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How psychological factor influence loss aversion behavior: A unique utility function explanation

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Abstract

How psychological factor influence loss aversion behavior of individuals under uncertainty has been mostly tested for years. However, there is no any specific utility function can describe the way how psychological influence loss aversion behavior of individuals under uncertainty realistically. In current study, a once switchable utility function is presented. This positive odd n power plus linear utility function allows negative cost of game *c*, proxy of cost of game, to measures the difference between current wealth and certainty equivalent. As the certainty equivalent is difference between expected utility value and risk premium, so, the variable *c* can reflect both changes on current wealth and certainty equivalent continuously and avoid the subjective weighted probability bias existed in most expected utility functions. Most important point is that this utility function can be used to measure the psychological influence (individual's speed of adjustment to unexpected events and risk as feeling hypothesis) on loss aversion behavior of individuals to risks, represented by *n* in the utility function. Finally, this unique utility function can be employed empirically to measure both loss aversion behavior and psychological factors under the heterogeneity among individuals towards the risky choices.

Keywords: Psychological factors, Switchable utility function, Loss aversion

Introduction

In real life, there are many factors influence individuals' income or wealth levels, so, how random stochastically in wealth change affect the individual utility function is an important topic in this field, also, expected utility function need to be coherent or can be continuously reflect individual's changing behavior. For example, if an individual subjective weighted probability is kept unchanged in each expected utility function or for every expected outcome, but, the current wealth level of this individual unexpectedly changed, either increased or decreased, based on prospect theory, these unexpected current wealth levels can be treated as new referent points, so, if an individual has loss aversion behavior, one emerged question is how these unexpectedly changes in the current wealth levels affect this individual's decision to risks. In fact, many researchers have found that various utility functions have some limitation to answer this question. For instance, based on Arrow-Pratt's risk aversion and expected utility functions, all the prior studies found that individuals made their choices under uncertainty mostly based on their risk aversion, or loss aversion, wealth levels and the difference between expected return and certain equivalent, so, there were heterogeneous risky choice behavior among individuals, they were either risk-averters or risk-takers depended on their risk preference, referent points and the curvatures under the utility functions applied in these studies. As the results, the need of a unique utility function which can be used to express both risk aversion and risk taking behaviors under specified conditions is more urgent. (Rothschild and Stiglitz, 1971; Gorden et al., 1972; Graves, 1979; Kihlstrom et al., 1980; Bell, 1988; Cox and Sadiraj, 2002; Abdellaoui et al. 2007; Norstad, 2011; Abbas and Bell, 2011; Martin von Gaudecker et al., 2011; Giorgi and Post, 2011; and Ray and Robson, 2012)

Contrast to the enriched researches on weighted probability under the expected utility functions, and loss aversion behavior under the prospect theory (Kahneman and Tversky, 1979), another important factor, rooted in the same theory: the psychological perspective, has not been emphasized in prior researches. This psychological factor has not been encapsulated into any utility function form to explain individuals' loss aversion behavior and risk taking behavior. Most researches which studied the relationship between psychological factor and risk aversion or risk taking behavior were tested either in labs, in field experiments, or just described in theoretical frameworks. For example, some prior researches stated that psychological principles or factors should be included into the studies on loss aversion behavior and decision making process under uncertainty for the subjective weighted probability in various expected utility functions (Loewenstein et al., 2001; Camerer, 2005; Novemsky and Kahneman, 2005; Shleifer, 2012 and Barberis, 2013). Their conclusions revealed that the psychological factors could affect not only the decision making process, but also influence the speed of these decision process. Furthermore, if risks are psychological feelings of individuals, then, these psychological feelings could affect the individuals' behavior under ambiguity.

In the current study, I present a positive odd n power plus linear utility function and try explain the individuals' loss aversion behavior under uncertainty by including psychological factor into the consideration so as to make the description of individual loss aversion behavior be more realistic and overcome some subjective weighted probability biases under the expected utility function forms.

Literature reviews

Expected utility function always be used to state the individuals' decision when the individuals must either maximize the expected utility value by choosing between the risky and riskless assets under mean-variance measurement, or minimize the variance for his or her portfolios subject to some constraints.

Pratt (1964) pointed out that individuals or decision makers have risk aversion properties. To avoid such risk, he or she would use cash or certain equivalent to exchange. The cash equivalent or certainty equivalent is the amount remained between expected value minus risk premium. Under the expected utility function theory, there are two risk aversions definitions, one is absolute risk aversion and another is relative risk aversion, what are expressed as - U"/U' and - wU"/U', here, the w is the initial wealth of individual. However, the expected utility was meaningful if and only if it had concavity properties with the U"<0, and U'>0. These properties implied that the expected utility function must be quadratic function. But, these properties may be wrong, for example, both the absolute risk aversion and relative risk aversion are increasing under some circumstances.

