Effect of All-Integer Quantization Decoding in LDPC Coded High Order Modulation Schemes

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Abstract. *There is strong evidence that LDPC codes under practical all-integer quantization decoding constraint can achieve very good BER performance over binary modulation scheme, e.g., BPSK. By focusing on higher order modulation scheme, the effect of all-integer quantization decoding is further investigated in this work. Our result reveals that LDPC coded M-QAM with only 3-bit integer quantization decoding can exhibit excellent performance, i.e., negligible performance loss comparing with non-quantized case.*

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1. Introduction

It is well known that low-density parity-check (LDPC) codes have been the primary channel coding scheme [1]. These codes can achieve the error-correction performance close to the Shannon limit [2]-[5]. The outstanding performance of LDPC codes is based on the decoding algorithms. The most popular decoding algorithm of LDPC is belief propagation (BP) based algorithm also known as sum-product algorithm (SPA) [6]. Nevertheless, one might consider that this algorithm requires high computational complexity for check node processing [7]. Alternatively, the approximation of SPA named min-sum algorithm (MSA) is extensively used in LDPC hardware decoders. Such algorithm can greatly reduce the computational complexity at the expense of the small performance degradation compared with SPA.

Typically, the LDPC decoding algorithms are represented by floating-point, i.e., infinite precision. As far as hardware implementation of LDPC decoder is concerned, the finite precision or quantization effects are an important issue to be considered. It is known that the decoding performance depends on the number of quantization bits. The LDPC decoding with the appropriate number of the quantization bits can provides a very good performance. On the other hand, the reduction of the quantization bits leads to an increase in speed and lower computational complexity [8] with sacrifice a performance loss. Therefore, the design of the decoding algorithm for LDPC coded QAM must consider a trade-off between the decoding performance and the computational complexity.

Many previous works investigate the quantization effect on the LDPC decoder using BPSK modulation [9–13]. Various decoding algorithms are presented to deal with this effect. However. most of the works present a quantization scheme with decimal. Our work aims to explore the effect of quantization without the number fractional part, i.e., all-integer quantization, on LDPC decoding.

There are a few studies of the quantization effect on the LDPC decoder with QAM modulation [14]. To the best of our knowledge, the effect of all-integer quantization on the LDPC coded M-QAM has never been explored. Our results show that the use of 3-bits integer quantized MSA for LDPC coded 16 and 64-QAM can provide the decoding performance approach to the infinite precision case.

The paper is organized as follows: Section 2 explains basic and background. Moreover, the system model is presented in the same section. The simulation results and discussion are described in Section 3. Finally, Section 4 presents the conclusions.

2. Basic and Background

Our work aims to explore the effect of quantization on LDPC coded QAM. The overall design diagram for the LDPC code is described in this section. The system in Fig. 1 can be explained as follows. First, the information vector m is generated and encoded to get the codeword c. After that, the codeword will be grouped into m-tuples and each m-tuple is mapped to a QAM constellation by Gray labeling. The groups of $t = \log_2 M$ code bits are transformed into a symbol of the bi-dimensional M-QAM constellation [14]. Then, the modulated signal will be transmitted over an additive white Gaussian noise (AWGN) channel.

At the receiver side, the demodulator extracts the log-likelihood ratios (LLR) for each bit in the m-tuples from the received signal, which are desired by the LDPC decoder. The min-sum algorithm decoder is used in this work. The processing of MSA consists of three steps. The first one is the check-to-variable message. The second step is the calculation of posterior LLR. The last one is the variable-check-step. The output of these procedures will be quantized.

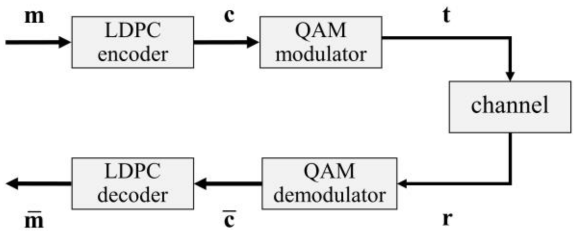

Fig. 1 Block diagram of LDPC coded QAM.

It is common to employ the infinite precision operations on the theoretical study of LDPC decoding. However, in hardware implementation, it is difficult to be calculated when the values have a larger number of decimals [15]. Hence, there is a great need to find a finite precision, i.e., quantization, scheme which gets an efficient implementation. The infinite values of MSA decoding will be quantized into integer values.

For a uniform quantization scheme, the notation $(q{:}f)$ is applied. Let q denotes the total number of quantization bits and f is the bit length of decimal part of the value [16-18]. To quantize the infinite precision into finite precision, we assign the lower bound and the upper bound of quantization as follows:

$$[-Q_J\,, Q_J] = \left[\frac{-(2^{q-1}-1)}{2^f}\,, \frac{(2^{q-1}-1)}{2^f}\right] \tag{1}$$

The interval between every two adjacent quantized values is expressed by $\frac{1}{2^f}$. The set of quantized values is expressed as:

$$\boldsymbol{Q} = [-Q_J \quad -Q_{J-1} \ldots Q_0 \quad Q_{J-1} \quad Q_J] \tag{2}$$

where $Q_J = J/2^f$ and $J = 2^{q-1} - 1$. For all-integer quantization, the fractional part is assigned as zero ($f = 0$) in this paper. For instance, the $(3{:}0)$ quantization scheme will produce the set of quantized values as $\boldsymbol{Q} = [-3 \ -2 \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3]$. Let x be the input value and y is the output or quantized value. The infinite values can be quantized under the parameters $(q{:}f)$ by calculating as follows:

$$y = \begin{cases} \frac{\lfloor x \cdot 2^f + 0.5 \rfloor}{2^f}, & -\frac{(2^{q-1}-1)}{2^f} \le x \le \frac{(2^{q-1}-1)}{2^f}, \\ -\frac{(2^{q-1}-1)}{2^f}, & x < -\frac{(2^{q-1}-1)}{2^f}, \\ \frac{(2^{q-1}-1)}{2^f}, & x > \frac{(2^{q-1}-1)}{2^f} \end{cases} \tag{3}$$

To the best of our knowledge, the LDPC code with the all-integer quantized MSA decoder over BPSK modulation can achieve a very good performance, i.e., the same decoding performance as the MSA with infinite precision decoding. This paper aims to study the quantization effect on the LDPC coded QAM. So, we expect that the MSA decoder for LDPC coded M-QAM will also be robust to the integer quantization.

3. Simulation Results

The bit error rate (BER) performances of LDPC coded M-QAM under all-integer quantization decoding are presented in this section. Regular LDPC codes and QAM-AWGN channel are assumed for all the simulations. LDPC decoder can be employed by SPA or MSA. The maximum iterations for LDPC decoder are fixed to be 50. For simplicity, the term 'infinite precision' is used to represent unquantized LDPC decoder whereas the term 'q bits integer' is used to denote quantized LDPC decoder with q-bits quantizer.

For all-integer quantization effect on the MSA decoder for LDPC code with BPSK modulation, the primary result is shown in Fig. 2. It can be observed that the LDPC code with 4-bit integer quantized MSA decoder can achieve the BER performance close to the MSA with infinite precision. The use of 3-bit integer quantization obtains the decoding performance loss about 0.5 dB.

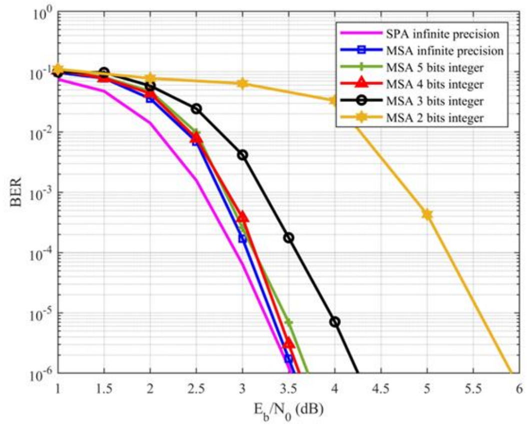

Fig. 2 Effect of all-integer quantization on LDPC decoder with BPSK modulation. The block length is 1152 bits and the code rate is 2/3.

In order to further present the robustness of MSA to quantization, Fig. 3 shows the performance comparison between SPA and MSA for the case of LDPC coded 16-QAM. The following observations can be made from the figure:

a) With infinite precision, the performances of SPA and MSA are almost the same.

b) MSA with 4 bits integer can provide the same performance as infinite precision.

c) SPA with 4 bits integer has very poor decoding performance.

d) MSA with 3 bits integer can still provide the same performance as infinite precision.

e) MSA with 3 bits integer has roughly 2 dB coding gain over MSA with 2 bits.

f) MSA with 2 bits integer can perform better than SPA with 4 bits integer.

This implies that SPA is very sensitive to the effect of integer quantization decoding. By contrast, MSA is more robust to such effect. Unlike binary modulation scheme that needs 4 bits integer quantization, MSA with only 3 bits integer is sufficient to provide good LDPC decoding performance in 16-QAM system. The reason is that, for 16-QAM system, the range of LLR involved in all decoding steps suits well with just 3 bits integer quantizer. From the results, it is sufficient to state that, for 16-QAM system, LDPC code that employs MSA with all integer quantization can provide excellent decoding performance.

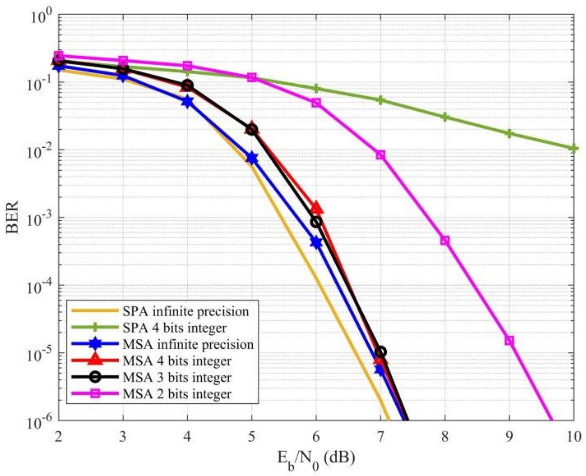

Fig. 3 BER performance of LDPC coded 16-QAM with various quantization schemes. The block length is 264 bits and the code rate is 1/2.

To further explore the effect of integer quantization in LDPC coded higher modulation scheme, the block length of LDPC code is increased from 264 bits (used in Fig.3) to be 1156 bits and 64-QAM is selected to conduct the simulation. Figure 4 shows the performance comparison between SPA and MSA for the case of LDPC coded 64-QAM. It can be seen from the figure that, like 16-QAM system, MSA with only 3 bits integer is still sufficient to provide excellent LDPC decoding performance. Therefore, the effect of integer quantization for MSA decoder in LDPC coded 64-QAM system can be made small by using at least 3-bit quantizer. The results also implies that the use of higher order modulation tends to leave wider performance gap between SPA and MSA. This can be observed from the figure that there is about 0.4 performance difference between SPA and MSA with integer quantization for 64-QAM system while there is no significant performance difference for 16-QAM system.

To guarantee these decoding performances, the evaluation matric is presented in Table I. The calculation of mean square error (MSE) is used to analyze the BER performance. The MSA can be expressed by

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{y}_i - y_i\right)^2 \tag{4}$$

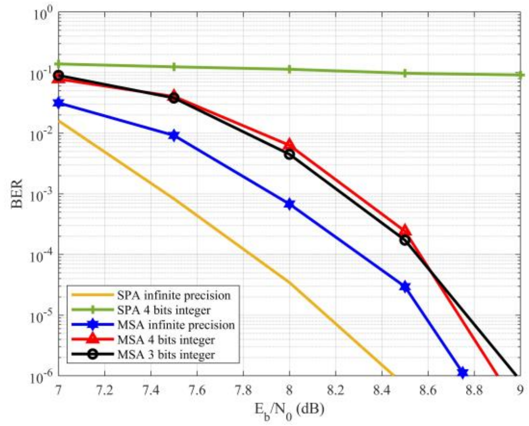

Fig. 4 BER performance of LDPC coded 64-QAM with various quantization schemes. The block length is 1152 bits and the code rate is 1/2.

Let $\hat{y}_i$ and y_i be the i-th quantized value and infinite precision value, respectively. The number of values is represented by N. As shown in figure 1, it is found that the MSE values of 4-bit integer quantized MSA decoding are very close to zero. Therefore, the quantized MSA provides the good BER performance as corresponds to the simulation results.

Decoding Algorithms	Block Length	
	1152	264
MSE	≈ 0.1	≈ 0.1

Table 1 The mean square error (MSE) of 4-bit integer quantized MSA LDPC decoder

4. Conclusions

The utilization of a low-resolution decoding algorithm for LDPC codes causes simplified computation and lower decoding delay. The effects of integer quantization on the LDPC decoder by using M-QAM modulation are demonstrated in the paper. All simulation results show that the MSA decoding with 3-bit integer quantization for LDPC coded QAM can attain good BER performance, i.e., a similar performance to the MSA decoding with infinite precision. As a result, under integer quantization, an efficient hardware implementation of high-throughput LDPC decoder can be realized.

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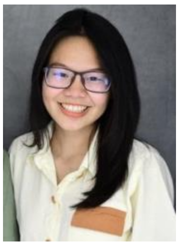
Biographies



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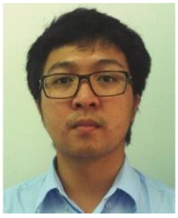

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