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Original Article

Quadripartitioned single valued neutrosophic sets with covering based rough sets and their matrix representation

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Abstract

The notion of a quadripartitioned single-valued neutrosophic set (in short QSVNS) is considered to be the more general mathematical framework of the neutrosophic set to model indeterminacy. In QSVNS, the indeterminacy component is divided into two parts, namely, “unknown” and “contradiction”. In a real-life scenario, while handling indeterminacy, we may have some hesitation about whether the indeterminacy occurs due to the belongingness or the non-belongingness of an object. This leads to the introduction of QSVNS. On the other hand, the theory of rough set (RS) is introduced to depict the incomplete data hidden in nature with an aid of equivalence relation. So, by combining the QSVNS and the RS, a new mathematical structure known as a quadripartitioned single-valued neutrosophic rough set is formed. The main purpose of this article is to present two types of quadripartitioned single-valued neutrosophic covering rough set models. Also, we have introduced QSVN β-covering approximation space and studying some of its properties. Based on QSVN β -covering approximation spaces, two types of QSVN covering rough set models are investigated. Furthermore, a matrix representation of the QSVN covering-based rough set model is developed. Finally, an algorithm-based model under QSVN covering-based rough set is developed and employed in a case study to diagnose a patient/s that is more likely suffering from a disease having certain symptoms.

Keywords: covering rough set, quadripartitioned single-valued neutrosophic set, approximation space, decision matrix

1. Introduction

Zadeh’s (Zadeh, 1965) notion of a fuzzy set (FS) is capable of dealing with different classes of imprecise objects that are influenced by human knowledge in various practical applications. A membership function is the main constituent of a fuzzy set where the value of the membership function for each element of a domain ranges between 0 and 1. So, the fuzzy set provides more information to the decision-makers to make their decisions more authentic and valid. It has attracted the attention of researchers from all over the globe and so they investigated the FS deeply and obtained many interesting results. To date, the FS has been used successfully in almost all branches of Mathematics. Some useful works based on FS are addressed in Guiffrida and Nagi, (1998), Jiang and Eastman, (2000), Yagar (1982), and Zimmermann (2011). To make the fuzzy set more functional and operational, it has been extended to intuitionistic fuzzy set (Atanassov, 1986), interval-valued fuzzy set (Gorzalczany, 1987), interval-valued intuitionistic fuzzy set (Atanassov & Gargov, 1989), spherical fuzzy set (Ashraf, Abdullah, Mahmood, Ghani & Mahmood, 2019), picture fuzzy set (Cuong & Kreinovich, 2013), hesitant fuzzy set (Torra, 2010), bipolar fuzzy set (Zhang, 1998), Pythagorean fuzzy set (Peng & Selvachandran, 2019), and linguistic approach to fuzzy set in decision-making (Tong & Bonissone, 1980), etc.

In 1982, Pawlak (Pawlak, 1982) introduced the concept of a rough set (RS) to handle the inconsistent, incomplete, imprecise information present in various fields of human knowledge. It can easily catch the attention of researchers, especially in the field of artificial intelligence, cluster analysis, measurement theory, classification theory, taxonomy, etc. In Pawlak, (1998) has shown a practical application of RS in data analysis. Bai and Sarkis (2010) introduced the RS theory in green supplier development. Chen, Miao, and Wang (2010) presented a rough set model motivated by ant colony optimization. Sharma, Kumari, and Kar (2020) proposed an RS model for forecasting, etc. The

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covering-based rough sets are introduced for covering data in knowledge management (Deer, Restrepo, Cornelis, & Gomez, 2016; Yao & Yao, 2012; Zhu, 2009). For the generalization purpose, the covering-based rough set is connected with the fuzzy set theory proposed in Jiang, Zhan, Sun and Alcantud (2020); Yang and Hu (2018), Zhan and Sun (2020), Zhan, Jiang and Yao (2020), Zhan, Zhang and Yao (2020).

In 1998, Smarandache introduced a new branch of Philosophy, known as "Neutrosophy" and it is used to study the neutralities of an object. Here an object means any item or axiom or postulate or theorem, etc. Later on, in 2005, Smarandache initiated another mathematical theory known as neutrosophic set (NS) (Smarandache, 2005). The NS can be viewed as an extension of the intuitionistic fuzzy set. In an NS environment, the decision-makers are capable to address the issues that contain indeterminacy, inconsistency, or incompleteness. So, NS is proved to be a more powerful and superior tool than the other existing ones. For scientific and technical support systems, Wang, Smarandache, Zhang and Sunderraman (2010) introduced the single-valued neutrosophic set (SVNS) as an instance of NS. Both RS and NS are capable to manage uncertain, incomplete, imprecise, and inconsistent information, which leads to the introduction of the rough neutrosophic set (RNS) proposed by Mondal and Pramanik (2015). In Wang and Zhang (2020) introduced the single-valued neutrosophic(SVN) covering-based rough sets in multi-criteria group decision-making problems. The two types of SVN covering rough sets and their application are proposed in Wang and Zhang (2018).

The NS is takes care of indeterminacy separately i.e it is independent of truthiness and falsity. In NS, each element of a universe is characterized by a truth-membership value ($T_A(x)$), an indeterminacy-membership value ($I_A(x)$), and a falsity-membership value ($F_A(x)$) in such a way that each membership value belongs to the non-standard interval $[^-0, 1^+]$ in such a manner that $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$. As the real non-standard interval $[^-0, 1^+]$ does not represent a precise interval, so to remove such an issue, the SVNS (Wang, Smarandache, Zhang & Sunderraman, 2010) is introduced by replacing $[^-0, 1^+]$ by $[0, 1]$ with the restriction $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. Some recent works associated to neutrosophic set are proposed in Rattana and Chinram (2020), Samad, Zulqarnain, Sermutlu, Ali, Siddique, Jarad and Abdeljawad (2021), Songsaeng, Shum, Chinram and Iampan (2021), Zulqarnain, Garg, Siddique, Alsubie, Hamadneh and Khan (2021), Zulqarnain, Saeed, Ahamad, Abdal, Zafar, and Aslam (2020), Zulqarnain, Xin, Saeed, Smarandache and Ahmad (2020). In our real-life scenario, we may come across such a situation where there is hesitation while handling with indeterminacy. It is quite natural from a practical point of view that indeterminacy can be divided into two parts, namely contradiction i.e. both true and false, and unknown i.e. neither true nor false. So, there is a serious need to generalize the notion of SVNS. Chatterjee, Majumder and Samanta (2016) introduced the notion of a quadripartitioned single-valued neutrosophic set (QSVNS) as an extension of SVNS. In addition, the QSVNS concept is mainly motivated by Smarandache's n-valued refined neutrosophic set, and to refined neutrosophic logic, and to refined neutrosophic probability (Smarandache, 2013), where the true value T is refined/split into types of sub-truths such as T_1, T_2, ... Similarly, indeterminacy I is refined/split into types of sub-indeterminacies I_1, I_2, … , and the falsehood F is refined/split into sub-falsehood F_1, F_2, … . Also, we may consider the QSVNS as a particular case of the refined neutrosophic set. Moreover, Ulucay presented the concept of interval-valued refined neutrosophic sets and their applications (Ulucay, 2021). Kandasamy *et al.* studied the refined neutrosophic sets for the sentiment analysis of tweets (Kandasamy, Vasantha, Obbineni & Smarandache, 2020). Abobala studied the matrix representation of the refined neutrosophic sets in the algebraic equations (Abobala, 2021). Freen *et al.* introduced the multi-objective non-linear four-valued refined neutrosophic optimization (Freen, Kousar, Khalil & Imran, 2020). In QSVNS, the component $I_A(x)$ is split into two components, namely, the contradiction component $C_A(x)$ and the unknown or ignorance component $U_A(x)$ where $T_A(x)$, $C_A(x)$, $U_A(x)$, $F_A(x) \in [0, 1]$ such that $0 \le T_A(x) + C_A(x) + U_A(x) + F_A(x) \le 4$. Matrix theory has captured more attention from researchers over the decades due to its unique structure of storing large data in a small space and it is easy to handle different types of aggregate operators on matrices for decision-making problems. Through matrix representations, calculations become easier and they can be easily evaluated through the computer. To handle uncertain information, the classical matrix theory has been expanded extensively in various forms and many new operators are introduced by the researchers to model the vagueness.