Based on Arrow-Pratt's risk aversion and expected utility functions, Rothschild and Stiglitz (1971), Gordon et.al. (1972), Kihlstrom et al. (1980), and Bell (1988) argued that the absolute and relative risk aversion under quadratic utility functions were not sufficient to explain some experimental results and individuals' behavior under the change of their wealth levels. For instance, individual could save more or less under the uncertainty of return of saving rate, or they may consumed less even their wealth levels increased. Supported by Abbas and Bell (2011), Graves (1979) pointed out an important concept "safety first principle". This principle meant that individuals tried to express their increasing relative risk aversion behavior to minimize the probability to fall below the "disaster" level of current wealth. As the result, the bet must be small enough relative to his or her "disaster wealth level", then, he or she may be indifferent between taking the bet or not. But, if the bet was larger relative to his or her "disaster wealth level", then, he or she might hesitate to take the bet. Cox and Sadiraj (2002) stated that expected utility theory could not provide a coherent positive theory of risk averse behavior and there was calibration problem based on assumed concavity and additive on initial wealth to income. The authors also cited that a concave expected utility model implied that people were approximately risk neutral when stakes were small, so, the expected utility function could not provide a plausible confirmation of risk aversion over modest stakes which was not consistent with expected utility theory. These results implied a possible switching preference when the wealth reached a certain level. Norstad (2011) focused on the initial wealth and expected utility theory under risk aversion condition. He stated that individuals evaluated an investment was attractive or not based on the difference between certainty equivalent and their current wealth. He concluded that there was no any of utility function could be used to describe all investors or even any individual's behavior. It was entirely reasonable for an investor's attitude to risk when such risk was varying with the amount of wealth. As, the investor's risk tolerance may change from time to time, but, these changes would likely be far more gradual than those with a constant risk aversion. However, in his article, Norstad did not give out the any general expression of relationship between the certainty equivalents and changing initial wealth levels. So, individual's assessment to risk was substantially influenced by their circumstances and conditions.

To address heterogeneous behaviors among individuals, Kahneman and Tversky (1979, 1992) developed and advanced the prospect theory, the authors allowed individual to choose between different assets with risk. The individual first evaluated the possible outcome of the decision, and arranged the outcomes under some rules, then, the individual would set a reference point to evaluate the possible worst losses and best gains. Therefore, they would behave to choose the higher utility as if they could calculate the value based on the possible outcomes according to their subjectively probability. The authors concluded that the interaction for overweight on small probability of gain and underweight on large probability of loss based on concavity-convexity of the value and weighting function implies four patterns of risk behaviors: risk averse for gain under moderate or high probability, risk seeking for loss under moderate or high probability, risk seeking of gain under small probabilities and risk aversion of loss under small probabilities. Hereafter, there were many researches focusing on four main parts under the prospect theory: reference dependence; loss aversion; diminishing sensitivity; and probability weighting (Barberis, 2013).

For example, Abdellaoui et al. (2007) pointed that there was about 40% of total subjects included their study with concave utility functions for losses. This fact had not been presented in any form of utility functions. Further, the convex utility in loss domain under the prospect theory was a bias when subjective probabilities for large loss just only above 1/3, this made risky loss to be more attractive when compared with the certain equivalent, as the result, the curvature was expressed as risk seeking under the loss domain. Moreover, the authors stated that the loss aversion definition under the prospect theory had a main drawback: the loss aversion cannot be separated from the curvature of utility functions, so, how to measure the individual’s attitude to the losses cannot be summarized generally. These conclusions were echoed by Martin von Gaudecker et al. (2011). They found that the curvature of utility function and loss aversion behavior were important for individual to make decision under risks, however, many individuals did switch their risk preferences during the test, while some other seems to be risk neutral. So, they concluded that risk preferences in their experiment were very heterogeneous.

According to the prospect theory, individuals do not evaluate their wealth at the end or final states, rather, they prefer to evaluate their gains or losses relative to their current wealth levels in each state. However, whether the reference points are constant or not, in other words, whether the reference points are exogenous or endogenous have been discussed in prior studies. For example, De Giorgi and Post (2011) argued that in financial field, the state-dependent preference structure was more reasonable, as investors usually compare their prospect outcomes with their reference points in the same state. The authors stated that if the reference point was endogenous, then, the loss aversion could not affect individual behavior, however, if the reference point has an exogenous element, for instance, the reference point included the rate of return on riskless assets, then, the loss aversion did influence individual behavior. Finally, the authors concluded that the state-dependent reference point model could cause different behavior among individuals in each state, such variety on behaviors was depended on the difference between the reference points and current wealth levels as well as the speed of individual's adjustment to new information or unexpected events.

