ANALOGY THEORY OF MATHEMATICS AND PHYSICS MODELING OF THE TERMINAL VELOCITY AND KINETIC ENERGY OF THE 5000 METRES SPRINT VIA NEWTON'S SECOND LAW

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Abstract

In this work, mathematics and physics modeling for the 5,000 meters sprint was developed. We established a model, velocity time-dependent and kinetic energy time-dependent, for the sprint in the 5,000 meters as a function of mathematics. Substitute force time-dependent for the sprint that 3 types of the runner in the 5,000 meters into Newton's second law. In general, we used the first-order linear in-homogeneous ordinary differential equation for evaluation of the velocity depending on time and kinetic energy depending on time for the sprint in the 5,000 meters. Then the best model of force time-dependent was the modeling of 3 types. The correlation coefficient by the experiment data and the motion model 3 was 0.93, thus more than model 1 (0.89) and model 2 (0.92). In terms of biological capacity of the long-distance sprinter's lungs, it tended to increase in the oxygen gas containment area and pulmonary capacity.

Keywords: Time dependent velocity, Time dependent Kinetic energy, Correlation coefficient

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Introduction

The 5,000 meters or 5,000-metre run (shorter specialties) is a common long-distance running or endurance running event in path and yard. A similar distance in road running is called a 5k run (Rattanawichai et al., 2021). The IAAF compiles official records for both outdoor and indoor 5,000-metre parth events (Ari et al., 2006). Fernando Alacid et al. (2020) creation modeling of the forecasting variables of the half-marathon is mainly anthropometric, the performance of velocity, but with moderate coefficients of the formulation. The variables of note in the marathon category are fundamentally those associated with training and those derived from physiological evaluation and anthropometric parameters. Nuuttila et al. (2021) showed the method of predicting the value of endurance performance in the 5,000 meters runner.

In this paper, we aimed to can show the evaluation of the velocity depends on time for runners of the 5,000 meters, the value of coefficient endurance performance, the performance of kinetic energy obtained from the experiment data, and mathematical and Newton's second law of motion (Atamp, 1990).

Materials and Methods

Determination of the velocity time-dependent for the 5000 meters mini marathon sprint

We firstly developed the model of mathematics and physics for 5000 meters running. We used Newton's second law of motion (Helene & Yamashita et al., 2009) as shown in Figure 1 and Equation (1)



Figure 1 Diagrammatic sketch of forces acting on a mass of runner on a plain plane.

We used general first - order linear ordinary differential equations and it could be written according to the pattern (Riley & Hobson, 2006) as shown in Equation (2):

$$\frac{dy}{dx} + p(x)y = h(x).$$
⁽²⁾

From figure 1, we could write equation of motion for runner 5000 meters as $f_{s1}(t) - \alpha_d v_{s1} = m_s \frac{dv_{s1}}{dt}$. This paper brings an idea of creating a mathematics and physics model of the sprint from the research called "The force, power, and energy of the 100 meters sprint" (Helen & Yamashita et al., 2009; Hutem et al, 2017), in part of $f(t) = F - \beta t$ which is a polynomial function. Then the creation of the mathematics and physics model was obtained from the research called "Comparison of the World and European Records in the 100m Dash by a Quasi-Physical Model" (Tiago et al., 2016), in the maintenance force terms (f_m) $f_m = f_1 e^{-Ct}$ which was an exponential function. We created the applied force timedependent model1 for runner 5,000 meters $f_{s1}(t) = \frac{f_0}{\beta_{ed}} t e^{-\beta_{ed}(t+2)}$ (Hutem et al., 2017), where f_0 is the initially applied force (N), β_{ed} is the coefficient endurance sprint (s⁻¹). The parameter α_d is the coefficient air resistance. The parameter m_s is mass for sprint and using the initial velocity $v_{s1}(t=0) = 0$ and runner's mass is m_s which provides support by the applied force time-dependent from liner first-order ordinary difference equation (Hutem, 2019) as shown in Equation (3):

$$\frac{dv_{s1}}{dt} + \frac{\alpha_d v_{s1}}{m_s} = \frac{f_0 t e^{-\beta_{ed}(t+2)}}{m_s \beta_{ed}},$$
(3)

where $v_{s1}(t)$ is the time depends velocity for runner 5,000 meters.