In Wang and Zhang (2018), studied two types of single-valued neutrosophic rough set models to solve decision-making problems. Getting motivation from this, in the present paper, we have combined QSVNSs and covering based rough sets. As such combination has not been found in any research article to date, it gives a valid reason to propose the present study. Also, we present the two types of QSVN covering rough set models. This new combination surely provides a new insight to the decision-makers or researchers to invent more interesting results relating to the current topic.

1.1 Novelty

The author believes that the covering-based rough set model under the quadripartitioned single-valued neutrosophic settings has not been introduced in any research work to date. This gives rise to the introduction of quadripartitioned single-valued covering-based rough set models and thus we use their matrix representation for the MCDM mechanism to solve a real-world problem by using a new algorithm in this paper. In our literature review, it has been observed that the covering-based rough set concept in various topics has been successfully applied under the fuzzy set (Yang & Hu, 2018; Zhan, Jiang & Yao, 2020; Zhan, Zhang & Yao, 2020), intuitionistic fuzzy set (Zhan & Sun, 2020), and single-valued neutrosophic set (Wang & Zhang, 2018, 2020). Although a lot of topics based on covering based rough set model have been explored, these are not relevant to our present topic. The single-valued neutrosophic set presented in Wang, Smarandache, Zhang & Sunderraman (2010) has been developed to model uncertainty that contains indeterminacy and it is the generalization of the fuzzy set (Zadeh, 1965; Zimmermann, 2011) and intuitionistic fuzzy set (Atanassov, 1986). As we know, the human brain is responsible for how to respond to an uncertain problem and there is no specific parameter to measure uncertainty. But we aim to reduce the uncertainty level to come close to the actual result. Indeterminacy is a complicated element to measure imprecise knowledge and the SVNS is a tool to handle it a bit. After the in-depth study of the SVNS, the researchers have been pointed out that, indeterminacy may occur due to contradictory and unknown information. To realize this, Chatterjee, Majumdar and Samanta (2016) coined the term "quadripartitioned single-valued neutrosophic set" to gather more information about indeterminacy, and so it captures more attention from the scientist because it influences human intelligence quite significantly. To fill up this knowledge gap, we have introduced the quadripartitioned single-valued neutrosophic covering-based rough set model. Moreover, by conducting this work, we try to extend the single-valued neutrosophic covering-based rough set model proposed in Wang and Zhang (2018). And we hope that the proposed study will provide more imprecise knowledge to the decision-makers which help them a lot to make a more accurate decision by reducing uncertainty. Therefore, the main advantage of the present study is that, due to the inherent structure of the proposed model, it can accommodate more hidden information that is useful for in-depth analysis. And to reduce the complex structure of the models, we introduce their matrix representation for manipulating and computing large data sets simply.

1.2 Structure of the paper

The rest of the paper is organized in the following manner: Section 2 includes the fundamental concepts that are relevant in the context of the present article. Section 3 contains the QSVN β-covering approximate space. In Section 4, we have studied two types of QSVN covering-based rough set models. Section 5 is based on the matrix representation of the QSVN rough set approximation model. An application of QSVN Covering Based Rough Set in Medical Diagnosis Problem is proposed in section 6. The last section i.e. Section 7, includes a conclusion with future direction.

2. Preliminaries

In this section, we give brief fundamental concepts that are useful for the subsequent sections of the present study.

Definition 2.1 (Bonikowski, Bryniarski, & Wybraniec-Skardowska, 1998; Pomykala, 1987) Let U be an initial universe and I^U denotes a family of subsets of U. If none of subsets of I^U is empty and $\bigcup I^U = U$, then I^U is called a covering of U.

Definition 2.2 (Pawlak, 1982) Let U be the set of the universe and ρ be an equivalence relation on U. Then the couple (U, ρ) is called a Pawlak approximation space. Here ρ generates a partition $U/\rho = \{\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n\}$ on U, where $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n$ are the equivalence classes generated by the equivalence relation ρ.

For any $X \subseteq U$, we can represent the lower and upper approximation of X by $\underset{*}{\rho}(X)$ and $\overset{*}{\rho}(X)$ respectively and they are defined as:

$$\underset{*}{\rho}(X) = \bigcup\{\Upsilon_i \in U/\rho : \Upsilon_i \subseteq X\},\ \overset{*}{\rho}(X) = \bigcup\{\Upsilon_i \in U/\rho : \Upsilon_i \cap X \neq \varnothing\}$$

If $\underset{*}{\rho}(X) \neq \overset{*}{\rho}(X)$, then X is called a rough set.

Definition 2.3 (Wang, Smarandache, Zhang & Sunderraman, 2010) Let U be a non-empty universal set. Then, a single-valued neutrosophic set (SVNS) A in U is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}$, where $T_A(x), I_A(x), F_A(x)$ denote the truth-membership, indeterminacy-membership, and falsity-membership values and $T_A(x), I_A(x), F_A(x) \in [0,1]$ such that $T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.4 (Chatterjee, Majumdar, & Samanta, 2016) Let X be a non-empty universal set. Then, a quadripartitioned single-valued neutrosophic set(QSVNS) A over U characterizes each element $x \in X$ by a truth-membership function T_A, a contradiction-membership function C_A, an unknown-membership function U_A, and a falsity-membership function F_A such that for each $x \in X$, T_A, C_A, U_A, $F_A \in [0, 1]$ and $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$. The family of all QSVNSs in X is denoted by $QSVNS^X$. For simplicity, a *QSVN* number is represented by $\alpha = \langle a, b, c, d\rangle$, where $a, b, c, d \in [0,1]$, and $a + b + c + d \leq 4$.

Definition 2.5 For any $A, B \in$ QSVNS^X, we have the following properties:

(1) For any $x \in X$, $A \subseteq B$

iff $T_A(x) \leq T_B(x), C_A(x) \leq C_B(x), U_A(x) \geq U_B(x)$, and $F_A(x) \geq F_B(x)$

(2) $A \cap B = \{\langle x, T_A(x) \wedge T_B(x), C_A(x) \wedge C_B(x), U_A(x) \vee U_B(x), F_A(x) \vee F_B(x)\rangle : x \in X\}$

(3) $A \bigcup B = \left\{ \left\langle x, T_A(x) \vee T_B(x), C_A(x) \vee C_B(x), U_A(x) \wedge U_B(x), F_A(x) \wedge F_B(x) \right\rangle : x \in X \right\}$

(4) $\overline{A}$ or $A' = \left\{ \left\langle x, F_A(x), U_A(x), C_A(x), T_A(x) \right\rangle : x \in X \right\}$

(5) $A \oplus B = \left\{ \left\langle x, T_A(x) + T_B(x) - T_A(x).T_B(x), C_A(x) + C_B(x) - C_A(x).C_B(x), U_A(x).U_B(x), F_A(x).F_B(x) \right\rangle : x \in X \right\}$

(6) $A \otimes B = \left\{ \left\langle x, T_A(x).T_B(x), C_A(x).C_B(x), U_A(x) + U_B(x) - U_A(x).U_B(x), F_A(x) + F_B(x) - F_A(x).F_B(x) \right\rangle : x \in X \right\}$

3. Quadripartitioned Single-Valued Neutrosophic *β*-Covering Approximation Space

In this section, we introduce a new approximation space known as quadripartitioned single-valued neutrosophic *β*-covering approximation space. To get insight into the new approximation space, we discuss the following.

