STOCK RETURN PREDICTABILITY IN EMERGING MARKETS

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A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy (Economics)
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ABSTRACT

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The stock return predictability still be a puzzle in the financial economics society that have not yet solved for several decades. Some of them believe that they are predictable, and that stakeholders are able to ensure opportunities to allocate their assets in advance while others disagree and believe in stock market efficiency. In spite of the fact that a number of researchers that have recognized the predictive model as fact, there are still some doubts in terms of econometric issues. Econometricians generally agree that the predictive variable has a local–to-unity property that has a significant correlation with stock returns in an infinite set of a given sample and the other issue is the near unit root characteristic of stock returns' stochastic volatility. Both of the issues end up with an over-reject characteristic of standard hypothesis testing form of predictive regression.

Due to the previously mentioned econometrics issues, the CJP approach will be applied to the predictability of stock returns as well as to testing to rectify the issues. The CJP approach will utilize the change of time method to rectify the nonstationary of stochastic volatility and the nonparametric instrumental variable estimator known as the Cauchy estimator to fix the endogeneity problem of covariates.

To investigate the stock return predictability with the mentioned econometrics issues, this research applies stock return data from Stock Exchange of Thailand (SET) to extract stochastic volatility and testing of local-to-unity property of it. Consequently, the time change method is applied to generate robustness hypothesis testing for unit root or near unit root stochastic volatility of stock returns. The last step applies the nonparametric Cauchy estimator to make a set of instrument variable of covariate for the stock return prediction

Empirical results show that the stock return significantly generates local-to-unity and near local-to-unity of stochastic volatility. The time change method can be applied to resolve the local-to-unity of stochastic volatility problem by using stochastic stopping time of the process or volatility time to replace the calendar time, which will make stochastic volatility in each new period become stationary across all observations. The Cauchy estimator was applied for random sampling based on volatility time and shows no support for predictability at any frequency for the selected predictors, such as the dividend–price and earnings–price ratios.

This research concludes that it seems clear that stock returns cannot be predicted by dividend–price and earnings–price ratios if the characteristics of the data are properly checked and managed.

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CHAPTER 1

INTRODUCTION

In the field of financial economics, the predictability of stock returns is one of the most prevalent problems that has not yet been solved. There is much debate surrounding the predictability of stock returns; some believe that they are predictable, and that stakeholders are able to ensure opportunities to allocate their assets in advance while others disagree and believe in stock market efficiency.

Campbell and Shiller (1988) were the first to report on this topic, along with Fama and French (1988), who reported that the dividend-price ratio has some predictive power. This idea was then further brought to light, becoming a given of the industry, by Cochrane (2005), who stated, "Price equals to expected discounted payoff." (p. xiii). The stochastic discounted factor (s.d.f.) concept comes from the fundamental uncertainty of macroeconomics, which reflects the price of assets via the mechanics of both consumption-based and general equilibrium models (Cochrane, 1999, 2008, 2011). Another important study in the literature is that Balvers, Cosimano, and McDonald (1990); their study indicated that the mechanics of stock returns cannot be explained by 20any current economic theories if they are not predictable. The predictability of stock returns has also been discussed by a number of econometricians and financial economists, including Lettau and Ludvigson (2001), Chan and Kogan (2002), Avramov (2004). In terms of implication, Ferson, Sarkissian, and Simin (2003) showed that strategic portfolio management, asset allocation strategy, performance evaluation, and market timing are all able to utilize predictive regression. McMillan and Wohar (2010) applied non-linear forecasting, in the context of the present value model, to the dividend-price ratio to increase its predictive power, while Kellard, Nankervis, and Papadimitriou (2010) used the dividend-price ratio to forecast the equity premium of FTSE100 for both in-sample and out-of-sample data, with an acceptable result.

The robustness of predictors has also been studied in addition to their predictive power. Campbell and Shiller (2001) offered evidence on the time variations of stock returns. McMillan and Wohar (2013) ran a forecast of the stock returns on international markets using panel data of dividend-price ratios; they achieved time-varying results, and found no evidence of predictability at the early stage and increasingly predictable results in the last two decades. Stambaugh (1999) proposed using predictive regression to study stock returns that regressed on a lagged stochastic regressor, or a dividend-price ratio, and found that the error term is correlated with the innovations, resulting in the non-robustness of OLS estimators. This result indicates that the OLS estimator may not be a proper method for parameters estimation and hypothesis testing. Campbell and Yogo (2006), to correct the non-robustness of the estimators, proposed both a pretest and an efficient test of predictability. In addition, predictive power has improved when some weak restrictions were imposed on the sign of coefficients and return forecasts, as proposed by Campbell and Thompson (2008); though the predictive level is not significantly improved by this method, it still offered some advantage to mean-variance investors. Chen and Deo (2009) indicated that the inference problem in predictive regression can be attributed to the persistence in the regressors series, specifically the poor estimation of the intercept parameter. Restricted likelihood has been suggested to be a new inferential base for rectifying the problem, especially for local-to-unity estimations.

In spite of the fact that a number of researchers that have recognized the predictive model as fact, there are still some doubts in terms of econometric issues, particularly when it comes to regressors' persistence and small sample size bias. Nelson and Kim (1993) reported two types of small sample size bias, the coefficient of the estimator and the asymptotic standard error of overlapping periods, which resulted in the t-ratio being too large in predictive regression. Wolf (2000) proposed a new statistical method for finding reliable confidence intervals called subsampling, which can be used for regression parameters with fat-tailed and persistent data, and found no signs of return predictability. In the same vein, Lanne (2002) suggested a new robust test for the persistent regressors in predictive regression, but also found no evidence of return predictability. Valkanov (2003) found some evidence of return predictability in long-horizon data after applying asymptotic distribution theory, even though the short one was unpredictable. Ang and Bekaert (2007) inserted an additional predictive variable, short-term rate, into the regression, and found a mild prediction power on the dividend-price ratio for short-horizon data, but no evidence for long-horizon data. Finally, Welch and Goyal (2008) found that a predictive regression using a standard ratio predictor performed badly in terms of the predictability of stock returns for both in-sample and out-of-sample data; they suggested using historical mean, which had more prediction power.

An emerging market was selected to evaluate prediction power and resolve the econometric issues. An emerging market is one that is developing but has not reached the level of the standard markets, such as the United States, Japan, and Western Europe. Kvint (2009) defined an emerging market country as one that is transitioning from a government controlled economy to a free-market-oriented economy, gradually integrating with the global marketplace, expanding the middle class, improving standards of living, social stability and tolerance, and experiencing an increase in cooperation with multilateral institutions. Marois (2012) suggested that financial constraints have become much more important in the emerging market. Vercueil (2015) proposed a new definition of the “emerging economy” as different from "emerging markets" based on financial criteria. Major emerging markets include Argentina, Brazil, China, India, Indonesia, Mexico, Poland, South Africa, South Korea, Turkey, Egypt, Iran, Nigeria, Pakistan, Russia, Saudi Arabia, Taiwan, and Thailand; some special characteristics of these markets include a smaller size, higher levels of risk, and increased liquidity as compared to standard markets. They are more likely to provide more returns on investments, but there are also more risks due to several economic factors.

This study was inspired by the research of Choi, Jacewitz, and Park (2016) to study the Thai stock markets in particular. The Choi et al. approach, or CJP, approach of return prediction and hypothesis testing using a two-step econometric methodology will be used to address the econometrics issues in predictive regression. There are a couple of major problems in predictive regression for stock returns, including endogeneity of the covariates and nonstationary of volatility, that will be addressed in this study.

Econometricians generally agree that the predictive variable has a local–to-unity property that has a significant correlation with stock returns in an infinite set of a given sample. Stambaugh (1999) showed that a standard hypothesis testing form of predictive regression will always have an over- reject characteristic due to the previously mentioned reasons. The other issue is the near unit root characteristic of stock returns' stochastic volatility. Schaller and Van Norden (1997) applied an extension of Markov switching to nonstationary stock returns data using a state-space model; the stock returns then showed clear evidence of being predictable Jacquier, Polson, and Rossi (2004) introduced the leverage effect, which is the correlation between volatility and mean innovation, for fat- tailed data in mean equation innovations, and found strong evidence in favor of fat-tails for daily exchange rates and equity indices with a difference in estimated volatility between extended models and basic models of stochastic volatility. Fat- tailed stochastic volatility may result in generous size distortions on standard tests that rely on a constant, unconditional variance, like stationary ARCH or GARCH. Additionally, Cavaliere (2004) overhauled the effects of permanent shifts in the variance of the errors of an autoregressive process on unit root tests for integrated and local-to-unity processes with heteroskedastic errors and found the regular unit root tests to be highly deviated, meaning that the predictability of stock returns hypothesis testing will be implicitly distorted by fat-tailed stochastic volatility. In terms of econometric analysis, typical issues with stock return data and predictive variables, such as deterministic trends, fat tails, jumps, and structural breaks may therefore seriously affect the critical value of hypothesis testing.

Due to the previously mentioned econometrics issues, the CJP approach will be applied to the predictability of stock returns as well as to testing to rectify the issues. The CJP approach will utilize the change of time method to rectify the nonstationary of stochastic volatility and the nonparametric instrumental variable estimator known as the Cauchy estimator to fix the endogeneity problem of covariates. This combination will improve on the model evaluation in terms of both efficiency and effectiveness. The sign transform of the predictive variables is used to be an instrument of the Cauchy estimator that effectively rectifies the issues induced by the latent of persistence and other related issues in predictive variables. The time change method differently generates robustness hypothesis testing for unit root or near unit root stochastic volatility of stock returns that have induced significant distortion on regular hypothesis testing in predictive regression. A stochastic stopping time of the process or volatility time will be used for the time change to replace the calendar time, which will make stochastic volatility in each new period become stationary across all observations. The predictive regression of stock returns can then be properly analyzed.

Given its characteristics, the CJP approach to stock returns predictions may be able to provide some valuable information to the investment industry, especially in emerging markets such as Thailand, so that stakeholders can make proper decisions to get optimal payoffs on their portfolios, which would be novel in this industry.

In terms of the objectives of the study, there are several questions that I want to answer in this research:

1) What are the current econometrics issues in stock returns data in emerging countries?

2) Can the alternative approach better predict stock returns for emerging countries?

3) What are the differences in characteristics of stock returns predictions between an emerging country like Thailand and the United States?

Answering the above questions by using the proposed methodology can help illuminate the characteristics and prediction behavior of the Thai stock market and others like it; the answers can also offer implications to stakeholders once there is a reasonable prediction number. This study focuses on Thailand as an example of an emerging country for several reasons; firstly, Thailand is the one emerging country in the world that is classified as middle income. Secondly, the value of the Thai stock market has developed in recent years and is enticing to foreign investors, as there is still some opportunity for profit.

The compulsory data for the study will be the stock market's data, which can be easily obtained from the Stock Exchange of Thailand.

The new methodology introduced in this study is expected to, at a base level, be able to provide stock returns prediction in the capital markets. In terms of academic purposes, some new methodologies in statistics and economics have been introduced and applied, such as the Cauchy estimator and the time change method. In the industrial world, active portfolio management, assets allocations, and some risk information will be made available by the results of stock returns predictions. For macroeconomists, the asset pricing and a new expected equilibrium will be reached and a set of policy implications may assist in stabilizing economic growth. Finally, this work will be a part of supportive information for financial economists in terms of whether stock returns are predictable in capital markets.

The structure of this study is as follows. Chapter 2 discusses stochastic volatility and how to estimate it in the Thai stock market as well as how to forecast conditional volatility in comparison with the GARCH (1,1) model. The study will use the estimated stochastic volatility from this chapter as the calculation basis for solving the econometric issues, especially the nonstationary volatility. Chapter 3 presents the new time change method to remedy nonstationary volatility in data and its impact on stock returns predictability. Chapter 4 focuses on the new nonparametric estimation method, which addresses the endogeneity issue in covariates; its influence on stock returns predictivity is also summarized. The last chapter includes conclusions and discussions thereof, as well as implications for future study.

CHAPTER 2

STOCHASTIC VOLATILITY ESTIMATION AND FORECASTING IN THE THAI STOCK MARKET

Volatility modelling is vital, not only in financial economics, asset pricing, and portfolio management as a proxy for risk measurement, but also in terms of trading. This section will introduce some basics of volatility modelling and estimation and discuss stochastic volatility in detail. The MCMC simulation with Gibb sampling will be used to estimate stochastic volatility.

2.1 Introduction

In the field of financial economics, volatility refers to the spread of all likely returns of an asset and is frequently measured as a sample standard deviation in point of statistics, as per Poon (2005). In recent years, volatility forecasting has become a well-known field of research with various applications in financial economics and investments. Bhowmik and Wang (2020) reviewed the effects of return and volatility analysis over the last 12 years, and most of them have worked for developing stock market. Audrino and Hu (2016) examined volatility forecasting and found empirical evidence of asymmetries in the dynamics of both the return and volatility processes. The proposed model was able to capture many empirical stylized facts while remaining cognizant of the parsimony concept of model estimation. Volatility forecasting of the Saudi stock market was conducted by Al Rahahleh and Kao (2018) and founded useful some basis information from the data collected period for investors and related parties to make decisions using this model to forecast the risks associated with investing in the Saudi stock market. Volatility modeling is also essential in return predictability modeling as a proxy for risk measurement and as key primary information for hypothesis testing.

The stochastic volatility model, a well-known multidisciplinary model, requires serious knowledge of probability theory, financial economics, and econometrics, but generates a reasonable model that aims to understand instantaneous volatility and provide a better understanding of asset pricing and investments in financial markets. Some studies also use sample variance as a volatility measure, but sample standard deviation is preferable because it has the same measurement unit as the returns. At this stage, volatility is not the same as asset risk - risk is only related to poor outcomes while volatility also considers good ones. One key fact about volatility is that it is not always constant over time; it is therefore more relevant to use conditional volatility to represent the volatility of assets.

The biggest issue in the early stage of this approach is the availability of the likelihood function of the model that compels it to predict distribution. In the 1990s, a new simulation technique was developed to increase the efficiency of the estimation of stochastic volatility (Jacquier, Polson, & Rossi, 1994, 2002), and then Barndorff-Nielsen and Shephard (2004) proved the success of this method and applied it to the analysis of volatility forecasting based on realized volatility; since then, it has become a well-known model used by econometricians around the world. Stochastic volatility models are commonly used to indicate values of unobservable conditional volatility, both in-sample and out-of-sample, for financial asset pricing applications, such as option pricing.

This chapter is organized as follows. The next section concerns basic background information on the estimation of volatility, the stochastic volatility model, and how to estimate volatility using MCMC simulation, followed by empirical results from the Thai stock market. It then discusses the simulation's ability to forecast volatility as compared to the best-known model, the generalized autoregressive conditional heteroscedasticity (GARCH(1,1)) model. The last section contains the conclusion and a discussion of the implications of this study.

2.2 Basic Background and Estimation of Volatility

Consider a series of returns, y_i, $i = 1, 2, 3, \ldots, T$, in which the sample standard deviation, $\hat{\sigma}$, is defined as

$$\hat{\sigma} = \left(\frac{1}{(T-1)}\sum_{i=1}^{T}(y_i - \mu)^2\right)^{\frac{1}{2}}, \qquad (2.1)$$

and which has unconditional volatility over the T period. One key stylized fact of volatility is that it is always not constant over a given period; it is therefore more relevant to use conditional volatility, h_t, to represent the volatility of asset returns at time t. There are a variety of types of volatility estimation algorithms depending on the reference period, e.g. intraday, daily, monthly. When longer period volatility is required and shorter period data is available, volatility can simply be calculated using Equation (2.1). These results become less accurate if the same period data is used to create the proxy volatility. Engle (1982) evolved the ARCH (autoregressive conditional heteroscedasticity) model, with major enhancements by Bollerslev (1986), into the GARCH (generalized autoregressive conditional heteroscedasticity) model, which is presented below to produce conditional volatility of returns at the same time scale of information y_t ;

$$\begin{gathered} y_t = \mu + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0, \sqrt{h_t}\right) \\ \varepsilon_t = z_t\sqrt{h_t} \\ h_t = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \beta_1 h_{t-1} + \beta_2 h_{t-2} + \cdots \end{gathered} \qquad (2.2)$$

The ARCH/GARCH model is estimated by the maximum likelihood method of (ε_t) and its results offer a better value for conditional volatility than the classic historical volatility from absolute returns, though this also depends on the important assumption that Equation (2.1) is return-generating with respect to ε_t which is white noise. The series must also be long enough to make this type of estimation, though most volatility of time series models are squared returns models, especially when it comes to the ARCH type of model.

Several estimation methods for volatility have been developed to lessen the impact of fat-tail in distribution, including the ARCH model; they all follow the assumption of a white noise variable or some specific probability distribution function for returns. One of the most well-known methods for minimizing the effects of this issue realized volatility. One of major advantage of this estimator is the minimal effect of fat-tail in distribution caused by market microstructure concern from intraday data. The term realized volatility was coined by Andersen and Bollerslev (1998) and is now used to indicate volatility estimates determined by using the sum of intraday squared returns at short intervals. This type of volatility estimation will converge to continuous time volatility based on the assumption that the returns are s set of continuous martingales such that

$$dY_t = \sigma_t dW_t \, , \tag{2.3}$$

where dW_t denotes a Wiener process. Let Δ be the sampling grid such that there are $n\Delta$ sample frequency of returns in one unit of time.

$$\begin{gathered} y_{\Delta,t} \equiv Y_t - Y_{t-\Delta} \\ RV_{t+1} = \sum_{j=1}^{n} y_{\Delta,t+j\Delta}^2 \end{gathered} \tag{2.4}$$

If asset returns have no serial correlation and σ_t has a continuous sampling path, then the quadratic variation theory of Karatzas and Shreve (1988) will be pursued such that:

$$p \lim_{n\Delta\to\infty} \left(\int_t^{t+1} \sigma_s^2 ds - \sum_{j=1}^{n} y_{\Delta,t+j\Delta}^2 \right) = 0 \quad . \tag{2.5}$$

The sampling path of returns from Equation (2.5) should be long and frequent enough to make the volatility at period t appreciable, however, one stylized fact of high frequency returns is that the sampling interval is shorter than five minutes. This generates a lurking serial correlation, which is caused by the market microstructure problem; Aït-Sahalia and Mykland (2009) proposed removing the microstructure problem by using realized volatility.

2.3 Stochastic Volatility

The notation SV will refer to stochastic volatility in this study. The SV model requires serious knowledge of probability theory, financial economics, and econometrics in order to generate a reasonable model that aims to understand instantaneous volatility, study the effects of time-varying variance, and provide a better understanding of asset pricing and investment in financial markets. Econometricians frequently use the ARCH process to explain the dynamics of volatility instead of using stochastic volatility, which is not suitable. Conditional volatility, which is normally geneKIrated from the ARCH process, is directly calculated from past returns; it is convenient to create a forecast in only one step and it is also useful to be able to obtain one-step-ahead likelihood functions to predict the returns distribution (Engle, 2001). The SV approach uses its model structure to create a predictive returns distribution, mrather than directly using previous returns. In general, the SV approach is more suitable for application to continuous time returns data that requires a model of volatility of asset prices. The major issue with this approach as of now is the difficulty in obtaining the likelihood function of the model, which is necessary for predictive distribution. In the 1990s, new simulation techniques were developed to make the estimations of SV models more efficient; Barndorff-Nielsen and Shephard (2002) then used this method to analyze volatility forecasting based on realized volatility, making it one of the most well-known models for econometricians. One of the most common applications of stochastic volatility models is predicting the values of unobservable conditional volatility, both in-sample and out-of-sample, for financial asset pricing applications, such as option-pricing applications.

