A TEST STATISTIC FOR SELECTION OF MULTIVARIATE LINEAR REGRESSION MODELS

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ABSTRACT

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In this study, a test statistic used to select a multivariate linear regression model based on Mallows's C_p with the same rationale as the SC_p criterion from the system of equations Vichit Lorchirachoonkul and Jirawan Jitthavech (2012: 2386-2394) proposed. The D statistic, which is the difference between the modified C_p statistics in the reduced model and in the full model, approximates to a standard normal distribution.

The modified C_p statistic without the hypothesis testing, MC, and the proposed test statistic T_D based on the percentage of selecting the model correctly were compared via a simulation study. Variable selection was carried out using backward elimination with five datasets consisting of 100 samples of size 200 and significance levels of 0.05 and 0.10, and the correlation between the equations was set at 0.3, 0.4, 0.5, 0.7, and 0.8, respectively. The multivariate linear regression full models consisted of two dependent variables, two relevant independent variables, and two irrelevant independent variables. In addition, the random disturbances were uncorrelated across observations in the same equation but contemporaneously correlated across equations. The results of the simulation study showed that the test statistic T_D was able to select the model more often than the modified C_p criterion in all datasets, and, for both criteria, no under-fit models were selected.

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CHAPTER 1

INTRODUCTION

This chapter is comprised of the following parts. In section 1.1, the statement of the problem and its importance in model selection of multivariate linear regression models is outlined. Sections 1.2-1.4 consist of the objectives, scope, and usefulness of the study, respectively. Finally, Section 1.5 describes the definitions used in this dissertation.

1.1 Statement of the Problem Statement and Its Importance

In regression analysis, model selection is a methodology to identify a "good" model among models each of which consists of a number of potentially independent and different relevant variables for predicting the dependent variables. Popular well-known criteria include the Akaike information criterion (AIC), the Bayesian information criterion (BIC), Mallows's C_p, and R^2. The AIC, BIC, and Mallows's C_p are criteria which show that the optimal model is the one with the lowest value. Conversely, R^2 is a criterion showing that the optimal model is the one with the highest value. However, R^2 may result in having too many independent variables in the model, which is not consistent with the principal of developing a regression model with the fewest number of independent variables possible, although it can explain dependent variables more fully. Therefore, of interested is using criteria that consider the optimal model from the lowest value. Mallows's C_p has been shown to perform at least as well as other reported criteria (Buchan, Grewal and Robson, 1993: 901-908; Lo, Yuan and Ni, 1999: 1383-1396; Miyashiro and Takano, 2015: 325-331) and is easy to understand because the value from the criterion is related to the mean squared prediction error

(MSPE). Therefore, any model with the lowest value of Mallows's C_p, or lowest value of MSPE, should be the optimal model. Many authors have proposed Mallows's C_p for model selection by considering its value for selecting the best model but they did not test for statistical significance. The Mallows's C_p is calculated from the ratio of the MSPE and the error variance based on the true model as (Mallows, 1973: 661-675, 1995: 362-372)

$$\Gamma_p = \frac{E\left((\hat{\mathbf{y}} - E(\mathbf{y}))'(\hat{\mathbf{y}} - E(\mathbf{y}))\right)}{\sigma^2}, \tag{1.1}$$

where $\hat{\mathbf{y}}$ is the predicted value of $\mathbf{y}$ and σ^2 is the unknown error variance based on the true model. In practice, the error variance in the full model is used to approximate the unknown error variance in the true model.

In general, when using Mallows's C_p for model selection, the residual sum of squares always becomes smaller when variables are added to the model. Thus, the best sub-model is the one where its C_p value is closest to the value of the full model.

By directly extending the definition of Mallows's C_p statistic in a single equation model to a multivariate regression model, Sparks, Troskie and Coutsourides (1983: 1775-1793) were the first to introduce C_p for variable selection in multivariate regression models. Fujikoshi and Satoh (1997:707-716) modified Mallows's C_p in a multivariate linear regression model, it was intended to reduce bias in the situation where the collection of candidate models includes both underspecified and overspecified models. Yanagihara and Satoh (2010: 1226-1238) proposed an unbiased C_p criterion for multivariate ridge regression, whereas Kubokawa and Sivastava (2012: 2465-2489) presented a modified C_p statistic based on the ridge-type estimator of covariance matrix under the asymptotic theory that both the sample size and the dimension of covariance matrix go to infinity. Vichit Lorchirachoonkul and Jirawan Jitthavech (2012: 2386-2394) modified Mallows's

C_p using a system of equations to change the unknown error covariance matrix of the true model to the covariance matrix of the dependent variables. Nagai (2013: 73-106) proposed the generalized C_p (GC_p) criterion for multivariate ridge regression based on the predicted mean squared error (PMSE) being minimized. In addition, Fujikoshi Sakura and Yanagihara (2014: 184-200) proposed a multivariate C_p when the sample size is large but the dimension p of the response variables is fixed, which also had a consistency property under $\mathbf{\Omega}_{\varepsilon} = O(n)$ as well as $\mathbf{\Omega}_{\varepsilon} = O(np)$ when $p/n \to c \in [0,1)$.

A test for the equality of Mallows's C_p between the full and reduced models was suggested by Gilmour (1996: 49-56). Previously, Sparks et al. (1983: 1775-1793) proposed a test of Mallows' s C_p in a multivariate linear regression model.

In this study, an alternative test statistic for selection among multivariate linear regression models is of interest, and the C_p criterion for use in the selection of a multivariate linear regression model based on a modified C_p statistic used in the system of equations proposed by Vichit Lorchirachoonkul and Jirawan Jitthavech (2012) is considered. Additionally, the performance of the proposed test was investigated using a simulation study.

1.2 Objectives of the Study

1) To propose a test statistic for model selection among multivariate linear regression models.

2) To investigate the performance of the proposed test statistic using a simulation study

1.3 Scope of the Study

In this study, a new test statistic for selection of multivariate linear regression models is developed based on Mallows's C_p in the system of equations derived by

Vichit Lorchirachoonkul and Jirawan Jitthavech (2012: 2386-2394) as follows:

1) The random disturbances are assumed normally distributed with conditional zero mean and covariance matrix $\mathbf{\Omega}_{\varepsilon}$.

2) The random disturbances have heteroscedasticity and are contemporaneously correlated across equationsbut are uncorrelated across observations.

3) The estimation of parameters in the model is by using generalized least squares.

1.4 Usefulness of the Study

The proposed test statistic presented could be useful in many research areas, such as economics, finance, management, marketing, and other related fields.

1.5 Definitions

1.5.1 Contemporaneous Correlation

Contemporaneous correlation in multivariate linear regression models is uncorrelated across observations but correlated across equations. Therefore, the covariance matrix of the disturbances of m equations is specified as

$$\mathbf{\Sigma}_{\varepsilon} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_{mm} \end{bmatrix}. \tag{1.2}$$

1.5.2 Fit Model

A fit model occur when only relevant variables are included in the model.

1.5.3 Over-Fit Model

An over-fit model contains irrelevant variables, which means that the model has poor predictive performance.

1.5.4 Under-Fit Model

An under-fit model has relevant variables excluded, which also means that the model has poor predictive performance.

1.5.5 Percentage of Correct Model Selection

The percentage of correct model selection is a measure used to determine the likelihood of choosing the correct model containing only relevant variables. The percentage of correct model selection is expressed as

$$\textbf{\% of correct model selection} = \frac{\text{number of having only the relevant variables in the model}}{\text{number of replications}} \times 100 \qquad (1.3)$$

CHAPTER 2

LITERATURE REVIEW

In this chapter, a literature review of academic publications relevant to the study is covered. A review of the system of equation is presented in section 2.1, and multivariate linear regression models are outlined in section 2.2, followed by a review of the selection of multivariate linear regression models in section 2.3. A review the distribution of a linear combination of gamma random variables is presented in section 2.4 followed by the testing of relevant criteria in section 2.5.

2.1 The System of Equations

The system of equations of interest in this study consist of m equations may be written as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{2.1}$$

where $\mathbf{y}$ is the $nm \times 1$ vector of dependent variables distributed as $N_{nm}(\mathbf{X}\boldsymbol{\beta}, \boldsymbol{\Omega}_{\varepsilon})$,

$\mathbf{X}$ is the $nm \times p_T$ diagonal matrix of independent variables,

$\boldsymbol{\beta} = (\boldsymbol{\beta}_1' \ \boldsymbol{\beta}_2' \ldots \boldsymbol{\beta}_m')'$ is the $p_T \times 1$ unknown parameter vector, and

$\boldsymbol{\varepsilon}$ is the $nm \times 1$ vector of disturbance distributed as $N_{nm}(\mathbf{0}, \boldsymbol{\Omega}_{\varepsilon})$.

A system of equations in matrix form can be written as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 & 0\cdots & 0 \\ 0\cdots & \mathbf{X}_2 & \cdots 0 \\ & \vdots & \\ 0\cdots & 0\cdots & \mathbf{X}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_m \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \vdots \\ \boldsymbol{\varepsilon}_m \end{bmatrix}, \tag{2.2}$$

where $\mathbf{y}_i$ is the $n \times 1$ dependent variables in equation $i, i = 1, 2, ..., m,$

$\mathbf{X}_i$ is the $n \times p_i$ matrix of independent variables including a constant unit vector in equation i with $rank(\mathbf{X}_i) = p_i, i = 1, 2, ..., m,$

$\boldsymbol{\beta}_i$ is the $p_i \times 1$ parameter vector in equation $i, i = 1, 2, ..., m,$

p_i is the number of parameter in equation $i, i = 1, 2, ..., m,$ and

n is the number of observation in each equation.

The total number of parameters in (2.1) is equal to $p_T = \sum_{i=1}^{m} p_i$. The $nm \times 1$ disturbance vector $\boldsymbol{\varepsilon}$ is assumed to be uncorrelated across observations in the same equations but contemporaneously correlated across observations in different equations. The distribution of $\boldsymbol{\varepsilon}$ is assumed to be multivariate normal with zero mean and the following $nm \times nm$ covariance matrix in the form of a Kronecker product as

$$E(\boldsymbol{\varepsilon\varepsilon}' \mid \mathbf{X}) = \boldsymbol{\Omega}_\varepsilon = \boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_n, i = 1, 2, ..., m \tag{2.3}$$

$$E(\varepsilon_{it}\varepsilon'_{js} \mid \mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_m) = \begin{cases} \sigma_{ij} & if \quad s = t \\ 0 & otherwise, \end{cases}$$

where $\boldsymbol{\Sigma}_\varepsilon$ is the $m \times m$ contemporaneous disturbance covariance matrix defined in (1.2), which gives

$$\boldsymbol{\Omega}_\varepsilon = \boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_n = \begin{bmatrix} \sigma_{11}\mathbf{I}_n & \sigma_{12}\mathbf{I}_n & \cdots & \sigma_{1m}\mathbf{I}_n \\ \sigma_{21}\mathbf{I}_n & \sigma_{22}\mathbf{I}_n & \cdots & \sigma_{2m}\mathbf{I}_n \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1}\mathbf{I}_n & \sigma_{m2}\mathbf{I}_n & \cdots & \sigma_{mm}\mathbf{I}_n \end{bmatrix}. \tag{2.4}$$

2.2 Multivariate Linear Regression Models

Multivariate linear regression models are the special case of the system of equations models in which the same sets of independent variables is used in all equations.

For multivariate linear regression models consisting of m equations, the dependent variables $\mathbf{y}$ can be expressed in two forms: the first being in vector form and the second in matrix form. The first form of multivariate linear regression models is the same as in (2.1) where $\mathbf{X}_i = \mathbf{X}_c,\ p_i = p,\ i = 1,2,...,m$ can be written as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{X}_c & 0 & \cdots & 0 \\ 0 & \mathbf{X}_c & \cdots & 0 \\ & \cdot & & \\ & \cdot & & \\ & \cdot & & \\ 0 & 0 & \cdots & \mathbf{X}_c \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{\beta}_m \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{\varepsilon}_m \end{bmatrix}, \tag{2.5}$$

where $\mathbf{y}_i$ is the $n \times 1$ dependent variable vector in equation $i, i = 1,2,...,m,$

$\mathbf{X}_c$ is the $n \times p$ matrix of independent variables including a constant unit vector in equation $i, i = 1,2,...,m,$

$\boldsymbol{\beta}_i$ is the $p \times 1$ parameter vector in equation $i, i = 1,2,...,m,$ and

$\boldsymbol{\varepsilon}_i$ is the $n \times 1$ random disturbance vector in equation $i, i = 1,2,...,m,$

p is the number of parameters in equation $i, i = 1,2,...,m,$ and
n is the number of observations in each equation.

The second form of a multivariate linear regression model, as defined by many authors (Johnson and Wichern, 2002: 384; Rencher, 2002: 337; Srivastava, 2002: 277), is as follows:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \qquad (2.6)$$

where $\mathbf{y}$ is the $n \times m$ dependent variable matrix distributed as $N_{nm}(\mathbf{X}\boldsymbol{\beta}, \boldsymbol{\Omega}_{\varepsilon})$,

$\mathbf{X}$ is the $n \times p$ matrix of independent variables including a constant unit vector,

$\boldsymbol{\beta}$ is the $p \times m$ unknown parameter,

$\boldsymbol{\varepsilon}$ is the $n \times m$ random disturbance matrix distributed as $N_{nm}(\mathbf{0}, \boldsymbol{\Omega}_{\varepsilon})$,

k is the number of independent variable in each equation,

p is the number of parameters in each equation, $p = k + 1$,

m is the number of equations, and

n is the number of observations in each equation.

The matrix form of (2.6) can be written as

$$\begin{bmatrix} y_{11} & y_{12} \cdots y_{1m} \\ y_{21} & y_{22} \cdots y_{2m} \\ & \vdots \\ y_{n1} & y_{n1} \cdots y_{nm} \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{12} \cdots X_{1k} \\ 1 & X_{21} & X_{22} \cdots X_{2k} \\ & & \vdots \\ 1 & X_{n1} & X_{n2} \cdots X_{nk} \end{bmatrix} \begin{bmatrix} \beta_{01} & \beta_{02} \cdots \beta_{0m} \\ \beta_{11} & \beta_{12} \cdots \beta_{1m} \\ & \vdots \\ \beta_{k1} & \beta_{k2} \cdots \beta_{km} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \cdots \varepsilon_{1m} \\ \varepsilon_{21} & \varepsilon_{22} \cdots \varepsilon_{2m} \\ & \vdots \\ \varepsilon_{n1} & \varepsilon_{n1} \cdots \varepsilon_{nm} \end{bmatrix}, \qquad (2.7)$$

The total number of parameters in a multivariate linear regression model is equal to $p_T = mp$ and the disturbance of $\boldsymbol{\varepsilon}$ is assumed to be multivariate normal with zero mean and the covariance matrix is the same as in (2.4)

2.3 Model Selection Criteria in Multivariate Linear Regression Models

With reference to the multivariate linear regression models in (2.6), before model selection in regression can be discussed, the structure and assumption of the model must be defined.

We define the *true* model in multivariate linear regression models as

$$\mathbf{y} = \mathbf{X}_* \boldsymbol{\beta}_* + \boldsymbol{\varepsilon}_*, \tag{2.8}$$

where $\mathbf{y}$ is the $n \times m$ dependent variable matrix distributed as $N_{nm}(\mathbf{X}_* \boldsymbol{\beta}_*, \boldsymbol{\Omega}_*)$,

$\mathbf{X}_*$ is $n \times p_*$ matrix of independent variables including a constant unit vector of the true model,

$\boldsymbol{\beta}_*$ is the $p_* \times m$ unknown parameter of the true model,

$\boldsymbol{\varepsilon}_*$ is the $n \times m$ random disturbance matrix distributed as $N_{nm}(\mathbf{0}, \boldsymbol{\Omega}_*)$,

p_* is the number of parameters in each equation of the true model,

m is the number of equations, and

n is the number of observations in equation.

We define the *candidate* model in multivariate linear regression models as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{2.9}$$

where $\mathbf{y}$ is the $n \times m$ dependent variable matrix distributed as $N_{nm}(\mathbf{X}\boldsymbol{\beta}, \boldsymbol{\Omega}_{\varepsilon})$,

$\mathbf{X}$ is the $n \times p$ matrix of independent variables including a constant unit vector in the candidate model,

$\boldsymbol{\beta}$ is the $p \times m$ unknown parameter in the candidate model,

$\boldsymbol{\varepsilon}$ is the $n \times m$ random disturbance matrix distributed as $N_{nm}(\mathbf{0}, \boldsymbol{\Omega}_{\varepsilon})$,

m is the number of equations, and

n is the number of observations in the equation.

Suppose that L_2 is the difference between the estimated model and the expectation of the *true* model, is defined as

$$L_2 = \frac{1}{n}(\mathbf{X}_* \boldsymbol{\beta}_* - \mathbf{X}\hat{\boldsymbol{\beta}})'(\mathbf{X}_* \boldsymbol{\beta}_* - \mathbf{X}\hat{\boldsymbol{\beta}}), \tag{2.10}$$

then we define the log-likelihood of the true model f_* as

$$\log(f_*) = -\frac{nm}{2}\log(2\pi) - \frac{n}{2}\log(|\boldsymbol{\Sigma}_*|) - \frac{1}{2}tr(\mathbf{y} - \mathbf{X}_*\boldsymbol{\beta}_*)'\boldsymbol{\Omega}_*^{-1}(\mathbf{y} - \mathbf{X}_*\boldsymbol{\beta}_*) . \tag{2.11}$$

The log-likelihood of the candidate model f is

$$\log(f) = -\frac{nm}{2}\log(2\pi) - \frac{n}{2}\log(|\boldsymbol{\Sigma}_\varepsilon|) - \frac{1}{2}tr(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'\boldsymbol{\Omega}_\varepsilon^{-1}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) . \tag{2.12}$$

The log-likelihood of the difference between the *true* model and the candidate model is

$$\log(f_*) - \log(f) = \frac{n}{2}\log(\frac{|\boldsymbol{\Sigma}_\varepsilon|}{|\boldsymbol{\Sigma}_*|}) + \frac{1}{2}tr\left((\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'\boldsymbol{\Omega}_\varepsilon^{-1}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})\right) - \frac{1}{2}tr\left((\mathbf{y} - \mathbf{X}_*\boldsymbol{\beta}_*)'\boldsymbol{\Omega}_*^{-1}(\mathbf{y} - \mathbf{X}_*\boldsymbol{\beta}_*)\right). \tag{2.13}$$

Finally, the expectation of $\frac{2}{n}\left(\log(f_*) - \log(f)\right)$ with respect to the *true* model is

$$K - L = \log(\frac{|\boldsymbol{\Sigma}_\varepsilon|}{|\boldsymbol{\Sigma}_*|}) + tr\left(\boldsymbol{\Sigma}_\varepsilon^{-1}\boldsymbol{\Sigma}_*\right) + \frac{1}{n}tr\left((\mathbf{X}_*\boldsymbol{\beta}_* - \mathbf{X}\boldsymbol{\beta})'\boldsymbol{\Omega}_\varepsilon^{-1}(\mathbf{X}_*\boldsymbol{\beta}_* - \mathbf{X}\boldsymbol{\beta})\right) - m. \tag{2.14}$$

2.3.1 The Akaike Information Criterion.

The Akaike information criterion (AIC) is a model selection criterion derived from minimizing the expected estimate of $K - L$ information, and is defined as

$$AIC = -2\log(likelihood) + 2(number\ of\ \ parameter), \tag{2.15}$$

where $-2\log(likelihood) = nm\log(2\pi) + n\log(|\hat{\boldsymbol{\Sigma}}_\varepsilon|) + mn,$

m is the number of equations,

$|\hat{\boldsymbol{\Sigma}}_\varepsilon|$ is the determinant of the sample error covariance matrix, and

n is the number of observations.

The number of parameters is equal to $mp+\frac{1}{2}m(m+1)$, where mp is the number of parameters for $\boldsymbol{\beta}$ and $\frac{1}{2}m(m+1)$ is the number of parameters for the sample error covariance matrix $\hat{\boldsymbol{\Sigma}}_\varepsilon$.Therefore, the AIC from equation (2.15) is given as

$$AIC = nm\log(2\pi) + n\log(|\hat{\boldsymbol{\Sigma}}_\varepsilon|) + nm + 2mp + m(m+1)\,. \tag{2.16}$$

The constant $nm\log(2\pi)+nm$ can be ignored (McQuarries and Tsai, 1998: 147), and so the AIC becomes

$$AIC = n\log(|\hat{\boldsymbol{\Sigma}}_\varepsilon|) + 2mp + m(m+1)\,. \tag{2.17}$$

The definition of AIC for multivariate linear regression models is defined as

$$AIC_p = \log(|\hat{\boldsymbol{\Sigma}}_\varepsilon|) + \frac{2mp + m(m+1)}{n}\,. \tag{2.18}$$

2.3.2 Bayesian Information Criterion

Similar to AIC, the Bayesian information criterion (BIC) is a model selection criterion derived from minimizing the expected estimated of $K-L$ information. The AIC is evaluated from a summation of $-2\log(likelihood)$ and twice the number of parameters, while the BIC is evaluated using a summation of $-2\log(likelihood)$ and the log of the number of observations multiplied by the number of parameters, and so is defined as

$$\begin{aligned} BIC &= -2\log(likelihood) + \log(n)(number\ of\ parameter), \qquad (2.19)\\ &= nm\log(2\pi) + n\log(|\hat{\boldsymbol{\Sigma}}_\varepsilon|) + nm + \log(n)\left(mp + \frac{1}{2}m(m+1)\right). \end{aligned}$$

The constant $nm\log(2\pi)+nm$ can be ignored (McQuarries and Tsai, 1998: 147), and so the BIC becomes

$$BIC = n\log(|\hat{\boldsymbol{\Sigma}}_{\varepsilon}|) + \log(n)\left(mp + \frac{1}{2}m(m+1)\right), \tag{2.20}$$

where p, m, $|\hat{\boldsymbol{\Sigma}}_{\varepsilon}|$, and n are defined in the above.

2.3.3 Mallows's C_p for the Selection of Multivariate Linear Regression Models

By directly extending the definition of Mallows's C_p criterion in a single equation model to a multivariate regression model, Sparks et al. (1983: 1775-1793) defined C_p for variable selection in multivariate regression as follows:

$$\mathbf{SPC} = (n - p_f)\mathbf{S}_e^{-1}\mathbf{S}_h + (2p_r - p_f)\mathbf{I}_m , \tag{2.21}$$

where $\mathbf{S}_e = \mathbf{y}'(\mathbf{I} - \mathbf{X}_f(\mathbf{X}'_f\mathbf{X}_f)^{-1}\mathbf{X}'_f)\mathbf{y}$ is the sum of squared errors in the full model, $\mathbf{S}_h = \mathbf{y}'(\mathbf{X}_f(\mathbf{X}'_f\mathbf{X}_f)^{-1}\mathbf{X}'_f - \mathbf{X}_r(\mathbf{X}'_r\mathbf{X}_r)^{-1}\mathbf{X}'_r)\mathbf{y}$ is the difference between the sum of squared errors in the reduced model and those in the full model, and p_f and p_r are the number of parameters in each equation of the full model and reduced model, respectively. They suggested a modified C_p for variable selection in terms of the trace of $\mathbf{SPC}$, the determinant of $\mathbf{SPC}$, and the largest root of $\mathbf{SPC}$. However, the determinant of $\mathbf{SPC}$ is not appropriate because it may be negative.

Fujikoshi and Satoh (1997: 707-716) modified C_p in a multivariate linear regression model to be defined as

$$FSC = ntr(\mathbf{S}_{FS}^{-1}\hat{\boldsymbol{\Sigma}}_r) + 2mp_r - (m+1)tr\left(\hat{\boldsymbol{\Sigma}}_f^{-1}\left(\hat{\boldsymbol{\Sigma}}_r - \hat{\boldsymbol{\Sigma}}_f\right)\right), \tag{2.22}$$

where $\mathbf{S}_{FS} = \dfrac{n\hat{\mathbf{\Sigma}}_f}{(n-p_f)}$,

$\hat{\mathbf{\Sigma}}_r$ is the estimate of the error covariance matrix in the reduced model,

$\hat{\mathbf{\Sigma}}_f$ is the estimate of the error covariance matrix in the full model,

p_r is the number of parameters of the reduced model in equation $i, i = 1, 2, ..., m$,

p_f is the number of parameters of the full model in equation $i, i = 1, 2, ..., m,$,

m is the number of equations, and

n is the number of observations in each equation.