The afterward is called the formula for linear first-order in-homogeneous differential equation (Susan, 2004). The solution to the equation is supplied as in Equation (4):

$$v_{s1}(t) = e^{-\frac{\alpha_d}{m_s}t} \left(\frac{f_0}{m_s \beta_{ed}} \int t e^{\left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)t} e^{-2\beta_{ed}} dt + C \right)$$
(4)

Using integration by part and the initial time depends velocity modeling 1 is zero, that is, if at $v_{s1}(t=0) = 0$, we obtain the time depends velocity modeling 1 complete solution of the runner as shown in Equation (5):

$$v_{s1}(t) = \frac{f_0 e^{-(\beta_{ed} t + 2\beta_{ed})}}{m_s \beta_{ed}} \left(\frac{t}{\left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)} - \frac{1}{\left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^2} \right) + \frac{f_0 e^{\left(-2\beta_{ed} - \frac{\alpha_d}{m_s} t\right)}}{m_s \beta_{ed} \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^2}$$
(5)

Thus the kinetic energy time-dependent modeling 1 for runner 5000 meters as can be defined as in Equation (6):

$$E_{ks1}(t) = \frac{1}{2}m_s \left(\frac{f_0 e^{-(\beta_{ed}t + 2\beta_{ed})}}{m_s \beta_{ed}} \left(\frac{t}{\left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)} - \frac{1}{\left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^2}\right) + \frac{f_0 e^{\left(-2\beta_{ed} - \frac{\alpha_d}{m_s}t\right)}}{m_s \beta_{ed} \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^2}\right)^2$$
(6)

To solve the kinematic parameter f_0 , β_{ed} , α_d of velocity timedependent modeling 1 in equation (5) and (6). We can be using the "Find Fit" command to find f_0 , β_{ed} and α_d parameter value. It is illustrated in the result section. We are concerned with studying the motion of the runner, present 5000 meters when the applied force is the function exponential and polynomial which depends on time, which is $f_{s2}(t) = \frac{f_0}{\beta_{ed}}(t^2 - t)e^{-\beta_{ed}(t+2)}$. Using the initial velocity $v_{s2}(t=0) = 0$ and the runner's mass is m_s provide support by the applied for time-dependent $f_{s2}(t) - \alpha_d v_{s2} = m_s \frac{dv_{s2}}{dt}$. From the equation, we used Newton's second law as shown in Equation (7):

$$\frac{dv_{s2}}{dt} + \frac{\alpha_d v_{s2}}{m_s} = \frac{f_0 \left(t^2 - t\right) e^{-\beta_{ed} \left(t+2\right)}}{m_s \beta_{ed}} \tag{7}$$

where $v_{s2}(t)$ is the time depends velocity of the runner 5,000 meters. The afterward is called the formula for the linear first-order inhomogeneous differential equation. The solution to the equation is shown in Equation (8):

$$v_{s2}(t) = e^{-\frac{\alpha_d}{m_s}t} \left(\frac{f_0 e^{-2\beta_{ed}}}{m_s \beta_{ed}} \int t^2 e^{\left(\frac{\alpha_d}{m} - \beta_{ed}\right)t} dt - \frac{f_0 e^{-2\beta_{ed}}}{m \beta_{ed}} \int t e^{\left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)t} dt + C \right)$$
(8)

Using integration by part and the initial time depends velocity modeling 2 is zero, that is, if at $v_{s2}(t=0) = 0$, we obtain the time depends velocity modeling 2 complete solutions of the runner as shown in Equation (9):

$$v_{s2}(t) = \frac{f_0 e^{-\beta_{ed}(t+2)}}{m_s \beta \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^3} \left\{ \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^2 t^2 - 2\left(\frac{\alpha_d}{m_s} - \beta_{ed}\right) t + 2 \right\} - \frac{f_0 e^{-\beta_{ed}(t+2)}}{m_s \beta_{ed} \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^2} \left\{ \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right) t - 1 \right\} - \left\{ \frac{2f_0 e^{-2\beta_{ed}}}{m_s \beta_{ed} \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^3} + \frac{f_0 e^{-2\beta_{ed}}}{m_s \beta_{ed} \left(\frac{\alpha_d}{m_s} - \beta_{ed}\right)^2} \right\} e^{-\frac{\alpha_d}{m_s}t}$$
(9)

