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EQUITY PREMIUM PUZZLE IN THAILAND: REVISITED

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Abstract

This paper empirically studies the standard consumption-based asset pricing model of Lucas (1978), Breeden (1979) and the consumption-based asset pricing model with recursive utility of Weil (1989) using quarterly data from Thailand from 2000 to 2016. The equity premium is calculated using FEDR returns, which includes dividend and right benefits and considers the effect of stock split. The result confirms that the models, which are the standard consumption-based asset pricing model of Lucas (1978), Breeden (1979) and the consumption-based asset pricing model with the recursive utility of Weil (1989), assuming consumption growth are iid and Markov processes, cannot explain observed expected equity premium and observed expected risk-free return in Thailand data. Thailand's quarterly financial data from 2000 to 2016 exhibit the equity premium and risk-free rate puzzles.

Keyword: Equity premium puzzle, Risk-free rate puzzle, FEDR return

Introduction

The study of asset returns is an important topic in finance and ecomomics. Many models have been developed to study the asset returns in the market. One of the most important model is the Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964) and Lintner(1965) which shows the relationship between asset returns and systematic risk, represented by each asset's market beta, the sensitivity of asset returns to the movement of the market premium. The Consumption Capital Asset Pricing Model (CCAPM) extends the CAPM model and uses consumption beta in place of the market beta. This consumption-based asset pricing model, introduced by Lucas (1978) and Breeden

(1979), is an important theory to explain the relationship between consumption and asset returns.

Asset pricing models are important in understanding asset retuens. Consumption-based asset pricing models, in particular, are important in linking macroeconomics and finance theories together. An empirical study of consumption-based asset pricing models can help in the understanding of the relationship between macroeconomic risk factors and asset prices with implications in policy development and economic decision making. Consumption-based asset pricing models have not been studied extensively in Thailand. This paper aims to provide a comprehensive empirical study of consumption-based asset pricing models with Thailand data for the purpose of providing an extensive study of the relationship between macroeconomics and finance, through consumption growth and asset growth, respectively.

This paper utilizes Financial and Economic Data for Research (FEDR) from the University of the Thai Chamber of Commerce (UTCC), which are research-quality financial data, to represent Thailand financial market return. As far as we know, currently, this is the only research-quality financial data in Thailand, therefore, utilizing FEDR financial data should provide more accurate results than raw market data.

Models that are studied in this paper include the standard consumption-based asset pricing model of Lucas (1978), Breeden (1979), and the consumption-based asset pricing model with recursive utility of Weil (1989). For each model we assume consumption growth are independent and identically distributed (i.i.d.) and Markov processes. Thus, there are four different models studied in this paper.

The remainder of the paper is organized as follows. We discuss the theoretical background of the consumption-based asset pricing models in section 2. Here we discuss the models as well as the results of other related studies and their findings. We also discuss an important result from the literature that are the equity premium puzzle and the risk-free rate puzzle. The empirical estimation of model parameters are done in in section 3. The findings are summarized in section 4 as well as the discussion of the results. We conclude our paper and discuss future research implications in section 5.

Theoretical background

The consumption-based asset pricing model of Lucas (1978) and Breeden (1979), which is based on a representative agent's preference maximization problem, is an important theory to explain the relationship between consumption and asset returns by linking macroeconomics and finance theories together. There have been many papers that test the consumption-based asset pricing model by using both financial and macroeconomic data. One seminal paper is Mehra and Prescott (1985) which investigated U.S. data from 1889 to 1978, using two-state Markov and i.i.d. log-normal consumption growth process, and found that the empirical equity premium is too large to be explained by reasonable preference parameter values, specifically the risk aversion parameter. Hansen and Singleton (1982) estimated preference parameters including risk aversion coefficient and the time discount factor by using the generalized method of moments (GMM) framework with U.S. data from 1959 to 1977. The result also showed that the equity premium cannot be explained by reasonable values of preference parameters. This problem came to be called the "equity premium puzzle".

Several papers attempted to solve the equity premium puzzle. Weil (1989) tried to solve the equity premium puzzle using the consumption-based asset pricing model with Epstein and Zin (1989) recursive preferences. These preferences allow for the separation between risk aversion and the intertemporal elasticity of substitution. As in Mehra and Prescott (1985), Weil (1989) considered both two-state Markov and i.i.d. log-normal consumption growth process and used the same data from U.S. and found that separating risk aversion and intertemporal elasticity of substitution is not sufficient to solve the equity premium puzzle. In addition, the paper found that the expected risk-free return from the model is too low which cannot be explained with reasonable values of model parameters. This is known as the "risk-free rate puzzle".

Sedthapinun (2000) studied the equity premium using the same methodology as Hansen and Singleton (1982) using Thailand quarterly data[1] from June 1986 to December 1996. The result showed that there is no equity premium puzzle with Thailand financial data. On the other hand, Duangthong (2014) used Thailand quarterly data[2] from 1993 to 2010 and showed that the equity premium puzzle exists and can be detected using Hansen–Jagannathan bound (Hansen and Jagannathan, 1991). Similarly, Harnphattananusorn (2014) showed that equity premium puzzle exists under the consumption-based asset pricing model with time-separable constant relative risk aversion (CRRA) utility (Lucas, 1978) and i.i.d. log-normal consumption growth using quarterly Thailand data from 2000 to 2011[3]

Consumption-based asset pricing models

Lucas (1978) proposed a model to explain the equilibrium relationship

1 The data include real aggregate consumption of non-durable goods, real Stock Exchange of Thailand (SET) index and real time deposit interest rate of Thai government saving bank.

2 The data include real aggregate consumption of non-durable and service goods, 21 business categories of financial market, which will be used to analyzed the financial market real return and real time deposit interest rate of Thai government saving bank.

3 This data includes real aggregate consumption of non-durable and service goods, real stock Exchange of Thailand (SET) index and the real 90-day government treasury bill.

between consumption and asset returns in terms of expected equity premium and expected risk-free return. Consider a representative agent who maximizes a certain utility function defined over consumption C_t, $U(C_t)$

$$\max_{C_t} E_0\left(\sum_{t=1}^{\infty} \beta^t U(C_t)\right), \quad \textbf{(1)}$$

subject to period by period budget constraints

$$W_{t+1} = (W_t - C_t)R_{t+1}, \quad \textbf{(2)}$$

where parameter β is the time discount factor such that $0 < \beta < 1$ and W_t and C_t denote beginning of period wealth of agent and agent's consumption in period t. This agent can invest his net wealth after taking out his consumption, $W_t - C_t$. $R_{t+1} = \frac{P_{t+1}+C_{t+1}}{P_t}$ is a total return of assets in complete set which agent can freely buy or sell as much of payoff which is price and consumption in period $t+1$, where P_t is the price in period t of the market portfolio.

The first-order condition with respect to C_t for the agent's utility maximization problem yields the following Euler equation:

$$\mathrm{E_t}\left[\beta \frac{\mathrm{U'(C_{t+1})}}{\mathrm{U'(C_t)}} \mathrm{R_{t+1}}\right] = 1. \quad \textbf{(3)}$$

This Euler equation is the same as presented by Hansen and Richard (1987) and is an "Asset pricing model". This model tries to explain the fundamental prices or values of uncertain cash flows. To value an asset, we have to consider expectations of payoff and discount rate, that is

$$E_t\left[M_{t+1}Y_{j,t+1}\right] = P_{j,t},$$

where $Y_{j,t+1}$ is the total payoff in next period, and M_{t+1} is called stochastic discount factor or pricing kernel. The expectation is conditional on time t information. We can write in term of return as following;

$$E_t\left[M_{t+1}R_{j,t+1}\right] = 1. \quad (4)$$

Where $R_{j,t+1} = \frac{Y_{j,t+1}}{P_{j,t}}$ is return on asset j from period t to $t+1$. In general, $R_{j,t+1}$ is uncertain so the expected time conditional risky asset return on asset j is

$$E_t\left[R_{j,t+1}\right] = \frac{1-\mathrm{Cov}_t\left(M_{t+1}, R_{j,t+1}\right)}{E_t(M_{t+1})}.$$

If we define a riskless asset as a security paying one unit ($Y_{j,t+1} = 1$) then the risk-free return, $R_{f,t+1} = \frac{1}{P_{f,t}}$, can be written as:

$$R_{f,t+1} = \frac{1}{E_t[M_{t+1}]}.$$

This model can be explained using stochastic discount factor, M_{t+1}. From Euler equation of Lucas (1978) (3) and pricing equation (4), the stochastic discount factor, M_{t+1} can be written as

$$M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)}.$$

Time-separable utility model (Lucas, 1978; Mehra and Prescott, 1985)

Mehra and Prescott (1985) proposed a model following Lucas (1978) to explain the equilibrium relationship between consumption and asset returns in terms of expected equity premium and expected risk-free return. The representative agent's preferences are represented by the following constant relative risk aversion (CRRA) utility function

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad (5)$$

where γ is the relative risk aversion coefficient. By maximizing the expected utility in $\max_{C_t} E_0(\sum_{t=1}^{\infty} \beta^t U(C_t))$, **(1)** with CRRA utility function (5), we get the following stochastic discount factor

$$M_{t+1} = \beta G_{c,t+1}^{-\gamma}, \quad (6)$$

where $G_{c,t+1}$ is the consumption growth $\left(\frac{C_{t+1}}{C_t}\right)$.