Yanagihara and Satoh (2010: 1226-1238) introduced an unbiased C_p criterion for multivariate ridge regression

$$MC_p(\theta) = (1 - \frac{m+1}{n-p})tr(\mathbf{W}_\theta \mathbf{S}^{-1}) + 2mtr(\mathbf{M}_\theta^{-1}\mathbf{M}_0) + m(m+1), \qquad (2.23)$$

where

$\mathbf{W}_\theta = (\mathbf{Y} - \hat{\mathbf{Y}}_\theta)'(\mathbf{Y} - \hat{\mathbf{Y}}_\theta)$, $\mathbf{S} = \dfrac{(\mathbf{Y} - \hat{\mathbf{Y}}_\theta)'(\mathbf{Y} - \hat{\mathbf{Y}}_\theta)}{(n-p)}$ is an unbiased estimator of $\mathbf{\Sigma}_\varepsilon$, $\mathbf{M}_\theta = (\mathbf{X}'\mathbf{X}) + \theta \mathbf{I}_k$,

θ is a non-negative value and referred to as a ridge parameter,

p is the number of parameters in each equation,

m is the number of equations, and

n is the number of observations.

Kubokawa and Sivastava (2012: 2465-2489) present a modified C_p statistic based on the ridge-type estimator of the covariance matrix under the asymptotic theory that both the sample size and the dimension of the covariance matrix go towards infinity:

$$C_{\lambda} = tr(\tilde{\boldsymbol{\Sigma}}_{\lambda}^{-1}\mathbf{S}) - \frac{nm(n-p_r-m-1)}{n-p_f-m-1} + mp_r + \tilde{\lambda}tr(\tilde{\boldsymbol{\Sigma}}^{-1}), \qquad (2.24)$$

where $\tilde{\boldsymbol{\Sigma}}_{\lambda} = \dfrac{\tilde{\mathbf{S}} + \tilde{\lambda}\mathbf{I}_p}{n}$ is the right type estimator of $\boldsymbol{\Sigma}_{\varepsilon}$,

$\mathbf{S} = \dfrac{(\mathbf{y}-\mathbf{X}\hat{\boldsymbol{\beta}})(\mathbf{y}-\mathbf{X}\hat{\boldsymbol{\beta}})'}{n}$ is the maximum likelihood estimators of $\boldsymbol{\Sigma}_{\varepsilon}$,

$\tilde{\mathbf{S}} = \dfrac{(\mathbf{y}-\tilde{\mathbf{X}}\tilde{\boldsymbol{\beta}})(\mathbf{y}-\tilde{\mathbf{X}}\tilde{\boldsymbol{\beta}})'}{n}$ is the maximum likelihood estimators of $\boldsymbol{\Sigma}_{\varepsilon}$ in the full model,

and $\tilde{\lambda} = c_n tr(\tilde{\mathbf{S}}/nm)$, for $c_n = 0(n^{-\delta}), \delta \geq 0$.

Nagai (2013: 73-106) proposed the generalized $C_p(GC_p)$ criterion by selecting the minimized predicting mean square error (PMSE) of the ridge regression after optimizing the ridge parameter, and can be defined as

$$GC_p(\theta,\lambda) = \lambda tr(\mathbf{W}_{\theta}\mathbf{S}^{-1}) + 2mtr(\mathbf{M}_{\theta}^{-1}\mathbf{M}_0), \qquad (2.25)$$

where λ is a non-negative parameter when considering the optimization of λ to obtain θ, and $\mathbf{W}_{\theta}, \mathbf{S}, \mathbf{M}_{\theta}, \textit{and } \theta$ are defined as in (2.23).

Vichit Lorchirachoonkul and Jirawan Jitthavech (2012: 2386-2394) modified Mallows's C_p for a system of equations model by changing the unknown error covariance matrix of the true model to the covariance matrix of the dependent variables. For a given set of dependent variables, the new weight (the covariance matrix of the dependent variables) is constant and independent of model specification. The criterion can be used for model selection in both nested and non-nested models, and is defined as

$$\begin{aligned} S\Gamma_p &= E\left((\hat{\mathbf{y}}-E(\mathbf{y}))'\,\boldsymbol{\Omega}_y^{-1}(\hat{\mathbf{y}}-E(\mathbf{y}))\right) \\ &= E(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}) - ntr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_{\varepsilon}) + 2tr\left(\mathbf{X}'\boldsymbol{\Omega}_y^{-1}\mathbf{X}\left(\mathbf{X}'\boldsymbol{\Omega}_{\varepsilon}^{-1}\mathbf{X}\right)^{-1}\right), \end{aligned} \qquad (2.26)$$

where $\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{H}\mathbf{y}$, $\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}}$, $\mathbf{H} = \mathbf{X}\left(\mathbf{X}'\boldsymbol{\Omega}_{\varepsilon}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\boldsymbol{\Omega}_{\varepsilon}^{-1}$,

$\hat{\boldsymbol{\beta}}$ is the generalized least squares estimate of $\boldsymbol{\beta}$,

$\boldsymbol{\Omega}_{\varepsilon}^{-1} = \boldsymbol{\Sigma}_{\varepsilon}^{-1} \otimes \mathbf{I}_n$, $\boldsymbol{\Sigma}_{\varepsilon}$ is the $m \times m$ contemporaneous disturbance covariance matrix and defined as in (2.4), and

$\boldsymbol{\Omega}_{y}^{-1} = \boldsymbol{\Sigma}_{y}^{-1} \otimes \mathbf{I}_n$, $\boldsymbol{\Sigma}_{y}$ is the $m \times m$ covariance matrix of dependent variables estimated directly by the covariance matrix of sampled dependent variables:

$$\hat{\boldsymbol{\Sigma}}_y = \begin{bmatrix} \hat{\sigma}_{y11} & \hat{\sigma}_{y12} & \cdots & \hat{\sigma}_{y1m} \\ \hat{\sigma}_{y21} & \hat{\sigma}_{y22} & \cdots & \hat{\sigma}_{y2m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{ym1} & \hat{\sigma}_{ym2} & \cdots & \hat{\sigma}_{ymm} \end{bmatrix}, \tag{2.27}$$

($\mathbf{y}, \mathbf{X}, m, and\, n$ are the same as in (2.1)).

Fujikoshi et al. (2014: 184-200) modified the C_p criterion for multivariate linear regression models where sample size n is large and have a consisting property under $\boldsymbol{\Omega} = \mathbf{0}(n)$ when $\frac{p}{n} \to [0,1)$. This modification can be expressed as

$$MC_p = C_p - (m+1)tr(\hat{\boldsymbol{\Sigma}}_f^{-1}(\hat{\boldsymbol{\Sigma}}_r - \hat{\boldsymbol{\Sigma}}_f)), \tag{2.28}$$

where $C_p = (n-p)tr(\hat{\boldsymbol{\Sigma}}_f^{-1}\hat{\boldsymbol{\Sigma}}_r) + 2mp_r$, $\hat{\boldsymbol{\Sigma}}_r = \frac{\mathbf{y}'(\mathbf{I}_n - \mathbf{P}_r)\mathbf{y}}{n}$ is the maximum likelihood estimator of $\boldsymbol{\Sigma}_{\varepsilon}$ of the reduced model, $\hat{\boldsymbol{\Sigma}}_f = \frac{\mathbf{y}'(\mathbf{I}_n - \mathbf{P}_f)\mathbf{y}}{n}$ is the maximum likelihood estimator of $\boldsymbol{\Sigma}_{\varepsilon}$ of the full model, $\mathbf{P}_r = \mathbf{X}_r(\mathbf{X}_r'\mathbf{X}_r)^{-1}\mathbf{X}_r'$, $\mathbf{P}_f = \mathbf{X}_f(\mathbf{X}_f'\mathbf{X}_f)^{-1}\mathbf{X}_f'$, $\mathbf{X}_r$ is the $n \times p_r$ independent variables matrix of the reduced model, and $\mathbf{X}_f$ is the $n \times p_f$ independent variables matrix of the full model.

2.3.4 R^2

The R^2 criterion in the system of equations suggested by McElroy (1977: 381-387) is defined as

$$R^2 = 1 - \frac{\mathbf{e}'\hat{\mathbf{\Omega}}_\varepsilon^{-1}\mathbf{e}}{\mathbf{y}'\hat{\mathbf{\Omega}}_\varepsilon^{-1}\mathbf{y}} , \tag{2.29}$$

where $\mathbf{y}$ is the $nm \times 1$ vector consisting of the subvector $\mathbf{y}_i$, the $n \times 1$ dependent variable vector in equation $i, i = 1, 2, ..., m,$

$\mathbf{e}$ is the $nm \times 1$ vectors of residuals in the model and distributed as $N_{nm}(\mathbf{0}, \mathbf{\Omega}_\varepsilon),$

$\hat{\mathbf{\Omega}}_\varepsilon = \hat{\mathbf{\Sigma}}_\varepsilon \otimes \mathbf{I}_n$, $\hat{\mathbf{\Sigma}}_\varepsilon$ is the estimated $m \times m$ cross-equation sample covariance matrix of errors, and $\mathbf{I}_n$ is the $n \times n$ identity matrix,

m is the number of equations in the model, and

n is the number of observations.

2.4 The Distribution of a Linear Combination of Gamma Random Variables

In this section, we consider the distribution of the sum of gamma random variables in the form

$$Y = X_1 + X_2 + ... + X_s, \tag{2.30}$$

where X_i, $i = 1, 2, ..., s$ are independent gamma random variables with X_i having density

$$f(x) = x^{\alpha_i - 1} \frac{e^{-x/\beta_i}}{\beta_i^{\alpha_i}\Gamma(\alpha_i)}, x > 0, \alpha_i > 0, \beta_i > 0. \tag{2.31}$$

Mathai (1982: 591-597) presented the distribution of the sum of independent gamma random variables with difference parameters in term of a confluent hypergeometric function as

$$f(y)=\left[\prod_{j=1}^{s}(-\beta_j)^{-\alpha_j}\right]\sum_{j=1}^{s}\sum_{r=1}^{\alpha_j}(-1)^r b_{jr} y^{r-1}\frac{e^{-y/\beta_j}}{(r-1)!},\ y>0 \qquad (2.32)$$

$$b_{jr}=\lim_{x\to 1/\beta_j}\left\{\frac{1}{(\alpha_j-r)!}\frac{\partial^{\alpha_j-r}}{\partial t^{\alpha_j-r}}\left[\left(t-\frac{1}{\beta_j}\right)^{\alpha_j}\prod_{i=1}^{s}\left(t-\frac{1}{\beta_i}\right)^{\alpha_i}\right]\right\}.$$

Moschopoulos (1985: 541-544) provided a formula for sum of independent gamma random variables with different parameters expressed as a single gamma-series as

$$f(y)=C\sum_{k=0}^{\infty}\delta_k\frac{y^{\alpha+k-1}e^{-y/\beta_1}}{\Gamma(\alpha+k)\beta_1^{\alpha+k}},\ y>0 \qquad (2.33)$$

where $\alpha=\sum_{i=1}^{s}\alpha_i$, $C=\prod_{i=1}^{s}\left(\frac{\beta_1}{\beta_i}\right)^{\alpha_i}$, $\delta_{k+1}=\frac{1}{k+1}\sum_{i=1}^{k+1}i\gamma_i\delta_{k+1-i}$, $k=0,1,2,\ldots$, $\delta_0=1$.

Efthymoglou and Aalo (1995: 1610-1612) derived a probability density function (pdf) of Y defined in (2.30) as

$$f(y)=\frac{1}{\pi}\int_0^{\infty}\frac{\cos\left(\sum_{k=1}^{s}\alpha_k\arctan(\beta_k t)-yt\right)}{\prod_{k=1}^{s}(1+t^2\beta_k^2)^{\alpha_k/2}}dt,\ y>0. \qquad (2.34)$$

Akkouchi (2005: 205-211) provided the pdf of random variable Y by means of the generalized beta function as

$$f(y)=Ct^{\alpha_1+...+\alpha_s-1}\int_0^1...\int_0^1 e^{-tC_{\beta_1,...,\beta_s}(u_1,...,u_{s-1})}B_{\alpha_1,...,\alpha_s}(u_1,...,u_{s-1})du_1...du_{s-1}, y>0. \quad (2.35)$$

where $C=\dfrac{\beta_1^{\alpha_1}\beta_2^{\alpha_2}...\beta_s^{\alpha_s}}{\Gamma(\alpha_1+\alpha_2+...+\alpha_s)}$,

$$C_{\beta_1,...,\beta_s}(u_1,...,u_{s-1}):=\beta_1\prod_{j=1}^{s-1}u_j+\sum_{i=2}^{s-1}\beta_i(1-u_i)\prod_{j=1}^{s-1}u_j+\beta_s(1-u_{s-1}),$$

$$B_{\alpha_1,...,\alpha_s}(u_1,...,u_{s-1}):=\frac{1}{\beta_s(\alpha_1,...,\alpha_s)}\prod_{j=1}^{s-1}u_j^{\alpha_1+...+\alpha_j-1}(1-u_j)^{\alpha_{j+1}-1}.$$

Aalo, Terawat Piboongungon and Efthymoglou (2005: 2002-2005) presented the pdf of random variable Y via the confluent Lauricella multivariate hypergeometric function as

$$f(y)=\frac{\beta_1^{-\alpha_1}...\beta_s^{-\alpha_s}y^{\sum_{k=1}^{s}\alpha_k-1}}{\Gamma(\sum_{k=1}^{s}\alpha_k)}\Phi_2^s\left(\alpha_1,...,\alpha_s;\sum_{k=1}^{s}\alpha_k;\frac{-y}{\beta_1},...,\frac{-y}{\beta_s}\right), y>0, \quad (2.36)$$

where $\Phi_2^s(...)$ denote the confluent Lauricella multivariate hypergeometric function.

Next, the sum of correlated gamma random variables is presented with reference to the linear combination of gamma random variables shown in (2.30). If X_i and X_j have correlation coefficient ρ_{ij}, $i\neq j$, and the pdf of X_i is distributed as in (2.31), then the distribution of a linear combination of the sum of correlated gamma random variables are given below.

Alouini, Abdi and Kaveh (2001: 1471-1480) proposed that the distribution sum of correlate gamma distribution and shape parameters are equal, as given by

$$f(y)=\prod_{j=1}^{s}\left(\frac{\lambda_1}{\lambda_j}\right)^{\alpha}\sum_{k=0}^{\infty}\delta_k\frac{y^{s\alpha+k-1}e^{-y/\lambda_1}}{\lambda_1^{s\alpha+k}\Gamma(s\alpha+k)}, y>0, \quad (2.37)$$

where $\lambda_1 = \min \lambda_j$, $\{\lambda_j\}$ are the eigenvalues of matrix $\mathbf{A} = \mathbf{DC}$, $\mathbf{D}$ is the $s \times s$ diagonal matrix with entries $\{\beta_j\}$, and C is the $s \times s$ positive definite matrix

$$\mathbf{C} = \begin{bmatrix} 1 & \sqrt{\rho_{12}} \cdots \sqrt{\rho_{1s}} \\ \sqrt{\rho_{21}} & 1 \quad \cdots \sqrt{\rho_{2s}} \\ \vdots & \cdots \quad \vdots \\ \sqrt{\rho_{s1}} & \sqrt{\rho_{s2}} \cdots \quad 1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \beta_1 & 0 \cdots 0 \\ 0 & \beta_2 \cdots 0 \\ \vdots & \\ 0 & 0 \cdots \beta_s \end{bmatrix},$$

$$\delta_{k+1} = \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j=1}^{s} \left(1 - \frac{\lambda_1}{\lambda_j} \right)^i \right] \delta_{k+1-i}, \delta_0 = 1.$$

Holm and Alouini (2004: 1367-1376) presented the pdf of the sum of two correlate gamma variables, $X = X_1 + X_2$, following a type I McKay distribution and pdf as

$$f(x) = \frac{\sqrt{\pi}}{\Gamma(\alpha)} \left(\frac{x}{2\tilde{\beta}} \right)^{\alpha - \frac{1}{2}} \left[\frac{1}{\beta_1 \beta_2 (1 - \rho_{x_{12}})} \right]^{\alpha} e^{-\frac{x(\beta_1 + \beta_2)}{2\beta_1\beta_2(1-\rho_{x_{12}})}} I_{\alpha - \frac{1}{2}}(x\tilde{\beta}), \qquad (2.38)$$

where $\tilde{\beta}^2 = \dfrac{(\beta_1 - \beta_2)^2 + 4\beta_1\beta_2\rho_{x_{12}}}{4\beta_1^2\beta_2^2(1-\rho_{x_{12}})^2}$, $X_1 \sim \Gamma(\alpha, \beta_1)$, $X_2 \sim \Gamma(\alpha, \beta_2)$, and $\rho_{x_{12}}$ is correlation coefficient between X_1 and X_2.

Zhang (2005: 273-285) approximate the weighted chi-squared distribution $\sum_{i=1}^{q} c_i A_i, A_i \sim \chi_1^2(\mu_i^2)$ of the form $\alpha\chi_d^2 + \beta$ via matching the first three cumulants where c_i, $i = 1,2,...,q$, are the nonzero real coefficients and μ_i^2 is the noncentral parameter of variate A_i.

Yuan and Bentler (2010: 273-291) proposed the distribution of the quadratic forms $T = \mathbf{q}'\mathbf{V}\mathbf{q}$, $\mathbf{q} \sim N(\mathbf{0}, \mathbf{\Gamma})$, which are equivalent to $T = \sum_{i=1}^{d} g_i t_i^2$ with t_i being independent and following $N(0,1)$. The matrix $\mathbf{\Gamma}$ is full rank while $\mathbf{V}$ is nonegative definite, and $\{g_i\}_{i=1}^{d}$ are the eigenvalues of $\mathbf{V\Gamma}$. The first approximation of T is $T \sim c\chi_d^2$ where $c = \sum_{i=1}^{d} g_i / d$ perform well when d is large. Otherwise, the second approximation of the distribution of T is given by $a\chi_b^2$ where a and b are determined by matching the first two moments of T; $a\chi_b^2$ perform well when λ_i's are small.

Zakaria, Boland and Moslim (2013: 408-414) showed that the Mckay distribution in (2.38) gives a better fit to the sum of two correlated gamma variables than Alouini et al. (2001: 1471-1480) performance criteria in (2.37) for the sum of more than two correlated gamma variables.

Chuang and Shih (2012: 457-472) proposed the weighted sum of correlated chi-squared random variables with two degrees of freedom; that is,

$$W = \sum_{i=1}^{r} a_i K_i \,, \tag{2.39}$$

where K_i's are correlated chi-squared variables with 2 degrees of freedom and a_i's are positive constant approximated by $R \sim \gamma\chi_f^2$, where γ and f are chosen so that the first two cumulants of the distribution are equal, as follows:

$$\gamma = \frac{4\sum_{i=1}^{r} a_i^2 + 8\sum_{i<j}^{r} \rho_{ij} a_i a_j}{4\sum_{i=1}^{r} a_i}, \text{and } f = \frac{8\left(\sum_{i=1}^{r} a_i\right)^2}{4\sum_{i=1}^{r} a_i^2 + 8\sum_{i<j}^{r} \rho_{ij} a_i a_j}.$$

In addition, in a review of the literature on the distribution of the difference between two independent gamma variates, we define

$$U = X_1 - X_2, \tag{2.40}$$

where X_i, $i=1,2$ are independent gamma random variables with X_i having density in (2.31). Following this, the possible distributions of difference between two independent gamma variables are given below.

Holm and Alouini (2004: 1367-1376) presented the difference of two independent gamma distributions U in Mckay's form with the pdf given by

$$f(u)=\frac{(1-c^2)^{a+\frac{1}{2}}|u|^a}{\sqrt{\pi}2^a b^{a+1}\Gamma(a+\frac{1}{2})}e^{-x\frac{c}{b}}K_a\left(\frac{|u|}{b}\right), u\neq 0 \qquad (2.41)$$

where $a=\alpha-\frac{1}{2}$, $b=\frac{2\beta_1\beta_2}{(\beta_1+\beta_2)}$, $c=-\frac{(\beta_1-\beta_2)}{(\beta_1+\beta_2)}$, and $K_a(.)$ is the modified Bessel function of the second kind and of order a.

Kuchler and Tappe (2008: 2478-2484) proposed the distribution of an independent gamma distribution in the form of a bilateral gamma distribution, the pdf being given by

$$f(u)=\frac{(\beta_1)^{\alpha_1}(\beta_2)^{\alpha_2}}{(\beta_1+\beta_2)^{\frac{1}{2}(\alpha_1+\alpha_2)}\Gamma(\alpha_1)}u^{\frac{1}{2}(\alpha_1+\alpha_2)-1}e^{-\frac{u}{2}(\beta_1+\beta_2)}W_{\frac{1}{2}(\alpha_1+\alpha_2),\frac{1}{2}(\alpha_1+\alpha_2-1)}(x(\beta_1+\beta_2)),$$

(2.42)

where the Whittaker function $W_{\lambda,\mu}(u)$ is written as

$$W_{\lambda,\mu}(u)=\frac{\Gamma(-2\mu)}{\Gamma(\frac{1}{2}-\mu-\lambda)}M_{\lambda,\mu}(u)+\frac{\Gamma(2\mu)}{\Gamma(\frac{1}{2}+\mu-\lambda)}M_{\lambda,-\mu}(u).$$

Klar (2015: 3708-3715) proposed the difference between two independent gamma distribution in Mckay's form as

$$f(u)=\begin{cases} ce^{\beta_2 u}\int u^{\alpha_1-1}(t-u)^{\alpha_2-1}e^{-(\beta_1+\beta_2)t}dt, & u>0 \\ ce^{\beta_1 u}\int u^{\alpha_2-1}(t+u)^{\alpha_1-1}e^{-(\beta_1+\beta_2)t}dt, & u<0 \end{cases}, \qquad (2.43)$$

where $c = \dfrac{\beta_1^{\alpha_1}\beta_2^{\alpha_2}}{\Gamma(\alpha_1)\Gamma(\alpha_2)}$.

2.5 Testing of the Criteria for Model Selection

In developing Mallows's C_p to test the hypotheses on a linear regression model, Mallows (1995: 362-372) proposed a test statistic for testing $H_0 : \beta_{p+1} = 0$.

$$C_p - C_{p+1} = 2 + \frac{(SSE_p - SSE_{p+1})}{\hat{\sigma}^2}, \tag{2.44}$$

where $\dfrac{(SSE_p - SSE_{p+1})}{\hat{\sigma}^2} \sim t^2$.

The $p+1^{st}$ variable included at level α if

$$\frac{(SSE_p - SSE_{p+1})}{\hat{\sigma}^2} > t_\alpha^2, \tag{2.45}$$

where t_α is the significance point of t with $p+1$ the appropriate number of degrees of freedom, SSE_{p+1} is the sum of squared errors for a model with $p+1$ regressors, and SSE_p is the sum of squared errors for a model with p regressors.

Gilmour (1996: 49-56), and Olive and Hawkins (2005: 43-50), presented Mallows's C_p in a linear regression model for testing $H_0 : \Gamma_r = \Gamma_f$; the null hypothesis is rejected if

$$C_p > (p_f - p_r + 1)F + 2p_r - p_f - 1, \tag{2.46}$$

where $F \sim F_{p_f - p_r + 1, n - p_f - 1}$,

p_r is the number of parameters in the reduced model, and

p_f is the number of parameters in the full model.

In a multivariate linear regression model, Sparks et al. (1983: 1775-1793) proposed a model selection using the C_p criterion in testing the hypothesis of $H_0 : SP\Gamma_r = SP\Gamma_f$; the null hypothesis is rejected if

$$tr(SPC_r) > \chi^2_{p_f m}(\delta) + (2p_r - p_f)m, \tag{2.47}$$

where $\delta = (n-p)tr(S_e^{-1}S_h)$, S_e^{-1}, S_h, p_f, p_r, $n\ and\ m$ are previously defined as in (2.21).