Definition 3.1 Let *X* be an initial universe and $QSVNS^X$ be the QSVN power set of *X*. For a QSVN number $\beta = \langle a, b, c, d \rangle$, the set $\bar{P} = \{P_1, P_2, ..., P_n\}$, where $P_i \in QSVN^X$ is called a QSVN *β*-covering of *X*, if for all *x* ϵ *X*, there exists $P_i \in P$ such that $P_i(x) \geq \beta$ and (*X, P*) is called a QSVN *β*-covering approximation space.

Definition 3.2 Suppose $\bar{P} = \{P_1, P_2, ..., P_n\}$ be a QSVN *β*-covering of *X*. Then, for any *x* ϵ *X*, the QSVN *β*-neighborhood $\hat{N}_x^\beta$ of *x* induced by *P* is defined as:

$$\hat{N}_x^\beta = \bigcap \left\{ P_i \in P : P_i(x) \geq \beta \right\}$$

More precisely, we can write

$$\hat{N}_x^\beta = \bigcap \left\{ P_i \in P : T_{P_i}(x) \geq a, C_{P_i}(x) \geq b, U_{P_i}(x) \leq c, F_{P_i}(x) \leq d \right\}$$

Example 3.2.1 Let $X = \{x_1, x_2, x_3, x_4\}$, $P = \{P_1, P_2, P_3, P_4\}$ and $\beta = \langle 0.5, 0.4, 0.6, 0.7 \rangle$. Then, the QSVN *β*-covering of *X* induced by *P* is represented by the following Table 1. By using definition 3.2**,** we have

$$\hat{N}_{x_1}^\beta = P_2 \bigcap P_4,\ \hat{N}_{x_2}^\beta = P_1 \bigcap P_4,\ \hat{N}_{x_3}^\beta = P_1 \bigcap P_2 \bigcap P_3 \bigcap P_4,\ \hat{N}_{x_4}^\beta = P_1 \bigcap P_2 \bigcap P_3$$

Table 1. The tabular representation of QSVN *β*-covering of *X* induced by *P*

X	P_1	P_2	P_3	P_4
X_1	$\langle 0.6, 0.3, 0.3, 0.4 \rangle$	$\langle 0.6, 0.5, 0.4, 0.3 \rangle$	$\langle 0.6, 0.5, 0.7, 0.6 \rangle$	$\langle 0.9, 0.6, 0.5, 0.5 \rangle$
X_2	$\langle 0.7, 0.5, 0.3, 0.6 \rangle$	$\langle 0.4, 0.4, 0.2, 0.8 \rangle$	$\langle 0.8, 0.3, 0.4, 0.2 \rangle$	$\langle 0.6, 0.8, 0.4, 0.5 \rangle$
X_3	$\langle 0.7, 0.6, 0.5, 0.6 \rangle$	$\langle 0.5, 0.4, 0.5, 0.1 \rangle$	$\langle 0.7, 0.7, 0.6, 0.5 \rangle$	$\langle 0.7, 0.6, 0.4, 0.5 \rangle$
X_4	$\langle 0.5, 0.5, 0.3, 0.4 \rangle$	$\langle 0.8, 0.7, 0.5, 0.4 \rangle$	$\langle 0.7, 0.5, 0.4, 0.5 \rangle$	$\langle 0.6, 0.3, 0.7, 0.5 \rangle$
$\hat{N}_{x_k}^\beta$	X_1	X_2	X_3	X_4
$\hat{N}_{x_1}^\beta$	$\langle 0.6, 0.5, 0.5, 0.5 \rangle$	$\langle 0.4, 0.5, 0.4, 0.3 \rangle$	$\langle 0.6, 0.3, 0.7, 0.6 \rangle$	$\langle 0.8, 0.6, 0.7, 0.5 \rangle$
$\hat{N}_{x_2}^\beta$	$\langle 0.4, 0.5, 0.7, 0.6 \rangle$	$\langle 0.6, 0.5, 0.4, 0.6 \rangle$	$\langle 0.5, 0.3, 0.5, 0.8 \rangle$	$\langle 0.5, 0.2, 0.4, 0.5 \rangle$
$\hat{N}_{x_3}^\beta$	$\langle 0.2, 0.4, 0.5, 0.6 \rangle$	$\langle 0.5, 0.6, 0.8, 0.1 \rangle$	$\langle 0.5, 0.4, 0.6, 0.6 \rangle$	$\langle 0.3, 0.1, 0.4, 0.5 \rangle$
$\hat{N}_{x_4}^\beta$	$\langle 0.2, 0.4, 0.7, 0.4 \rangle$	$\langle 0.4, 0.5, 0.7, 0.4 \rangle$	$\langle 0.3, 0.4, 0.3, 0.5 \rangle$	$\langle 0.5, 0.5, 0.5, 0.5 \rangle$

Therefore, we represent all QSVN *β*-neighborhood $\hat{N}_{x_k}^\beta$ (k=1, 2, 3, 4) in Table 2 given as:

Table 2. The tabular representation of $\hat{\mathrm{N}}_{x_k}^{\beta}$ (k=1, 2, 3, 4)

Algorithm: Decision-making algorithm based on QSVN rough set model for medical diagnosis

Input: The decision-maker provided the QSVN information system $\left\langle X, P, \beta, \mathrm{A} \right\rangle$

Output: Choose the best alternative according to the ranking score ordering of all the alternatives.
Computations:

- Compute the QSVN β-neighborhood $\hat{\mathrm{N}}_x^{\beta}$ of x induced by P (See Definition 3.2)
- Compute the QSVN covering upper approximation $\overset{*}{P}(A)$ and lower approximation $\underset{*}{P}(A)$ of A (See Definition 4.1)
- Calculate the aggregate value by using $\hat{\Theta}(A) = \overset{*}{P}(A) \oplus \underset{*}{P}(A)$ OR $\hat{\Theta}(A) = \overset{*}{P}(A) \otimes \underset{*}{P}(A)$ (See Definition 2.5)
- Compute the score value
$$\Lambda(x) = \frac{T_{\hat{\Theta}(A)}(x) + C_{\hat{\Theta}(A)}(x)}{\sqrt{\left(T_{\hat{\Theta}(A)}(x)\right)^2 + \left(C_{\hat{\Theta}(A)}(x)\right)^2 + \left(U_{\hat{\Theta}(A)}(x)\right)^2 + \left(F_{\hat{\Theta}(A)}(x)\right)^2}}$$
- Rank all the alternatives according to their score values and choose the best possible patient for diagnosis

X	X_1	X_2	X_3	X_4
$\Lambda(x_k)$	1.39	1.40	0.4	1.3

OR

X	X_1	X_2	X_3	X_4
$\Lambda(x_k)$	0.71	0.63	0.4	0.45

We present the properties of the QSVN β-neighborhood in the following theorem.

Theorem 3.3 Let $\bar{P} = \{P_1, P_2, ..., P_n\}$ be a QSVN β-covering of X. Then, we claim that the following statements hold:

(i) $\hat{\mathrm{N}}_x^{\beta}(x) \geq \beta$, for each $x \in X$

(ii) For all $x, y, z \in X$, $\hat{\mathrm{N}}_x^{\beta}(y) \geq \beta$ and $\hat{\mathrm{N}}_y^{\beta}(z) \geq \beta \Rightarrow \hat{\mathrm{N}}_x^{\beta}(z) \geq \beta$

(iii) For any two QSVN numbers β_1, β_2, if $\beta_1 \leq \beta_2 \leq \beta$, then $\hat{\mathrm{N}}_x^{\beta_1} \subseteq \hat{\mathrm{N}}_x^{\beta_2}$ for all $x \in X$.

Proof. All proofs are straightforward.

Theorem 3.4 Let $\bar{P} = \{P_1, P_2, ..., P_n\}$ be a QSVN β-covering of X. For any x, y $\in X$, $\hat{\mathrm{N}}_x^{\beta}(y) \geq \beta$ if and only if $\hat{\mathrm{N}}_y^{\beta} \subseteq \hat{\mathrm{N}}_x^{\beta}$.