2.3.1 Model Specification and Estimation

This section will introduce the SV model and specify the notation. Even though the current time development of stochastic volatility is typically continuous time, a number of past contributors used discrete-time models. Bayesian estimation using the Markov chain Monte Carlo (MCMC) method has been applied to estimate the volatility of the model.

1) The Stochastic Volatility Model

Stochastic volatility development began with knowledge of returns prediction approximated by mixture distribution; the treatment of asset prices as a stochastic process controlled by the increments of a primary activity variable was proposed by Clark (1973). He studied many properties of log-price (Y_t) and introduced:

$$Y_t = W_{\tau_t}, t \geq 0, \tag{2.6}$$

where W is Brownian motion, t is continuous time, and τ is a time-change. A time-change is defined as a non-negative process with a non-decreasing sample path. W and τ are independent processes following the concept of Brownian motion, that $Y_t|\tau_t \sim N(0, \tau_t)$. The increment of Y_t are a normal mixture distribution and in the further study Y_t is a martingale under some assumptions.

Taylor (1982) was another primary contributor in terms of the volatility clustering of stochastic volatility. His paper used a discrete-time model and introduced daily returns as a difference of log-prices:

$$y_i = Y_i - Y_{i-1}, \;\; i = 1, 2, 3, \dots \;\; , \tag{2.7}$$

where y_i is both an asset return and martingale process. He also defined the risky part of returns as a multiplication process:

$$y_i = \sigma_i \varepsilon_i \tag{2.8}$$

where ε_i is a standard normal distribution and σ_i is some nonnegative process. In this case, ε_i and σ_i are independent. From Equation (2.8), the signs of y are controlled by ε and σ determines the dynamic of volatility and the fat-tailed effect in the peripheral distribution of the returns y. Taylor also proposed ε as an autoregressive process and defined:

$$\sigma_i = \exp\,(h_i/2), \tag{2.9}$$

where h_i is a non-zero mean Gaussian linear process. The most well-known example of h_i is a first-order autoregressive model such that:

$$h_i = \mu + \phi(h_{i-1} - \mu) + \eta_i \tag{2.10}$$

where η is a Gaussian white noise process and ϕ is the AR coefficient of h_i. In the current stochastic volatility study, ε is relaxing to a simple i.i.d. process for dealing with returns predictability in predictive regression.

In summary, the stochastic volatility model can be shown in the ordered form of its center parameterization in continuous time formatted as follows:

$$\begin{aligned} y_t|h_t &\sim \mathbb{N}(0, \exp(h_t))\,, \\ h_t|h_{t-1}, \mu, \phi, \sigma_\eta &\sim \mathbb{N}\big(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2\big)\,, \\ h_0|\mu, \phi, \sigma_\eta &\sim \mathbb{N}\big(\mu, \sigma_\eta^2/(1-\phi^2)\big)\,, \end{aligned} \tag{2.11}$$

where $\mathbb{N}(\mu, \sigma_\eta^2)$ represents the normal distribution with mean μ and variance σ_η^2 and $\theta = (\mu, \phi, \sigma_\eta)'$ denotes a vector of parameters; the mean of log variance μ, the coefficient of autoregression ϕ, and the volatility of log variance σ_η. The SV process $h = (h_0, h_1, h_2, \cdots, h_n)'$ appearing in Equation (2.11) is either unobservable or the latent time-varying volatility process. Note that the initial state h_0 is assumed to be a stationary distribution of the autoregressive process of order one. A Bayesian estimation of stochastic volatility models used Markov chain Monte Carlo (MCMC) samplers to perform an inferential statistic by obtaining samples from the posterior distribution of parameters and unobserved variables. Its results can be used for future volatility prediction.

2) Prior Distribution

From the Bayesian estimation method, a prior distribution for the parameter vector $\theta = (\mu, \phi, \sigma_\eta)'$ must be defined; the distribution of $\mu, \phi, and\ \sigma_\eta$ are assumed to be independent from each other such that $p(\theta) = p(\mu)p(\phi)p(\sigma_\eta)$.

Generally, the level of the returns parameter μ is assumed to be normal prior to distribution such that $\mu \sim \mathbb{N}(b_\mu, B_\mu)$, and it will always be chosen from the real number set, e.g. $b_\mu = 0,$ and $B_\mu \geq 50.$, Kastner (2016) indicated that data choice does not have a significant impact on estimation results.

For the autoregressive coefficient or persistence parameter, $\phi \in (-1,1)$, Kim, Shephard, and Chib (1998) suggested using $(\phi + 1)/2 \sim B(a_{0,} b_0)$ and finding the distribution function as:

$$p(\phi) = \frac{1}{2B(a_{0,} b_0)} \left(\frac{1+\phi}{2}\right)^{a_{0,}-1} \left(\frac{1-\phi}{2}\right)^{b_0-1} \tag{2.12}$$

where $a_{0,}$ and $b_{0,}$ are positive hyperparameters and the beta function is $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$. The interval of (-1,1) holds the answers for the selected distribution; the expectation and variance of this distribution will be:

$$E(\phi) = \frac{2a_0}{a_0 + b_0} - 1\,,$$
$$V(\phi) = \frac{4a_0 b_0}{(a_0+b_0)^2(a_0+b_0+1)} \quad . \tag{2.13}$$

From the above expression, Kastner (2016) suggested that previous expectations of ϕ depend only on the term $\left(\frac{a_0}{b_0}\right)$. It becomes a positive value if and only if $a_0 > b_0$ and becomes a negative value if and only if $a_0 < b_0$. When the number of a data point is high and the volatility of the log-variance σ_η is close to zero, the data point is likely to contain little to no information about ϕ. Kim et al. (1998) also chose $a_0 =$

20 and $b_0 = 1.5$ to imply a prior mean of 0.86 with a prior standard deviation of 0.11, and thus a significantly small likelihood of nonpositive values for ϕ. Finally, the hyper parameter $(a_{0,} b_0)$ is potentially highly influential on the results and should be paid attention to during performing simulation.

For the volatility $\sigma_\eta \in \mathbb{R}^+$, Kastner (2016) chose $\sigma_\eta^2 \sim \mathrm{B}_{\sigma_\eta} \times \chi_1^2 = \mathcal{G}(1/2\,, 1/2\mathrm{B}_{\sigma_\eta})$, which is not conjugate in the usual sampling scheme in terms of the common Inverse-Gamma prior for σ_η^2. Kastner (2016) also learned that the choice of the hyperparameter B_{σ_η} is of minor influence in empirical applications, provided that it is not too small. The parameter B_{σ_η} also has considerably small impact in empirical study.

3) Simulation-based Sampling Using MCMC

The MCMC approach to estimating stochastic volatility was contributed to by Jacquier et al. (1994) and Tsay (2010). This method is capable of simulating the n-dimension posterior densities of the unobserved conditional volatility in Equation (2.11). Estimation of Equation (2.11) is most likely complicated because of its involvement of the n-dimensional h distribution, as shown below:

$$f(y_t|\theta) = \int f(y_t|h_t).\, f(h_t|\theta) dh_t$$

The goal of this approach is to maximize the likelihood function of (y_t); the probability density of (y_t) is determined by (h_t) which can also be examined by θ.

In this study, the sample will be drawn from the posterior distribution of the unobserved conditional variance h_t and the vector parameter θ via the MCMC algorithm. To reduce the chance of correlation within the draws, the joint sampling of all instantaneous volatility will be obtained using the all without a loop (AWOL) technique discussed in McCausland, Miller, and Pelletier (2011) as used in Kastner's (2016) empirical application. This also requires the auxiliary finite mixture approximation of the errors, as mentioned in Kim et al. (1998).

2.4 Empirical Results

In this section, we test for the SV estimation using the MCMC algorithm and forecasting accuracy and compare the results from this approach with the results from the GARCH (1,1) to demonstrate SV modelling. Details of the data used in our empirical analysis are also provided.

2.4.1 Data Description

Stochastic volatility was estimated using an MCMC simulation; this section describes the estimation and compares the results with the classic GARCH (1,1) model.

We considered a series of stock returns based on the value-weighted index of the Stock Exchange of Thailand (SET) from 1 September 1997 to 31 January 2019 available in daily (5246 data points), weekly (1118 data points), monthly (258 data points), quarterly (86 data points), and yearly (22 data points) frequencies of compounding daily returns. This period was chosen because there was no structural data change after the major economic crisis in 1997. The input of the demeaned stock returns data (y_t) was calculated from the difference of the logarithm of the SET index. The pattern of the data for each frequency of the study period is presented in Figure 2.1.

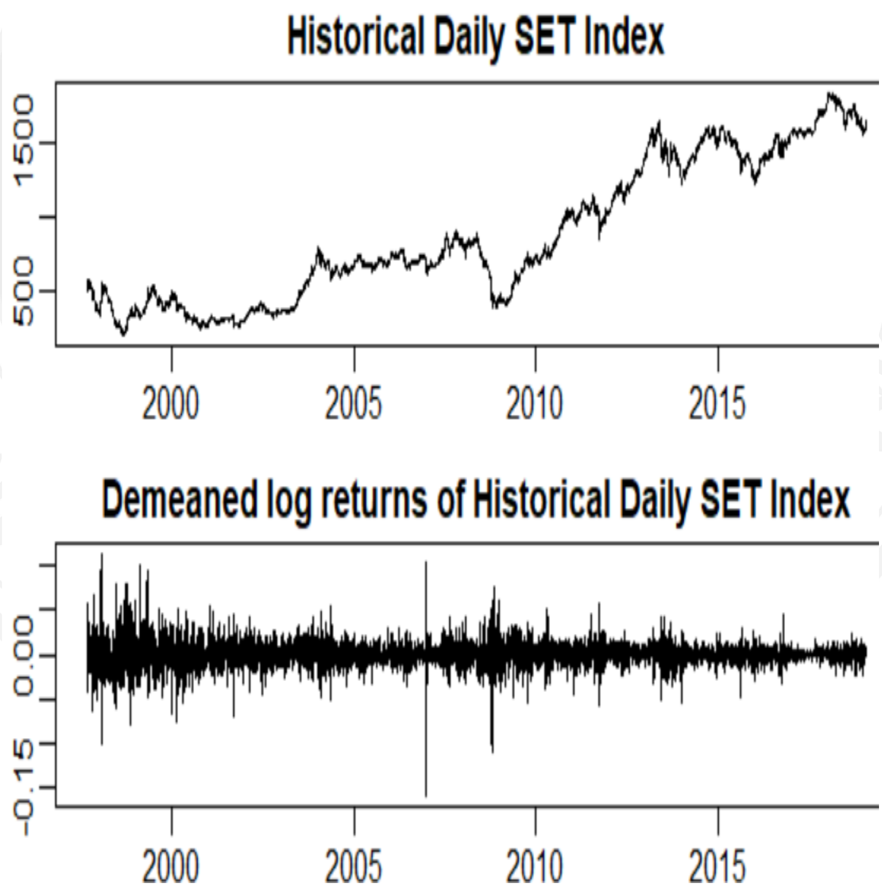
To test the stochastic volatility modelling and estimation using the MCMC approach, we compare the estimation results with GARCH (1,1) results as well as the conditional volatility forecasting accuracy of the Mean Square Error (MSE). The input of the demeaned stock returns data (y_t) is calculated by the difference of logarithmic of SET index. The pattern of the data during the study period is presented in Figure 2.1. In the figure, some volatility clusters can be observed in the demeaned log returns of the daily data, reflecting the behavior of the Thai stock market during the study period.

2.4.2 Identifying Prior Probability Distributions and Configuration Parameters

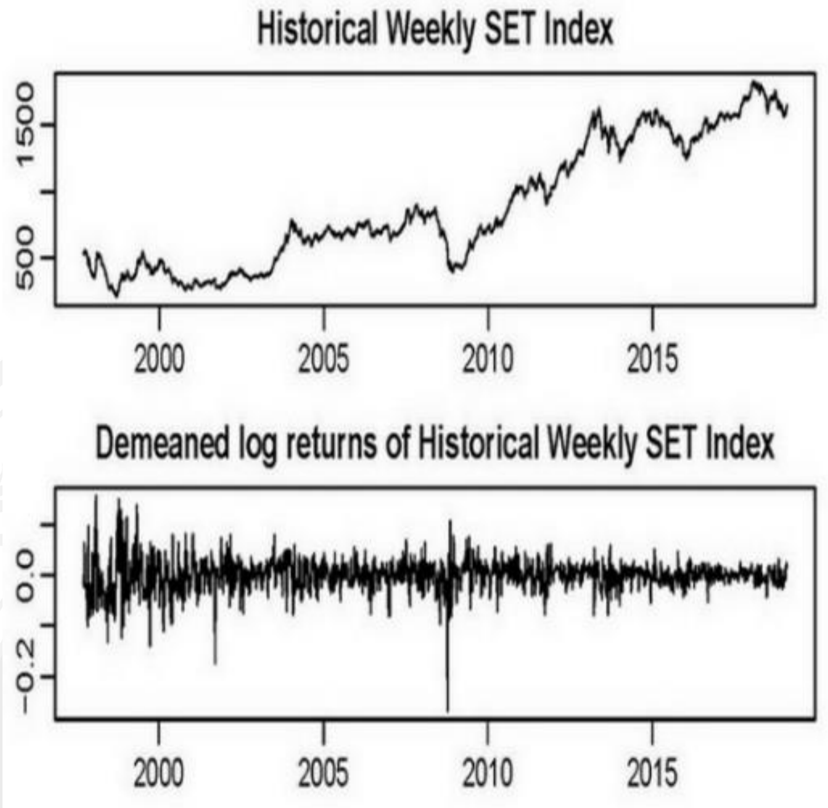
Appropriate identification of prior hyperparameters for the distribution of the vector parameter θ is required to run the simulation, with some configuration parameters. The R-programming has been coded to run the Bayesian inference using the MCMC approach. The prior vector μ is a vector of length 2, containing the mean and standard deviation of the normal prior for the level of the conditional variance μ. We choose a prior c(-5,1) because previous studies suggest that the likelihood normally bring enough information about this parameter.

For specifying, the prior vector ϕ has, again, a vector of length 2 containing a_0 and b_0, as specified in Equation (2.12). The default is currently given through c(20, 2.5) which is it should not be set too small and it should beware of as this hyperparameter have a high possibility influential.

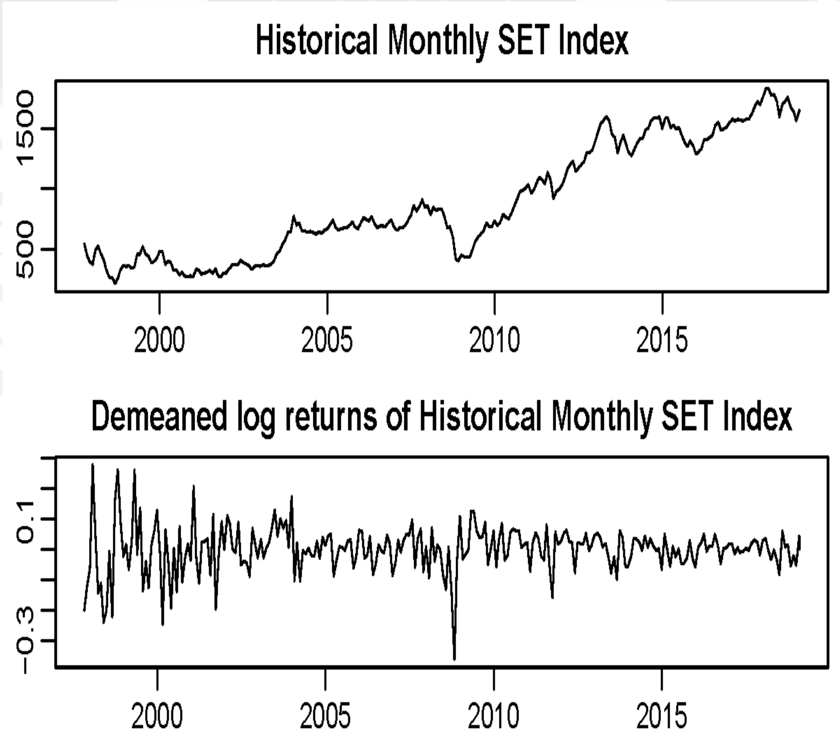
The prior vector σ_η is used to control the prior variance of conditional variance. The value 0.1 is used in this study as it is usually not very influential in general utilization.



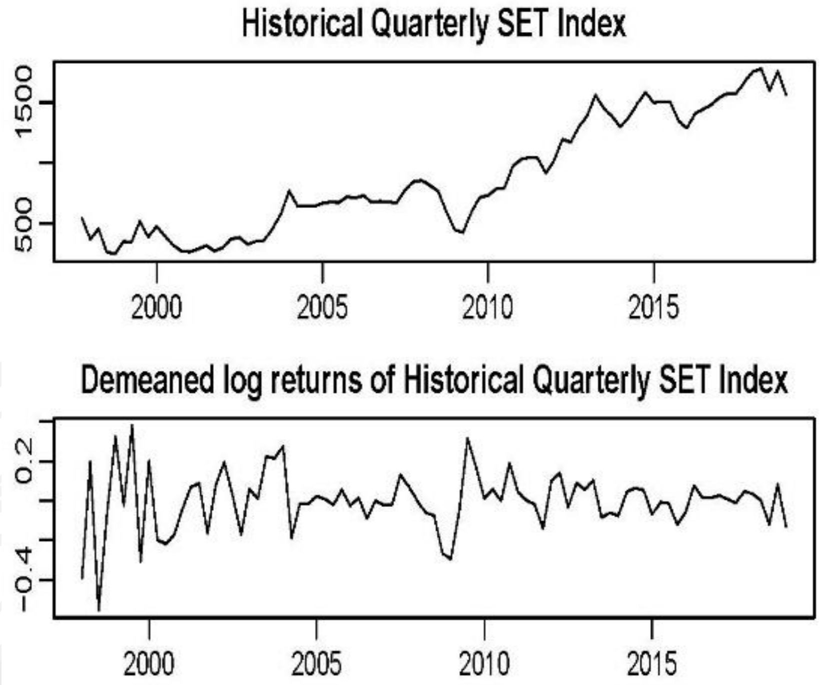
(a)


Historical Weekly SET Index
1500
500
2000
2005
2010
2015
Demeaned log returns of Historical Weekly SET Index
0.0
−0.2
2000
2005
2010
2015

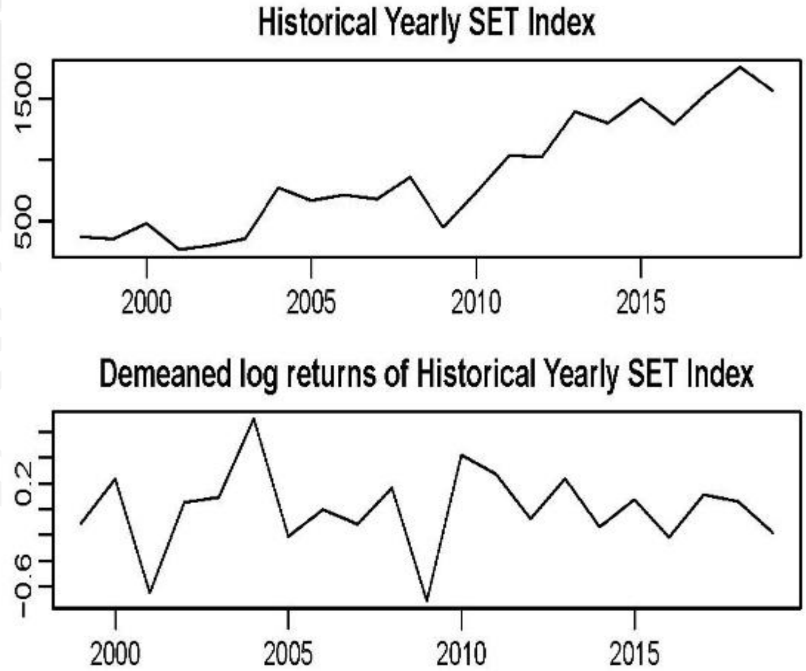
(b)


Historical Monthly SET Index
1500
500
2000
2005
2010
2015
Demeaned log returns of Historical Monthly SET Index
0.1
−0.3
2000
2005
2010
2015

(c)



(d)



(e)

Figure 2.1 Visualization of the Stock Exchange of Thailand (SET) Index (Top Graphs) and the Demeaned Returns (Bottom Graphs): (a) Daily Data, (b) Weekly Data, (c) Monthly Data, (d) Quarterly Data, and (e) Yearly Data.