From psychological perspective, Loewenstein et al. (2001) and Shleifer (2012) studied the speed of individual responses to the new information or unexpected events based on different hypotheses. Loewenstein et al. (2001) presented the risk as feelings hypothesis. Under this hypothesis, the feelings as one factor was included into the decision process under uncertainty. By explaining the interaction between cognitive evaluation and feelings towards risks, the author argued that people usually treated risks at two steps: evaluate the risk cognitively and react emotionally, also some people might learn quickly to some types of risk, while others did not. These results were underpinned by Shleifer (2012). Derived from "Heuristics and Biases" under the prospect theory, the author reviewed and classified the thinking systems into fast (system 1) and slow (system 2) types, and pointed out that the heuristics (anchoring and representative heuristics) and biases (overweight on small probabilities of gain and underweight on large probabilities of loss) could accelerate people's thinking, but the results were always incorrect. So, to some extent, the risk as feelings hypothesis or fast thinking system could explained risk taking behavior under certain situations. The phenomenon of risk taking behavior could be explored more deeply by the relationship between the psychological response and loss aversion. Novemsky and Kahneman (2005) and Camerer (2005) analyzed the influence of psychological response on loss aversion. Novemsky and Kahneman (2005) stated that there were no loss aversion behavior under three situations: if the goods that provide same benefit but with different properties could be exchanged without loss aversion; if the goods could be exchanged intendedly (endogenous), such behavior could not be categorized as loss aversion; if the loss aversion in balanced risks, then, there was no risk aversion, for example, when risky willingness to accept equals to the willingness to accept. For example, if the income effect, which was the difference between risky selling to the selling was too small to be negligible, then, there is no risk aversion, so, the loss aversion to risk could not be applied entirely in concavity under expected utility functions.

Theoretical framework

As there was a bias when subjective weighted probability under the expected utility functions (Abdellaoui et al., 2007), and the individual risk aversion behavior may switch between risk averter, risk neutral and risk taker according to the difference between their current wealth levels and certain equivalent (Graves, 1979; Cox and Sadiraj, 2002; Norstad, 2010, Abbas and Bell, 2011; Martin von Gaudecker et al., 2011), so, any theoretical or empirical study on individual risk aversion behavior should not only focus on absolute, relative risk aversion and subjective outcomes, but also, take the psychological responses into the study, because any unexpected event, such as news or information what may cause unexpected impact on their incomes, could psychologically affect the individual risk aversion behavior either in fast, slow responses or even no responses under the risk neutral or under no loss aversion conditions (De Giorgi and Post, 2011; Loewenstein et al., 2001; Novemsky and Kahneman; 2005; Camerer, 2005; Shleifer, 2012; Barberis, 2013).

The model in current study is not only built on several previous studies about the utility functions and investor's risk aversion behavior, but also included psychological responses, for example, under the risk as feelings hypothesis there are two parts: the degree of psychological impact on unexpected change of income (evaluate the risk cognitively) and psychological response (react emotionally), so, how these two parts can be used to explain the individual's risk behavior more realistically under the heterogeneous referent points and risk preferences of individuals?

The hypothesis:

The psychological responses do influence the individual loss aversion behavior in both loss and gain domains.

The Model of this study is:

$U(c) = a(c)^n + b(c),\ a>0,\ b>0,\ n=3, 5, 7, 9, 11$, *and n will not equals to 1, 2, or even numbers*; (1)

Where c = *current wealth – certainty equivalent*

Where *a*, *b* are real number. *a* represent the psychological impact degree on unexpected events (income effects hereafter), which is an absolute value of the difference between expected value of outcome on subjective estimation of individuals and the fair or objectively outcomes, or each initial wealth level (in this study), big value of *a* means potential high psychological impact exists cognitively, which implies that the difference between the expected value of outcome and actual value of outcome is higher; *c* means "cost of game" of individual; *n* stands for individual's psychological factor, it is measurement for psychological factor to risk changes (react emotionally): the slower adjustment to unexpected events emotionally, the less score he or she has.

Property 1: for every rational individual:

$U'(c) = an(c)^{n-1} + b > 0,\ an(c)^{n-1} > -b;\ U''(c) = an(n-1)(c)^{n-2} < 0$, only for $c<0$; (2)

Property 2: Arrow-Pratt Absolute Risk Aversion and Relative Risk Aversion:

$R_A(c) = -U''(c)/U'(c) = -\ [an\ (n-1)*(c)^{n-2}/an*c^{n-1}+b]$ (3)

$R_R(c) = -c*U''(c)/U'(c) = -\ [c*an\ (n-1)\ (c)^{n-2}/an*c^{n-1}+b]$ (4)

Property 3:

$dR_A(c)/d(c) = -\ \{an\ (n-1)(c)^{n-3} * [b(n-2)-an(c)^{n-1}]/(an(c)^{n-1}+b)^2\}$ (5)

When n = positive odd integer number, $c<0$, then,

$dR_A(c)/d(c)$ <0, =0, >0, will be depended on $[b(n-2)-an(c)^{n-1}]$ >0, =0, <0; (DARA, Indifferent, or IARA);

When n = positive odd integer number, $c>0$, then,

$dR_A(c)/d(c)$ <0, =0, >0 will be depended on $[b(n-2)-an(c)^{n-1}]$>0, =0, <0; (DARA, Indifferent, or IARA);

Property 4:

$dR_R(c)/d(c) = -[b*an*(n-1)^2*(c)^{n-2}/(an*(c)^{n-1}+b)^2]$; (6)

When n = positive odd integer number, $c<0$, then, $d\ R_R(c)/d(c) > 0$, (IRRA);

When n = positive odd integer number, $c>0$, then, $d\ R_R(c)/d(c) < 0$, (DRRA);

When $c=0$, all would be Constant Absolute and Relative Risk Aversion (CARA, CRRA, or Indifference);

Property 5:

When $c<0$, and n = odd positive integer, an individual with such utility function is risk averter, when, $c>0$, there no any risk averter whenever n is odd or even positive integer, so, only n = odd positive integer is feasible for both negative c and positive c.