Proof.

$(\Rightarrow)$ Suppose $\beta = \langle a, b, c, d \rangle$ be a QSVN number. For $\hat{\mathrm{N}}_x^{\beta}(y) \geq \beta$,

$$T_{\hat{\mathrm{N}}_x^{\beta}(y)} = T \bigcap_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} P_i(y) = \bigwedge_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} T_{P_i(y)} \geq a, \quad C_{\hat{\mathrm{N}}_x^{\beta}(y)} = C \bigcap_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} P_i(y) = \bigwedge_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} C_{P_i(y)} \geq b$$

$$U_{\hat{\mathrm{N}}_x^{\beta}(y)} = U \bigcap_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} P_i(y) = \bigwedge_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} U_{P_i(y)} \leq c, \quad F_{\hat{\mathrm{N}}_x^{\beta}(y)} = F \bigcap_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} P_i(y) = \bigwedge_{\substack{T_{P_i}(x) \geq a \\ C_{P_i}(x) \geq b \\ U_{P_i}(x) \leq c \\ F_{P_i}(x) \leq d}} F_{P_i(y)} \leq d$$

Now, for each $z \in X$,

$$T_{\hat{N}_x^\beta(z)} = \bigwedge_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} T_{P_i(z)} \ge \bigwedge_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} T_{P_i(z)} = T_{\hat{N}_y^\beta(z)},\ C_{\hat{N}_x^\beta(z)} = \bigwedge_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} C_{P_i(z)} \ge \bigwedge_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} C_{P_i(z)} = C_{\hat{N}_y^\beta(z)},$$

$$U_{\hat{N}_x^\beta(z)} = \bigvee_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} U_{P_i(z)} \le \bigwedge_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} U_{P_i(z)} = U_{\hat{N}_y^\beta(z)},\ F_{\hat{N}_x^\beta(z)} = \bigvee_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} F_{P_i(z)} \le \bigwedge_{\substack{T_{P_i}(x)\ge a\\ C_{P_i}(x)\ge b\\ U_{P_i}(x)\le c\\ F_{P_i}(x)\le d}} F_{P_i(z)} = F_{\hat{N}_y^\beta(z)}$$

Hence, $\hat{N}_y^\beta \subseteq \hat{N}_x^\beta$.

$(\Leftarrow)$ For any $x, y \in X$, $\hat{N}_y^\beta \subseteq \hat{N}_x^\beta$. Then,

$T_{\hat{N}_x^\beta(y)} \ge T_{\hat{N}_y^\beta(y)} \ge a$, $C_{\hat{N}_x^\beta(y)} \ge C_{\hat{N}_y^\beta(y)} \ge b$, $U_{\hat{N}_x^\beta(y)} \le U_{\hat{N}_y^\beta(y)} \le c$, and $F_{\hat{N}_x^\beta(y)} \le F_{\hat{N}_y^\beta(y)} \le d$.

Next, we give the notion of QSVN β-neighborhood in the QSVN β-covering approximation space in the following.

Definition 3.5 Let *(X, P)* be a QSVN β-covering approximation space where $P = \{P_1, P_2, \ldots, P_n\}$. Then, for each $x \in X$, we define the QSVN β-neighborhood $\hat{N}_x^\beta$ of *x* as:

$$\hat{N}_x^\beta = \left\{ y \in X : \hat{N}_x(y) \ge \beta \right\}, \text{ where } \hat{N}_x(y) = \left\langle T_{\hat{N}_x^\beta(y)}, C_{\hat{N}_x^\beta(y)}, U_{\hat{N}_x^\beta(y)}, F_{\hat{N}_x^\beta(y)} \right\rangle.$$

Theorem 3.6 Let $P = \{P_1, P_2, \ldots, P_n\}$ be a QSVN β-covering of *X*. Then the following statements hold:

(i) $x \in \hat{N}_x^\beta$ for each $x \in X$

(ii) For all $x, y, z \in X$, if $x \in \hat{N}_y^\beta$, $y \in \hat{N}_z^\beta$, then $x \in \hat{N}_z^\beta$.

Proof. Proofs are obvious.

Theorem 3.7 Let $P = \{P_1, P_2, \ldots, P_n\}$ be a QSVN β-covering of *X*. Then, for all $x \in X$, $x \in \hat{N}_y^\beta$ *iff* $\hat{N}_x^\beta \subseteq \hat{N}_y^\beta$.

Proof. $(\Rightarrow)$ For any $z \in \hat{N}_x^\beta$, we have $\hat{N}_x^\beta(z) \ge \beta$. Again, $x \in \hat{N}_y^\beta, \hat{N}_y^\beta(x) \ge \beta$.

By using (ii) of theorem 3.3, we have $\hat{N}_x^\beta(z) \ge \beta$. Hence, $z \in \hat{N}_y^\beta$.

Therefore, $\hat{N}_x^\beta \subseteq \hat{N}_y^\beta$.

$(\Leftarrow)$ By using (i) of theorem 3.6, for each $x \in X$, $x \in \hat{N}_x^\beta$. If $\hat{N}_x^\beta \subseteq \hat{N}_y^\beta$ then $x \in \hat{N}_y^\beta$.

4. Two Types of Quadripartitioned Single Valued Neutrosophic Covering Rough Set Models

In this section, we introduce the two types of QSVN covering rough set models based on QSVN β-neighborhoods. Then we investigate their lower and upper approximate operators.

Definition 4.1

Let *(X, P)* be a QSVN β-covering approximation space. For any $A \in QSVN^X$, where $A = \{\langle x, T_A(x), C_A(x), U_A(x), F_A(x)\rangle : x \in X\}$ we define the QSVN covering upper approximation $\overset{*}{P}(A)$ and lower approximation $\underset{*}{P}(A)$ of *A* as follows:

$$\overset{*}{P}(A) = \left\{ \left\langle \begin{matrix} x, \vee_{y\in X}[T_{\hat{N}_x^\beta(y)} \wedge T_{A(Y)}], \vee_{y\in X}[C_{\hat{N}_x^\beta(y)} \wedge C_{A(Y)}], \wedge_{y\in X}[U_{\hat{N}_x^\beta(y)} \vee U_{A(Y)}], \\ \wedge_{y\in X}[F_{\hat{N}_x^\beta(y)} \vee F_{A(Y)}] \end{matrix} \right\rangle : x \in X \right\}$$

$$\underset{*}{P}(A)=\left\{\left\langle \begin{array}{l} x,\wedge_{y\in X}[F_{\hat{N}_x^{\beta}(y)}\vee T_{A(Y)}],\wedge_{y\in X}[U_{\hat{N}_x^{\beta}(y)}\vee C_{A(Y)}],\vee_{y\in X}[C_{\hat{N}_x^{\beta}(y)}\wedge U_{A(Y)}],\\ \vee_{y\in X}[T_{\hat{N}_x^{\beta}(y)}\wedge F_{A(Y)}] \end{array}\right\rangle : x\in X\right\}$$

If $\overset{*}{P}(A)\neq \underset{*}{P}(A)$, then *A* is called the first type of QSVN covering rough set.

