# THESIS APPROVAL

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THESIS

# SYSTEMATIC APPROACH OF SIMULATION AND CONTROL OF HEAT EXCHANGER NETWORKS USING PARAMETRIC PROGRAMMING AND SPLIT-RANGE CONTROL

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Heat exchanger networks (HENs) are the heart of heat-integrated plants that guarantees the optimum energy recovery. This thesis developed a systematic approach of an optimal operation for HENs. Formulating mixed-integer non-linear programming (MINLP) from given hot and cold stream data, a total annual cost (TAC) can be minimized. Flexibility test is performed using linear programming (LP) in order to propose the final flexible HENs for design the control structure. Next, a parametric programming is used in this step to find active constraint regions. Up to this point, an optimal split-range control structure can be determined by integer linear program (ILP). Finally, a proposed operation and its configuration are verified and studied to understand their dynamics behavior. Four case studies were used to illustrate the proposed procedure. Case study 1 from Zamora and Grossmann (1998), the total annual is 285,916 \$/year distributed among the 8 units. Two active constraint regions are found. Biegler *et al.* (1997) as a case study 2 has the total annual costs at 85,290 \$/year for 6 unit operations with 3 active constraint regions. For case study 3, from Aaltola (2003), the total annual costs for 4 units is 27,743 \$/year, no active constraint can be found. Case study 4 from Yerramsetty (2008) shows the total annual costs at $2.942 \times 10^6$ \$/year. There are 15 operations with 5 active constraint regions. In all cases, additional periodical data were performed in order to extend operation regions. We noticed that the superstructure cannot be improved upon to the number of periodical data. For dynamics test, all cases show the performance of control structure to keep all target temperatures at the desired values.

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Sarun Porsom
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| | | |
|---|---|---|
| CV | = | controlled variable |
| DOF(s) | = | degree(s) of freedom |
| HEN(s) | = | heat exchanger network(s) |
| ILP | = | integer linear programming |
| LMTD | = | logarithmic mean temperature difference |
| LP | = | linear programming |
| MINLP | = | mixed-integer linear programming |
| MPT | = | multi parametric toolbox |
| MV | = | manipulated variable |
| NLP | = | nonlinear programming |
| RI | = | resilience index |
| TAC | = | total annual cost |

**Parameters**

| | | |
|---|---|---|
| $A$ | = | area of exchangers ($m^2$) |
| $Ccu_i$ | = | cost of cold utility ($/kW) |
| $Chu_j$ | = | cost of hot utility ($/kW) |
| $C_p$ | = | heat capacity (kW/kg.$^oC$) |
| $DT_{min}$ | = | minimum allowable temperature difference ($^oC$) |
| $HU_{up}$ | = | upper bound of total hot utility available (kW) |
| $N_c$ | = | number of cold process streams |
| $N_{CV}$ | = | number of controlled variables |
| $N_{DOF}$ | = | number of degrees of freedom |
| $N_{DOF,U}$ | = | number of degrees of freedom with respect to utility cost optimization |
| $N_h$ | = | number of hot process streams |
| $N_{MV}$ | = | number of manipulated variables |

# LIST OF ABBREVIATIONS (Continued)

| | | |
|---|---|---|
| $N_{MV}$ | = | number of manipulated variables |
| $N_s$ | = | number of process streams |
| $N_t$ | = | number of targets |
| $N_U$ | = | number of utility types |
| $N_{unit}$ | = | number of exchanger units (matches) |
| $N_{unit,min}$ | = | minimum number of units (matches) |
| $DS$ | = | dimension space spanned by the manipulated variables in the inner HEN to the outer HEN |
| $RO$ | = | relative order |
| $U$ | = | overall heat transfer coefficient (kW/$^o$C.m$^2$) |
| $m$ | = | mass flowrate (kg/s) |
| $nok$ | = | number of stages |
| $r_{k,j}$ | = | relative order between controlled variable *k* and manipulated variable *j* |

**Variables**

| | | |
|---|---|---|
| $Q_i$ | = | duty of heat exchanger *i* (kW) |
| $Q_{c,i}$ | = | duty of cold utility exchanger *i* (kW) |
| $Q_{h,i}$ | = | duty of hot utility exchanger *i* (kW) |
| $T$ | = | temperature ($^o$C) |
| $ub_i$ | = | bypass fraction of exchanger *i* |

**Binary variables**

| | | |
|---|---|---|
| $x_{i,j}$ | = | existence of relationship between manipulated variable *i* and manipulated variable *j* |
| $z_{k,j}$ | = | existence of control pair between controlled variable *k* and manipulated variable *j* |

# SYSTEMATIC APPROACH OF SIMULATION AND CONTROL OF HEAT EXCHANGER NETWORKS USING PARAMETRIC PROGRAMMING AND SPLIT-RANGE CONTROL

## INTRODUCTION

For certain classes of chemical processes, optimal operation can be achieved at input constraint vertices. However, under the changes of operating conditions, optimal vertices could be affected and shifted. Hence, appropriate manipulations of inputs for tracking optimal vertices are required. A promising procedure based on split-range control technique is proposed in this work.

Split-range control is a simple technique that can handle constraint problems on manipulated variables, (Marlin, 2000). In split-range control, two manipulated variables can be used to adjust one controlled output, that is, when one manipulated variable is saturated, the other will take over task of the saturated one. However, the objective here is focusing on the application of split-range control to implement optimal operation. Glemmestad *et al.* (1996) and Giovanini *et al.* (2003) pointed out that in most cases optimal operation of HENs can be implemented using split-range control. A systematic procedure for finding an optimal split-range control structure of HENs can be found in Lersbamrungsuk *et al.* (2008).

This work develops a systematic approach of an optimal operation for heat exchanger networks, in a step-by-step as follows. Firstly, HENs are synthesized using multi-period mixed-integer-non-linear programming (MINLP) model to minimize the total annual cost (TAC). Secondly, we check flexibility of HEN. Thirdly, identify active constrain regions using parametric programming (e.g. with MPT toolbox Kvasnica *et al*. (2004)). Fourthly, generate an optimal control structure using split-range control. Finally, check an optimal control structure using Aspen Dynamics.

# OBJECTIVES

This thesis aims to develop a systematic approach of an optimal operation for heat exchanger networks. The proposed operation and configuration is verified and studied to cover its dynamics behavior.

## Scopes of work

1. Using multi-parametric toolbox to solve set of active constraints for additional information by using integer linear programming (ILP).

2. Four cases studies proposed in this work. Case study 1, from Zamora and Grossmann (1998), contains three hot and two cold streams. Case study 2, from Biegler *et al.* (1997), has two hot and two cold process streams. Case study 3, from Aaltola (2003), covers two hot and two cold process streams. Case study 4, Yerramsetty (2008), contains four hot and five cold streams.

# LITERATURE REVIEW

In this section, the detail information of flexible HEN design, multi-period HEN design, parametric programming and optimal operation of heat exchanger networks were shown as follows:

## 1. Flexible HEN design

While good design methodologies exist for the design of HENs with fixed parameters, changes in operating parameters might cause deviation from optimality or even cause control and safety problems. Therefore, a good HEN should be optimal for a set of nominal conditions as well as provide the flexibility to handle a range of operating conditions. Changes in operating conditions might originate from uncertainty in parameters during operation or from different operating conditions, which then refers to multi-period operation.

This section reviews the literatures on the design of flexible HENs under variable operating conditions, both on uncertainty as well as multi-period approaches. The literature presented is limited to grassroots design of HENs.

Marselle *et al.* (1982) firstly developed the concept of resilient HENs which can tolerate uncertainties in temperatures and flow rates. This method, however, needs a manual combination of a series of optimal designs under different worst-case scenarios, making the application of this method to large-size problems practically impossible. To measure the degree of flexibility of a HEN, Saboo et al. (1985) proposed the resilience index (RI). The RI can be used to compare between different options on a quantitave basis.

Swaney and Grossmann (1985a,b) introduced a flexibility index, which represents a maximum deviation of an uncertain parameter while still remaining within the feasible region. The flexibility index provides a basis for different designs to be compared as well as providing the necessary information about which critical points in the feasible region that limit the design. This concept is illustrated in Fig. 1. The point in the middle represents the nominal operating conditions for parameters. The surrounding constraints mark the region R for which there is a feasible solution. The rectangles inside show the maximum deviation of the two parameters for which the network remains feasible.

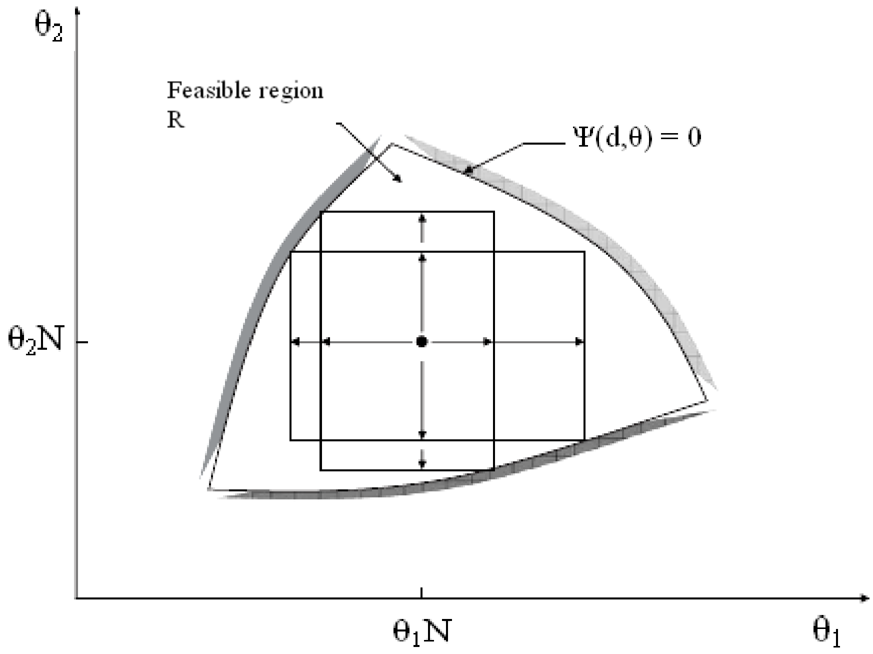



**Figure 1** Feasible region of operation
**Source:** Zhang *et al.* (2006)

Tantimuratha et al. (2001) proposed a two-stage design procedure, where the first stage consists of area targeting using area target models developed by Briones and Kokossis (1999). For a selected set of matches from the first stage, the network is optimised with an iterative procedure to minimise annual costs. This method, however, could still lead to sub-optimal solutions because it relies on decomposition.

**2. Multi-period HEN design**

For multi-period operations, the different operating modes resemble the corner points of the feasible region. Floudas and Grossmann (1986) introduced a multi-period MINLP model based on the transhipment model of Papoulias and Grossmann (1983). This model for minimum utilities and minimum number of matches uses pinch points at each operating period. The next stage is a reformulated NLP model that develops the network configuration, presented by Floudas and Grossmann (1987). Here again, the decomposition of the problem into different stages significantly reduces the size of the problem, but does not take into account the trade-off between area, number of units, and energy rigorously. The model proposed by Papalexandri and Pistikopoulos (1994a,b) simultaneously explores alternatives in an MINLP problem. This of course limits the size of the problem to relatively small-scale problems. Aaltola (2003) proposed a model which simultaneously optimizes the multi-period MINLP problem for minimum costs and flexibility, without relying on sequential decomposition. This model, based on the stage-wise HEN superstructure representation of Yee et al. (1990), has been successfully applied to industrial HEN problems. Since the design of multi-period heat exchangers involves the use of the same heat exchangers for different heat loads, this will generally involve bypassing part of a stream or even an entire stream from the heat exchanger unit. The formulation of Aaltola (2003) allows the elimination of bypass modeling, which would introduce non-linear constraints into the formulation. Chen and Hung (2004) proposed a three-step approach for designing flexible multi-period HEN, which is based on the stage-wise HEN superstructure representation of Yee et al. (1990) and the mathematical formulation of Aaltola (2003). The authors introduced the maximum area consideration in the objective function, and decomposed the problem into three main iterative steps: simultaneous HEN synthesis, flexibility analysis, and removal of infeasible networks. The number of iterations required in this approach is of concern for industrial problems. Zhang *et al.* (2006) were proposed modifications to Aaltola (2003)'s model include the use of maximum area per period in the area cost calculation of the MINLP objective function, and the removal of slack variables and weighed parameters from the existing NLP improvement model. The new model has been applied to one industrial case study, demonstrating that the new combined MINLP–NLP model can obtain better solutions by not relying on the average area assumption in the MINLP stage.

**3. Parametric programming**

In an optimization framework, where the objective is to minimize or maximize a performance criterion subject to a given set of constraints and where some of the parameters in the optimization problem vary between specified lower and upper bounds, parametric programming is a technique for obtaining (i) the objective function and the optimization variables as a function of these parameters and (ii) the regions in the space of the parameters where these functions are valid (Fiacco, 1983; Gal, 1995; Acevedo and Pistikopoulos, 1996, 1997; Pertsinidis *et al.*, 1998; Papalexandri, 1998; Acevedo, 1999; Dua and Pistikopoulos, 1999)

The main advantage of using the parametric programming techniques to address such problems is that for problems pertaining to plant operations, such as for process planning Pistikopoulos *et al.* (1998) and scheduling, one can obtain a complete map of all the optimal solutions. Moreover, as the operating conditions fluctuate, one does not have to re-optimize for the new set of conditions since the optimal solution as a function of parameters (or the new set of conditions) is already available. Mathematically, such problems can be posed as multi-parametric mixed-integer nonlinear programming problems of the following form:

$$
\begin{aligned}
& z(\theta) = \min d^T y + f(x) \\
& \text{s.t. } Ey + g(x) \le b + F\theta \\
& \theta_{\min} \le \theta \le \theta_{\max} \\
& x \in X \subseteq \Re^n \\
& y \in Y = \{0,1\}^m \\
& \theta \in \Theta \subseteq \Re^s, \qquad (1)
\end{aligned}
$$

where $y$ is vector of 0–1 binary variables, $x$ a vector of continuous variables, $f$ a scalar, continuously differentiable and convex function of $x$, $g$ a vector of continuously differentiable and convex functions of $x$, $b$ and $d$ are constant vectors, $E$ and $F$ are constant matrices, $\theta$ is a vector of parameters, $\theta_{\min}$ and $\theta_{\max}$ are the vectors of lower and upper bounds on $\theta$, and $X$ and $\Theta$ are compact and convex polyhedral sets of dimensions $n$ and $s$, respectively. While the detailed theory and algorithms for solving Eq. (1) was proposed by Dua *et al.,* (1999, 2000), the engineering significance of solving Eq. (1) by using parametric programming.

## 4. Optimal operation of heat exchanger networks

To operate the plant in an optimal manner, one should first answer the two questions: (Q1) are there enough degrees of freedom (DOFs, or manipulated variables) for control? and (Q2) are there extra degrees of freedom for an optimization? The question (Q1) is used to check the possibility to control each controlled variable independently, while the question (Q2) is used to check whether except the control design for setpoint satisfaction, we also need to consider the economic objective or not. For heat exchanger networks, Marselle *et al.* (1982) proposed the definition of the number of degrees of freedom ($N_{DOF}$) by

$$N_{DOF} = N_{units} - N_t \qquad (2)$$

where $N_{units}$ is the number of exchanger units or manipulated variables (degrees of freedom) and $N_t$ is the number of target temperatures.

The condition $N_{DOF} > 0$ is necessary for the operation to be feasible and utility cost optimizable. However, this is not enough to answer the questions (Q1) and (Q2). The more precise definition of the number of degrees of freedom with respect to utility cost optimization ($N_{DOF,U}$) that was sufficient to answer the two questions was proposed by Glemmestad (1997) as shown in the following:

$$N_{DOF,U} = DS + N_U - N_t \qquad (3)$$

where $DS$ is the dimensional space spanned by the manipulated variables in the inner HEN to the outer HEN and $N_U$ is the number of utility types.

The implication of $N_{DOF,U}$ can be summarized as shown in Table 1. The operation will be structurally feasible (question Q1) if and only if the condition $N_{DOF,U} \geq 0$ can be satisfied. Furthermore, there will be extra degrees of freedom for utility cost optimization (question Q2) if and only if $N_{DOF,U} > 0$.

**Table 1** The implication of $N_{DOF,U}$

| Case | (Q1) | (Q2) |
|---|---|---|
| $N_{DOF,U}$<0 | No | No |
| $N_{DOF,U}$=0 | Yes | No |
| $N_{DOF,U}$>0 | Yes | Yes |

It is obvious that the operation of HENs will be more challenging when $N_{DOF,U} > 0$ because aside from the setpoint satisfaction, the utility cost should also be minimized. This means that a common heuristic rule for the control design such as manipulating the last heat exchanger on a stream for a direct effect (Marselle *et al.*, 1982; Calandranis and Stephanopoulos, 1988; Mathisen, 1994) may not be preferred from an energy point of view.

Several strategies were proposed to handle optimal operation of HENs. The early researches were techniques based on structural information. Marselle *et al.* (1982) applied a graph theory to suggest a control structure and developed a control policy to adjust flow distributions in HENs to meet target temperatures with minimum utility usage. Calandranis and Stephanopoulos (1988) used the structural characteristics of HENs to identify routes to allocate loads to available sinks and developed a knowledge-based concept to select the best route. The methods based on structural information using a sign matrix (directional effect among manipulated variables and controlled variables) were proposed by Mathisen (1994) and Glemmestad *et al.* (1996).

However, the control design based on structural information cannot guarantee the optimality in some cases, such as when HENs contain heat load loops, hence a conventional online-optimization may be recommended. The researches based on online and periodic optimizations for optimal operation of HENs can be found in Aguilera and Marchetti (1998), Glemmestad *et al.* (1999) and González *et al.* (2006). Nevertheless, online-optimization requires a rather complex approach for the implementation. Hence, some recent researches for the optimal operation have been devoted to simple ways of implementing (economic) optimal operation. For example, Skogestad (2000) proposed a concept of 'self-optimizing control' that is, finding a magic variable to keep constant and then resulting in optimality. Pistikopoulos *et al.* (2002) used offline parametric optimization to simplify the task of online optimization.