In Figure 2.1, some volatility clusters can be observed in the demeaned log returns of each frequency, which reflect the behavior of the Thai stock market during the study period. High volatility clusters were found in 1997–2000 and 2008–2009 due to the Thai economic crisis and the hamburger crisis, respectively.

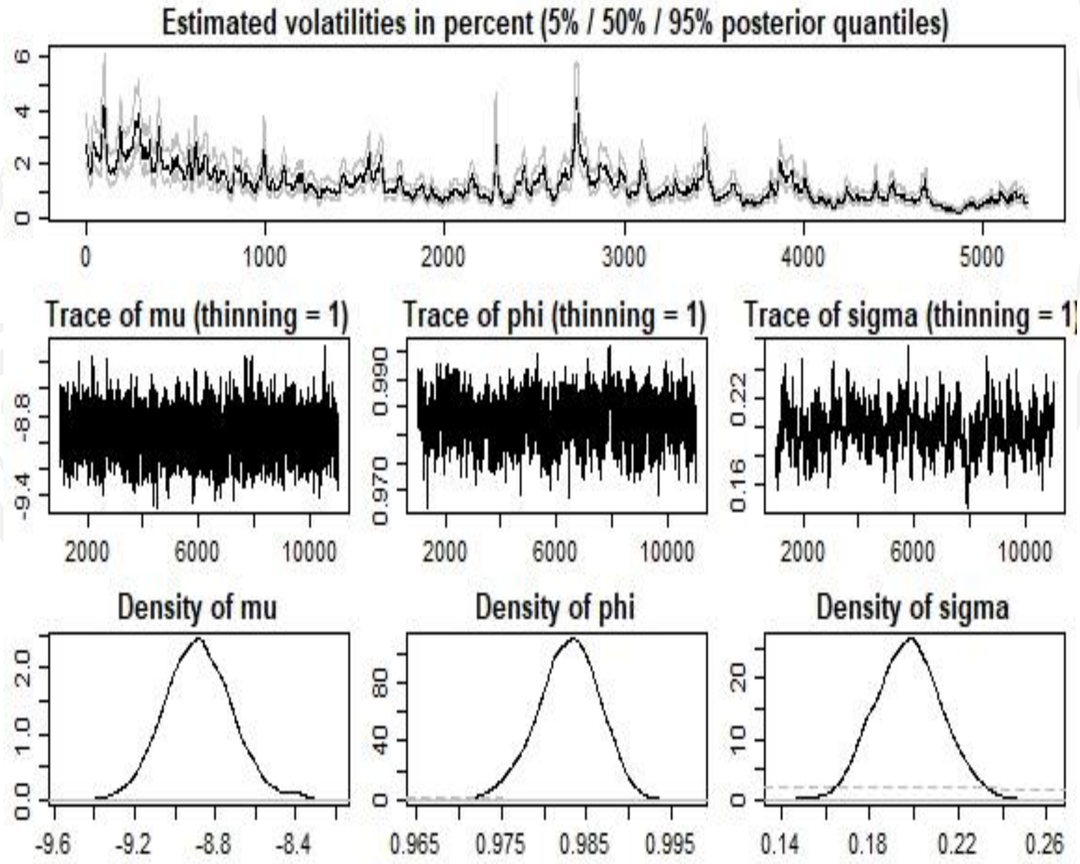
In terms of identifying the size of the burn-in and the number of iterations, the values for these parameters are 1,000 and 10,000, respectively, which should suffice to carry out the simulation. Three thinning parameters are also available, all of which are by the default of algorithm.

2.4.3 The Empirical Results of SV Model

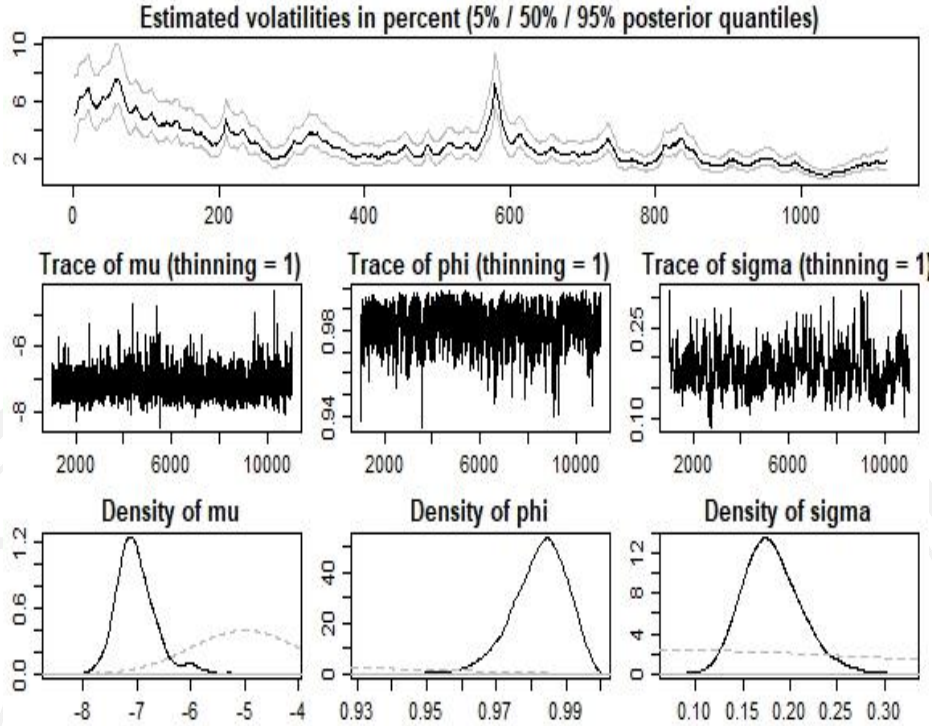
Appropriate identification of prior hyperparameters for the vector parameter θ distribution was necessary to run the simulation with configuration parameters. R programming was coded to run the Bayesian inference using the MCMC approach. The prior vector μ had a length of 2 with the mean and standard deviation of the normal prior to the level of the conditional variance μ. Prior c(–5,1) was chosen because previous studies have suggested that the likelihood normally brings enough information on this parameter. The prior vector ϕ was again specified as a vector of a length of 2 containing a0 and b0, as specified in Equation (2.2). The default was, at this point, given through c(20, 2.5), as it could not be set to be too small; awareness of this hyperparameter as a significant potential influence is crucial. The prior vector $\sigma\eta$ was used to control the prior variance of the conditional variance. The exact choice of 0.1 was used, as this value is usually not very influential in general use. For identification of the size of the burn-in and the number of iterations, the values of these parameters were set at 1000 and 10,000, respectively, which was considered sufficient to carry out the simulation. Three thinning parameters were available, all of which were given by default of the algorithm.

An observation of the collected samples revealed that the number of samples was reduced when their frequency increased. Subsequently, all samples were fitted to the demeaned stock returns using the stochastic volatility model given in Equation (2.1). We considered the log- AR stochastic volatility model, which is well-known and often applied, but some studies have used the exponential- AR stochastic volatility model for their applications as well.

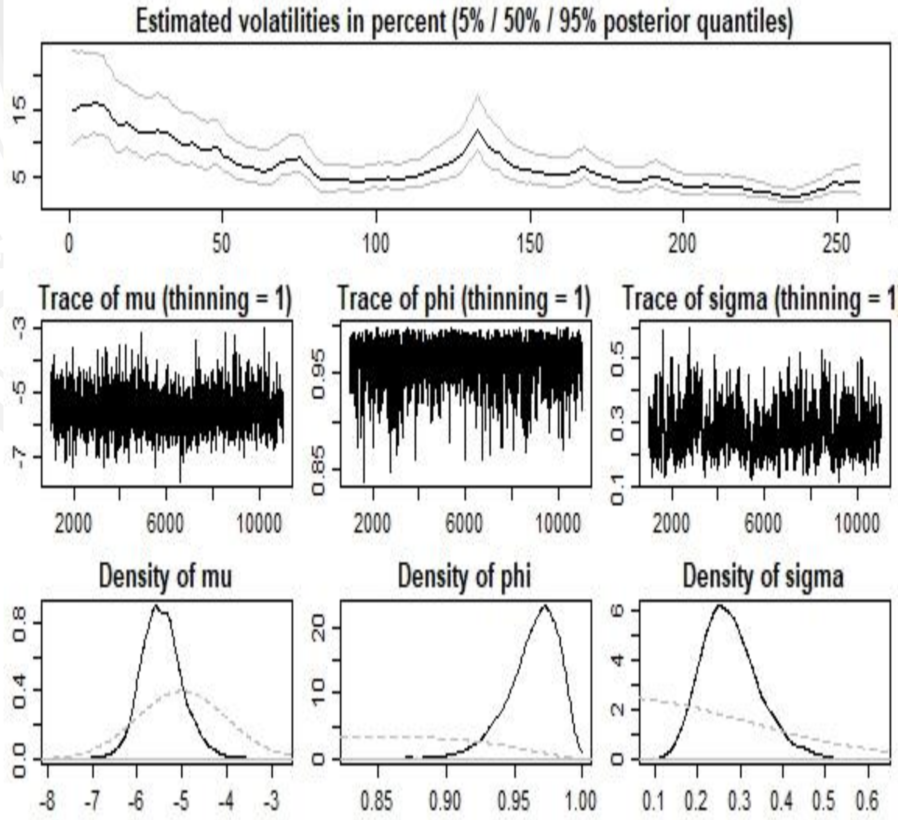
The visualization of the extracted stochastic volatility from the model is presented in Figures 2.1-2.3 in terms of daily, weekly, monthly, quarterly, and yearly frequencies. From Figure 2.2, stochastic volatility can be estimated by MCMC simulation. The results indicate that the model can extract better stochastic volatility with a higher frequency of data; the same is true for general volatility models. However, the stochastic volatility model performs better with a lower frequency of data as compared to other volatility models, such as GARCH (1,1). The in-sample and out-of-sample volatility forecasting of both models were compared to a proxy for true volatility, with the square of SET index returns using mean square error (MSE) were employed to examine accuracy. The details of the analysis are shown in Figure 2.3; conditional volatility from GARCH (1,1) was calculated using EViews software.



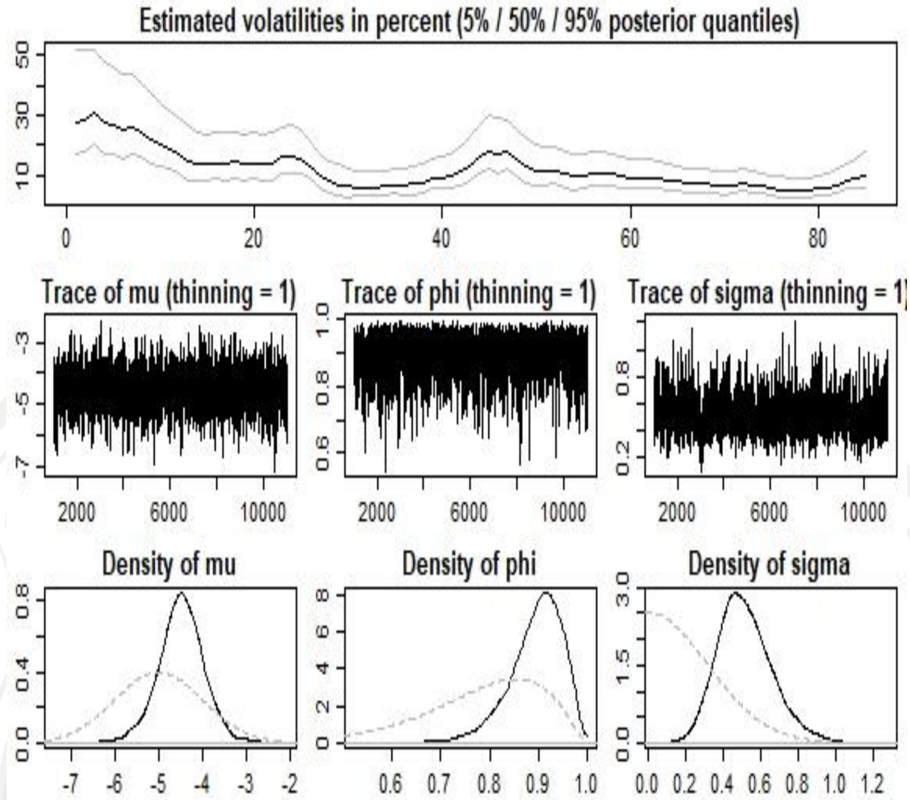
(a)


Estimated volatilities in percent (5% / 50% / 95% posterior quantiles)
Trace of mu (thinning = 1)
Trace of phi (thinning = 1)
Trace of sigma (thinning = 1)
Density of mu
Density of phi
Density of sigma

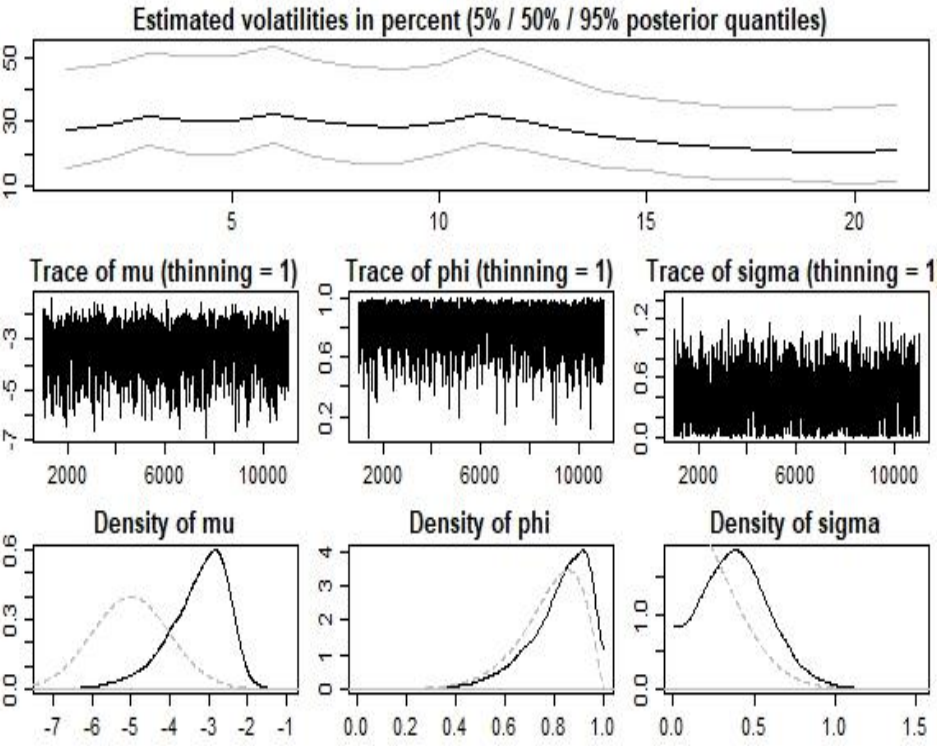
(b)


Estimated volatilities in percent (5% / 50% / 95% posterior quantiles)
Trace of mu (thinning = 1)
Trace of phi (thinning = 1)
Trace of sigma (thinning = 1)
Density of mu
Density of phi
Density of sigma

(c)


Estimated volatilities in percent (5% / 50% / 95% posterior quantiles)
Trace of mu (thinning = 1)
Trace of phi (thinning = 1)
Trace of sigma (thinning = 1)
Density of mu
Density of phi
Density of sigma

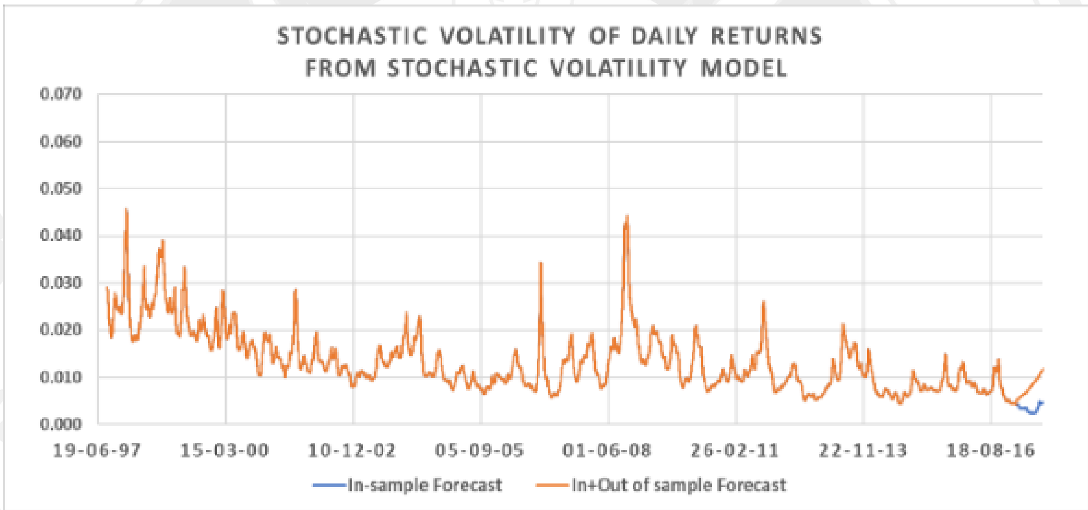
(d)


Estimated volatilities in percent (5% / 50% / 95% posterior quantiles)
Trace of mu (thinning = 1)
Trace of phi (thinning = 1)
Trace of sigma (thinning = 1)
Density of mu
Density of phi
Density of sigma

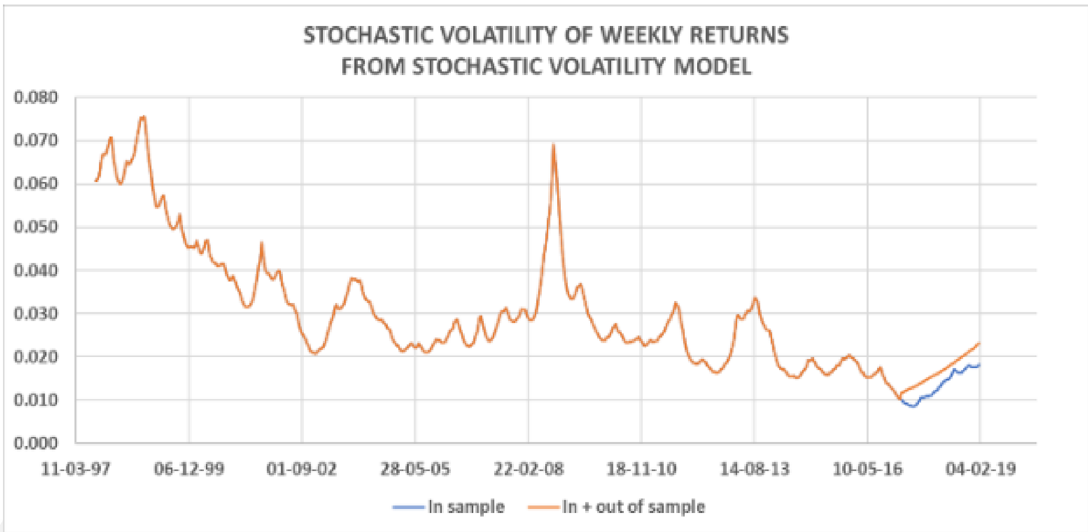
(e)

Figure 2.2 Visualization of the Estimated Stochastic Volatility of the SET Index, the Posterior Medians, and the 5% / 95% Quantiles (Top Graphs), of the Trace Plots of the Posterior Draws for Parameters μ, ϕ, and σ (Middle Graphs) where the Dotted Lines on the Right Indicate Predicted Future Volatility, and of the Kernel Density Estimate for the Parameters Contained in Parameters μ, ϕ, and σ (Bottom Graphs). (a) Daily Data, (b) Weekly Data, (c) Monthly Data, (d) Quarterly Data, and (e) Yearly Data.