Proposition 4.2

Let *P* be a QSVN *β*-covering of *X*. Then for any $A,B\in QSVN^X$, the QSVN upper and lower approximation operators have the following properties:

(i) $\overset{*}{P}\left(\overline{A}\right)=\left(\overline{\underset{*}{P}(A)}\right), \underset{*}{P}\left(\overline{A}\right)=\left(\overline{\overset{*}{P}(A)}\right)$

(ii) If $A\subseteq B$, then $\underset{*}{P}(A)\subseteq \underset{*}{P}(B)$, $\overset{*}{P}(A)\subseteq \overset{*}{P}(B)$

(iii) $\underset{*}{P}(A\cap B)=\underset{*}{P}(A)\cap \underset{*}{P}(B)$, $\overset{*}{P}(A\cup B)=\overset{*}{P}(A)\cup \overset{*}{P}(B)$

(iv) $\underset{*}{P}(A\cup B)\supseteq \underset{*}{P}(A)\cup \underset{*}{P}(B)$, $\overset{*}{P}(A\cap B)\subseteq \overset{*}{P}(A)\cap \overset{*}{P}(B)$

Proof. (i) We have,

$$\overset{*}{P}\left(\overline{A}\right)=\left\{\left\langle \begin{array}{l} x,\vee_{y\in X}[T_{\hat{N}_x^{\beta}(y)}\wedge T_{\overline{A}(Y)}],\vee_{y\in X}[C_{\hat{N}_x^{\beta}(y)}\wedge C_{\overline{A}(Y)}],\wedge_{y\in X}[U_{\hat{N}_x^{\beta}(y)}\vee U_{\overline{A}(Y)}],\\ \wedge_{y\in X}[F_{\hat{N}_x^{\beta}(y)}\vee F_{\overline{A}(Y)}] \end{array}\right\rangle : x\in X\right\}$$

$$=\left\{\left\langle \begin{array}{l} x,\vee_{y\in X}[T_{\hat{N}_x^{\beta}(y)}\wedge F_{A(Y)}],\vee_{y\in X}[C_{\hat{N}_x^{\beta}(y)}\wedge U_{A(Y)}],\wedge_{y\in X}[U_{\hat{N}_x^{\beta}(y)}\vee C_{A(Y)}],\\ \wedge_{y\in X}[F_{\hat{N}_x^{\beta}(y)}\vee T_{A(Y)}] \end{array}\right\rangle : x\in X\right\}$$

$$=\left(\overline{\underset{*}{P}(A)}\right)$$

Similarly, $\underset{*}{P}\left(\overline{A}\right)=\left(\overline{\overset{*}{P}(A)}\right)$.

(ii) As $A\subseteq B$, so $T_A(x)\le T_B(x), C_A(x)\le C_B(x), U_A(x)\ge U_B(x)$, and $F_A(x)\ge F_B(x)$, $\forall x\in X$.

$$T_{\underset{*}{P}(A)}(x)=\wedge_{y\in X}\left[F_{\hat{N}_x^{\beta}(y)}\vee T_A(y)\right]\le \wedge_{y\in X}\left[F_{\hat{N}_x^{\beta}(y)}\vee T_B(y)\right]=T_{\underset{*}{P}(B)}(x),$$

$$C_{\underset{*}{P}(A)}(x)=\wedge_{y\in X}\left[U_{\hat{N}_x^{\beta}(y)}\vee C_A(y)\right]\le \wedge_{y\in X}\left[U_{\hat{N}_x^{\beta}(y)}\vee C_B(y)\right]=C_{\underset{*}{P}(B)}(x),$$

$$U_{\underset{*}{P}(A)}(x)=\vee_{y\in X}\left[C_{\hat{N}_x^{\beta}(y)}\wedge U_A(y)\right]\ge \vee_{y\in X}\left[C_{\hat{N}_x^{\beta}(y)}\wedge U_B(y)\right]=U_{\underset{*}{P}(B)}(x),$$

$$F_{\underset{*}{P}(A)}(x)=\vee_{y\in X}\left[T_{\hat{N}_x^{\beta}(y)}\wedge F_A(y)\right]\ge \vee_{y\in X}\left[T_{\hat{N}_x^{\beta}(y)}\wedge F_B(y)\right]=F_{\underset{*}{P}(B)}(x).$$

Hence, $\underset{*}{P}(A)\subseteq \underset{*}{P}(B)$

In a similar way, $\overset{*}{P}(A)\subseteq \overset{*}{P}(B)$.

(iii)We have,

$$\underset{*}{P}\left(A\cap B\right)=\left\{\left\langle \begin{array}{l} x, \wedge_{y\in X}[F_{\hat{N}_x^\beta(y)} \vee T_{A\cap B(Y)}], \wedge_{y\in X}[U_{\hat{N}_x^\beta(y)} \vee C_{A\cap B(Y)}], \\ \vee_{y\in X}[C_{\hat{N}_x^\beta(y)} \wedge U_{A\cap B(Y)}], \vee_{y\in X}[T_{\hat{N}_x^\beta(y)} \wedge F_{A\cap B(Y)}] \end{array}\right\rangle : x\in X\right\}$$

$$=\left\{\left\langle \begin{array}{l} x, \wedge_{y\in X}[F_{\hat{N}_x^\beta(y)} \vee \left(T_{A(Y)} \wedge T_{B(Y)}\right)], \wedge_{y\in X}[U_{\hat{N}_x^\beta(y)} \vee \left(C_{A(Y)} \wedge C_{B(Y)}\right)], \\ \vee_{y\in X}[C_{\hat{N}_x^\beta(y)} \wedge \left(U_{A(Y)} \vee U_{B(Y)}\right)], \vee_{y\in X}[T_{\hat{N}_x^\beta(y)} \wedge \left(F_{A(Y)} \vee F_{B(Y)}\right)] \end{array}\right\rangle : x\in X\right\}$$

$$=\left\{\left\langle \begin{array}{l} x, \wedge_{y\in X}[\left(F_{\hat{N}_x^\beta(y)} \vee T_{A(Y)}\right) \wedge \left(F_{\hat{N}_x^\beta(y)} \vee T_{B(Y)}\right)], \wedge_{y\in X}[\left(U_{\hat{N}_x^\beta(y)} \vee C_{A(Y)}\right) \wedge \left(U_{\hat{N}_x^\beta(y)} \vee C_{B(Y)}\right)], \\ \vee_{y\in X}[\left(C_{\hat{N}_x^\beta(y)} \wedge U_{A(Y)}\right) \vee \left(C_{\hat{N}_x^\beta(y)} \wedge U_{B(Y)}\right)], \vee_{y\in X}[\left(T_{\hat{N}_x^\beta(y)} \wedge F_{A(Y)}\right) \vee \left(T_{\hat{N}_x^\beta(y)} \wedge F_{B(Y)}\right)] \end{array}\right\rangle : x\in X\right\}$$

$$=\underset{*}{P}\left(A\right)\cap \underset{*}{P}\left(B\right)$$

Similarly, we can show that $\overset{*}{P}\left(A\bigcup B\right)=\overset{*}{P}\left(A\right)\bigcup \overset{*}{P}\left(B\right)$.

(iv) $A\subseteq A\bigcup B$, $B\subseteq A\bigcup B$, $A\bigcap B\subseteq A$, and $A\bigcap B\subseteq B$

$\underset{*}{P}(A)\subseteq \underset{*}{P}(A\bigcup B), \underset{*}{P}(B)\subseteq \underset{*}{P}(A\bigcup B)\Rightarrow \underset{*}{P}(A\bigcup B)\supseteq \underset{*}{P}(A)\bigcup \underset{*}{P}(B)$

Again, $\overset{*}{P}(A\bigcap B)\subseteq \overset{*}{P}(A)$ and $\overset{*}{P}(A\bigcap B)\subseteq \overset{*}{P}(B)\Rightarrow \overset{*}{P}(A\bigcap B)\subseteq \overset{*}{P}(A)\bigcap \overset{*}{P}(B)$

We now propose another QSVN covering rough set model, which deals with the crisp lower and upper approximations of each set under the QSVN environment.

Definition 4.3

Let *(X, P)* be a QSVN *β*-covering approximation space. For each crisp subset $Y\in QSVN^X$ (where $QSVN^X$ denotes the power set of *X*), we define the QSVN covering upper approximation $\overset{*}{P}(Y)$ and lower approximation $\underset{*}{P}(Y)$ of *Y*.

$$\overset{*}{P}(Y)=\left\{y\in X:\hat{N}_y^\beta\bigcap Y\neq\varnothing\right\},\ \underset{*}{P}(Y)=\left\{y\in X:\hat{N}_y^\beta\subseteq Y\right\}$$

If $\overset{*}{P}(Y)\neq \underset{*}{P}(Y)$, then *Y* is called the second type of QSVN covering rough set.