This thesis is to develop a systematic approach of an optimal operation for heat exchanger networks. First, HENs are synthesized using multi-period mixed-integer-non-linear programming (MINLP) model to minimize the total annual cost (TAC) was proposed by Aaltola (2003). Second, we check flexibility of HEN was proposed by Aaltola (2003). Third, check degrees of freedom were proposed by Glemmestad (1997). Fourth, determine set of active constraint for information by using integer linear program (ILP) for optimal split-range control structure was proposed by Lersbamrungsuk (2008). Finally, check an optimal control structure using Aspen Dynamics.

# MATERIALS AND METHODS

## Materials

1. Laptop
   a) CPU (Intel Core2Duo CPU 2.0 GHz)
   b) 2.00 GB of RAM
   c) 120 GB of hard disk
2. Operating System: Microsoft Window Vista Home Premium service pack 1
3. Software
   a) GAMS versions 22.8 beta
   b) MATLAB version 2007b
   c) ASPEN PLUS version 2006.5
   d) ASPEN DYNAMICS version 2006.5

## Methods

### 1. Simultaneous MINLP model

Aaltola (2003) was proposed to develop a method to solve flexible HEN synthesis problems so that the trade-offs between energy costs, fixed charges for units and costs for the exchanger area can be simultaneously optimized. In this part is a simultaneous optimization model with necessary simplifications to make the formulation robust and efficient enough to solve industrial size problems.

1.1 Model formulation from Aaltola (2003)

The existence of each potential heat exchanger in the superstructure is represented by binary variables. This means that if a heat exchanger exists in any period, it must exist in every period, and the fixed cost is charged accordingly. The formulation includes continuous variables assigned to temperatures and to the heat loads of each period. The model is solved to minimize the annualized cost of the equipment plus the annual cost of utilities. This model provides matches that take place ($z_{i,j,k}$, $z_i^{cu}$, $z_j^{hu}$) and for every period, areas of each exchanger ($A_{i,j,k,p}$, $A_{i,p}^{cu}$ and $A_{j,p}^{hu}$), corresponding exchanger loads ($q_{i,j,k,p}$, $q_{i,p}^{cu}$ and $q_{j,p}^{hu}$) and possible bypass flow for each exchanger.

In order to formulate the MINLP model for total annual cost comprising of utility cost, fixed charges for exchanger units and areas, the following definitions are necessary:

i) Indices:

*i* = hot process or utility stream,

*j* = cold process or utility stream,

*k* = index for stage 1,…, NOK

and temperature location 1,…, NOK + 1,

*p* = operation period;

ii) Sets:

HP = set of a hot process stream *i*,

CP = set of a cold process stream *j*,

HU = hot utility,

CU = cold utility,

ST = set of a stage in the superstructure, *k* = 1,…, NOK,

PR = set of a operation period, *p*= 1,…, NOP;

iii) Parameters:

$T^{IN}$ = inlet temperature of stream,

$T^{OUT}$ = outlet temperature of stream,

F = heat capacity flow rate,

U = overall heat transfer coefficient,

$C^{CU}$ = per unit cost for cold utility,

$C^{HU}$ = per unit cost for hot utility,

$C^{F}$ = fixed charge for heat exchanger unit,

C = area cost coefficient for heat exchanger,

B = exponent for area cost,

AF = annualization factor,

NOK = number of stages,

NOP = number of periods,

DOP = duration of period,

$Q^{UP}$ = an upper bound on heat exchange,

$DT^{UP}$ = an upper bound on temperature difference,

$HU^{UP}$ = an upper bound on total hot utility available,

$\varepsilon$ = exchanger minimum approach temperature;

iv) Positive variables:

$t_{i,k,p}$ = temperature of hot stream *i* at hot end of stage *k* in period *p*,

$t_{j,k,p}$ = temperature of cold stream *j* at hot end of stage *k* in period *p*,

$dt_{i,j,k,p}$ = temperature difference for match (*i,j*) at temperature location *k* in period *p*,

$dt^{hu}_{i,k,p}$ = temperature difference for match of cold stream *j* and hot utility in period *p*,

$dt^{cu}_{j,k,p}$ = temperature difference for match of hot stream *i* and cold utility in period *p*,

$q_{i,j,k,p}$ = heat exchanged between hot stream *i* and cold stream *j* in period *p*,

$q^{hu}_{j,p}$ = heat exchanged between cold stream *j* and hot utility in period *p*,

$q^{cu}_{i,p}$ = heat exchanged between hot stream *i* and cold utility in period *p*;

v) Binary variables:

$z_{i,j,k}$ = existence of match ($i,j$) in stage $k$,

$z_j^{hu}$ = existence of match between cold stream $j$ and hot utility,

$z_i^{cu}$ = existence of match between hot stream $i$ and cold utility;

vi) Variables:

*TAC* = total annual costs for the network.

The set of constraints consists of:

Overall heat balances are to ensure sufficient heating and cooling of each process stream in each period. These equality constraints specify that the heat content of each stream equals the sum of heat exchanged with other streams at each stage plus the exchange with the utility.

$$(T_{i,p}^{IN} - T_{i,p}^{OUT})F_{i,p} = \sum_{k \in ST} \sum_{j \in CP} q_{i,j,k,p} + q_{i,p}^{cu}, \qquad i \in HP, \quad p \in PR \tag{4}$$

$$(T_{j,p}^{OUT} - T_{j,p}^{IN})F_{j,p} = \sum_{k \in ST} \sum_{i \in HP} q_{i,j,k,p} + q_{j,p}^{hu}, \qquad j \in CP, \quad p \in PR \tag{5}$$

Heat balance is required for each stream at each stage of the superstructure in each period to determine temperatures. To properly define the temperature variables and stages, the index $k$ is used. The set $k$ = 1,…,*NOK* is used to represent the NOK stages while the set $k$ = 1,…,*NOK*+1 is for temperature locations in the superstructure. The heat balances are as follows:

$$(t_{i,k,p} - t_{i,k+1,p})F_{i,p} = \sum_{j \in CP} q_{i,j,k,p}, \qquad k \in ST, \quad i \in HP, \quad p \in PR \tag{6}$$

$$(t_{j,k,p} - t_{j,k+1,p})F_{j,p} = \sum_{i \in HP} q_{i,j,k,p}, \qquad k \in ST, \quad j \in CP, \quad p \in PR \tag{7}$$

Assignment of inlet temperatures in each period. The superstructure inlet corresponds to temperature location $k$ = 1 for hot streams, while for cold streams to location $k$ = $NOK$ + 1:

$$T_{i,p}^{IN} = t_{i,1,p}, \qquad i \in HP, \quad p \in PR \tag{8}$$

$$T_{j,p}^{IN} = t_{j,NOK+1,p}, \qquad j \in CP, \quad p \in PR \tag{9}$$

Constraints for feasibility of temperatures in each period are needed to specify a monotonic decrease of temperature at each stage. Also, an upper bound for the outlet temperature of each stage is set at the respective stream’s outlet temperature. Note that the outlet temperature does not necessarily correspond to the stream’s target temperature, since heating and cooling using utilities may take place at the superstructure outlet.

$$t_{i,k,p} \geq t_{i,k+1,p}, \qquad k \in ST, \quad i \in HP, \quad p \in PR \tag{10}$$

$$t_{j,k,p} \geq t_{j,k+1,p}, \qquad k \in ST, \quad j \in CP, \quad p \in PR \tag{11}$$

$$T_{i,p}^{OUT} \leq t_{i,NOK+1,p}, \qquad i \in HP, \quad p \in PR \tag{12}$$

$$T_{j,p}^{OUT} \geq t_{i,l,p}, \qquad j \in CP, \quad p \in PR \tag{13}$$

Energy balances for utility matches are determined for each process stream and period in terms of the corresponding outlet temperature in the last stage and the corresponding target temperature.

$$(t_{i,NOK+1,p} - T_{i,p}^{OUT})F_{i,p} = q_{i,p}^{cu}, \qquad i \in HP, \quad p \in PR \tag{14}$$

$$(T_{j,p}^{OUT} - t_{j,1,p})F_{j,p} = q_{j,p}^{hu}, \qquad j \in CP, \quad p \in PR \tag{15}$$

Logical constraints are used for existence of matches (i,j) in stage $k$ and for utilities. In addition, the upper bound on heat exchange capacity $Q^{UP}$ for each period can be set to the smallest heat content of corresponding period of the two streams involved in the match.

$$q_{i,j,k,p} - Q_p^{UP} z_{i,j,k} \le 0, \qquad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR \qquad (16)$$

$$q_{i,p}^{cu} - Q_p^{UP} z_i^{cu} \le 0, \qquad i \in HP, \quad p \in PR \qquad (17)$$

$$q_{j,p}^{hu} - Q_p^{UP} z_j^{hu} \le 0, \qquad j \in CP, \quad p \in PR \qquad (18)$$

$$z_{i,j,k}, \; z_i^{cu}, \; z_j^{hu} \in \{0,1\} \qquad (19)$$

Calculation of temperature differences for each temperature location in each period are used to ensure feasible driving forces for exchangers. The binary variables are used to activate these constraints. When a match (*i*,*j*) in stage *k* occurs, $z_{i,j,k}$ equals one and the constraint becomes active, so that the temperature difference is properly calculated. However, when the match does not take place ($z_{i,j,k}$ equals zero), $DT_p^{UP}$ sets the upper bound for temperature approach. Here $DT_p^{UP}$ is defined as a maximum positive temperature difference between hot and cold stream for each period. Similar constraints are also used for utilities:

$$dt_{i,j,k,p} \le t_{i,k,p} - t_{j,k,p} + DT_p^{UP}(1 - z_{i,j,k}), \qquad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR \qquad (20)$$

$$dt_{i,j,k+1,p} \le t_{i,k+1,p} - t_{j,k+1,p} + DT_p^{UP}(1 - z_{i,j,k}), \qquad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR \qquad (21)$$

$$dt_{i,p}^{cu} \le t_{i,NOK+1,p} - T_{CU}^{OUT} + DT_p^{UP}(1 - z_i^{cu}), \qquad i \in HP, \quad p \in PR \qquad (22)$$

$$dt_{j,p}^{hu} \le T_{HU}^{OUT} - t_{j,1,p} + DT_p^{UP}(1 - z_j^{hu}), \qquad j \in CP, \quad p \in PR \qquad (23)$$

These constraints can be expressed as inequalities, since the minimization of the objective function, where the exchanger areas are calculated using the temperature approaches, drives the temperature approaches upwards.

The objective function is non-linear and non-convex and hence, despite the linear set of constraints, the solution of the resulting optimization model represents a local optimum. However, it is possible to generate a pool of the local optima by performing several runs with different upper bounds on the hot utility availability. Also, a value for the exchanger minimum approach temperature (EMAT) can be defined, setting the upper bound for a crisscross heat transfer i.e. non vertical heat transfer on the composite curves.

The lowest allowable exchanger minimum approach temperature is defined as:

$$dt_{i,j,k,p} \geq \varepsilon, \qquad (24)$$

Total hot utility availability is limited by following constraints:

$$\sum_{j \in CP} q_{j,p}^{hu} \leq HU_p^{UP}, \qquad j \in CP, \quad p \in PR \qquad (25)$$

In order to avoid the modeling of bypasses in the MINLP model, the objective function considers the area of one match to be the mean value of areas in different periods, hence the model under-estimates total area costs and over-estimates exchanger areas. The exact method of calculating the real area costs is to add up the costs related to the maximum match areas. The methods for searching for maximum areas and obtaining real area investment costs in an objective function, introduce non-linearities in constraints or nonlinearities with discontinuous derivatives in the objective function. A model with these methods would be more difficult to solve and less robust. Despite the area cost underestimation, the multi-period model finds good partial solutions for separate periods, which can be seen when the results of the multi-period model are compared to the corresponding results of the single period model. The real total annual costs are calculated after the optimization, within a separate part of the model.

TAC (total annual costs for the network) can be defined as the summation of:

i) Unit costs for all matches,
ii) mean area costs for matches (*i*,*j*,*k*),
iii) mean area costs for cold utility matches,
iv) mean area costs for hot utility matches,
v) weighted cold utility costs and
vi) weighted hot utility costs.

Thus, the objective function is

$$\min TAC =$$

$$AF\left[\sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST} C^F_{i,j} Z_{i,j,k} + \sum_{i\in HP}\sum_{CU} C^F_{i,CU} z^{cu}_i + \sum_{j\in CP}\sum_{HU} C^F_{i,HU} z^{hu}_j\right] \quad \text{(i)}$$

$$+AF\sum_{p\in PR}\frac{1}{NOP}\sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST} C_{i,j}\left[\frac{q_{i,j,k,p}}{LMTD_{i,j,k,p}U_{i,j}}\right]^{B_{i,j}} \quad \text{(ii)}$$

$$+AF\sum_{p\in PR}\frac{1}{NOP}\sum_{i\in HP} C_{i,CU}\left[\frac{q^{cu}_{i,p}}{LMTD_{i,CU,p}U_{i,CU,p}}\right]^{B_{i,CU}} \quad \text{(iii)}$$

$$+AF\sum_{p\in PR}\frac{1}{NOP}\sum_{j\in CP} C_{j,HU}\left[\frac{q^{hu}_{j,p}}{LMTD_{j,HU,p}U_{i,HU,p}}\right]^{B_{j,HU}} \quad \text{(iv)}$$

$$+\sum_{p\in PR}\frac{DOP_p}{NOP}\sum_{i\in HP} C^{CU} q^{cu}_{i,p} \quad \text{(v)}$$

$$+\sum_{p\in PR}\frac{DOP_p}{NOP}\sum_{j\in CP} C^{HU} q^{hu}_{j,p} \quad \text{(vi)} \qquad (26)$$

This optimization formulation is able to take into account weighted periods, so that the most common operating condition can dominate, while the uncommon one is still considered. This can be done by defining the duration of periods $DOP_p$ for utility costs.

1.2 Illustrative example for MINLP stage

Next, the basic example, consisting of two hot and two cold streams, Saboo and Morari (1984) is considered where the heat capacity flowrate $F_{H2}$ is an uncertain parameter. The problem data for the multi-period problem is selected as is shown in Table 2. In addition to data from the original example:

i) the annual costs of unit duty for hot utility is 115.2 €/(kW·a) and for cold utility 1.3 €/(kW·a),

ii) the costs equation for exchangers is 8333.3 · unit+641.7 · Area (€),

iii) lifetime used is 3 a and rate of interest 18%,

iv) overall heat transfer coefficients for all matches are 4 $kW·m^{-2}·K^{-1}$.

**Table 2** Operating conditions for example.

| Stream | Period 1 | | | Period 2 | | |
|---|---|---|---|---|---|---|
| | $T^{IN}$ (K) | $T^{OUT}$ (K) | $FC_p$ (kW/K) | $T^{IN}$ (K) | $T^{OUT}$ (K) | $FC_p$ (kW/K) |
| H1 | 723 | 553 | 2 | 723 | 553 | 2 |
| H2 | 583 | 323 | 1 | 583 | 323 | 1.8 |
| C1 | 388 | 563 | 2 | 388 | 563 | 2 |
| C2 | 313 | 393 | 3 | 313 | 393 | 3 |

**Source:** Aaltola (2003)

The number of stages and hot utility limits were defined by the initialization procedure. In solving the multi-period MINLP model by using 3 stages, setting the hot utility limits ( $HU_1^{UP}$ and $HU_2^{UP}$ ) to less than zero and forbidding stream splitting, the results are shown in Table 3.

**Table 3** Results of MINLP model.

| Match | Period 1 | | | | Period 2 | | | |
|---|---|---|---|---|---|---|---|---|
| i,j,k | 1.1.3 | 2.1.1 | 2.2.2 | CU.1 | 1.1.3 | 2.1.1 | 2.2.2 | CU.1 |
| $A_{i,j,k,p}$[m$^2$] | 0.97 | 0.69 | 2.09 | | 0.22 | 9.03 | 4.14 | |
| $q_{i,j,k,p}$ [kW] | 330 | 20 | 240 | 10 | 122 | 228 | 240 | 218 |
| $F^h_{i,j,k,p}$ [kW/K] | 2 | 1 | 1 | | 2 | 1.8 | 1.8 | |
| $F^c_{i,j,k,p}$ [kW/K] | 2 | 2 | 3 | | 2 | 2 | 3 | |
| $T^{INh}_{i,k,p}$ [$^o$C] | 723 | 583 | 563 | | 723 | 583 | 456 | |
| $T^{OUTh}_{i,k,p}$ [$^o$C] | 558 | 563 | 323 | | 662 | 456 | 323 | |
| $T^{Inc}_{j,k,p}$ [$^o$C] | 388 | 553 | 313 | | 388 | 449 | 313 | |
| $T^{OUTc}_{j,k,p}$ [$^o$C] | 553 | 563 | 393 | | 449 | 563 | 393 | |

**Source:** Aaltola (2003)

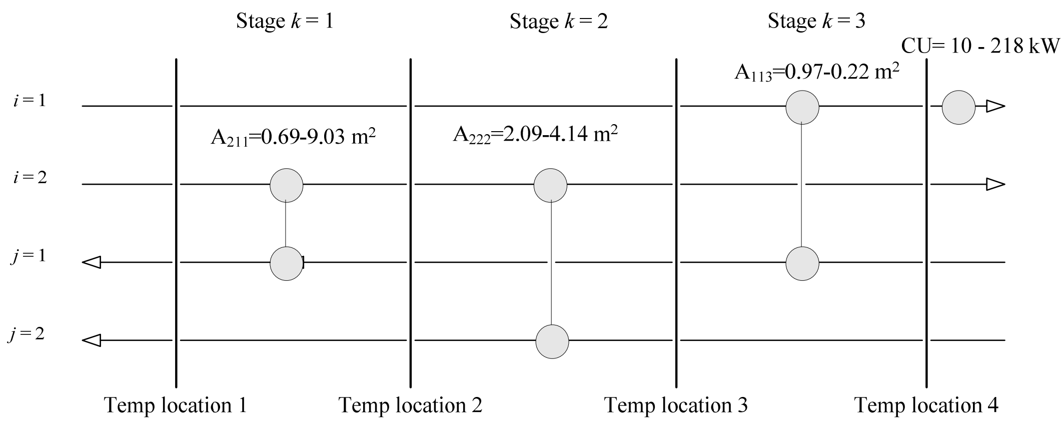



**Figure 2** The resulting HEN after the first MINLP.
**Source:** Aaltola (2003)

The HEN resulting from the MINLP optimization stage consists of three process to process heat exchangers and one cold utility. For each exchanger there are two different areas and for the cold utility there is a heat load for both periods. If it was possible to increase and decrease the areas of the exchangers and regulate the cold utility load, this configuration would be able to operate under these two periods with the set up values shown in Figure 2.