CHAPTER 3

A PROPOSED TEST STATISTIC FOR MODEL SELECTION

In this section, the focus is on a multivariate linear regression models of m equations when the random disturbance are uncorrelated across observations in the same equation but contemporaneously correlated across equations. The general multivariate regression models can be written in a vector form as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{3.1}$$

where $\mathbf{y}$ is the $nm \times 1$ vector consisting of the subvector $\mathbf{y}_i$, the $n \times 1$ dependent variable vector in equation $i, i = 1, 2, ..., m,$

$\mathbf{X}$ is the $nm \times mp$ diagonal matrix consisting of diagonal submatrices $\mathbf{X}_c$, the $n \times p$ matrix of independent variables including a constant unit vector in equation $i, i = 1, 2, ..., m,$

$\boldsymbol{\beta}$ is the $mp \times 1$ parametric vector consisting of the subvector $\boldsymbol{\beta}_i$, the $p \times 1$ parameter vector in equation $i, i = 1, 2, ..., m,$

$\boldsymbol{\varepsilon}$ is an $nm \times 1$ random disturbance vectors having a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Omega}_\varepsilon = \boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_n$, where $\boldsymbol{\Sigma}_\varepsilon$ is the $m \times m$ random disturbance covariance matrix, and $\mathbf{I}_n$ is the $n \times n$ identity matrix,

p is the number of parameters in equation $i, i = 1, 2, ..., m,$

m is the number of equations, and

n is the number of observations in each equation.

In this study, the full model of a multivariate linear regression model is assumed to be an over-fit model of the true unknown model but is a good approximation of the true model. It can be expressed in the form

$$\mathbf{y}_f = \mathbf{X}_f \boldsymbol{\beta}_f + \boldsymbol{\varepsilon}_f \,, \tag{3.2}$$

where $\mathbf{y}_f$ is the $nm \times 1$ vector consisting of the subvector $\mathbf{y}_{f_i}$, the $n \times 1$ dependent variables vector of the full model in equation $i, i = 1, 2, ..., m,$

$\mathbf{X}_f$ is the $nm \times mp_f$ diagonal matrix consisting of diagonal submatrices $\mathbf{X}_{f_i}$, the $n \times p_f$ matrix of independent variables including a constant unit vector of the full model in equation $i, i = 1, 2, ..., m,$

$\boldsymbol{\beta}_f$ is the $mp_f \times 1$ parameter vector consisting of the subvector $\boldsymbol{\beta}_{f_i}$, the $p_f \times 1$ parameter vector of the full model in equation $i, i = 1, 2, ..., m,$

$\boldsymbol{\varepsilon}_f$ is the $nm \times 1$ random disturbance vectors having a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Omega}_f$, and

p_f is the number of parameters of the full model in equation $i, i = 1, 2, ..., m.$

When some of the independent variables are omitted from the full model, the latter becomes a reduced model, which can be similarly written as

$$\mathbf{y}_r = \mathbf{X}_r \boldsymbol{\beta}_r + \boldsymbol{\varepsilon}_r \,, \tag{3.3}$$

where $\mathbf{y}_r$ is the $nm \times 1$ vector consisting of the subvector $\mathbf{y}_{r_i}$, the $n \times 1$ dependent variables vector of the reduced model in equation $i, i = 1, 2, ..., m,$

$\mathbf{X}_r$ is the $nm \times mp_r$ diagonal matrix consisting of diagonal submatrices $\mathbf{X}_{r_i}$, the $n \times p_r$ matrix of independent variables including a constant unit vector of the reduced model in equation $i, i = 1, 2, ..., m,$

$\boldsymbol{\beta}_r$ is the $mp_r \times 1$ parameter vector consisting of the subvector $\boldsymbol{\beta}_{r_i}$, the $p_r \times 1$ parameter vector of the reduced model in equation $i, i = 1, 2, ..., m,$

$\boldsymbol{\varepsilon}_r$ is the $nm \times 1$ random disturbance vectors having a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Omega}_r$, and

p_r is the number of parameters of the reduced model in equation $i, i = 1, 2, \ldots, m.$

3.1 The Proposed Test Statistic

According to the 14th century philosopher William of Ockam, Ockham's Razor states that, *"More things should not be used than are necessary*". With this in mind, the number of the independent variables in a linear multivariate model should therefore be as small as necessary to explain the variations of dependent variables. The systematic selection of variables in multivariate linear regression models is one of the most studied topics in the statistical academic community. In this study, a statistic based on the modified C_p derived in the system of equations proposed by Vichit Lorchirachoonkul and Jirawan Jitthavech (2012: 2386-2394) is developed for variable selection in a group of multivariate linear regression models and the distribution of the proposed test statistic is also derived.

It can be easily shown that, in a multivariate linear regression model, the statistic $S\Gamma_p$ in (2.26) can be written as

$$M\Gamma = E\left((\hat{\mathbf{y}} - E(\mathbf{y}))' \, \mathbf{\Omega}_y^{-1} \left(\hat{\mathbf{y}} - E(\mathbf{y}) \right) \right). \tag{3.4}$$

In the remainder of Chapter 3, the statistical properties of $M\Gamma$ in (3.4) are studied in detail and a test statistic is also derived for use in variable selection in multivariate linear regression models.

Theorem 1. For multivariate linear regression models given in (3.1), the statistic $M\Gamma$ can be expressed as

$$M\Gamma = E\left(\mathbf{e}' \mathbf{\Omega}_y^{-1} \mathbf{e} \right) - (n - 2p) tr(\mathbf{\Sigma}_y^{-1} \mathbf{\Sigma}_\varepsilon), \tag{3.5}$$

where $\mathbf{e}$ is the $nm \times 1$ vectors of residuals in the model and distributed as $N_{nm}(\mathbf{0}, \mathbf{\Omega}_\varepsilon),$

$\mathbf{\Omega}_y^{-1} = \mathbf{\Sigma}_y^{-1} \otimes \mathbf{I}_n$, where $\mathbf{\Sigma}_y$ is the $m \times m$ covariance matrix of dependent variables,

$\mathbf{\Sigma}_\varepsilon$ is the $m \times m$ contemporaneous disturbance covariance matrix,

p is the number of parameters in equation $i, i = 1, 2, ..., m$,

m is the number of equations, and

n is the number of observations in each equation.

Proof.

$$M\Gamma = E\left(\left(\hat{\mathbf{y}} - E(\mathbf{y})\right)' \mathbf{\Omega}_y^{-1} \left(\hat{\mathbf{y}} - E(\mathbf{y})\right)\right)$$

$$= E\left(\left(\mathbf{Hy} - E(\mathbf{y})\right)' \mathbf{\Omega}_y^{-1} \left(\mathbf{Hy} - E(\mathbf{y})\right)\right), \qquad (3.6)$$

where

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{\Omega}_\varepsilon^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{\Omega}_\varepsilon^{-1} = (\mathbf{I}_m \otimes \mathbf{X}_c(\mathbf{X}_c'\mathbf{X}_c)^{-1}\mathbf{X}_c'),$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_c & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_c & \dots & \mathbf{0} \\ & \vdots & & \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{X}_c \end{bmatrix},$$

$\mathbf{X}_c$ is the $n \times p$ matrix of independent variables including a constant unit vector in equation $i, i = 1, 2, ..., m$.

Expanding the right hand side of (3.6) yields

$$M\Gamma = E\left(\left(\mathbf{Hy} - E(\mathbf{y})\right)' \mathbf{\Omega}_y^{-1} \left(\mathbf{Hy} - E(\mathbf{y})\right)\right)$$

$$= E\left(\mathbf{y}'\mathbf{H}'\mathbf{\Omega}_y^{-1}\mathbf{Hy} - \mathbf{y}'\mathbf{H}'\mathbf{\Omega}_y^{-1}E(\mathbf{y}) - E(\mathbf{y})'\mathbf{\Omega}_y^{-1}\mathbf{Hy} + E(\mathbf{y})'\mathbf{\Omega}_y^{-1}E(\mathbf{y})\right)$$

$$= E\left(\mathbf{y}'\mathbf{H}'\mathbf{\Omega}_y^{-1}\mathbf{Hy}\right) - E(\mathbf{y}')\mathbf{H}'\mathbf{\Omega}_y^{-1}E(\mathbf{y}) - E(\mathbf{y}')\mathbf{\Omega}_y^{-1}\mathbf{Hy}E(\mathbf{y}) + E(\mathbf{y}')\mathbf{\Omega}_y^{-1}E(\mathbf{y})$$

$$= tr(\mathbf{H}'\mathbf{\Omega}_y^{-1}\mathbf{H}\mathbf{\Omega}_\varepsilon) + E(\mathbf{y}')\mathbf{H}'\mathbf{\Omega}_y^{-1}\mathbf{H}E(\mathbf{y}) - E(\mathbf{y}')\mathbf{H}'\mathbf{\Omega}_y^{-1}E(\mathbf{y})$$

$$-E(\mathbf{y}')\mathbf{\Omega}_y^{-1}\mathbf{Hy}E(\mathbf{y}) + E(\mathbf{y}')\mathbf{\Omega}_y^{-1}E(\mathbf{y})$$

$$= tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon) + E(\mathbf{y}')\left(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\mathbf{H} - \mathbf{H}'\boldsymbol{\Omega}_y^{-1} - \boldsymbol{\Omega}_y^{-1}\mathbf{H} + \boldsymbol{\Omega}_y^{-1}\right)E(\mathbf{y})$$

$$= tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon) + E(\mathbf{y}')\left((\mathbf{I}_{nm} - \mathbf{H})'\boldsymbol{\Omega}_y^{-1}(\mathbf{I}_{nm} - \mathbf{H})\right)E(\mathbf{y})$$

$$= tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon) + E\left(\mathbf{y}'(\mathbf{I}_{nm} - \mathbf{H})'\boldsymbol{\Omega}_y^{-1}(\mathbf{I}_{nm} - \mathbf{H})\mathbf{y}\right)$$

$$- tr\left((\mathbf{I}_{nm} - \mathbf{H})'\boldsymbol{\Omega}_y^{-1}(\mathbf{I}_{nm} - \mathbf{H})\boldsymbol{\Omega}_\varepsilon\right)$$

$$= tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon) + E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - tr\left((\mathbf{I}_{nm} - \mathbf{H})'\boldsymbol{\Omega}_y^{-1}(\mathbf{I}_{nm} - \mathbf{H})\boldsymbol{\Omega}_\varepsilon\right)$$

$$= tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon) + E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - tr(\boldsymbol{\Omega}_y^{-1}\boldsymbol{\Omega}_\varepsilon) + tr(\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon)$$

$$+ tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\boldsymbol{\Omega}_\varepsilon) - tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon)$$

$$= E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - tr(\boldsymbol{\Omega}_y^{-1}\boldsymbol{\Omega}_\varepsilon) + tr(\boldsymbol{\Omega}_y^{-1}\mathbf{H}\boldsymbol{\Omega}_\varepsilon) + tr(\mathbf{H}'\boldsymbol{\Omega}_y^{-1}\boldsymbol{\Omega}_\varepsilon)$$

$$= E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - tr\left((\boldsymbol{\Sigma}_y^{-1} \otimes \mathbf{I}_n)(\boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_n)\right)$$

$$+ tr\left((\boldsymbol{\Sigma}_y^{-1} \otimes \mathbf{I}_n)(\mathbf{I}_m \otimes \mathbf{X}_c(\mathbf{X}_c'\mathbf{X}_c)^{-1}\mathbf{X}_c')(\boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_n)\right)$$

$$+ tr\left((\mathbf{I}_m \otimes \mathbf{X}_c(\mathbf{X}_c'\mathbf{X}_c)^{-1}\mathbf{X}_c')(\boldsymbol{\Sigma}_y^{-1} \otimes \mathbf{I}_n)(\boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_n)\right)$$

$$= E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - tr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_n) + tr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{X}_c(\mathbf{X}_c'\mathbf{X}_c)^{-1}\mathbf{X}_c')$$

$$+ tr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{X}_c(\mathbf{X}_c'\mathbf{X}_c)^{-1}\mathbf{X}_c')$$

$$= E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - ntr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon) + 2tr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{X}_c(\mathbf{X}_c'\mathbf{X}_c)^{-1}\mathbf{X}_c')$$

$$= E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - ntr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon) + 2\left(tr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon) \times tr(\mathbf{X}_c(\mathbf{X}_c'\mathbf{X}_c)^{-1}\mathbf{X}_c')\right)$$

$$= E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - ntr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon) + 2ptr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon)$$

$$= E\left(\mathbf{e}'\boldsymbol{\Omega}_y^{-1}\mathbf{e}\right) - (n - 2p)tr(\boldsymbol{\Sigma}_y^{-1}\boldsymbol{\Sigma}_\varepsilon) . \qquad \square$$

After this, from Theorem 1, the estimate of the modified C_p in a multivariate linear regression model $M\Gamma$ can be expressed as

$$MC = \mathbf{e}'\hat{\mathbf{\Omega}}_y^{-1}\mathbf{e} - (n-2p)tr(\hat{\mathbf{\Sigma}}_y^{-1}\hat{\mathbf{\Sigma}}_\varepsilon), \tag{3.7}$$

where $\mathbf{e}$ is the $nm \times 1$ vector of residuals in the model and distributed as $N_{nm}(\mathbf{0}, \mathbf{\Omega}_\varepsilon)$,

$\hat{\mathbf{\Omega}}_y^{-1} = \hat{\mathbf{\Sigma}}_y^{-1} \otimes \mathbf{I}_n$, $\hat{\mathbf{\Sigma}}_y$ is the $m \times m$ covariance matrix of sampled dependent variables estimated directly by the covariance matrix of sampled dependent variables,

$\hat{\mathbf{\Sigma}}_\varepsilon$ is the $m \times m$ sample error covariance matrix of the model,

p is the number of parameters in equation $i, i = 1, 2, ..., m$, and

n is the number of observation in each equation.

Let p_f be the number of parameters in the full model and assume that the estimated covariance matrix of the residuals in the full model $\mathbf{S}_{e_f}$ is a good approximation of $\mathbf{\Sigma}_\varepsilon$. From (3.7), the estimate of the modified C_p in the full model can be expressed as

$$MC_f = \mathbf{e}'_f\hat{\mathbf{\Omega}}_y^{-1}\mathbf{e}_f - (n-2p_f)tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f}), \tag{3.8}$$

where $\mathbf{e}_f$ is the $nm \times 1$ vector of residuals in the full model and distributed as $N_{nm}(\mathbf{0}, \mathbf{\Omega}_f)$,

$\mathbf{S}_{e_f}$ is the $m \times m$ sample error covariance matrix of the full model, and

p_f is the number of parameters of the full model in equation $i, i = 1, 2, ..., m$.

Similarly, the estimate of the modified C_p in the reduced model is given by

$$MC_r = \mathbf{e}'_r\hat{\mathbf{\Omega}}_y^{-1}\mathbf{e}_r - (n-2p_r)tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f}), \tag{3.9}$$

where $\mathbf{e}_r$ is the $nm \times 1$ vector of residuals in the reduced model and distributed as $N_{nm}(0, \mathbf{\Omega}_r)$, and

p_r is the number of parameters of the reduced model in equation i, $i = 1, 2, ..., m$.

In any reduced model, the number of parameters p_r is less than the number of parameters in the full model, i.e. $p_r < p_f$.

When selecting variables in the multivariate linear regression model in (3.1) under the $M\Gamma$ criterion, an explanatory variable can be eliminated from the model specification if discarding it does not significantly increase the value of the statistic, otherwise it is retained in the model. In other words, a variable or a number of variables can be eliminated from the model specification if the null hypothesis is not rejected at a significance level α.

$$H_0 : M\Gamma_r = M\Gamma_f \text{ against } H_A : M\Gamma_r > M\Gamma_f, \tag{3.10}$$

where $M\Gamma_f$ and $M\Gamma_r$ are the values of the modified C_p statistic of the full model and reduced model, respectively. In order to test the hypothesis (3.10), it is sufficient to consider whether the difference

$$D = MC_r - MC_f,$$

is significant from zero. From (3.8) and (3.9), it is obvious that the difference D can be expressed as

$$D = MC_r - MC_f = (\mathbf{e}_r - \mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r - \mathbf{e}_f) - 2dtr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f}), \tag{3.11}$$

where d is the difference in the number of parameters between in the full and reduced models, $d = p_f - p_r$, and the sample variance of $\mathbf{e}_f$, $\mathbf{S}_{ef}$, may be capable of replacing $\mathbf{\Sigma}_\varepsilon$ provided that the full model is a “good” approximation of the unknown “true” model.

With the concept of variable selection in multivariate linear regression models, the null hypothesis would be rejected when the different between the modified C_p in the full model and the reduced model is high.

The D statistic in (3.11) consists of two terms, the first term being of quadratic form and the second term the trace of a matrix. The first term of the statistic D, $(\mathbf{e}_r-\mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r-\mathbf{e}_f)$, is distributed as central chi-squared if and only if $\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}$ is idempotent (Searle, 1971: 57; Graybill, 1976: 135 and Rencher, 2000: 103), where $(\mathbf{e}_r-\mathbf{e}_f)\sim N_{nm}(\mathbf{0},\mathbf{\Omega}_{rf})$. In this case, the above well-known conclusion cannot be applied because $\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}$ is not idempotent, as shown in Appendix B.

The first term in the quadratic form can be expressed in terms of a linear combination of the products of two correlated normal vectors as

$$(\mathbf{e}_r-\mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r-\mathbf{e}_f)=\left[(\mathbf{e}_{r_1}-\mathbf{e}_{f_1})' \quad (\mathbf{e}_{r_2}-\mathbf{e}_{f_2})'\cdots(\mathbf{e}_{r_m}-\mathbf{e}_{f_m})'\right]\begin{bmatrix}\hat{\sigma}_y^{11}\mathbf{I}_n & \hat{\sigma}_y^{12}\mathbf{I}_n & \cdots & \hat{\sigma}_y^{1m}\mathbf{I}_n\\ \hat{\sigma}_y^{21}\mathbf{I}_n & \hat{\sigma}_y^{22}\mathbf{I}_n & \cdots & \hat{\sigma}_y^{2m}\mathbf{I}_n\\ \vdots & \vdots & \ddots & \vdots\\ \hat{\sigma}_y^{m1}\mathbf{I}_n & \hat{\sigma}_y^{m2}\mathbf{I}_n & \cdots & \hat{\sigma}_y^{mm}\mathbf{I}_n\end{bmatrix}\begin{bmatrix}(\mathbf{e}_{r_1}-\mathbf{e}_{f_1})\\(\mathbf{e}_{r_2}-\mathbf{e}_{f_2})\\ \vdots\\(\mathbf{e}_{r_m}-\mathbf{e}_{f_m})\end{bmatrix}$$

$$=\left[\sum_{i=1}^{m}\hat{\sigma}_y^{i1}(\mathbf{e}_{r_i}-\mathbf{e}_{f_i})'\mathbf{I}_n \quad \sum_{i=1}^{m}\hat{\sigma}_y^{i2}(\mathbf{e}_{r_i}-\mathbf{e}_{f_i})'\mathbf{I}_n \quad \cdots \quad \sum_{i=1}^{m}\hat{\sigma}_y^{im}(\mathbf{e}_{r_i}-\mathbf{e}_{f_i})'\mathbf{I}_n\right]\begin{bmatrix}(\mathbf{e}_{r_1}-\mathbf{e}_{f_1})\\(\mathbf{e}_{r_2}-\mathbf{e}_{f_2})\\ \vdots\\(\mathbf{e}_{r_m}-\mathbf{e}_{f_m})\end{bmatrix}$$

$$=\left[\sum_{i=1}^{m}\hat{\sigma}_y^{i1}(\mathbf{e}_{r_i}-\mathbf{e}_{f_i})'(\mathbf{e}_{r_1}-\mathbf{e}_{f_1})+\sum_{i=1}^{m}\hat{\sigma}_y^{i2}(\mathbf{e}_{r_i}-\mathbf{e}_{f_i})'(\mathbf{e}_{r_2}-\mathbf{e}_{f_2})+\cdots+\sum_{i=1}^{m}\hat{\sigma}_y^{im}(\mathbf{e}_{r_i}-\mathbf{e}_{f_i})'(\mathbf{e}_{r_m}-\mathbf{e}_{f_m})\right]$$

$$=\sum_{i=1}^{m}\sum_{j=1}^{m}\hat{\sigma}_y^{ij}(\mathbf{e}_{r_i}-\mathbf{e}_{f_i})'(\mathbf{e}_{r_j}-\mathbf{e}_{f_j})$$

$$=\sum_{i=1}^{m}\sum_{j=1}^{m}\hat{\sigma}_y^{ij}\mathbf{d}_i'\mathbf{d}_j, \qquad (3.12)$$

where $\mathbf{d}_i=\mathbf{e}_{r_i}-\mathbf{e}_{f_i}$ and $\mathbf{d}_j=\mathbf{e}_{r_j}-\mathbf{e}_{f_j}$.

However, the result in equation (3.12) is in terms of a linear combination of two correlated variables with the same probability distribution. Next, the $\mathbf{d}_j$ vector is replaced by a new vector with the same probability distribution of $\mathbf{d}_j$ but expressed in terms of $\mathbf{d}_i$.

Lemma 1. Let $\mathbf{d}_i$ and $\mathbf{d}_j$ be the $n \times 1$ correlated vectors distributed as $N_n(\mathbf{0}, \sigma_{d_i}^2 \mathbf{I}_n)$ and $N_n(\mathbf{0}, \sigma_{d_j}^2 \mathbf{I}_n)$ respectively, $\rho_{d_{ij}}$ be a correlation coefficient between $\mathbf{d}_i$ and $\mathbf{d}_j$, $\mathbf{z}_{d_i}$ be an $n \times 1$ vector distributed as $N_n(\mathbf{0}, \sigma_{d_i}^2 \mathbf{I}_n)$, independent of $\mathbf{d}_i$, and $\tilde{\mathbf{d}}_j$ be defined as

$$\tilde{\mathbf{d}}_j = \rho_{d_{ij}} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{d}_i + \sqrt{(1 - \rho_{d_{ij}}^2)} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{z}_{d_i}. \tag{3.13}$$

Then $\mathbf{d}_j$ and $\tilde{\mathbf{d}}_j$ are identically distributed as $N_n(\mathbf{0}, \sigma_{d_j}^2 \mathbf{I}_n)$.

Proof. Since $\mathbf{d}_i$ and $\mathbf{z}_{d_i}$ are independent normal random vectors with the same zero mean and variance $\sigma_{d_i}^2 \mathbf{I}_n$, the expectation of $\tilde{\mathbf{d}}_j$ is equal to zero, that is

$$\begin{aligned} E(\tilde{\mathbf{d}}_j) &= E\left(\rho_{d_{ij}} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{d}_i + \sqrt{(1 - \rho_{d_{ij}}^2)} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{z}_{d_i} \right) \\ &= \mathbf{0}, \end{aligned}$$

and the variance of $\tilde{\mathbf{d}}_j$ can be expressed as

$$Var(\tilde{\mathbf{d}}_j) = Var\left(\rho_{d_{ij}} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{d}_i + \sqrt{(1 - \rho_{d_{ij}}^2)} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{z}_{d_i} \right)$$

$$= \rho_{d_{ij}}^2 \frac{\sigma_{d_j}^2}{\sigma_{d_i}^2} Var(\mathbf{d}_i) + (1-\rho_{d_{ij}}^2)\frac{\sigma_{d_j}^2}{\sigma_{d_i}^2} Var(\mathbf{z}_{d_i}). \tag{3.14}$$

By definition, $Var(\mathbf{z}_{d_i})$ and $Var(\mathbf{d}_i)$ are both equal to $\sigma_{d_i}^2 \mathbf{I}_n$. Then $Var(\tilde{\mathbf{d}}_j)$ becomes

$Var(\tilde{\mathbf{d}}_j) = \sigma_{d_j}^2 \mathbf{I}_n$.

