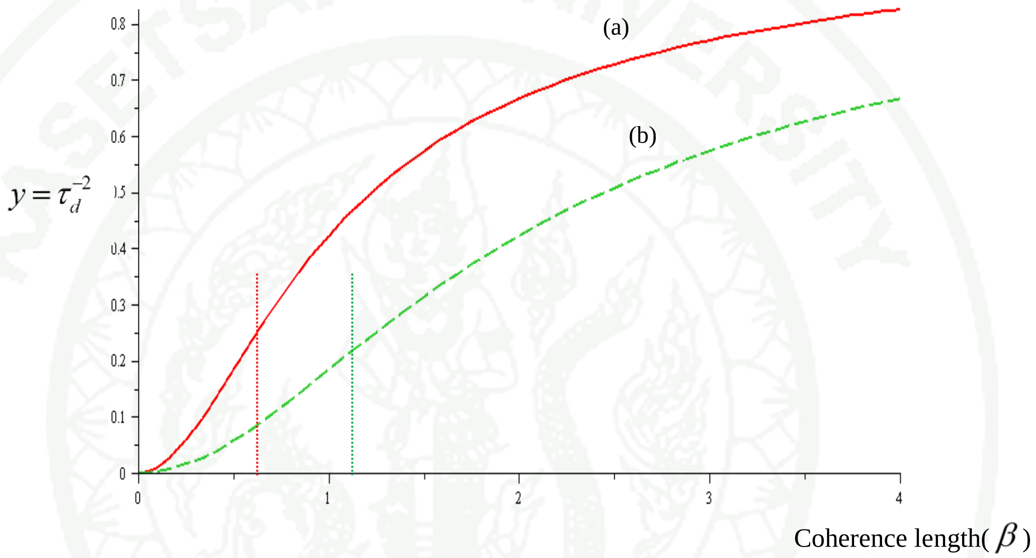


Figure 12 Plot of the decoherence time scale corresponding to matrix elements given by Eq.(122). Critical value of $\beta = L/2$ (two dash line) splits the decrease rate into two regions where the correlation length $L = 1$ (a) and $L = 2$ (b) respectively.

Two analytical asymptotic expressions for the evolution of the off-diagonal density matrix in the coordinate representation are given by

Case I. Short-range region $|x - x'| \prec\prec \frac{L}{2}$

$$\tau_d^{-2} = 2\gamma^{-2}\left(\frac{x - x'}{L}\right)^2 \qquad (123)$$

Case II. Long-range region $|x-x'| \succ\succ \frac{L}{2}$

$$\tau_d^{-2} = \gamma^{-2}\left(1-\sqrt{\frac{1}{2}}\frac{L}{|x-x'|}\right) \tag{124}$$

Where, the relaxation time scale (γ), $\gamma^{-2} = \frac{\xi_L}{\hbar^2}$

From Eq.(123) and Eq.(124), asymptotically, there are two regions with different decoherence rates plotting in Figure 13. The loss of quantum coherence is induced by the interference of waves scattered by a Gaussian random potential. Wherever the correlation length *L* is much larger than the coherence length β, there is localized particle (the short wave scattering¸ λ < *L*) and the scattering would occur by nearest neighbor interaction. The quantum particle recognizes only the nearest neighbor random potentials. We anticipate that a significant interference leading to this decoherence is due to a large number of scattering events. On the other hand if $\beta \succ 2$, the long wavelength is the main cause that induces the decoherence. This is because the wave-particle can scatter with each random potential, even though it is far from the source of potential (the long wave scattering ¸ λ > *L*). Thus the scattering-induced decoherence is due to the time decrease rate that the coherence length β is less than *L*/2. Next, we shall show the numerical results which fairly in agreement with the above discussion.