To be able to utilize the consumption-based asset pricing model presented above with empirical data, we need to specify the stochastic process of the consumption growth $G_{c,t+1}$ in equation (6).

Consumption growth process as a two-state Markov chain

Mehra and Prescott (1985) assumed that consumption growth, $G_{c,t+1}$, follows a two-state Markov chain. The two states are high (h) and low (l). Let $G_{c,s}$ be the consumption growth in states $s = \{h, l\}$.

We can write the vector of consumption growth across states as

$$G_{c,t+1} = \begin{pmatrix} G_{c,h} \\ G_{c,l} \end{pmatrix} \tag{7}$$

Accordingly, the stochastic discount factor in each state can be written as

$$M_{t+1} = \begin{pmatrix} M_h \\ M_l \end{pmatrix} = \begin{pmatrix} \beta G_{c,h}^{-\gamma} \\ \beta G_{c,l}^{-\gamma} \end{pmatrix}. \tag{8}$$

The transition matrix of the Markov chain is given by

$$\boldsymbol{\pi} = \begin{pmatrix} \pi_{h,h} & \pi_{h,l} \\ \pi_{l,h} & \pi_{l,l} \end{pmatrix}, \tag{9}$$

where $\pi_{s,s'}$ is the transition probability of being in state $s' = \{h, l\}$ after being in state $s = \{h, l\}$ in the previous period. The steady state probability for state $s = \{h, l\}$ is denoted by Π_s, and the vector of the probability is given by

$$\boldsymbol{\Pi} = \begin{pmatrix} \Pi_h \\ \Pi_l \end{pmatrix}, \tag{10}$$

where $\Pi_h + \Pi_l = 1$. The risk-free return is

$$R_{f,t+1} = \frac{1}{E_t[M_{t+1}]}. \tag{11}$$

Therefore, the expected risk-free return is

$$E(R_f) = \Pi_h R_{f,h} + \Pi_l R_{f,l}, \tag{12}$$

where the risk-free return in states $s = h, l$ are

$$R_{f,h} = \left[\pi_{h,h}\, \beta\, G_{c,h}^{-\gamma} + \pi_{h,l}\, \beta\, G_{c,l}^{-\gamma}\right]^{-1},$$

$$R_{f,l} = \left[\pi_{l,h}\, \beta\, G_{c,h}^{-\gamma} + \pi_{l,l}\, \beta\, G_{c,l}^{-\gamma}\right]^{-1}.$$

Similar to the risk-free asset, the expected return of the market portfolio can be derived. Following Lucas (1978), the model assumes the market clearing condition. This implies that the agent consumes only dividend benefit (D_t) after agent invests in financial assets ($C_t = D_t$).

Therefore, the payoff of the market portfolio in period $t+1$ is $P_{t+1} + C_{t+1}$, and its return is given by

$$R_{m,t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} = R_{t+1}, \tag{13}$$

which can be rewritten in terms of consumption growth, $G_{c,t+1}$, and price-consumption ratio, $Z_t = \frac{P_t}{C_t}$, as follows:

$$R_{m,t+1} = \frac{1+Z_{t+1}}{Z_t} G_{c,t+1}. \tag{14}$$

As a result, the expected return on the financial market portfolio or the market return is

$$E(R_m) = \Pi_h R_{m,h} + \Pi_l R_{m,l} \, , \tag{15}$$

where the market return in each state $s = \{h, l\}$ are

$$R_{m,h} = \pi_{h,h}\left(\frac{1+Z_h}{Z_h} G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_h} G_{c,l}\right),$$

$$R_{m,h} = \pi_{h,h}\left(\frac{1+Z_h}{Z_h} G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_h} G_{c,l}\right). \tag{16}$$

The pricing equation, or expected equity premium, is defined by the difference of the expected net market return and the expected net return on the risk-free asset:

$$E(r_m - r_f) = \Pi_h\left(R_{m,h} - R_{f,h}\right) + \Pi_l\left(R_{m,l} - R_{f,l}\right) , \tag{17}$$

where $r_{f,t} = R_{j,t} - 1$ denotes the net return of asset j. The derivation of this condition is in Appendix A.

Consumption growth process as i.i.d. log-normal process

An alternative formulation in the literature is to assume that the consumption growth process is independent and identically distributed log-normal (i.i.d. log-normal) as in Breeden (1986); Hansen and Singleton (1982). Particularly in Thailand, all previous works regarding equity premium puzzle employ the i.i.d. log-normal assumption. Another benefit of this model is the ability to identify the market price of risk and the consumption risk. The pricing equation with the i.i.d. log-normal consumption growth and the asset return can be written as:

$$E(m_{t+1}) + E\left(r_{j,t+1}\right) + \frac{1}{2}\mathrm{Var}(m_{t+1}) + \frac{1}{2}\mathrm{Var}\left(r_{j,t+1}\right) + \mathrm{Cov}\left(m_{t+1}, r_{j,t+1}\right) = 0 \, , \tag{18}$$

where $m_{t+1} = \log M_{t+1}$ and $r_{j,t+1} = \log R_{j,t+1}$ are the logarithm of the stochastic discount factor, and the logarithm of returns of asset j, respectively. With the CRRA utility function (3),

$$m_{t+1} = \log \beta G_{c,t+1}^{-\gamma} = \log \beta - \gamma g_{c,t+1}. \tag{19}$$

Where $g_{c,t+1} = \log G_{c,t+1}$ is the logarithm of the consumption growth rate. As a result, the expected risk-free rate can be written as

$$E\left(r_{f,t+1}\right) = -\log\beta + \gamma E\left(g_{c,t+1}\right) - \frac{1}{2}\gamma^2 \mathrm{Var}\left(g_{c,t+1}\right), \quad (20)$$

where $r_{f,t+1} = \log R_{f,t+1}$. Similarly, the expected equity premium, which assumes $r_{f,t+1} = r_{m,t+1}$, is

$$E\left(r_{m,t+1} - r_{f,t+1}\right) = \gamma \mathrm{Var}\left(g_{c,t+1}\right) - \frac{1}{2}\mathrm{Var}\left(r_{m,t+1}\right). \quad (21)$$

This expected equity premium (21) is called the "Consumption Capital Asset Pricing Model (CCAPM)". The detailed derivation of the pricing equation for the expected risk-free return (20) and the expected equity premium (21) are in appendix B.

Epstein and Zin recursive utility model (Kreps and Porteus, 1978; Epstein and Zin, 1989; Weil, 1989)

Following Weil (1989), a new class of preferences under constant elasticity of intertemporal substitution and a constant risk aversion, which are independent of each other, are used to study the equity premium puzzle (Kreps and Porteus, 1978; Epstein and Zin, 1989; Weil, 1989). In particular, the representative agent is to maximize the following recursive utility

$$V(W_t) = \max_{C_t} U(C_t, E_t[V(W_{t+1})]), \quad (22)$$

where $V(\cdot)$ is the value function for a representative agent with wealth, W_t, that maximizes the utility function

$$U(C,V) = \frac{\left((1-\beta)C^{1-\frac{1}{\psi}} + \beta(1+(1-\beta)(1-\gamma)V)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{(1-\beta)(1-\gamma)}, \quad (23)$$

and ψ is the elasticity of intertemporal substitution. This model is subject to the period-by-period budget constraint (2). The first-order condition with respect to C_t and $V(W_t)$ then yields the following Euler equation:

$$E_t\left[\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} R_{t+1}^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right] = 1. \tag{24}$$

As a result, the stochastic discount factor (M_{t+1}) is

$$M_{t+1} = \left(\beta G_{c,t+1}^{-\frac{1}{\psi}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} R_{t+1}^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}. \tag{25}$$

This equation emphasizes that both the complete market returns R_{t+1} and consumption growth $G_{c,t+1}$ affect the stochastic discount factor M_{t+1}. The separable utility model is a special case of the recursive utility model. If we set $\gamma = \frac{1}{\psi}$, the stochastic discount factor formula will be the same as the one from (4). Following Weil (1989), the model assumes the market clearing condition ($C_t = D_t$). Thus, the stochastic discount factor (25) can be written in term of price-consumption ratio as follows:

$$M_{t+1} = \left(\beta G_{c,t+1}^{-\frac{1}{\psi}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \left(\frac{1+Z_{t+1}}{Z_t}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}. \tag{26}$$

Again, to be able to utilize the consumption-based asset pricing model presented above with empirical data, we need to specify the stochastic process of the consumption growth, $G_{c,t+1}$.

