

## Research article

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## On Some Identities of the $(s, t)$ -Pell and $(s, t)$ -Pell-Lucas Polynomial Sequences

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### Abstract

In this paper, we establish some identities of the relations between the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences. Moreover, we obtain some identities of limits for the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences.

**Keywords:**  $(s, t)$ -Pell polynomial,  $(s, t)$ -Pell-Lucas polynomial, recursive sequence

### 1. Introduction

For the recursive sequences, there are many forms of the recursive sequences that have been widely studied and appeared in various fields of sciences (1-3). In 1883, the Belgian mathematician Eugene Charles Catalan introduced the polynomials sequence which are defined by

$$F_{n+1}(x) = xF_n(x) + F_{n-1}(x), n \geq 1$$

where  $F_0(x)=0$  and  $F_1(x) = 1$ . Subsequently, the Fibonacci polynomials studied by the German mathematician E. Jacobsthal are defined by

$$J_{n+1}(x) = J_n(x) + 2xJ_{n-1}(x), n \geq 1$$

where  $J_0(x)=1$  and  $J_1(x) = 1$ .

In 1970, the Lucas polynomial studied by Bicknell, are defined by.

$$L_{n+1}(x) = xL_n(x) + L_{n-1}(x), n \geq 1$$

where  $L_0(x)=2$  and  $L_1(x) = x$ . For more details can found in (4-5).

In 1985, A.F. Horadam and J.M. Mahon (6) introduced the Pell polynomial

sequence and the Pell-Lucas polynomial sequence which are defined by

$$P_n(x) = 2xP_{n-1}(x) + P_{n-2}(x),$$

$$Q_n(x) = 2xQ_{n-1}(x) + Q_{n-2}(x)$$

for  $n \geq 2$ , with initial conditions  $P_0(x) = 0$ ,  $P_1(x) = 1$ ,  $Q_0(x) = 2$  and  $Q_1(x) = 2x$ .

In 2012, Gulec and Taskara (7) introduced the  $(s, t)$ -Pell sequence and  $(s, t)$ -Pell-Lucas sequence which are defined by

$$P_n(s, t) = 2sP_{n-1}(s, t) + tP_{n-2}(s, t),$$

$$Q_n(s, t) = 2sQ_{n-1}(s, t) + tQ_{n-2}(s, t)$$

for  $n \geq 2$ , where  $s$  and  $t$  are any real numbers with  $s^2 + t > 0$ ,  $s > 0$  and  $t \neq 0$  with the initial conditions

$$P_0(s, t) = 0, \quad P_1(s, t) = 1,$$

$$Q_0(s, t) = 2 \quad \text{and} \quad Q_1(s, t) = 2s.$$

Later, Srisawat and Sripad (8-9) introduced the matrix methods and some more identities for the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas numbers.

In 2021, S. Srisawat and W. Sripad (10) introduced the new generalizations of the  $(s, t)$ -Pell polynomial sequence and the  $(s, t)$ -Pell-Lucas polynomial sequence which are as following definition.

**Definition 1.1** Let  $s$  and  $t$  be any real numbers with  $s^2 + t > 0, s > 0$  and  $t \neq 0$ . Then the  $(s, t)$ -Pell polynomial sequence  $\{P_n(s, t)(x)\}_{n=0}^\infty$  and the  $(s, t)$ -Pell-Lucas polynomial sequence  $\{Q_n(s, t)(x)\}_{n=0}^\infty$  are defined respectively by  $P_n(s, t)(x) = 2sxP_{n-1}(s, t)(x) + tP_{n-2}(s, t)(x)$ ,  $Q_n(s, t)(x) = 2sxQ_{n-1}(s, t)(x) + tQ_{n-2}(s, t)(x)$  for  $n \geq 2$ , with the initial conditions  $P_0(s, t)(x) = 0, P_1(s, t)(x) = 1, Q_0(s, t)(x) = 2$  and  $Q_1(s, t)(x) = 2sx$ .

The first few terms of the  $(s, t)$ -Pell polynomial sequence  $\{P_n(s, t)(x)\}_{n=0}^\infty$  are  $0, 1, 2sx, 4s^2x^2 + t, 8s^3x^3 + 4tsx, \dots$  and so on. Also, the first few terms of the  $(s, t)$ -Pell-Lucas polynomial sequence  $\{Q_n(s, t)(x)\}_{n=0}^\infty$  are  $2, 2sx, 4s^2x^2 + 2t, 8s^3x^3 + 6tsx, 16s^4x^4 + 16ts^2x^2 + 2t^2, \dots$  and so on.