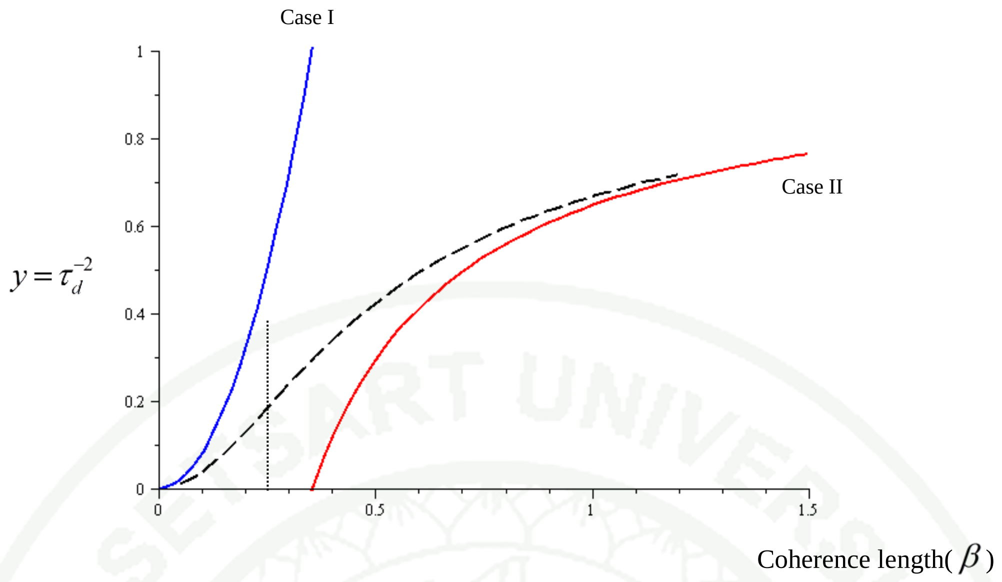


Figure 13 Plot The decoherence time scale Eq.(122)(dash line) and two asymptotic characteristics of decoherence time scale.

Decays of Off-Diagonal Density Matrix

As an example we compare the break down of coherence in 2 cases: (I) a heavy mass and (II) a light mass. If the harmonic motion particle has a very large mass then the wave packet is narrow in space. In this case, the short wavelength scattering is the main cause of decoherence. In another case, the light mass has a wave packet which spreading out. This spreading indicates that a wider range of position is more probable as the particle moves. The long-wavelength components increase. This means the long wave scattering causes decoherence.

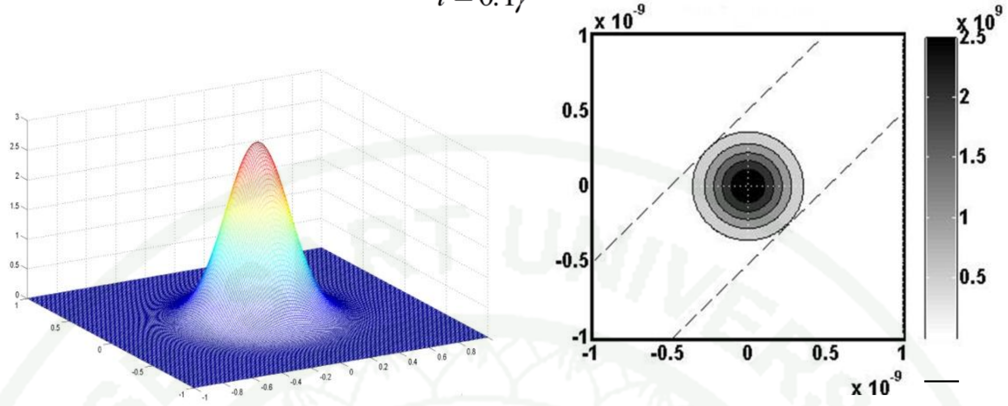
To visualize the decrease of coherence, we plot the magnitude of the matrix elements of density matrix Eq.(121) in 3D and 2D on a plane in Figure(14,15).

The decohering of simple harmonic oscillators with light and heavy mass are initially in their ground states. We plot the off-diagonals elements of density matrix at difference values of mass , for the light mass, $m = m_e$ and for the heavy mass, $m = 20m_e$ all the plots are ¸ $\lambda = 10^4 (nm)^{-1}$ = 0.4 , L = 0.5 nm , E_0 = 1 eV and $\gamma = \frac{\hbar}{\sqrt{\xi_L}}$. The components of the density matrix (the light mass particle) rapidly decay from their initial values in the interval time (0,5γ). In contrary, in Figure 15, the components of the density matrix (the heavy mass particle) scarcely decrease in the interval time (0, 5γ).