Table 1 Individual risk aversion property

	ARA	RRA	Individual RA property
When c<0, n=odd	~~DARA~~,CARA,IARA	IRRA	Risk Averter
When c>0, n=odd	DARA,~~CARA~~,IARA	DRRA	Risk Lover

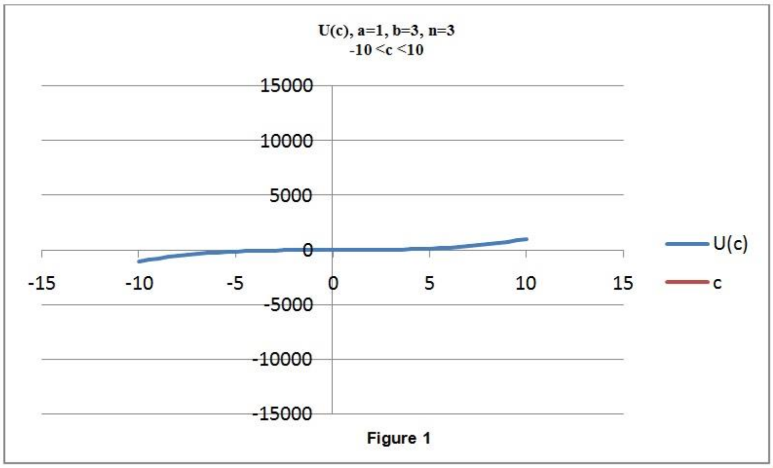
Note: see proof in Appendix A

Property 6:

From the equation (1) and (2), *a*, *b* are determinants of slope of utility function, the "*n*" stands for psychological factor to risk changes. *a*, *b*, *n* together will show properties of each individual's response to cost of game level in utility function. For example, If an individual, expected utility value is much different with his or her each initial wealth level, he or she may or may not exhibits obvious sensitivity to cost of games changes, which means that he or she may or may not take a bet even if the bet is fair to him or her given that he or she response slowly on unexpected changes. However, if his or her adjustment to unexpected events is fast, such individual will take the bet even if it is not fair game to them based on same difference between the expected value and the each initial wealth levels.

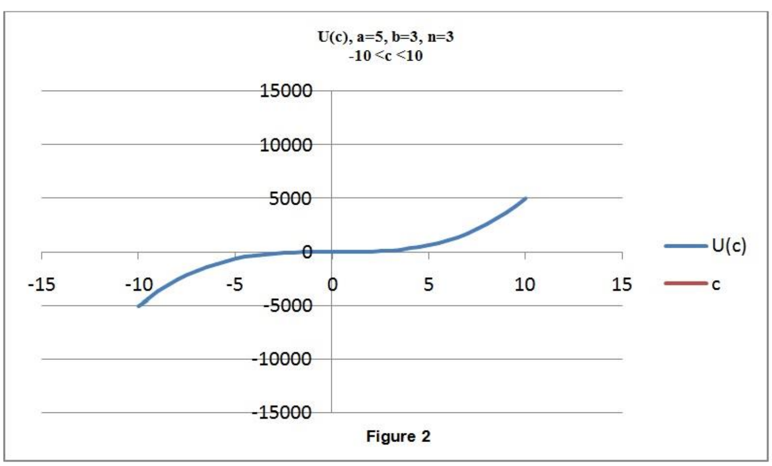
Discussions

Figure 1 shows that when the cost of game is small enough and the individual with lower degree of psychological impact *a* and slower response to unexpected events *n*, his or her utility function looks like a straight line, this means that such individual is indifferent between taking the bet or doing nothing (Norstad, 2010; Loewenstein et al., 2001; Shleifer, 2012). Figure 2 reveals that when an individual with higher degree psychological impact (*a* equals to 5), other conditions remained), then, his or her utility function becomes more volatile: when the cost of game is negative, his or her utility function is a concave curve, and the utility value is near to or equal to zero when the cost of game c changed between -5 and +5, so, the higher degree psychological impact shorten the span of indifference on cost of game for an individual.