The characteristic equation for the recurrence relation of the  $(s, t)$ -Pell polynomial sequence  $\{P_n(s, t)(x)\}_{n=0}^\infty$  and the  $(s, t)$ -Pell-Lucas polynomial sequence  $\{Q_n(s, t)(x)\}_{n=0}^\infty$  in Definition 1.1 is

$$r^2 - 2srx - t = 0, \tag{1.1}$$

where  $\alpha$  and  $\beta$  are the roots of the equation (1.1), where  $\alpha = sx + \sqrt{s^2x^2 + t}$  and  $\beta = sx - \sqrt{s^2x^2 + t}$ . Note that  $\alpha + \beta = 2sx, \alpha - \beta = 2\sqrt{s^2x^2 + t}$  and  $\alpha\beta = -t$ .

To convenience, we will use the symbol  $P_n(x)$  and  $Q_n(x)$  instead of the  $n^{th}$  term of  $(s, t)$ -Pell polynomial  $\{P_n(s, t)(x)\}_{n=0}^\infty$  and the  $n^{th}$  term of the  $(s, t)$ -Pell-Lucas polynomial  $\{Q_n(s, t)(x)\}_{n=0}^\infty$ , respectively.

**Theorem 1.2** (Binet's formulas) The  $n^{th}$   $(s, t)$ -Pell and the  $n^{th}$   $(s, t)$ -Pell-Lucas polynomials are given by

$$P_n(x) = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 0 \tag{1.2}$$

and

$$Q_n(x) = \alpha^n + \beta^n, n \geq 0, \tag{1.3}$$

respectively, where  $\alpha$  and  $\beta$  are the roots of the characteristic equation  $r^2 - 2srx - t = 0$  and  $\alpha > \beta$ .

**2. Main Results**

**Theorem 2.1** For  $n$  and  $r$  are positive integers with  $n \geq r$ . Let  $\{P_n(x)\}_{n=0}^\infty$  and  $\{Q_n(x)\}_{n=0}^\infty$  be the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences, respectively Then

- (i)  $P_{n+r}(x) + t^r P_{n-r}(x) = \begin{cases} P_n(x)Q_r(x), & r \text{ is even} \\ P_r(x)Q_n(x), & r \text{ is odd} \end{cases}$
- (ii)  $Q_{n+r}(x) + t^r Q_{n-r}(x) = \begin{cases} Q_r(x)Q_n(x), & r \text{ is even} \\ 4(s^2x^2 + t)P_r(x)P_n(x), & r \text{ is odd.} \end{cases}$

**Proof.** (i) Firstly, we assume that  $r$  is an even number. By using Binet's formulas for the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences, we have

$$\begin{aligned} &P_{n+r}(x) + t^r P_{n-r}(x) \\ &= \frac{\alpha^{n+r} - \beta^{n+r}}{\alpha - \beta} + (-\alpha\beta)^r \left( \frac{\alpha^{n-r} - \beta^{n-r}}{\alpha - \beta} \right) \\ &= \frac{1}{\alpha - \beta} (\alpha^{n+r} - \beta^{n+r} + \alpha^n \beta^r - \alpha^r \beta^n) \\ &= \frac{1}{\alpha - \beta} (\alpha^r (\alpha^n - \beta^n) + \beta^r (\alpha^n - \beta^n)) \\ &= \frac{1}{\alpha - \beta} (\alpha^n - \beta^n) (\alpha^r + \beta^r) \\ &= P_n(x)Q_r(x), \end{aligned}$$

and we obtain

$$\begin{aligned} &Q_{n+r}(x) + t^r Q_{n-r}(x) \\ &= (\alpha^{n+r} + \beta^{n+r}) + (-\alpha\beta)^r (\alpha^{n-r} + \beta^{n-r}) \\ &= (\alpha^{n+r} + \beta^{n+r}) + (\alpha\beta)^r (\alpha^{n-r} + \beta^{n-r}) \\ &= \alpha^{n+r} + \beta^{n+r} + \alpha^n \beta^r + \alpha^r \beta^n \\ &= \alpha^r (\alpha^n + \beta^n) + \beta^r (\alpha^n + \beta^n) \\ &= (\alpha^r + \beta^r) (\alpha^n + \beta^n) \\ &= Q_r(x)Q_n(x). \end{aligned}$$