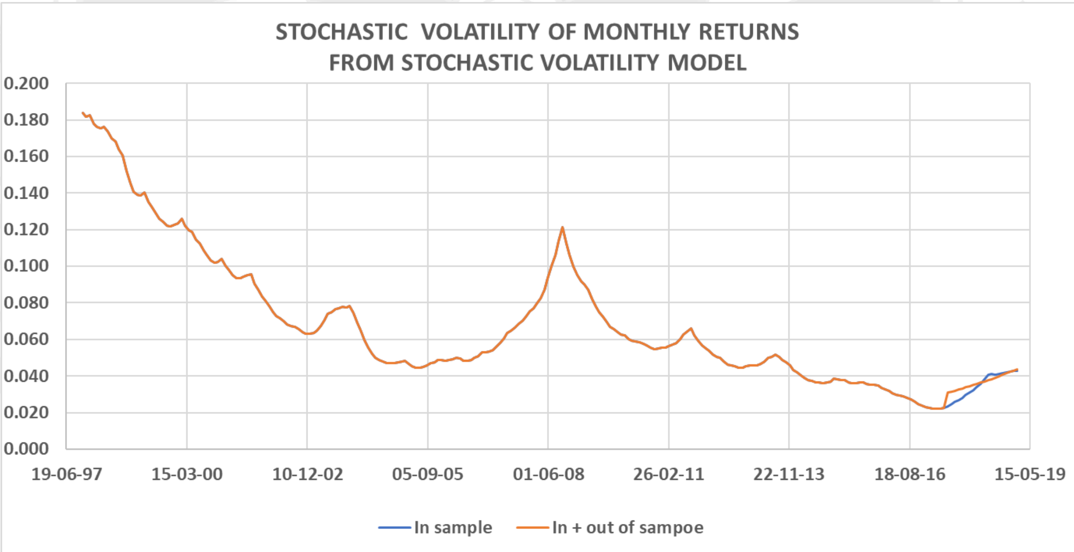
Panel (i)



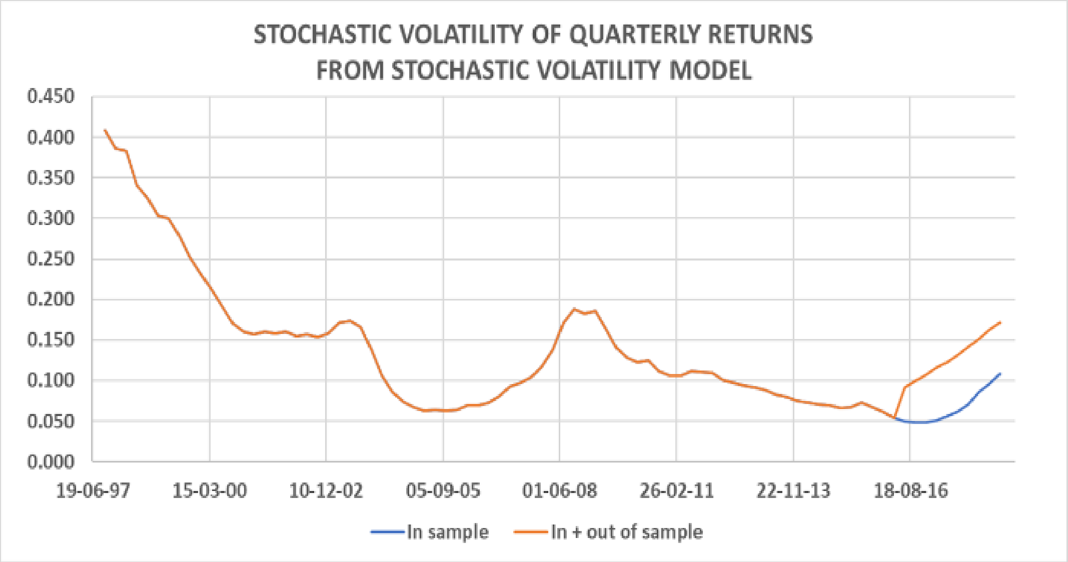
(a)


STOCHASTIC VOLATILITY OF WEEKLY RETURNS
FROM STOCHASTIC VOLATILITY MODEL
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
11-03-97
06-12-99
01-09-02
28-05-05
22-02-08
18-11-10
14-08-13
10-05-16
04-02-19
In sample
In + out of sample

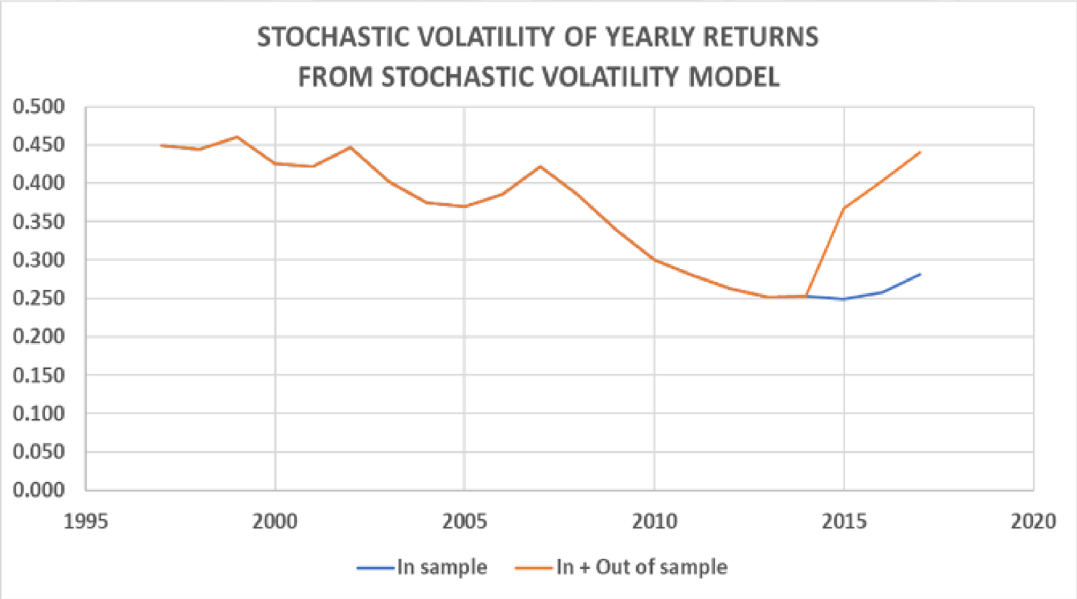
(b)


STOCHASTIC VOLATILITY OF MONTHLY RETURNS
FROM STOCHASTIC VOLATILITY MODEL
0.200
0.180
0.160
0.140
0.120
0.100
0.080
0.060
0.040
0.020
0.000
19-06-97
15-03-00
10-12-02
05-09-05
01-06-08
26-02-11
22-11-13
18-08-16
15-05-19
In sample
In + out of sampoe

(c)

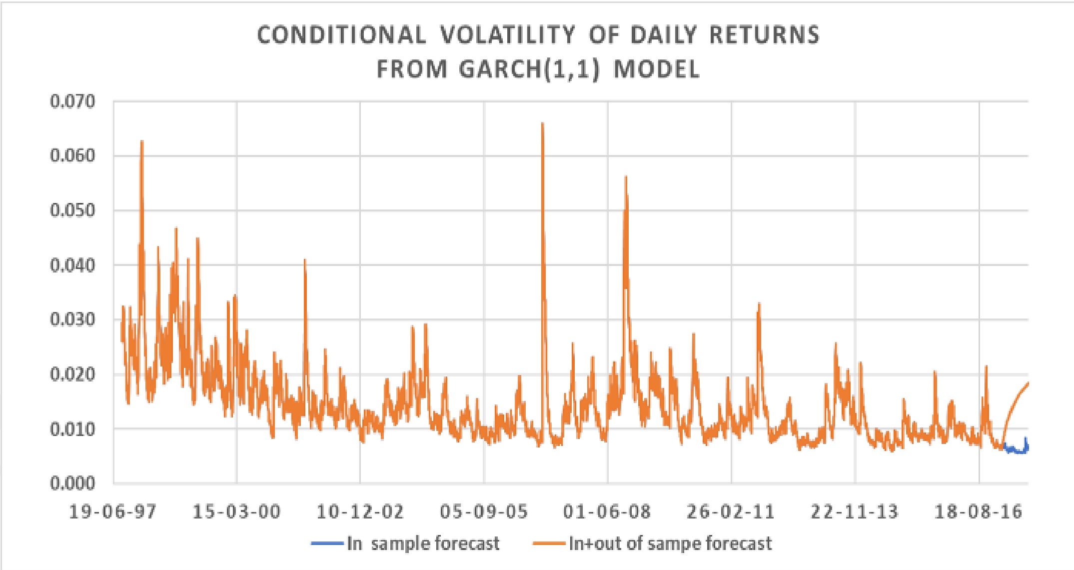

STOCHASTIC VOLATILITY OF QUARTERLY RETURNS
FROM STOCHASTIC VOLATILITY MODEL
0.450
0.400
0.350
0.300
0.250
0.200
0.150
0.100
0.050
0.000
19-06-97
15-03-00
10-12-02
05-09-05
01-06-08
26-02-11
22-11-13
18-08-16
In sample
In + out of sample

(d)

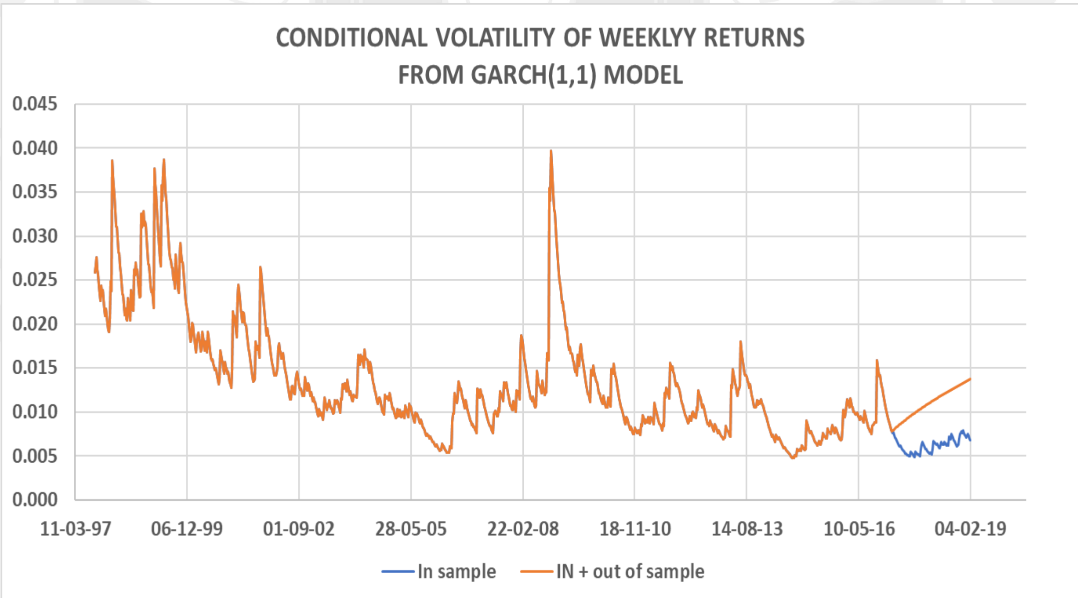

STOCHASTIC VOLATILITY OF YEARLY RETURNS
FROM STOCHASTIC VOLATILITY MODEL
0.500
0.450
0.400
0.350
0.300
0.250
0.200
0.150
0.100
0.050
0.000
1995
2000
2005
2010
2015
2020
In sample
In + Out of sample

(e)

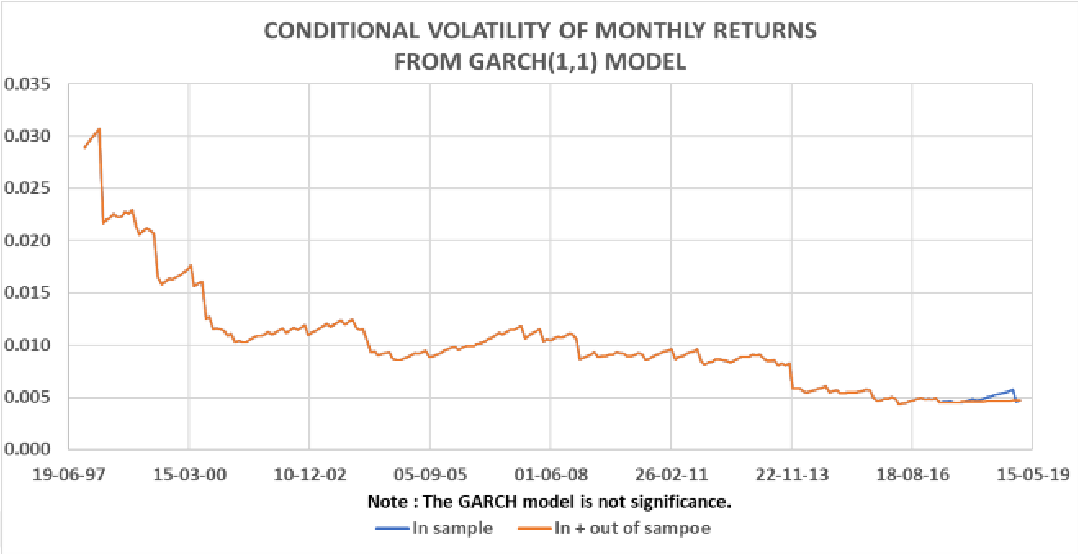
Panel (ii)


CONDITIONAL VOLATILITY OF DAILY RETURNS
FROM GARCH(1,1) MODEL
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
19-06-97
15-03-00
10-12-02
05-09-05
01-06-08
26-02-11
22-11-13
18-08-16
In sample forecast
In+out of sampe forecast

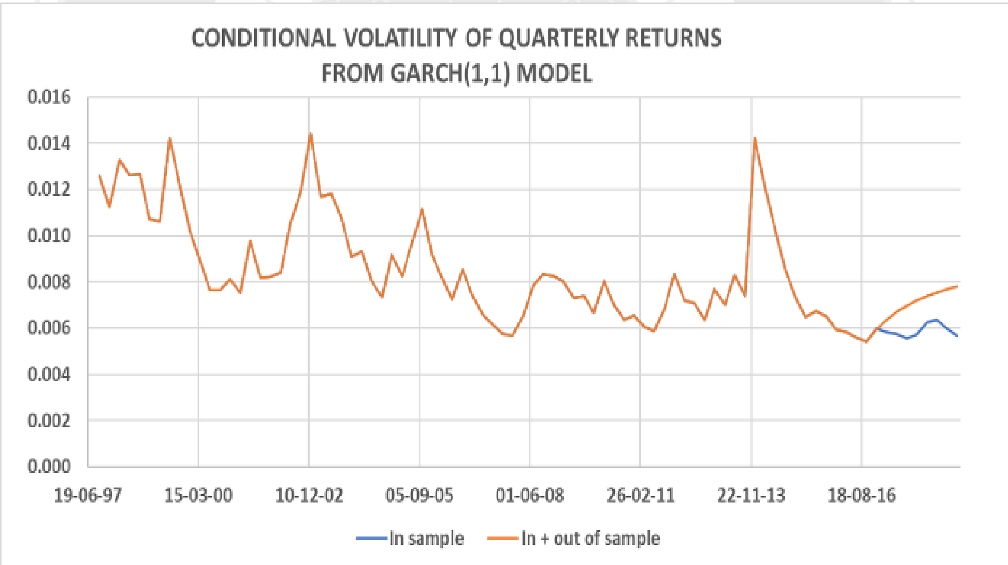
(a)


CONDITIONAL VOLATILITY OF WEEKLYY RETURNS
FROM GARCH(1,1) MODEL
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
11-03-97
06-12-99
01-09-02
28-05-05
22-02-08
18-11-10
14-08-13
10-05-16
04-02-19
In sample
IN + out of sample

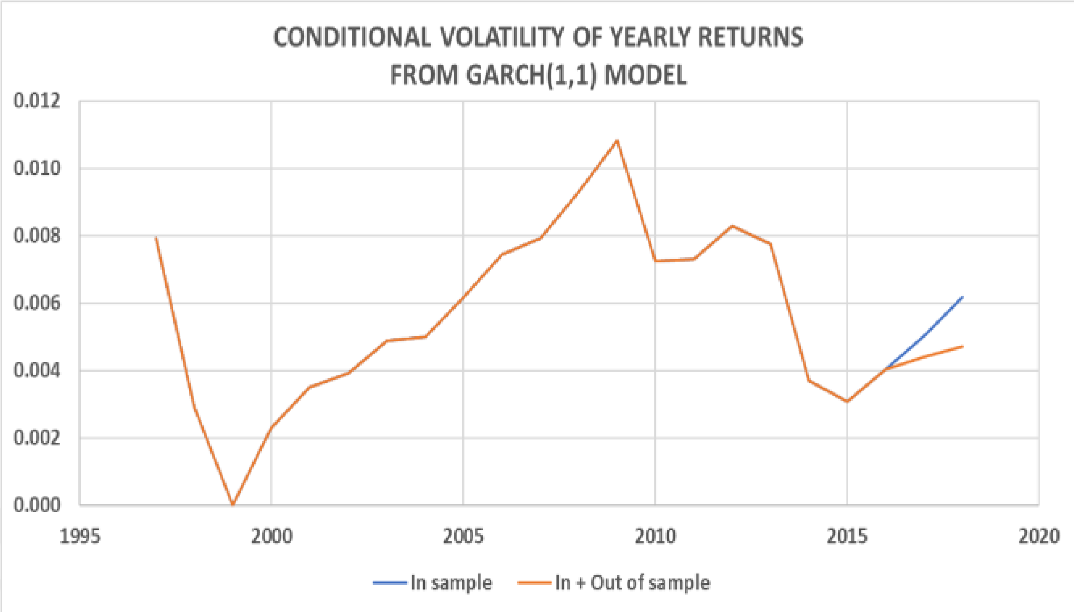
(b)


CONDITIONAL VOLATILITY OF MONTHLY RETURNS
FROM GARCH(1,1) MODEL
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
19-06-97
15-03-00
10-12-02
05-09-05
01-06-08
26-02-11
22-11-13
18-08-16
15-05-19
Note : The GARCH model is not significance.
In sample
In + out of sampoe

(c)


CONDITIONAL VOLATILITY OF QUARTERLY RETURNS
FROM GARCH(1,1) MODEL
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
19-06-97
15-03-00
10-12-02
05-09-05
01-06-08
26-02-11
22-11-13
18-08-16
In sample
In + out of sample

(d)



(e)

Figure 2.3 Visualization of the Extracted Volatility Comparing the Stochastic Volatility Model in Panel (i) and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH (1,1)) Model in Panel (ii). (a) Daily Data, (b) Weekly Data, (c) Monthly Data, (d) Quarterly Data, and (e) Yearly Data.