Proposition 4.4

Let *P* be a QSVN *β*-covering of *X*. Then, The QSVN covering upper and lower approximation operators defined in Definition 4.3 have the following properties: For all $A,B\in QSVN^X$,

(i) $\underset{*}{P}(\varnothing)=\varnothing, \overset{*}{P}(X)=X; \underset{*}{P}(X)=X, \overset{*}{P}(\varnothing)=\varnothing$

(ii) $\underset{*}{P}\left(\overline{A}\right)=\left(\overline{\overset{*}{P}(A)}\right), \overset{*}{P}\left(\overline{A}\right)=\left(\overline{\underset{*}{P}(A)}\right)$

(iii) If $A\subseteq B$, then $\underset{*}{P}(A)\subseteq \underset{*}{P}(B), \overset{*}{P}(A)\subseteq \overset{*}{P}(B)$

(iv) $\underset{*}{P}(A\bigcap B)=\underset{*}{P}(A)\bigcap \underset{*}{P}(B), \overset{*}{P}(A\bigcup B)=\overset{*}{P}(A)\bigcup \overset{*}{P}(B)$

(v) $\underset{*}{P}(A\bigcup B)\supseteq \underset{*}{P}(A)\bigcup \underset{*}{P}(B), \overset{*}{P}(A\bigcap B)\subseteq \overset{*}{P}(A)\bigcap \overset{*}{P}(B)$

(vi) $\underset{*}{P}\left(\underset{*}{P}(A)\right)\subseteq \underset{*}{P}(A), \overset{*}{P}\left(\overset{*}{P}(A)\right)\supseteq \overset{*}{P}(A)$

(vii) $\underset{*}{P}(A)\subseteq A\subseteq \overset{*}{P}(A)$

Proof. All proofs are directly followed.

5. Matrix Representations of QSVN Covering Rough Set Models

In this section, we introduce the matrix representation of the proposed model by introducing some new matrices and their operations.

Definition 5.1

Let *P* = *{P₁, P₂, …, Pₘ}* be a QSVN *β*-covering of *X* = *{x₁, x₂, …, xₙ}*. Then, $M_{P} = \left(P_j\left(x_i\right)\right)_{n\times m}$ is a matrix representation of *P*, and $M_{P}^{\beta} = \left(k_{ij}\right)_{n\times m}$ is called a *β*-matrix representation of *P*, where

$$k_{ij} = \begin{cases} 1, P_j\left(x_i\right)\geq \beta \\ 0, otherwise \end{cases}$$

Example 5.1.1 (Considering the Example 3.2.1) Let $\beta = \langle 0.5, 0.4, 0.6, 0.7\rangle$

$$M_{P} = \begin{pmatrix} \langle 0.6,0.3,0.3,0.4\rangle & \langle 0.6,0.5,0.4,0.3\rangle & \langle 0.6,0.5,0.7,0.6\rangle & \langle 0.9,0.6,0.5,0.5\rangle \\ \langle 0.7,0.5,0.3,0.6\rangle & \langle 0.4,0.4,0.2,0.8\rangle & \langle 0.8,0.3,0.4,0.2\rangle & \langle 0.6,0.8,0.4,0.5\rangle \\ \langle 0.7,0.6,0.5,0.6\rangle & \langle 0.5,0.4,0.5,0.1\rangle & \langle 0.7,0.7,0.6,0.5\rangle & \langle 0.7,0.6,0.4,0.5\rangle \\ \langle 0.5,0.5,0.3,0.4\rangle & \langle 0.8,0.7,0.5,0.4\rangle & \langle 0.7,0.5,0.4,0.5\rangle & \langle 0.6,0.3,0.7,0.5\rangle \end{pmatrix}$$

$$M_{P}^{\beta} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Definition 5.2

Let $A = \left(a_{ik}\right)_{n\times m}$ and $B = \left(\left\langle b_{kj}^{+}, c_{kj}^{+}, c_{kj}^{-}, b_{kj}^{-}\right\rangle\right)_{n\times m}$ $(k = 1,2,..,m;\ j = 1,2,..,l)$ are two matrices. We define

$A * B = \left(\left\langle d_{ij}^{+}, e_{ij}^{+}, e_{ij}^{-}, d_{ij}^{-}\right\rangle\right)(i = 1,2,..,n;\ j = 1,2,...,l)$, where

$$\left\langle d_{ij}^{+}, e_{ij}^{+}, e_{ij}^{-}, d_{ij}^{-}\right\rangle = \left\langle \begin{matrix} \wedge_{k=1}^{m}\left[\left(1-a_{ik}\right)\vee b_{kj}^{+}\right], \wedge_{k=1}^{m}\left[\left(1-a_{ik}\right)\vee c_{kj}^{+}\right], \\ 1-\wedge_{k=1}^{m}\left[\left(1-a_{ik}\right)\vee\left(1-c_{kj}^{-}\right)\right], 1-\wedge_{k=1}^{m}\left[\left(1-a_{ik}\right)\vee\left(1-b_{kj}^{-}\right)\right] \end{matrix}\right\rangle$$

Proposition 5.3

Let $P = \{P_1, P_2, ..., P_m\}$ be a QSVN *β*-covering of $X = \{x_1, x_2, ..., x_n\}$. Then,

$M_{P}^{\beta} * M_{P}^{T} = \left(\hat{N}_{x_i}^{\beta}\left(x_j\right)\right)(i = 1,2,..,n;\ j = 1,2,...,n)$, where M_{P}^{T} is the transpose of M_{P}.

Proof.

Let $M_{P}^{T} = \left(P_k\left(x_j\right)\right)_{m\times n}$, $M_{P}^{\beta} = \left(s_{ik}\right)_{n\times m}$ and $M_{P}^{\beta} * M_{P}^{T} = \left(\left\langle d_{ij}^{+}, e_{ij}^{+}, e_{ij}^{-}, d_{ij}^{-}\right\rangle\right)(i, j = 1,2..,n)\cdot$

Since $\tilde{P}$ is a QSVN *β*-covering of *X*, for each $i(1\leq i\leq n)$, there exist $k(1\leq k\leq m)$ such that $s_{ik} = 1$. Then,

$$\left\langle d_{ij}^{+}, e_{ij}^{+}, e_{ij}^{-}, d_{ij}^{-} \right\rangle = \left\langle \begin{array}{l} \wedge_{k=1}^{m}\left[(1-s_{ik}) \vee T_{P_k}(x_j)\right], \wedge_{k=1}^{m}\left[(1-s_{ik}) \vee C_{P_k}(x_j)\right], \\ 1-\wedge_{k=1}^{m}\left[(1-s_{ik}) \vee \left(1-U_{P_k}(x_j)\right)\right], 1-\wedge_{k=1}^{m}\left[(1-s_{ik}) \vee \left(1-F_{P_k}(x_j)\right)\right] \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \wedge_{s_{ik}=1}\left[(1-s_{ik}) \vee T_{P_k}(x_j)\right], \wedge_{s_{ik}=1}\left[(1-s_{ik}) \vee C_{P_k}(x_j)\right], \\ 1-\wedge_{s_{ik}=1}\left[(1-s_{ik}) \vee \left(1-U_{P_k}(x_j)\right)\right], 1-\wedge_{s_{ik}=1}\left[(1-s_{ik}) \vee \left(1-F_{P_k}(x_j)\right)\right] \end{array} \right\rangle$$

$$= \left\langle \wedge_{s_{ik}=1} T_{P_k}(x_j), \wedge_{s_{ik}=1} C_{P_k}(x_j), 1-\wedge_{s_{ik}=1}\left(1-U_{P_k}(x_j)\right), 1-\wedge_{s_{ik}=1}\left(1-F_{P_k}(x_j)\right) \right\rangle$$

$$= \left\langle \wedge_{P_k(x_i)\geq\beta} T_{P_k}(x_j), \wedge_{P_k(x_i)\geq\beta} C_{P_k}(x_j), 1-\wedge_{P_k(x_i)\geq\beta}\left(1-U_{P_k}(x_j)\right), 1-\wedge_{P_k(x_i)\geq\beta}\left(1-F_{P_k}(x_j)\right) \right\rangle$$

$$= \left(\bigcap_{P_k(x_i)\geq\beta} P_k(x_j) \right)$$

$$= \hat{N}_{x_i}^{\beta}(x_j), 1 \leq i, j \leq n.$$

Thus, $M_{\breve{P}}^{\beta} * M_{\breve{P}}^{T} = \left(\hat{N}_{x_i}^{\beta}(x_j) \right) (i = 1, 2, .., n;\ j = 1, 2, ..., \mathrm{n})$