Secondly, if  $r$  is an odd number, then the result are as follows:

$$\begin{aligned} &P_{n+r}(x) + t^r P_{n-r}(x) \\ &= \frac{\alpha^{n+r} - \beta^{n+r}}{\alpha - \beta} + (-\alpha\beta)^r \left( \frac{\alpha^{n-r} - \beta^{n-r}}{\alpha - \beta} \right) \\ &= \frac{1}{\alpha - \beta} (\alpha^{n+r} - \beta^{n+r} - \alpha^n \beta^r + \alpha^r \beta^n) \\ &= \frac{1}{\alpha - \beta} (\alpha^r (\alpha^n + \beta^n) - \beta^r (\alpha^n + \beta^n)) \\ &= \frac{1}{\alpha - \beta} (\alpha^r - \beta^r) (\alpha^n + \beta^n) \\ &= P_r(x)Q_n(x), \end{aligned}$$

and

$$\begin{aligned} & Q_{n+r}(x) + t^r Q_{n-r}(x) \\ &= (\alpha^{n+r} + \beta^{n+r}) + (-\alpha\beta)^r (\alpha^{n-r} + \beta^{n-r}) \\ &= (\alpha^{n+r} + \beta^{n+r}) - (\alpha\beta)^r (\alpha^{n-r} + \beta^{n-r}) \\ &= \alpha^{n+r} + \beta^{n+r} - \alpha^n \beta^r - \alpha^r \beta^n \\ &= \alpha^r (\alpha^n - \beta^n) - \beta^r (\alpha^n - \beta^n) \\ &= (\alpha^r - \beta^r)(\alpha^n - \beta^n) \\ &= (\alpha - \beta)^2 P_r(x) P_n(x) \\ &= 4(s^2 x^2 + t) P_r(x) P_n(x). \quad \square \end{aligned}$$

**Corollary 2.2** For any positive integer  $n$ . Let  $\{P_n(x)\}_{n=0}^\infty$  and  $\{Q_n(x)\}_{n=0}^\infty$  be the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences, respectively. Then

- (i)  $P_{2n}(x) = P_n(x)Q_n(x)$ ,
- (ii)  $Q_{2n}(x) + 2t^n$   

$$= \begin{cases} Q_n^2(x) & , n \text{ is even} \\ 4(s^2 x^2 + t)P_n^2(x) & , n \text{ is odd.} \end{cases}$$

**Proof.** Taking  $r = n$  in Theorem 2.1, the proof completed.  $\square$

**Remark 2.3** For  $r = 1$  in Theorem 2.1, then we have the following identities (Theorem 2.7 in (10)),

$$\begin{aligned} & P_{n+1}(x) + tP_{n-1}(x) = Q_n(x) \\ & \text{and} \\ & Q_{n+1}(x) + tQ_{n-1}(x) = 4(s^2 x^2 + t)P_n(x). \end{aligned}$$

**Theorem 2.4** For  $m, n$  and  $r$  are positive integers with  $m \geq n$ . Let  $\{P_n(x)\}_{n=0}^\infty$  and  $\{Q_n(x)\}_{n=0}^\infty$  be the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences, respectively. Then