In Figure 2.3, the conditional volatility of the GARCH (1,1) plot shows more fluctuation than the stochastic volatility model due to the means of estimation. However, the peaks of both plots are at almost the same position across the sampling period, and most of the peaks from the stochastic volatility model seem to be lower than those from the GARCH (1,1) model. For the lower frequency data (i.e., monthly, quarterly, and yearly), the pattern of the volatility clusters is totally different in the stochastic volatility model, which is more accurate. For out-of-sample forecasting, as shown on the far right of each panel in Figure 2.3, the forecasting accuracy of the stochastic volatility model is lower than that of the GARCH (1,1) model if we use the square of return as the proxy for volatility for comparison with the forecast numbers. This also corresponds with the concept of the GARCH method, which is a process of return squared, and contrasts with the work of Hsieh (1991) and Danielsson (1994). Finally, the results of the forecasting accuracy for all frequencies are summarized in Table 2.1, where the results are also replicated for other sample frequencies.

Table 2.1 Summary of the Power of the Forecasting Accuracy of Conditional Volatility.

Frequency	In-Sample	Out-of-Sample	Total	Mean Square Error (MSE)	
				Stochastic Volatility Model	GARCH(1,1)
Daily	4771	475	5246	1.4459×10^{-6}	1.1913×10^{-7}
Weekly	1018	100	1118	8.4628×10^{-8}	1.1581×10^{-8}
Monthly	233	25	258	1.5524×10^{-6}	N/A [1]
Quarterly	76	10	86	3.4957×10^{-4}	N/A [1]
Yearly	19	3	22	2.7394×10^{-2}	N/A [1]

[1] The GARCH(1,1) model was not statistically significant, and thus it could not forecast volatility.

As shown in Table 2.1, the forecasting accuracy of the SV model is better than the GARCH (1,1) model for model forecasting for daily, weekly, and monthly returns data. On the other hand, the quarterly and yearly data contrasted.

Table 2.2 Summary of the Persistence Parameter of Volatility (ϕ)

Frequency	Total sample	Persistence parameter of volatility (ϕ) of SV model	
		Simulation summary	Calculation
Daily	5246	0.983	0.999**
Weekly	1118	0.981	0.999**
Monthly	258	0.962	0.999**
Quarterly	86	0.890	0.999**
Yearly	22	0.810	0.999**

The above table shows that the study of time change seem to carry out in the next stage to relieve the high persistence of volatility, then follows with the stocks returns prediction and testing in the next after chapter as well.

2.5 Conclusion

Stochastic volatility is one of the most useful ways to estimate unobserved conditional volatility, which has been important in financial economics for decades. This model aims to understand instantaneous volatility and to carry out better asset pricing and investment in financial markets by utilizing a structure for predictive return distribution and volatility forecasting rather than by directly using past returns, as a GARCH-type model does. It is important to note that knowledge of probability theory and stochastic processes is required to understand this approach. Estimation of the stochastic volatility model using simulation-based inference has been common in the industry for more than two decades, and so a Bayesian estimation with the MCMC approach was used, with R programing calculation included, in this study. Even though conditional volatility models like GARCH (1,1) models provide better forecasting accuracy than stochastic volatility given high frequency data (daily and monthly data in particular), it is not sufficient in terms of modeling low frequency data (monthly, quarterly, and yearly). Hence, the stochastic volatility models provide a viable alternative method to modeling and forecasting volatility in those cases. The results support the application of stochastic volatility to volatility modeling and forecasting, which are crucial parts of risk management and derivative pricing. One key observation from this study is that the persistence parameter of the volatility (ϕ) of the model is quite close to the unit root, and will affect stock returns' predictability as detailed in Table 2.2.

CHAPTER 3

ECONOMETRIC ISSUES AND THE TIME CHANGE METHOD FOR RETURN PREDICTION

This chapter reviews the stock returns' predictability and introduces the CJP approach to remedy a major econometrics issue. It will focus on the major recognized research on return prediction based on predictive regression with the near-unit root property of covariate and econometric analysis thereof.

3.1 Return Prediction Models

Many empirical studies of financial economics have focused on return prediction; one of the most recognized is Stambaugh (1999), look at predictive regression, in which he determined:

$$y_i = \alpha + \beta x_{i-1} + u_i \tag{3.1}$$

for $i = 1, 2, 3, \ldots, N$, where (y_i) is a column vector of the stock return and (x_i) is a column vector of some predictive variable with conceivable prediction power. The study of the regression of a return on a lagged stochastic regressor shows strong evidence that the regression disturbance is correlated with the innovation of the regressor. PÁStor and Stambaugh (2009) developed a new predictive system that allows the predictor to be imperfectly correlated with the conditional return. The results from Equation (3.1) show that a stock return is predictable whenever α and β are statistically significant. The most recognized predictive variables are dividend-price ratio and earning-price ratio, as per Campbell and Shiller (1988) and Goyal and Welch (2003). For the sake of simplicity, we allow (x_i) be a univariate time series across this study.

In most of the literature, β is the slope parameter of the regression equation in Equation (3.1) and is estimated by the ordinary least square method, resulting in $\hat{\beta}$ as an estimator. Hypothesis testing has also been applied to check for statistical significance based on a t-test denoted by $\tau(\hat{\beta}_n)$. We get:

$$\tau(\hat{\beta}_N) \longrightarrow_d \mathbb{N}(0,1) \tag{3.2}$$

when $N \longrightarrow \infty$ under the basic assumption of regression. Significant amounts of research in the literature mention that certain data descriptions may cause the abnormal distribution of the standard t-statistic, resulting in a substantial bias to the hypothesis testing critical value and an estimator of greater than one (Dickey & Fuller, 1979; White, 1958). The details of these studies have been justified as the following details.

3.1.1 Predictors' Persistence and Endogeneity

The predictive variables(x_i) normally expressed in the predictive regression have a high level of persistence, as mentioned in Goyal and Welch (2003), Torous, Valkanov, and Yan (2004), and Choi et al. (2016). In term of modelling, the autoregression model is always adapted such that the autoregressive parameter is either near explosion or close to one, which is called a local-to-unity process. Let Equation (3.3) become a local-to-unity process such that:

$$x_i = \left(1 - \frac{c}{N}\right) x_{i-1} + v_i \tag{3.3}$$

for some $c \geq 0$. In the sequel, let x_0 be equal to 0 and (v_i) be a martingale difference sequence with respect to an increasing sequence of filtration $(\mathscr{F}_i)$ which, following the two assumptions, means:

$$\begin{aligned}
&1.\ \sum_{i=1}^{N} E(v_i^2 \,|\, \mathscr{F}_{i-1}) \longrightarrow_p 1 \text{ as } N \longrightarrow \infty \text{ and} \\
&2.\ \frac{1}{N}\sum_{i=1}^{N} E\left(v_i^2 I_{(|v_i| > N^{\frac{1}{2}}\epsilon)} \middle| \mathscr{F}_{i-1}\right) \longrightarrow_p 0 \text{ as } N \longrightarrow \infty
\end{aligned} \tag{3.4}$$

for all $\epsilon > 0$.

The limit distribution of Equation (3.3) can be considered in several cases. For $\delta = \left|1 - \frac{c}{N}\right| < 1$ (stationary regime), Mann and Wald (1943) showed that:

$$\tau(\hat{\delta}_N) = \left(\sum_{i=1}^{N} x_{i-1}^2\right)^{\frac{1}{2}} (\hat{\delta}_N - \delta) \longrightarrow_d \mathbb{N}(0,1) \tag{3.5}$$

when $N \longrightarrow \infty$ and $\mathbb{N}(0,1)$ is a standard normal random variable. For the explosive case, $\delta = \left|1 - \frac{c}{N}\right| > 1$, Equation (3.5) holds only v_i as a sequence of independent standard normal random variables; however, in most cases of v_i such that $|\delta| > 1$, the limit distribution of $\tau(\hat{\delta}_N)$ may not exist, as mentioned in Anderson (1959). However, when $\delta = \left|1 - \frac{c}{N}\right| = 1$, Equation (3.7) is no longer applicable, even if the random variable v_i is independent standard normally distributed; White (1958) and Rao (1978) showed that:

$$\tau(\hat{\delta}_N) \longrightarrow_d \frac{1}{2}(W^2(1) - 1)\left(\int_0^1 W^2(t)\,dt\right)^{-\frac{1}{2}}, \tag{3.6}$$

where $(W(t): 0 \leq t \leq 1)$ is a Weiner process. For the case of $\delta = 1 - \frac{c}{N}$ lies in a neighborhood of one, Chan and Wei (1987) showed that:

$$\tau(\hat{\delta}_N) \longrightarrow_d \int_0^1 (1 + ht)^{-1} W(t) dW(t) \left(\int_0^1 (1 + ht)^{-2} W^2(t)\,dt\right)^{-\frac{1}{2}} \tag{3.7}$$

as $N \longrightarrow \infty$ and $h = e^{-2c} - 1$ where $(W(t): 0 \leq t \leq 1)$ is again pa Weiner process. In practice, $(\frac{c}{N})$ must be small so that δ stays near one.

In conjunction with the predictive regression in Equation (3.1), Choi et al. (2016) proposed that $(\xi_i)_{i>1}$ be a random variable with an independently and identically distributed (i.i.d.) sequence of mean zero. The asymptotic behavior of

$\sum_{i=1}^{n}(\xi_i)$ is normalized by some of the deterministic sequence (b_n) when ξ_i is integrable. The standard choice of $b_n = \sqrt{n}$ is used to obtain central limit theorems and invariance principles in a vector Brownian motion in terms of the continuous function $\mathbb{C}(0,1)$. Let Z_n be a normalization of a partial sum process characterized as $Z_n(r) = N^{-\frac{1}{2}}\sum_{i=1}^{[Nr]}\xi_i$ for $r \in [0,1]$ where $[z]$ is the integer part of any real number z. From this point, the invariance principle holds for (ξ_i) if $Z_N \longrightarrow_d \textit{Brownian motion process}$ on [0,1] as $N \longrightarrow \infty$ when the limit distribution of the summation of independent random variables does not depend on the distribution of random variables in certain conditions. The long run covariance of (ξ_i) will then be the asymptotic vector for the Brownian motion of the variance–covariance matrix. Indeed, they let $\xi_i = (u_i, v_i)'$, which holds the invariance principle with the bivariate limit Brownian motion $B = (U, V)'$ when the variance covariance matrix is given by:

$$\Omega = \begin{pmatrix} \omega_u^2 & \omega_{uv} \\ \omega_{vu} & \omega_v^2 \end{pmatrix}, \tag{3.8}$$

where (u_i) is the sequence of martingale difference.

Eventually, we can also get $\frac{\sum_{i=1}^{N} u_i^2}{N} \longrightarrow_p \omega_u^2$ under mild conditions as result of probability convergence. The asymptotic distribution of $\tau(\hat{\beta}_n)$ can then be deduced and is given by:

$$\tau(\hat{\beta}_N) \longrightarrow_d \frac{1}{\omega_u}\left(\int_0^1 V_c(r)^2\, dr\right)^{-\frac{1}{2}} \int_0^1 V_c(r) dU(r) \tag{3.9}$$

as $N \longrightarrow \infty$ where $V_c(r)$ defines an Ornstein-Uhlenbeck process or the mean reversion process driven by the limited Brownian motion of V where $V_c(r)$ is defined as a solution to the stochastic differential equation $dV_c(r) = -cV_c(r)dr + dV(r)$. In the same study, Choi et al. also introduced the relationship between the limited Brownian motion of U and V such that:

$$W = U - \frac{\omega_{uv}}{\omega_v^2} V, \tag{3.10}$$

and they get $U = \frac{\omega_{uv}}{\omega_v^2} V + W$ by construction of a Brownian motion independent of V. The asymptotic distribution of Equation (3.9) may then be written as the cumulative probability of:

$$P = \frac{\omega_{uv}}{\omega_u \omega_v^2} \left(\int_0^1 V_c(r)^2 \, dr \right)^{-\frac{1}{2}} \int_0^1 V_c(r) dV(r) \tag{3.11}$$

$$Q = \frac{1}{\omega_u} \left(\int_0^1 V_c(r)^2 \, dr \right)^{-\frac{1}{2}} \int_0^1 V_c(r) dW(r) \tag{3.12}$$

At this point, the limit distribution of Q is normal due to the independence of V and W. The limit distribution of P is essentially that of the t-ratio of the autoregressive coefficient in the local-to-unity model proposed by Phillips (1987). The actual limit distribution of $\tau(\hat{\beta}_N)$ is a mixture of normal distribution and local-to-unity distribution with the ratio determined by the long-run correlation coefficient $\rho_{uv} = \omega_{uv}/\omega_u \omega_v$ of (u_i) and (v_i). Indeed, it can be deduced that $\tau(\hat{\beta}_N) \longrightarrow_d \mathbb{N}(0,1)$ if $\rho_{uv}^2 \longrightarrow 0$. As $\rho_{uv}^2 \longrightarrow 1$, the limit distribution of $\tau(\hat{\beta}_N)$ moves away from standard normal distribution in the long run.

For these reasons, the asymptotic distribution of test statistics seriously depends on the parameter c, which performs an inconsistence estimator as $\frac{c}{N}$ always gets smaller when the sample size N increases.

Campbell and Yogo (2006) extended their study and found that, if there is a perfect cointegration in the predictive variable and a perfect long-run correlation between the innovation of the predictive variable (v_i) and the regression errors(u_i), the limit size of the one-sided t-test at 95% confidence is as large as 46% from their simulation results; these results did not deviate much from their empirical study. Indeed, well-known predictors, such as dividend-price ratio and price-earnings ratio, may be rejected in a unit root test, but these predictors are highly persistent, meaning that an autoregressive parameter is close to unity. Furthermore, stock returns always correlate with predictor innovations in the long run.

3.1.2 Nonstationary Stochastic Volatility in Returns

Choi et al. (2016) reported that stock returns (y_i) are broadly assumed to have time-varying stochastic volatility that resolves many econometric issues when following the usual specification of the volatility model such that:

$$u_i = \sigma_{i-1}\varepsilon_i, \tag{3.13}$$

where (ε_i) is the martingale difference with respect to filtration $(\mathscr{F}_i)$ that will result in $\mathrm{E}(\varepsilon_i^2 \big| \mathscr{F}_{i-1}) = 1$ for all $i \geq 1$. From the basic martingale concept, they got $\mathrm{E}(u_i^2 \big| \mathscr{F}_{i-1}) = \sigma_{i-1}$, which means that σ_{i-1} tends to be the conditional variance of u_i given the information at time $i-1, i \geq 1$. Jacquier et al. (2004) contributed the notion that the logarithmic of the volatility process takes the place of a local-to-unity process for a variety of stock and foreign exchange rate returns data. The autoregressive parameter of the stochastic volatility process then becomes close to one under some suitable functional transformations. This finding is in line with those of Geweke (1994) and Gallant, Hsieh, and Tauchen (1997). In addition, Choi et al. (2016) concluded that the exact stochastic volatility process has a high chance of getting a unit root since it is the exponential of a local-to-unity process. They then define the conditioning volatility fiction necessary to contain this nonstationary process as follows:

$$\sigma_i = \varpi(z_i) \tag{3.14}$$

where $\varpi: \mathbb{R} \longrightarrow \mathbb{R}_+$ and z_i is some local-to-unity process defined as:

$$z_i = \left(1 - \frac{c}{N}\right) z_{i-1} + w_i \,. \tag{3.15}$$

For some $c \geq 0$, with ϖ assumed as having asymptotic homogeneity as well as being a deterministic or stochastic function independent of another randomness model, as also reported by Cavaliere (2004) and Cavaliere and Taylor (2007). At that point, Choi et al. (2016) proposed the volatility model specification given by:

$$\sigma_i = \varpi\left(\frac{z_i}{\sqrt{N}}\right) \tag{3.16}$$

The limit distribution of $\tau(\hat{\beta}_N)$ under the specifications of the above volatility model can be obtained from the results based on Equation (3.16) and can be easily obtained from our results using Equation (3.14). Chung and Park (2007) developed a theory of predictive regression in Equation (3.1) with the disturbance term (u_i), which is specified in Equations (3.13) and (3.14); Choi et al. (2016) extended this knowledge by using (v_i), (ε_i), and (w_i) in Equations (3.1), (3.13), and (3.15) and redefined $\xi_i = (\varepsilon_i, v_i, w_i)'$ based on the assumption of the invariance principle and the limit Brownian motion denoted by $B = (U, V, W)'$. They also defined W_c as the Ornstein-Uhlenbeck process with a mean reversion parameter of $c \geq 0$ such that:

$$\tau(\hat{\beta}_N) \rightarrow_d \frac{\int_0^1 V_c(r)\varpi\big(W_c(r)\big)dU(r)}{\left(\int_0^1 (\varpi(W_c(r))^2 dr\right)^{1/2}\left(\int_0^1 V_c(r)^2\, dr\right)^{1/2}}, \tag{3.17}$$

under the volatility model specified in Equation (3.16). The limit distribution of $\tau(\hat{\beta}_N)$ is not standard normal distribution, even though the innovations of the predictive variable (v_i) and innovations for the volatility factor (w_i) are independent of the innovation of errors (ε_i). Factors that affect this include the volatility function, the asymptotic covariance of innovation, and the near-unit root parameter.