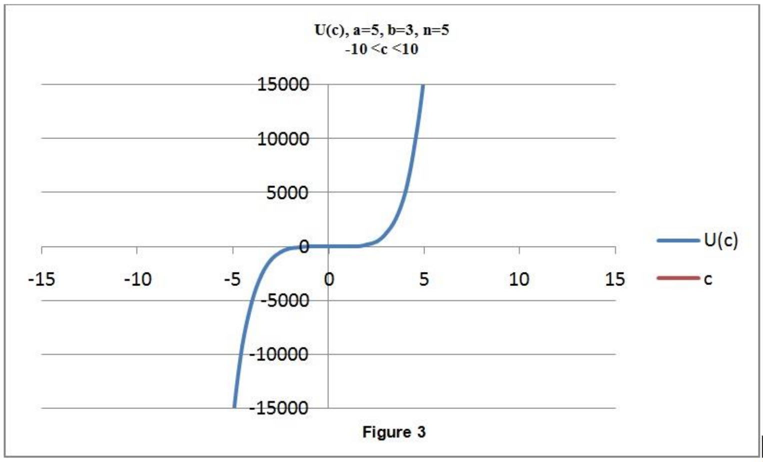


Figures 1 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)

When compare the Figure 3 to Figure 2, there is not only psychological impact, but also psychological factor to risk changes. When an individual with higher adjustment speed to the risk changes (*n* equals to 5, not 3 in Figure 2), this individual's utility function become more sharp, and the span for switch points from concave (he or she is a risk averter) in negative region domain to convex in positive region (he or she is a risk lover) is much shorter than that

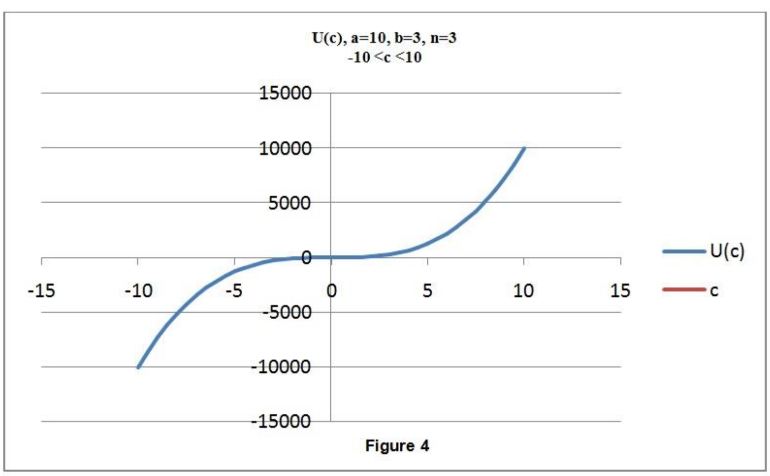


Figures 2 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)



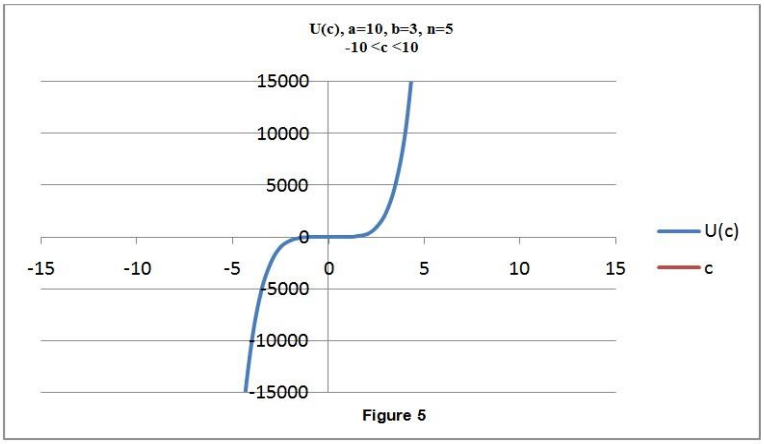
Figures 3 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)

In Figure 2. This result suggest that an individual adjust his or her "disaster wealth level" higher when his or her current wealth is less than the cost of game, but such "disaster wealth level" is adjusted lower when his or her current wealth is more than the cost of game (Graves, 1979; De Giorgi and Post, 2011).

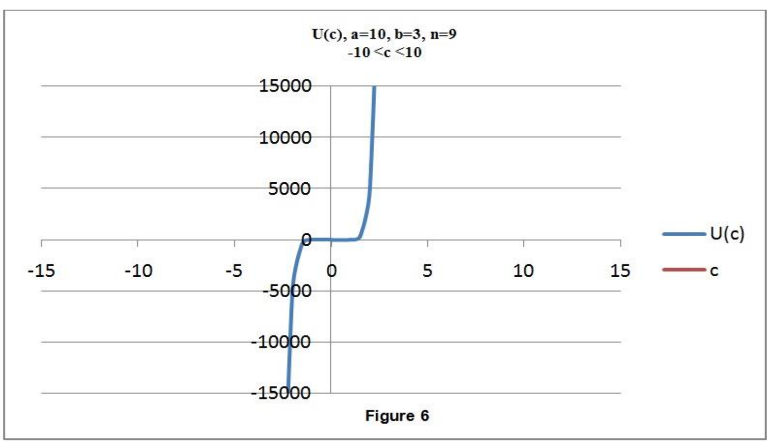


Figures 4 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)

From Figure 4 and Figure 2, when an individual with same adjustment speed on psychological response, but the psychological impact doubles, which implies that the individual cognizes the unexpected events more seriously). Under this condition, this individual's utility function changed to be more volatile in slopes either in negative region or in positive region of the utility functions, the range of switch points also become shorter in Figure 4 than that in Figure 2.