## 2. Feasibility test

After the multi-period simultaneous MINLP model has provided the optimal HEN structure for certain periods, one may ask if the configuration between the defined conditions is feasible. Alatola (2003) was proposed how to ensure that the network is feasible in operating not only over these specified periods, but also over the whole range of the specified parameters, is discussed. This is referred to the task of keeping the outlet temperatures in the network, defined by the MINLP model, at their target values during a short and long time horizon. Note that there is an assumption of perfect control, i.e., control can be adjusted depending on the realization of uncertain parameters and no delays in the measurements, or adjustments in the control are considered.

## 2.1 Multi-period LP feasibility test model

The LP formulation has been developed by Alatola (2003) to analyze the structural and final flexibility of the HEN. The isothermal mixing assumption is used in the formulation to maintain the linear constraints and make the model compatible with MINLP model, so that critical conditions found with the LP model are suitable for further MINLP optimization. The proposed model is especially suitable for determining network feasibility in the cases where, in addition to a large number of correlated uncertain parameters, only a few independent variations take place. Therefore, it is a practical tool for cases featuring large long term variations with only a few short term disturbances occurring at the same time e.g., the integration of waste heat streams of pulp mills and district heating systems. Note that this LP model takes account of not only structural feasibility but also feasibility depending on individual exchanger conductance.

The LP formulation for minimum temperature approach violations of a given network configuration is formulated by using the same indices and sets as used by the MINLP model. In addition to definitions of the MINLP model the following ones are necessary:

i) Parameters:

$DT^{UP}$ = an upper bound on temperature difference,

$CN^{UP}$ = an upper bound on conductance,

Z = existence of match ($i,j$) in stage $k$ (MINLP results),

$Z^{CU}$ = existence of match between cold stream $j$ and hot utility (MINLP results),

$Z^{HU}$ = existence of match between hot stream $i$ and cold utility (MINLP results);

ii) Positive variables:

$sdt_{i,j,k,p}$ = slack variable for temperature approach violations related to match ($i,j$) at temperature location $k$ in period $p$;

iii) Variables:

$z$ = the summation of temperature approach violations.

With these additional definitions, the model can now be formulated. The set of constraints consists of:

$$(T_{i,p}^{IN} - T_{i,p}^{OUT})F_{i,p} = \sum_{k \in ST} \sum_{j \in CP} q_{i,j,k,p} + q_{i,p}^{cu}, \qquad i \in HP, \quad p \in PR \tag{27}$$

$$(T_{j,p}^{OUT} - T_{j,p}^{IN})F_{j,p} = \sum_{k \in ST} \sum_{i \in HP} q_{i,j,k,p} + q_{j,p}^{hu}, \qquad j \in CP, \quad p \in PR \tag{28}$$

Heat balances:

$$(t_{i,k,p} - t_{i,k+1,p})F_{i,p} = \sum_{j \in CP} q_{i,j,k,p}, \qquad k \in ST, \quad i \in HP, \quad p \in PR \tag{29}$$

$$(t_{j,k,p} - t_{j,k+1,p})F_{j,p} = \sum_{i \in HP} q_{i,j,k,p}, \qquad k \in ST, \quad j \in CP, \quad p \in PR \tag{30}$$

Assignment of inlet temperatures:

$$T_{i,p}^{IN} = t_{i,1,p}, \qquad i \in HP, \quad p \in PR \tag{31}$$

$$T_{j,p}^{IN} = t_{j,NOK+1,p}, \qquad j \in CP, \quad p \in PR \tag{32}$$

Feasibility of temperatures:

$$t_{i,k,p} \geq t_{i,k+1,p}, \qquad k \in ST, \quad i \in HP, \quad p \in PR \tag{33}$$

$$t_{j,k,p} \geq t_{j,k+1,p}, \qquad k \in ST, \quad j \in CP, \quad p \in PR \tag{34}$$

$$T_{i,p}^{OUT} \leq t_{i,NOK+1,p}, \qquad i \in HP, \quad p \in PR \tag{35}$$

$$T_{j,p}^{OUT} \geq t_{i,l,p}, \qquad j \in CP, \quad p \in PR \tag{36}$$

Energy balances for utility matches:

$$(t_{i,NOK+1,p} - T^{OUT}_{i,p})F_{i,p} = q^{cu}_{i,p}, \qquad i \in HP, \quad p \in PR \tag{37}$$

$$(T^{OUT}_{j,p} - t_{j,1,p})F_{j,p} = q^{hu}_{j,p}, \qquad j \in CP, \quad p \in PR \tag{38}$$

Logical constraints for existence of matches (*i,j*) in stage *k* and utilities:

$$q_{i,j,k,p} - Q^{UP}_p Z_{i,j,k} \le 0, \qquad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR \tag{39}$$

$$q^{cu}_{i,p} - Q^{UP}_p Z^{cu}_i \le 0, \qquad i \in HP, \quad p \in PR \tag{40}$$

$$q^{hu}_{j,p} - Q^{UP}_p Z^{hu}_j \le 0, \qquad j \in CP, \quad p \in PR \tag{41}$$

Calculation of temperature differences for each temperature location in each period are used to ensure feasible driving forces for existing exchangers. The fixed binary variables Z, $Z^{CU}$ and $Z^{HU}$ are used to define whether the constraint is involved or not (exchangers exist if the parameter $Z_{i,j,k}$ = 1). The temperature differences are expressed as equalities to avoid the miscalculation of the exchanger areas, since the objective function does not have strong terms in relation to the temperature differences. Additional slack variables are added to temperature approach Equations (42) and (43). The slack variables are added to allow violations for feasible driving forces for existing exchangers.

$$dt_{i,j,k,p} = t_{i,k,p} - t_{j,k,p} + sdt_{i,j,k,p}, \qquad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR \tag{42}$$

Only when ($Z_{i,j,k}$ = 1).

$$dt_{i,j,k+1,p} = t_{i,k+1,p} - t_{j,k+1,p} + sdt_{i,j,k+1,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR \tag{43}$$

Only when ($Z_{i,j,k}$ = 1).

$$dt^{cu}_{i,p} = t_{i,NOK+1,p} - T^{OUT}_{CU}, \qquad i \in HP, \quad p \in PR \tag{44}$$

Only when ($Z^{CU}_i$ = 1).

$$dt^{hu}_{j,p} \le T^{OUT}_{HU} - t_{j,1,p}, \qquad j \in CP, \quad p \in PR \tag{45}$$

Only when ($Z^{HU}_j = 1$).

The maximum allowed conductance for each exchanger $CN^{UP}_{i,j,k}$ is limited by the inequality constraints (46) where the arithmetic means of temperature differences are used instead of LMTD to maintain the linear nature. In Equation (47), $q^{MINLP}_{i,j,k,p}$ and $dt^{MINLP}_{i,j,k,p}$ are parameters, obtained from the MINLP model. These parameters are connected with matches( $Z^{MINLP}_{i,j,k,p}$ = 1) and periods with maximum exchanger area, thus the conductance of each match in the LP model is limited so as to be smaller than or equal to conductance related to the maximum areas of each match in the MINLP model.

$$q_{i,j,k,p} \leq \left( dt_{i,j,k,p} + dt_{i,j,k+1,p} \right) CN^{UP}_{i,j,k}, \quad i \in HP, \; j \in CP, \; k \in ST, \; p \in PR \quad (46)$$

Only when ($Z_{i,j,k} = 1$), where,

$$CN^{UP}_{i,j,k} = \frac{q^{MINLP}_{i,j,k,p}}{dt^{MINLP}_{i,j,k,p} + dt^{MINLP}_{i,j,k+1,p}}, \qquad i \in HP, \; j \in CP, \; k \in ST \qquad (47)$$

After this, the objective function is written in the following manner to minimize the summation of additional slack variables i.e., temperature approach violations:

$$\text{Min } z = \sum_{p \in PR} \sum_{i \in HP} \sum_{j \in CP} \sum_{k \in ST} \left( sdt_{i,j,k,p} + sdt_{i,j,k+1,p} \right) \qquad (48)$$

The idea of this LP formulation is to minimize temperature approach violations, which can be solved with multi-period data representing information, something that is required to expose infeasibilities. This data is formed by making denser discretization of the problem data for correlated parameters and the addition of periodical data for short term disturbances.

After solving the LP feasibility test, the multi-period MINLP model is resolved with data that includes additional periods, representing the worst temperature approach violation i.e., critical conditions that limit the flexibility of a design. This loop, including the MINLP stage and LP stage, as is shown in Figure 3, continues until the resulting network is feasible for the whole specified range of parameter variations. If the problem involves many streams with uncorrelated disturbances, an active set strategy for the automated solution of the flexibility test (Grossmann and Floudas (1987)) should be used to identify the potential active constraints that limit the flexibility of a design.

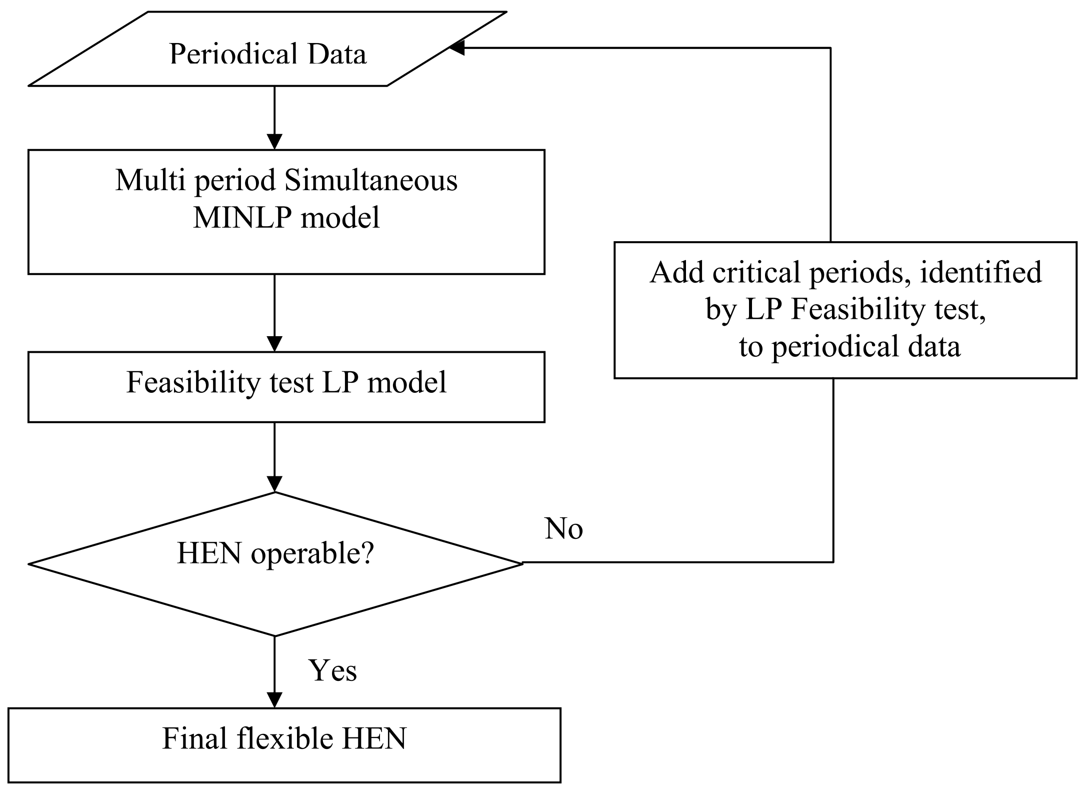



**Figure 3** Description of overall method identifying the required periods for multi-period MINLP stage and removing simplifications related to MINLP model.

**Source:** Aaltola (2003)

2.2 Illustrative example for feasibility test

As an illustrative example this feasibility test is applied into the example presented first in Subsection 1.1. The new multi-period data for the LP model is produced by dividing the parameter changes into ten steps. In this small example it means that data is similar for each period, except when $F_{H2}$ increases from 1 to 1.8 with the steps of 0.089 kW/K. Solving this LP model yields the following values for slack variables $sdt_{i,j,k,p}$ representing temperature approach violations related to match *i,j*, temperature interval *k* and period *p* [K],

$sdt_{2.1.1.8} = 1.792$

$sdt_{2.1.1.9} = 1.859$

$sdt_{2.1.2.3} = 3.115$

$sdt_{2.1.2.4} = 5.860$

$sdt_{2.1.2.5} = 6.729$

$sdt_{2.1.2.6} = 6.068$

$sdt_{2.1.2.7} = 4.145$

$sdt_{2.1.2.8} = 1.166$

The worst temperature approach violation is in period p = 5 being 6.7 K corresponding to flow rate $F_{H2}$ = 1.35 kW. This period is added to the data of the MINLP model, which after addition involves three periods as is shown in Table 4.

**Table 4** Operating conditions after added critical period.

| Stream | Period 1 | | | Period 2 | | | Period 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $T^{IN}$ (K) | $T^{OUT}$ (K) | $FC_p$ (kW/K) | $T^{IN}$ (K) | $T^{OUT}$ (K) | $FC_p$ (kW/K) | $T^{IN}$ (K) | $T^{OUT}$ (K) | $FC_p$ (kW/K) |
| H1 | 723 | 553 | 2 | 723 | 553 | 2 | 723 | 553 | 2 |
| H2 | 583 | 323 | 1 | 583 | 323 | 1.8 | 583 | 323 | 1.35 |
| C1 | 350 | 563 | 2 | 350 | 563 | 2 | 350 | 563 | 2 |
| C2 | 313 | 393 | 3 | 313 | 393 | 3 | 313 | 393 | 3 |

**Source:** Aaltola (2003)

Resolving the multi-period MINLP model gives the results shown in Table 5. After this, analysis of the resulting network feasibility with the proposed LP model yields zero for all slack variables ($sdt_{i,j,k,p}$), which means that proper adjustment of bypasses (over matches 212 and 223) and cold utility load (in stream H2) leads to a feasible operation of the network over the specified range of uncertain parameters.

**Table 5** Results of MINLP model with additional critical period.

| Match | Period 1 | | | | Period 2 | | | | Period 3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| i.j.k | 1.1.1 | 2.1.2 | 2.2.3 | CU.2 | 1.1.1 | 2.1.2 | 2.2.3 | CU.2 | 1.1.1 | 2.1.2 | 2.2.3 | CU.2 |
| $A_{i,j,k,p}$ [$m^2$] | 1.06 | 0.03 | 1.64 | | 1.06 | 0.03 | 0.77 | | 1.06 | 0.03 | 0.94 | |
| $Q_{i,j,k,p}$ [kW] | 340 | 10 | 240 | 10 | 340 | 10 | 240 | 218 | 340 | 10 | 240 | 102.6 |
| $F^{h}_{i,j,k,p}$ [kW/K] | 0.48 | 0.24 | 0.24 | | 0.48 | 0.43 | 0.43 | | 0.48 | 0.32 | 0.32 | |
| $F^{c}_{i,j,k,p}$ [kW/K] | 0.48 | 0.48 | 0.71 | | 0.48 | 0.48 | 0.71 | | 0.48 | 0.48 | 0.71 | |
| $T^{INh}_{i,k,p}$ [$^{o}$C] | 723 | 583 | 573 | | 723 | 583 | 577.4 | | 723 | 583 | 575.6 | |
| $T^{OUTh}_{i,k,p}$ [$^{o}$C] | 553 | 573 | 333 | | 553 | 577.4 | 444.1 | | 553 | 575.6 | 398.6 | |
| $T^{INC}_{j,k,p}$ [$^{o}$C] | 393 | 388 | 313 | | 393 | 388 | 313 | | 393 | 388 | 313 | |
| $T^{OUTc}_{j,k,p}$ [$^{o}$C] | 563 | 393 | 393 | | 563 | 393 | 393 | | 563 | 393 | 393 | |

**Source:** Aaltola (2003)

# 3. Optimal operation of heat exchanger networks

This part will describe the overall idea of implementing optimal operation of HENs in this thesis.

## 3.1 Switching between active constraints

This section describes possible methods to implement the optimal policy by tracking the changing set of active constraints. To extend the application of split-range control to implement optimal operation of chemical processes, let us show a motivation example on a furnace as shown in Figure 4. There are two types of fuel (fuel A and fuel B) for using in the furnace. Assume that fuel A is cheaper than fuel B. Hence, to operate the furnace in an optimal manner (minimizing fuel cost), fuel A should be used in the nominal condition while fuel B should be a supplementary (e.g. the flow of fuel A reaches the upper limit). When fuel A is in use, the flow of fuel B presents the active constraint (saturated) at the lower bound. Likewise, when fuel B is in use (i.e. the flow of fuel A reaches the maximum limit), the flow of fuel A present at the active constraint (saturated) at the upper bound. This optimal operation can be implemented by the split-range control as shown in Figure 4.

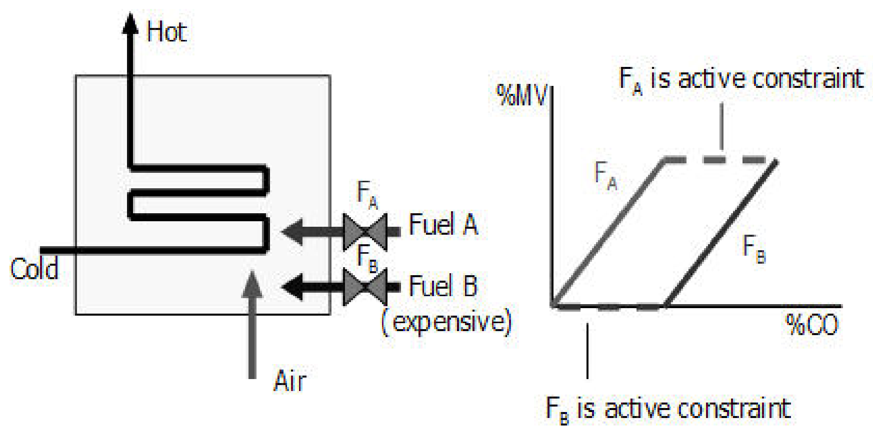



**Figure 4** A furnace system

For the furnace system in Figure 4, suppose that optimal operation always lies at the constraints of $F_A$ or $F_B$ depending on the variation of the desired outlet temperature. Hence, the active constraint regions can be divided to two regions as summarized in Table 6. In the two regions, $F_A$ and $F_B$ switch alternately as active constraint. For this example, the set of active constraints can be found obviously by inspection. However, for more complicated systems, a systematic method is required. In general, parametric programming (Kvasnica *et al.*, 2004) can be served for this task.

**Table 6** Set of active constraints for furnace system

| Region | $F_A$ | $F_B$ |
|---|---|---|
| 1 | U | S |
| 2 | S | U |

U - Unsaturated manipulated variable (inactive constraint) to be used for control a system

S - Saturated manipulated variable (active constraint)

The information of set of active constraints will be used to design optimal split-range control structure as discussed in the next section.

3.2 Determination of optimal split-range control structure

From Table 6, to operate the furnace in an optimal manner, $F_A$ and $F_B$ should be switched alternately between being manipulated variables for and active constraints. Clearly, this can be implemented using a split-range controller with the combination of $F_A$ and $F_B$. However, for more complicated system such as more number of manipulated variables and more number of active constraint regions, an integer linear programming (ILP) formulation (Lersbamrungsuk *et al.*, 2008) should be used in the design of optimal split-range control structure. The problem formulation can be shown as Problem P1.

Definition 1: Set of controlled and manipulated variables

*CV*: set of controlled variables, $CV = \{ CV_1, CV_2, \ldots, CV_{N_{CV}-1}, CV_{N_{CV}} \}$

*MV*: set of manipulated variables, $MV = \{ MV_1,\ MV_2, \ldots\ MV_{N_m-1},\ MV_{N_m} \}$

*MVAAT*: subset of *MV* with manipulated variables which are always active constraints (saturated at upper or lower bounds)

*MVINAT*: subset of *MV* with manipulated variables which are always inactive constraints (never saturated)

*MVAT*: subset of *MV* with manipulated variables which change between being active and inactive constraints

Definition 2: Primary and secondary manipulated variables

Primary manipulated variable is a manipulated variable that is used for controlling an output (target), except when it is saturated. Secondary manipulated variable is a manipulated variable that is used to take over the task of a saturated primary manipulated variable.

Definition 3: Relationship between primary and secondary manipulated variables

Let $x_{i,j}$ (where $i, j \in MV$) be a binary variable which represents the relationship between manipulated variable $MV_i$ and manipulated variable $MV_j$

for $i$=$j$, $x_{i,i}$ = 1 implies $MV_i$ is a primary manipulated variable and $x_{i,i}$ = 0 implies $MV_i$ is a secondary manipulated variable or unused

for $i \neq j$, $x_{i,j}$ = 1 implies $MV_j$ is a secondary manipulated variable for $MV_i$ and $x_{i,j}$ = 0 implies $MV_j$ is not a secondary manipulated variable for $MV_i$

Definition 4: Relative order between manipulated variables and controlled variables

Let $r_{k,j}$ be relative order between controlled variable $CV_k$ and manipulated variable $MV_j$. Relative order is a structural measure of how direct an effect an input has on an output (Daoutidis and Kravaris, 1992). However, we here assume $r_{k,j}$ as the number of exchangers between controlled variable $CV_k$ and manipulated variable $MV_j$

Definition 5: Relationship between controlled variables and manipulated variables

Let $z_{k,j}$ (where $k \in CV, j \in MV$) be a binary variable that represents the relationship between controlled variable $CV_k$ and manipulated variable $MV_j$

$z_{k,j}$ = 1 implies controlled variable $CV_k$ is paired with manipulated variable $MV_j$ and $z_{k,j}$ = 0 implies controlled variable $CV_k$ is not paired with manipulated variable $MV_j$

**Problem P1**

Objective Function:

$J_I$ : Minimizing the number of ‘‘inter-connection’’ or ‘‘complexity’’ of control structure (unnecessary relationships between primary and secondary manipulated variables).

$J_{II}$ : Minimizing the sum of relative order of the control pairs.

$$J = \min(wJ_I + J_{II})$$

$$J_I = \sum_{i \in MV} \sum_{j \in MV, j \neq i} x_{i,j}\ , \ J_{II} = \sum_{k \in CV} \sum_{j \in MV} r_{k,j} z_{k,j}$$

Subject to

Constraint 1: Assign one primary manipulated variables to each control objective.

$$\sum_{i \in MV} x_{i,i} = N_{CV} \tag{49}$$

Constraint 2: Manipulated variables $MV_i$ that always is an active constraint should not be used for other purposes

$$x_{i,i} = 0 \qquad i \in MVAAT \tag{50}$$

$$\sum_{j \in MV, j \neq i} x_{i,j} = 0 \qquad i \in MVAAT \tag{51}$$

$$\sum_{j \in MV, j \neq i} x_{j,i} = 0 \qquad i \in MVAAT \tag{52}$$

Constraint 3: Manipulated variables $MV_i$ that is never an active constraint is used as a primary manipulated variable with no need of a secondary manipulated variable.

$$x_{i,i} = 1 \qquad i \in MVINAT \tag{53}$$

$$\sum_{j \in MV, j \neq i} x_{i,j} = 0 \qquad i \in MVINAT \tag{54}$$

$$\sum_{j \in MV, j \neq i} x_{j,i} = 0 \qquad i \in MVINAT \tag{55}$$

Constraint 4: Manipulated variables $MV_i$ that changes between being an active and inactive constraint may be a primary or secondary manipulated variables.

$$-x_{i,i} + \sum_{j \in MVAT, j \neq i} x_{i,j} = 0 \qquad i \in MVAT \tag{56}$$

$$x_{j,j} + \sum_{i \in MVAT, i \neq j} x_{i,j} \geq 1 \qquad j \in MVAT \tag{57}$$

$$M(x_{j,j} - 1) + \sum_{i \in MVAT, i \neq j} x_{i,j} \leq 0 \qquad j \in MVAT \tag{58}$$

Constraint 5: Possible and impossible split-range combination of manipulated variables (these constraints are obtained from the information of active constraint regions).

Constraint 5A: Impossible split-range combination of manipulated variables

$$\sum_{i \in MVAT^{A,R}} \sum_{j \in MVAT^{A,R}, j \neq i} x_{i,j} = 0 \qquad R \in RS \tag{59}$$

Constraint 5B: Possible split-range combination of manipulated variables

$$x_{j,j} + \sum_{i \in MVAT^{I,R}} x_{i,j} \geq 1 \qquad j \in MVAT^{A,R},\ R \in RS \tag{60}$$

$$x_{i,i} + \sum_{j \in MVAT^{A,R}} x_{j,i} \geq 1 \qquad i \in MVAT^{I,R},\ R \in RS \tag{61}$$

Constraint 6: Assign one manipulated variables to each control objective

$$\sum_{j \in MV} z_{k,j} = 1 \qquad k \in CV \tag{62}$$

Constraint 7: Only primary manipulated variables are paired with controlled variables.

$$-x_{j,j} + \sum_{k \in CV} z_{k,j} = 0 \qquad j \in MV \qquad (63)$$

It is possible for the ILP to have no feasible solution, that is, no optimal split-range control structure can be found. This may happen when there are conflicts among the equations in constraint 5. In this case, an online optimization may be suggested for implementing optimal operation.

**4. Procedure for design of optimal split-range control structure**

The procedure for design of optimal split-range control structure consists of two steps:

Step 1: Check degrees of freedom (DOFs). Only the system with some degrees of freedom for optimization will be considered.

- For systems in which no degree of freedom is available for optimization, the optimal operation can be reduced as control problems. (e.g. control pairing problems that can be handled using direct effect or RGA input saturation problem or input constraint problems (Giovanini *et al.,*2003) )

- For systems in which some degrees of freedom are available for optimization, go further to step 2.

Step 2: Find active constrain regions using parametric programming. (e.g. with MPT toolbox (Kvasnica *et al.,*2004 ) )

- For a number of regions: an optimal control structure can be found using Problem P1.

Example 1: A trivial HEN

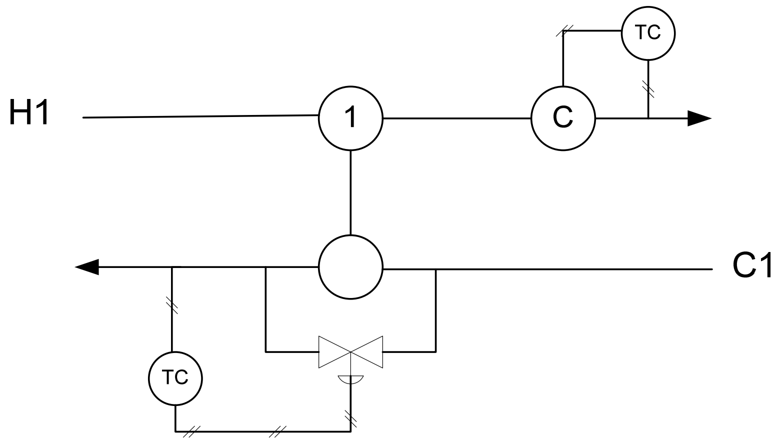



**Figure 5** A trivial HEN

The HEN in Figure 5 contains one process exchanger and one utility types ($N_U$=1). Two outlet stream temperatures have targets ($N_t$ =2). The dimensional space spanned by the manipulated variables in the inner HEN to the outer HEN (*DS*) is equal to 1 (see the calculation in Glemmestad, 1997). Using equation (3), we have $N_{DOF,U}=1+1-2=0$. Hence there is no need of an optimal operation strategy. If direct effect rules (Daoutidis *et al.*, 1992 and Soroush, 1996) are applied, then the resulting control structure can be shown in Figure 5.

In this work, we proposed systematic approach of an optimal operation for HENs to flowchart shown in Figure 6. First, HENs are synthesized using multi-period mixed-integer-non-linear programming (MINLP) model to minimize the total annual cost (TAC) was proposed by Aaltola (2003). Second, we check flexibility of HEN was proposed by Aaltola (2003). Third, check degrees of freedom were proposed by Glemmestad (1997). Fourth, determine set of active constraint for information by using integer linear program (ILP) for optimal split-range control structure was proposed by Lersbamrungsuk (2008). Finally, check an optimal control structure using Aspen Dynamics.

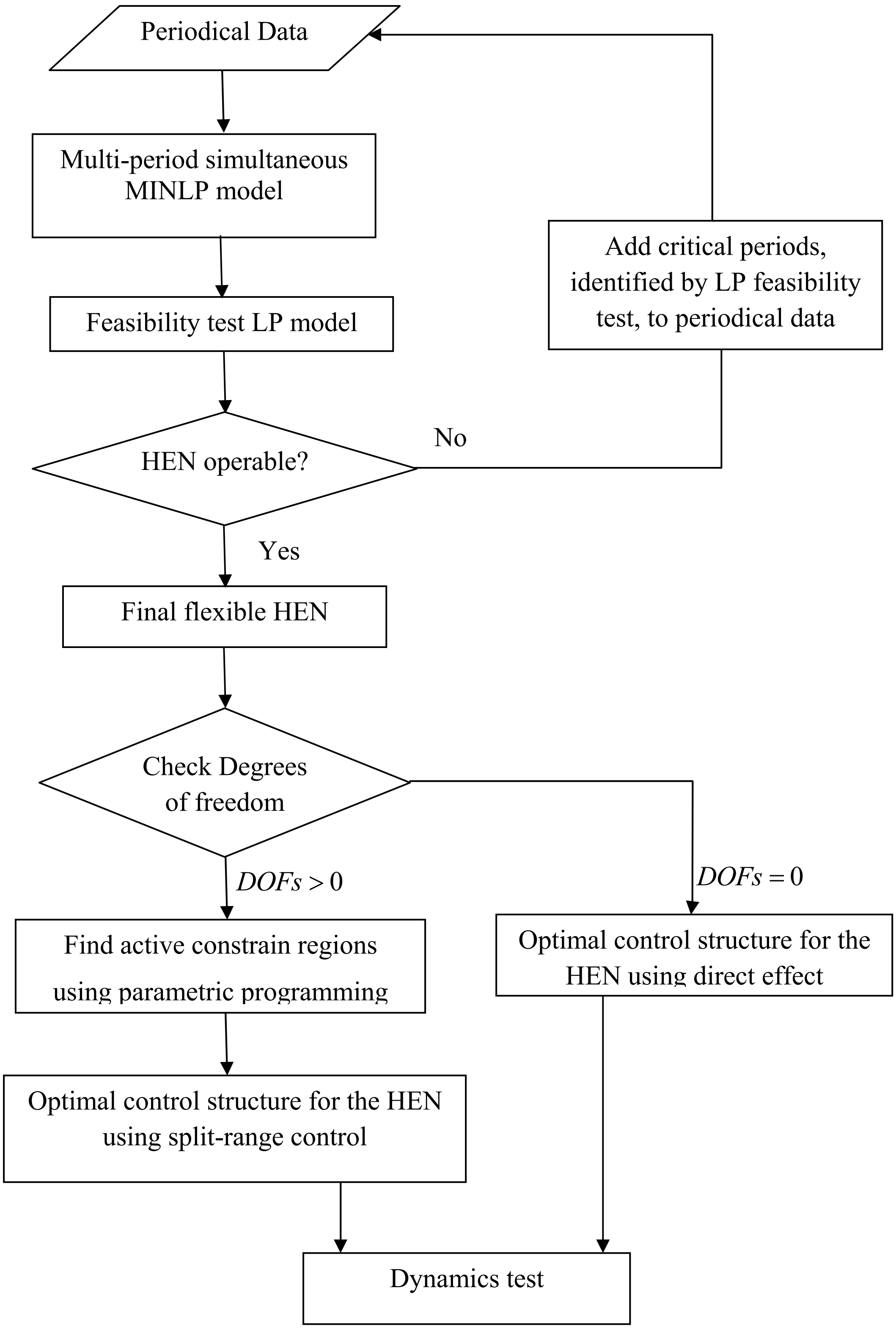



**Figure 6** Systematic approach of an optimal operation for HENs

# RESULTS AND DISCUSSION

The results can be divided into three parts. The first part shows MINLP model and LP feasibility for obtain superstructure of HENs with flexibility. The second part is set of active constraints using parametric programming to find optimal control structure. And, the third part is dynamics test to check target temperature can handle. Four cases studies are proposed in this work. Case study 1, from Zamora and Grossmann (1998), contains three hot and two cold streams. Case study 2, from Biegler *et al.* (1997), has two hot and two cold process streams. Case study 3, from Aaltola (2003), covers two hot and two cold process streams. Case study 4, from Yerramsetty (2008), contains four hot and five cold streams.

## Case study 1

The HEN information from Zamora and Grossmann (1998) as shown in Table 7 is studied here. The disturbances are the inlet temperatures of all streams, with the expected vary ±5% $^{o}$C for streams H1, H2, H3, C1 and C2. The costs equation for exchangers is 7,400 · unit cost (\$) +80 · Area (m$^2$).

**Table 7** Problem data for case study 1

| Period | Stream | $T_{in}$ (°C) | $T_{out}$ (°C) | $FC_p$ (kW/°C) | $h$ (kW/m²K) | Cost (US$/ kW year) |
|---|---|---|---|---|---|---|
| Period 1 | H1 | 159 | 77 | 2.285 | 0.10 | |
| | H2 | 267 | 80 | 0.204 | 0.04 | |
| | H3 | 343 | 90 | 0.538 | 0.50 | |
| | C1 | 26 | 127 | 0.933 | 0.01 | |
| | C2 | 118 | 265 | 1.961 | 0.5 | |
| Period 2 | H1 | 166.95 | 77 | 2.285 | 0.10 | |
| | H2 | 280.35 | 80 | 0.204 | 0.04 | |
| | H3 | 343 | 90 | 0.538 | 0.50 | |
| | C1 | 27.3 | 127 | 0.933 | 0.01 | |
| | C2 | 118 | 265 | 1.961 | 0.5 | |
| Period 3 | H1 | 159 | 77 | 2.285 | 0.10 | |
| | H2 | 280.35 | 80 | 0.204 | 0.04 | |
| | H3 | 325.85 | 90 | 0.538 | 0.50 | |
| | C1 | 26 | 127 | 0.933 | 0.01 | |
| | C2 | 112.1 | 265 | 1.961 | 0.5 | |
| Period 4 | H1 | 151.05 | 77 | 2.285 | 0.10 | |
| | H2 | 253.65 | 80 | 0.204 | 0.04 | |
| | H3 | 343 | 90 | 0.538 | 0.50 | |
| | C1 | 24.7 | 127 | 0.933 | 0.01 | |
| | C2 | 118 | 265 | 1.961 | 0.5 | |
| Period 5 | H1 | 159 | 77 | 2.285 | 0.10 | |
| | H2 | 253.65 | 80 | 0.204 | 0.04 | |
| | H3 | 360.15 | 90 | 0.538 | 0.50 | |
| | C1 | 26 | 127 | 0.933 | 0.01 | |
| | C2 | 123.9 | 265 | 1.961 | 0.5 | |
| | Steam | 300 | 300 | | 0.05 | 110 |
| | Water | 20 | 60 | | 0.20 | 10 |

Results from the multi-period simultaneous MINLP model are shown in Table 8. The model was solved on a mobile Intel Core2Duo CPU 2.0 GHz using DICOPT (to solve MINLP), CONOPT (to solve NLP sub problem) and CPLEX (to solve MIP sub problem) ® via GAMS. The total annual cost, after the MINLP optimization, is 285,916 $/year when the maximum area is 80 $m^2$ distributed among the 8 units. (e.g. H2.C2.1 stands for match H2-C1 in stage 1, H3.C2.2 stands for match H3-C2 in stage 2, H2.C1.3 stands for match H2-C1 in stage 3, C1 stand for stream of H1, C2 stand for stream of H2, C3 stand for stream of H3, H1 stand for stream of C1 and H2 stand for stream of C2)

**Table 8** Results of MINLP model about existence of match ($Z_{i,j,\mathrm{k}}$) in stage k for case study 1

| Match(i,j) | Stage(k) | | | Utility |
|---|---|---|---|---|
| | 1 | 2 | 3 | |
| H2.C2 | 1 | | | |
| H3.C2 | | 1 | | |
| H2.C1 | | | 1 | |
| C1 | | | | 1 |
| C2 | | | | 1 |
| C3 | | | | 1 |
| H1 | | | | 1 |
| H2 | | | | 1 |

The resulting network was checked with the LP feasibility test model to ensure that the network could be operated feasibly under the specified conditions. In addition to the five periods uncertain parameters, the disturbances of the district heating stream in the LP model were modeled, allowing the inlet temperature to change ±5% in five periods. The final network configuration from MINLP is shown Figure 7.

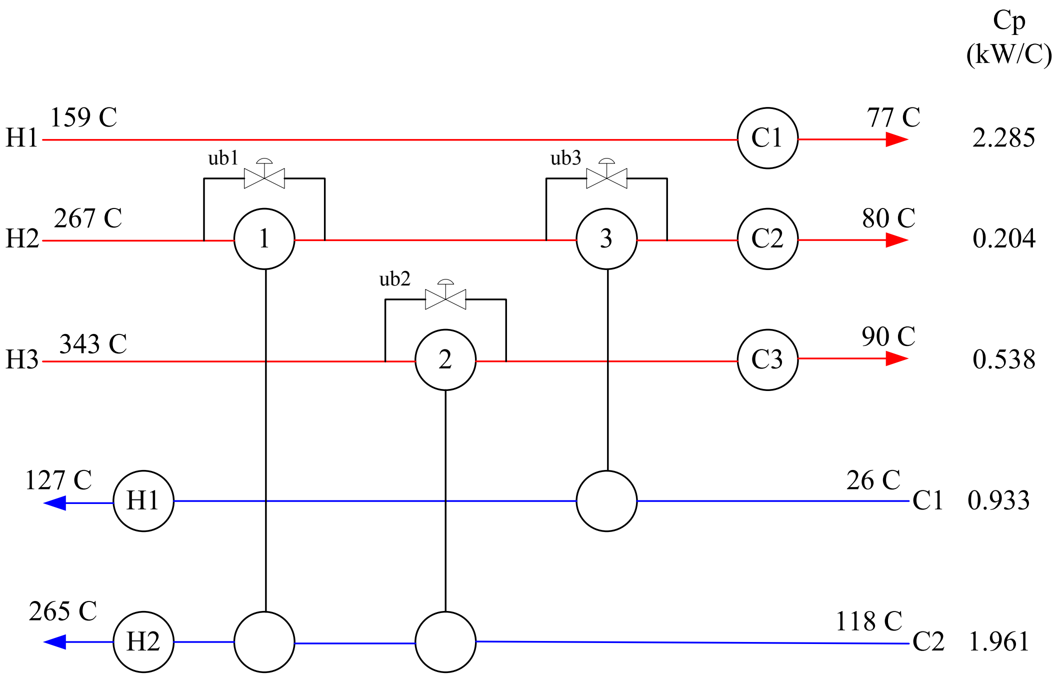



**Figure 7** Final network configurations for case study 1

From the strategy, degrees of freedom (DOFs) should be first checked. The dimensional spaces spanned by the manipulated variables in the inner HEN to the outer HEN (DS) are 3. Number of utility are 5 and number of target temperatures are 5.Using equation (3), we have $N_{DOF,U}$ = 3+5-5 = 3. This case have three remaining degree of freedom for utility cost optimization, hence, a strategy for optimal operation is needed.

The additional information required is the sets of active constraint regions that can be obtained using multi-parametric toolbox (MPT) (Kvasnica *et al.*, 2004). Two active constraint regions found are shown in Table 9.

**Table 9** Set of active constraints in case study 1

| Active constraints region | Manipulated variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $Q_{c1}$ | $Q_{c2}$ | $Q_{c3}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
| 1 | U | $S_L$ | U | U | U | $S_L$ | $S_L$ | U |
| 2 | U | $S_L$ | $S_L$ | U | U | $S_L$ | U | U |

U - Unsaturated manipulated variable (inactive constraint),
$S_L$ - Saturated manipulated variable (active constraint) at the lower bound

Table 9 demonstrates that manipulated variables $Q_{c3}$ and $u_{b2}$ can become active constraint implies that these two manipulated variables should be combined as a split-range pair. The manipulated variable $Q_{c1}$, $Q_{h1}$, $Q_{h2}$ and $u_{b3}$ are never become active constraints (never saturate), hence, there is no need of split-range combinations. $Q_{c2}$ and $u_{b1}$ are saturated in both regions, hence, there should not be used for any purpose. The control structure should be $Q_{c1}$-$T_{H1}^{out}$, $u_{b3}$-$T_{H2}^{out}$, $Q_{c3}$-$T_{H3}^{out}$, $Q_{h1}$-$T_{C1}^{out}$ and $Q_{h2}$-$T_{C2}^{out}$ as shown in Figure 8.

For this case study, the ILP Problem P1 will be used to suggest an optimal split-range control structure. The software "GAMs" with the solver "CPLEX" was used to solve the ILP. The solution from Problem P1 (minimizing complexity in optimal split-range pairs and controllability purpose in terms of minimizing the sum of relative orders). The additional information of relative orders is shown in Table 10. The values of binary variables $x_{i,j}$ and $z_{k,j}$ from solving Problem P1 are shown in Tables 11 and 12, respectively. In Table 11 shows that $Q_{c1}$, $Q_{h1}$, $Q_{h2}$, $u_{b2}$ and $u_{b3}$ are chosen as primary manipulated variables (see diagonal elements with $x_{i,i} = 1$) while $Q_{c3}$ is the secondary manipulated variable for $u_{b2}$ ($x_{4,2} = 1$). Table 12 shows the appropriate control pairing, $T_{H1}^{out}$ -$Q_{c1}$, $T_{H2}^{out}$ - $u_{b3}$, $T_{H3}^{out}$ -$u_{b2}$, $T_{C1}^{out}$ - $Q_{h1}$ and $T_{C2}^{out}$ - $Q_{h2}$ (see $z_{1,1} = z_{2,5} = z_{3,4} = z_{4,2} = z_{5,3} = 1$). The resulting control structure is shown in Figure 8.

**Table 10** Relative orders of the HEN in case study 1

| CV \ MV | $Q_{c1}$ | $Q_{c2}$ | $Q_{c3}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|---|---|---|---|---|---|---|---|---|
| $T_{H1}^{out}$ | 1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| $T_{H2}^{out}$ | ∞ | 1 | ∞ | ∞ | ∞ | 3 | 4 | 2 |
| $T_{H3}^{out}$ | ∞ | ∞ | 1 | ∞ | ∞ | ∞ | 2 | ∞ |
| $T_{C1}^{out}$ | ∞ | ∞ | ∞ | 1 | ∞ | 3 | 4 | 2 |
| $T_{C2}^{out}$ | ∞ | ∞ | ∞ | ∞ | 1 | 2 | 3 | ∞ |

**Table 11** The values of $x_{i,j}$ after solving Problem P1 for case study 1

| Primary MV \ Secondary MV | $Q_{c1}$ | $Q_{c3}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|---|---|---|---|---|---|---|---|
| $Q_{c1}$ | 1 | | | | | | |
| $Q_{h1}$ | | | 1 | | | | |
| $Q_{h2}$ | | | | 1 | | | |
| $u_{b2}$ | | 1 | | | | 1 | |
| $u_{b3}$ | | | | | | | 1 |

(the remaining entries are zero)

**Table 12** The values of $z_{k,j}$ after solving Problem 1 for case study 1

| MV / CV | $Q_{c1}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b2}$ | $u_{b3}$ |
|---|---|---|---|---|---|
| $T_{H1}^{out}$ | 1 | | | | |
| $T_{H2}^{out}$ | | | | | 1 |
| $T_{H3}^{out}$ | | | | 1 | |
| $T_{C1}^{out}$ | | 1 | | | |
| $T_{C2}^{out}$ | | | 1 | | |

(the remaining entries are zero)

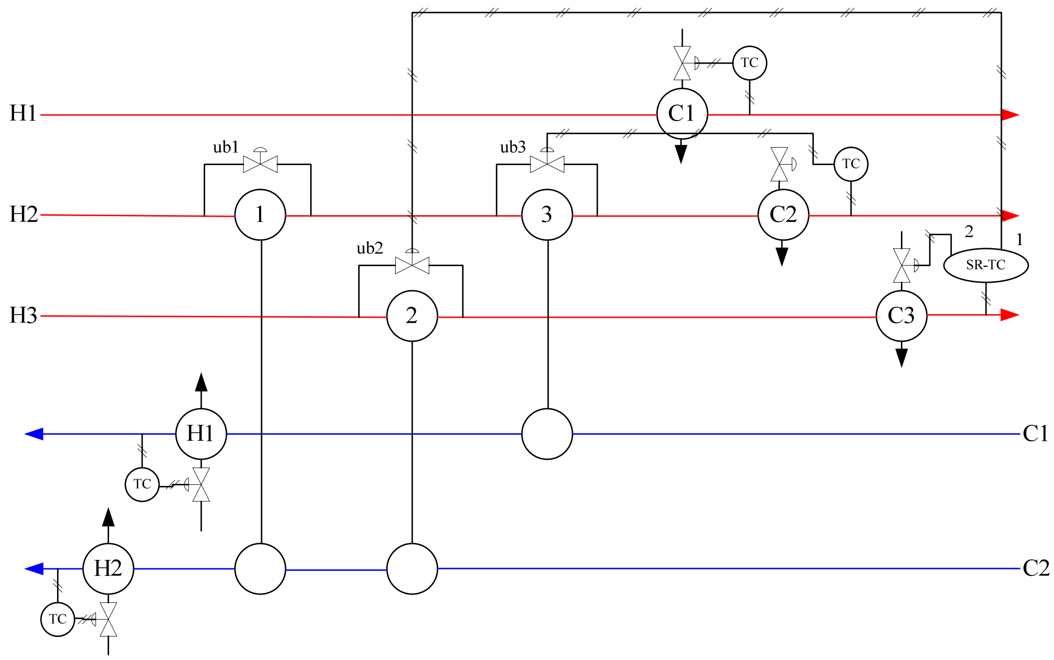


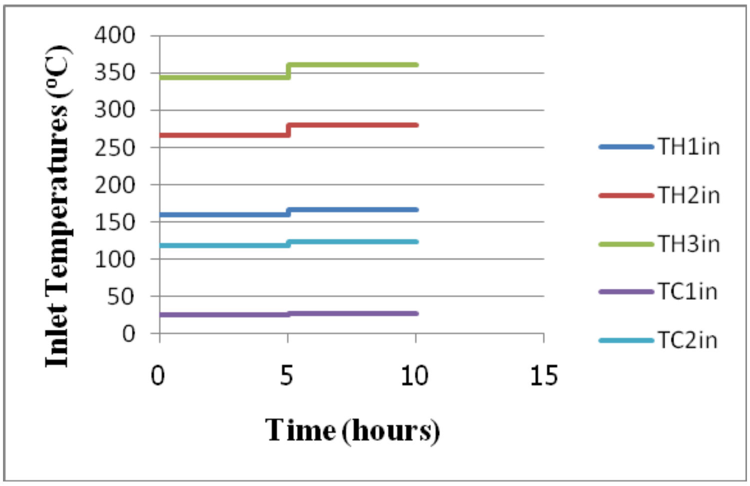

**Figure 8** The control structure for case study 1

Split-range signal of a split-range controller can be obtained by considering the information of active constraint regions in Table 9. The split-range signal of the pair of $u_{b2}$ and $Q_{c3}$ they switch alternately to their lower constraints (SR-TC is split-range temperature control).

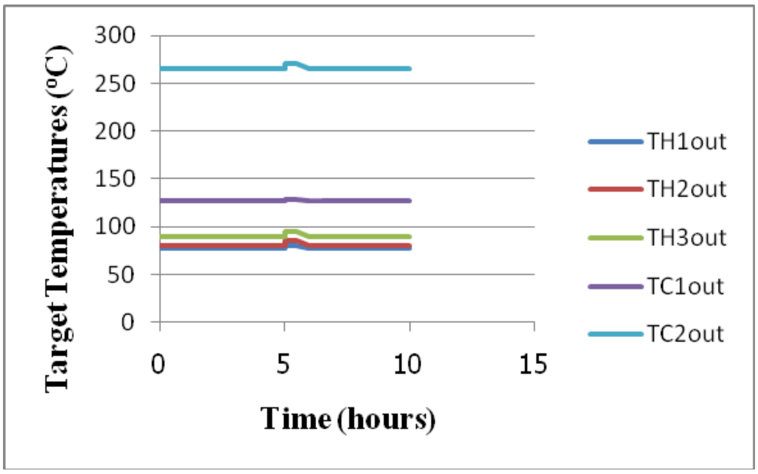
The HEN in the case study 1 with the resulting control structures is tested by performing dynamic simulation on Aspen Dynamics v2006.5. The information of disturbances and active constraints of the system in case study 1 are shown in Table 13. The dynamic results show that the control structures can provide optimality. Figure 9 shows the dynamic result of the HEN with the control structure. Figure 9b shows the ability of the control structure to keep all target temperatures at the desired values even under the saturation of some manipulated variables. At more than 5 hours, split-range control gives the response to the disturbance as seen in the Figure 9c. $u_{b2}$ is primary manipulate variable switch to $Q_{c3}$ is secondary manipulate variable to protect stream of H3. This consequence comes from the ability of the control structure to track the right active constraint during the operation (see Figure 9c, set of active constraints in Table 13).

**Table 13** Disturbances and active constraints in case study 1

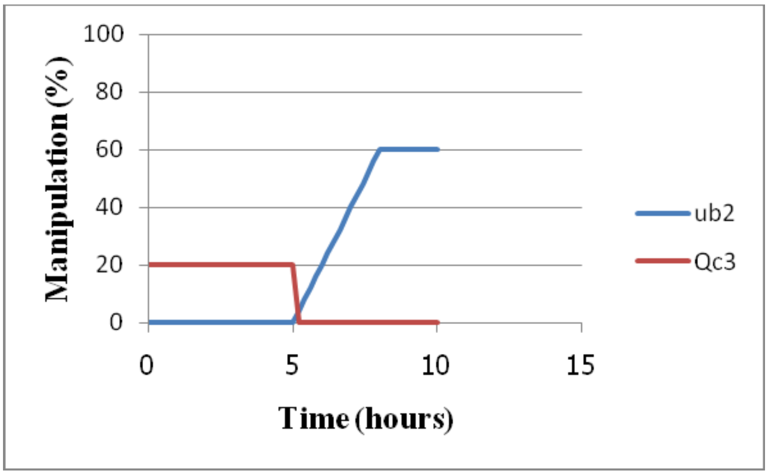
| Time | Disturbance | | | | | Active constraint | |
|---|---|---|---|---|---|---|---|
| (hour) | $\Delta T_{H1}^{in}$ | $\Delta T_{H2}^{in}$ | $\Delta T_{H3}^{in}$ | $\Delta T_{C1}^{in}$ | $\Delta T_{C2}^{in}$ | $u_{b2}$ | $Q_{c3}$ |
| < 5 | 0 | 0 | 0 | 0 | 0 | $S_L$ | |
| > 5 | +7.95 | +13.35 | +17.15 | +1.3 | +5.9 | | $S_L$ |




(a) Inlet temperatures




(b) Target temperatures




(c) Manipulated variable ($Q_{c3}$ and $u_{b2}$)

**Figure 9** Dynamic simulation of the HEN in case study 1

**Case study 2**

The HEN information from Biegler *et al.* (1997) as shown in Table 14 is studied. There are two hot and two cold process streams with target outlet temperatures. The disturbances are the inlet temperatures of all streams, with the expected variation ±5% $^{o}C$ for streams H1, H2, C1 and C2. The costs equation for exchangers is 150 · unit cost ($) +5,500 · Area ($m^2$).

**Table 14** Problem data for case study 2

| Periods | Stream | $T_{in}$($^{o}C$) | $T_{out}$($^{o}C$) | $FC_p$ (kW/$^{o}C$) | $h$ (kW/$m^{2o}C$) | Cost ($ /kW year) |
|---|---|---|---|---|---|---|
| Period 1 | H1 | 650 | 370 | 10 | 1 | |
| | H2 | 590 | 370 | 20 | 1 | |
| | C1 | 410 | 650 | 15 | 1 | |
| | C2 | 353 | 500 | 13 | 1 | |
| Period 2 | H1 | 682.5 | 370 | 10 | 1 | |
| | H2 | 560.5 | 370 | 20 | 1 | |
| | C1 | 430.5 | 650 | 15 | 1 | |
| | C2 | 335.35 | 500 | 13 | 1 | |
| Period 3 | H1 | 682.5 | 370 | 10 | 1 | |
| | H2 | 619.5 | 370 | 20 | 1 | |
| | C1 | 389.5 | 650 | 15 | 1 | |
| | C2 | 370.65 | 500 | 13 | 1 | |
| Period 4 | H1 | 617.5 | 370 | 10 | 1 | |
| | H2 | 619.5 | 370 | 20 | 1 | |
| | C1 | 389.5 | 650 | 15 | 1 | |
| | C2 | 335.35 | 500 | 13 | 1 | |
| Period 5 | H1 | 682.5 | 370 | 10 | 1 | |
| | H2 | 560.5 | 370 | 20 | 1 | |
| | C1 | 430.5 | 650 | 15 | 1 | |
| | C2 | 370.65 | 500 | 13 | 1 | |

**Table 14** (Continued)

| Periods | Stream | $T_{in}$(°C) | $T_{out}$(°C) | $FC_p$ (kW/°C) | $h$ (kW/m$^2$°C) | Cost ($ /kW year) |
|---|---|---|---|---|---|---|
| | Steam | 680 | 680 | | 5 | 80 |
| | Water | 300 | 320 | | 1 | 10 |

Results from the multi-period simultaneous MINLP model are shown in Table 15. The total annual costs, after the MINLP optimization, is 85,290 $/year when the maximum area is 5,550 m$^2$ distributed among the 6 units.

**Table 15** Results of MINLP model about existence of match ($Z_{i,j,k}$) in stage k for case study 2

| Match(i,j) | Stage(k) | | | Utility |
|---|---|---|---|---|
| | 1 | 2 | 3 | |
| H1.C1 | 1 | | | |
| H1.C2 | | 1 | | |
| H2.C1 | | | 1 | |
| C1 | | | | 1 |
| C2 | | | | 1 |
| H1 | | | | 1 |

The resulting network was checked with the LP feasibility test model to ensure that the network could be operated feasibly under the specified conditions. In addition to the five periods uncertain parameters, the disturbances of the district heating stream in the LP model were modeled, allowing the inlet temperature to change ±5% in five periods. The final network configuration from MINLP is shown Figure 10.

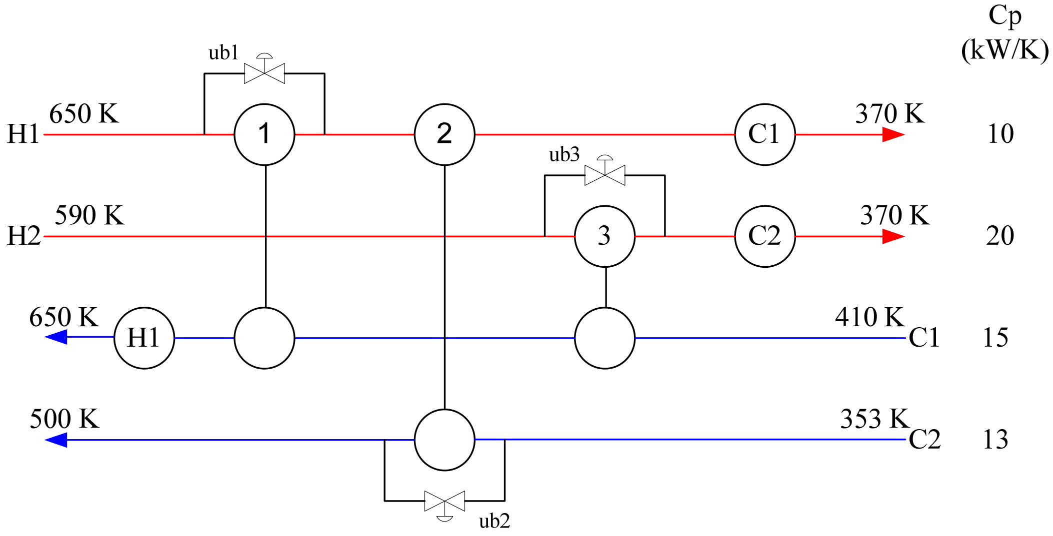



**Figure 10** Final network configurations for case study 2

From direction, degrees of freedom (DOFs) should be first checked. The dimensional spaces spanned by the manipulated variables in the inner HEN to the outer HEN (*DS*) are 3. Number of utility are 3 and number of target temperatures are 4. Using equation (3), we have $N_{DOF,U}$ = 3+3-4 = 2. This case have two remaining degree of freedom for utility cost optimization, hence, a strategy for optimal operation is needed.

The additional information required is the set of active constraint regions that can be obtained using multi-parametric toolbox (Kvasnica *et al.*, 2004). Three active constraint regions found are shown in Table 16.

**Table 16** Set of active constraints in case study 2

| Active constraints region | Manipulated variables | | | | | |
|---|---|---|---|---|---|---|
| | $Q_{c1}$ | $Q_{c2}$ | $Q_{h1}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
| 1 | U | U | U | $S_L$ | U | $S_L$ |
| 2 | $S_L$ | U | U | U | U | $S_L$ |
| 3 | U | U | $S_L$ | U | U | $S_L$ |

U - Unsaturated manipulated variable (inactive constraint),
$S_L$ - Saturated manipulated variable (active constraint) at the lower bound

Table 16 demonstrates that manipulated variables $Q_{c1}$, $Q_h$ and $u_{b1}$ can become active constraints at the lower bounds (i.e. zero utility duties or fully close of bypasses). The manipulated variable $Q_{c2}$ and $u_{b2}$ is never an active constraint (never saturated), hence, there is no need of split-range combinations and $u_{b3}$ are saturated in both regions, hence, there should not be used for any purpose.

For this case study, the ILP Problem P1 will be used to suggest an optimal split-range control structure. The software "GAMs" with the solver "CPLEX" was used to solve the ILP. The solution from Problem P1 (minimizing complexity in optimal split-range pairs and controllability purpose in terms of minimizing the sum of relative orders). The additional information of relative orders is shown in Table 17. The values of binary variables $x_{i,j}$ and $z_{k,j}$ from solving Problem P1 are shown in Tables 18 and 19, respectively. In Table 18 shows that $Q_{c1}$, $Q_{c2}$, $Q_{h1}$ and $u_{b2}$ are chosen as primary manipulated variables (see diagonal elements with $x_{i,i} = 1$) while $u_{b1}$ is the secondary manipulated variable for $Q_{c1}$ ($x_{1,4} = 1$)and $Q_{h1}$ ($x_{3,4} = 1$). Table 19 shows the appropriate control pairing, $T_{H1}^{out}$ -$Q_{c1}$, $T_{H2}^{out}$ -$Q_{c2}$, $T_{C1}^{out}$ - $Q_{h1}$ and $T_{C2}^{out}$ -$u_{b2}$ (see $z_{1,1} = z_{2,2} = z_{3,3} = z_{4,4} = 1$). The resulting control structure is shown in Figure 11.

**Table 17** Relative orders of the HEN in case study 2

| MV / CV | $Q_{c1}$ | $Q_{c2}$ | $Q_{h1}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|---|---|---|---|---|---|---|
| $T_{H1}^{out}$ | 1 | ∞ | ∞ | 3 | 2 | 4 |
| $T_{H2}^{out}$ | ∞ | 1 | ∞ | ∞ | ∞ | 2 |
| $T_{C1}^{out}$ | ∞ | ∞ | 1 | 2 | ∞ | 3 |
| $T_{C1}^{out}$ | ∞ | ∞ | ∞ | 2 | 1 | 3 |

**Table 18** The values of $x_{i,j}$ after solving Problem P1 for case study 2

| Secondary MV / Primary MV | $Q_{c1}$ | $Q_{c2}$ | $Q_{h1}$ | $u_{b1}$ | $u_{b2}$ |
|---|---|---|---|---|---|
| $Q_{c1}$ | 1 | | | 1 | |
| $Q_{c2}$ | | 1 | | | |
| $Q_{h1}$ | | | 1 | 1 | |
| $u_{b2}$ | | | | | 1 |

(the remaining entries are zero)

**Table 19** The values of $z_{k,j}$ after solving Problem P1 for case study 2

| MV / CV | $Q_{c1}$ | $Q_{c2}$ | $Q_{h1}$ | $u_{b2}$ |
|---|---|---|---|---|
| $T_{H1}^{out}$ | 1 | | | |
| $T_{H2}^{out}$ | | 1 | | |
| $T_{C1}^{out}$ | | | 1 | |
| $T_{C2}^{out}$ | | | | 1 |

(the remaining entries are zero)

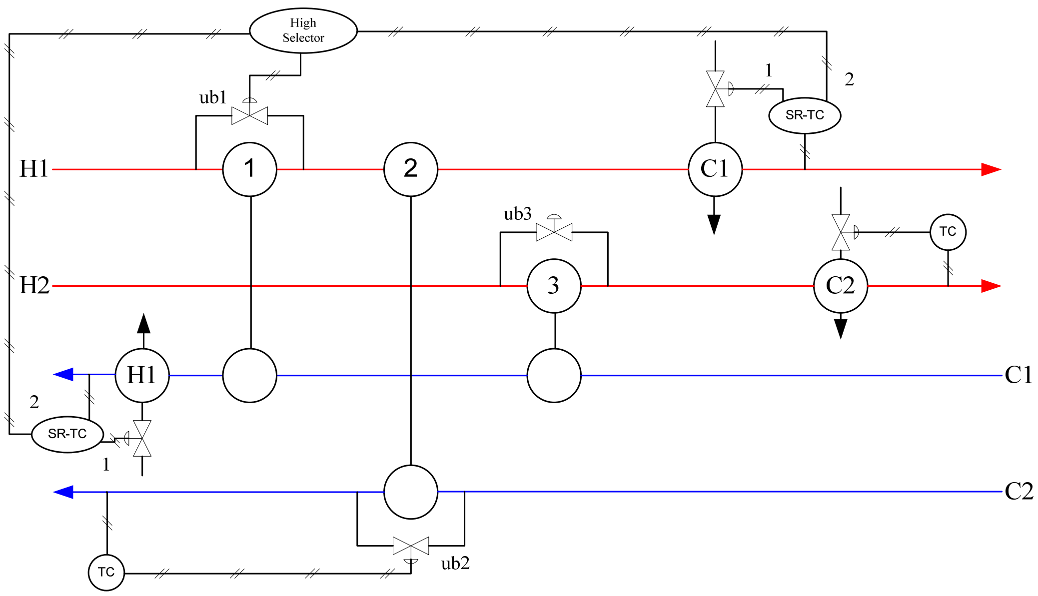
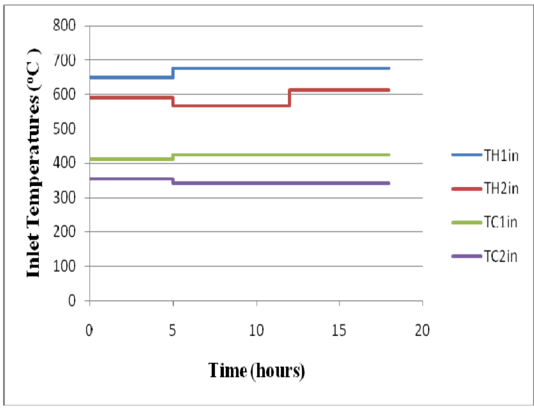



**Figure 11** The control structure for case study 2

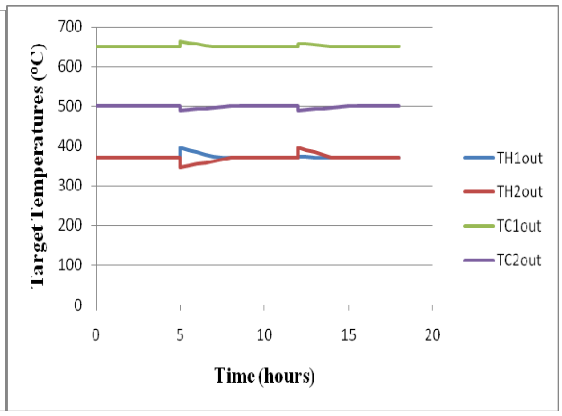
The HEN in the case study 2 with the resulting control structures is tested by performing dynamic simulation on Aspen Dynamics v2006.5. The information of disturbances and active constraints of the system at each period are shown in Table 20. The dynamic results show that the control structures can provide optimality. Figure 12 shows the dynamic result of the HEN with the control structure. Figure 12b shows the ability of the control structure to keep all target temperatures at the desired values even under the saturation of some manipulated variables. At 5-12 hours, we have disturbed all inlet temperature (see in Table 20) but system can control target temperature. At more than 12 hours, split-range control gives the response to the disturbance as seen in the Figure 12c. $Q_{h1}$ is primary manipulate variable handle target temperature of C1. The input saturation problem is solved by switching ability to use a secondary manipulated variable when a primary manipulated variable is saturated. This consequence comes from the ability of the control structure to track the right active constraint during the operation (see Figure 12c and set of active constraints in Table 20).

**Table 20** Disturbances and active constraints in case study 2

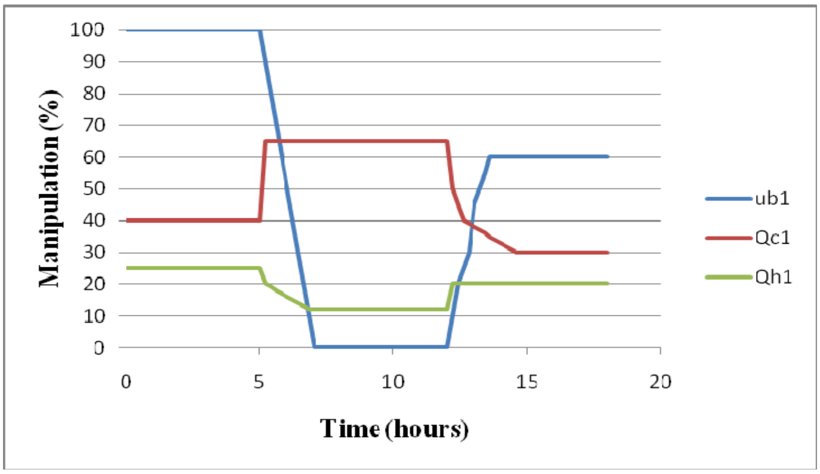
| Time (hour) | Disturbance | | | | Active constraint | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta T_{H1}^{in}$ | $\Delta T_{H2}^{in}$ | $\Delta T_{C1}^{in}$ | $\Delta T_{C2}^{in}$ | $Q_{c1}$ | $Q_{c2}$ | $Q_{h1}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
| less than 5 | 0 | 0 | 0 | 0 | | | | $S_L$ | | $S_L$ |
| 5-12 | 26.52 | -23.52 | 14.52 | -11.67 | $S_L$ | | | | | $S_L$ |
| more than 12 | 25.91 | 22.91 | 13.91 | -11.06 | | | $S_L$ | | | $S_L$ |




(a) Inlet temperatures




(b) Target temperatures




(c) Manipulated variable ($Q_{c1}$, $Q_{h1}$ and $u_{b1}$)

**Figure 12** Dynamic simulation of the HEN in case study 2

**Case study 3**

The HEN information from Aaltola (2003) as shown in Table 21 is studied. There are two hot and two cold process streams with target outlet temperatures. The disturbances are the inlet temperatures of all streams, with the expected variation ±5% K for streams H1, H2, C1 and C2. The costs equation for exchangers is 8,333.3·unit cost ($) +641.7 · Area ($m^2$).

**Table 21** Problem data for case study 3

| Periods | Stream | $T_{in}$(K) | $T_{out}$(K) | $FC_p$ (kW/K) | $h$ (kW/$m^2$K) | Cost ($ /kW year) |
|---|---|---|---|---|---|---|
| Period 1 | H1 | 759.15 | 553 | 2 | 4 | |
| | H2 | 583 | 323 | 1 | 4 | |
| | C1 | 320 | 495 | 2 | 4 | |
| | C2 | 328.65 | 393 | 3 | 4 | |
| Period 2 | H1 | 723 | 553 | 2 | 4 | |
| | H2 | 612.15 | 323 | 1 | 4 | |
| | C1 | 336 | 495 | 2 | 4 | |
| | C2 | 313 | 393 | 3 | 4 | |
| Period 3 | H1 | 759.15 | 553 | 2 | 4 | |
| | H2 | 583 | 323 | 1 | 4 | |
| | C1 | 320 | 495 | 2 | 4 | |
| | C2 | 297.44 | 393 | 3 | 4 | |
| Period 4 | H1 | 723 | 553 | 2 | 4 | |
| | H2 | 553.85 | 323 | 1 | 4 | |
| | C1 | 336 | 495 | 2 | 4 | |
| | C2 | 313 | 393 | 3 | 4 | |
| Period 5 | H1 | 686.85 | 553 | 2 | 4 | |
| | H2 | 583 | 323 | 1 | 4 | |
| | C1 | 304 | 495 | 2 | 4 | |
| | C2 | 313 | 393 | 3 | 4 | |

**Table 21** (Continued)

| Periods | Stream | $T_{in}$(K) | $T_{out}$(K) | $FC_p$ (kW/K) | $h$ (kW/m$^2$K) | Cost ($ /kW year) |
|---|---|---|---|---|---|---|
| | Steam | 500 | 500 | | 4 | 6220.8 |
| | Water | 5 | 30 | | 4 | 70.2 |

Results from the multi-period simultaneous MINLP model are shown in Table 22. The total annual costs, after the MINLP optimization, is 27,743 $/year when the maximum area is 641.7 m$^2$ distributed among the 4 units.

**Table 22** Results of MINLP model about existence of match ($Z_{i,j,\mathrm{k}}$) in stage k for case study 3

| Match(i,j) | Stage(k) | | | Utility |
|---|---|---|---|---|
| | 1 | 2 | 3 | |
| H2.C1 | 1 | | | |
| H2.C2 | | 1 | | |
| H1.C1 | | | 1 | |
| C1 | | | | 1 |

The resulting network was checked with the LP feasibility test model to ensure that the network could be operated feasibly under the specified conditions. In addition to the five periods uncertain parameters, the disturbances of the district heating stream in the LP model were modeled, allowing the inlet temperature to change ±5%K in five periods. However, three period of first part is flexible but we obtained a new superstructure by add four and five period data. Nevertheless, the superstructure did not change but the widely operation can be performed. The final network configuration from MINLP is shown Figure 13.

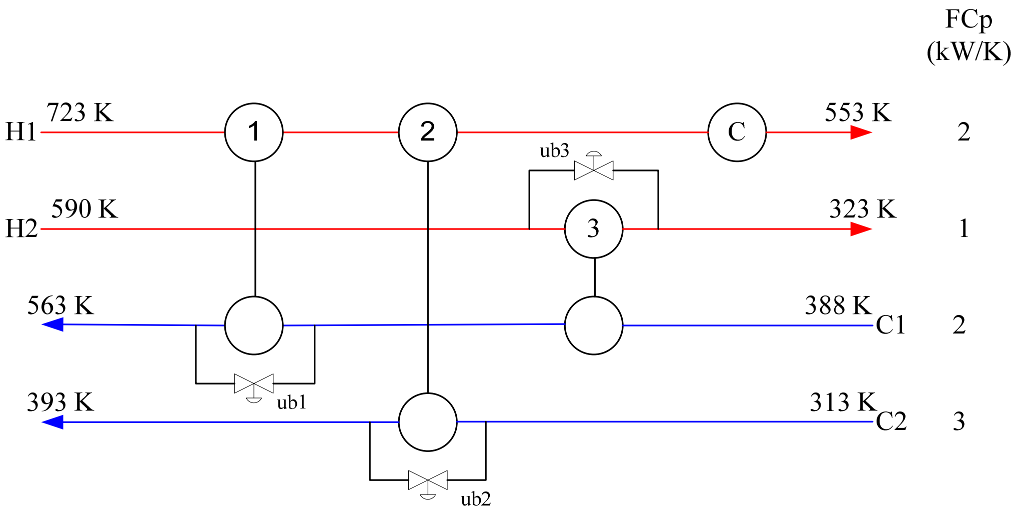



**Figure 13** Final network configurations for case study 3

From direction, degrees of freedom (DOFs) should be first checked. The dimensional spaces spanned by the manipulated variables in the inner HEN to the outer HEN (DS) are 3. Number of utility is 1 and number of target temperatures are 4. Using equation (3), we obtain $N_{DOF,U}$ = 3+1-4 = 0. In this case there is no remaining degree of freedom for utility cost optimization, using direct effect.

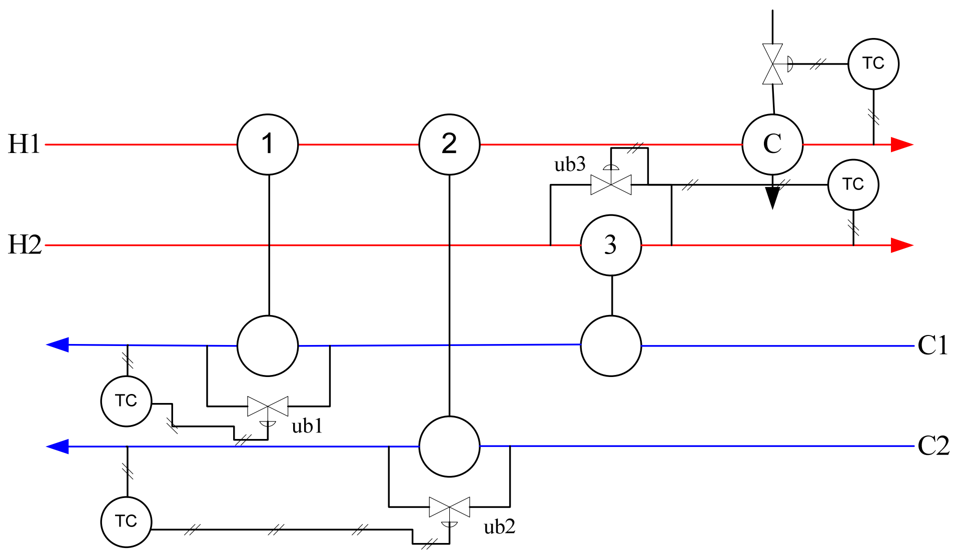
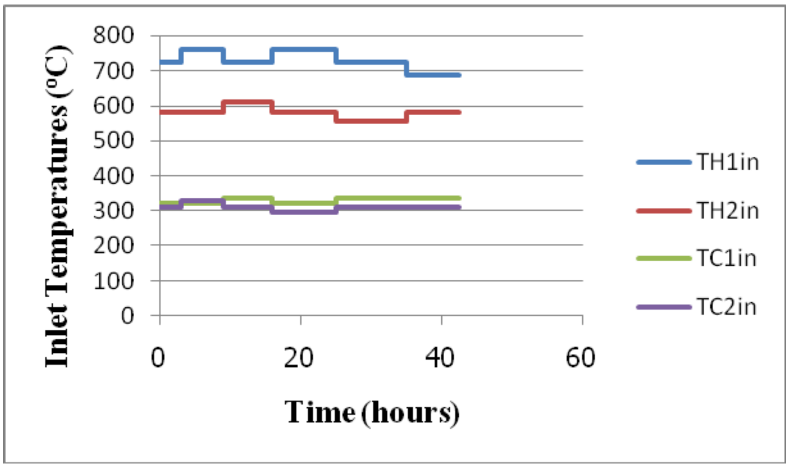



**Figure 14** The control structure for case study 3

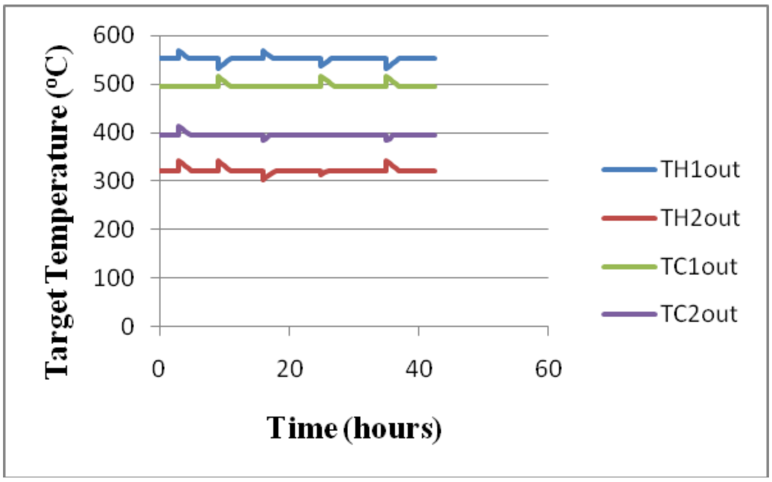
The HEN in the case study 3 with the resulting control structures is tested by performing dynamic simulation on Aspen Dynamics v2006.5. The dynamic results show that the control structures can provide optimality. The information of disturbances of the system at each period is shown in Table 23. Figure 15 shows the dynamic result of the HEN with the control structure. Figure 15b shows the ability of the control structure to keep all target temperatures at the desired values even under the saturation of some manipulated variables. In this case, we concern target temperature are satisfy only. At 3-9 hours, systems have disturbed $\Delta T_{H1}^{in}$ and $\Delta T_{C2}^{in}$ raise effect to $T_{H1}^{out}$ and $T_{C2}^{out}$ rise as well. Nerveless, cold utility and $u_{b2}$ can control the target temperature to steady.

**Table 23** Disturbances in case study 3

| Time (hours) | Disturbance | | | |
|---|---|---|---|---|
| | $\Delta T_{H1}^{in}$ | $\Delta T_{H2}^{in}$ | $\Delta T_{C1}^{in}$ | $\Delta T_{C2}^{in}$ |
| less than 3 | 0 | 0 | 0 | 0 |
| 3-9 | +36.15 | 0 | 0 | +15.65 |
| 9-16 | 0 | +29.15 | +19.4 | 0 |
| 16-25 | +36.15 | 0 | 0 | -15.65 |
| 25-35 | 0 | -29.15 | +19.4 | 0 |
| more than 35 | -36.15 | 0 | +19.4 | 0 |




(a) Inlet temperatures




(b) Target temperatures

**Figure 15** Dynamic simulation of the HEN in case study 3

**Case study 4**

The HEN information Yerramsetty (2008) as shown in Table 24 is studied here. The disturbances are the inlet temperatures of all streams, with the expected vary ±5% °C for streams H1, H2, H3, H4, C1, C2, C3, C4 and C5. The costs equation for exchangers is 10,000 · unit cost ($) +350 · Area ($m^2$). Cost of hot oil is 60 US$kW$^{-1}$ year$^{-1}$ and cost of water 6 US$kW$^{-1}$ year$^{-1}$.

**Table 24** Problem data for case study 4

| Periods | Stream | $T_{in}$ (°C) | $T_{out}$ (°C) | $FC_p$ (kW/K) | $h$ (kW/m²K) | Cost ($ /kW year) |
|---|---|---|---|---|---|---|
| Period 1 | H1 | 327 | 40 | 100 | 0.5 | |
| | H2 | 220 | 160 | 160 | 0.4 | |
| | H3 | 220 | 60 | 60 | 0.14 | |
| | H4 | 160 | 45 | 400 | 0.3 | |
| | C1 | 100 | 300 | 100 | 0.35 | |
| | C2 | 35 | 164 | 70 | 0.7 | |
| | C3 | 85 | 138 | 350 | 0.5 | |
| | C4 | 60 | 170 | 60 | 0.14 | |
| | C5 | 140 | 300 | 200 | 0.6 | |
| Period 2 | H1 | 343.35 | 40 | 100 | 0.5 | |
| | H2 | 231 | 160 | 160 | 0.4 | |
| | H3 | 231 | 60 | 60 | 0.14 | |
| | H4 | 152 | 45 | 400 | 0.3 | |
| | C1 | 95 | 300 | 100 | 0.35 | |
| | C2 | 33.25 | 164 | 70 | 0.7 | |
| | C3 | 89.25 | 138 | 350 | 0.5 | |
| | C4 | 57 | 170 | 60 | 0.14 | |
| | C5 | 147 | 300 | 200 | 0.6 | |
| Period 3 | H1 | 310.65 | 40 | 100 | 0.5 | |
| | H2 | 231 | 160 | 160 | 0.4 | |
| | H3 | 209 | 60 | 60 | 0.14 | |
| | H4 | 152 | 45 | 400 | 0.3 | |
| | C1 | 105 | 300 | 100 | 0.35 | |
| | C2 | 36.75 | 164 | 70 | 0.7 | |
| | C3 | 80.75 | 138 | 350 | 0.5 | |
| | C4 | 63 | 170 | 60 | 0.14 | |
| | C5 | 133 | 300 | 200 | 0.6 | |

**Table 24** (Continued)

| Periods | Stream | $T_{in}$ (°C) | $T_{out}$ (°C) | $FC_p$ (kW/K) | $h$ (kW/m$^2$K) | Cost ($ /kW year) |
|---|---|---|---|---|---|---|
| Period 4 | H1 | 310.65 | 40 | 100 | 0.5 | |
| | H2 | 231 | 160 | 160 | 0.4 | |
| | H3 | 209 | 60 | 60 | 0.14 | |
| | H4 | 168 | 45 | 400 | 0.3 | |
| | C1 | 95 | 300 | 100 | 0.35 | |
| | C2 | 36.75 | 164 | 70 | 0.7 | |
| | C3 | 80.75 | 138 | 350 | 0.5 | |
| | C4 | 63 | 170 | 60 | 0.14 | |
| | C5 | 147 | 300 | 200 | 0.6 | |
| Period 5 | H1 | 343.35 | 40 | 100 | 0.5 | |
| | H2 | 209 | 160 | 160 | 0.4 | |
| | H3 | 231 | 60 | 60 | 0.14 | |
| | H4 | 152 | 45 | 400 | 0.3 | |
| | C1 | 95 | 300 | 100 | 0.35 | |
| | C2 | 36.75 | 164 | 70 | 0.7 | |
| | C3 | 89.25 | 138 | 350 | 0.5 | |
| | C4 | 57 | 170 | 60 | 0.14 | |
| | C5 | 133 | 300 | 200 | 0.6 | |
| | Hot oil | 330 | 250 | - | 0.5 | 60 |
| | Water | 15 | 30 | - | 0.5 | 6 |

Results from the multi-period simultaneous MINLP model are shown in Table 24. The total annual costs, after the MINLP optimization, is 2.942 × $10^6$ $/year when the maximum area is 350 m$^2$ distributed among the 15 units.

**Table 25** Results of MINLP model about existence of match ($Z_{i,j,\mathrm{k}}$) in stage k for case study 4

| Match | Stage (k) | | | Utility |
|---|---|---|---|---|
| | 1 | 2 | 3 | |
| H4.C3 | 1 | | | |
| H1.C1 | 1 | | | |
| H1.C5 | 1 | | | |
| H1.C2 | 1 | | | |
| H1.C2 | | 1 | | |
| H2.C5 | | 1 | | |
| H2.C2 | | 1 | | |
| H1.C1 | | | 1 | |
| H1.C2 | | | 1 | |
| H3.C4 | | | 1 | |
| H1 | | | | 1 |
| H3 | | | | 1 |
| H4 | | | | 1 |
| C1 | | | | 1 |
| C5 | | | | 1 |

The resulting network was checked with the LP feasibility test model to ensure that the network could be operated feasibly under the specified conditions. In addition to the five periods uncertain parameters, the disturbances of the district heating stream in the LP model were modeled, allowing the inlet temperature to change ±5% in five periods. The final network configuration from MINLP is shown Figure 16.

From systematic approach of an optimal operation for HENs flowchart, degrees of freedom (DOFs) should be first checked. The dimensional spaces spanned by the manipulated variables in the inner HEN to the outer HEN (*DS*) are 10. Number of utility are 5 and number of target temperatures are 9. Using equation (3), we have

$N_{DOF,U}$ = 10+5-9 = 6. This case have six remaining degree of freedom for utility cost optimization, hence, a strategy for optimal operation is needed.

The additional information required is the set of active constraint regions that can be obtained using multi-parametric toolbox (Kvasnica *et al.*, 2004). Five active constraint regions found are shown in Table 26.

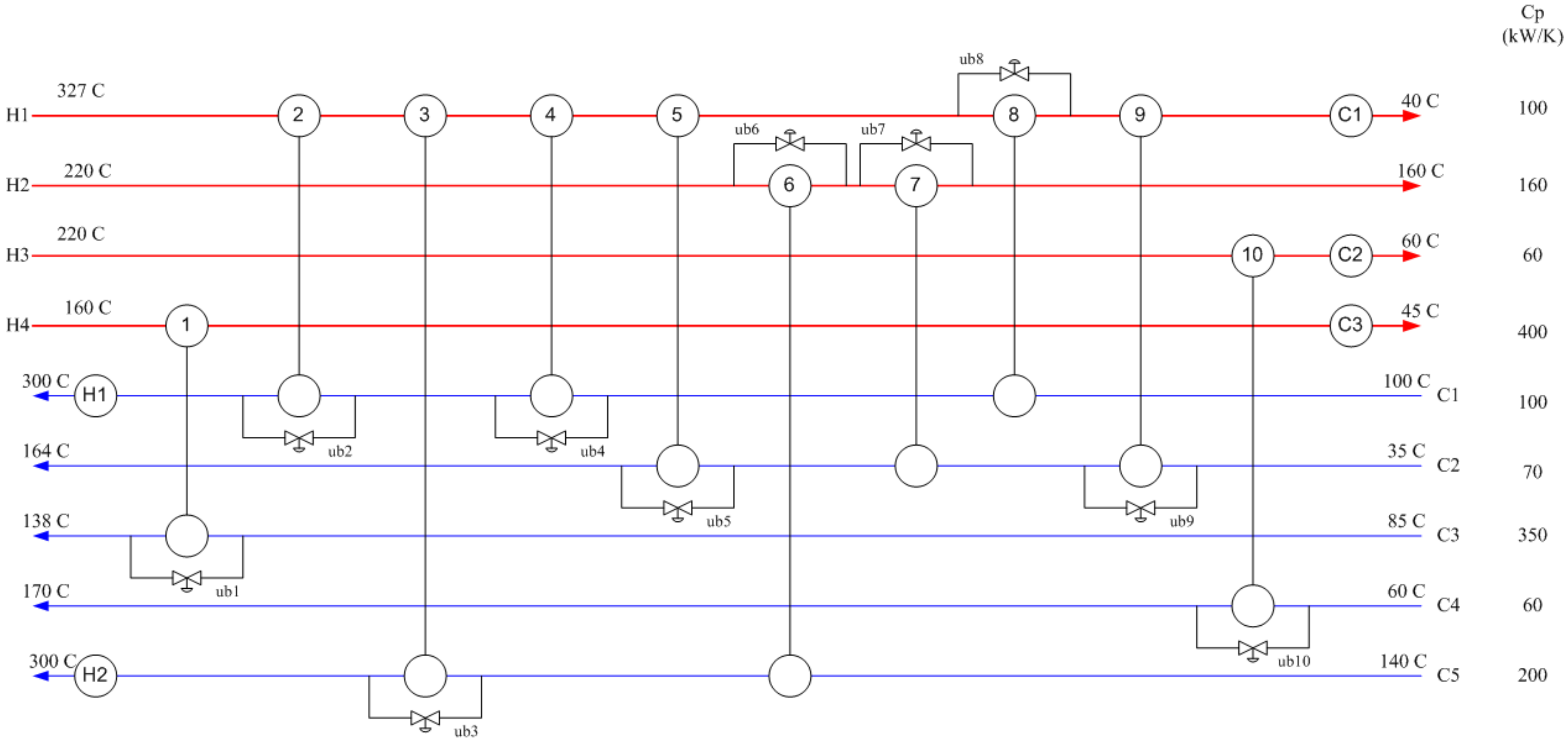



**Figure 16** Final network configurations for case study 4

**Table 26** Set of active constraints in case study 4

| Active constraints region | Manipulated variables | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $Q_{c1}$ | $Q_{c2}$ | $Q_{c3}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ | $u_{b4}$ | $u_{b5}$ | $u_{b6}$ | $u_{b7}$ | $u_{b8}$ | $u_{b9}$ | $u_{b10}$ |
| 1 | $S_L$ | U | U | $S_L$ | $S_L$ | U | U | U | $S_L$ | U | $S_L$ | U | U | $S_L$ | U |
| 2 | U | U | U | U | U | U | $S_L$ | $S_L$ | $S_L$ | U | $S_L$ | U | $S_L$ | $S_L$ | U |
| 3 | $S_L$ | U | U | $S_L$ | U | U | U | $S_L$ | $S_L$ | U | $S_L$ | U | U | $S_L$ | U |
| 4 | U | U | U | $S_L$ | $S_L$ | U | U | U | $S_L$ | U | $S_L$ | U | $S_L$ | $S_L$ | U |
| 5 | U | U | U | U | $S_L$ | U | $S_L$ | U | $S_L$ | U | $S_L$ | U | $S_L$ | $S_L$ | U |

U - Unsaturated manipulated variable (inactive constraint),

$S_L$ - Saturated manipulated variable (active constraint) at the lower bound

Table 26 demonstrates that manipulated variables $Q_{c1}$, $Q_{h1}$, $Q_{h2}$, $u_{b2}$, $u_{b3}$ and $u_{b8}$ can become active constraint at the lower bounds (i.e., zero utility duties or fully close of bypasses). The manipulated variable $Q_{c2}$, $Q_{c3}$, $u_{b1}$, $u_{b5}$, $u_{b7}$ and $u_{b10}$ are never become active constraints (never saturate), hence, there is no need of split-range combinations. $u_{b4}$, $u_{b6}$ and $u_{b10}$ are saturated all regions, hence, there should not be used for any purpose.