3.1.3 Other Econometric Issues

Under the statistical shock of stock return data, the limit distribution of $\tau(\hat{\beta}_N)$ is not robust and depends on several key factors in the stochastic process, such as deterministic trends, heteroscedasticity in innovation, jumps, and structure breaks of time series data, as well as a variety issues from the predictors. These factors tend to the increase the amount of size distortion in the limit distribution of $\tau(\hat{\beta}_N)$.

The deterministic trends and structure breaks have the same effect on the asymptotic distribution of $\tau(\hat{\beta}_N)$. A small structure break in the mean offers some reason for the common characteristics of predictive regression, as reported by Lettau and Van Nieuwerburgh (2008). Additionally, Kim, Leybourne, and Newbold (2004) found that the structure break in the innovation's variance can extensively distort the standard normal distribution of the OLS t-ratio distribution in a prediction test. Recently, Choi et al. (2016) found that, if there is a deterministic trend or a structure break in the data set and the researchers apply a general type of critical value with no-trend or no-break, the inference is proved invalid.

Another factor that can affect the limit distribution of $\tau(\hat{\beta}_N)$ is a finite sample set. The sample should be relatively large enough for a good approximation of the limit distribution of stock returns prediction; previous research has attempted to connect finite sample distribution and limit distribution by using nonstationary processes to generate data for the predictive variable.

3.2 The CJP Approach Model

The CJP approach was proposed by Choi et al. (2016) to develop a new methodology to rectify the previously mentioned problems. This method is composed of two subroutines, including a time scale change, a type of nonparametric method, and a Cauchy estimation, which is a type of instrumental variable. The time change clearly accounts for the nonstationary stochastic volatility of stock returns while the Cauchy estimator clears up other econometric issues caused by the nonstationary and endogeneous aspects of the predictive variable. When combining both methods, the t-ratio of the Cauchy estimator and the time-changed data will converge to achieve standard normal distribution. In this chapter, the time change method will be reviewed and discussed, while the Cauchy estimator will be discussed in the next chapter.

3.2.1 The Change of Time Method: Volatility (Generated) Time

The key concept of the change of time method involves finding a simpler way to represent the stochastic process of a complicated structure by combining a simple stochastic process with a change of time, e.g., changing time over volatility. Swishchuk (2016) mentioned that this method was introduced to the stochastic process by Wolfgang Doeblin in 2000 and considers the analysis of path of a heterogeneous real-valued diffusion process thru the concept of the martingale. The implication of the change of time method in finance directly involves volatility, the measure for variation or fluctuation of the stock prices in the financial markets; volatility is particularly important now as so many financial markets are trying to use the Brownian motion with change of time to represent stock prices, which is called operating time or business time. Indeed, many stochastic differential equations can be solved by using the change of time method, including the Ornstein-Uhlenbeck process. The most typical example of the connection between the Brownian motion and stochastic volatility is presented as follows.

Let $M_t = \int_0^t \sigma(s,w)d(B_s,t) \geq 0$, where B_s is the Brownian motion, $\sigma(s,w)$ is a positive process of stochastic volatility, and M_t is a martingale such that $\int_0^t \sigma_s^2 ds < +\infty$. M_t is then able to be presented such that $M_t = \hat{B}_{T_t}$ where $T_t \coloneqq \int_0^t \sigma_s^2 ds$, $\hat{B}_t \coloneqq M_{\hat{T}_t}$, and $\hat{T}_t = \inf\{s \geq 0 \mid \int_0^t \sigma_s^2 ds > t\}$. This also denotes that $\hat{B}_t \coloneqq M_{\hat{T}_t}$ is a martingale with respect to filtration $\widehat{\mathscr{F}}_t \coloneqq \mathscr{F}_{\hat{T}_t}$.

Choi et al. (2016) also proposed applying this method to remedy the nonstationary stochastic volatility issue, which relies on the continuous time stochastic process. Let (Y_t), the sequence of logarithmic stock price, be a stochastic process in a continuous time with respect to filtration $(\mathscr{F}_t)$. The null hypothesis of the returns predictability with respect to filtration $(\mathscr{F}_t)$ is given by

$$\mathrm{E}(dY_t|\mathscr{F}_t) = 0 \tag{3.18}$$

which means that:

$$\mathrm{E}(Y_{t+h} - Y_t|\mathscr{F}_t) = 0 \tag{3.19}$$

for all t and $h > 0$. Note that $(\mathscr{F}_t)$ provides more information than is provided by the realized value of (Y_s) up to time $t > 0$ alone.

The continuous regression with a predictor X has been considered to test stock price returns predictability (dY). Therefore, we get:

$$dY_t = (\alpha + \beta X_t)dt + dU_t \,, \tag{3.20}$$

where both (X_t) and (U_t), a martingale, are assumed with respect to filtration $(\mathscr{F}_t)$. At this point, if the above continuous time regression is not predictable for dY with respect to filtration $(\mathscr{F}_t)$, it suggests that $\alpha = \beta = 0$. In conjunction with the proposed predictive regression in Equation (3.1), the new equation can be constructed as:

$$y_i = Y_{i\Delta} - Y_{(i-1)\Delta} \,, \tag{3.21}$$

for $i = 1, 2, \ldots, N$ with some choice of time intervals Δ and $T = N\Delta$. On the other hand, let $x_i = X_{(i-1)\Delta}$, and $u_i = U_{i\Delta} - U_{(i-1)\Delta}$ for $i = 1, 2, \ldots, N$.

To correspond with the change of time method concept, the quadratic variation $[Y]_t$ must be introduced and defined as:

$$[Y]_t = \operatorname*{plim}_{\lfloor \pi_t \rfloor \to 0} \sum_{i=1}^{m} (Y_{ti} - Y_{ti-1})^2 \,, \tag{3.22}$$

where $\lfloor \pi_t \rfloor$ is the mesh of partition $0 = t_0 < \cdots < t_m = t$ of the interval $[0, t]$. It is also well known that dt in Equation (3.20) represents the bounded component of dY and does not provide any information for $[Y]$ such that $[Y] = [U]$ where $[U]$ denotes the quadratic variation of a martingale U defined similarly to $[Y]$ in Equation (3.22). At this point, $[Y]$ is also independent from the values of α and β and U is assumed to be a continuous martingale. Following the theoretical approach of the change of time method and the Brownian motion, we get:

$$dU_t = \sigma_t dW_t \,, \tag{3.23}$$

where σ is a stochastic volatility process and W is a Wiener process (Dambis, 1965) and Dubins and Schwarz (1965) (the notation DDS will hereafter be used refer to Dambis, Dubins, and Schwarz) independently contributed one of the most well-defined theories on the change of time method by introducing the Brownian motion process B such that:

$$U_t = B_{[U]_t} = B_{[Y]_t} , \tag{3.24}$$

Or, equivalently, that:

$$U_{T_t} = B_t , \tag{3.25}$$

where is T is time change as defined by:

$$T_t = \inf\{s \geq 0 \,|[U]_s = [Y]_s > t\} \tag{3.26}$$

The Brownian motion B is also referred to as the DDS Brownian motion of U which give some sense of time deformation, a Brownian motion in continuous time, and it also contributes that all continuous martingales are a Brownian motion that refer to dealing with a different clock. Therefore, if $[U] = [Y]$ is a strictly increasing function then T is the time inverse from $[U] = [Y]$, which is defined as stopping time in a stochastic volatility process.

The concept of DDS Brownian motion is applied to deal with inconsistencies in the stochastic volatility of the predictive regression errors in Equation (3.1). Under the null hypothesis that stock returns are not predictable, it gets $u_i = y_i$ which is the same as the continuous time regression set up in Equation (3.20). The regression errors (u_i) become a sequence of martingale differences and, based on martingale property, it does not provide any additional information on the conditional and unconditional of higher moments.

Consider a sequence of stopping time:

$$0 = T_0 \le T_\Delta \le T_{2\Delta} \le \cdots \le T_{N\Delta} = T \tag{3.27}$$

for fixed Δ, and let $N = [Y]_T/\Delta$ such that:

$$y_i^* = Y_{T_{i\Delta}} - Y_{T_{(i-1)\Delta}} \tag{3.28}$$

for $i = 1, 2, \dots, N$, can be obtained. Furthermore, Equation (3.28) can lead to:

$$Y_{T_{i\Delta}} - Y_{T_{(i-1)\Delta}} = W_{i\Delta} - W_{(i-1)\Delta} =_d \mathbb{N}(0, \Delta) \tag{3.29}$$

where W is DDS Brownian motion of Y. Finally, let $x_{i-1}^* = X_{T_{(i-1)\Delta}}$ and the regression in Equation (3.1) can be compared with the following:

$$y_i^* = \alpha c_i^* + \beta x_{i-1}^* + u_i^* , \tag{3.30}$$

where $c_i^* = T_{i\Delta} - T_{(i-1)\Delta}$. The variable (c_i^*) is compulsory as it is a number between observation rather than a constant interval of regression in Equation (3.1).
The continuous time regression of Equation (3.21) is given by:

$$dY_{T_t} = \left(\alpha + \beta X_{T_t}\right) dT_t + dU_{T_t} = \left(\alpha + \beta X_{T_t}\right) dT_t + dB_t \tag{3.31}$$

which is comparable with Equation (3.20) for the discrete time predictive regression in Equation (3.1). It also well-known that Equation (3.20) is the predictive equation without time transformation. Following the assumption of the change of time method, σ is strictly positive, and indeed T is provided directly by the inverse of $[U]$ with $d[U]_t = \sigma_t^2 dt$, resulting in $\sigma_{T_t}^2 dT_t = dt$. Therefore, it follows that:

$$dY_{T_t} = \frac{\alpha + \beta X_{T_t}}{\sigma_{T_t}^2} dT_t + dB_t , \tag{3.32}$$

The predictive regression in volatility time again aligns with the continuous time predictive regression in Equation (3.20) in real time. Predictive regression in Equation (3.32) shows that stock returns are predictable by X/σ^2 in volatility time after real time is transformed into volatility time; it is also predictable by X in real time.

The problem of data containing various kinds of nonstationary volatility from the original predictive regression in Equation (3.1) has been remedied by using the newly proposed continuous time regression in Equation (3.32), which has generated errors that are independent normal under the null hypothesis. The stock returns data collected following the volatility time concept will be independent normal due to the DDS theorem if the logarithmic stock price follows a continuous martingale. For the sake of consistency, real time return predictability will be the focus here.

Choi et al. (2016) also suggested considering a more general martingale process such that U is given as:

$$dU_t = \sigma_t \left(dW_t + \int_{\mathbb{R}} x\Lambda(dt, dx) \right), \tag{3.33}$$

where σ and W are defined in Equation (3.26) and Λ is a Poisson random measure independent of σ and W. For $t > 0$ and $A \subset \mathbb{R}$ given, $\Lambda([0,t], A)$ defines a Poisson process representing the number of jumps with a size in a set A that occurs before time t. The specification of Y in Equation (3.19) includes general Levy process with jumps as well. Let

$$E\big(\Lambda(dt, dx)\big) = \lambda(dx)dt \tag{3.34}$$

where λ is the Levy measure associated with A. In particular, $\lambda(A)$ for $A \subset \mathbb{R}$ counts the expected number of jumps with size in A during a time of unit length. Equation (3.33) is then integrable and assumed to exist such that:

$$\int_{\mathbb{R}} \lambda(dx)dt \ = 0 \tag{3.35}$$

so U in Equation (3.33) becomes a martingale.

For the process Y with U in Equation (3.33), Choi et al. (2016) proposed its bipower variation:

$$\{Y\}_t = \operatorname*{plim}_{\lfloor\pi_t\rfloor\to 0} \sum_{i=2}^{m} \left|Y_{ti} - Y_{t(i-1)}\right|\left|Y_{t(i-1)} - Y_{t(i-2)}\right| \tag{3.36}$$

in place of its quadratic variation $[Y]$ in Equation (3.22) to obtain the required time change where $\lfloor\pi_t\rfloor$ is defined in Equations (3.22). In general, jump processes have vanishing bipower variation such that:

$$d\{Y\}_t = \frac{2}{\pi}\sigma_t^2 dt\ , \tag{3.37}$$

for U defined in Equation (3.33), as shown by Barndorff-Nielsen and Shephard (2004). The time change T is then defined from $\{Y\}$ such that:

$$T_t = \inf\{s \geq 0\ |\{U\}_s = \{Y\}_s > t\}\ , \tag{3.38}$$

reduces the continuous martingale part of U to a Brownian motion. The time change corrects for nonstationary factors in the volatility process for the jump martingale part of $U,$ since its instantaneous variance is proportional to $\{U\} = \{Y\}$.

3.3 Empirical Results of the Time Change Method and Its Effect on Stochastic Volatility

As discussed in Chapter 3, the goal of the change of time method is to find a new, simpler representation for the stochastic process of a complicated structure by combining simple stochastic processes with a change of time. To visualize this, the data from Chapter 2 has been further analyzed using the time change method to improve the complicated structure of the unobserved conditional volatility; the effect on return predictability after using the time change method has also been analyzed.

3.3.1 Data Transformation Method

The quadratic variation $[Y]$ must first be calculated as the initial step. The estimate for $[Y]$ is:

$$[Y]_t^{\delta} = \sum\nolimits_{i\delta<t} \left(Y_{i\delta} - Y_{(i-1)\delta}\right)^2 \tag{3.39}$$

which is referred to as the realized variance or the realized bipower variation.

The consequence step, the time change, or the volatility time must be calculated such that:

$$T_t^{\delta} = \inf\left\{s \geq 0 \,\middle|\, [Y]_s^{\delta} > t\right\} \tag{3.40}$$

For a fixed $\Delta > 0$, the new $y_i^{*\delta}$ and $c_i^{*\delta}$ must be defined such that:

$$y_i^{*\delta} = Y_{T_{i\Delta}^{\delta}} - Y_{T_{(i-1)\Delta}^{\delta}} \tag{3.41}$$

$$c_i^{*\delta} = T_{i\Delta}^{\delta} - T_{(i-1)\Delta}^{\delta} \tag{3.42}$$

and $x_{i-1}^{*\delta} = X_{T_{(i-1)\Delta}^{\delta}}$, the predictive regression in Equation (3.30), can be:

$$y_i^{*\delta} = \alpha c_i^{*\delta} + \beta x_{i-1}^{*\delta} + u_i^{*\delta}, \tag{3.43}$$

3.3.2 Empirical Results

As an further step of the chaos stochastic volatility structure in SET data, the time change method have been applied to adjust the classical calendar time, will be referred as FT, become the new random time scale, will be referred as RT, based on the collection of quadratic variation adjustment per instruction in early of this chapter. Each frequency of the collected data has been analyzed to get the stochastic volatility of RT as compared to the stochastic volatility from the FT time scale. The details on the data used in the empirical analysis are also provided.

1) Data Description

The input of the demeaned stock returns data (y_t) is calculated by the difference in logarithmic of SET index. Data patterns during the study period are presented in Figure 3.1. Some volatility clusters can be observed in the demeaned log returns of the daily data, reflecting the behavior of the Thai stock market during the study period. 5242 total data points are used in this study. We again consider a series of stock returns; the returns on the SET value-weighted index were obtained from the Stock Exchange of Thailand from 1 September 1997 to 31 January 2019 and are available in daily, weekly, monthly, quarterly, and yearly frequencies, which compound daily returns. The demeaned returns for the SET index at each frequency are computed by deducting the average of return at the same frequency. Visualization of the SET index and returns is presented in Figure 3.1. The top panel is a series of SET indices while the other is a series of returns.

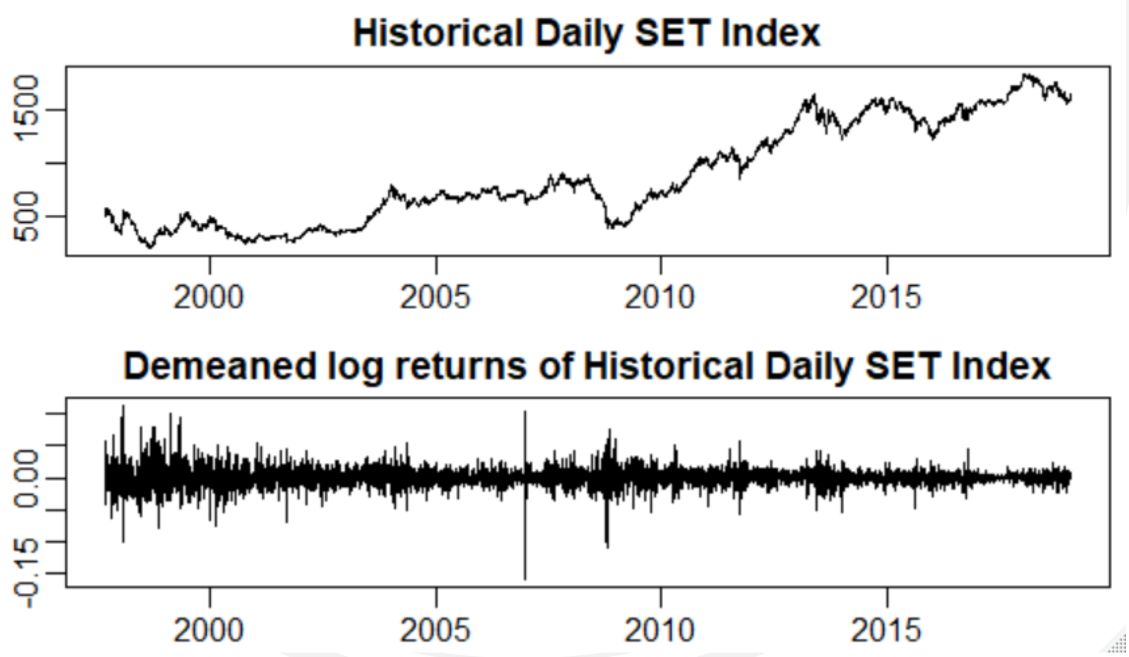


Figure 3.1 Visualization of SET Index and Its Demeaned Return Volatility Persistence was Calculated and is Summarized in the Following Table.

Table 3.1 The Summary of the Persistence Parameter of Stochastic Volatility

Frequency	**Total sample**	**Persistence parameter of stochastic volatility (ϕ)**	
		Simulation summary	**Calculation**
Daily	5246	0.983	0.9994**
Weekly	1118	0.981	0.9991**
Monthly	258	0.962	0.9995**
Quarterly	86	0.890	0.9963**
Yearly	22	0.810	0.9991**

The calculation results in Table 3.1 indicate that the stochastic volatility of all data frequencies have local-to-unity properties, which may affect the test of the null hypothesis of Equation (3.1). The time change method must therefore be utilized to compensate in the next step.

2) Effect of Time Change on Stochastic Volatility

To analyze the impact of changes in the time scale on the unobserved conditional volatility of the stock returns, finding the volatility time from quadratic variation using Equations (3.39) and (3.40) and then transforming the covariates from the chronological time scale to the new volatility time scale is essential. The stock returns data from the Thai stock exchange database was used and the collection of weekly, monthly, quarterly, and yearly FT data has been considered according to the chronological time scale time interval. For the samples collected in the stochastic time interval, we set the level of increment in each sampling interval to be the approximate average for weekly, monthly, quarterly, and yearly integrated volatility, as measured by the summation of quadratic variation, to make both the sample interval plans equivalent in terms of the number of data points. Again, we used the stock returns (y_i) to run the SV model with the key parameters mentioned in 2.2.2. The daily frequency of RT cannot be obtained as the number of integrated volatilities is too small.