- (i)  $P_m(x)P_{n+r}(x) - P_{m+r}(x)P_n(x)$   

$$= (-t)^n P_r(x) P_{m-n}(x)$$
- (ii)  $Q_m(x)Q_{n+r}(x) - Q_{m+r}(x)Q_n(x)$   

$$= (\alpha^r - \beta^r)(-t)^n (Q_{m-n}(x) - 2\alpha^{m-n}).$$

**Proof.** By using Binet's formulas for the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences, we have

$$\begin{aligned} & P_m(x)P_{n+r}(x) - P_{m+r}(x)P_n(x) \\ &= \left(\frac{\alpha^m - \beta^m}{\alpha - \beta}\right) \left(\frac{\alpha^{n+r} - \beta^{n+r}}{\alpha - \beta}\right) \\ &\quad - \left(\frac{\alpha^{m+r} - \beta^{m+r}}{\alpha - \beta}\right) \left(\frac{\alpha^n - \beta^n}{\alpha - \beta}\right) \\ &= \frac{1}{(\alpha - \beta)^2} (-\beta^m \alpha^{n+r} - \alpha^m \beta^{n+r} + \alpha^n \beta^{m+r} \\ &\quad \quad \quad + \alpha^{m+r} \beta^n) \\ &= \frac{1}{(\alpha - \beta)^2} (-\alpha^n \beta^m (\alpha^r - \beta^r) \\ &\quad \quad \quad + \alpha^m \beta^n (\alpha^r - \beta^r)) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{(\alpha - \beta)^2} (-\alpha^n \beta^m + \alpha^m \beta^n)(\alpha^r - \beta^r) \\ &= \frac{1}{(\alpha - \beta)^2} (\alpha\beta)^n (\alpha^{m-n} - \beta^{m-n})(\alpha^r - \beta^r) \\ &= (-t)^n P_r(x) P_{m-n}(x), \end{aligned}$$

and

$$\begin{aligned} & Q_m(x)Q_{n+r}(x) - Q_{m+r}(x)Q_n(x) \\ &= (\alpha^m + \beta^m)(\alpha^{n+r} + \beta^{n+r}) \\ &\quad - (\alpha^{m+r} + \beta^{m+r})(\alpha^n + \beta^n) \\ &= \alpha^{n+r} \beta^m - \alpha^n \beta^{m+r} + \alpha^m \beta^{n+r} - \alpha^{m+r} \beta^n \\ &= \alpha^n \beta^m (\alpha^r - \beta^r) - \alpha^m \beta^n (\alpha^r - \beta^r) \\ &= (\alpha^r - \beta^r)(\alpha^n \beta^m - \alpha^m \beta^n) \\ &= (\alpha^r - \beta^r)(\alpha\beta)^n (\beta^{m-n} - \alpha^{m-n}) \\ &= (\alpha^r - \beta^r)(\alpha\beta)^n (\alpha^{m-n} + \beta^{m-n} - 2\alpha^{m-n}) \\ &= (\alpha^r - \beta^r)(-t)^n (Q_{m-n}(x) - 2\alpha^{m-n}). \quad \square \end{aligned}$$

**Remark 2.5** If we take  $r = 1$  in Theorem 2.4. Then we have (Theorem 2.6 in (10))

$$\begin{aligned} & P_m(x)P_{n+1}(x) - P_{m+1}(x)P_n(x) \\ &= (-t)^n P_{m-n}(x) \end{aligned}$$

and

$$\begin{aligned} & Q_m(x)Q_{n+1}(x) - Q_{m+1}(x)Q_n(x) \\ &= 2(-t)^n \sqrt{s^2 x^2 + t} (Q_{m-n}(x) \\ &\quad - 2(sx + \sqrt{s^2 x^2 + t})^{m-n}). \end{aligned}$$

**Remark 2.6** In Theorem 2.4 (ii), we have the relation between the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences as follow,

$$\begin{aligned} & Q_m(x)Q_{n+r}(x) - Q_{m+r}(x)Q_n(x) \\ &= (\alpha^r - \beta^r)(-t)^n (Q_{m-n}(x) - 2\alpha^{m-n}) \\ &= (\alpha^r - \beta^r)(-t)^n (\beta^{m-n} - \alpha^{m-n}) \\ &= -(\alpha^r - \beta^r)(-t)^n (\alpha^{m-n} - \beta^{m-n}) \\ &= -(\alpha - \beta)^2 (-t)^n P_r(x) P_{m-n}(x) \\ &= -4(-t)^n (s^2 x^2 + t) P_r(x) P_{m-n}(x). \end{aligned}$$

Therefore,

$$\begin{aligned} & Q_m(x)Q_{n+r}(x) - Q_{m+r}(x)Q_n(x) \\ &= -4(-t)^n (s^2 x^2 + t) P_r(x) P_{m-n}(x). \end{aligned}$$

If  $r = 1$ , we have

$$\begin{aligned} & Q_m(x)Q_{n+1}(x) - Q_{m+1}(x)Q_n(x) \\ &= -4(-t)^n (s^2 x^2 + t) P_{m-n}(x). \end{aligned}$$

**Theorem 2.7** Let sequences  $\{P_n(x)\}_{n=0}^\infty$  and  $\{Q_n(x)\}_{n=0}^\infty$  be the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences, respectively. If  $sx > 0, s^2 + t > 0$  and  $t \neq 0$ . Then