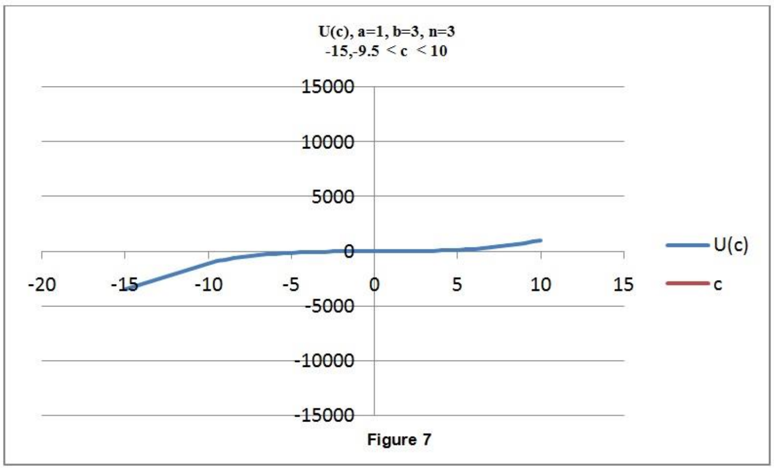
Figures 5 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)



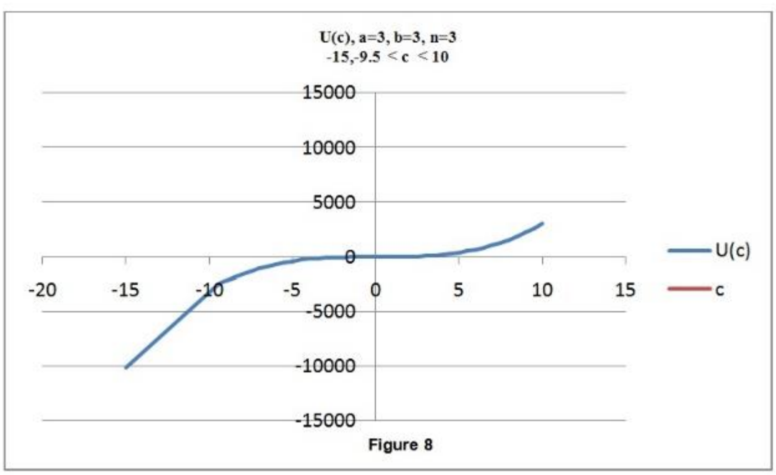
Figures 6 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)

Figure 5 and 6 discover that when the degree of psychological impact doubled, but associated with different psychological response speed of adjustment to the risk changes, the utility function of this individual become much more fluctuated when the cost of game still switched between -10 and +10.

Figure 7 and Figure 8 demonstrate that when an individual's current wealth suddenly changed in negative way, so, the cost of game suddenly become more negative than that in prior figures. For example, an individual the cost of game suddenly change from -10 and +10 to be -15 to +10. Compare the Figure 7 and Figure 1 (with same degree of psychological impact and same speed of adjustment to risk changes), his or her utility function become



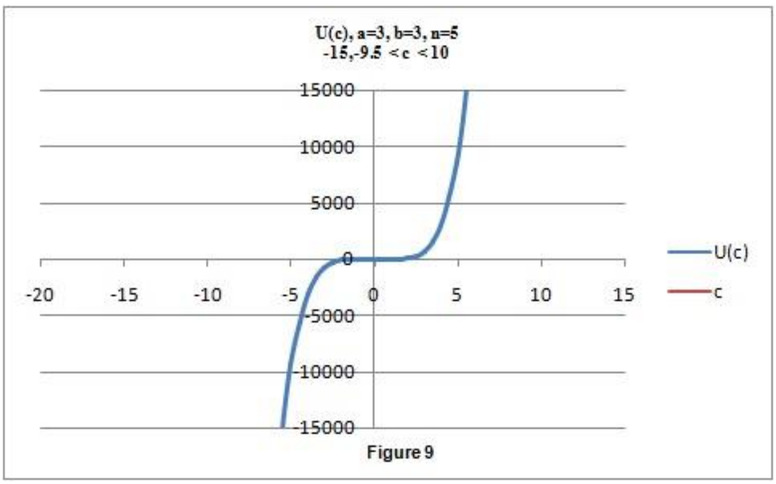
Figures 7 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)