For this case study, the ILP Problem P1 will be used to suggest an optimal split-range control structure. The software "GAMs" with the solver "CPLEX" was used to solve the ILP. The solution from Problem P1 (minimizing complexity in optimal split-range pairs and controllability purpose in terms of minimizing the sum of relative orders). The additional information of relative orders is shown in Table 27. The values of binary variables $x_{i,j}$ and $z_{k,j}$ from solving Problem P1 are shown in Tables 28 and 29, respectively. In Table 28 shows that $Q_{c1}$, $Q_{c2}$, $Q_{c3}$, $Q_{h1}$, $Q_{h2}$, $u_{b1}$, $u_{b5}$, $u_{b7}$ and $u_{b10}$ are chosen as primary manipulated variables (see diagonal elements with $x_{i,i} = 1$) while $u_{b2}$ is the secondary manipulated variable for $Q_{h1}$ ($x_{4,7} = 1$), $u_{b3}$ is the secondary manipulated variable for $Q_{h2}$ ($x_{5,8} = 1$) and $u_{b8}$ is the secondary manipulated variable for $Q_{c1}$ ($x_{1,11} = 1$). Table 29 shows the appropriate control pairing, $T_{H1}^{out}$ - $Q_{c1}$, $T_{H2}^{out}$ - $u_{b7}$, $T_{H3}^{out}$ - $Q_{c2}$, $T_{H4}^{out}$ - $Q_{c3}$, $T_{C1}^{out}$ - $Q_{h1}$, $T_{C2}^{out}$ - $u_{b5}$, $T_{C3}^{out}$ - $u_{b1}$, $T_{C4}^{out}$ - $u_{b1}$ and $T_{C5}^{out}$ - $Q_{h1}$ (see $z_{1,1} = z_{2,8} = z_{3,2} = z_{4,3} = z_{5,4} = z_{6,7} = z_{7,6} = z_{8,9} = z_{9,5} = 1$). The resulting control structure is shown in Figure 17.

**Table 27** Relative orders of the HEN in case study 4

| MV / CV | $Q_{c1}$ | $Q_{c2}$ | $Q_{c3}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ | $u_{b4}$ | $u_{b5}$ | $u_{b6}$ | $u_{b7}$ | $u_{b8}$ | $u_{b9}$ | $u_{b10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_{H1}^{out}$ | 1 | ∞ | ∞ | ∞ | ∞ | ∞ | 7 | 6 | 5 | 4 | 6 | 5 | 3 | 2 | ∞ |
| $T_{H2}^{out}$ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 7 | 6 | 5 | 4 | 2 | 1 | 3 | 2 | ∞ |
| $T_{H3}^{out}$ | ∞ | 1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 2 |
| $T_{H4}^{out}$ | ∞ | ∞ | 1 | ∞ | ∞ | 2 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| $T_{C1}^{out}$ | ∞ | ∞ | ∞ | 1 | ∞ | ∞ | 2 | 4 | 3 | 5 | ∞ | ∞ | 4 | ∞ | ∞ |
| $T_{C2}^{out}$ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 4 | 3 | 2 | 1 | 3 | 2 | 4 | 3 | ∞ |
| $T_{C3}^{out}$ | ∞ | ∞ | ∞ | ∞ | ∞ | 1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| $T_{C4}^{out}$ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 1 |
| $T_{C5}^{out}$ | ∞ | ∞ | ∞ | ∞ | 1 | ∞ | 3 | 2 | ∞ | ∞ | 3 | ∞ | ∞ | ∞ | ∞ |

**Table 28** The values of $x_{i,j}$ after solving Problem P1 for case study 4

| Secondary MV / Primary MV | $Q_{c1}$ | $Q_{c2}$ | $Q_{c3}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ | $u_{b5}$ | $u_{b7}$ | $u_{b8}$ | $u_{b10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Q_{c1}$ | 1 | | | | | | | | | | 1 | |
| $Q_{c2}$ | | 1 | | | | | | | | | | |
| $Q_{c3}$ | | | 1 | | | | | | | | | |
| $Q_{h1}$ | | | | 1 | | | 1 | | | | | |
| $Q_{h2}$ | | | | | 1 | | | 1 | | | | |
| $u_{b1}$ | | | | | | 1 | | | | | | |
| $u_{b5}$ | | | | | | | | | 1 | | | |
| $u_{b7}$ | | | | | | | | | | 1 | | |
| $u_{b10}$ | | | | | | | | | | | | 1 |

(the remaining entries are zero)

**Table 29** The values of $z_{k,j}$ after solving Problem P1 for case study 4

| MV / CV | $Q_{c1}$ | $Q_{c2}$ | $Q_{c3}$ | $Q_{h1}$ | $Q_{h2}$ | $u_{b1}$ | $u_{b5}$ | $u_{b7}$ | $u_{b10}$ |
|---|---|---|---|---|---|---|---|---|---|
| $T_{H1}^{out}$ | 1 | | | | | | | | |
| $T_{H2}^{out}$ | | | | | | | | 1 | |
| $T_{H3}^{out}$ | | 1 | | | | | | | |
| $T_{H4}^{out}$ | | | 1 | | | | | | |
| $T_{C1}^{out}$ | | | | 1 | | | | | |
| $T_{C2}^{out}$ | | | | | | | 1 | | |
| $T_{C3}^{out}$ | | | | | | 1 | | | |
| $T_{C4}^{out}$ | | | | | | | | | 1 |
| $T_{C5}^{out}$ | | | | | 1 | | | | |

(the remaining entries are zero)

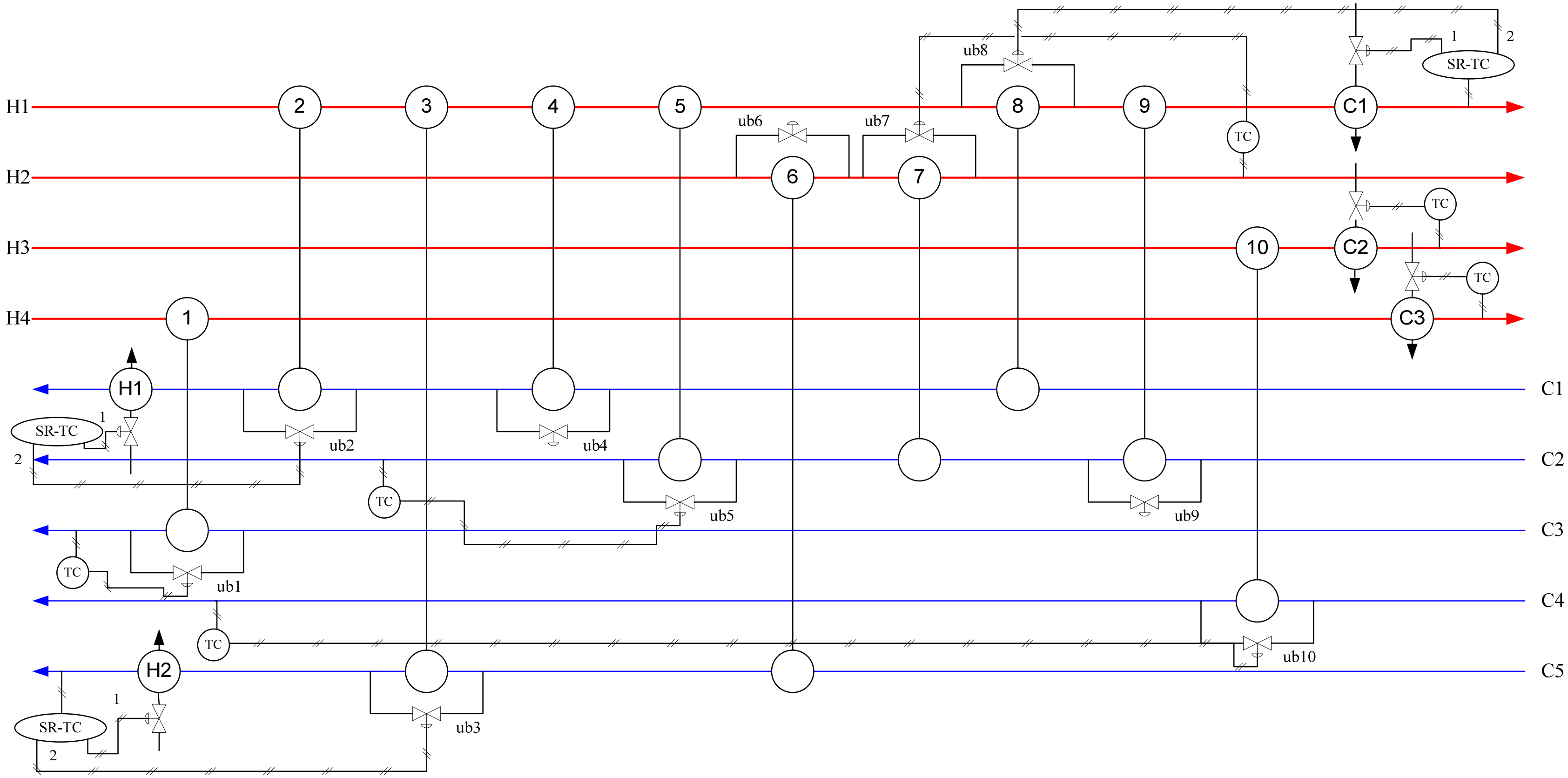
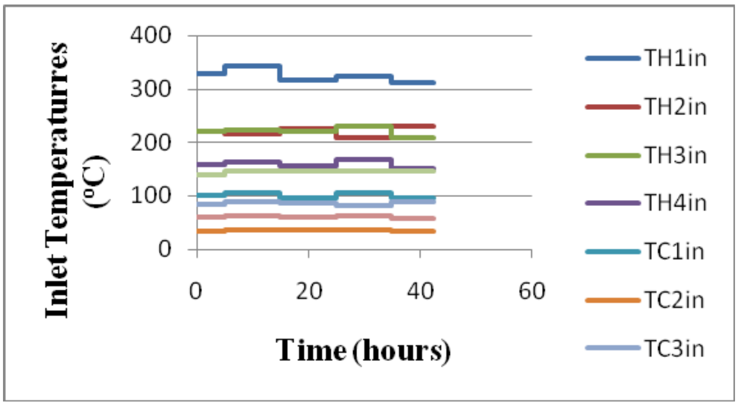



**Figure 17** The control structure for case study 4

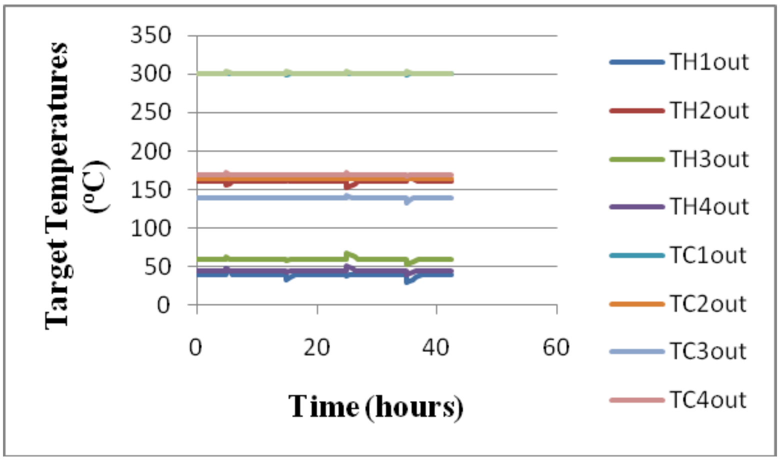
The HEN in the case study 4 with the resulting control structures is tested by performing dynamic simulation on Aspen Dynamics v2006.5. The information of disturbances and active constraints of the system at each period are shown in Table 30. The dynamic results show that the control structures can provide optimality. Figure 18 shows the dynamic result of the HEN with the control structure. Figure 18b shows the ability of the control structure to keep all target temperatures at the desired values even under the saturation of some manipulated variables. At 5-15 hours, we have disturbed all inlet temperature (see in Table 30) $u_{b2}$, $u_{b3}$ and $u_{b8}$ can become active constraint at the lower bounds (fully close of bypasses) while secondary manipulated variable are used for control target temperature. At 15-25 hours, split-range control gives the response to manage the disturbance as seen in the Figure 18c. $Q_{c1}$ is become active constraint at the lower bounds (zero utility duties) while $u_{b8}$ is used to handle target temperature of H1. In the same way, the input saturation problem is solved by switching ability to use a secondary manipulated variable when a primary manipulated variable is saturated. Therefore, $Q_{h1}$ and $u_{b3}$ are used to control target temperature of C1 and C5. This consequence comes from the ability of the control structure to track the right active constraint during the operation (see Figure 18c and set of active constraints in Table 30).

**Table 30** Disturbances and active constraints in case study 4

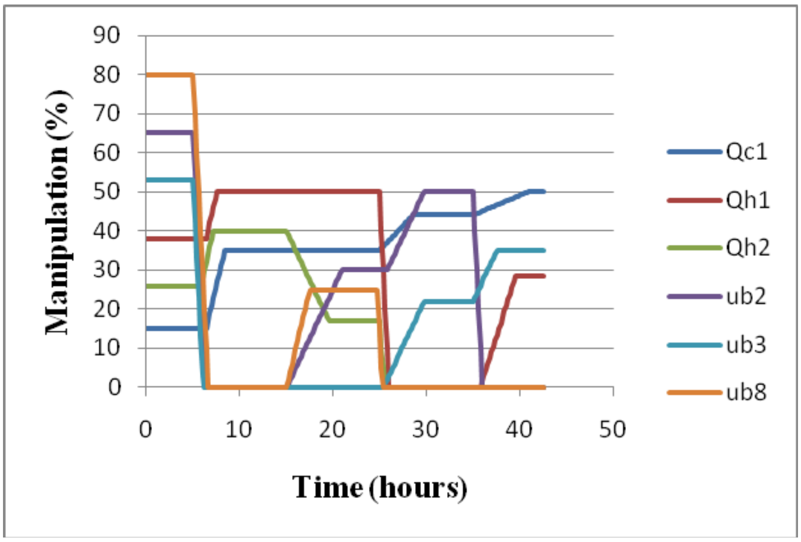
| | | Time (hour) | | | | |
|---|---|---|---|---|---|---|
| | | less than 5 | 5-15 | 15-25 | 25-35 | more than 35 |
| Disturbance | $\Delta T_{H1}^{in}$ | 0 | +15 | -11 | -5 | -16.35 |
| | $\Delta T_{H2}^{in}$ | 0 | -4.3 | +5 | -11 | +11 |
| | $\Delta T_{H3}^{in}$ | 0 | +3.3 | +2 | +11 | -11 |
| | $\Delta T_{H4}^{in}$ | 0 | +4.3 | -3 | +8 | -8 |
| | $\Delta T_{C1}^{in}$ | 0 | +5 | -4 | +5 | -5 |
| | $\Delta T_{C2}^{in}$ | 0 | +1.75 | +1 | +1.75 | -1.75 |
| | $\Delta T_{C3}^{in}$ | 0 | +3 | +2 | -4.25 | +4.25 |
| | $\Delta T_{C4}^{in}$ | 0 | +3 | -1 | +3 | -3 |
| | $\Delta T_{C5}^{in}$ | 0 | +6.5 | +7 | +7 | +7 |
| Active constraint | $Q_{c1}$ | $S_L$ | | $S_L$ | | |
| | $Q_{c2}$ | | | | | |
| | $Q_{c3}$ | | | | | |
| | $Q_{h1}$ | $S_L$ | | $S_L$ | $S_L$ | |
| | $Q_{h2}$ | $S_L$ | | | $S_L$ | $S_L$ |
| | $u_{b1}$ | | | | | |
| | $u_{b2}$ | | $S_L$ | | | $S_L$ |
| | $u_{b3}$ | | $S_L$ | $S_L$ | | |
| | $u_{b4}$ | $S_L$ | $S_L$ | $S_L$ | $S_L$ | $S_L$ |
| | $u_{b5}$ | | | | | |
| | $u_{b6}$ | $S_L$ | $S_L$ | $S_L$ | $S_L$ | $S_L$ |
| | $u_{b7}$ | | | | | |
| | $u_{b8}$ | | $S_L$ | | $S_L$ | $S_L$ |
| | $u_{b9}$ | $S_L$ | $S_L$ | $S_L$ | $S_L$ | $S_L$ |
| | $u_{b10}$ | | | | | |




(a) Inlet temperatures




(b) Target temperatures




(c) Manipulated variable ($Q_{c1}$, $Q_{h1}$, $Q_{h2}$, $u_{b2}$, $u_{b3}$ and $u_{b8}$)

**Figure 18** Dynamic simulation of the HEN in case study 4

# CONCLUSIONS AND RECOMMENDATION

## Conclusions

Systematic approach was developed in this work to obtain an optimal control of heat exchanger networks. Firstly, we started with obtaining the superstructure of HENs with MINLP model and the minimum total annual cost (TAC) by feasibility test with LP model. Secondly, the degree of freedom (*DOFs*) must be checked. If *DOFs* are equal to zero, the direct effect strategy will be employed orelse ( $DOFs > 0$ ) the split-range control technique will perform. Thirdly, the active constrain regions can be formulated using parametric programming. Fourthly, optimal split-range control structure can be determined by integer linear program (ILP). Finally, the proposed control structure is dynamically tested to ensure the optimal conditions.

The information of active constraint regions in a given disturbance window and relative orders are used. Case study 1 from Zamora and Grossmann (1998), the total annual costs is 285,916 $/year distributed among the 8 units. Two active constraint regions are found. Three remaining degree of freedom for utility cost optimization, used ILP to determine an optimal split-range control structure. Biegler *et al.* (1997) as a case study 2 has the total annual costs at 85,290 $/year for 6 unit operating with 3 active constraint regions. Two remaining degree of freedom for utility cost optimization, used ILP to determine an optimal split-range control structure. For case study 3, from Aaltola (2003), the total annual costs for 4 units is 27,743 $/year, no active constraints can be found. No remaining degree of freedom for utility cost optimization, no need optimal control, for case study 3 using direct effect. Case study 4 from Yerramsetty (2008), the total annual costs is $2.942 \times 10^6$ $/year. There are 15 operating with 5 active constraint regions. In all cases, additional periodical data were performed in order to extend operation regions. We noticed that the superstructure cannot be improved upon to the number of periodical data. Six remaining degree of freedom for utility cost optimization, used ILP to determine an optimal split-range control structure. For dynamics test, all cases show the performance of control structure to keep all target temperatures at the desired values.

**Recommendation**

1. In this work, we adjusted inlet temperatures only. In practical we can have several ways to disturb the system of HENs such as flowrate change and setpoint change.

2. If we have many loop of split-range control, faults tolerance are used to continue operating properly in the event of the failure of (or one or more faults within) some of its loops.

3. Advance control would be of interest instead of only split-range control.

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