- (i)  $\lim_{n \rightarrow \infty} \frac{P_{n+r}(x)}{P_n(x)} = \alpha^r$ ,
- (ii)  $\lim_{n \rightarrow \infty} \frac{Q_{n+r}(x)}{Q_n(x)} = \alpha^r$ ,
- (iii)  $\lim_{n \rightarrow \infty} \frac{P_n(x)}{Q_n(x)} = \frac{1}{\alpha - \beta}$ ,
- (iv)  $\lim_{n \rightarrow \infty} \frac{P_{n+r}(x)}{Q_n(x)} = \frac{\alpha^r}{\alpha - \beta}$

and

$$(v) \lim_{n \rightarrow \infty} \frac{P_n(x)}{Q_{n+r}(x)} = \frac{1}{(\alpha - \beta)\alpha^r},$$

where  $r$  is a non-negative integer.

**Proof.** By using Binet's formulas, we have:

$$(i) \lim_{n \rightarrow \infty} \frac{P_{n+r}(x)}{P_n(x)} = \lim_{n \rightarrow \infty} \frac{\alpha^{n+r} - \beta^{n+r}}{\alpha^n - \beta^n} \\ = \lim_{n \rightarrow \infty} \frac{1 - \left(\frac{\beta}{\alpha}\right)^{n+r}}{1 - \frac{1}{\alpha^r} \left(\frac{\beta}{\alpha}\right)^{n+r}}$$

Since  $\left|\frac{\beta}{\alpha}\right| < 1$ , then  $\lim_{n \rightarrow \infty} \left(\frac{\beta}{\alpha}\right)^{n+r} = 0$ .

Therefore,  $\lim_{n \rightarrow \infty} \frac{P_{n+r}(x)}{P_n(x)} = \alpha^r$ .

Next for (ii), we consider

$$\lim_{n \rightarrow \infty} \frac{Q_{n+r}(x)}{Q_n(x)} = \lim_{n \rightarrow \infty} \frac{\alpha^{n+r} + \beta^{n+r}}{\alpha^n + \beta^n} \\ = \lim_{n \rightarrow \infty} \frac{1 + \left(\frac{\beta}{\alpha}\right)^{n+r}}{1 + \frac{1}{\alpha^r} \left(\frac{\beta}{\alpha}\right)^{n+r}} \\ = \alpha^r.$$

Similarly,

$$\lim_{n \rightarrow \infty} \frac{P_n(x)}{Q_n(x)} = \lim_{n \rightarrow \infty} \frac{\alpha^n - \beta^n}{(\alpha - \beta)(\alpha^n + \beta^n)} \\ = \lim_{n \rightarrow \infty} \frac{1 - \left(\frac{\beta}{\alpha}\right)^n}{(\alpha - \beta)\left(1 + \left(\frac{\beta}{\alpha}\right)^n\right)} \\ = \frac{1}{\alpha - \beta}.$$

Then, we have

$$\lim_{n \rightarrow \infty} \frac{P_{n+r}(x)}{Q_n(x)} \\ = \frac{1}{\alpha - \beta} \lim_{n \rightarrow \infty} \frac{\alpha^{n+r} - \beta^{n+r}}{\alpha^n + \beta^n} \\ = \frac{1}{\alpha - \beta} \lim_{n \rightarrow \infty} \alpha^r \left( \frac{1 - \left(\frac{\beta}{\alpha}\right)^{n+r}}{1 + \left(\frac{\beta}{\alpha}\right)^n} \right) \\ = \frac{\alpha^r}{\alpha - \beta}.$$

Finally,

$$\lim_{n \rightarrow \infty} \frac{P_n(x)}{Q_{n+r}(x)} = \frac{1}{\alpha - \beta} \lim_{n \rightarrow \infty} \frac{\alpha^n - \beta^n}{\alpha^{n+r} + \beta^{n+r}} \\ = \frac{1}{\alpha - \beta} \lim_{n \rightarrow \infty} \frac{1}{\alpha^r} \left( \frac{1 - \left(\frac{\beta}{\alpha}\right)^n}{1 + \left(\frac{\beta}{\alpha}\right)^{n+r}} \right) \\ = \frac{1}{(\alpha - \beta)\alpha^r}.$$

### 3. Conclusions

In this paper, we obtain some more identities of relations between the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences by using the Binet formulas. Furthermore, some identities of limits for the  $(s, t)$ -Pell and  $(s, t)$ -Pell-Lucas polynomial sequences are obtained.

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