Example 5.4 (From Example 5.1.1)

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} * \begin{pmatrix} \langle 0.6, 0.3, 0.3, 0.4 \rangle & \langle 0.6, 0.5, 0.4, 0.3 \rangle & \langle 0.6, 0.5, 0.7, 0.6 \rangle & \langle 0.9, 0.6, 0.5, 0.5 \rangle \\ \langle 0.7, 0.5, 0.3, 0.6 \rangle & \langle 0.4, 0.4, 0.2, 0.8 \rangle & \langle 0.8, 0.3, 0.4, 0.2 \rangle & \langle 0.6, 0.8, 0.4, 0.5 \rangle \\ \langle 0.7, 0.6, 0.5, 0.6 \rangle & \langle 0.5, 0.4, 0.5, 0.1 \rangle & \langle 0.7, 0.7, 0.6, 0.5 \rangle & \langle 0.7, 0.6, 0.4, 0.5 \rangle \\ \langle 0.5, 0.5, 0.3, 0.4 \rangle & \langle 0.8, 0.7, 0.5, 0.4 \rangle & \langle 0.7, 0.5, 0.4, 0.5 \rangle & \langle 0.6, 0.3, 0.7, 0.5 \rangle \end{pmatrix}^{T}$$

$$M_{\breve{P}}^{\beta} * M_{\breve{P}}^{T} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} * \begin{pmatrix} \langle 0.6, 0.3, 0.3, 0.4 \rangle & \langle 0.7, 0.5, 0.3, 0.6 \rangle & \langle 0.7, 0.6, 0.5, 0.6 \rangle & \langle 0.5, 0.5, 0.3, 0.4 \rangle \\ \langle 0.6, 0.5, 0.4, 0.3 \rangle & \langle 0.4, 0.4, 0.2, 0.8 \rangle & \langle 0.5, 0.4, 0.5, 0.1 \rangle & \langle 0.8, 0.7, 0.5, 0.4 \rangle \\ \langle 0.6, 0.5, 0.7, 0.6 \rangle & \langle 0.8, 0.3, 0.4, 0.2 \rangle & \langle 0.7, 0.7, 0.6, 0.5 \rangle & \langle 0.7, 0.5, 0.4, 0.5 \rangle \\ \langle 0.9, 0.6, 0.5, 0.5 \rangle & \langle 0.6, 0.8, 0.4, 0.5 \rangle & \langle 0.7, 0.6, 0.4, 0.5 \rangle & \langle 0.6, 0.3, 0.7, 0.5 \rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle 0.6, 0.5, 0.5, 0.5 \rangle & \langle 0.4, 0.4, 0.4, 0.8 \rangle & \langle 0.5, 0.4, 0.5, 0.5 \rangle & \langle 0.6, 0.3, 0.7, 0.5 \rangle \\ \langle 0.6, 0.3, 0.5, 0.5 \rangle & \langle 0.6, 0.5, 0.4, 0.6 \rangle & \langle 0.7, 0.6, 0.5, 0.6 \rangle & \langle 0.5, 0.3, 0.7, 0.5 \rangle \\ \langle 0.6, 0.3, 0.7, 0.6 \rangle & \langle 0.4, 0.3, 0.6, 0.2 \rangle & \langle 0.5, 0.4, 0.4, 0.4 \rangle & \langle 0.5, 0.3, 0.3, 0.5 \rangle \\ \langle 0.6, 0.3, 0.3, 0.4 \rangle & \langle 0.4, 0.3, 0.6, 0.2 \rangle & \langle 0.5, 0.4, 0.4, 0.4 \rangle & \langle 0.5, 0.5, 0.5, 0.5 \rangle \end{pmatrix}$$

$$= \left(\hat{N}_{x_i}^{\beta}(x_j) \right) (1 \leq i, j \leq 4)$$

Definition 5.5

Let $A = \left(\langle b_{ij}^{+}, c_{ij}^{+}, c_{ij}^{-}, b_{ij}^{-} \rangle \right)_{m \times n}$ and $B = \left(\langle d_j^{+}, e_j^{+}, e_j^{-}, d_j^{-} \rangle \right)_{n \times 1}$ be two matrices. Then, we define the following:

$$D = A \circ B = \left(\langle f_i^{+}, g_i^{+}, g_i^{-}, f_i^{-} \rangle \right)_{m \times 1} \text{ and } E = A \Diamond B = \left(\langle h_i^{+}, k_i^{+}, k_i^{-}, h_i^{-} \rangle \right)_{m \times 1} \text{ where}$$

$$\langle f_i^{+}, g_i^{+}, g_i^{-}, f_i^{-} \rangle = \left\langle \vee_{j=1}^{n}\left(b_{ij}^{+} \wedge d_j^{+}\right), \vee_{j=1}^{n}\left(c_{ij}^{+} \wedge e_j^{+}\right), \wedge_{j=1}^{n}\left(c_{ij}^{-} \vee e_j^{-}\right), \wedge_{j=1}^{n}\left(b_{ij}^{-} \vee d_j^{-}\right) \right\rangle$$

$$\langle h_i^{+}, k_i^{+}, k_i^{-}, h_i^{-} \rangle = \left\langle \wedge_{j=1}^{n}\left(b_{ij}^{-} \vee d_j^{+}\right), \wedge_{j=1}^{n}\left(\left(1-c_{ij}^{+}\right) \vee e_j^{+}\right), \vee_{j=1}^{n}\left(\left(1-c_{ij}^{-}\right) \wedge e_j^{-}\right), \vee_{j=1}^{n}\left(b_{ij}^{+} \wedge d_j^{-}\right) \right\rangle$$

Theorem 5.6

Let $\breve{P} = \{P_1, P_2, ..., P_m\}$ be a QSVN β-covering of $X = \{x_1, x_2, ..., x_n\}$. Then, for any $A \in QSVN^{X}$,

$\overset{*}{P}(A) = \left(M_{\breve{P}}^{\beta} * M_{\breve{P}}^{T} \right) \circ A$ and $\underset{*}{P}(A) = \left(M_{\breve{P}}^{\beta} * M_{\breve{P}}^{T} \right) \Diamond A$, where $A = (a_i)_{n \times 1}$ with $a_i = \langle T_A(x_i), C_A(x_i), U_A(x_i), F_A(x_i) \rangle$.