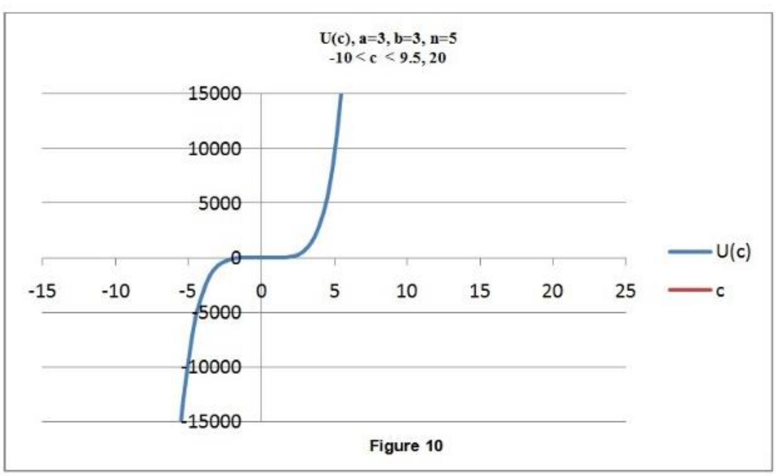
Figures 8 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)

More concave in the negative domain, which implies that this individual is not willing to take an investment more when his current wealth is much less than the cost of game (Gordon, 1972, Rothschild and Stiglitz, 1971; Graves, 1979;

Abdellaoui et al., 2007; Abbas and Bell, 2011). Figure 8 shows that when there is a higher degree psychological impact, the curve of utility function is changed in both side: his or her utility function become more in concave in negative side and more in convex in positive side. But, the slopes are different between that in negative and that in positive sides: the slope in negative domain is higher than that in positive domain, such results reveal that an individual take unexpected change in wealth or income more seriously and become more conservative.



Figures 9 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)



Figures 10 Simulation of interactions among a, *b*, *n* and *c* (Cost of game)

Figure 9 and Figure 10 disclose the "extreme" effects on suddenly loss and suddenly gain in current wealth respectively. For instance, if other condition are controlled, when an individual become suddenly worse in his or her current wealth, he or she may become more aggressive on his or her investment decision, so as to recover his or her loss to the "disaster wealth" point as soon as possible, this feature could be reflect by the degree of slopes in both Figures. In Figure 9 and Figure 10, the costs of game are almost same in negative side, but lower in positive side in Figure 10, this phenomenon express that when an individual with sudden gain in current wealth, then, the individual may become more aggressive in investment decisions. These results also support the findings of Graves (1979) that if an individual's wealth and bet size are double, such individual may be more likely to take the bet, because he or she would have more left even if the outcome is lose (Loewenstein et al., 2001; De Giorgi and Post, 2011; Shleifer, 2012).

Conclusions

The current paper presents an unique utility function to fill the gaps existed in prior studies that: even there is at least 40% of individual with concave utility functions under loss domain, no specified utility function can be used to represent these populations (Abdellaoui et al., 2007); moreover, there is no utility function which includes psychological perspectives when analyze the loss aversion behavior (Loewenstein et al., 2001; Novemsky and Kahneman, 2005; Camerer, 2005; Shleifer, 2012).

The utility function in current study discovers that the psychological factors (evaluate the risk cognitively and react to risk emotionally) do influence loss aversion behavior. For example, when other conditions are controlled, an individual who cognize the unexpected events more seriously, the slope of utility function become more vertical in the negative domain, which means that this individual want to recover his or her loss to the "disaster wealth" point as soon as possible. Meanwhile, if other condition are kept, the faster response to risk changes, the more vertical of slope of utility function will be, this result shows that individual react more faster emotionally to risk changes, so, he or she become more risk taking. Generally, the higher degree of psychological impact (how seriously individual cognize the risks) and higher level of psychological response (how fast individual react to risk changes emotionally) tend to switch individuals' risk averter to be risk taker under a certain difference between current wealth and certain equivalent, or vice versa.

Finally, the simulation results by using the utility function in current study are aligned with many prior researches, for instance, when current wealth levels fall below exogenous referent points, individuals may switch their risk aversion behaviors so as to follow the "safety first principle" (Gordon et al., 1972; Graves, 1979; Abdellaoui et al., 2007; De Giorgi and Post, 2011; Abbas and Bell, 2011). At the same time, the current study avoid the problems existed in prior studies, such as the increment simultaneously for both absolute risk and relative risk aversion under quadratic functions, and the biases from subjective weighted probabilities under expected utility functions.

Furthermore, the utility function in current study is also aligned with risk lover properties under the prospect theory (See the Appendix B), and this utility function can be employed empirically to measure both loss aversion behavior and psychological factors under the heterogeneity among individuals towards the risky choices. For instance, theoretically, future researches could focus on the measurements of psychological factors (*a* and *n*), while, empirically, financial institutions, including banks could use this utility function to evaluate each parameter for different classes of clients, which are categorized by their incomes, ages, and habits, so as to estimate their risk preference and psychological factors more realistically.