Therefore, $\tilde{\mathbf{d}}_j$ is distributed as $N_n(\mathbf{0}, \sigma_{d_j}^2 \mathbf{I}_n)$, which is identical to $\mathbf{d}_j$. □

Lemma 2. $(\mathbf{e}_r - \mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r - \mathbf{e}_f) = \sum_{i=1}^{m} w_{1i} \frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$, (3.15)

where $\hat{\mathbf{\Omega}}_y^{-1}$, $\mathbf{e}_f$, $\mathbf{e}_r$ are defined in 3.7, 3.8, 3.9 respectively, $w_{1i} = \sum_{j=1}^{m} \hat{\sigma}_y^{ij} \rho_{d_{ij}} \sigma_{d_i} \sigma_{d_j}$, $\hat{\sigma}_y^{ij}$ is the ij^{th} element of $\hat{\mathbf{\Sigma}}_y^{-1}$, and $\mathbf{d}_i$ is defined in Lemma1.

Proof. The quadratic form $(\mathbf{e}_r - \mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r - \mathbf{e}_f)$ can be expressed as

$$(\mathbf{e}_r - \mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r - \mathbf{e}_f) = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \mathbf{d}_i'\mathbf{d}_j. \tag{3.16}$$

By Lemma 1, it is evident that (3.12) can be re-written as

$$(\mathbf{e}_r - \mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r - \mathbf{e}_f) = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \left(\rho_{d_{ij}} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{d}_i'\mathbf{d}_i + \sqrt{(1-\rho_{d_{ij}}^2)} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{d}_i'\mathbf{z}_{d_i} \right)$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \rho_{d_{ij}} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{d}_i'\mathbf{d}_i + \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \sqrt{(1-\rho_{d_{ij}}^2)} \frac{\sigma_{d_j}}{\sigma_{d_i}} \mathbf{d}_i'\mathbf{z}_{d_i}.$$

Since $\mathbf{z}_{d_i}$ is independent of $\mathbf{d}_i$, we get

$$(\mathbf{e}_r - \mathbf{e}_f)'\hat{\mathbf{\Omega}}_y^{-1}(\mathbf{e}_r - \mathbf{e}_f) = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij}\rho_{d_{ij}}\sigma_{d_i}\sigma_{d_j}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$$

$$= \sum_{i=1}^{m} w_{1i}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2},$$

where $w_{1i} = \sum_{j=1}^{m} \hat{\sigma}_y^{ij}\rho_{d_{ij}}\sigma_{d_i}\sigma_{d_j}$. (3.17) □

Similarly, the second term of the test statistic in equation (3.11), $tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{ef})$, can be written in terms of a linear combination of the product of two correlated normal vectors as

$$tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{ef}) = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij}\hat{\sigma}_{ef_{ij}} = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij}\frac{\mathbf{e}_{f_i}'\mathbf{e}_{f_j}}{(n-p_f)}, i,j = 1,2,...,m, \quad (3.18)$$

where $\hat{\sigma}_{ef_{ij}} = \frac{\mathbf{e}_{f_i}'\mathbf{e}_{f_j}}{(n-p_f)}, i = 1,2,...,m; j = 1,2,...,m$,is the unbiased estimator of σ_{ij} (Greence, 2012: 258).

The transformation of $\mathbf{e}_{f_i}'\mathbf{e}_{f_j}$ to $\mathbf{e}_{f_i}'\mathbf{e}_{f_i}$ can be carried out in a similar way as in the case $\mathbf{d}_i'\mathbf{d}_j$, as shown in Lemmas 1 and 2.

Lemma3. Let $\mathbf{e}_{f_i}$ and $\mathbf{e}_{f_j}$ be the $n \times 1$ correlated vectors distributed as $N_n(\mathbf{0}, \sigma_{e_{f_i}}^2\mathbf{I}_n)$ and $N_n(\mathbf{0}, \sigma_{e_{f_j}}^2\mathbf{I}_n)$, respectively, $\rho_{e_{f_{ij}}}$ is the correlation coefficient between $\mathbf{e}_{f_i}$ and $\mathbf{e}_{f_j}$, $\mathbf{z}_{e_{f_i}}$ be the $n \times 1$ vector distributed as $N_n(\mathbf{0}, \sigma_{e_{f_i}}^2\mathbf{I}_n)$, independent of $\mathbf{e}_{f_i}$, and $\tilde{\mathbf{e}}_{f_j}$ defined as

$$\tilde{\mathbf{e}}_{f_j} = \frac{\rho_{e_{f_{ij}}}\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{e}_{f_i} + \sqrt{(1-\rho_{e_{f_{ij}}}^2)}\frac{\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{z}_{e_{f_i}}. \quad (3.19)$$

Then $\mathbf{e}_{f_j}$ and $\tilde{\mathbf{e}}_{f_j}$ are distributed as $N_n(\mathbf{0},\sigma^2_{e_{f_j}}\mathbf{I}_n)$.

Proof. Since $\mathbf{e}_{f_i}$ and $\mathbf{z}_{e_{f_i}}$ are independent normal random vectors with the same zero mean and variance $\sigma^2_{e_{f_i}}\mathbf{I}_n$, the expectation of $\tilde{\mathbf{e}}_{f_j}$ is equal to zero, that is

$$\begin{aligned} E(\tilde{\mathbf{e}}_{f_j}) &= E\left(\frac{\rho_{e_{f_{ij}}}\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{e}_{f_i} + \sqrt{(1-\rho^2_{e_{f_{ij}}})}\frac{\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{z}_{e_{f_i}} \right) \\ &= \mathbf{0}, \end{aligned}$$

and the variance of $\tilde{\mathbf{e}}_{f_j}$ can be expressed as

$$\begin{aligned} Var(\tilde{\mathbf{e}}_{f_j}) &= Var\left(\frac{\rho_{e_{f_{ij}}}\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{e}_{f_i} + \sqrt{(1-\rho^2_{e_{f_{ij}}})}\frac{\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{z}_{e_{f_i}} \right) \\ &= \frac{\rho^2_{e_{f_{ij}}}\sigma^2_{e_{f_j}}}{\sigma^2_{e_{f_i}}} Var\left(\mathbf{e}_{f_i}\right) + (1-\rho^2_{e_{f_{ij}}})\frac{\sigma^2_{e_{f_j}}}{\sigma^2_{e_{f_i}}} Var\left(\mathbf{z}_{e_{f_i}}\right). \end{aligned} \tag{3.20}$$

By definition, $Var(\mathbf{z}_{e_{f_i}})$ and $Var(\mathbf{e}_{f_i})$ are both equal to $\sigma^2_{e_{f_i}}\mathbf{I}_n$, and so $Var(\tilde{\mathbf{e}}_{f_j})$ becomes

$$Var(\tilde{\mathbf{e}}_{f_j}) = \sigma^2_{e_{f_j}}\mathbf{I}_n .$$

Therefore, $\tilde{\mathbf{e}}_{f_j}$ is distributed as $N_n(\mathbf{0},\sigma^2_{e_{f_j}}\mathbf{I}_n)$, which is identical to $\mathbf{e}_{f_j}$. □

Lemma 4.
$$tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f}) = \sum_{i=1}^{m} w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}, \tag{3.21}$$

where $w_{2i} = \sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\rho_{e_{f_{ij}}} \sigma_{e_{f_j}} \sigma_{e_{f_i}}}{(n-p_f)}$, and $\mathbf{e}_{f_i}$ is defined in Lemma 3.

Proof. The $tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f})$ can be expressed as

$$tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f}) = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij}\hat{\sigma}_{e_{f_{ij}}} = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_j}}{(n-p_f)}. \tag{3.22}$$

By Lemma 3, the random vector $\mathbf{e}_{f_j}$ in (3.22) can be replaced by the random vector $\tilde{\mathbf{e}}_{f_j}$ in (3.19), and so (3.22) becomes

$$\begin{aligned} tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f}) &= \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_j}}{(n-p_f)} \\ &= \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\mathbf{e}'_{f_i}}{(n-p_f)} \left(\frac{\rho_{e_{f_{ij}}}\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{e}_{f_i} + \sqrt{(1-\rho^2_{e_{f_{ij}}})}\frac{\sigma_{e_{f_j}}}{\sigma_{e_{f_i}}}\mathbf{z}_{e_{f_i}} \right) \\ &= \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\rho_{e_{f_{ij}}}\sigma_{e_{f_j}}\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{(n-p_f)\sigma_{e_{f_i}}} + \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \sqrt{(1-\rho^2_{e_{f_{ij}}})} \frac{\sigma_{e_{f_j}}}{(n-p_f)\sigma_{e_{f_i}}} \mathbf{e}'_{f_i}\mathbf{z}_{e_{f_i}} . \end{aligned}$$

Since $\mathbf{z}_{e_{f_i}}$ is independent of $\mathbf{e}_{f_i}$, we get

$$\begin{aligned} tr(\hat{\mathbf{\Sigma}}_y^{-1}\mathbf{S}_{e_f}) &= \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\rho_{e_{f_{ij}}}\sigma_{e_{f_j}}\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{(n-p_f)\sigma_{e_{f_i}}} \\ &= \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\rho_{e_{f_{ij}}}\sigma_{e_{f_j}}\sigma_{e_{f_i}}\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{(n-p_f)\sigma^2_{e_{f_i}}} \\ &= \sum_{i=1}^{m} w_{2i} \frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}, \end{aligned}$$

where $w_{2i} = \sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\rho_{e_{f_{ij}}} \sigma_{e_{f_j}} \sigma_{e_{f_i}}}{(n - p_f)}$. (3.23) □

Substituting (3.15) and (3.21) into (3.11) yields

$$D = \sum_{i=1}^{m} w_{1i} \frac{\mathbf{d}_i' \mathbf{d}_i}{\sigma_{d_i}^2} - 2d \sum_{i=1}^{m} w_{2i} \frac{\mathbf{e}_{f_i}' \mathbf{e}_{f_i}}{\sigma_{e_{f_i}}^2}. \quad (3.24)$$

3.2 The Distribution of the Test Statistic

Both in (3.24) are in the forms of a weighted sum of correlated chi-squared variables with n degrees of freedom, or are the forms of the sum of correlated gamma random variables. Many authors have presented the distribution of the sum of independent gamma random variables in different forms (Mathai, 1982: 591-597; Mochopoulos, 1985: 541-544; Akkouchi, 2005: 205-211; Aalo et al., 2005: 2002-2005) but cannot apply to D statistic in (3.24). Alouini et al. (2001: 1471-1480) proposed that the distribution of sum of correlate gamma distributions and shape parameters are equal. After this, Holm and Alouini (2004: 1367-1376) provided the distribution of the sum of two correlated gamma random variables in terms of a Type I McKay distribution. The probability density function of the above are not in the form of a well-known distribution, which complicates the process and makes its application more difficult. Thus, a simple approximation of the distribution of the statistic in terms of the weighted sum of a correlated chi-squared variable is considered in this study, as the following Theorem shows.

Theorem 2. Let $\mathbf{d}_i$ be distributed as $N_n(\mathbf{0}, \sigma_{d_i}^2 \mathbf{I}_n)$ and w_{1i} be as defined in (3.17). Then the distribution of $\sum_{i=1}^{m} w_{1i} \frac{\mathbf{d}_i' \mathbf{d}_i}{\sigma_{d_i}^2}$ can be approximated by $R_1 \sim \gamma_1 \chi_{f_1}^2$, with the first two cumulants being equal, where

$$\gamma_1 = \frac{\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2\right)}{\sum_{i=1}^{m} w_{1i}}, \quad f_1 = \frac{n\left(\sum_{i=1}^{m} w_{1i}\right)^2}{\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2\right)},$$

$\rho_{d_{ij}}^2$ is the correlation coefficient of $\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$ and $\frac{\mathbf{d}_j'\mathbf{d}_j}{\sigma_{d_j}^2}$.

Proof. Since $\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$ is distributed as chi-squared with n degrees of freedom, it is discernible that the expectation of $\sum_{i=1}^{m} w_{1i}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$ is

$$E\left(\sum_{i=1}^{m} w_{1i}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}\right) = \sum_{i=1}^{m} w_{1i}\left(E\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}\right)$$

$$= n\sum_{i=1}^{m} w_{1i}.$$

The variance of $\sum_{i=1}^{m} w_{1i}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$ can be written as

$$Var\left(\sum_{i=1}^{m} w_{1i}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}\right) = \sum_{i=1}^{m} Var\left(w_{1i}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}\right) + 2\sum_{i=1}^{m}\sum_{j>i}^{m} Cov(w_{1i}\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}, w_{1j}\frac{\mathbf{d}_j'\mathbf{d}_j}{\sigma_{d_j}^2})$$

$$= \sum_{i=1}^{m} w_{1i}^2 Var\left(\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}\right) + 2\sum_{i=1}^{m}\sum_{j>i}^{m} w_{1i}w_{1j}Cov(\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}, \frac{\mathbf{d}_j'\mathbf{d}_j}{\sigma_{d_j}^2})$$

$$= 2n\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}Cov(\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}, \frac{\mathbf{d}_j'\mathbf{d}_j}{\sigma_{d_j}^2})\right)$$

$$= 2n\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2\right),$$

since $Var\left(\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}\right) = Var\left(\frac{\mathbf{d}_j'\mathbf{d}_j}{\sigma_{d_j}^2}\right) = 2n$, and $\rho_{d_{ij}}^2$ is the correlation coefficient of $\frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$ and $\frac{\mathbf{d}_j'\mathbf{d}_j}{\sigma_{d_j}^2}$ (Isserlis, 1981: 134-139; Joarder, 2009: 581-592).

The distribution of the weighted sum of correlated chi-squared variables $\sum_{i=1}^{m} w_{1i} \frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$ can be approximated by $R_1 \sim \gamma_1 \chi_{f_1}^2$ (Brown, 1975: 987-992; Makambi, 2003: 225-234; Hou, 2005: 179-187; Chuang and Shih, 2012: 457-472) where the first two cumulants of the two distributions are equal. Thus, equating the first two cumulants of $\sum_{i=1}^{m} w_{1i} \frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2}$ and R_1 yields

$$\gamma_1 f_1 = n \sum_{i=1}^{m} w_{1i}, \tag{3.25}$$

$$2\gamma_1^2 f_1 = 2n \sum_{i=1}^{m} \left(w_{1i}^2 + 2 \sum_{j>i}^{m} w_{1i} w_{1j} \rho_{d_{ij}}^2 \right). \tag{3.26}$$

Solving (3.25) and (3.26) gives rise to

$$\gamma_1 = \frac{\sum_{i=1}^{m} \left(w_{1i}^2 + 2 \sum_{j>i}^{m} w_{1i} w_{1j} \rho_{d_{ij}}^2 \right)}{\sum_{i=1}^{m} w_{1i}}, \tag{3.27}$$

$$f_1 = \frac{n \left(\sum_{i=1}^{m} w_{1i} \right)^2}{\sum_{i=1}^{m} \left(w_{1i}^2 + 2 \sum_{j>i}^{m} w_{1i} w_{1j} \rho_{d_{ij}}^2 \right)}. \tag{3.28}$$ □

Theorem 3. Let $\mathbf{e}_{f_i}$ be distributed as $N_n(\mathbf{0},\sigma^2_{e_{f_i}}\mathbf{I}_n)$ and w_{2i} be as defined in equation (3.23), then the distribution of $\sum_{i=1}^{m} w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}$ can be approximated by $R_2 \sim \gamma_2\chi^2_{f_2}$ with the first two cumulants being equal, where

$$\gamma_2 = \frac{\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho^2_{ef_{ij}}\right)}{\sum_{i=1}^{m} w_{2i}}, \quad f_2 = \frac{n\left(\sum_{i=1}^{m} w_{2i}\right)^2}{\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho^2_{ef_{ij}}\right)},$$

$\rho^2_{ef_{ij}}$ is the correlation coefficient of $\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}$ and $\frac{\mathbf{e}'_{f_j}\mathbf{e}_{f_j}}{\sigma^2_{e_{f_j}}}$.

Proof. Since $\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}$ is distributed as chi-squared with n degrees of freedom, the expectation of $\sum_{i=1}^{m} w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}$ can be written as

$$E\left(\sum_{i=1}^{m} w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}\right) = \sum_{i=1}^{m} w_{2i}E\left(\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}\right)$$

$$= n\sum_{i=1}^{m} w_{2i}\,.$$

The variance of $\sum_{i=1}^{m} w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}$ can be written as

$$Var\left(\sum_{i=1}^{m} w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}\right) = \sum_{i=1}^{m} Var\left(w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}\right) + 2\sum_{i=1}^{m}\sum_{j>i}^{m} Cov(w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}, w_{2j}\frac{\mathbf{e}'_{f_j}\mathbf{e}_{f_j}}{\sigma^2_{e_{f_j}}})$$

$$= \sum_{i=1}^{m} w_{2i}^2 Var\left(\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}\right) + 2\sum_{i=1}^{m}\sum_{j>i}^{m} w_{2i}w_{2j}Cov(\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}, \frac{\mathbf{e}'_{f_j}\mathbf{e}_{f_j}}{\sigma^2_{e_{f_j}}})$$

$$= 2n\sum_{i=1}^{m} w_{2i}^2 + 2\sum_{i=1}^{m}\sum_{j>i}^{m} w_{2i}w_{2j}Cov(\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}, \frac{\mathbf{e}'_{f_j}\mathbf{e}_{f_j}}{\sigma^2_{e_{f_j}}})$$

$$= 2n\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho^2_{ef_{ij}} \right),$$

since $Var\left(\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}\right) = Var\left(\frac{\mathbf{e}'_{f_j}\mathbf{e}_{f_j}}{\sigma^2_{e_{f_j}}}\right) = 2n$, and $\rho^2_{ef_{ij}}$ is the correlation coefficient of $\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}$ and $\frac{\mathbf{e}'_{f_j}\mathbf{e}_{f_j}}{\sigma^2_{e_{f_j}}}$ (Isserlis, 1981: 134-139; Joarder, 2009: 581-592).

The distribution of the weighted sum of correlated chi-squared variable $\sum_{i=1}^{m} w_{2_i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{ef_i}}$ can be approximated by $R_2 \sim \gamma_2\chi^2_{f_2}$ (Brown, 1975: 987-992; Makambi, 2003: 225-234; Hou, 2005: 179-187; Chuang and Shih, 2012: 457-472) where the first two cumulants of the two distributions are equal. Thus, equating the first two cumulants of $\sum_{i=1}^{m} w_{2i}\frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma^2_{e_{f_i}}}$ and R_2 yields

$$\gamma_2 f_2 = n\sum_{i=1}^{m} w_{2i}, \tag{3.29}$$

$$2\gamma_2^2 f_2 = 2n\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho^2_{ef_{ij}} \right). \tag{3.30}$$

Solving (3.29) and (3.30) gives rise to

$$\gamma_2 = \frac{\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho^2_{ef_{ij}} \right)}{\sum_{i=1}^{m} w_{2i}}, \tag{3.31}$$

$$f_2 = \frac{n\left(\sum_{i=1}^{m} w_{2i}\right)^2}{\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho_{ef_{ij}}^2\right)}. \tag{3.32} \square$$

Next, proof that the two quadratic terms in the D statistic in (3.24) are independent is shown.

Lemma 5. $\mathbf{d}'\mathbf{W}_1\mathbf{d}$ and $\mathbf{e}'_f\mathbf{W}_2\mathbf{e}_f$ are independent where $\mathbf{d}$ and $\mathbf{e}_f$ are the $nm \times 1$ vectors consisting of m subvectors $\mathbf{d}_i$ distributed as $N_n(\mathbf{0}, \sigma_{d_i}^2\mathbf{I}_n)$ and m subvectors $\mathbf{e}_{f_i}$ distributed as $N_n(\mathbf{0}, \sigma_{e_{f_i}}^2\mathbf{I}_n)$, $i = 1, 2, ..., m$, respectively, and $\mathbf{W}_1$ and $\mathbf{W}_2$ are $nm \times nm$ diagonal matrices with the i^{th} diagonal elements $\frac{w_{1i}}{\sigma_{d_i}^2}$ and $\frac{w_{2i}}{\sigma_{e_{f_i}}^2}$, respectively.

Proof. The residual vectors $\mathbf{e}_{f_i}$ and $\mathbf{e}_{r_i}$ in the full and reduced models can be respectively expressed in term of $\mathbf{y}_i$ as (Graybill, 1976: 174)

$$\mathbf{e}_{f_i} = (\mathbf{I}_n - \mathbf{M}_f)\mathbf{y}_i, \tag{3.33}$$

$$\mathbf{e}_{r_i} = (\mathbf{I}_n - \mathbf{M}_r)\mathbf{y}_i, \tag{3.34}$$

where $\mathbf{M}_f = \mathbf{X}_f(\mathbf{X}'_f\mathbf{X}_f)^{-1}\mathbf{X}'_f$ and $\mathbf{M}_r = \mathbf{X}_r(\mathbf{X}'_r\mathbf{X}_r)^{-1}\mathbf{X}'_r$. From (3.33) and (3.34), the difference vectors $\mathbf{d}_i$ can be written as

$$\mathbf{d}_i = (\mathbf{M}_f - \mathbf{M}_r)\mathbf{y}_i.$$

Consider the term $\mathbf{M}_r\mathbf{M}_f$ by partitioning the matrix $\mathbf{X}_f$ into $\mathbf{X}_r$ and $\mathbf{X}_d$.

$$\mathbf{M}_r\mathbf{M}_f = \mathbf{X}_r(\mathbf{X}_r'\mathbf{X}_r)^{-1}\mathbf{X}_r'\mathbf{X}_f(\mathbf{X}_f'\mathbf{X}_f)^{-1}\mathbf{X}_f'$$

$$= \mathbf{X}_r(\mathbf{X}_r'\mathbf{X}_r)^{-1}\mathbf{X}_r'\begin{bmatrix}\mathbf{X}_r & \mathbf{X}_d\end{bmatrix}\left(\begin{bmatrix}\mathbf{X}_r \\ \mathbf{X}_d\end{bmatrix}\begin{bmatrix}\mathbf{X}_r & \mathbf{X}_d\end{bmatrix}\right)^{-1}\begin{bmatrix}\mathbf{X}_r \\ \mathbf{X}_d\end{bmatrix}$$

$$= \mathbf{X}_r(\mathbf{X}_r'\mathbf{X}_r)^{-1}\mathbf{X}_r'\begin{bmatrix}\mathbf{X}_r & \mathbf{X}_d\end{bmatrix}\begin{bmatrix}\mathbf{X}_r'\mathbf{X}_r & \mathbf{X}_r'\mathbf{X}_d \\ \mathbf{X}_d'\mathbf{X}_r & \mathbf{X}_d'\mathbf{X}_d\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{X}_r \\ \mathbf{X}_d\end{bmatrix}$$

$$= \mathbf{M}_r, \tag{3.35}$$

since $\mathbf{X}_r$ and $\mathbf{X}_d$ are independent. Similarly,

$$\mathbf{M}_r\mathbf{M}_f = \mathbf{M}_r. \tag{3.36}$$

Thus, by (3.35), (3.36), and the independence properties of $\mathbf{M}_f$ and $\mathbf{M}_r$, the quadratic forms $\mathbf{d}'\mathbf{W}_1\mathbf{d}$ and $\mathbf{e}_f'\mathbf{W}_2\mathbf{e}_f$ can be expressed in terms of $\mathbf{y}$ as

$$\mathbf{d}'\mathbf{W}_1\mathbf{d} = \sum_{i=1}^{m}\frac{w_{1i}}{\sigma_{d_i}^2}\mathbf{y}_i'(\mathbf{M}_f - \mathbf{M}_r)(\mathbf{M}_f - \mathbf{M}_r)\mathbf{y}_i$$

$$= \sum_{i=1}^{m}\frac{w_{1i}\sigma_{y_i}^2}{\sigma_{d_i}^2}\tilde{\mathbf{y}}_i'(\mathbf{M}_f - \mathbf{M}_r)\tilde{\mathbf{y}}_i, \tag{3.37}$$

$$\mathbf{e}_f'\mathbf{W}_2\mathbf{e}_f = \sum_{i=1}^{m}\frac{w_{2i}}{\sigma_{e_{f\,i}}^2}\mathbf{y}_i'(\mathbf{I}_n - \mathbf{M}_f)(\mathbf{I}_n - \mathbf{M}_f)\mathbf{y}_i$$

$$= \sum_{i=1}^{m}\frac{w_{2i}\sigma_{y_i}^2}{\sigma_{e_{f\,i}}^2}\tilde{\mathbf{y}}_i'(\mathbf{I}_n - \mathbf{M}_f)\tilde{\mathbf{y}}_i, \tag{3.38}$$

where $\tilde{\mathbf{y}}_i$ is a standardized form of $\mathbf{y}_i$ with variance $\mathbf{I}_n$. From (3.37) and (3.38), it can be said that, if $\tilde{\mathbf{y}}_i'(\mathbf{M}_f - \mathbf{M}_r)\tilde{\mathbf{y}}_i$ and $\tilde{\mathbf{y}}_i'(\mathbf{I}_n - \mathbf{M}_f)\tilde{\mathbf{y}}_i$ are independent, then $\mathbf{d}'\mathbf{W}_1\mathbf{d}$ and $\mathbf{e}_f'\mathbf{W}_2\mathbf{e}_f$ are independent. Once again, by (3.38) and the idempotent property of $\mathbf{M}_f$, we can be conclude that

$$\begin{aligned}(\mathbf{M}_f - \mathbf{M}_r)(\mathbf{I}_n - \mathbf{M}_f) &= \left(\mathbf{M}_f - \mathbf{M}_f\mathbf{M}_f - \mathbf{M}_r + \mathbf{M}_r\mathbf{M}_f\right)\\ &= \left(\mathbf{M}_f - \mathbf{M}_f - \mathbf{M}_r + \mathbf{M}_r\right)\\ &= \mathbf{0},\end{aligned}$$

which leads to the supposition that $\tilde{\mathbf{y}}_i'(\mathbf{M}_f - \mathbf{M}_r)\tilde{\mathbf{y}}_i$ and $\tilde{\mathbf{y}}_i'(\mathbf{I}_n - \mathbf{M}_f)\tilde{\mathbf{y}}_i$ are independent (Graybill, 1976: 139). □

The standardized form of the D statistic, which is similar to the standardized form of the AIC in Yanagihara and Ohmoto (2005: 417-433), is derived in the following theorem.