Consumption growth process as a two-state Markov chain

Using the same specification as section 2.2.1, the stochastic discount factor (26) of being in a state $s' \in \{h, l\}$ after being in state $s \in \{h, l\}$ in the previous period is

$$M_{s,s'} = \left(\beta G_{c,s'}^{-\frac{1}{\psi}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \left(\frac{1+Z_{s'}}{Z_s}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}. \tag{27}$$

Similarly, the expected risk-free return is

$$E(R_f) = \Pi_h R_{f,h} + \Pi_l R_{f,l}, \tag{28}$$

where the risk-free return in each state $s \in \{h, l\}$ are

$$R_{f,h} = \left[\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\left(\pi_{h,h}\, G_{c,h}^{-\gamma}\left(\frac{Z_h+1}{Z_h}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} + \pi_{h,l}\, G_{c,l}^{-\gamma}\left(\frac{Z_l+1}{Z_h}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\right)\right]^{-1}, \tag{29}$$

$$R_{f,l} = \left[\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\left(\pi_{l,h}\, G_{c,h}^{-\gamma}\left(\frac{Z_h+1}{Z_l}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} + \pi_{l,l}\, G_{c,l}^{-\gamma}\left(\frac{Z_l+1}{Z_l}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\right)\right]^{-1}. \qquad (30)$$

The expected market return is

$$E(R_m) = \Pi_h R_{m,h} + \Pi_l R_{m,l} \qquad (31)$$

where the market return in each state $s \in \{h, l\}$ are

$$R_{m,h} = \pi_{h,h}\left(\frac{1+Z_h}{Z_h} G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_h} G_{c,l}\right),$$

$$R_{m,l} = \pi_{l,h}\left(\frac{1+Z_h}{Z_l} G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_l} G_{c,l}\right), \qquad (32)$$

Following Weil (1989), the expected equity premium is

$$E\left(\frac{R_m}{R_f}\right) = \Pi_h \frac{R_{m,h}}{R_f,h} + \Pi_l \frac{R_{m,l}}{R_{f,l}}. \qquad (33)$$

The detailed derivations of the pricing equations for the expected risk-free return (28) and the expected equity premium (33) are in appendix C.

Consumption growth process as i.i.d. log-normal process

If the consumption growth process ($G_{c,t+1}$) is assumed to be i.i.d. log-normal, as in section 2.2.2, the stochastic discount factor (26) can be written as:

$$m_{t+1} = \frac{1-\gamma}{1-\frac{1}{\psi}}\log\beta - \gamma g_{c,t+1} + \frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}\log\left(\frac{1+Z_{t+1}}{Z_t}\right). \qquad (34)$$

Similar to the asset pricing equation in Section 2.2.2, the expected logarithm of risk-free return ($r_{f,t+1} = \log R_{f,t+1}$) is

$$E(r_{f,t+1}) = -\log\beta + \frac{1}{\psi}E(g_{c,t+1}) - \frac{\gamma\left(1+\frac{1}{\psi}\right)-\frac{1}{\psi}}{2}\mathrm{Var}(g_{c,t+1}), \qquad (35)$$

and the expected equity premium, which assumes that $r_{t+1} = r_{m,t+1}$, is

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma\mathrm{Var}(g_{c,t+1}) - \frac{1}{2}\mathrm{Var}(r_{m,t+1}), \qquad (36)$$

where $g_{c,t+1}$ is the logarithm of consumption growth and $r_{m,t+1}$ is the logarithm of market returns. The equity premium (36) is the same formula as the equity premium (21) in Section 2.2.2. The detailed derivations of the asset pricing equations are in appendix D.

In addition, if we set $\gamma = \frac{1}{\psi}$, then the expected logarithm of risk-free return

(35) is the same as expected logarithm of risk-free return (20) under time-separable utility model in Section 2.2.2

Equity premium puzzle and risk-free rate puzzle

The equity premium puzzle was discovered by Mehra and Prescott (1985) when they used the standard consumption-based asset pricing model with time-separable CRRA utility and two-state Markov of consumption growth. The Equity premium model and risk-free rate model are shown in equation (17) and equation (10) which are

$$E(r_m - r_f) = \Pi_h(R_{m,h} - R_{f,h}) + \Pi_l(R_{m,l} - R_{f,l}),$$

$$E(R_f) = \Pi_h R_{f,h} + \Pi_l R_{f,l}.$$

They tried to compare the equity premium and risk-free rates above with actual observed equity premium and risk-free return of U.S. annually data from 1889 to 1978. They claimed that the reasonable risk aversion value should be between 1 and 10. For example, they found that the largest average equity premium from the model is 0.35 % using risk aversion ($\gamma = 10$), or using risk aversion ($\gamma = 2$), the average risk-free return from the model is at least 3.7 % While the actual average observed equity premium and risk-free return during this period were around 6 % and 0.8 %, respectively. The results show that the models cannot simultaneously explain why actual observed average equity premium was so high while the actual observed average risk-free return was so low. They call this problem "equity premium puzzle".

Mehra and Prescott (1985) suggested that the reason for the puzzle could be because of the restriction of time-separable CRRA utility in which the elasticity of intertemporal substitution must be a reciprocal of the risk aversion. Weil (1989) tried to solve the equity premium puzzle using the consumption-based asset pricing model with the Epstein and Zin (1989) recursive preferences in which the risk aversion and elasticity of intertemporal substitution are independent of each other and considered both two-state Markov and i.i.d. log-normal consumption growth processes. The result showed that the equity premium puzzle still exists. However, the puzzle can be separated into an equity premium puzzle and a risk-free rate puzzle.

Weil (1989) showed that the equity premium from the model was still much lower than the actual observed equity premium. This discrepancy was the result of only the risk aversion coefficient, similar to the result of Mehra and Prescott (1985). In addition, Weil (1989) showed that the risk-free rate from the model was much higher than the actual observed risk-free rate. However, this discrepancy was not related to the high risk aversion coefficient but rather the elasticity of intertemporal substitution coefficient.

The result shows that the equity premium puzzle still exists even after the separation of risk aversion and elasticity of intertemporal substitution. However, the puzzle can now be separated into the equity premium puzzle and risk-free rate puzzle. The equity premium puzzle shows that a higher-than-reasonable risk aversion parameter must be used for the equity premium from the model to be close to the actual observed equity premium. For the risk-free rate puzzle, Weil (1989) could not find any value of the elasticity of intertemporal substitution coefficient that can produce a risk-free rate from the model that is reasonably close to the actual observed risk-free rate. The risk-free rate from the model is consistently much higher than the actual observed rate.

Calibration of asset pricing models

We can fit the parameters of the consumption-based asset pricing models discussed in section 2 with empirical data. In this section, we calibrate the model parameter using FEDR market returns.

Data

Data used to study the equity premium, and risk-free rate puzzle in Thailand in this paper is the Financial and Economic Data for Research (FEDR) from the University of the Thai Chamber of Commerce (UTCC) which adjusts financial data from the Stock Exchange of Thailand (SET). The FEDR market returns are constructed using a similar framework to the CRSP market returns from the Center for Research in Security Prices at the University of Chicago. In particular, the returns for each stock are calculated as the sum of the returns from the capital gains, cash dividends, stock dividends, and right benefits, taking into account stock split/reverse. The FEDR market returns are the returns of the value-weighted portfolio of all stocks in the SET.

The key advantage of the FEDR market returns over the SET Total Returns Index provided by SET is that the former returns are available since April 1975, while the latter is available only after January 2002. Figure 1 shows the quarterly FEDR market returns, quarterly SET total returns, and risk-free quarterly returns. To avoid the complication of the financial crisis of 1997, the main analyses in this paper use quarterly data from the first quarter of 2000 onwards.

The observed risk-free returns, $R_{f,t+1}$, are the 3-month time deposits average returns of main Thai Commercial

Banks[4] , taken from the Bank of Thailand (BOT). The main reason for using the deposit rates as risk-free returns is the availability of the data. This choice is different from the literature (e.g., Mehra and Prescott, 1985), which used the ninety-day government Treasury bill. The ninety-day Treasury bill returns of Thailand are available only after 2005, while the deposit rates are available since 1978.

Consumption data are taken from the seasonally-adjusted, real private consumption expenditure (PCE) on domestic non-durable goods and services, developed by the Office of the National Economic and Social Development Board (NESDB). They are converted into per capita consumption, C_t using Thai population data, N_t collected by Department of Provincial Administration and are converted from nominal to real using the consumption Price Index (CPI), CPI_t which is taken from the Bureau of Trade and Economic Indices (BTEI). Real consumption growth is calculated using year-over-year measurements of quarterly data to adjust for seasonal effect as following

$$G_{c,t} = \frac{C_t}{CPI_t \cdot N_t} \cdot \frac{CPI_{t-4} \cdot N_{t-4}}{C_{t-4}} \quad (37)$$

This data is presented in Figure 3

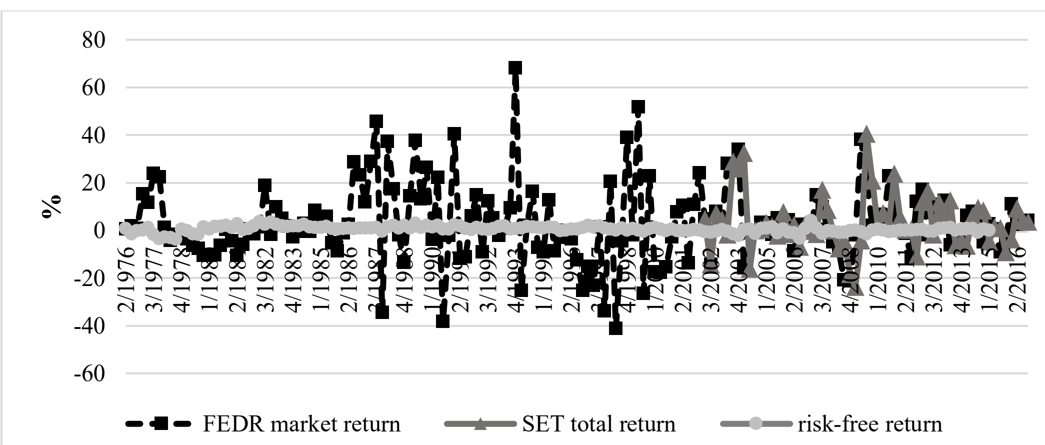

Figure 1 the quarterly observed data of the FEDR real market returns (1976-2016), SET real total returns (2002-2016), and real 3-month time deposits average rate of the Thai Commercial Banks (1978-2016).

4 the Thai Commercial Banks includes Bangkok Bank, Krungthai Bank, The siam Commercial Bank, Kasikorn Bank and Bank of Ayudhya.

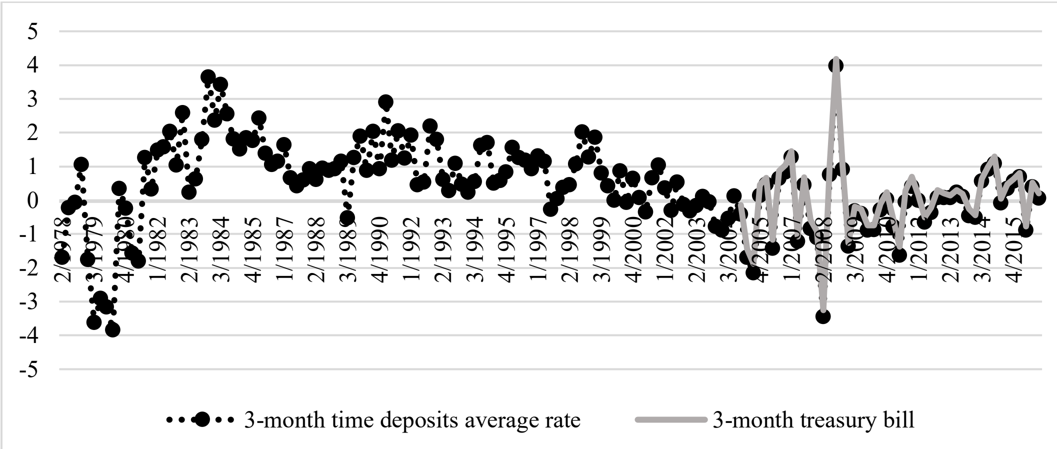

Figure 2 the quarterly observed data of the real 3-month time deposits average rate of the Thai Commercial Banks (1978-2016) and ninety-day treasury bills (2005-2016).

Table 1 presents the summary statistics of the observed real market returns, the observed real risk-free returns, the observed real equity premium, and observed real consumption growth.

Table 1 the summary statistics of quarterly observed data from 2000-2016.

	R_m	R_f	$R_m - R_f$	G_c
Mean (%)	2.74	-0.8	2.83	4.06
Standard deviation(%)	12.15	0.97	12.37	2.02
Autocorrelation	0.3143	0.2349	0.3148	0.5129

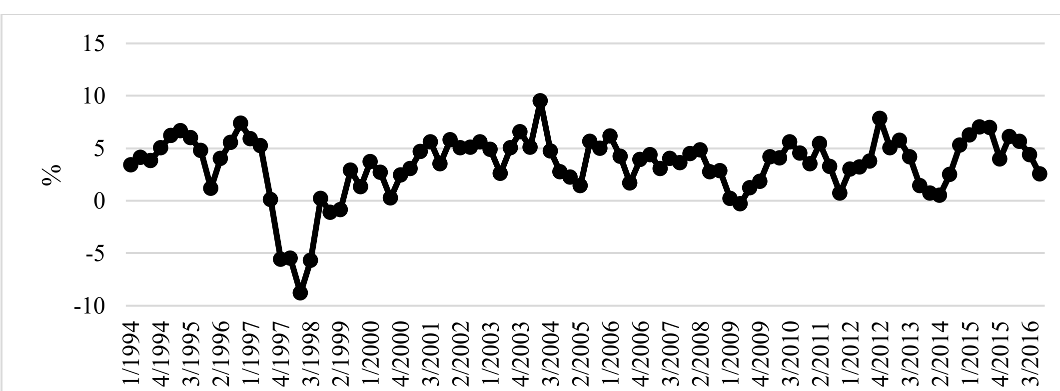

Figure 3 the real observed consumption growth, $G_{c,t}$ since 1993-2016

The standard asset pricing model

Calibration of asset pricing model when consumption growth process is a two-states Markov chain

We calibrate the preference parameter from the standard consumption-based asset pricing model with the two-state Markov chain consumption growth process which follows Mehra and Prescott (1985). From

$$E(r_m - r_f) = \Pi_h(R_{m,h} - R_{f,h}) + \Pi_l(R_{m,l} - R_{f,l}),$$

where the market returns in each state $s = h, l$ are

$$R_{m,h} = \pi_{h,h}\left(\frac{1+Z_h}{Z_h}G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_h}G_{c,l}\right),$$

$$R_{m,l} = \pi_{l,h}\left(\frac{1+Z_h}{Z_l}G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_l}G_{c,l}\right).$$

The risk-free returns in each state $s = h, l$ are

$$R_{f,h} = \left[\pi_{h,h}\,\beta\,G_{c,h}^{-\gamma} + \pi_{h,l}\,\beta\,G_{c,l}^{-\gamma}\right]^{-1},$$

$$R_{f,l} = \left[\pi_{l,h}\,\beta\,G_{c,h}^{-\gamma} + \pi_{l,l}\,\beta\,G_{c,l}^{-\gamma}\right]^{-1}.$$

The price-consumption ratio is

$$\begin{pmatrix} Z_h \\ z_l \end{pmatrix} = [1 - \beta\boldsymbol{\Gamma}]^{-1}\beta\boldsymbol{\Gamma}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ and } \boldsymbol{\Gamma} = \begin{pmatrix} \pi_{h,h}G_{c,h}^{-\gamma} & \pi_{h,l}G_{c,l}^{-\gamma} \\ \pi_{l,h}G_{c,l}^{-\gamma} & \pi_{l,l}G_{c,l}^{-\gamma} \end{pmatrix}.$$

We calibrate the risk aversion coefficient, γ, by minimizing the error distance between the equity premium equation and the observed expected equity premium for a given value of time discount factor β.

$$\min_{\gamma}\sqrt{\left[E(r_m - r_f) - \left(\Pi_h(R_{m,h} - R_{f,h}) + \Pi_l(R_{m,l} - R_{f,l})\right)\right]^2}$$

We assume that consumption growth process $G_{c,t+1}$ is

$$G_{c,h} = \mu + \sigma, \tag{38}$$

$$G_{c,l} = \mu - \sigma. \tag{39}$$

where μ is the mean of consumption growth and σ is the standard derivation of consumption growth which are shown in Table 1.

We also assume that the transition matrix $\boldsymbol{\pi}$ is symmetric, i.e., $\pi_{s,s'} = \pi_{s',s}$ for all $s, s' \in \{h, l\}$. The symmetry implies that the matrix can be completely determined by the autocorrelation of consumption growth ρ alone:

$$\boldsymbol{\pi} = \begin{pmatrix} \pi_{h,h} & \pi_{h,l} \\ \pi_{l,h} & \pi_{l,l} \end{pmatrix} = \begin{pmatrix} \frac{1+\rho}{2} & \frac{1-\rho}{2} \\ \frac{1-\rho}{2} & \frac{1+\rho}{2} \end{pmatrix}. \quad (40)$$

The observed autocorrelation of consumption growth ρ is 0.5129, as shown in Table 1. As a result, the symmetry also implies that the steady state probability Π_s is 0.5 for every state $s \in \{h, l\}$.

Similar to Weil (1989), we set time discount factor, β, to be 0.95; 0.96; 0.97; 0.98. This is to be consistent with the estimated beta from the literature where the value is confirmed to be a high value close to 1.00 (Hansen and Singleton (1983).) Table 2 shows the calibration results. Calibrated risk aversion coefficient γ in Table 2 is higher than 10 for all chosen levels of time discount factor β.

Table 2 the calibrated risk aversion γ which matches expected equity premium from Equation (17) using the observed equity premium in Table 1 and implied expected risk-free return from Equation (10)

$\boldsymbol{\beta}$	**$\boldsymbol{\gamma}$ from equation (17)**	**Net expected risk-free return from equation (10) (%)**
0.95	127.9820	3669.80
0.96	128.5036	3671.47
0.97	129.0197	3673.08
0.98	127.5304	3674.65

Calibration of asset pricing model when consumption growth process is i.i.d. Log-normal

The standard consumption-based asset pricing model assumes i.i.d. Log-normal of consumption growth which follows Breeden (1986) and Hansen and Singleton (1982). We calibrate the asset pricing model using the expect equity premium (21) and the expected logarithm of risk-free return (20)

$$E\left(r_{m,t+1} - r_{f,t+1}\right) = \gamma \mathrm{Var}\left(g_{c,t+1}\right) - \tfrac{1}{2}\mathrm{Var}\left(r_{m,t+1}\right),$$

$$E(r_{f,t+1}) = -\log\beta + \gamma E(g_{c,t+1}) - \frac{1}{2}\gamma^2 \mathrm{Var}(g_{c,t+1}).$$

Now we can calibrate risk aversion γ using only expected equity premium (21) as follows:

$$\gamma = \frac{E(r_{m,t+1} - r_{f,t+1}) + \frac{1}{2}\mathrm{Var}(r_{m,t+1})}{\mathrm{Var}(g_{c,t+1})}. \quad (41)$$

The calibrated risk aversion coefficient γ from the equation (41) is 75.4422. This value of risk aversion coefficient γ contradicts the literature as in the case of two-states Markov chain presented earlier.

Table 3 The summary statistics of quarterly real observed logarithm data of financial market returns ($r_{m,t+1}$), risk-free returns ($r_{f,t+1}$), equity premia ($r_{m,t+1} - r_{f,t+1}$) and consumption growth ($g_{c,t+1}$) since 2000-2016.

	$g_{c,t+1}$	$r_{m,t+1}$	$r_{f,t+1}$	$r_{m,t+1} - r_{f,t+1}$
Mean (%)	3.97	2.03	-0.089	2.12
stand derivation (%)	1.94	11.72	0.97	11.95
Autocorrelation	0.5177	0.3237	0.2335	0.3244
Covariance with $g_{c,t+1}$	0.0004	-0.00003	-0.00001	-0.00002

Table 4 The result of the logarithm of expected risk-free return equation (20) when the observed logarithm of mean risk-free returns $E(r_{f,t+1}) = -0.089$ %.

β	γ from equation (41)	$E(r_{f,t+1})$ from equation (20) (%)
0.95	75.4422	197.55
0.96	75.4422	196.50
0.97	75.4422	195.46
0.98	75.4422	194.44

The recursive consumption-based asset pricing model

Calibration of asset pricing model when consumption growth process is two-states Markov chain

The recursive consumption-based asset pricing model assumes a two-states Markov chain of the consumption growth process, which follows Weil (1989). We simultaneously calibrate the asset pricing model using the expected equity premium (33) and the expected risk-free return (28) as

$$E\left(\frac{R_m}{R_f}\right) = \Pi_h \frac{R_{m,h}}{R_f, h} + \Pi_l \frac{R_{m,l}}{R_f, l},$$

$$E(R_f) = \Pi_h R_{f,h} + \Pi_l R_{f,l},$$

where the risk-free returns in each state $s \in \{h, l\}$ are

$$R_{f,h} = \left[\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\left(\pi_{h,h}\, G_{c,h}^{-\gamma}\left(\frac{Z_h+1}{Z_h}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} + \pi_{h,l}\, G_{c,l}^{-\gamma}\left(\frac{Z_l+1}{Z_h}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\right)\right]^{-1},$$

$$R_{f,l} = \left[\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\left(\pi_{l,h}\, G_{c,h}^{-\gamma}\left(\frac{Z_h+1}{Z_l}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} + \pi_{l,l}\, G_{c,l}^{-\gamma}\left(\frac{Z_l+1}{Z_l}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\right)\right]^{-1}.$$

The market return in each state $s \in \{h, l\}$ are

$$R_{m,h} = \pi_{h,h}\left(\frac{1+Z_h}{Z_h} G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_h} G_{c,l}\right),$$

$$R_{m,l} = \pi_{l,h}\left(\frac{1+Z_h}{Z_l} G_{c,h}\right) + \pi_{h,l}\left(\frac{1+Z_l}{Z_l} G_{c,l}\right),$$

and price-consumption ratio in each state $s \in \{h, l\}$ are

$$Z_h = \beta\left(\pi_{h,h} G_{c,h}^{1-\gamma}(Z_h+1)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} + \pi_{h,l} G_{c,l}^{1-\gamma}(Z_l+1)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}},$$

$$Z_l = \beta\left(\pi_{l,h} G_{c,h}^{1-\gamma}(Z_h+1)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} + \pi_{l,l} G_{c,l}^{1-\gamma}(Z_l+1)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}.$$

We calibrate these asset pricing models with our data to find preference parameters which are both risk aversion, γ, and the elasticity of intertemporal substitution, ψ. We follow section 3.1.1 and use Thailand quarterly financial data from 2000-2016. The observed real equity premia, $\frac{R_{m,t+1}}{R_{f,t+1}}$ and the logarithm of observed real equity premia, $r_{m,t+1} - r_{f,t+1}$ are shown in Table 5.

Table 5 The summary statistics of quarterly real observed equity premia ($\frac{R_{m,t+1}}{R_{f,t+1}}$) and logarithm of equity premia ($r_{m,t+1} - r_{f,t+1}$) since 2000-2016.

	$\frac{R_{m,t+1}}{R_{f,t+1}}$	$r_{m,t+1} - r_{f,t+1}$
Mean	1.0286	2.12%
standard derivation	0.1235	11.95%
Autocorrelation	0.3125	0.3244
Covariance with $G_{c,t+1}$	-0.00006	N/A
Covariance with $g_{c,t+1}$	N/A	-0.00002

We find both the risk aversion coefficient, γ, and the elasticity of intertemporal substitution, ψ, by minimizing the error distance of the equity premium equation (33) and the expected risk-free return (28)

$$\min_{\gamma,\psi} \sqrt{\left(E\left(\frac{R_m}{R_f}\right) - (\Pi_h \frac{R_{m,h}}{R_{f,h}} + \Pi_l \frac{R_{m,l}}{R_{f,l}})\right)^2 + \left(E\left(R_f\right) - (\Pi_h R_{f,h} + \Pi_l R_{f,l})\right)^2}. \quad (42)$$

Table 6 The result of risk aversion coefficient, γ, and elasticity of intertemporal substitution, ψ are given time discount factor, β.

β	γ	ψ
0.95	79.8870	-0.9444
0.96	-53.8815	3.6950
0.97	-45.4243	2.1916
0.98	47.6275	3.4705

Calibration of asset pricing model when consumption growth process is i.i.d. Log-normal

The recursive consumption-based asset pricing model assumes i.i.d. log-normal of the consumption growth process which follows Weil (1989). We calibrate the asset pricing model using the expected logarithm of equity premium (36) and expected logarithm of risk-free return (35) as

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma \mathrm{Var}(g_{c,t+1}) - \frac{1}{2}\mathrm{Var}(r_{m,t+1}),$$

$$E(r_{f,t+1}) = -\log\beta + \frac{1}{\psi}E(g_{c,t+1}) - \frac{\gamma\left(1+\frac{1}{\psi}\right)-\frac{1}{\psi}}{2}\mathrm{Var}(g_{c,t+1}).$$

The risk aversion coefficient, γ, can be calibrated with

$$\gamma = \frac{E(r_{m,t+1}-r_{f,t+1})+\frac{1}{2}\mathrm{Var}(r_{m,t+1})}{\mathrm{Var}(g_{c,t+1})} \quad (43)$$

Therefore, we can find an elasticity of intertemporal substitution coefficient, ψ using only the expected logarithm of equity premium (36) for any given risk aversion coefficient, γ, and time discount factor, β, as follows:

$$\psi = \frac{E(g_{c,t+1})-\frac{\gamma-1}{2}\mathrm{Var}(g_{c,t+1})}{E(r_{f,t+1})+\log\beta+\frac{\gamma}{2}\mathrm{Var}(g_{c,t+1})}. \quad (44)$$

We now represent these observed equity premia, and risk-free returns are the same in Section 3.1.2 which are shown in Table 5.

As a result, the equity premium from equation (36) and the equity premium from equation (21) are identical when the consumption growth process as i.i.d. log-normal. As a result, the calibrated risk aversion coefficient, γ is 75.4422, the same as in Section 3.1.2. This value is too high to be consistent with the empirical evidence available in the literature. The only benefit of this Epstein and Zin (1989) recursive utility is to be able to calibrate the elasticity of intertemporal substitution, ψ independently from the calibrated value of risk aversion, γ. Therefore, we still cannot solve the equity premium puzzle because the model still requires an unreasonably low value of risk aversion.

Table 7 The result of elasticity of intertemporal substitution, ψ match the expected riskfree return equation (35) with mean of the logarithm of risk-free return in Table 5 when given discount factor, β.

β	γ	ψ
0.95	75.4422	-0.6755
0.96	75.4422	-0.9322
0.97	75.4422	-1.4938
0.98	75.4422	-3.7006

Summary and discussion

The summary result

We summarize the result of all four models studied in this paper.

Standard asset pricing model with a time-separable utility model

The result in table 8 shows that both calibrated risk aversion coefficient, γ, for each time discount rates β =0.95, 0.96, 0.97, 0.98 are not in the acceptable range of this value (between 1 and 10.) In addition, we cannot find the value of the average risk-free return that is close to the actual observed rate which is -0.8%.

Table 8 The calibrated risk aversion, γ and expected risk-free return under both two-state Markov chain assumption and i.i.d. log-normal assumption

	Two-state Markov chain		**i.i.d. Log-normal**	
β	γ (from eq. (17))	$E(R_f)$ (from eq.(10))	γ (from eq. (21))	$E(r_f)$ (from eq.(20))
0.95	127.9820	3769.80 %	75.4422	197.55 %
0.96	128.5036	3771.47 %	75.4422	196.50 %
0.97	129.0197	3773.08 %	75.4422	195.46 %
0.98	127.5304	3774.65 %	75.4422	194.44 %

The resulting risk aversion coefficients, γ, which are shown in the second and fourth columns of Table 8, contradict the literature (Mehra and Prescott, 1985), which suggested that the plausible value for the relative risk aversion coefficient, γ, should be between 1 and 10. Even if we could accept a large value of risk aversion coefficient, γ, as the true value, the model would have predicted an infeasibly large level of the average risk-free return as shown in the third and fifth columns of Table 8. This shows that the equity premium puzzle (according to Mehra and Prescott (1985)) exists in Thailand quarterly financial data from 2000-2016 with both the two-state Markov chain assumption and i.i.d. log-normal assumption under the time separable preference.