Thus the kinetic energy time-dependent modeling 2 $(E_{ks2}(t))$ for runner 5,000 meters as can be defined as in Equation (10):

$$E_{ks2}(t) = \frac{1}{2}m_{s} \left(\frac{f_{0}e^{-\beta_{ed}(t+2)}}{m_{s}\beta_{ed}\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}\right)^{3}} \left(\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}\right)^{2} t^{2} - 2\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}\right) t + 2 \right) - \frac{f_{0}e^{-\beta_{ed}(t+2)}}{m_{s}\beta_{ed}\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}\right)^{2}} \left(\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}\right) t - 1 \right) - \left(\frac{2f_{0}e^{-2\beta_{ed}}}{m_{s}\beta_{ed}\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}\right)^{3}} + \frac{f_{0}e^{-2\beta_{ed}}}{m_{s}\beta_{ed}\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}\right)^{2}} \right) e^{-\frac{\alpha_{d}}{m_{s}}} \right)^{2}$$
(10)

To solve the kinematic parameter f_0 , β_{ed} , α_d of velocity time-dependent modeling 2 in equation (9) and (10). We can be using the "Find Fit" command to find f_0 , β_{ed} and α_d parameter value. It is illustrated in the result section. We are concerned with studying the motion of the runner, present 5,000 meters when the applied force is time-dependent, which is $f_{s3}(t) = \frac{f_0}{\beta_{ed}}(t^2 + 4t + 2)e^{-\beta_{ed}^2(t+2)}$. The given initial velocity $v_{s3}(t=0) = 0$ and the runner's mass is *m* provide support by the applied force of the function exponential and polynomial in time-dependent $f_{s3}(t) - \alpha_d v_{s3} = m_s \frac{dv_{s3}}{dt}$. From the equation, we used the second law of Newton as shown in Equation (11):

$$\frac{dv_{s3}}{dt} + \frac{\alpha_d v_{s3}}{m_s} = \frac{f_0}{m_s \beta_{ed}} \left(t^2 + 4t + 2 \right) e^{-\beta_{ed}^2 (t+2)} , \qquad (11)$$

where $v_{s3}(t)$ is the velocity time-dependent of sprinting. The afterward is called the formula for the linear first-order in-homogeneous differential equation. The solution to the equation is shown in Equation (12):

$$v_{s3}(t) = e^{-\frac{\alpha_d}{m_s}t} \left(\frac{f_0 e^{-2\beta_{ed}^2}}{m_s \beta_{ed}} \int (t^2 + 4t + 2) e^{\left(\frac{\alpha_d}{m_s} - \beta_{ed}^2\right)t} dt + C \right),$$
(12)

Using integration by part and the initial velocity time-dependent modeling 3 is zero, that is, if at $v_{s3}(t=0) = 0$, where $\lambda = \frac{\alpha_d}{m_s} - \beta_{ed}^2$, we obtained the velocity time-dependent modeling 3 as shown in Equation (13):

$$v_{s3}(t) = \frac{f_{0}e^{-\beta_{ed}^{2}(t+2)}}{m_{s}\beta_{ed}\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{3}} \left(\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} t^{2} + \left(4\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} - 2\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)\right) t + \left(2\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} - 4\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right) + 2\right) - \frac{f_{0}e^{-\left(2\beta_{ed}^{2} + \frac{\alpha_{d}}{m_{s}}\right)}}{m_{s}\beta_{ed}\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{3}} \left(2\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} - 4\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right) + 2\right)$$
(13)