Proof. We have,

$$\left(\left(M_{P}^{\beta} * M_{P}^{T}\right) \circ A\right)\left(x_i\right)=\left\langle \begin{array}{l} \vee_{j=1}^{n}\left(T_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\wedge T_A\left(x_j\right)\right), \vee_{j=1}^{n}\left(C_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\wedge C_A\left(x_j\right)\right), \\ \wedge_{j=1}^{n}\left(U_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\vee U_A\left(x_j\right)\right), \wedge_{j=1}^{n}\left(F_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\vee F_A\left(x_j\right)\right) \end{array}\right\rangle$$

$$=\left(\overset{*}{P}(A)\right)\left(x_i\right)$$

$$\left(\left(M_{P}^{\beta} * M_{P}^{T}\right) \lozenge A\right)\left(x_i\right)=\left\langle \begin{array}{l} \wedge_{j=1}^{n}\left(F_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\vee T_A\left(x_j\right)\right), \wedge_{j=1}^{n}\left(U_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\vee C_A\left(x_j\right)\right), \\ \vee_{j=1}^{n}\left(C_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\wedge U_A\left(x_j\right)\right), \vee_{j=1}^{n}\left(T_{\hat{N}_{x_i}^{\beta}}\left(x_j\right)\wedge F_A\left(x_j\right)\right) \end{array}\right\rangle$$

$$=\left(\underset{*}{P}(A)\right)\left(x_i\right)$$

Therefore, $\overset{*}{P}(A)=\left(M_{P}^{\beta} * M_{P}^{T}\right)\circ A$ and $\underset{*}{P}(A)=\left(M_{P}^{\beta} * M_{P}^{T}\right)\lozenge A$.

6. Application of QSVN Covering Based Rough Set in Medical Diagnosis Problem

In this section, we construct a new algorithm for solving a medical diagnosis problem using QSVN rough set model. Also, with the help of a case study, the significance of the proposed algorithm is justified. Then, a comparative study with the existing method is exhibited.

6.1 Case study

Let $X=\left\{x_j : j=1,2,...,n\right\}$ be a set of patients under consideration. The set of patients is going to a hospital for medical treatment and having symptoms denoted by the set $A=\left\{y_i : i=1,2,...,m\right\}$. Let y_1 =fever, y_2 =vomiting, y_3 =headache, y_4 =muscle pain, y_5 =skin rash, etc are the major symptoms of a certain disease D. A doctor is available for the treatment of every patient having the symptoms mentioned here. During medical diagnosis, for every patient $x_j \in X\,(j=1,2,...,n)$, the doctor provides a symptom value $V_i\left(x_j\right)(i=1,2,...,m)$. The symptom value is defined as $V_i\left(x_j\right)=\left(\left\langle T_{V_i}\left(x_j\right), C_{V_i}\left(x_j\right), U_{V_i}\left(x_j\right), F_{V_i}\left(x_j\right)\right\rangle\right)$, where the doctor believes that $T_{V_i}\left(x_j\right)\in[0,1]$ denotes the degree of confirmation that the patient x_j surely has the symptom y_i, $C_{V_i}\left(x_j\right)\in[0,1]$ denotes the degree that the doctor is undecided whether the patient x_j having the symptom y_i or not, $U_{V_i}\left(x_j\right)\in[0,1]$ denotes the degree that the doctor has no knowledge about the symptoms, and $F_{V_i}\left(x_j\right)\in[0,1]$ denotes the degree that the doctor confirms that the patient x_j does not have any symptoms y_i such that $0\le T_{V_i}\left(x_j\right)+C_{V_i}\left(x_j\right)+U_{V_i}\left(x_j\right)+F_{V_i}\left(x_j\right)\le 4$.

Furthermore, .due to immediate health risk to the patient, the doctor assumes the critical value $\beta=\langle a,b,c,d\rangle$ in such a way that for any patient x_j with at least one symptom y_i, having a symptom value V_i cannot be less than β. If such condition holds, then $P=\left\{P_1,P_2,....,P_m\right\}$ is a QSVN β-covering of X.

Example: Let $X=\left\{x_1,x_2,x_3,x_4\right\}$ be a set of patients and $S=\left\{y_1=\text{fever}, y_2=\text{vomiting}, y_3=\text{headache}, y_4=\textit{muscle pain}\right\}$ denotes the four major symptoms for dengue disease and $\beta=\langle 0.5,0.4,0.6,0.7\rangle$ be a critical value. Then the doctor's (decision-makers) evaluation of every patient $x_j\,(j=1,2,3,4)$ is known as QSVN β-covering induced by X is shown in Table1. The QSVN β-neighborhood $\hat{N}_{x_k}^{\beta}$ (k=1, 2, 3, 4) is shown in Table 2. Suppose the diagnosis value of dengue disease for each patient is represented by

$$A=\left\{\frac{x_1}{\langle 0.7,0.6,0.4,0.2\rangle},\frac{x_2}{\langle 0.5,0.6,0.3,0.4\rangle},\frac{x_3}{\langle 0.4,0.6,0.3,0.8\rangle},\frac{x_4}{\langle 0.6,0.4,0.6,0.4\rangle}\right\}.$$

Then we compute the following:

$$\overset{*}{P}(A)=\left\{\frac{x_1}{\langle 0.6,0.5,0.5,0.4\rangle},\frac{x_2}{\langle 0.5,0.6,0.4,0.4\rangle},\frac{x_3}{\langle 0.4,0.4,0.3,0.8\rangle},\frac{x_4}{\langle 0.6,0.4,0.6,0.5\rangle}\right\}$$

$$\underset{*}{P}(A)=\left\{\frac{x_1}{\langle 0.7,0.6,0.4,0.2\rangle},\frac{x_2}{\langle 0.5,0.6,0.3,0.4\rangle},\frac{x_3}{\langle 0.5,0.6,0.3,0.6\rangle},\frac{x_4}{\langle 0.6,0.4,0.6,0.4\rangle}\right\}$$

Then,

$$\hat{\Theta}(A)=\overset{*}{P}(A)\oplus\underset{*}{P}(A)$$

$$=\left\{\frac{x_1}{\langle 0.88,0.8,0.2,0.08\rangle},\frac{x_2}{\langle 0.75,0.84,0.12,0.16\rangle},\frac{x_3}{\langle 0.2,0.24,0.51,0.92\rangle},\frac{x_4}{\langle 0.84,0.64,0.36,0.2\rangle}\right\}$$

OR

$$\hat{\Theta}(A)=\overset{*}{P}(A)\otimes\underset{*}{P}(A)$$

$$=\left\{\frac{x_1}{\langle 0.42,0.3,0.7,0.52\rangle},\frac{x_2}{\langle 0.25,0.36,0.58,0.64\rangle},\frac{x_3}{\langle 0.2,0.24,0.51,0.92\rangle},\frac{x_4}{\langle 0.36,0.16,0.84,0.7\rangle}\right\}$$

Now we obtain the score value $\Lambda(x_k), k=1,2,3,4$ as follows:

Ranking the score values of all the alternatives according to the size of their numerical values, we have

$\Lambda(x_3)<\Lambda(x_4)<\Lambda(x_1)<\Lambda(x_2)$ **OR** $\Lambda(x_3)<\Lambda(x_4)<\Lambda(x_2)<\Lambda(x_1)$

Therefore, either patient x_2 or patient x_1 is more likely to suffer from the disease dengue. So, the doctor diagnoses either patient x_2 or patient x_1. The other patients namely x_3 and x_4 are out of danger.

7. Conclusions and Future Scope

The proposed study is a gateway to making a connection between QSVNS and the covering-based rough set. Then QSVN β-covering based neighborhood and its properties are discussed. By initiating some properties and definitions in QSVN β-covering approximation spaces, we give two types of QSVN covering- based rough set models. To enhance the applicability of the present work, matrix representations of approximation spaces are investigated. The matrix representation of QSVN covering-based rough lower and upper approximation is introduced which is very much helpful in decision-making problems. Also, with the help of an algorithm, a case study has been investigated for medical diagnosis.

In the future, researchers can investigate the comparative analysis between the proposed studies with the other existing covering rough set models. Moreover, the proposed theory provides more information to study the incomplete information in weather forecasting, machine learning, data mining, development of application software, etc. Besides these, the present study can be investigated over two universes; it can be applicable in multi-criteria group decision-making (MCGDM) problems. The QSVN rough set model can be further generalized by introducing QSVN rough soft set, QSVN rough hypersoft set, and their matrix representation.

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