Epstein and Zin recursive utility model

This section shows that both calibrated risk aversion coefficient, γ, and calibrated elasticity of intertemporal substitution coefficient ψ for time discount rate β =0.95, 0.96, 0.97, 0.98 are not in a simultaneously acceptable length of value between 1 and 10 and more than 0, respectively. These results can conclude that the consumption-based asset pricing model with both the two-state Markov chain assumption and i.i.d. log-normal assumption under the Epstein and Zin recursive preference cannot explain observed equity premia and logarithm of risk-free returns.

Table 9 The result of risk aversion coefficient, γ, and elasticity of intertemporal substitution, ψ, given time discount factor β.

	Two-state Markov chain		i.i.d. Log-normal	
β	γ (from eq. (42))	ψ (from eq. (42))	γ (from eq. (41))	ψ (from eq.(44))
0.95	79.8870	-0.9444	75.4422	-0.6755
0.96	-53.8815	3.6950	75.4422	-0.9322
0.97	-45.4243	2.1916	75.4422	-1.4938
0.98	47.6275	3.4704	75.4422	-3.7006

The result from Table 9 still shows that we cannot find both reasonable values of the risk aversion coefficient, γ, as shown in the second and fourth columns of Table 9 which are not between 1 to 10. That is, the asset pricing model with Epstein and Zin (1989) recursive utility when the consumption growth process is two-states Markov chain and i.i.d. log-normal, cannot solve the equity premium puzzle and the risk-free rate puzzle. Even if, we could accept such a high value of risk aversion coefficient, γ as the true value. For time discount factors, β = 0.95, 0.96, 0.97, 0.98, the values of elasticity of intertemporal substitution are infeasible (negative value), which are shown the third and fifth columns in Table 9.

Discussion

We first calibrated the standard consumption-based model with time-separable utility function assuming that the consumption growth is both i.i.d. log-normal and a two-state Markov process. The result shows that the calibrated parameters were not in their acceptable ranges. We then calibrated the model with Epstein and Zin recursive utility function which relaxes certain conditions, more specifically, the elasticity of intertemporal substitution and risk aversion parameters are independent of each other, assuming that the consumption growth is both i.i.d. log-normal and a two-state Markov process. The result also shows that the calibrated parameters were not in their acceptable ranges. This result shows that neither modeling the consumption growth as i.i.d. log-normal or two-state Markov process, or relaxing certain conditions by utilizing a time-separable utility function can help the models in explaining equity premium puzzle and risk-free rate puzzle.

We also used FEDR market return data which is a Thai financial market dataset that includes dividend and right benefits and considers the effect of stock split and correspond to the theory of market returns which are different from data used in Duangthong (2014) who used the total return (includes dividend and right benefits and considers the effect of stock split) of only some companies in SET, and Sedthapinun (2000) and Harnphattananusorn (2014) who used the return of SET index (does not include dividend benefit but includes right benefit and the effect of stock split). Our results are consistent with Duangthong (2014) and Harnphattananusorn (2014) which is that equity premium puzzle exists in Thailand financial data.

The empirical results with FEDR financial data show that we cannot explain the relationship between consumption risk and equity premium with the models used. This is shown in the resulting parameters that are outside the acceptable range. The implication here is that the four models used cannot explain the relationship between consumption growth and asset growth. The result is consistent with that of Duangthong (2014) which used data of some firms in market and Harnphattananusorn (2014) which used SET index. We conclude then that consumption growth data are not enough to explain equity premium and risk-free rate in Thailand data for the four models used in this paper, thus, confirming that the puzzles do exist in Thailand financial data. This result is also consistent with the result in the literature with US data as well (Mehra and Prescott (1985), Weil (1989).)

Conclusion

The consumption-based asset pricing model is an important theory to explain the relationship between consumption risk and asset returns by linking macroeconomics and finance theories together. The model can be used to explain the relationship between equity premium and consumption risk, that is, investors in risky assets should be compensated for bearing consumption risk, as well as how much the price of consumption risk should be.

There have been many empirical papers that study the consumption-based asset pricing model. Studies using US data show that the resulting parameters from the calibrated models should not be possible based on the literature. These results are known to be called the equity premium puzzle and the risk-free rate puzzle and imply that some consumption-based asset pricing models can not model the relationship between equity premium and consumption risk. Studies using certain Thailand data show that there are no equity premium and risk-free rate puzzles while other studies show that the puzzles do exist.

Our result with Thailand data, shows that none of the models can explain the observed equity premia and the risk-free returns using reasonable values of parameters. This important result shows that these models may not be appropriate when used as a basis to develop economic or financial policies for Thailand. Therefore, we suggest to investigate other models that may be more appropriate for Thailand.

It would be interesting to investigate whether long-run risk models can explain the equity premium in Thailand. This model has been used successfully in the literature for solving the equity premium puzzle and the risk-free rate puzzle in US data from 1930 to 2006 (Bansal et al, 2008).

Another important issue for this paper is that we did not consider the impact of the global financial crisis in 2008 on the financial market return used in this paper. The crisis could affect the credit premium and liquidity premium of risk-free return, which could explain why the average value of equity premium is low. A more thorough investigation of this effect may provide interesting results.

As more data become available, data-driven models are becoming more important. These models rely on models that perform well under reasonable assumptions. Therefore, it is imperative to confirm the validity of models before using them as a base to develop policy or decision-making models. Our result shows that standard consumption-based asset pricing models, even when relaxing certain restrictions, cannot explain the relationship between consumption risk and equity premium in Thailand financial data. Therefore, it is important to investigate and develop better models for Thailand financial data.

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A. The asset pricing equation in two-state Markov chain (Mehra and Prescott, 1985)

The consumption growth $G_{c,t+1}$ follows a two-state Markov chain which is shown in Section 2.2.1. We found risk-free return in state $s = h, l$ as following

$$R_{f,s} = \frac{1}{\sum_{s'=h,} \pi_{s,s'} M_{s'}} \tag{45}$$

We substitute stochastic discount factor in state $s = h, l$ in equation (6), then the risk-free return in state $s = h, l$ follows

$$R_{f,h} = \left[\pi_{h,h}\, \beta\, G_{c,h}^{-\gamma} + \pi_{h,l}\, \beta\, G_{c,l}^{-\gamma}\right]^{-1}$$

$$R_{f,l} = \left[\pi_{l,h}\, \beta\, G_{c,h}^{-\gamma} + \pi_{l,l}\, \beta\, G_{c,l}^{-\gamma}\right]^{-1}.$$

And expected risk-free return can be written as

$$E(R_f) = \Pi_h R_{f,h} + \Pi_l R_{f,l}.$$

In next step, we follow market return which we have to find price-consumption ratio in state $s = h, l$ by following Euler equation in Section 2.1 of being in state $s' = h, l$ after being in state $s = h, l$ in the previous period which is written as

$$\beta \sum_{s'=h,l} \pi_{s,s'}\, G_{c,s'}^{-\gamma} R_{c,s'} = 1. \tag{46}$$

The market return $R_{c,s'}$ follows equation (14) then Euler equation can be rewritten as

$$\beta \sum_{s'=h,l} \pi_{s,s'}\, G_{c,s'}^{-\gamma} \frac{1+Z_{s'}}{Z_s} = 1,$$

$$\beta \sum_{s'=h,l} \pi_{s,s'}\, G_{c,s'}^{-\gamma} (1 + Z_{s'}) = Z_s. \tag{47}$$

This Euler equation can be rewritten in term of matrix as

$$\begin{pmatrix} Z_h \\ Z_l \end{pmatrix} = \begin{pmatrix} \beta\pi_{h,h}\, G_{c,h}^{-\gamma}(1 + Z_h) + \beta\pi_{h,l}\, G_{c,l}^{-\gamma}(1 + Z_l) \\ \beta\pi_{l,h}\, G_{c,h}^{-\gamma}(1 + Z_h) + \beta\pi_{l,l}\, G_{c,l}^{-\gamma}(1 + Z_l) \end{pmatrix},$$

$$\begin{pmatrix} Z_h \\ Z_l \end{pmatrix} = \beta\boldsymbol{\Gamma} \begin{pmatrix} 1 + Z_h \\ 1 + Z_l \end{pmatrix} = \beta\boldsymbol{\Gamma} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \beta\boldsymbol{\Gamma} \begin{pmatrix} Z_h \\ Z_l \end{pmatrix}, \tag{48}$$

where

$$\boldsymbol{\Gamma} = \begin{pmatrix} \pi_{h,h}\, G_{c,h}^{-\gamma} + \pi_{h,l}\, G_{c,l}^{-\gamma} \\ \pi_{l,h}\, G_{c,h}^{-\gamma} + \pi_{l,l}\, G_{c,l}^{-\gamma} \end{pmatrix}.$$

Therefore, we found pricing consumption ratio in state $s = h, l$ which can be written as

$$\begin{pmatrix} Z_h \\ Z_l \end{pmatrix} = [I - \beta\boldsymbol{\Gamma}]^{-1} \beta\boldsymbol{\Gamma} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{49}$$