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Appendix A

$U(c) = a(c)^n + b(c)$ a>0, b>0, n=3,5,7,9, 11, and N=/1, 2, even numbers;
(1)

$U'(c) = an(c)^{n-1} + b > 0$, $an(c)^{n-1} > -b$; $U''(c) = an(n-1)(c)^{n-2} < 0$ only for c<0;
(2)

$R_A(c) = -U''(c)/U'(c) = -\ [an\ (n-1)*(c)^{n-2}/an*c^{n-1}+b]$
(3)

$R_R(c) = -c*U''(c)/U'(c) = -\ [c*an\ (n-1)\ (c)^{n-2}/an*c^{n-1}+b]$
(4)

$d\ R_A(c)/d(c) = -\ \{an\ (n-1)(c)^{n-3} * [b\ (n-2)-an\ (c)^{n-1}]/(an(c)^{n-1}+b)^2\}$
(5)

When n = positive odd integer number, $c<0$, then,

d $R_A(c)/d(c)$ <0, =0, >0, will be depended on $[b\,(n-2)-an\,(c)^{n-1}] > 0, =0, <0$; (DARA, Indifferent, or IARA);

When n = positive odd integer number, $c>0$, then,

d $R_A(c)/d(c)$ <0, =0, >0 will be depended on $[b\,(n-2)-an\,(c)^{n-1}]>0, =0, <0$; (DARA, Indifferent, or IARA);

Proof:

$$d\,R_A(c)/d(c) = \{ - [an(n-1)*(c)^{n-2}/(an*c^{n-1}+b)]\}'$$

$$= - \frac{[an\,(n-1)\,(n-2)\,(c)^{(n-3)}*(an(c)^{(n-1)}+b)]-[an\,(n-1)\,(c)^{(n-2)}*an\,(n-1)\,(c)^{(n-2)}]}{[(an(c)^{(n-1)}+b)]^{2}}$$

$$= - \frac{[an\,(n-1)\,(c)^{(n-3)}*(b\,(n-2)-an(c)^{(n-1)})]}{[(an(c)^{(n-1)}+b)]^{2}}$$

As $[an\,(n-1)\,(c)^{(n-3)}$ always > 0, so, the sign of d $R_A(c)/d(c)$ will be dependent on $[(b\,(n-2)-an(c)^{(n-1)})]$

$$d\,R_R(c)/d(c) = -[b*an*(n-1)^2*(c)^{n-2}/(an*(c)^{n-1}+b)^2];$$
(6)

Proof:

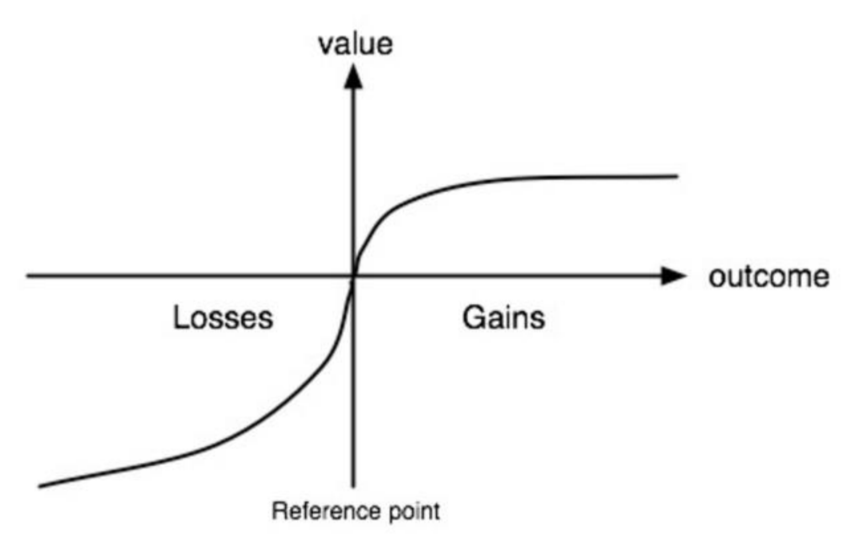
$$d\,R_R(c)/d(c) = \{ - c * [an(n-1)*(c)^{n-2}/(an*c^{n-1}+b)]\}'$$

$$= - \frac{[an\,(n-1)\,(n-1)\,(c)^{(n-2)}*(an(c)^{(n-1)}+b) - an\,(n-1)*(c)^{(n-2)}*an\,(n-1)\,(c)^{(n-1)}]}{[(an(c)^{(n-1)}+b)]^{2}}$$

$$= - \frac{[b*an*(n-1)^{2}*(c)^{(n-2)}]}{[(an(c)^{(n-1)}+b)]^{2}}$$

n = 3, 5, 7, 9, 11, When $c<0$, then, d $R_R(c)/d(c) > 0$, when, $c>0$, then, d $R_R(c)/d(c) < 0$

Appendix B



1) Vertical flip and
2) Turn 90 degree to the left, get the following: