

CHAPTER 5 CONCLUSIONS

5.1 Outputs

ISI International Journal

Wiyada Kumam, Chaichana Jaiboon, Poom Kumam, and Akarate Singta, A shrinking projection method for generalized mixed equilibrium problems, variational inclusion problems and a finite family of quasi-nonexpansive mappings, Journal of Inequalities and Applications, Volume 2010, Article ID 458247, 25 pages.

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International Conference

Wiyada Kumam and Poom Kumam, "A system of equilibrium problems and variational inclusion problems", The 4th International Conference on Fixed Point Theory, Variational Inequality and Its Approximation Algorithms, July 16-19, 2010, Chengdu, China.

National Conference

W. Kuman, C. Jaiboon, P. Kumam and A. Singta.2010, "A new hybrid projection method for variational inclusion problems and generalized mixed equilibrium problems." The 4 th Conference on Fixed Point Theory and Applications (FPTA 2010) July 23-24,2010 Department of Mathematics Faculty of Science king Mongkut's University of Technology Thonburi (KMUTT) Bangkok Thailand. Page 65-66.

5.2 Conclusions

In this research, we establish the following results.

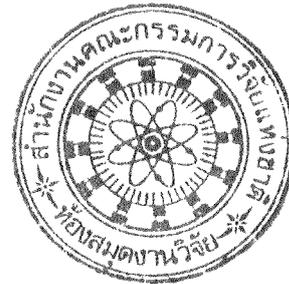
Theorem 5.2.1. *Let C be a nonempty closed convex subset of a real Hilbert space H , let F be a bifunction from $C \times C$ to \mathbb{R} satisfying (A1)-(A4) and let $\varphi : C \rightarrow \mathbb{R} \cup \{+\infty\}$ be a proper lower semicontinuous and convex function with assumption (B1) or (B2).*

Let $\{T_i\}_{i=1}^N$ a finite family of quasi-nonexpansive and L_i -Lipschitz mappings of C into itself, let A be a β -inverse-strongly monotone mapping of C into H , let B be a ξ -inverse-strongly monotone mapping of C into H and $M : H \rightarrow 2^H$ be a maximal monotone mapping. Assume that

$$\Theta := \bigcap_{i=1}^N F(T_i) \cap \text{GMEP}(F, \varphi, A) \cap I(B, M) \neq \emptyset.$$

Let K_n be the K -mapping generated by T_1, T_2, \dots, T_N and $\gamma_{n,1}, \gamma_{n,2}, \dots, \gamma_{n,N}$. Let $\{x_n\}$, $\{y_n\}$, $\{v_n\}$, $\{z_n\}$ and $\{u_n\}$ be sequences generated by $x_0 \in H$, $C_1 = C$, $x_1 = P_{C_1}x_0$, $u_n \in C$ and let

$$\left\{ \begin{array}{l} F(u_n, y) + \varphi(y) - \varphi(u_n) + \langle Ax_n, y - u_n \rangle + \frac{1}{r_n} \langle y - u_n, u_n - x_n \rangle \geq 0, \quad \forall y \in C, \\ y_n = J_{M, \delta_n}(u_n - \delta_n B u_n), \\ v_n = J_{M, \lambda_n}(y_n - \lambda_n B y_n), \\ z_n = \alpha_n x_n + (1 - \alpha_n) K_n v_n, \\ C_{n+1} = \left\{ z \in C_n : \|z_n - z\| \leq \|x_n - z\| \right\}, \\ x_{n+1} = P_{C_{n+1}} x_0, \quad n \in \mathbb{N}, \end{array} \right. \quad (5.2.1)$$



where $\{\alpha_n\} \subset (0, 1)$ satisfy the following conditions:

- (i) $\{\alpha_n\} \subset [0, e]$ for some e with $0 \leq e < 1$,
- (ii) $\{\delta_n\}, \{\lambda_n\} \subset [a, b]$ for some a, b with $0 < a < b < 2\xi$,
- (iii) $\{r_n\} \subset [c, d]$ for some c, d with $0 < c < d < 2\beta$.

Then, $\{x_n\}$ and $\{u_n\}$ converge strongly to $z = P_{\Theta}x_0$.

Corollary 5.2.2. Let C be a nonempty closed convex subset of a real Hilbert space H , let F be a bifunction from $C \times C$ to \mathbb{R} satisfying (A1)-(A4) and let $\varphi : C \rightarrow \mathbb{R} \cup \{+\infty\}$ be a proper lower semicontinuous and convex function with assumption (B1) or (B2). Let $\{T_i\}_{i=1}^N$ a finite family of nonexpansive mappings of C into itself, let A be a β -inverse-strongly monotone mapping of C into H , let B be a ξ -inverse-strongly monotone mapping of C into H and $M : H \rightarrow 2^H$ be a maximal monotone mapping. Assume that

$$\Theta := \bigcap_{i=1}^N F(T_i) \cap \text{GMEP}(F, \varphi, A) \cap I(B, M) \neq \emptyset.$$

Let K_n be the K -mapping generated by T_1, T_2, \dots, T_N and $\gamma_{n,1}, \gamma_{n,2}, \dots, \gamma_{n,N}$. Let $\{x_n\}$, $\{y_n\}$, $\{v_n\}$, $\{z_n\}$ and $\{u_n\}$ be sequences generated by (5.2.1) satisfy the following conditions in Theorem 5.2.1. Then, $\{x_n\}$ and $\{u_n\}$ converge strongly to $z = P_\Theta x_0$.