We use price-consumption ratio to find (conditional expected) market return in state $s = h, l$ as follows:

$$R_{m,s} = \sum_{s'=h,l} \pi_{s,s'} G_{c,s'}^{-\gamma} \frac{1+Z_{s'}}{Z_s}, \tag{50}$$

or

$$R_{m,h} = \pi_{h,h} \left(\frac{1+Z_h}{Z_h} G_{c,h} \right) + \pi_{h,l} \left(\frac{1+Z_l}{Z_h} G_{c,l} \right),$$

$$R_{m,l} = \pi_{l,h} \left(\frac{1+Z_h}{Z_l} G_{c,h} \right) + \pi_{h,l} \left(\frac{1+Z_l}{Z_l} G_{c,l} \right),$$

and the expected market return is

$$E(R_m) = \Pi_h R_{m,h} + \Pi_l R_{m,l}.$$

The expected equity premium of Mehra and Prescott (1985) is

$$E(r_m - r_f) = \Pi_h (R_{m,h} - R_{f,h}) + \Pi_l (R_{m,l} - R_{f,l}).$$

B. The asset pricing equation in I.I.D. (Mehra and Prescott, 1985)

The consumption growth $G_{c,t+1}$ follows i.i.d. log-normal which is shown in Section 2.2.2. It affects the stochastic discount factor M_{t+1} as i.i.d. log-normal and we assume return on asset j Rj;t+1 is i.i.d. log-normal. Therefore the conditional expectations are the same as unconditional expectations which the pricing equation can be rewritten in terms of unconditional expectations as:

$$E(M_{t+1} R_{j,t+1}) = 1. \tag{51}$$

This pricing equation is simplified in term of logarithm which can be rewritten as:

$$E(m_{t+1}) + E(r_{j,t+1}) + \frac{1}{2}\mathrm{Var}(m_{t+1}) + \frac{1}{2}\mathrm{Var}(r_{j,t+1}) + \mathrm{Cov}(m_{t+1}, r_{j,t+1}) = 0,$$

where $m_{t+1} = \log M_{t+1}$ and $r_{j,t+1} = \log R_{j,t+1}$ are the logarithm of the stochastic discount factor, and the logarithm of returns on asset j make that $r_{j,t+1}$ can be approximately net returns on asset j, respectively. Under the CRRA utility function, the logarithm of stochastic discount factor is

$$m_{t+1} = \log(\beta G_{c,t+1}^{-\gamma}) = \log \beta - \gamma g_{c,t+1}.$$

We found expected and variance of the logarithm of stochastic discount factor m_{t+1}

$$E(m_{t+1}) = \log \beta - \gamma E(g_{c,t+1}), \tag{52}$$

$$\mathrm{Var}(m_{t+1}) = \gamma^2 \mathrm{Var}(g_{c,t+1}), \tag{53}$$

where $E(g_{c,t+1})$ is the mean of the logarithm of consumption growth, $\text{Var}(g_{c,t+1})$ is the variance of the logarithm of consumption growth, $\text{Var}(r_{m,t+1})$ is the variance of the logarithm of the market returns.

We found the expected logarithm of risk-free return ($r_{f,t+1} = \log R_{f,t+1}$) from equation (9) as

$$R_{f,t+1} = \frac{1}{E_t(M_{t+1})},$$

$$E(r_{f,t+1}) = -E(m_{t+1}) - \frac{1}{2}\text{Var}(m_{t+1}) + \frac{1}{2}Var(E_t(m_{t+1})).$$

If m_{t+1} is i.i.d. then $E_t(m_{t+1})$ is constant and $Var(E_t(m_{t+1})) = 0$. Therefore,

$$E(r_{f,t+1}) = -E(m_{t+1}) - \frac{1}{2}\text{Var}(m_{t+1}),$$

$$E(r_{f,t+1}) = -\log\beta + \gamma E(g_{c,t+1}), -\gamma^2\text{Var}(g_{c,t+1}). \tag{54}$$

Under asset pricing equation 18, the risk premium on asset j can be written as

$$E(m_{t+1}) + E(r_{j,t+1}) + \frac{1}{2}\text{Var}(m_{t+1}) = -\text{Cov}(m_{t+1}, r_{j,t+1}) - \frac{1}{2}\text{Var}(r_{j,t+1}),$$

$$+E(r_{j,t+1}) - (-E(m_{t+1}) - \frac{1}{2}\text{Var}(m_{t+1})) = -\text{Cov}(m_{t+1}, r_{j,t+1}) - \frac{1}{2}\text{Var}(r_{j,t+1}),$$

$$E(r_{j,t+1} - r_{f,t+1}) = -\text{Cov}(m_{t+1}, r_{j,t+1}) - \frac{1}{2}\text{Var}(r_{j,t+1}), \tag{55}$$

where $\text{Cov}(m_{t+1}, r_{j,t+1})$ is the covariance between the logarithm of stochastic discount factor m_{t+1} and the logarithm of return on asset *j*, $r_{j,t+1}$ which can be written as:

$$\text{Cov}(m_{t+1}, r_{j,t+1}) = \text{Cov}(\log\beta - \gamma g_{c,t+1}, r_{j,t+1}),$$

$$\text{Cov}(m_{t+1}, r_{j,t+1}) = -\gamma\text{Cov}(g_{c,t+1}, r_{j,t+1}), \tag{56}$$

where $\text{Cov}(g_{c,t+1}, r_{j,t+1})$ is the covariance between the logarithm of consumption growth and the logarithm of returns on asset *j*. Therefore, the risk premium on asset *j* is

$$E(r_{j,t+1} - r_{f,t+1}) = \gamma\text{Cov}(g_{c,t+1}, r_{j,t+1}) - \frac{1}{2}\text{Var}(r_{j,t+1}). \tag{57}$$

Also, the risk premium on market return is the equity premium which can be written as

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma\text{Cov}(g_{c,t+1}, r_{m,t+1}) - \frac{1}{2}\text{Var}(r_{m,t+1}), \tag{58}$$

where the logarithm of market return can be written as:

$$r_{m,t+1} = g_{c,t+1} + \log\left(\frac{1+Z_{t+1}}{Z_t}\right). \tag{59}$$

Note that, the price-consumption ratio is constant ($Z_{t+1} = Z_t$) when consumption growth is i.i.d. log-normal. Therefore we can rewrite equity premium as

$$E(r_{m,t+1}-r_{f,t+1})=\gamma \mathrm{Cov}\left(g_{c,t+1},g_{c,t+1}+\log\left(\frac{1+Z_{t+1}}{Z_t}\right)\right)-\frac{1}{2}\mathrm{Var}(r_{m,t+1}),$$

$$E(r_{m,t+1}-r_{f,t+1})=\gamma \mathrm{Cov}(g_{c,t+1},g_{c,t+1})-\frac{1}{2}\mathrm{Var}(r_{m,t+1}),$$

$$E(r_{m,t+1}-r_{f,t+1})=\gamma \mathrm{Var}(g_{c,t+1})-\frac{1}{2}\mathrm{Var}(r_{m,t+1}). \quad (60)$$

C. The asset pricing equation in two-state Markov chain Weil (1989)

We follow consumption growth $G_{c,t+1}$ in equation (7) and the stochastic discount factor M_{t+1} in equation (26) which are shown in Section 2.3.1. Euler equation in equation (24) is

$$\sum_{s'=h,l}\left[\pi_{s,s'}\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}G_{c,s'}^{\frac{1}{\psi}\frac{1-\gamma}{1-\frac{1}{\psi}}}R_{c,s'}^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]=1$$

$$\sum_{s'=h,l}\left[\pi_{s,s'}\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}G_{c,s'}^{\frac{1}{\psi}\frac{1-\gamma}{1-\frac{1}{\psi}}}\left(G_{c,s'}\frac{1+Z_{s'}}{Z_s}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]=1$$

$$\sum_{s'=h,l}\left[\pi_{s,s'}\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}G_{c,s'}^{1-\gamma}(1+Z_{s'})^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]=Z_s^{\frac{1-\gamma}{1-\frac{1}{\psi}}}$$

$$Z_s=\beta\left[\sum_{s'=h,l}\left[\pi_{s,s'}G_{c,s'}^{1-\gamma}(1+Z_{s'})^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \quad (61)$$

The price-consumption ratio in state $s=h,l$, Z_s are

$$Z_h=\beta\left[\pi_{h,h}G_{c,h}^{1-\gamma}(1+Z_h)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}+\pi_{h,l}G_{c,l}^{1-\gamma}(1+Z_l)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}},$$

$$Z_l=\beta\left[\pi_{l,h}G_{c,h}^{1-\gamma}(1+Z_h)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}+\pi_{l,l}G_{c,l}^{1-\gamma}(1+Z_l)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}.$$

Now, the market return in state $s=h,l$, $R_{m,s}$ is

$$R_{m,s} = \sum_{s'=h,l} \pi_{s,s'} G_{c,s'} \frac{1+Z_{s'}}{Z_s} \tag{62}$$

or

$$R_{m,h} = \pi_{h,h} G_{c,h} \frac{1+Z_h}{Z_h} + \pi_{h,l} G_{c,l} \frac{1+Z_l}{Z_h}$$

$$R_{m,l} = \pi_{l,h} G_{c,h} \frac{1+Z_h}{Z_l} + \pi_{l,l} G_{c,l} \frac{1+Z_l}{Z_l}$$

and the expected market return is

$$E(R_m) = \Pi_h R_{m,h} + \Pi_l R_{m,l}.$$

We found (conditional expected) risk-free return in state $s = h, l$ as following

$$R_{f,s} = \frac{1}{\sum_{s'=h,l} [\pi_{s,s'} M_{s'}]}$$

$$R_{f,s} = \frac{1}{\sum_{s'=h,l} \left[\pi_{s,s'} \beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}} G_{c,s'}^{-\gamma} \left(\frac{1+Z_{s'}}{Z_s}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} \right]} \tag{63}$$