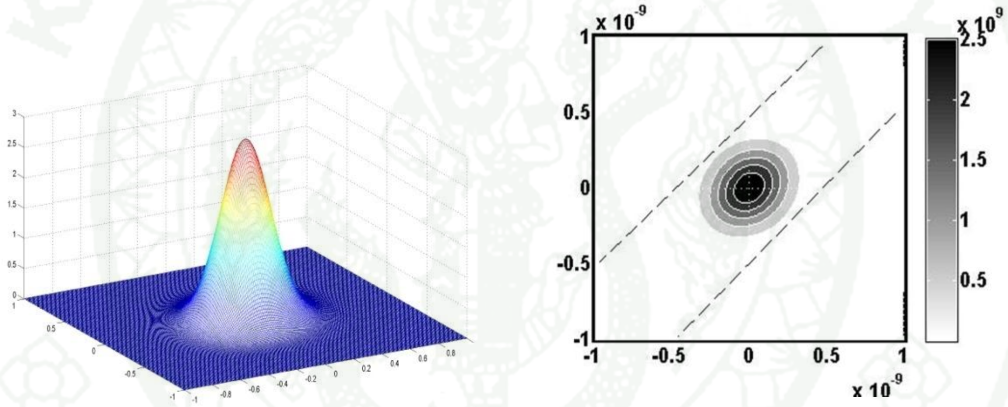
In each diagram the shade region represents the Gaussian wave. The peak is at the darkest. The peak is lower and the wave is spreading out as time goes by. In analogy with Eq.(121), the off-diagonal elements decrease. Two dash lines are curves of the equation $y = \beta \pm L/2$.

Case I. Heavy mass ($m = 20m_e$ **)**

$t = 0.1\gamma$



$t = 2\gamma$



$t = 5\gamma$

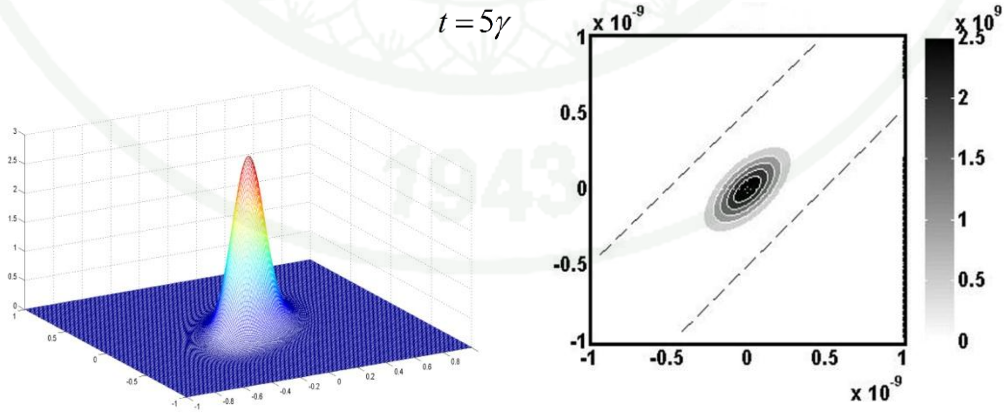
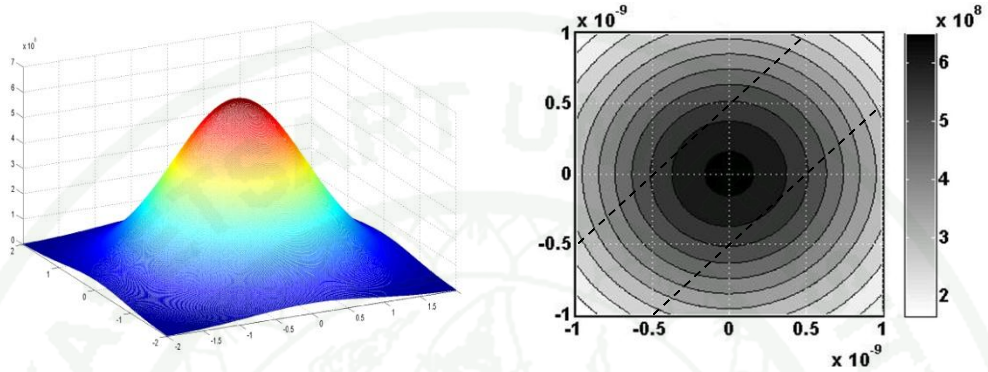