Theorem 5.2.3. Let C be a nonempty closed convex subset of a real Hilbert space H , let F be a bifunction from $C \times C$ to \mathbb{R} satisfying (A1)-(A4) and let $\varphi : C \rightarrow \mathbb{R} \cup \{+\infty\}$ be a proper lower semicontinuous and convex function with assumption (B1) or (B2). Let $\{T_i\}_{i=1}^N$ a finite family of quasi-nonexpansive and L_i -Lipschitz mappings of C into itself, let A be a β -inverse-strongly monotone mapping of C into H and let B be a ξ -inverse-strongly monotone mapping of C into H . Assume that

$$\Theta := \bigcap_{i=1}^N F(T_i) \cap \text{GMEP}(F, \varphi, A) \cap \text{VI}(C, B) \neq \emptyset.$$

Let K_n be the K -mapping generated by T_1, T_2, \dots, T_N and $\gamma_{n,1}, \gamma_{n,2}, \dots, \gamma_{n,N}$. Let $\{x_n\}$, $\{y_n\}$, $\{v_n\}$, $\{z_n\}$ and $\{u_n\}$ be sequences generated by $x_0 \in H$, $C_1 = C$, $x_1 = P_{C_1} x_0$, $u_n \in C$ and let

$$\left\{ \begin{array}{l} F(u_n, y) + \varphi(y) - \varphi(u_n) + \langle Ax_n, y - u_n \rangle + \frac{1}{r_n} \langle y - u_n, u_n - x_n \rangle \geq 0, \quad \forall y \in C, \\ y_n = P_C(u_n - \delta_n B u_n), \\ v_n = P_C(y_n - \lambda_n B y_n), \\ z_n = \alpha_n x_n + (1 - \alpha_n) K_n v_n, \\ C_{n+1} = \{z \in C_n : \|z_n - z\| \leq \|x_n - z\|\}, \\ x_{n+1} = P_{C_{n+1}} x_0, \quad n \in \mathbb{N}, \end{array} \right.$$

where $\{\alpha_n\} \subset (0, 1)$ satisfy the following conditions:

- (i) $\{\alpha_n\} \subset [0, e]$ for some e with $0 \leq e < 1$,
- (ii) $\{\delta_n\}, \{\lambda_n\} \subset [a, b]$ for some a, b with $0 < a < b < 2\xi$,
- (iii) $\{r_n\} \subset [c, d]$ for some c, d with $0 < c < d < 2\beta$.

Then, $\{x_n\}$ and $\{u_n\}$ converge strongly to $z = P_\Theta x_0$.

Theorem 5.2.4. Let C be a nonempty closed convex subset of a real Hilbert space H , let F be a bifunction from $C \times C$ to \mathbb{R} satisfying (A1)-(A4) and let $\varphi : C \rightarrow \mathbb{R} \cup \{+\infty\}$ be a proper lower semicontinuous and convex function with assumption (B1) or (B2).

Let $\{T_i\}_{i=1}^N$ a finite family of quasi-nonexpansive and L_i -Lipschitz mappings of C into itself, let S_A be an κ_β -strictly pseudo-contraction mapping of C into C and let S_B be an κ_ξ -strictly pseudo-contraction mapping of C into C . Assume that

$$\Theta := \bigcap_{i=1}^N F(T_i) \cap GMEP(F, \varphi, I - S_A) \cap F(S_B) \neq \emptyset.$$

Let K_n be the K -mapping generated by T_1, T_2, \dots, T_N and $\gamma_{n,1}, \gamma_{n,2}, \dots, \gamma_{n,N}$. Let $\{x_n\}$, $\{y_n\}$, $\{v_n\}$, $\{z_n\}$ and $\{u_n\}$ be sequences generated by $x_0 \in H$, $C_1 = C$, $x_1 = P_{C_1}x_0$, $u_n \in C$ and let

$$\left\{ \begin{array}{l} F(u_n, y) + \varphi(y) - \varphi(u_n) + \langle (I - S_A)x_n, y - u_n \rangle + \frac{1}{r_n} \langle y - u_n, u_n - x_n \rangle \geq 0, \quad \forall y \in C, \\ y_n = (1 - \delta_n)u_n + \delta_n S_B u_n, \\ v_n = (1 - \lambda_n)y_n + \lambda_n S_B y_n, \\ z_n = \alpha_n x_n + (1 - \alpha_n)K_n v_n, \\ C_{n+1} = \{z \in C_n : \|z_n - z\| \leq \|x_n - z\|\}, \\ x_{n+1} = P_{C_{n+1}}x_0, \quad n \in \mathbb{N}, \end{array} \right.$$

where $\{\alpha_n\} \subset (0, 1)$ satisfy the following conditions:

- (i) $\{\alpha_n\} \subset [0, e]$ for some e with $0 \leq e < 1$,
- (ii) $\{\delta_n\}, \{\lambda_n\} \subset [a, b]$ for some a, b with $0 < a < b < 1 - \kappa_\xi$,
- (iii) $\{r_n\} \subset [c, d]$ for some c, d with $0 < c < d < 1 - \kappa_\beta$.

Then, $\{x_n\}$ and $\{u_n\}$ converge strongly to $P_{\Theta}x_0$.