The results of time changing method have been summary in Table 3.2.

Table 3.2 The Summary of the Persistence Parameter of Random Time Stochastic Volatility

Frequency	**Total sample**	**Persistence parameter of stochastic volatility (ϕ)**	
		Simulation summary	**Calculation**
Daily	5246	N/A	N/A
Weekly	1118	0.894	0.9954**
Monthly	258	0.751	0.9488**
Quarterly	86	0.756	0.9277**
Yearly	22	0.783	0.7770**

As shown in Table 3.2, the persistence coefficient of RT stochastic volatility is obviously reduced as compared to that of the FT in Table 3.1. The change of time method clearly demonstrates its effectiveness in terms of adjusting the level of persistence in stochastic volatility from a local-to-unity level to a normal one. A visual comparison of the stochastic volatility between the chronological time scale and the volatility time scale is presented in Figures 3.2 to 3.5.

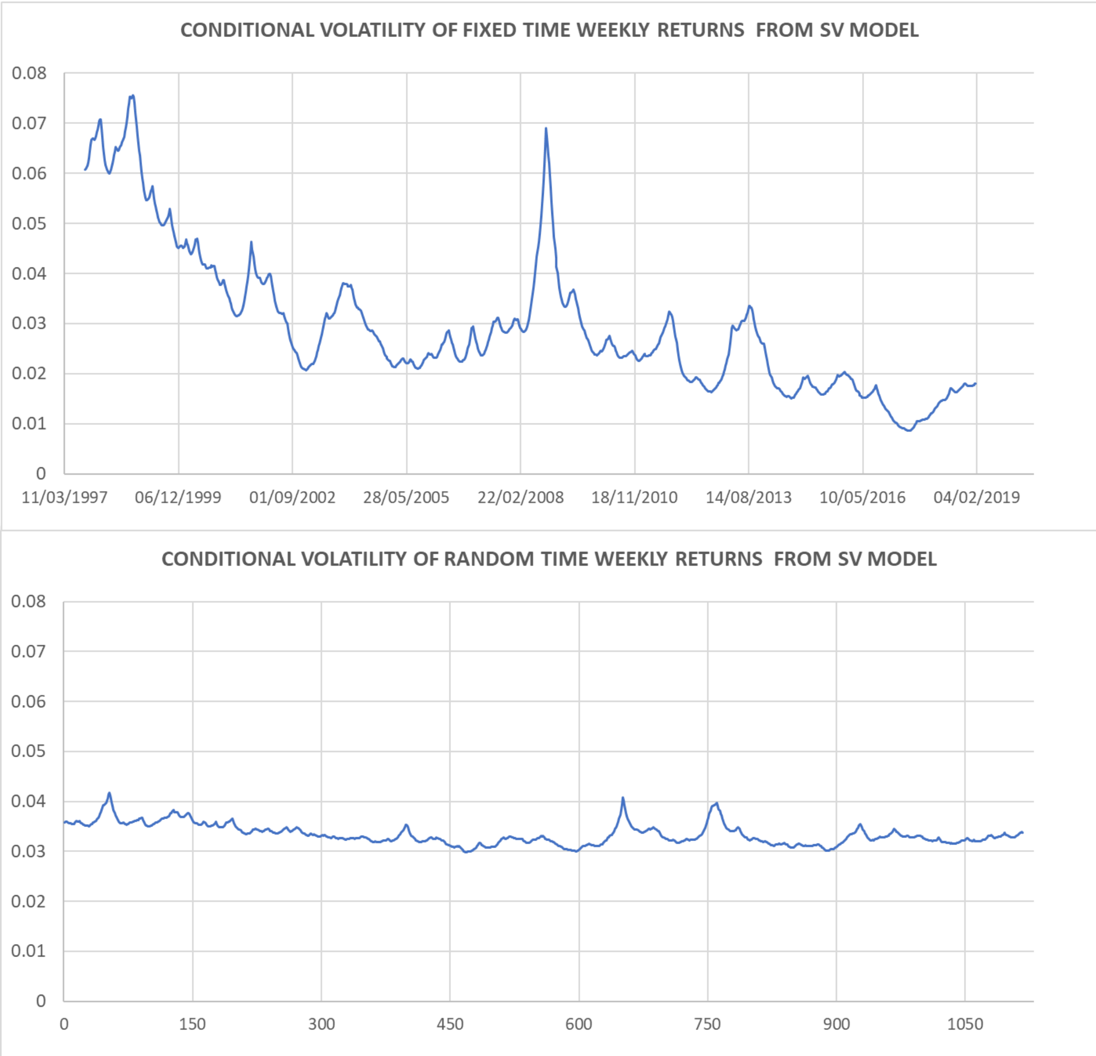
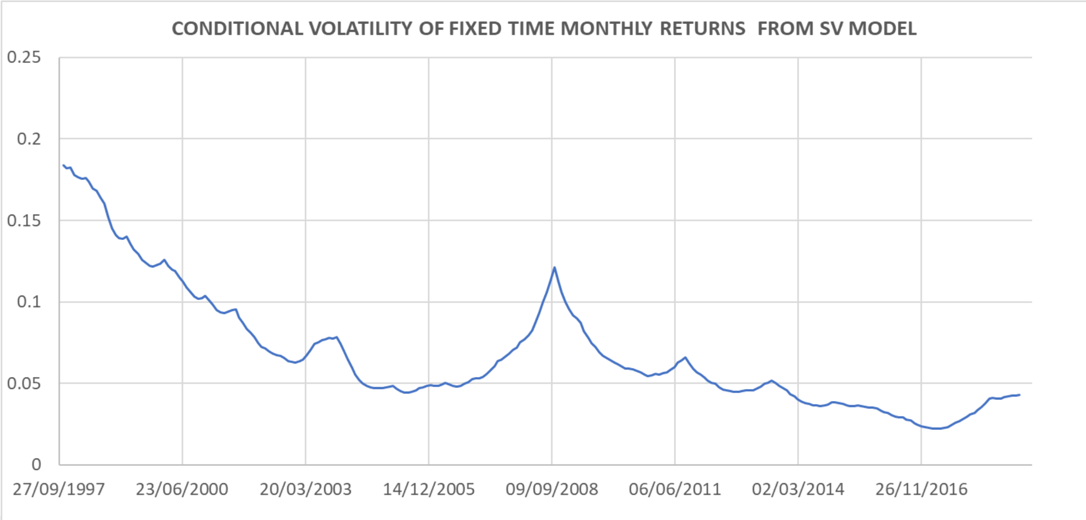


Figure 3.2 Extracted Conditional Volatility of FT and RT, Weekly Returns Data.



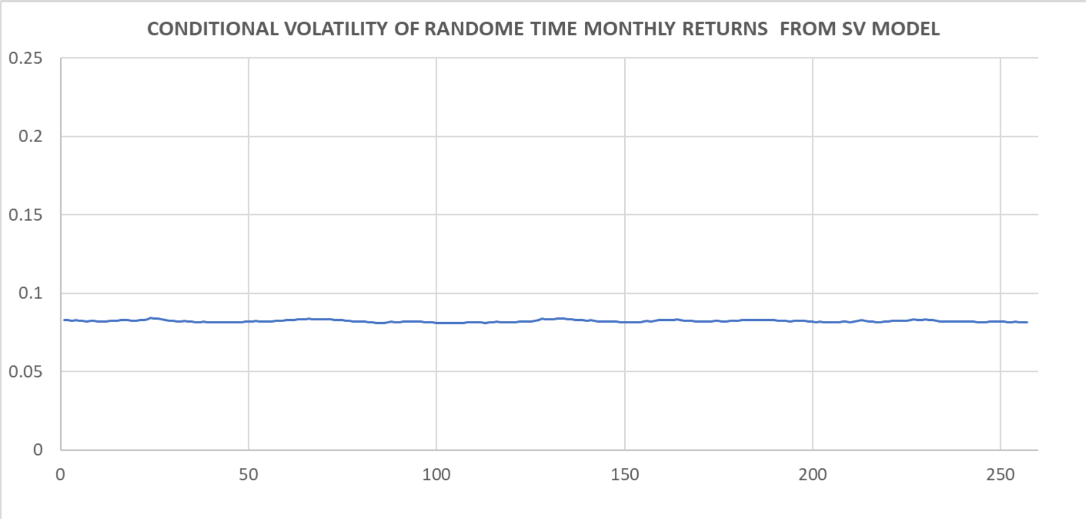


Figure 3.3 Extracted Conditional Volatility of FT and RT, Monthly Returns Data.

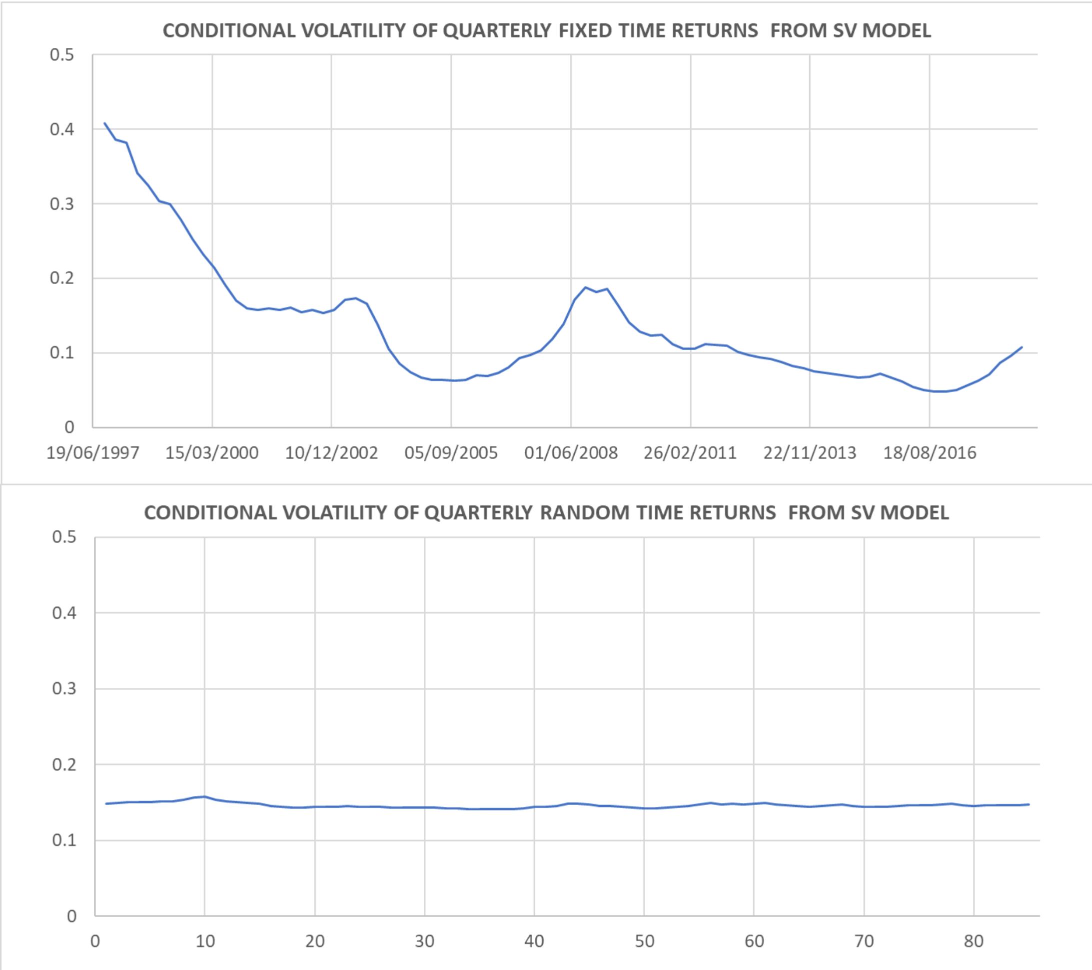


Figure 3.4 Extracted Conditional Volatility of FT and RT, Quarterly Returns Data.

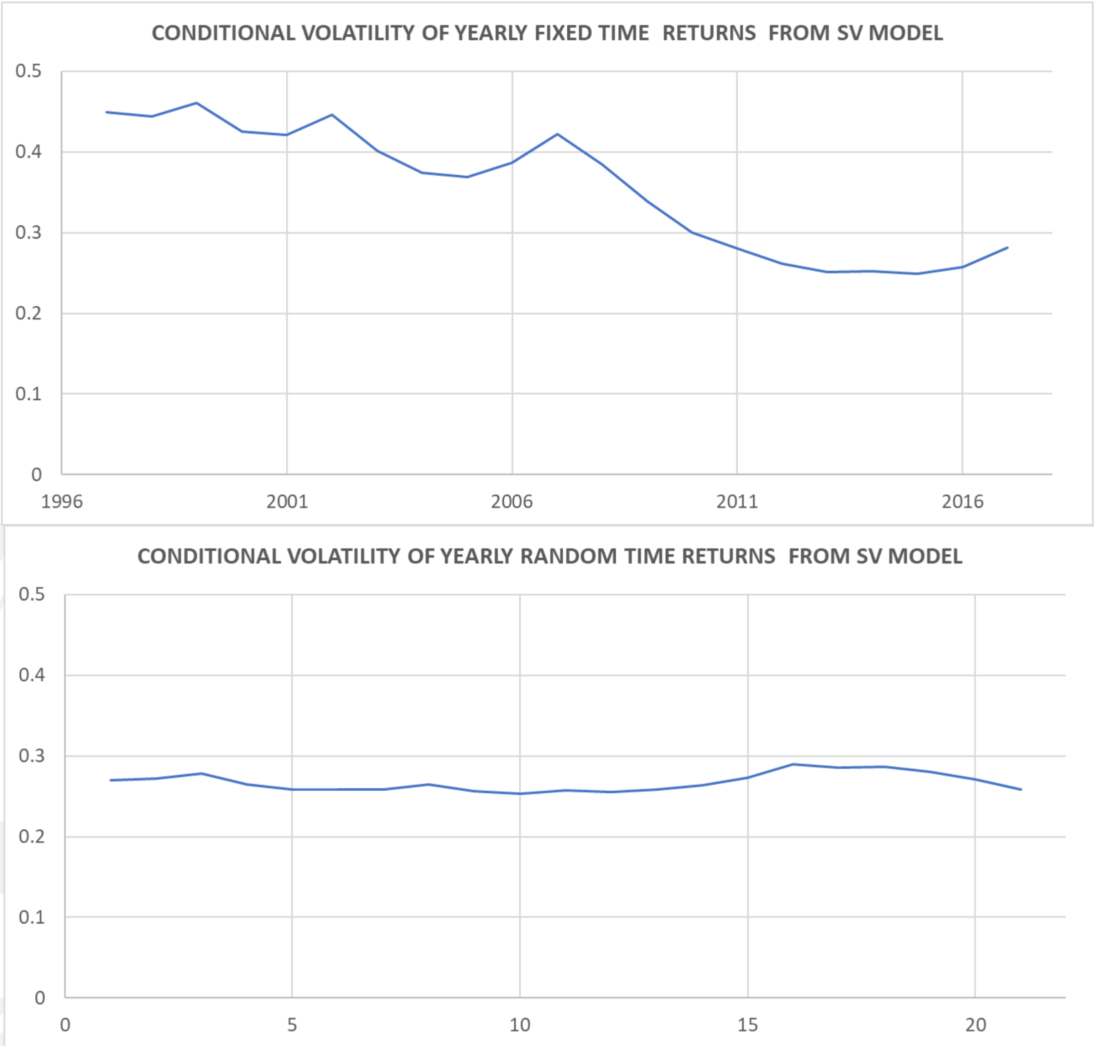


Figure 3.5 Extracted Conditional Volatility of FT and RT, Yearly Returns Data.

A visualization of unobserved conditional volatility is presented in Figures 3.2 to 3.5 for each collected data frequency. All of the fixed time interval data analyses are presented in the top panel of each figure while the random time interval analyses are presented in the bottom panel of each figure. The same axis scale is used for each graph to ensure accurate comparison of the magnitude of fluctuations inside each figure. In Figures 3.2 through 3.5, the fluctuation of the conditional volatilities of stock returns are gigantically reduced by the random time scale as derived from the time changing method.

3.4 Conclusion

This chapter reviewed the theoretical aspects of the econometric issues of predictive regression. Two main econometric issues concerning the persistence and endogeneity of covariates and the nonstationary of stochastic volatility were discussed, starting with key assumptions and ending with their impact on predictive regression model estimation and testing.

In terms of the persistence and endogeneity of covariates, the concepts of limit distribution of standard normal distribution, log-normal distribution, and student t distribution were examined. The local-to-unity process of covariates was introduced with a derivation to study the impact of this issue on the limit distribution of student t statistics of the slope coefficient. The results indicate that its limit distribution deviated from standard normal distribution, affecting the critical value of hypothesis testing, which could cause an incorrect decision based on over-rejection in hypothesis testing. The empirical study in this case will be conducted in the next chapter with the new instrumental variable.

To solve the nonstationary stochastic volatility issue, the new methodology of time changing was introduced. The backgrounds of this method are based on the knowledge and advantage of the martingale process of stochastic calculus. The new concept of DDS Brownian motion, invented by Dambis, Durbins, and Schwarz in 1965, was used to change the time scale form chronological time (t) to the new random time scale, or volatility time (T). The mechanics of this time change technique involve switching between the martingale difference of error terms of predictive regression and the other martingale from the Brownian motion with an exact time. If it is integrable and the errors term can be estimated by the quadratic variation, then the new volatility time can be selected and the samples can be picked with the selected volatility time interval. The empirical study was carried out with the SET index data from the previous chapter. The results of the empirical analysis found that the stochastic volatility of the SET index returns is persistent at all frequencies with a confidence level above 99 %. The time change method was applied to rectify this problem with an acceptable result on weekly SET index data and an even better one when the data frequency increased to monthly, quarterly, and yearly. The results are in line with those of Choi et al. (2016), who used U.S. stock market data. The impact of this issue on returns predictability will be discussed in the next chapter.

Finally, the time change method can be also applied in financial economics and mathematical finance. One well-known application involves using the time change method to derive the Black-Scholes formula, even though there are several ways to proof the Black-Scholes formula, such as the PDE and martingale methods. The other well-known application is the pricing of financial and energy derivative of multifactor Levy-based models as well as option pricing formula for mean-reverting commodity pricing model.

CHAPTER 4

STOCK RETURNS PREDICTABILITY AND THE CAUCHY ESTIMATOR

This chapter will review the predictors persistence and endogeneity issues discussed in the last chapter using the CJP model. Specifically, the focus will be on the empirical analysis of stock returns predictability in terms of both chronological time and volatility time and testing as well as the new type of instrumental variable (IV) from the Cauchy estimator.