Theorem 4. The expected and variance of the D statistic can be expressed as

$$\mu_D = n\sum_{i=1}^{m}\left[\hat{\sigma}_y^{ii}\left(\sigma_{d_i}^2 - \frac{2d}{n-p_f}\sigma_{ef_i}^2\right) + 2\sum_{j>i}^{m}\hat{\sigma}_y^{ij}\left(\rho_{d_{ij}}\sigma_{d_i}\sigma_{d_j} - \frac{2d}{n-p_f}\rho_{ef_{ij}}\sigma_{ef_i}\sigma_{ef_j}\right)\right]$$

$$\sigma_D^2 = 2n\sum_{i=1}^{m}\left[\sigma_{d_i}^2 S_{d_i}^2 + \left(\frac{2d\sigma_{e_{f_i}}}{n-p_f}\right)^2 S_{ef_i}^2 + 2\sum_{j>i}^{m}\left(\sigma_{d_i}\sigma_{d_j}\rho_{d_{ij}}^2 S_{d_i}S_{d_j} + \left(\frac{2d}{n-p_f}\right)^2\sigma_{e_{f_i}}\sigma_{e_{f_j}}\rho_{ef_{ij}}^2 S_{ef_i}S_{ef_j}\right)\right],$$

where $S_{d_i} = \sum_{k=1}^{m}\hat{\sigma}_y^{ik}\rho_{d_{ik}}\sigma_{d_k}$, $S_{d_j} = \sum_{k=1}^{m}\hat{\sigma}_y^{jk}\rho_{d_{jk}}\sigma_{d_k}$, $S_{ef_i} = \sum_{k=1}^{m}\hat{\sigma}_y^{ik}\rho_{ef_{ik}}\sigma_{ef_k}$, and

$S_{ef_j} = \sum_{k=1}^{m}\hat{\sigma}_y^{jk}\rho_{ef_{jk}}\sigma_{ef_k}$.

Proof. From Theorems 2 and 3, the D statistic in (3.24) can be expressed as the difference of R_1 and R_2, that is

$$D = R_1 - 2dR_2 \tag{3.39}$$

where $R_1 = \sum_{i=1}^{m} w_{1i} \frac{\mathbf{d}_i{}'\mathbf{d}_i}{\sigma_{d_i}^2}$, $R_2 = \sum_{i=1}^{m} w_{2i} \frac{\mathbf{e}'_{f_i}\mathbf{e}_{f_i}}{\sigma_{e_{f_i}}^2}$, and $d = p_f - p_r$.

The expectation of the D statistic is

$$E(D) = \mu_D = E(R_1 - 2dR_2)$$

Since from Theorem 2, $R_1 \sim \gamma_1 \chi_{f_1}^2$, and from Theorem 3, $R_2 \sim \gamma_2 \chi_{f_2}^2$, the expectation of the D statistic can be written as

$$E(D) = \gamma_1 f_1 - 2d\gamma_2 f_2 .$$

Because,

$$\gamma_1 f_1 = n\sum_{i=1}^{m} w_{1i} \text{ , and } \gamma_2 f_2 = n\sum_{i=1}^{m} w_{2i} \text{ ,}$$

then

$$\gamma_1 f_1 - 2d\gamma_2 f_2 = n\left[\sum_{i=1}^{m} w_{1i} - 2d\sum_{i=1}^{m} w_{2i}\right]. \tag{3.40}$$

Substituting (3.17) and (3.23) into (3.40) yields

$$\mu_D = n\left[\sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \rho_{d_{ij}} \sigma_{d_i} \sigma_{d_j} - 2d\sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\rho_{e_{f_{ij}}} \sigma_{e_{f_i}} \sigma_{e_{f_j}}}{n - p_f}\right]$$

$$= n\sum_{i=1}^{m}\left[\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \rho_{d_{ij}} \sigma_{d_i} \sigma_{d_j} - 2d\sum_{j=1}^{m} \hat{\sigma}_y^{ij} \frac{\rho_{e_{f_{ij}}} \sigma_{e_{f_i}} \sigma_{e_{f_j}}}{n - p_f}\right]$$

$$= n\sum_{i=1}^{m}\left[\hat{\sigma}_y^{ii}\sigma_{d_i}^2 + 2\sum_{j>i}^{m} \hat{\sigma}_y^{ij} \rho_{d_{ij}} \sigma_{d_i} \sigma_{d_j} - \frac{2d}{n - p_f}\left(\hat{\sigma}_y^{ii}\sigma_{ef_i}^2 + 2\sum_{j>i}^{m} \hat{\sigma}_y^{ij} \rho_{ef_{ij}} \sigma_{ef_i} \sigma_{ef_j}\right)\right]$$

$$= n\sum_{i=1}^{m}\left[\hat{\sigma}_y^{ii}\left(\sigma_{d_i}^2 - \frac{2d}{n-p_f}\sigma_{ef_i}^2\right) + 2\sum_{j>i}^{m}\hat{\sigma}_y^{ij}\left(\rho_{d_{ij}}\sigma_{d_i}\sigma_{d_j} - \frac{2d}{n-p_f}\rho_{ef_{ij}}\sigma_{ef_i}\sigma_{ef_j}\right)\right]. \tag{3.41}$$

The variance of D statistic can be written as

$$\sigma_D^2 = Var(R_1 - 2dR_2),$$

and, since $R_1 \sim \gamma_1\chi_{f_1}^2$ and $R_2 \sim \gamma_2\chi_{f_2}^2$ are independent, the variance of the D statistic can be written as

$$\sigma_D^2 = 2f_1\gamma_1^2 + 8d^2 f_2\gamma_2^2 \tag{3.42}$$

Once again, substituting γ_1 and f_1 from (3.27) and (3.28), respectively, and γ_2 and f_2 from (3.31) and (3.32), respectively, into (3.42) yields

$$f_1\gamma_1^2 = \frac{n\left(\sum_{i=1}^{m} w_{1i}\right)^2}{\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2\right)}\left(\frac{\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2\right)}{\sum_{i=1}^{m} w_{1i}}\right)^2$$

$$= n\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2\right) \tag{3.43}$$

$$f_2\gamma_2^2 = \frac{n\left(\sum_{i=1}^{m} w_{2i}\right)^2}{\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho_{ef_{ij}}^2\right)}\left(\frac{\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho_{ef_{ij}}^2\right)}{\sum_{i=1}^{m} w_{2i}}\right)^2$$

$$= n\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho_{ef_{ij}}^2\right). \tag{3.44}$$

Substituting (3.43) and (3.44) into (3.42) gives

$$\sigma_D^2 = 2n\sum_{i=1}^{m}\left(w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2 \right) + 8nd^2\sum_{i=1}^{m}\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho_{ef_{ij}}^2 \right)$$

$$= 2n\sum_{i=1}^{m}\left[w_{1i}^2 + 2\sum_{j>i}^{m} w_{1i}w_{1j}\rho_{d_{ij}}^2 + 4d^2\left(w_{2i}^2 + 2\sum_{j>i}^{m} w_{2i}w_{2j}\rho_{ef_{ij}}^2 \right)\right]$$

$$= 2n\sum_{i=1}^{m}\left[w_{1i}^2 + 4d^2 w_{2i}^2 + 2\sum_{j>i}^{m}\left(w_{1i}w_{1j}\rho_{d_{ij}}^2 + 4d^2 w_{2i}w_{2j}\rho_{ef_{ij}}^2 \right)\right] \qquad (3.45)$$

Substituting (3.17) and (3.23) into (3.45) yields

$$\sigma_D^2 = 2n\sum_{i=1}^{m}\left[\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\rho_{d_{ik}}\sigma_{d_i}\sigma_{d_k}\right)^2 + 4d^2\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\frac{\rho_{e_{f_{ik}}}\sigma_{e_{f_i}}\sigma_{e_{f_k}}}{n-p_f}\right)^2\right]$$

$$+4n\sum_{i=1}^{m}\left[\sum_{j>i}^{m}\left(\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\rho_{d_{ik}}\sigma_{d_i}\sigma_{d_k}\right)\left(\sum_{k=1}^{m} \hat{\sigma}_y^{jk}\rho_{d_{jk}}\sigma_{d_j}\sigma_{d_k}\right)\rho_{d_{ij}}^2\right)\right]$$

$$+4n\sum_{i=1}^{m}\left[\sum_{j>i}^{m} 4d^2\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\frac{\rho_{e_{f_{ik}}}\sigma_{e_{f_i}}\sigma_{e_{f_k}}}{n-p_f}\right)\left(\sum_{k=1}^{m} \hat{\sigma}_y^{jk}\frac{\rho_{e_{f_{jk}}}\sigma_{e_{f_j}}\sigma_{e_{f_k}}}{n-p_f}\right)\rho_{ef_{ij}}^2\right].$$

$$= 2n\sum_{i=1}^{m}\left[\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\rho_{d_{ik}}\sigma_{d_i}\sigma_{d_k}\right)^2 + \left(\frac{2d}{n-p_f}\right)^2\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\rho_{e_{f_{ik}}}\sigma_{e_{f_i}}\sigma_{e_{f_k}}\right)^2\right]$$

$$+4n\sum_{i=1}^{m}\left[\sum_{j>i}^{m}\left(\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\rho_{d_{ik}}\sigma_{d_i}\sigma_{d_k}\right)\left(\sum_{k=1}^{m} \hat{\sigma}_y^{jk}\rho_{d_{jk}}\sigma_{d_j}\sigma_{d_k}\right)\rho_{d_{ij}}^2\right)\right]$$

$$+4n\sum_{i=1}^{m}\left[\sum_{j>i}^{m}\left(\left(\frac{2d}{n-p_f}\right)^2\left(\sum_{k=1}^{m} \hat{\sigma}_y^{ik}\rho_{e_{f_{ik}}}\sigma_{e_{f_i}}\sigma_{e_{f_k}}\right)\left(\sum_{k=1}^{m} \hat{\sigma}_y^{jk}\rho_{e_{f_{jk}}}\sigma_{e_{f_j}}\sigma_{e_{f_k}}\right)\rho_{ef_{ij}}^2\right)\right]$$

Let

$$S_{d_i} = \sum_{k=1}^{m} \hat{\sigma}_y^{ik} \rho_{d_{ik}} \sigma_{d_k}, S_{d_j} = \sum_{k=1}^{m} \hat{\sigma}_y^{jk} \rho_{d_{jk}} \sigma_{d_k}, S_{ef_i} = \sum_{k=1}^{m} \hat{\sigma}_y^{ik} \rho_{ef_{ik}} \sigma_{ef_k},$$

$$S_{ef_j} = \sum_{k=1}^{m} \hat{\sigma}_y^{jk} \rho_{ef_{jk}} \sigma_{ef_k}.$$

Therefore,

$$\sigma_D^2 = 2n \sum_{i=1}^{m} \left[\sigma_{d_i}^2 S_{d_i}^2 + \left(\frac{2d\sigma_{e_{f_i}}}{n - p_f} \right)^2 S_{ef_i}^2 \right]$$

$$+4n \sum_{i=1}^{m} \sum_{j>i}^{m} \left(\sigma_{d_i} \sigma_{d_j} \rho_{d_{ij}}^2 S_{d_i} S_{d_j} + \left(\frac{2d}{n - p_f} \right)^2 \sigma_{e_{f_i}} \sigma_{e_{f_j}} \rho_{ef_{ij}}^2 S_{ef_i} S_{ef_j} \right). \qquad (3.46) \ \square$$

By direct transformation, it can be shown that the distribution of $R_1 \sim \gamma_1 \chi_{f_1}^2$ and $R_2 \sim \gamma_2 \chi_{f_2}^2$ are gamma distributions with shape parameter $\frac{f_1}{2}$ and scale parameter $2\gamma_1$, and shape parameter $\frac{f_2}{2}$ and scale parameter $2\gamma_2$, respectively. Since f_1 in (3.28) and f_2 in (3.32) increase linearly as n increases, it follows from the central limit theorem that the asymptotic distribution of R_1 and R_2 are normal with mean $\gamma_1 f_1$ and variance $2 f_1 \gamma_1^2$, and with mean $\gamma_2 f_2$ and variance $2 f_2 \gamma_2^2$, respectively, as n becomes large. Therefore, the D statistic, which is the difference between two normal variables R_1 and $2dR_2$, asymptotically approaches a normal distribution with mean

$$n \sum_{i=1}^{m} \left[\hat{\sigma}_y^{ii} \left(\sigma_{d_i}^2 - \frac{2d}{n - p_f} \sigma_{ef_i}^2 \right) + 2 \sum_{j>i}^{m} \hat{\sigma}_y^{ij} \left(\rho_{d_{ij}} \sigma_{d_i} \sigma_{d_j} - \frac{2d}{n - p_f} \rho_{ef_{ij}} \sigma_{ef_i} \sigma_{ef_j} \right) \right]$$

and variance

$$2n\sum_{i=1}^{m}\left[\sigma_{d_i}^2 S_{d_i}^2+\left(\frac{2d\sigma_{e_{f_i}}}{n-p_f}\right)^2 S_{ef_i}^2\right]+2n\sum_{i=1}^{m}\left[2\sum_{j>i}^{m}\left(\sigma_{d_i}\sigma_{d_j}\rho_{d_{ij}}^2 S_{d_i}S_{d_j}+\left(\frac{2d}{n-p_f}\right)^2\sigma_{e_{f_i}}\sigma_{e_{f_j}}\rho_{ef_{ij}}^2 S_{ef_i}S_{ef_j}\right)\right].$$

Corollary 1.For testing $H_0: M\Gamma_r = M\Gamma_f$ against $H_A: M\Gamma_r > M\Gamma_f$, if Theorems 1-4 and Lemma 1-5 are satisfied, then

$$T_D = \frac{D-\mu_D}{\sigma_D} \to N(0,1) \text{ in distribution,}$$

where σ_D is the standard deviation of the D statistic, $\sigma_D = \sqrt{Var(D)}$.

Therefore, the proposed test at significance level α will reject H_0 if

$T_D > Z_\alpha$,

where Z_α denotes the upper quantile of a standard normal distribution.

CHAPTER 4

SIMULATION STUDY

In this chapter, details of a simulation study carried out to compare the efficiency of the modified C_p criterion (MC) (Vichit Lorchirachoonkul and Jirawan Jitthavech, 2012: 2386-2394) and the proposed test statistic T_D in Chapter 3, Corollary 1 are given. The backward elimination procedure, which consists of two simultaneous equations with two relevant variables and two irrelevant variables, was used for variable selection in the multivariate linear regression models in (3.2). The simulation study is comprised of the following parts. Section 4.1 describes of the generation of five datasets used to select appropriate variables in the multivariate linear regression models is presented. The model selection process is considered using both the C_p criterion values and the testing of model selection. The variable selection process is outlined in Section 4.2 and finally, the result of the simulation study are detailed in Section 4.3.

4.1 Dataset Generation

Five datasets in the simulation were generated under the following specifications:

1) The multivariate linear regression full models consists of two dependent variables: two relevant independent variables and two irrelevant independent variables.

2) The four independent variables were generated based on uniform distributions $x_1 \sim U(235,250)$, $x_2 \sim U(35,65)$, $x_3 \sim U(190,200)$, and $x_4 \sim U(260,285)$ by the CALL STREAMINIT routine in SAS version 9.3 with seed

numbers of 763,275, 65,394,773, 18,753 and 9,827,247, respectively. Variables x_1 and x_2 were relevant, and variables, x_3 and x_4 irrelevant.

3) The random errors in the full model were normally distributed with zero means and the following covariance matrices and the corresponding correlation coefficients:

$$\Sigma_1 = \begin{bmatrix} 400.0000 & 84.8528 \\ 84.8528 & 200.0000 \end{bmatrix};\ \rho_{12} = 0.3,$$

$$\Sigma_2 = \begin{bmatrix} 400.0000 & 113.1371 \\ 113.1371 & 200.0000 \end{bmatrix};\ \rho_{12} = 0.4,$$

$$\Sigma_3 = \begin{bmatrix} 400.0000 & 141.4214 \\ 141.4214 & 200.0000 \end{bmatrix};\ \rho_{12} = 0.5,$$

$$\Sigma_4 = \begin{bmatrix} 400.0000 & 197.9899 \\ 197.9899 & 200.0000 \end{bmatrix};\ \rho_{12} = 0.7,$$

$$\Sigma_5 = \begin{bmatrix} 400.0000 & 226.2742 \\ 226.2742 & 200.0000 \end{bmatrix};\ \rho_{12} = 0.8.$$

The correlated random errors ε_1 and ε_2 with the covariance matrix Σ_{f_i} were generated from two independent standardized normal random variables, z_1 and z_2 by Cholesky decomposition of the covariance matrix Σ_{f_i}:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \Sigma_{f_i}^{1/2} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \tag{4.1}$$

where

$$\Sigma^{1/2} = \begin{bmatrix} \sigma_1 & 0 \\ \rho_{12}\sigma_2 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix}, \tag{4.2}$$

σ_1 and σ_2 are the standard deviations of ε_1 and ε_2, respectively, and correspondingly and ρ_{12} is the correlation coefficient of ε_1 and ε_2.

The standardized normal random variables, z_1 and z_2 were generated by the CALL STREAMINIT routine in SAS version 9.3 with seed numbers 498,753 and 45,327, respectively.

The two dependent variables in the multivariate regression full models were generated from the following equations:

$$y_1 = \beta_{10} + \beta_{11}x_1 + \beta_{12}x_2 + \beta_{13}x_3 + \beta_{14}x_4 + \varepsilon_1, \tag{4.3}$$

$$y_2 = \beta_{20} + \beta_{21}x_1 + \beta_{22}x_2 + \beta_{23}x_3 + \beta_{24}x_4 + \varepsilon_2, \tag{4.4}$$

where

$$\boldsymbol{\beta} = \begin{bmatrix} 5 & 2 & 5 & 0 & 0 & 4 & 6 & 11 & 0 & 0 \end{bmatrix}' . \tag{4.5}$$

A population consisting of two dependent variables, four independent variables, and two correlated random errors, 1,000,000 of each, was created as described above using the SAS program Population Generation, as shown in Appendix A. The population statistics of the four independent variables x_1, x_2, x_3, and x_4 are shown in Table 4.1, and their Pearson's correlation coefficients with corresponding p-values are shown in Table 4.2. Thus, it can be concluded that the population statistics are very close to the theoretical values of the corresponding distributions.

The population statistics of random errors ε_1 and ε_2 calculated from (4.1) are shown in Table 4.3. The Pearson's coefficients of correlation of ε_1 and ε_2 are 0.2995, 0.3992, 0.4990, 0.6989 and 0.7991 for populations 1-5 with the p-value <.0001, which corresponding to the specifications. From (4.1), it can be seen that the values of ε_1 in each population were unchanged; only the values of ε_2 in each population were slightly changed owing to the different values of ρ_{12}, as shown in Table 4.3.

For a given ρ_{12}, 100 samples of size 200 were selected from the populations of independent variables and random errors by SAS procedure PROC SURVEYSELECT using seed number 759,473 with equal probability and without replacement to calculate the dependent variables, y_1 and y_2 in (4.3) and (4.4), respectively. Using the same reasons given for the values of ε_1 and ε_2 in the previous paragraph, the statistics of y_1 were unchanged in the five datasets but those of y_2 were slightly changed because of small changes in ε_2 in the five datasets, as shown in Table 4.4.

Table 4.1 Population Statistics of the Independent Variables in the Simulation: Population Size, N=1,000,000

Variable	Mean	SD.	Min	Max
x_1	242.4980	4.3342	235.0000	250.0000
	(242.5000)	(4.3301)	(235.0000)	(250.0000)
x_2	50.0171	8.6600	35.0000	65.0000
	(50.0000)	(8.6603)	(35.0000)	(65.0000)
x_3	194.9997	2.8868	190.0000	200.0000
	(195.0000)	(2.8868)	(190.0000)	(200.0000)
x_4	272.5081	7.2197	260.0000	285.0000
	(272.5000)	(7.2169)	(260.0000)	(285.0000)

Note: The numbers in brackets are theoretical values.

Table 4.2 Pearson's Population Coefficients of Correlations for x_1, x_2, x_3 and x_4

Variable	x_1	x_2	x_3	x_4
x_1	1	0.0011	0.0010	-0.0004
		0.2585	0.3442	0.7008
x_2	0.0011	1	-0.0009	-0.0015
	0.2585		0.3870	0.1265
x_3	0.0010	-0.0009	1	0.0015
	0.3442	0.3870		0.1479
x_4	-0.0004	-0.0015	0.0015	1
	0.7008	0.1265	0.1479	

Table 4.3 Population Statistics of Random Errors ε_1 and ε_2 for Datasets 1-5

Dataset	Random error	Mean	SD	Min	Max
1	ε_1	0.0231	19.9747	-94.5026	95.9087
	ε_2	0.0148	14.1724	-75.3575	68.8949
2	ε_1	0.0231	19.9747	-94.5026	95.9087
	ε_2	0.0160	14.1695	-75.5795	66.3289
3	ε_1	0.0231	19.9747	-94.5026	95.9087
	ε_2	0.0171	14.1655	-74.8861	64.7686
4	ε_1	0.0231	19.9747	-94.5026	95.9087
	ε_2	0.0188	14.1538	-69.9335	68.7633
5	ε_1	0.0231	19.9747	-94.5026	95.9087
	ε_2	0.0193	14.1461	-69.1651	69.0808

Table 4.4 Population Statistics of Dependent Variables y_1 and y_2 for Datasets 1-5

Dataset	Dependent Variable	Mean	SD	Min	Max
1	y_1	740.1045	48.4861	578.7444	892.9211
	y_2	2,009.0000	99.7920	1,759.0000	2,262.0000
2	y_1	740.1045	48.4861	578.7444	892.9211
	y_2	2,009.0000	99.7923	1,758.0000	2,262.0000
3	y_1	740.1045	48.4861	578.7444	892.9211
	y_2	2,009.0000	99.7923	1,759.0000	2,261.0000
4	y_1	740.1045	48.4861	578.7444	892.9211
	y_2	2,009.0000	99.7918	1,758.0000	2,258.0000
5	y_1	740.1045	48.4861	578.7444	892.9211
	y_2	2,009.0000	99.7911	1,757.0000	2,257.0000

4.2 Variable Selection

Two criteria, the modified C_p criterion(MC) (Vichit Lorchirachoonkul and Jirawan Jitthavech, 2012: 2386-2394) and the proposed test statistic T_D in Chapter 3, Corollary 1, were used in variable selection employing the backward elimination method, as described in Chapter 3. In an iteration of the selection process using the MC, if its minimum value for all possible reduced models with one independent variable omission is not greater than the MC of the reference model, then the reduced model with the MC equal to the minimum value becomes the reference model in the next iteration and the omitted variable in the new reference model is permanently discarded from the model specification, otherwise the iteration is terminated. However, in the variable selection process using the proposed test statistic T_D, the full model is used as a reference in the entire variable selection process. In iterations of this process, the omitted variable in the model with the minimum T_D was permanently discarded from the model specification if the null hypothesis (3.10) was not rejected, otherwise the iteration was terminated.