Thus the kinetic energy time-dependent modeling 3 $(E_{ks3}(t))$ for runner 5,000 meters as can be defined as in Equation (14):

$$E_{is3}(t) = \frac{1}{2}m_{s} \left[\frac{f_{0}e^{-\beta_{ed}^{2}(t+2)}}{m\beta_{ed}\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{3}} \left[\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} t^{2} + \left[4\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} - 2\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right) \right] t + \left[2\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} - 4\left(\frac{\alpha_{d}}{m_{s}} - \beta_{ed}^{2}\right)^{2} - 4\left(\frac{\alpha$$

To solve the kinematic parameter f_0 , β_{ed} , α_d of time-dependent velocity modeling 3 in equation (13) and (14), We can be using the "Find Fit"

command to find f_0 , β_{ed} and α_d parameter value of the time dependent velocity numerical. It is illustrated in the result section.

Results

We studied the model of 3 types of running of time dependent force. We showed the evaluation of detail of velocity for $f_{s1}(t)$, $f_{s2}(t)$, and $f_{s3}(t)$ in section 2 of materials and methods. Tables and graphs of velocity are shown in Table 1 and Figure 2, Figure 3, and Figure 4 respectively.

Velocity of model (m/s) Velocity Total time experiment Model1 Model2 Model3 (s) (m/s) 25.97 4.62 4.62 4.62 4.62 51.28 5.07 5.06 5.06 5.06 77.82 5.14 5.06 5.06 5.06 109.58 5.05 5.05 5.05 5.11 137.79 5.08 5.04 5.04 5.04 169.96 5.06 5.03 5.03 5.03 193.29 5.07 5.03 5.03 5.03 221.34 5.06 5.02 5.01 5.02 254.98 5.02 5.01 5.01 5.01 284.00 5.00 5.00 5.00 5.00 312.62 4.99 4.99 4.99 4.99 342.05 4.97 4.98 4.98 4.98 373.98 4.92 4.97 4.97 4.97 409.83 4.88 4.96 4.96 4.96 439.42 4.87 4.95 4.95 4.95 468.43 4.94 4.94 4.94 4.91 496.94 4.93 4.93 4.91 4.93

 Table 1
 The relationship between velocity for sprint and time variable of 5000 meters sprint running

527.60	4.89	4.92	4.92	4.92
559.67	4.86	4.91	4.91	4.91
586.06	4.88	4.90	4.91	4.90
613.49	4.89	4.89	4.89	4.89

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Table 1The relationship between velocity for sprint and time variable of 5000meters sprint running (cont.)

Total time (s)	Velocity	Velocity of model (m/s)		
	experiment	Model1	Model2	Model3
	(m/s)			
651.54	4.85	4.88	4.88	4.88
681.72	4.87	4.87	4.87	4.87
706.61	4.84	4.86	4.86	4.86
741.20	4.83	4.85	4.85	4.85
767.63	4.82	4.84	4.84	4.84
793.38	4.84	4.84	4.84	4.84
826.44	4.84	4.83	4.83	4.83
853.60	4.85	4.82	4.82	4.82
880.65	4.86	4.81	4.81	4.82
911.34	4.85	4.80	4.80	4.81
948.24	4.83	4.79	4.79	4.80
965.23	4.89	4.78	4.78	4.80
correlation	correlation coefficient		0.92	0.93

From table 1 we show personification of the time dependent velocity obtained from the experiment data and mathematical and physics in the time-dependent applied force modeling 1, the time-dependent applied force modeling 2, and the time-dependent applied force modeling 3 of the applied force is the function exponential and polynomial which depends on time for neuromuscular factor determining 5 km running kinematic performance and running economy in well-trained athletes. The velocity experiment (Ari, 2006) and physics modeling 1 and modeling 2 differ by a large percentage in time dependent velocity from velocity data obtained

from the experiments. This is consistent with correlation coefficient modeling 1 = 0.89 and modeling 2 = 0.92. However, experiment and physics modeling 3 have a low percent difference in time dependent velocity from velocity data obtained from the experiment. This is consistent with correlation coefficient modeling 3 which was 0.93.