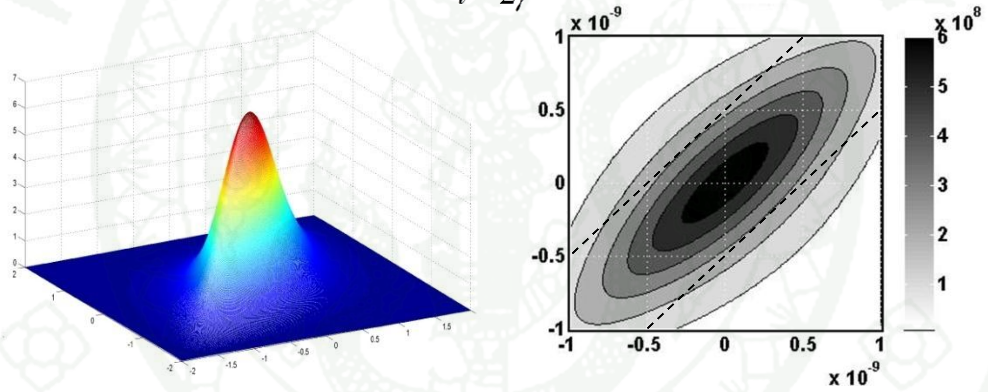
Figure 14 Plot of the off-diagonals elements of density matrix for heavy mass

<u>Case II. Light mass (</u> $m = m_e$ **)**

$t = 0.1\gamma$



$t = 2\gamma$



$t = 5\gamma$

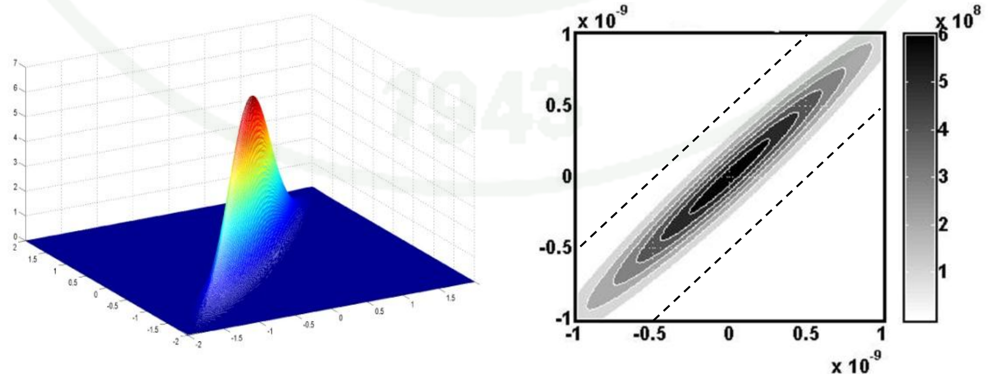


Figure 15 Plot of the off-diagonals elements of density matrix for light mass

CONCLUSION

In this work, the path integral technique is applied to calculate the time evolution of density matrix of the harmonic oscillator particle (1D) moving in disorder Gaussian random potential. In the approximation that we neglected multiple scattering, we got the impurity-averaged density matrix by tying all the potential vertices pairwise together. The asymptotic form of decoherence time scale was considered.

The characteristic of the decoherence time scale was studied. From the plot of the decoherence time scale as a function of the coherence length, varying within the interval (0,1-5*nm*) there are lines representing critical value, which split the decoherence time scale in two characteristics, i.e. the short-range region and the long-range region, comparing with the correlation length. In the first case, the short wavelength scattering induces decoherence. The scattering would occur by the nearest neighbor interaction. The significant interference leading to decoherence come from a large number of scattering events. In the second case, the long wavelength scattering is the main cause that destroys the coherence.

The analysis on the decrease in magnitude of density matrix in cases of the heavy and the light particles is introduced. For the heavy particle, the initial density matrix is localized in the short-range region. It can be seen from Figure(14) that the decrease of the off-diagonal matrix elements is from the long wavelength-scattering. In Figure(15), for the heavy particle, the decoherence effect can be explicitly characterized by two time scale rates corresponding to Eq.(123-124). This can be interpreted that it is due to the contribution of interaction from both short and long wavelength scatterings. This means that the wave-particle is able to scatter with each random potential, even though it is far from it. Finally we can conclude that the scattering-induced decoherence is from both long and short wavelength scatterings. It depends on the magnitude of the coherence length.

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