4.1 The Cauchy Estimator

The Cauchy estimator, first proposed in 1836, is a method for dealing with a set of problems in the predictive variable of stock returns predictive regression that can remain even after applying the time change method, including persistence, endogeneity, and the other data issues. The Cauchy estimator $(\hat{\beta}_N)$ can be applied to predictive Equation (3.1) without a constant term, $\alpha = 0$, which is given by:

$$\hat{\beta}_N = \left(\sum_{i=1}^{N} |x_{i-1}|\right)^{-1} \sum_{i=1}^{N} sgn(x_{i-1}) y_i \,, \qquad (4.1)$$

where $sgn(.)$ is the sign function defined as $sgn(x) = 1$ if $x \geq 0$ and $sgn(x) = -1$ if $x < 0$. $\hat{\beta}_N$ is an instrumental variable (IV) estimator that uses $sgn(x_{i-1})$ as the instrument. Some recent studies have applied Cauchy estimator to deal with issues from abnormal data set in predictive variables; So and Shin (1999) and Chang (2002), for example, used it to test for a unit root.

From the regression theory, the associated Cauchy t-ratio $\tau(\hat{\beta}_N)$ for β is given by:

$$\tau(\hat{\beta}_N) = \frac{\hat{\beta}_N}{s(\hat{\beta}_N)}. \tag{4.2}$$

The term $s(\hat{\beta}_N)$ is the standard error of the Cauchy estimator, which is given by:

$$s(\hat{\beta}_N) = \hat{\sigma}_N^2 \sqrt{N} \left(\sum_{i=1}^{N} \lceil x_{i-1} \rceil \right)^{-1} \tag{4.3}$$

with any consistent estimator $\hat{\sigma}_N^2$ for the asymptotic variance σ^2 of regression errors (u_i). The sufficient assumption to study the limit theory of the Cauchy estimator $(\hat{\beta}_N)$ is given by:

Assumption 4.1.1 Let $(u_i, \mathscr{F}_i)$ be a martingale difference sequence such that

(a) $\frac{1}{N}\sum_{i=1}^{N} E(u_i^2 | \mathscr{F}_{i-1}) \longrightarrow_p 1 \ as \ N \longrightarrow \infty$, and

(b) $\frac{1}{N}\sum_{i=1}^{N} E\left(u_i^2 I_{(|u_i| > N^{\frac{1}{2}}\epsilon)} \middle| \mathscr{F}_{i-1}\right) \longrightarrow_p 0 \ as \ N \longrightarrow \infty$ and for any $\epsilon > 0$.

The above assumption is expected to hold for a general class of martingale difference sequences.

Assumption 4.1.2 There exists a sequence K_N of the number such that

$$\left(\mathrm{K}_N^{-1} \sum_{i=1}^{N} \lceil x_i \rceil \right)^{-1} = 0_p(1)$$

for all large N.

The required conditions for Assumption 2.3.2 are mild, and it should therefore hold for a wide variety of the predictive variable (x_i). If (x_i) is a nonconstant, stationary, and ergodic time series, then the condition is satisfied with $\mathrm{K}_N = N$. On the other hand, if (x_i) has a near-to-unit root and its innovations satisfy the invariance principle, then:

$$N^{-3/2}\sum_{i=1}^{N}|x_{i-1}| \longrightarrow_d \int_0^1 |V_c(r)|dr \qquad (4.4)$$

where $V_c(r)$ is the Ornstein-Uhlenbeck process. The condition holds for $\mathrm{K}_N = N^{3/2}$. The condition in Assumption 4.1.2 allows for various other kinds of data anomalies in (x_i). Jumps and structural breaks in (x_i) are generally permitted if their numbers are finite.

Choi et al. (2016) proposed the asymptotic theory of the Cauchy estimator $\hat{\beta}_N$ and the Cauchy t-ratio $\tau(\hat{\beta}_N)$ as follows:

Lemma 4.1.1

a) if Assumptions 4.1.1 and 4.1.2 hold, then $\tilde{\beta}_N = \beta + 0_p\left(N^{-\frac{1}{2}}/\mathrm{K}_N\right)$ for all large N.

b) if Assumption 2.3.1 holds and $\beta = 0$, then $\tau(\hat{\beta}_N) \longrightarrow_d \mathbb{N}(0,1)$ as $N \longrightarrow \infty$.

At this point, the Cauchy estimator is generally consistent and offers several incredibly useful properties for next step application. First, its convergence rate is generally the same as the OLS estimator $\hat{\beta}_N$. Second, the Cauchy t-ratio has the standard normal limit distribution when imposing the conditions necessary for the central limit theory to hold for (u_i). Finally, the Cauchy t-statistic will have a limit normal distribution in long run regardless of persistence in the predictive variable or correlation between innovations in the predictive variables and stocks returns.

In terms of the quality of the estimator, Choi et al. (2016) reported that the Cauchy estimator is less efficient than the OLS estimator in the predictive regression Equation (3.1). If (x_i) is stationary and ergodic, then the asymptotic variance of the former is $\sigma^2(E|x_i|)^{-2}$ while that of the latter is $(\sigma^2(E(x_i)^2)^{-1})$. On the other hand, if (x_i) is a local-to-unity process with innovations (v_i) that are asymptotically independent of (u_i), then the limit distributions of the Cauchy and OLS estimators are normal mixtures with mixing variates given respectively by $\sigma\left(\int_0^1 |V_c(r)|dr\right)^{-1}$ and $\sigma\left(\int_0^1 V_c(r)^2 dr\right)^{-1/2}$. It can, therefore, easily deduced from the Cauchy-Schwarz inequality $\int_0^1 |V_c(r)|dr \leq \int_0^1 V_c(r)^2 dr$ a.s. that the OLS estimator is more efficient than the Cauchy estimator. However, in the context of predictive regressions, none of the above standard comparisons between the OLS and Cauchy estimators apply. Their relative efficiency will instead be dependent on the asymptotic correlation between the regression errors (u_i) and the innovations of predictive ratios (v_i), as well as on the realization of the predictive ratios (x_i) themselves.

4.2 Empirical Results of Return Predictability

Chapters 3 and 4 discussed the problems of endogeneity, persistence of covariate, and nonstationary stochastic volatility; the new IV estimator has been proposed to remedy the endogeneity and persistence issues while the time change method has been applied to rectify nonstationary volatility. To demonstrate, the same data from Chapters 2 and 3 has been further analyzed using the predictive regression model, starting with the OLS estimator with fixed time and random time interval data and testing, followed by the RLM estimator and testing as well as the Cauchy estimator and testing. The effect of using each technique on return predictability has also been reviewed and discussed.

4.2.1 Data Transformation Method

After getting $y_i^{*\delta}$ from Equation (3.43), in order to make an actual test following the null hypothesis of unpredictable in stock returns, the excess returns have been applied to use for vanishing intercept parameter in regression ($\alpha = 0$). Therefore, assuming $\alpha = 0$ in the regression Equation (3.30), the first test of the hypothesis should consider that $\beta = 0$. The Equation (3.36) will be changed as:

$$y_i^{*\delta} = \beta x_{i-1}^{*\delta} + u_i^{*\delta} \tag{4.5}$$

To use the Cauchy t-ratio $\tau(\hat{\beta}_N^{*\delta})$ for β, which is given by:

$$\tau(\hat{\beta}_N^{*\delta}) = \frac{\hat{\beta}_N^{*\delta}}{s(\hat{\beta}_N^{*\delta})} \tag{4.6}$$

$\hat{\beta}_N^{*\delta}$ is the Cauchy estimator for β, i.e.,

$$\hat{\beta}_N^{*\delta} = \left(\sum_{i=1}^{N} \left|x_{i-1}^{*\delta}\right|\right)^{-1} \sum_{i=1}^{N} sgn(x_{i-1}^{*\delta}) y_i^{*\delta} \,, \tag{4.7}$$

and $s\left(\hat{\beta}_N^{*\delta}\right)$ is the standard error of the Cauchy estimator $\hat{\beta}_N^{*\delta}$, which is given as:

$$\left(\hat{\beta}_N^{*\delta}\right) = \hat{\sigma}_N^2 \sqrt{N} \left(\sum_{i=1}^{N} \left[x_{i-1}^{*\delta}\right]\right)^{-1} \tag{4.8}$$

with any consistent estimator $\hat{\sigma}_N^2$ for the variance σ^{*2} of (u_i^*). Note that, by construction, we know that $\sigma^{*2} = \Delta$ if $u_i^* = y_i^*$ for all $i = 1, \ldots, N$.

4.2.2 Empirical Results of Persistence and Endogeneity of Covariate

Two types of covariates were used for the predictive regression, the dividend–price ratio and the earnings–price ratio, which was used as a predictor. Both have been checked for the level of persistence (δ) presented in Equation (3.3) when $\delta = (1 - \frac{c}{N})$. The results are summarized in Table 4.1.

Table 4.1 The Summary of the Persistence Parameter of Covariates

Frequency	**Predictors**	**The persistence parameter of covariates (δ)**	
		FT	**RT**
Daily	D/P	0.9985**	N/A
	P/E	0.9978**	N/A
Weekly	D/P	0.9928**	0.9956**
	P/E	0.9908**	0.9891**
Monthly	D/P	0.9590**	0.9777**
	P/E	0.9472**	0.9360**
Quarterly	D/P	0.8045	0.9341**
	P/E	0.7892**	0.7744**
Yearly	D/P	0.2616**	0.7321**
	P/E	0.2106**	0.1884**

The results indicate that all the covariates have a local-to-unity issue at the high data frequencies and have the same level of persistence, even though the time change method was applied.

4.2.3 Effect of Stock Returns Predictability with and Without the Time Change Method

This section looks at stock returns predictability after the demonstration of the time change method in the last chapter. At this point, we consider the OLS t-test, which will be referred to as simply OLS estimation, to be the primary estimation and testing method for testing return productivity. All OLS tests are fully evaluated and compared with both FT interval and RT return data. The restricted likelihood ratio test (RLRT) (Chen and Deo, 2009) method of estimation and testing is also used for comparison.

Table 4.2 The Summary of Return Predictability and Testing

[redacted]		Fixed time interval (y)				Random time interval (y*)			
Freq.	**Predictors**	**Beta (OLS)**	**t-statistics**	**Beta (RLRT)**	χ_1^2	**Beta (OLS)**	**t-statistics**	**Beta (RLRT)**	χ_1^2
Weekly	D/P	-9.2335×10^{-5}	-2.40*	-4.8577×10^{-4}	4.28*	-9.6759×10^{-5}	-0.47	-1.1385×10^{-4}	0.10
	E/P	-1.4995×10^{-5}	-1.81	-1.4647×10^{-4}	8.44**	-1.0270×10^{-4}	-0.50	-8.9095×10^{-5}	0.06
Monthly	D/P	2.4567×10^{-3}	3.37**	2.4045×10^{-3}	235.75**	3.8510×10^{-3}	1.18	2.4991×10^{-3}	0.10
	E/P	-1.8921×10^{-4}	-1.19	-1.8891×10^{-4}	30.37 **	-7.4293×10^{-4}	-0.68	-6.8892×10^{-4}	0.37
Quarterly	D/P	-1.8057×10^{-4}	-0.21	-1.5610×10^{-4}	7.87**	7.6080×10^{-4}	0.77	4.9024×10^{-3}	0.05
	E/P	-1.8045×10^{-4}	-1.00	-1.7450×10^{-4}	206.69 **	-1.7388×10^{-3}	-0.51	-1.9173×10^{-3}	0.80
Yearly	D/P	-1.3663×10^{-3}	-1.54	-1.4799×10^{-3}	785.67**	2.1604×10^{-2}	0.56	1.3849×10^{-2}	0.11
	E/P	-1.4739×10^{-4}	-0.68	-1.7293×10^{-4}	822.41**	-2.8321×10^{-3}	-0.20	-7.2232×10^{-4}	0.29

To compare the results from using chronological time and random sampling based on volatility time, we compare the OLS with those from other testing procedures, namely Chen and Deo's (2009) restricted maximum likelihood-based ratio test and the OLS t-test, which will be referred to as the RLRT and OLS tests, respectively. Table 4.2 summarizes the results for stock return predictability and testing; we first present the OLS estimates for the slope coefficient (β) of predictors and their t-statistics. In the following columns, we present the REML-based estimations and their likelihood ratio test statistics. The Cauchy estimates and their t-statistics will be discussed in the next section. To specify the results, the test values that are significant at the 95% and 99% confidence intervals are marked with the superscripts “∗” and “∗∗”, respectively.

For the predictive regression using the dividend-price ratio, the OLS test strongly accepted the alternative hypothesis of predictability$(H_1 : \beta \neq 0)$ at the 95% confidence level for the monthly frequency of FT interval while other OLS tests of RT interval time presented strong evidence of accepting the null hypothesis of no predictability. This might be one indicator of the unpredictability of stock returns. For the predictive regression using the dividend–price ratio in the chronological time scale, the OLS test strongly rejects the null hypothesis of no predictability at weekly and monthly frequencies. This result might lead some researchers to use the D/P ratio to predict the return using the OLS estimator. The RLRT estimators and test also provide strong evidence of return predictability at all frequencies. Generally, it seems correct to use both test results to show that there is some evidence of return predictability that appears to be relatively stronger at high frequencies. On the other hand, if we look at the test results based on the random sampling based on volatility time, the null hypothesis of no predictability is not rejected at all frequencies; all the test values are far below the critical value of the 95% confidence level. Indeed, the results show that, if we consider the econometric issues, particularly nonstationary stochastic volatility, in input data and take proper action to rectify them, no evidence of predictability remains. The results from the predictive regression using the price-earnings ratio are similar.

4.2.4 Empirical Results and Effect of the Stock Returns Predictability of the Cauchy Estimator

This section examines stock returns predictability after using the Cauchy estimator is comparison with the previous section’s results. By the definition of the Cauchy estimator, we only consider random sampling based on the volatility time scale in this analysis.

Table 4.3 The Summary of the Cauchy Estimator of Return Predictability

Cauchy estimator		Fixed time interval		Random time interval	
Frequency	**Predictors**	**Beta**	**t-statistics**	**Beta**	**t-statistics**
Weekly	D/P	-3.2758 x10^{-4}	-0.60	1.3870 x10^{-3}	1.95
	E/P	-2.8640 x10^{-4}	-2.09*	-8.0066 x10^{-5}	-0.41
Monthly	D/P	1.7874 x10^{-3}	1.73	5.8724 x10^{-3}	1.83
	E/P	-1.4957 x10^{-4}	-0.78	-4.6587 x10^{-4}	-0.43
Quarterly	D/P	-7.7320 x10^{-5}	0.21	7.0432 x10^{-3}	0.74
	E/P	-2.8742 x10^{-5}	-0.02	--9.6973 x10^{-4}	-0.29
Yearly	D/P	-1.8606 x10^{-4}	-0.70	-2.7394 x10^{-2}	-0.77
	E/P	-5.9881 x10^{-4}	-0.00	-3.8558 x10^{-3}	-0.30

Table 4.3 shows that the results from Cauchy estimator method are similar to those from other estimation and test methods post proper data management and support the findings of Choi et al. (2016).

4.3 Conclusion

The Cauchy estimator positively shows the improvement of the endogeneity and persistence of the predictors. This is due to its robustness in terms of improving the limit distribution to close to normal distribution.

The econometric issues discussed in the previous chapter, including nonstationary stochastic volatility and the persistence and endogeneity of predictors, could lead to predictive regression results that result in incorrect decisions based on the over-rejection in the statistical testing. Proper data management techniques including econometric parameter checking with some rectification to make the input data more reliable. The time change method and the Cauchy estimator can be used to improve the econometric issue in empirical analysis; the time change method helps remove nonstationary stochastic volatility while Cauchy estimator improves the persistence and endogeneity of predictors. A key takeaway from this empirical study is that there is still no strong evidence of stock return predictability based on the selected data from the Thai stock market. Some new predictors and/or larger amounts of data may be needed to further clarify this, and knowledge of the application of stochastic volatility could be further expanded regarding the new asset pricing formulas, such as the derivative of commodities pricing and the new derivative products from volatility.

CHAPTER 5

CONCLUSION AND DISCUSSION

This chapter will draw some conclusions from the results of this study and offer suggestions for potential additional topics of study based on the examination of stochastic volatility, econometric issues in predictive regression, and the CJP approach.

5.1 Conclusion and Discussion

The goal of this study was to look for econometric issues in stock returns data and use the new Choi approach to remedy them before comparing the results with foreign market data. To deal with econometric issues, stochastic volatility is required as a medium to link between the process of study. Therefore, the stochastic volatility model was introduced and estimated using the Bayesian simulation-based approach for statistical inference. One major difference between the stochastic volatility model and the ARCH/GARCH model is the variation source; the stochastic volatility model collects the variation from the stochastic process while the ARCH/GARCH model uses return data process to generate its variation. Of course, estimating the ARCH/GARCH model is simpler, but the volatility forecasting of the Thai stock market based on the stochastic volatility model was better than that of the GARCH (1,1) model in this study, showing better forecast accuracy at all frequencies of returns data.

The next step involved studying stock return predictability, one of the most important topics in financial economics, by using a predictive regression model. The most common predictors are the dividend–price and earnings–price ratios, and both are incredibly useful predictors of future stock returns. However, from the econometric perspective, there is some recognition evidence that returns and predictors data can seriously distort standard hypothesis testing. The persistence of predictors and a correlation between innovations of predictive regression has been extensively pointed out, though no definitive solution for this issue has been determined to date. Another recognition of the returns data characteristic is time varying stochastic volatility, which has never before been traced in a predictive regression. These two characteristics can explain the missing pieces in stock return predictability. The Choi approach is a new way to test for predictability by using both a time change and the robust Cauchy estimator. This new technique has proven that the t-ratio will always have a standard normal limiting distribution even with the previously mentioned econometric issues present in the data. The approach involves constructing a volatility time using realized volatility and then estimating the model using the Cauchy estimator, provided that the given error is independent and normally distributed. The random time sampling ensures that this will always be true.

Findings from this empirical study are presented as follows:

1) The input data contains nonstationary volatility and persistence of predictors.

2) If we use the current input data without any data management, then the predictive regression seems to be permissible.

3) Once we removed the nonstationary volatility using the time change method, we found no evidence to support stock return predictability, even though there has been tremendous support in previous studies for return predictability and it has become a stylized fact of stock returns.

4) The Cauchy estimator was applied for random sampling based on volatility time. This study showed no support for predictability at any frequency for the selected predictors, such as the dividend–price and earnings–price ratios.

5) It seems clear that stock returns cannot be predicted by dividend–price and earnings–price ratios if the characteristics of the data are properly checked and managed.

Finally, the results of this study also presented in the same direction with the results of Choi et al. (2016). They also found the same econometrics issues in New York Stock Exchange (NYSE) and using time change method as well as Cauchy estimator to rectify the issues. The ultimate results also indicated that stock returns cannot be predicted by dividend–price and earnings–price ratio which is similar to this study based on data from Stock Exchange of Thailand (SET).

5.2 The Suggested Topic for the Further Study

The applications of stochastic volatility could be further investigated, in terms of the derivative pricing of financial-energy markets or the option pricing of commodities, for instance. Additionally, the predictors without econometric issues should be explored and used as new predictors for predictive regression. The use of a new instrumental variable for predictive regression estimation, such as the other nonparametric indicator, would also be interesting to examine.

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