Five datasets selected by SAS procedure PROC SURVEYSELECT described in Section 4.1 are used in the simulation study, with each dataset consists of 100 samples of size 200. The models selected by each variable selection criterion were classified into three categories: fit, over-fit and under-fit. As mentioned previously, all dependent variables in the models in the fit category were relevant only, i.e. x_1 and x_2, the dependent variables in the models in the over-fit category consisted of all relevant variables and some irrelevant variables, and the dependent variables in the models in the under-fit category consisted of some or no relevant variables and some or no irrelevant variables.

4.3 Results of the Simulation Study

In this section, model selection results for the MC and T_D statistics for a multivariate linear regression models described in Section 4.2 are given.

Table 4.5 The Pearson's Population Coefficients of Correlation for $y_1, y_2, x_1, x_2, x_3, x_4, \varepsilon_1, \varepsilon_2$ and $y_1, y_2, \varepsilon_1, \varepsilon_2$

Dataset 1	y_1	y_2	x_1	x_2	x_3	x_4	ε_1	ε_2
y_1	1	0.9174	0.1799	0.8935	-0.0008	-0.0013	0.4125	0.1238
		<.0001	<.0001	<.0001	0.4314	0.1864	<.0001	<.0001
y_2	0.9174	1	0.2616	0.9550	-0.0006	-0.0016	0.0431	0.1424
	<.0001		<.0001	<.0001	0.5194	0.1007	<.0001	<.0001
ε_1	0.4125	0.0431	0.0002	0.0006	-0.0004	0.0003	1	0.2995
	<.0001	<.0001	0.8636	0.5799	0.6565	0.7860		<.0001
ε_2	0.1238	0.1424	-0.0007	0.0006	-0.0005	-0.0006	0.2995	1
	<.0001	<.0001	0.5093	0.5412	0.6478	0.5593	<.0001	
Dataset 2	y_1	y_2	x_1	x_2	x_3	x_4	ε_1	ε_2
y_1	1	0.9232	0.1799	0.8935	-0.0008	-0.0013	0.4125	0.1649
		<.0001	<.0001	<.0001	0.4314	0.1864	<.0001	<.0001
y_2	0.9232	1	0.2616	0.9550	-0.0007	-0.0016	0.0573	0.1425
	<.0001		<.0001	<.0001	0.5165	0.1023	<.0001	<.0001
ε_1	0.4125	0.0573	0.0002	0.0006	-0.0004	0.0003	1	0.3992
	<.0001	<.0001	0.8636	0.5799	0.6565	0.7860		<.0001
ε_2	0.1649	0.1425	-0.0006	0.0007	-0.0005	-0.0005	0.3992	1
	<.0001	<.0001	0.5385	0.5164	0.6251	0.5956	<.0001	

Table 4.5 (Continued)

Dataset 3	y_1	y_2	x_1	x_2	x_3	x_4	ε_1	ε_2
y_1	1	0.9290	0.1799	0.8935	-0.0008	-0.0013	0.4125	0.2061
		<.0001	<.0001	<.0001	0.4314	0.1864	<.0001	<.0001
y_2	0.9290	1	0.2616	0.9550	-0.0007	-0.0016	0.0714	0.1425
	<.0001		<.0001	<.0001	0.5140	0.1041	<.0001	<.0001
ε_1	0.4125	0.0714	0.0002	0.0006	-0.0004	0.0003	1	0.4990
	<.0001	<.0001	0.8636	0.5799	0.6565	0.7860		<.0001
ε_2	0.2061	0.1425	-0.0006	0.0007	-0.0005	-0.0005	0.4990	1
	<.0001	<.0001	0.5752	0.4961	0.6059	0.6393	<.0001	
Dataset 4	y_1	y_2	x_1	x_2	x_3	x_4	ε_1	ε_2
y_1	1.0000	0.9407	0.1799	0.8935	-0.0008	-0.0013	0.4125	0.2885
		<.0001	<.0001	<.0001	0.4314	0.1864	<.0001	<.0001
y_2	0.9407	1.0000	0.2616	0.9550	-0.0007	-0.0016	0.0997	0.1424
	<.0001		<.0001	<.0001	0.5106	0.1091	<.0001	<.0001
ε_1	0.4125	0.0997	0.0002	0.0006	-0.0004	0.0003	1.0000	0.6989
	<.0001	<.0001	0.8636	0.5799	0.6565	0.7860		<.0001
ε_2	0.2885	0.1424	-0.0004	0.0007	-0.0006	-0.0003	0.6989	1.0000
	<.0001	<.0001	0.6795	0.4711	0.5799	0.7575	<.0001	

Table 4.5 (Continued)

Dataset 5	y_1	y_2	x_1	x_2	x_3	x_4	ε_1	ε_2
y_1	1.0000	0.9465	0.1799	0.8935	-0.0008	-0.0013	0.4125	0.3298
		<.0001	<.0001	<.0001	0.4314	0.1864	<.0001	<.0001
y_2	0.9465	1.0000	0.2616	0.9550	-0.0007	-0.0016	0.1139	0.1424
	<.0001		<.0001	<.0001	0.5101	0.1124	<.0001	<.0001
ε_1	0.4125	0.1139	0.0002	0.0006	-0.0004	0.0003	1.0000	0.7991
	<.0001	<.0001	0.8636	0.5799	0.6565	0.7860		<.0001
ε_2	0.3298	0.1424	-0.0003	0.0007	-0.0006	-0.0002	0.7991	1.0000
	<.0001	<.0001	0.7558	0.4697	0.5759	0.8398	<.0001	

From Tables 4.1-4.3, it can be seen that the independent variables x_1, x_2, x_3 and x_4, and random errors ε_1 and ε_2, are in accordance with the specifications in Section 4.1. Pearson's coefficients of correlation for $y_1, y_2, x_1, x_2, x_3, x_4, \varepsilon_1, \varepsilon_2$ and $y_1, y_2, \varepsilon_1, \varepsilon_2$ with corresponding p-values are shown in Table 4.5. From Table 4.2, it can be concluded that no multicollinearity occurred in x_1, x_2, x_3, and x_4. No significant correlation was observed between the dependent variables y_1 and y_2, and the irrelevant variables x_3 and x_4, as shown in Table 4.5.

One example of variable selection by backward variable elimination by the *MC* criterion and another by the criterion of hypothesis testing on the test statistic T_D are illustrated in Tables 4.6-4.7 to demonstrate the variable selection process iteration by iteration. Details of simulation results under both criteria are shown in Appendix D

Table 4.6 An Example of the Variable Selection Process by the Modified *Cp* Criterion: $\rho_{12} = 0.4$

Iteration	**Model**	**Variables in the Model**	MC
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0773
1.2	Reduced Model	x_2, x_3, x_4	20.9237
1.3	Reduced Model	x_1, x_3, x_4	188.2746
1.4	Reduced Model	x_1, x_2, x_4	4.5347
1.5	Reduced Model	x_1, x_2, x_3	4.3493*
2.1	Reference Model	x_1, x_2, x_3	4.3493
2.2	Reduced Model	x_2, x_3	20.0889
2.3	Reduced Model	x_1, x_3	188.1150
2.4	Reduced Model	x_1, x_2	3.5913*
3.1	Reference Model	x_1, x_2	3.5913
3.2	Reduced Model	x_2	19.3870
3.3	Reduced Model	x_1	187.3595

Table 4.7 An Example of the Variable Selection Process When Hypothesis Testing by the Test Statistic, T_D: $\rho_{12} = 0.4$

Iteration	**Model**	**Variables in the Model**	T_D
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-
1.2	Reduced Model	x_2, x_3, x_4	8.7590
1.3	Reduced Model	x_1, x_3, x_4	9.8403
1.4	Reduced Model	x_1, x_2, x_4	-2.0928
1.5	Reduced Model	x_1, x_2, x_3	-2.9241*
2.1	Reference Model	x_1, x_2, x_3, x_4	-
2.2	Reduced Model	x_2, x_3	7.4864
2.3	Reduced Model	x_1, x_3	9.7318
2.4	Reduced Model	x_1, x_2	-2.9916*
3.1	Reference Model	x_1, x_2, x_3, x_4	-
3.2	Reduced Model	x_2	6.4673
3.3	Reduced Model	x_1	9.6235

The simulation results are summarized in Table 4.8. Under both criteria, no under-fit models were selected. Under the *MC* criterion, the percentage of fit models is in the range of 75-81% and the rest were over-fit models. When hypothesis testing using the T_D test statistic, the percentage of fit models is in the range of 95-97% at the significance level of 0.05 and 0.10, respectively. The better selection observed under the hypothesis testing can be explained thus. Under the *MC* criterion, the variable selection process is terminated if the minimum of the *MC* values of a candidate models in an iteration is greater than the *MC* value of the reference model, regardless of the value of difference. However, under the hypothesis testing criterion, the variable selection process is terminated if the test rejects the null hypothesis at a given significance level.

Table 4.8 Percentage of Over-Fit, Fit and Under-Fit Models under the Two Criteria for Size n = 200 and 100 Replications

Results for Variable Selection	$\rho_{12} = 0.3$	$\rho_{12} = 0.4$	$\rho_{12} = 0.5$	$\rho_{12} = 0.7$	$\rho_{12} = 0.8$
Under the *MC* criterion					
Over-fit	19.00	24.00	23.00	25.00	19.00
Fit	81.00	76.00	77.00	75.00	81.00
Under-fit	-	-	-	-	-
Under the Hypothesis Testing at Significance Level 0.05					
Over-fit	4.00	5.00	4.00	3.00	3.00
Fit	96.00	95.00	96.00	97.00	97.00
Under-fit	-	-	-	-	-
Under the Hypothesis Testing at Significant Level 0.10					
Over-fit	5.00	5.00	4.00	3.00	3.00
Fit	95.00	95.00	96.00	97.00	97.00
Under-fit	-	-	-	-	-

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Conclusion

In this study, a comparison of the efficiency of variable selection in multivariate linear regression models between the modified C_p criterion and the proposed test statistic T_D, with both criteria having been developed based on Mallows C_p in the system of equations derived by Vichit Lorchirachoonkul and Jirawan Jitthavech (2012: 2386-2394), was presented. The multivariate linear regression full models consisted of two dependent variables, two relevant independent variables, and two irrelevant independent variables. The random disturbances were shown to be uncorrelated across observations in the same equation but contemporaneously correlated across equations. Data were simulated in five datasets with correlation between the equations equal to 0.3, 0.4, 0.5, 0.7, and 0.8, respectively (see dataset generation in 4.1), with each dataset consisting of 100 samples of size 200. The properties of both criteria were evaluated by the percentage of correct model selection via the backward elimination procedure for variable selection in the models.

Four theorems and five lemmas were used to provide the statistical properties of the D statistic, which can be used in variable selection in the contemporaneous multivariate linear regression models. This is another alternative of variable selection by testing the hypothesis (3.10) instead of direct comparison with the modified C_p statistics.

Without the hypothesis testing, the backward procedure of variable elimination suggests the retaining of any variable in the model specification if its elimination results in the modified C_p statistic being higher than MC_f, even in the case of a slight increase. However, by using hypothesis testing, a variable can be eliminated from the model specification provided its elimination does not cause the rejection of the null hypothesis.

The simulation results show that the test statistic T_D was able to select the model more correctly than the modified C_p criterion in all of datasets. The percentage of fit models based on the MC criterion was in the range of 75-81% whereas, under the hypothesis testing of the test statistic T_D , the range was 95-97%. Moreover, for both criteria, no under-fit models were selected; in other words, none of the relevant variables were removed from the model. The percentage of the T_D test statistic at significance levels of 0.05 and 0.10 were the same except in the case where correlation between the equations was equal to 0.3, when it was slightly different.

In additional, the proposed concept is easily extended toward variable selection by the forward and stepwise procedures of variable selection. However, in the proposed test statistic in (3.24) , $D = \sum_{i=1}^{m} w_{1i} \frac{\mathbf{d}_i'\mathbf{d}_i}{\sigma_{d_i}^2} - 2d\sum_{i=1}^{m} w_{2i} \frac{\mathbf{e}_{f_i}'\mathbf{e}_{f_i}}{\sigma_{e_{f\,i}}^2}$, both terms are in the form of a weighted sum of correlated chi-squared variables with n degrees offreedom where w_{1i}'s and w_{2i}'s are positive constants, which mean that, if this was not the case, then other methods would need to be used for derive the distribution of the test statistic.

5.2 Future Work

In this study, a modified C_p test statistic for variable selection in the contemporaneous multivariate linear regression model in which the errors terms are both heteroscedastic and contemporaneous correlated across the equationsbut are uncorrelated across the observations. Future studies could encompass using the modified C_p test statistic for selection among system of equations models.

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APPENDICES

APPENDIX A

SAS Syntax for Population Generation When $\rho_{12} = 0.3$

```
data x0;
        do j=1 to 1000000;
                        x0=1;
                output;
        end;
run;

data x1;
        do j=1 to 1000000;
                call streaminit(763275);
                        r1=rand('uniform');
                        x1=r1*(250-235)+235;
                output;
        end;
run;

data x2;
        do j=1 to 1000000;
                call streaminit(65394773);
                        r2=rand('uniform');
                        x2=r2*(65-35)+35;
                output;
        end;
run;

data x3;
```

```
        do j=1 to 1000000;
                call streaminit(18753);
                        r3=rand('uniform');
                        x3=r3*(200-190)+190;
                output;
        end;
run;

data x4;
        do j=1 to 1000000;
                call streaminit(9827247);
                        r4=rand('uniform');
                        x4=r4*(285-260)+260;
                output;
        end;
run;

data error1;
        do j=1 to 1000000;
                call streaminit(498753);
                        z1=rand('normal');
                output;
        end;
run;

data error2;
        do j=1 to 1000000;
                call streaminit(45327);
                        z2=rand('normal');
                output;
        end;
run;
```

```
data error;
merge  error1 error2;
run;

data eroh;
set error;
stdev1 = 20;
stdev2 = 14.1421356;
roh12 = 0.3;
cov12 = roh12*stdev1*stdev2;
b11 = stdev1;
b21 = cov12/stdev1;
b22 = sqrt(stdev2*stdev2-b21*b21);
e1 = b11*z1;
e2 = b21*z1+b22*z2;
run;

data pop_xe;
  merge x0 x1 x2 x3 x4 eroh;
run;

data data_gen.data03_pop(keep=j x0 x1 x2 x3 x4 e1 e2 y1 y2);
   set pop_xe;
   y1 = 5+2*x1+5*x2+0*x3+0*x4+e1;
   y2 = 4+6*x1+11*x2+0*x3+0*x4+e2;
run;
```

APPENDIX B

Proof for an Idempotent Matrix Independent of the Quadratic Form

Theorem B: The distribution of the quadratic forms

Let $(\mathbf{e}_r - \mathbf{e}_f)$ be distributed as $N_{nm}(\mathbf{0}, \mathbf{\Omega}_{rf})$ and $\mathbf{\Omega}_y^{-1}$ be a $nm \times nm$ symmetrical matrix of constants of rank r, then $(\mathbf{e}_r - \mathbf{e}_f)'\mathbf{\Omega}_y^{-1}(\mathbf{e}_r - \mathbf{e}_f)$ is χ_r^2 if and only if $\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}$ is idempotent.

In our case we can provide a proof by using Theorem B as follows:

Proof. Let $\mathbf{\Omega}_y^{-1}$ be a $mn \times nm$ symmetric matrix and $\mathbf{\Omega}_{rf}$ be a $mn \times nm$ covariance matrix of $(\mathbf{e}_r - \mathbf{e}_f)$ by definition of an idempotent matrix if

$\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf} = \mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}$ then $\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}$ is an idempotent.

Since,

$$\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf} = (\mathbf{\Sigma}_y^{-1} \otimes \mathbf{I}_n)(\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)$$

$$= (\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)$$

$$\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf} = (\mathbf{\Sigma}_y^{-1} \otimes \mathbf{I}_n)(\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)(\mathbf{\Sigma}_y^{-1} \otimes \mathbf{I}_n)(\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)$$

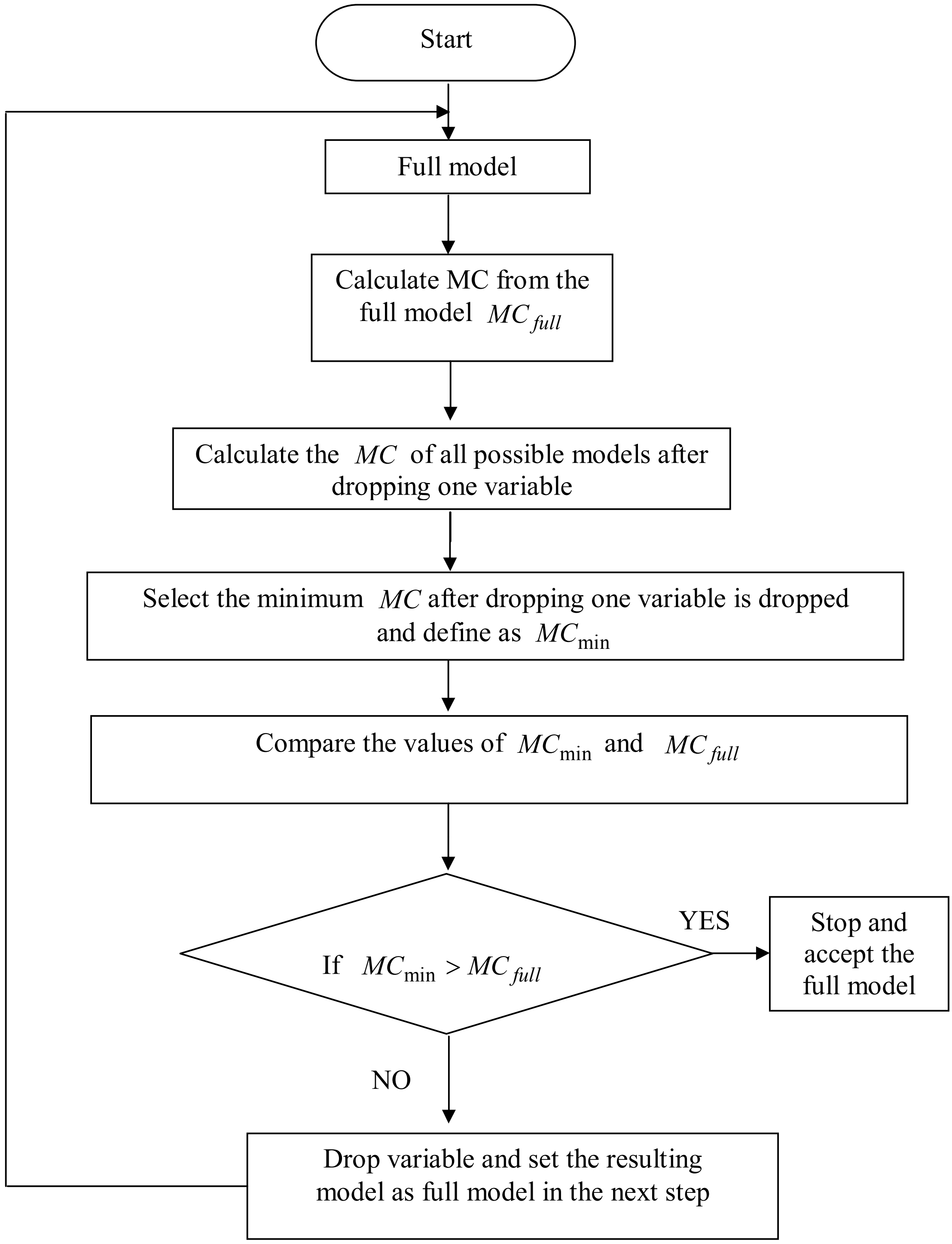
$$= (\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)(\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)$$

$$= (\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf}\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)$$

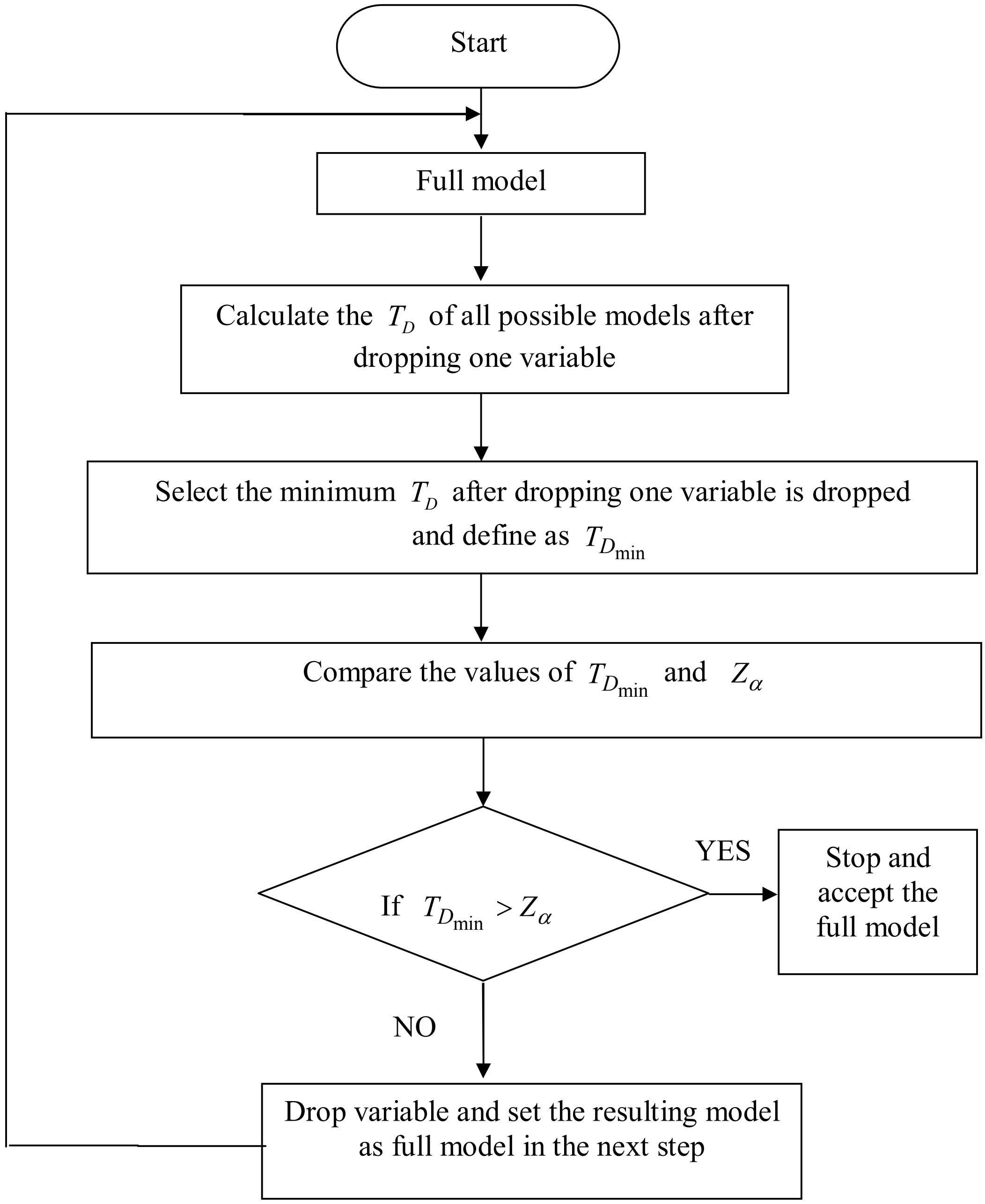
We can see that $(\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n) \neq (\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf}\mathbf{\Sigma}_y^{-1}\mathbf{\Sigma}_{rf} \otimes \mathbf{I}_n)$, and so $\mathbf{\Omega}_y^{-1}\mathbf{\Omega}_{rf}$ is not an idempotent matrix.