Figure 2 A) The parameter for performance running of using the "Find Fit" command in program computer is: $f_0 = 2.29$, $\beta_{ed} = 0.15$, $\alpha_d = 0.01$. Comparison of the timedependent velocity obtained from the experimental data and motion of model 1 of the applied force time-dependent decreasing function $(f_{s1}(t))$ for 5,000-meters running in research of Ari T. et.al. 2006 study investigated the effects of the neuromuscular and force-velocity characteristics in distance running performance and running economy.

B) Comparison of the time dependent kinetic energy obtained from the experimental data and motion of model 1 of the applied force time-dependent decreasing function $(f_{s1}(t))$ for 5,000-meters runner in research of Ari T. et.al.



Figure 3 A) The parameter for performance running of using the "Find Fit" command in program computer is : $f_0 = 0.84$, $\beta_{ed} = 0.21$, $\alpha_d = 0.01$ Comparison of the time-dependent velocity obtained from the experimental data and motion of

model 2 of the applied force time-dependent decreasing function $(f_{s2}(t))$ for 5,000-meters runner in research of Ari T. et.al.

B) Comparison of the time dependent kinetic energy obtained from the experimental data and motion of model 2 of the applied force time-dependent decreasing function $(f_{s2}(t))$ for 5,000-meters runner in research of Ari T. et.al.



Figure 4 A) The parameter for performance running of using the "Find Fit" command in program computer is $f_0 = 0.80$, $\beta_{ed} = 0.44$, $\alpha_d = 0.01$ Contrast of the time-dependent velocity obtained from the experimental data and motion of model 3 of the applied force time-dependent decreasing function $(f_{s3}(t))$ for 5,000-meters runner in research of Ari T. et.al..

B) Similitude of the time dependent kinetic energy obtained from the experimental data and motion of model 3 of the applied force time-dependent decreasing function $(f_{s3}(t))$ for 5,000-meters runner in research of Ari T. et.al.

Discussions

From Figure 2 personification of the time dependent velocity could be obtained from the experiment data and mathematical and physics model1 (Figure 2A) modeling 2 (Figure 3A) and modeling 3 (Figure 4A) of the applied force timedependent. From Figure 1 we compared the velocity which obtained from the sprinter's speed in the 5,000 meters distance (Alexander et al., 2019). Model 3 is the closest to 1. The biological capacity of the lung for long-distance sprinters tended to have an increase in the oxygen gas containment area and pulmonary capacity (Tanji et al., 2017). Table 1 shows the personification of the quantity velocity of the 5,000-metre sprinters in the article of Ari et al., 2006. This is consistent with the correlation coefficient value of model = 0.89 but the velocity of 5,000-metre sprinters from the applied force is the function exponential and polynomial decreasing function of Mathematics and Newton's second law of motion for modeling 2 has less difference of commission (%) from the experiment. This is consistent with the correlation coefficient value of the model = 0.92. The velocity of 5,000-metre sprinters from the applied force is a time-dependent decreasing function of Mathematics and Newton's second law of motion for modeling 3 has less difference of commission (%) from the experiment. This is consistent with the correlation coefficient value of the model = 0.92. The velocity of 5,000-metre sprinters from the applied force is a time-dependent decreasing function of Mathematics and Newton's second law of motion for modeling 3 has less difference of commission (%) from the experiment. This is consistent with the correlation coefficient value of the model = 0.93. From figure 2A, 3A, and 4A show the endurance coefficient value, $\beta_{ed} = 0.15 s^{-1}$ in model 1, $\beta_{ed} = 0.21 s^{-1}$ in model 2, and $\beta_{ed} = 0.44 s^{-1}$ in model3, in order. The endurance coefficient of model 3 is mostly affected by increased lung oxygen capacity.

Conclusions

The correlation coefficient value for the 5,000-metre runner in model 2 and model 3 is more than the correlation coefficient value of model 1. An initial force (f_0) that refined from model 1 is higher than that from models 2 and 3. The endurance coefficient value (β_{ed}) is lowest in model 1, and the coefficient drag force (α_d) from model 3 is less than models 2 and 1.

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