As a result, we substitute stochastic discount factor in state $s = h, l$ in equation (26) to found risk-free return in state $s = h, l$, as follows

$$R_{f,h} = \left[\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \left(\pi_{h,h} G_{c,h}^{-\gamma} \left(\frac{1+Z_h}{Z_h}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} + \pi_{h,l} G_{c,l}^{-\gamma} \left(\frac{1+Z_l}{Z_h}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} \right) \right]^{-1} \tag{64}$$

$$R_{f,l} = \left[\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \left(\pi_{l,h} G_{c,h}^{-\gamma} \left(\frac{1+Z_h}{Z_l}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} + \pi_{h,l} G_{c,l}^{-\gamma} \left(\frac{1+Z_l}{Z_l}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} \right) \right]^{-1}, \tag{65}$$

and the expected risk-free return is

$$E(R_f) = \Pi_h R_{f,h} + \Pi_l R_{f,l}.$$

Therefore, the equity premium in state $s = h, l$ from Weil (1989) is

$$\left(\frac{R_{m,s}}{R_{f,s}}\right) = \left[\sum_{s'=h,l} \pi_{s,s'} \beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}} G_{c,s'}^{-\gamma} (1+Z_{s'})^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} \right] \left[\sum_{s'=h,l} \pi_{s,s'} G_{c,s'} \frac{1+Z_{s'}}{Z_s} \right],$$

$$\left(\frac{R_{m,s}}{R_{f,s}}\right) = \frac{\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}}{Z_s^{\frac{1-\gamma}{1-\frac{1}{\psi}}}}\left[\sum_{s'=h,l}\pi_{s,s'}G_{c,s'}^{-\gamma}(1+Z_{s'})\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\left[\sum_{s'=h,l}\pi_{s,s'}G_{c,s'}(1+Z_{s'})\right], \quad (66)$$

and we substitute price-consumption ratio in state $s = h, l$ which is

$$\left(\frac{R_{m,s}}{R_{f,s}}\right) = \frac{\left[\sum_{s'=h,l}\pi_{s,s'}G_{c,s'}^{-\gamma}(1+Z_{s'})\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\left[\sum_{s'=h,l}\pi_{s,s'}G_{c,s'}(1+Z_{s'})\right]}{\left(\sum_{s'=h,l}\pi_{s,s'}G_{c,s'}^{1-\gamma}\left(1+Z_{s'}^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right)\right)}, \quad (67)$$

then the expected equity premium is

$$E\left(\frac{R_m}{R_f}\right) = \Pi_h\left(\frac{R_{m,h}}{R_{f,h}}\right) + \Pi_l\left(\frac{R_{m,l}}{R_{f,l}}\right). \quad (68)$$

D. The asset pricing equation in I.I.D. Weil (1989)

Following Section 2.3.2 and Appendix B, consumption growth $G_{c,t+1}$ follows i.i.d. log-normal then logarithm of stochastic discount factor is

$$m_{t+1} = \log M_{t+1} = \log\left[\left(\beta G_{c,t+1}^{-\frac{1}{\psi}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} R_{t+1}^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\right],$$

$$m_{t+1} = \frac{1-\gamma}{1-\frac{1}{\psi}}\log\beta - \gamma g_{c,t+1} + \left(\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}\right)\log\left(\frac{1+Z_{t+1}}{Z_t}\right). \quad (69)$$

Again, the price-consumption ratio Z_t is constant when consumption growth $g_{c,t+1}$ is i.i.d. log-normal. Therefore we found price-consumption ratio from Euler equation (24) as

$$E_t\left[\left(\beta G_{c,t+1}^{-\frac{1}{\psi}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} R_{t+1}^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right] = 1,$$

$$E_t\left[\left(\beta G_{c,t+1}^{-\frac{1}{\psi}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\left(G_{c,t+1}\frac{1+Z_{t+1}}{Z_t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right] = 1,$$

$$E_t\left[\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}G_{c,t+1}^{1-\gamma}\left(\frac{1+Z_{t+1}}{Z_t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]=1,$$

from price-consumption ratio is constant then

$$\left(\frac{1+Z_{t+1}}{Z_t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}=\frac{1}{\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}E\left(G_{c,t+1}^{1-\gamma}\right)}. \tag{70}$$

We can rewrite in term of logarithm as follows:

$$\log\left(\frac{1+Z_{t+1}}{Z_t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}=\log\frac{1}{\beta^{\frac{1-\gamma}{1-\frac{1}{\psi}}}E\left(G_{c,t+1}^{1-\gamma}\right)},$$

$$\log\left(\frac{1+Z_{t+1}}{Z_t}\right)=-\log\beta-\left(1-\frac{1}{\psi}\right)E\left(g_{c,t+1}\right)-\frac{1}{2}(1-\gamma)\left(1-\frac{1}{\psi}\right)\mathrm{Var}\left(g_{c,t+1}\right).$$

We found the moment of the logarithm of stochastic discount factor m_{t+1} including: expected and variance of logarithm of stochastic discount factor as

$$E(m_{t+1})=\frac{1-\gamma}{1-\frac{1}{\psi}}\log\beta-\gamma E\left(g_{c,t+1}\right)+\left(\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}\right)\log\left(\frac{1+Z_{t+1}}{Z_t}\right), \tag{71}$$

$$\mathrm{Var}(m_{t+1})=\mathrm{Var}\left(\frac{1-\gamma}{1-\frac{1}{\psi}}\log\beta-\gamma g_{c,t+1}+\left(\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}\right)\log\left(\frac{1+Z_{t+1}}{Z_t}\right)\right),$$

$$\mathrm{Var}(m_{t+1})=\gamma^2\mathrm{Var}\left(g_{c,t+1}\right). \tag{72}$$

We found the expected logarithm of risk-free returns $r_{f,t+1}$from equation (35) as

$$E\left(R_{f,t+1}\right)=R_{f,t}=\frac{1}{E(M_{t+1})},$$

$$E\left(r_f\right)=-E(m_{t+1})-\frac{1}{2}\mathrm{Var}(m_{t+1}), \tag{73}$$

$$E\left(r_{f,t+1}\right)=-\log\beta+\frac{1}{\psi}E\left(g_{c,t+1}\right)-\frac{1}{2}\left(\gamma\left(1+\frac{1}{\psi}\right)-\frac{1}{\psi}\right)\mathrm{Var}\left(g_{c,t+1}\right) \tag{74}$$

We found risk premium on asset *j* from equation (18) as

$$E\left(r_{j,t+1}-r_{f,t+1}\right)=-\mathrm{Cov}\left(m_{t+1},r_{j,t+1}\right)-\frac{1}{2}\mathrm{Var}(r_{j,t+1}). \tag{75}$$

We found that $\mathrm{Cov}\left(m_{t+1},r_{j,t+1}\right)$ is the covariance between the logarithm of stochastic discount factor m_{t+1} and the logarithm of return on asset *j* $r_{j,t+1}$ which can be written as:

$$\mathrm{Cov}(m_{t+1}, r_{j,t+1}) = \mathrm{Cov}\left(\frac{1-\gamma}{1-\frac{1}{\psi}}\log\beta - \gamma g_{c,t+1} + \left(\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}\right)\log\left(\frac{1+Z_{t+1}}{Z_t}\right), r_{j,t+1}\right),$$

$$\mathrm{Cov}(m_{t+1}, r_{j,t+1}) = -\gamma\mathrm{Cov}(g_{c,t+1}, r_{j,t+1}). \tag{76}$$

Therefore, the logarithm of returns on asset j. The risk premium on asset j is

$$E(r_{j,t+1} - r_{f,t+1}) = \gamma\mathrm{Cov}(g_{c,t+1}, r_{j,t+1}) - \frac{1}{2}\mathrm{Var}(r_{j,t+1}). \tag{77}$$

In Addition, the risk premium on market return is the equity premium which can be written as

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma\mathrm{Cov}(g_{c,t+1}, r_{m,t+1}) - \frac{1}{2}\mathrm{Var}(r_{m,t+1}). \tag{78}$$

In the assumption of the market return of Mehra and Prescott (1985); Weil (1989), market return $r_{m,t+1}$ follows equation (14) then we can rewrite equity premium as

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma\mathrm{Cov}\left(g_{c,t+1}, g_{c,t+1} + \log\left(\frac{1+Z_{t+1}}{Z_t}\right)\right) - \frac{1}{2}\mathrm{Var}(r_{m,t+1}), \tag{79}$$

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma\mathrm{Cov}(g_{c,t+1}, g_{c,t+1}) - \frac{1}{2}\mathrm{Var}(r_{m,t+1}), \tag{80}$$

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma\mathrm{Var}(g_{c,t+1}) - \frac{1}{2}\mathrm{Var}(r_{m,t+1}). \tag{81}$$