Appendix C

Flow Chart of Variable Selection Using MC



Flow Chart of Variable Selection Using T_D



Appendix D

Variable Selection Using Modified C_p (MC) and the C_p Test Statistic (T_D)

In this study, model selection is carried out by backward elimination procedure for variable selection in the multivariate linear regression model for five datasets. In all cases, correlation between the equations was 0.3, 0.4, 0.5, 0.7 and 0.8 respectively. Data in Table D.1 to Table D.5 are the results of variable selection using modified C_p (MC). Data in Table D.6 to Table D.10 are the results of variable selection using C_p test statistic (T_D).

Table D.1 The *MC* Values for Variable Selection When $\rho_{12} = 0.3$

Iteration	Model	Variables in the Model	Replicate									
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1074	5.1270	5.0320	5.0182	4.9814	5.0638	5.0244	4.9483	5.0117	5.0087
1.2	Reduce Model	x_2, x_3, x_4	17.6980	17.4340	19.8389	21.6077	19.2568	18.7152	19.1186	24.3355	24.7332	20.6094
1.3	Reduce Model	x_1, x_3, x_4	177.5557	179.2131	184.2187	184.8774	171.8315	171.9829	180.1775	182.0275	183.5898	186.8401
1.4	Reduce Model	x_1, x_2, x_4	3.1549*	3.9863	3.4869*	3.0135*	4.5806	3.3230*	6.5366	4.0110	3.7640	5.0205
1.5	Reduce Model	x_1, x_2, x_3	3.4888	3.0878*	5.8693	4.3844	3.5036*	5.7649	4.5276*	3.1530*	3.0588*	3.6865*
2.1	Reference Model		3.1549	3.0878	3.4869	3.0135	3.5036	3.3230	4.5276	3.1530	3.0588	3.6865
2.2	Reduce Model	x_2, x_4	15.7456	-	18.2283	19.6049	-	17.1495	-	-	-	-
2.3	Reduce Model	x_1, x_4	176.5047	-	184.3978	184.2602	-	170.2517	-	-	-	-
2.4	Reduce Model	x_2, x_3	-	15.3875	-	-	18.4780	-	18.2566	22.6691	22.9191	19.2781
2.5	Reduce Model	x_1, x_3	-	177.3176	-	-	170.6331	-	181.4838	181.9384	181.6580	185.5793
2.6	Reduce Model	x_1, x_2	1.5533*	1.9418*	4.2224	2.3825*	3.1549*	3.9582	5.6850	2.1494*	1.7908*	3.5822*
3.1	Reference Model		1.5533	1.9418		2.3825	3.1549			2.1494	1.7908	3.5822
3.2	Reduce Model	x_2	14.5442	14.1386		18.6987	18.2133			21.9211	21.3855	19.3181
3.3	Reduce Model	x_1	176.3583	176.2332		183.7983	170.2543			181.9773	181.6085	185.6296

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9064	4.8854	4.7826	5.0079	5.0903	5.0132	5.0269	4.8232	5.1024	4.9322
1.2	Reduce Model	x_2, x_3, x_4	24.8148	25.8161	26.2206	25.5232	16.2771	18.4919	22.6233	28.7609	20.6475	27.1471
1.3	Reduce Model	x_1, x_3, x_4	187.1874	176.8383	173.9015	178.6724	187.7670	178.4373	181.1450	180.7314	186.4474	184.1692
1.4	Reduce Model	x_1, x_2, x_4	3.2747*	2.9325*	4.5447*	3.0270*	4.6925	3.3937*	3.0653*	3.0957*	4.3728	4.2079
1.5	Reduce Model	x_1, x_2, x_3	7.8963	4.0656	5.8982	3.2538	3.5457*	4.3261	3.5734	3.5419	3.0643*	3.0436*
2.1	Reference Model		3.2747	2.9325	4.5447	3.0270	3.5457	3.3937	3.0653	3.0957	3.0643	3.0436
2.2	Reduce Model	x_2, x_4	23.0407	23.9031	25.3066	23.6247	-	16.7304	20.6953	26.9270	-	-
2.3	Reduce Model	x_1, x_4	185.6965	175.1729	173.6502	177.0494	-	177.5617	182.0340	179.0234	-	-
2.4	Reduce Model	x_2, x_3	-	-	-	-	14.7634	-	-	-	18.6089	25.8495
2.5	Reduce Model	x_1, x_3	-	-	-	-	187.8921	-	-	-	184.6862	182.3395
2.6	Reduce Model	x_1, x_2	6.2364	2.1138*	5.9453	1.2834*	2.9744*	2.6227*	1.6368*	1.7938*	2.3374*	2.2918*
3.1	Reference Model			2.1138		1.2834	2.9744	2.6227	1.6368	1.7938	2.3374	2.2918
3.2	Reduce Model	x_2		23.2170		21.7693	14.0073	16.6979	19.1332	25.3529	17.7043	24.9946
3.3	Reduce Model	x_1		174.5031		176.6919	187.3209	176.8017	181.4545	178.4043	185.0807	182.7623

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1577	5.0887	5.0163	5.0085	5.1424	5.0904	5.0274	4.9661	4.9733	4.8230
1.2	Reduce Model	x_2, x_3, x_4	16.4719	18.4997	19.3425	22.0599	14.9741	18.4421	20.1443	24.6952	21.9393	29.8760
1.3	Reduce Model	x_1, x_3, x_4	187.7409	185.4487	185.3771	179.0773	179.9067	178.5888	178.3782	178.6924	185.2778	178.2158
1.4	Reduce Model	x_1, x_2, x_4	3.2582	3.1460*	4.6347	3.0205*	3.3382	3.9765*	3.7058	3.0507*	3.1436*	4.4621
1.5	Reduce Model	x_1, x_2, x_3	3.1152*	3.6559	3.1674*	4.6569	3.1730*	4.1889	3.0986*	3.7486	6.5146	3.5853*
2.1	Reference Model		3.1152	3.1460	3.1674	3.0205	3.1730	3.9765	3.0986	3.0507	3.1436	3.5853
2.2	Reduce Model	x_2, x_4	-	16.7244	-	20.1834	-	17.3150	-	22.7918	20.0631	-
2.3	Reduce Model	x_1, x_4	-	185.0332	-	179.8510	-	177.5368	-	177.0588	183.6746	-
2.4	Reduce Model	x_2, x_3	14.4110	-	17.4643	-	13.0149	-	18.4358	-	-	28.5077
2.5	Reduce Model	x_1, x_3	186.2042	-	183.8680	-	178.5250	-	176.6246	-	-	177.0607
2.6	Reduce Model	x_1, x_2	1.2135*	1.7401*	2.7413*	2.6832*	1.3605*	3.2126*	1.7397*	1.8966*	4.6542	3.4226*
3.1	Reference Model		1.2135	1.7401	2.7413	2.6832	1.3605	3.2126	1.7397	1.8966		3.4226
3.2	Reduce Model	x_2	12.5086	15.6862	17.1275	19.7666	11.2149	16.6375	16.8824	21.4091		28.2457
3.3	Reduce Model	x_1	184.3944	183.6085	183.5139	179.5163	177.8072	179.2030	175.3413	175.8861		176.9526

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	32	33	34	35	36	37	38	39	40
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9800	4.9367	5.0005	4.9823	4.9115	5.0234	5.0659	4.9992	5.0527	5.1777
1.2	Reduce Model	x_2, x_3, x_4	19.7991	24.4213	18.0932	22.4259	22.7056	21.9052	17.5363	24.4816	20.4207	16.7062
1.3	Reduce Model	x_1, x_3, x_4	185.2202	178.9041	177.8069	181.0173	179.7808	178.3989	188.4450	176.0178	187.8261	185.0277
1.4	Reduce Model	x_1, x_2, x_4	6.3739	3.2276*	3.5962*	3.0081*	6.0996	3.1258*	4.1168	4.6250	4.4695	4.3081
1.5	Reduce Model	x_1, x_2, x_3	3.0854*	4.5159	7.7391	5.0107	4.7188*	4.3603	3.0584*	3.3044*	3.0631*	3.2956*
2.1	Reference Model		3.0854	3.2276	3.5962	3.0081	4.7188	3.1258	3.0584	3.3044	3.0631	3.2956
2.2	Reduce Model	x_2, x_4	-	22.5140	16.6403	20.4429	-	19.9207	-	-	-	-
2.3	Reduce Model	x_1, x_4	-	177.1472	176.4792	181.9222	-	177.8844	-	-	-	-
2.4	Reduce Model	x_2, x_3	18.1225	-	-	-	22.4475	-	15.7389	22.5332	18.8262	14.9271
2.5	Reduce Model	x_1, x_3	184.2494	-	-	-	179.5086	-	186.5933	174.2925	186.0172	183.5037
2.6	Reduce Model	x_1, x_2	4.4892	2.7773*	6.0219	3.0496	5.6253	2.4640*	2.1125*	2.9388*	2.4778*	2.3313*
3.1	Reference Model			2.7773				2.4640	2.1125	2.9388	2.4778	2.3313
3.2	Reduce Model	x_2		22.0766				18.8852	14.7909	22.5022	18.0167	14.0483
3.3	Reduce Model	x_1		179.2041				178.1054	187.7002	174.2093	185.5570	183.8686

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1242	5.1343	4.8421	5.0907	4.9337	5.0417	5.0582	5.1601	5.1580	5.0042
1.2	Reduce Model	x_2, x_3, x_4	19.2832	17.8596	29.1027	20.4752	25.3974	23.1945	22.9894	18.1675	20.7051	23.7412
1.3	Reduce Model	x_1, x_3, x_4	173.4440	187.2677	180.1819	188.8880	179.8810	184.5593	181.2760	189.6933	187.8898	179.0948
1.4	Reduce Model	x_1, x_2, x_4	3.0902*	5.1354	3.1156*	3.0826*	2.9951*	3.4593*	3.3457	3.4990	3.2204*	3.4038
1.5	Reduce Model	x_1, x_2, x_3	3.4573	3.1383*	4.1681	3.1808	3.0498	3.9330	3.0415*	3.1706*	3.7371	3.2194*
2.1	Reference Model		3.0902	3.1383	3.1156	3.0826	2.9951	3.4593	3.0415	3.1706	3.2204	3.2194
2.2	Reduce Model	x_2, x_4	17.2667	-	27.4541	18.5186	23.4375	21.4026	-	-	18.8429	-
2.3	Reduce Model	x_1, x_4	171.4580	-	179.4070	186.9606	177.9976	183.7217	-	-	186.0324	-
2.4	Reduce Model	x_2, x_3	-	16.0019	-	-	-	-	21.0445	16.1852	-	22.0075
2.5	Reduce Model	x_1, x_3	-	185.2765	-	-	-	-	179.7678	187.7085	-	178.4960
2.6	Reduce Model	x_1, x_2	1.4188*	3.2265	2.4359*	1.1727*	1.1113*	2.4158*	1.3337*	1.4708*	1.8193*	1.6349*
3.1	Reference Model		1.4188		2.4359	1.1727	1.1113	2.4158	1.3337	1.4708	1.8193	1.6349
3.2	Reduce Model	x_2	15.7100		26.4088	17.0577	21.8655	20.5212	19.3308	14.5037	17.4499	20.5246
3.3	Reduce Model	x_1	169.7806		178.8494	185.9941	176.7140	183.0381	178.1450	187.9599	186.9659	178.6244

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	52	53	54	55	56	57	58	59	60
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0418	5.0743	5.1746	5.1300	5.0718	4.9290	5.1816	4.7511	5.0334	5.0679
1.2	Reduce Model	x_2, x_3, x_4	19.2244	20.3736	18.6029	21.3129	22.3604	29.4702	16.2249	35.0080	22.1074	16.3591
1.3	Reduce Model	x_1, x_3, x_4	183.0778	186.9980	163.7221	182.9631	181.6997	183.8748	187.0888	181.1043	174.6045	187.3547
1.4	Reduce Model	x_1, x_2, x_4	3.4277*	4.2241	3.6172*	3.0860*	3.1193*	3.9108	3.2122*	2.8690*	3.2400*	4.2359
1.5	Reduce Model	x_1, x_2, x_3	5.6027	3.1813*	3.8095	3.7970	3.9046	3.0873*	3.9481	2.8832	3.2503	3.8503*
2.1	Reference Model		3.4277	3.1813	3.6172	3.0860	3.1193	3.0873	3.2122	2.8690	3.2400	3.8503
2.2	Reduce Model	x_2, x_4	17.5369	-	16.9717	19.2738	20.4913	-	14.2499	33.1185	20.3188	-
2.3	Reduce Model	x_1, x_4	181.6510	-	164.4656	184.8017	182.7127	-	185.2118	179.2254	175.5839	-
2.4	Reduce Model	x_2, x_3	-	18.5439	-	-	-	27.7058	-	-	-	15.5431
2.5	Reduce Model	x_1, x_3	-	185.1106	-	-	-	182.5382	-	-	-	188.2113
2.6	Reduce Model	x_1, x_2	3.9040	2.2622*	2.1514*	1.7523*	1.9656*	2.0699*	1.9554*	1.0032*	1.4667*	2.9331*
3.1	Reference Model			2.2622	2.1514	1.7523	1.9656	2.0699	1.9554	1.0032	1.4667	2.9331
3.2	Reduce Model	x_2		17.4835	15.5094	17.8962	19.0882	26.3928	13.0234	31.2524	18.5926	14.3709
3.3	Reduce Model	x_1		186.0233	165.7332	183.9362	183.3522	181.5301	183.9475	177.6306	174.9650	187.9973

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	62	63	64	65	66	67	68	69	70
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0777	5.1359	5.0545	5.0999	5.1553	5.1438	5.1237	4.9948	5.0182	5.1243
1.2	Reduce Model	x_2, x_3, x_4	22.2040	19.3787	22.6583	20.0993	17.8153	16.5709	15.1823	25.3375	19.8370	15.1143
1.3	Reduce Model	x_1, x_3, x_4	181.5872	180.3302	188.0692	175.9929	172.3081	186.2650	185.1678	185.0053	179.9955	173.3326
1.4	Reduce Model	x_1, x_2, x_4	3.3440*	3.1537*	3.4385*	3.2458	3.4626*	3.1550*	3.8118*	3.8167*	3.5367	3.4155*
1.5	Reduce Model	x_1, x_2, x_3	4.2102	3.1599	3.7238	3.1327*	3.8653	3.5938	3.9557	3.9609	3.2472*	5.0011
2.1	Reference Model		3.3440	3.1537	3.4385	3.1327	3.4626	3.1550	3.8118	3.8167	3.2472	3.4155
2.2	Reduce Model	x_2, x_4	20.4102	17.3956	21.0081	-	16.1214	14.7120	13.8671	24.0808	-	13.5311
2.3	Reduce Model	x_1, x_4	179.8557	178.5727	187.4435	-	170.7070	184.3206	184.3002	184.8867	-	173.1780
2.4	Reduce Model	x_2, x_3	-	-	-	18.1489	-	-	-	-	18.0345	-
2.5	Reduce Model	x_1, x_3	-	-	-	174.1688	-	-	-	-	178.8765	-
2.6	Reduce Model	x_1, x_2	2.4457*	1.1922*	2.0450*	1.2778*	2.1611*	1.6394*	2.6250*	2.6239*	1.7928*	3.1423*
3.1	Reference Model		2.4457	1.1922	2.0450	1.2778	2.1611	1.6394	2.6250	2.6239	1.7928	3.1423
3.2	Reduce Model	x_2	19.5273	15.4077	19.3779	16.5984	14.8022	13.7019	12.8605	22.6945	16.5231	13.3386
3.3	Reduce Model	x_1	179.1533	176.7297	186.0281	172.2963	169.8207	182.8278	183.4854	183.7525	178.5964	172.9371

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	72	73	74	75	76	77	78	79	80
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0495	5.0421	5.0121	5.1880	5.0472	5.0154	5.0676	5.1059	4.9400	5.0825
1.2	Reduce Model	x_2, x_3, x_4	21.9035	19.5663	23.4539	14.1396	17.7980	18.8320	18.5612	17.2230	23.9262	16.9661
1.3	Reduce Model	x_1, x_3, x_4	169.5981	178.7926	169.8979	184.7686	180.3037	189.3395	172.3368	185.1382	184.5259	189.7944
1.4	Reduce Model	x_1, x_2, x_4	3.7847	4.8094	3.3922	3.3674	3.3277*	3.0295*	3.0510*	4.5037	2.9724*	3.4904*
1.5	Reduce Model	x_1, x_2, x_3	3.2096*	3.1896*	3.2787*	3.2897*	5.2409	7.5029	3.4162	3.0940*	3.5515	5.6282
2.1	Reference Model		3.2096	3.1896	3.2787	3.2897	3.3277	3.0295	3.0510	3.0940	2.9724	3.4904
2.2	Reduce Model	x_2, x_4	-	-	-	-	16.1007	16.8436	16.7778	-	22.2685	15.2589
2.3	Reduce Model	x_1, x_4	-	-	-	-	181.6902	187.6557	171.3079	-	183.0536	188.3461
2.4	Reduce Model	x_2, x_3	20.0757	17.6512	21.6278	12.3841	-	-	-	15.3913	-	-
2.5	Reduce Model	x_1, x_3	168.3460	179.3590	168.2568	182.8694	-	-	-	183.7795	-	-
2.6	Reduce Model	x_1, x_2	1.9138*	3.0550*	1.7062*	1.4519*	3.5388	5.4969	1.3958*	2.4924*	1.6004*	4.0257
3.1	Reference Model		1.9138	3.0550	1.7062	1.4519			1.3958	2.4924	1.6004	
3.2	Reduce Model	x_2	18.5165	17.5621	20.0119	10.6037			15.3594	15.1116	21.5617	
3.3	Reduce Model	x_1	167.1914	179.9139	166.7348	181.0303			169.6475	183.1702	184.5356	

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	82	83	84	85	86	87	88	89	90
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9003	5.0523	5.0275	4.9882	4.9817	5.0976	4.9560	5.0807	5.0741	5.0782
1.2	Reduce Model	x_2, x_3, x_4	25.8269	21.1575	21.1710	23.0818	23.0520	16.9255	23.9680	20.9519	20.5060	18.6705
1.3	Reduce Model	x_1, x_3, x_4	176.8336	185.6219	188.7288	180.8243	182.0853	185.5112	183.4467	175.9847	178.7063	187.8873
1.4	Reduce Model	x_1, x_2, x_4	4.2969*	3.5655*	4.9421*	3.0496*	3.1238	3.4122	2.9844*	3.2142*	3.0792*	3.3638
1.5	Reduce Model	x_1, x_2, x_3	4.5789	3.8685	5.0802	3.1849	3.0821*	3.0864*	3.1251	3.7485	7.3616	3.1064*
2.1	Reference Model		4.2969	3.5655	4.9421	3.0496	3.0821	3.0864	2.9844	3.2142	3.0792	3.1064
2.2	Reduce Model	x_2, x_4	25.0245	19.9852	20.8026	21.1024	-	-	22.0122	19.1832	18.5475	-
2.3	Reduce Model	x_1, x_4	176.2332	184.5453	188.9140	179.1980	-	-	181.7830	175.9207	176.7965	-
2.4	Reduce Model	x_2, x_3	-	-	-	-	21.2867	14.9343	-	-	-	16.6738
2.5	Reduce Model	x_1, x_3	-	-	-	-	181.7875	183.6210	-	-	-	185.9112
2.6	Reduce Model	x_1, x_2	3.9813*	2.3201*	4.9146*	1.2609*	1.2270*	1.3995*	1.1506*	1.9206*	5.3657	1.3895*
3.1	Reference Model		3.9813	2.3201	4.9146	1.2609	1.2270	1.3995	1.1506	1.9206		1.3895
3.2	Reduce Model	x_2	24.6499	18.5519	20.4831	19.2773	19.4034	13.2369	20.1780	17.9641		15.2251
3.3	Reduce Model	x_1	176.1450	184.2793	189.1876	177.3979	182.4957	182.1820	180.2311	174.6260		184.3059

Table D.1 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
1.3	Reduce Model	x_1, x_3, x_4	183.9486	182.9128	188.0850	175.9898	177.6144	187.4203	187.2232	183.4529	189.5395	177.6589
1.4	Reduce Model	x_1, x_2, x_4	3.0871*	8.0127	3.6732*	3.0198*	3.0833*	3.1142*	3.4829*	3.0704*	3.1433*	6.2852*
1.5	Reduce Model	x_1, x_2, x_3	4.1447	7.3773*	6.1605	3.4430	3.3834	3.1240	3.9498	4.1753	3.6904	8.3136
2.1	Reference Model		3.0871	7.3773	3.6732	3.0198	3.0833	3.1142	3.4829	3.0704	3.1433	6.2852
2.2	Reduce Model	x_2, x_4	16.7559	-	18.4922	18.0926	22.3764	16.6431	18.1040	19.6633	13.8067	22.4608
2.3	Reduce Model	x_1, x_4	181.9872	-	186.9031	174.3143	176.5800	187.0303	185.8204	181.6671	187.5002	179.9519
2.4	Reduce Model	x_2, x_3	-	18.0341	-	-	-	-	-	-	-	-
2.5	Reduce Model	x_1, x_3	-	186.2159	-	-	-	-	-	-	-	-
2.6	Reduce Model	x_1, x_2	2.1165*	10.1830	4.9305	1.4469*	1.4429*	1.1431*	2.3277*	2.1364*	1.6577*	9.1737
3.1	Reference Model		2.1165			1.4469	1.4429	1.1431	2.3277	2.1364	1.6577	
3.2	Reduce Model	x_2	15.9694			16.4201	20.7624	14.6154	17.0579	19.0407	12.3197	
3.3	Reduce Model	x_1	181.0931			173.2156	177.0248	185.0435	184.7181	181.1146	186.1423	

Table D.2 The MC Values for Variable Selection When $\rho_{12} = 0.4$

Iteration	**Model**	**Variables in the Model**	**Replicate**									
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0773	5.0743	5.1357	5.0070	4.9370	5.0564	5.0157	4.9357	4.8715	5.1124
1.2	Reduce Model	x_2, x_3, x_4	20.9237	21.9656	20.2265	22.2279	17.0417	19.1894	19.6851	24.9994	26.4953	16.2050
1.3	Reduce Model	x_1, x_3, x_4	188.2746	177.7452	179.9783	184.6971	186.0886	171.7130	179.9809	181.8294	176.6401	182.4133
1.4	Reduce Model	x_1, x_2, x_4	4.5347	3.7358	3.1019*	3.0068*	6.3592	3.3168*	6.5249	3.9986	2.9242*	3.8251
1.5	Reduce Model	x_1, x_2, x_3	4.3493*	3.2230*	3.6849	4.3773	6.0763*	5.7599	4.5186*	3.1455*	4.0586	3.3347*
2.1	Reference Model		4.3493	3.2230	3.1019	3.0068	6.0763	3.3168	4.5186	3.1455	2.9242	3.3347
2.2	Reduce Model	x_2, x_4	-	-	18.1872	20.2296	-	17.6308	-	-	24.5887	-
2.3	Reduce Model	x_1, x_4	-	-	177.9317	184.0828	-	169.9837	-	-	174.9798	-
2.4	Reduce Model	x_2, x_3	20.0889	20.2234	-	-	19.3398	-	18.8068	23.3426	-	14.6714
2.5	Reduce Model	x_1, x_3	188.1150	175.8205	-	-	188.2962	-	181.2783	181.7452	-	180.6243
2.6	Reduce Model	x_1, x_2	3.5913*	1.9558*	1.6512*	2.3799*	7.7267	3.9546	5.6737	2.1422*	2.1124*	2.1159*
3.1	Reference Model		3.5913	1.9558	1.6512	2.3799				2.1422	2.1124	2.1159
3.2	Reduce Model	x_2	19.3870	18.8985	16.7530	19.3156				22.6009	23.9133	13.3486
3.3	Reduce Model	x_1	187.3595	175.8320	177.1091	183.6257				181.7882	174.3176	179.6281

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1041	4.8638	5.0835	5.0710	4.9846	4.8087	5.0940	4.9180	5.1494	5.0470
1.2	Reduce Model	x_2, x_3, x_4	19.1071	28.5559	16.7458	18.1457	23.8974	29.4724	21.2015	27.8729	16.9586	21.4241
1.3	Reduce Model	x_1, x_3, x_4	173.9575	179.8560	187.5984	181.6408	187.7067	180.5457	186.2577	183.9788	187.5854	188.5055
1.4	Reduce Model	x_1, x_2, x_4	4.2664	4.6738*	4.6832	3.6657*	4.1161	3.0863*	4.3651	4.2002	3.2539	3.4401
1.5	Reduce Model	x_1, x_2, x_3	3.3576*	5.1845	3.5373*	3.7250	3.6695*	3.5322	3.0592*	3.0350*	3.1098*	3.2033*
2.1	Reference Model		3.3576	4.6738	3.5373	3.6657	3.6695	3.0863	3.0592	3.0350	3.1098	3.2033
2.2	Reduce Model	x_2, x_4	-	28.1468	-	16.7019	-	27.6427	-	-	-	-
2.3	Reduce Model	x_1, x_4	-	181.4286	-	180.8061	-	178.8430	-	-	-	-
2.4	Reduce Model	x_2, x_3	17.2121	-	15.2317	-	22.6934	-	19.1662	26.5980	14.9006	19.6195
2.5	Reduce Model	x_1, x_3	172.5246	-	187.7166	-	186.4294	-	184.4995	182.1542	186.0520	186.6811
2.6	Reduce Model	x_1, x_2	2.6199*	4.8552	2.9647*	2.4120*	2.8104*	1.7893*	2.3330*	2.2897*	1.2123*	1.5866*
3.1	Reference Model		2.6199		2.9647	2.4120	2.8104	1.7893	2.3330	2.2897	1.2123	1.5866
3.2	Reduce Model	x_2	16.4242		14.4632	15.2823	21.7909	26.0702	18.2513	25.7460	13.0017	18.0566
3.3	Reduce Model	x_1	171.7872		187.1441	179.5635	185.5558	178.2301	184.8979	182.5849	184.2461	185.0785

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9975	5.0835	5.0674	4.9545	4.9628	5.0281	5.0379	4.8081	4.9694	5.1496
1.2	Reduce Model	x_2, x_3, x_4	22.6577	18.9173	21.5208	25.3176	22.5541	24.8944	19.0983	30.6188	20.3985	20.5700
1.3	Reduce Model	x_1, x_3, x_4	178.8820	178.3547	180.2505	178.4847	185.1055	187.0622	186.0191	178.0208	185.0145	187.9197
.4	Reduce Model	x_1, x_2, x_4	3.0139*	3.9725*	3.0718*	3.0436*	3.1366*	4.7561	4.2819	4.4512	6.3615	3.3368*
1.5	Reduce Model	x_1, x_2, x_3	4.6503	4.1844	6.6198	3.7412	6.5012	3.0619*	3.3525*	3.5767*	3.0784*	3.4323
2.1	Reference Model		3.0139	3.9725	3.0718	3.0436	3.1366	3.0619	3.3525	3.5767	3.0784	3.3368
2.2	Reduce Model	x_2, x_4	-	-	-	-	-	24.3671	18.3814	30.1885	21.9894	-
2.3	Reduce Model	x_1, x_4	-	-	-	-	-	187.4960	185.6980	177.7450	187.2284	-
2.4	Reduce Model	x_2, x_3	22.2425	18.1441	22.8687	23.8878	23.5540	-	-	-	-	18.8303
2.5	Reduce Model	x_1, x_3	178.5740	179.9347	181.8179	177.2585	187.3660	-	-	-	-	186.3689
2.6	Reduce Model	x_1, x_2	2.6812*	3.2108*	4.6577	1.8937*	4.6445	2.7751*	2.5922*	3.4178*	4.4803	1.5530*
3.1	Reference Model		2.6812	3.2108		1.8937		2.7751	2.5922	3.4178		1.5530
3.2	Reduce Model	x_2	20.3724	17.1114		22.0345		22.4171	16.8521	28.9934		16.9451
3.3	Reduce Model	x_1	179.3272	178.9760		175.6869		185.5065	184.1378	176.7651		185.1669

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.8736	4.9705	5.0548	4.9002	4.9729	5.1743	5.0354	5.1309	4.8459	5.0316
1.2	Reduce Model	x_2, x_3, x_4	26.8922	23.0563	21.3497	23.3550	22.8178	17.0227	20.4410	18.2488	25.4475	23.8027
1.3	Reduce Model	x_1, x_3, x_4	183.0870	180.8292	185.1227	179.5565	173.6376	184.8389	174.8047	187.0909	177.4115	184.3650
1.4	Reduce Model	x_1, x_2, x_4	2.9731*	3.0010*	3.0741*	6.0812	3.3920*	4.3071	5.5693	5.1321	3.7387*	3.4491*
1.5	Reduce Model	x_1, x_2, x_3	4.3819	5.0005	3.1941	4.7134*	4.5223	3.2940*	3.0817*	3.1355*	6.2505	3.9247
2.1	Reference Model		2.9731	3.0010	3.0741	4.7134	3.3920	3.2940	3.0817	3.1355	3.7387	3.4491
2.2	Reduce Model	x_2, x_4	24.9771	21.0788	19.4782	-	21.1657	-	-	-	24.3905	22.0073
2.3	Reduce Model	x_1, x_4	181.1873	181.7346	183.4873	-	172.2515	-	-	-	176.3552	183.5241
2.4	Reduce Model	x_2, x_3	-	-	-	23.1001	-	15.2541	18.4499	16.3952	-	-
2.5	Reduce Model	x_1, x_3	-	-	-	179.2890	-	183.3158	174.3704	185.1008	-	-
2.6	Reduce Model	x_1, x_2	2.4625*	3.0444	1.2130*	5.6131	2.8438*	2.3318*	3.5710	3.2233	5.0246	2.4071*
3.1	Reference Model		2.4625		1.2130		2.8438	2.3318				2.4071
3.2	Reduce Model	x_2	24.1620		17.8991		21.1317	14.3901				21.1328
3.3	Reduce Model	x_1	180.5650		182.4411		172.0631	183.6842				182.8392

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	42	43	44	45	46	47	48	49	50
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0189	5.0472	5.0912	4.9227	4.9636	5.0441	4.9722	5.0324	5.0937	4.8724
1.2	Reduce Model	x_2, x_3, x_4	24.0415	23.6288	19.1774	25.1855	25.9871	22.3155	22.8866	19.7752	17.3798	28.5190
1.3	Reduce Model	x_1, x_3, x_4	181.1760	181.0435	187.5453	177.2541	181.0579	185.6691	183.2266	182.8887	181.6260	179.1695
1.4	Reduce Model	x_1, x_2, x_4	4.2621	3.3403	3.0905*	3.1053*	3.1605*	3.1049*	3.2491*	3.4211*	4.4503	3.1725*
1.5	Reduce Model	x_1, x_2, x_3	3.0357*	3.0349*	5.6272	3.7943	3.5961	3.1876	3.7423	5.5912	3.5580*	5.4735
2.1	Reference Model		3.0357	3.0349	3.0905	3.1053	3.1605	3.1049	3.2491	3.4211	3.5580	3.1725
2.2	Reduce Model	x_2, x_4	-	-	17.2166	23.4233	24.2883	20.4352	21.3913	18.0874	-	26.8103
2.3	Reduce Model	x_1, x_4	-	-	185.6397	175.9212	180.2316	183.8332	181.5636	181.4656	-	178.9419
2.4	Reduce Model	x_2, x_3	22.1717	21.6897	-	-	-	-	-	-	16.3926	-
2.5	Reduce Model	x_1, x_3	181.2401	179.5392	-	-	-	-	-	-	180.2839	-
2.6	Reduce Model	x_1, x_2	2.2696*	1.3328*	3.6444	2.0229*	1.8078*	1.2483*	2.0557*	3.8955	2.7745*	3.8620
3.1	Reference Model		2.2696	1.3328		2.0229	1.8078	1.2483	2.0557		2.7745	
3.2	Reduce Model	x_2	21.3410	19.9806		22.3666	22.8078	18.9748	21.3567		16.1974	
3.3	Reduce Model	x_1	180.4575	177.9221		176.7242	180.1153	183.1119	180.5160		181.1926	

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	52	53	54	55	56	57	58	59	60
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9890	5.1482	5.0630	4.9699	4.9155	5.1756	4.9279	5.0685	5.1272	4.9591
1.2	Reduce Model	x_2, x_3, x_4	22.5716	18.2183	21.0158	18.5339	30.2018	16.6400	29.6091	22.7718	19.8883	24.1670
1.3	Reduce Model	x_1, x_3, x_4	182.0770	186.8921	186.8215	180.1941	183.6720	186.9164	166.8193	181.3667	180.1301	180.5584
1.4	Reduce Model	x_1, x_2, x_4	3.0127*	3.9797	4.2175	5.8831*	3.9011	3.2085*	3.1397	3.3379*	3.1486*	2.9976*
1.5	Reduce Model	x_1, x_2, x_3	3.4006	3.1787*	3.1743*	6.3150	3.0798*	3.9451	3.0904*	4.2021	3.1541	6.8975
2.1	Reference Model		3.0127	3.1787	3.1743	5.8831	3.0798	3.2085	3.0904	3.3379	3.1486	2.9976
2.2	Reduce Model	x_2, x_4	20.6512	-	-	19.8245	-	14.6668	-	20.9783	17.9088	22.2164
2.3	Reduce Model	x_1, x_4	183.7267	-	-	182.8119	-	185.0414	-	179.6381	178.3757	182.0484
2.4	Reduce Model	x_2, x_3	-	16.2630	19.1930	-	28.4478	-	27.7445	-	-	-
2.5	Reduce Model	x_1, x_3	-	185.0007	184.9384	-	182.3409	-	165.6544	-	-	-
2.6	Reduce Model	x_1, x_2	1.4558*	1.9691*	2.2599*	7.2438	2.0661*	1.9546*	1.2828*	2.4408*	1.1901*	4.9553
3.1	Reference Model		1.4558	1.9691	2.2599		2.0661	1.9546	1.2828	2.4408	1.1901	
3.2	Reduce Model	x_2	18.9860	15.1362	18.1282		27.1329	13.4463	25.9970	20.0972	15.9233	
3.3	Reduce Model	x_1	183.1036	183.7998	185.8575		181.3368	183.7799	164.4662	178.9385	176.5351	

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	62	63	64	65	66	67	68	69	70
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0909	4.7763	5.1488	5.1092	5.0767	4.9224	5.1796	5.0365	4.9819	5.0063
1.2	Reduce Model	x_2, x_3, x_4	20.6376	29.9055	18.2678	18.7369	19.4353	22.6382	17.1673	21.4072	26.0580	20.4660
1.3	Reduce Model	x_1, x_3, x_4	175.7453	182.7380	172.0341	183.4777	182.0059	172.1408	191.2009	184.7823	184.8121	179.7845
1.4	Reduce Model	x_1, x_2, x_4	3.2413	6.2865	3.4594*	3.6521	4.1710	3.5017*	3.2798	3.0378*	3.8107*	3.5309
1.5	Reduce Model	x_1, x_2, x_3	3.1274*	2.9737*	3.8629	3.3588*	3.6870*	4.1969	3.1128*	4.5422	3.9516	3.2395*
2.1	Reference Model		3.1274	2.9737	3.4594	3.3588	3.6870	3.5017	3.1128	3.0378	3.8107	3.2395
2.2	Reduce Model	x_2, x_4	-	-	16.5770	-	-	21.7572	-	19.4106	24.8020	-
2.3	Reduce Model	x_1, x_4	-	-	170.4352	-	-	172.3086	-	182.7935	184.6961	-
2.4	Reduce Model	x_2, x_3	18.6906	28.1777	-	17.2517	18.1089	-	15.1389	-	-	18.6667
2.5	Reduce Model	x_1, x_3	173.9251	181.7252	-	181.7772	180.7105	-	189.4423	-	-	178.6705
2.6	Reduce Model	x_1, x_2	1.2770*	4.4747	2.1620*	1.8782*	2.8010*	2.6674*	1.2131*	2.5553*	2.6214*	1.7911*
3.1	Reference Model		1.2770		2.1620	1.8782	2.8010	2.6674	1.2131	2.5553	2.6214	1.7911
3.2	Reduce Model	x_2	17.1472		15.2580	15.5593	17.1037	21.2567	13.2363	18.7606	23.4109	17.1583
3.3	Reduce Model	x_1	172.0570		169.5539	180.4761	180.9656	173.8199	187.5408	182.9448	183.5661	178.3924

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0391	4.7757	5.0317	5.0016	5.1872	5.1088	4.8930	5.1282	5.1494	5.0386
1.2	Reduce Model	x_2, x_3, x_4	22.4913	32.0235	20.1491	24.0408	14.3839	20.6740	24.7895	18.8344	20.7243	18.3000
1.3	Reduce Model	x_1, x_3, x_4	169.3552	180.3845	178.5843	169.6529	184.5401	175.7461	183.9163	181.7424	175.2240	180.1169
1.4	Reduce Model	x_1, x_2, x_4	3.7752	3.0237*	4.8030	3.3861	3.3663	4.4849	4.9434*	3.8198	3.2951	3.3208*
1.5	Reduce Model	x_1, x_2, x_3	3.2043*	3.2650	3.1823*	3.2709*	3.2896*	3.1741*	6.5754	3.1544*	3.1539*	5.2342
2.1	Reference Model		3.2043	3.0237	3.1823	3.2709	3.2896	3.1741	4.9434	3.1544	3.1539	3.3208
2.2	Reduce Model	x_2, x_4	-	30.3204	-	-	-	-	24.9553	-	-	16.6050
2.3	Reduce Model	x_1, x_4	-	180.6238	-	-	-	-	184.1790	-	-	181.4975
2.4	Reduce Model	x_2, x_3	20.6672	-	18.2354	22.2158	12.6246	18.7765	-	16.8576	18.7937	-
2.5	Reduce Model	x_1, x_3	168.1065	-	179.1477	168.0155	182.6414	176.3848	-	181.5462	173.2419	-
2.6	Reduce Model	x_1, x_2	1.9094*	1.5106*	3.0515*	1.7027*	1.4513*	2.5267*	6.4263	1.8166*	1.2917*	3.5337
3.1	Reference Model		1.9094	1.5106	3.0515	1.7027	1.4513	2.5267		1.8166	1.2917	
3.2	Reduce Model	x_2	19.1021	28.6112	18.1524	20.6027	10.8383	17.9001		15.8769	16.9393	
3.3	Reduce Model	x_1	166.9547	179.2358	179.7092	166.4981	180.8020	177.6910		180.2843	171.7490	

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0576	4.8865	4.9689	4.9418	5.0672	5.0677	4.9296	5.1063	5.0797	4.8436
1.2	Reduce Model	x_2, x_3, x_4	19.1033	26.5400	23.7157	24.6741	20.9820	19.2614	15.4575	20.7753	18.9678	21.2874
1.3	Reduce Model	x_1, x_3, x_4	172.0921	176.6313	181.9064	183.2625	178.4689	187.6931	182.7059	184.9308	187.2588	177.4256
1.4	Reduce Model	x_1, x_2, x_4	3.0451*	4.2825*	3.1160	2.9759*	3.0753*	3.3568	7.9973	3.3702*	3.1080*	6.2722*
1.5	Reduce Model	x_1, x_2, x_3	3.4070	4.5669	3.0745*	3.1145	7.3498	3.0994*	7.3691*	4.4588	3.1166	8.2916
2.1	Reference Model		3.0451	4.2825	3.0745	2.9759	3.0753	3.0994	7.3691	3.3702	3.1080	6.2722
2.2	Reduce Model	x_2, x_4	17.3344	25.7326	-	22.7246	19.0260	-	-	18.9792	17.2220	23.0479
2.3	Reduce Model	x_1, x_4	171.0660	176.0300	-	181.6043	176.5618	-	-	183.3120	186.8708	179.7212
2.4	Reduce Model	x_2, x_3	-	-	21.9598	-	-	17.2677	18.4088	-	-	-
2.5	Reduce Model	x_1, x_3	-	-	181.6112	-	-	185.7207	186.0123	-	-	-
2.6	Reduce Model	x_1, x_2	1.3906*	3.9687*	1.2244*	1.1457*	5.3568	1.3861*	10.1653	2.7567*	1.1401*	9.1498
3.1	Reference Model		1.3906	3.9687	1.2244	1.1457		1.3861		2.7567	1.1401	
3.2	Reduce Model	x_2	15.9234	25.3582	20.0807	20.8941		15.8342		18.3701	15.1976	
3.3	Reduce Model	x_1	169.4074	175.9396	182.3238	180.0576		184.1194		182.6912	184.8873	

Table D.2 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	92	93	94	95	96	97	98	99	100
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1334	4.9674	5.1199	5.0171	4.8432	5.0656	5.1090	5.0404	5.0367	5.1738
1.2	Reduce Model	x_2, x_3, x_4	19.9613	22.9232	22.2009	20.3841	28.5650	22.1051	19.9526	18.9047	23.5910	17.2842
1.3	Reduce Model	x_1, x_3, x_4	179.1065	184.0588	183.0721	184.6114	185.2402	178.5543	183.4762	181.3332	175.4541	174.5131
1.4	Reduce Model	x_1, x_2, x_4	3.3276*	3.9875	3.1907*	3.6716	4.1041	3.3495	3.5660	5.6205	3.1012*	3.1372
1.5	Reduce Model	x_1, x_2, x_3	3.7982	3.8661*	4.2909	3.5500*	3.9066*	3.2069*	3.0684*	4.7815*	3.2336	3.1112*
2.1	Reference Model		3.3276	3.8661	3.1907	3.5500	3.9066	3.2069	3.0684	4.7815	3.1012	3.1112
2.2	Reduce Model	x_2, x_4	18.3648	-	20.3566	-	-	-	-	-	22.1631	-
2.3	Reduce Model	x_1, x_4	177.3418	-	181.1785	-	-	-	-	-	175.0942	-
2.4	Reduce Model	x_2, x_3	-	22.1334	-	18.9073	28.0908	20.1818	17.9101	18.7668	-	15.2367
2.5	Reduce Model	x_1, x_3	-	183.3702	-	183.4772	184.3898	179.7611	182.1365	181.8496	-	172.4543
2.6	Reduce Model	x_1, x_2	1.9789*	2.7600*	2.3654*	2.2790*	3.4893*	1.4783*	1.5243*	5.2255	1.2814*	1.0748*
3.1	Reference Model		1.9789	2.7600	2.3654	2.2790	3.4893	1.4783	1.5243		1.2814	1.0748
3.2	Reduce Model	x_2	17.0029	21.4318	19.8120	17.3097	27.4258	19.0193	16.1536		20.2952	13.4650
3.3	Reduce Model	x_1	176.7728	182.5814	180.6275	182.9339	184.5056	179.3221	182.7409		173.2410	173.3437

Table D.3 The MC Values for Variable Selection When $\rho_{12} = 0.5$

Iteration	Model	Variables in the Model	Replicate									
			1	2	3	4	5	6	7	8	9	10
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0661	4.9592	5.0590	4.8377	5.1234	4.9918	5.1213	4.9258	5.0454	5.0022
1.2	Reduce Model	x_2, x_3, x_4	21.4935	25.5301	22.6611	26.8789	20.8332	22.9478	17.7138	17.5488	19.7304	20.3899
1.3	Reduce Model	x_1, x_3, x_4	188.1998	179.5388	177.6099	182.1276	179.8164	184.5700	180.9705	185.9854	171.4977	179.8395
1.4	Reduce Model	x_1, x_2, x_4	4.5242	2.9807*	3.7164	3.4831*	3.0931*	2.9977*	4.2320	6.3376	3.3040*	6.5055
1.5	Reduce Model	x_1, x_2, x_3	4.3388*	3.0009	3.2189*	5.1094	3.6877	4.3752	3.1002*	6.0502*	5.7615	4.5014*
2.1	Reference Model		4.3388	2.9807	3.2189	3.4831	3.0931	2.9977	3.1002	6.0502	3.3040	4.5014
2.2	Reduce Model	x_2, x_4	-	23.6522	-	25.7421	18.8006	20.9557	-	-	18.1754	-
2.3	Reduce Model	x_1, x_4	-	177.6028	-	181.0747	177.7740	183.9608	-	-	169.7680	-
2.4	Reduce Model	x_2, x_3	20.6544	-	20.9249	-	-	-	15.7104	19.8782	-	19.4906
2.5	Reduce Model	x_1, x_3	188.0388	-	175.6984	-	-	-	181.1945	188.1845	-	181.1248
2.6	Reduce Model	x_1, x_2	3.5820*	1.0213*	1.9493*	3.5860	1.6583*	2.3839*	2.2072*	7.6888	3.9551	5.6524
3.1	Reference Model		3.5820	1.0213	1.9493		1.6583	2.3839	2.2072			
3.2	Reduce Model	x_2	19.9562	21.7101	19.5960		17.3736	20.0393	14.7302			
3.3	Reduce Model	x_1	187.2847	176.4480	175.7009		176.9642	183.5169	180.3103			

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9196	4.8534	5.1021	4.9966	5.0932	4.8336	4.8458	5.0733	5.0425	5.0600
1.2	Reduce Model	x_2, x_3, x_4	25.7499	27.2540	16.6810	14.1974	19.6358	32.0543	29.3905	17.3193	23.3496	18.7203
1.3	Reduce Model	x_1, x_3, x_4	181.6780	176.4959	182.2704	182.2406	173.7904	181.9442	179.7347	187.4788	180.8573	181.4656
1.4	Reduce Model	x_1, x_2, x_4	3.9763	2.9133*	3.8184	5.5497*	4.2442	2.9064*	4.6611*	4.6609	3.0474*	3.6566*
1.5	Reduce Model	x_1, x_2, x_3	3.1377*	4.0621	3.3334*	6.6534	3.3415*	2.9644	5.1419	3.5147*	3.3481	3.7164
2.1	Reference Model		3.1377	2.9133	3.3334	5.5497	3.3415	2.9064	4.6611	3.5147	3.0474	3.6566
2.2	Reduce Model	x_2, x_4	-	25.3553	-	14.9395	-	30.2656	28.9809	-	21.3745	17.2766
2.3	Reduce Model	x_1, x_4	-	174.8422	-	182.8342	-	180.2860	181.3149	-	179.0471	180.6298
2.4	Reduce Model	x_2, x_3	24.1072	-	15.1754	-	17.7356	-	-	15.7936	-	-
2.5	Reduce Model	x_1, x_3	181.6000	-	180.4905	-	172.3576	-	-	187.5737	-	-
2.6	Reduce Model	x_1, x_2	2.1281*	2.1231*	2.1189*	6.9461	2.5901*	1.0394*	4.8189	2.9340*	1.3415*	2.4051*
3.1	Reference Model		2.1281	2.1231	2.1189		2.5901	1.0394		2.9340	1.3415	2.4051
3.2	Reduce Model	x_2	23.3647	24.7070	13.8494		16.9327	28.3496		15.0050	19.7273	15.8500
3.3	Reduce Model	x_1	181.6430	174.2010	179.4992		171.6085	179.0875		186.9930	179.0181	179.3889

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9702	4.7899	5.0820	4.8994	5.1363	5.1170	5.0317	4.9822	5.0729	5.0550
1.2	Reduce Model	x_2, x_3, x_4	24.6027	30.2873	21.8308	28.6933	17.5678	15.2359	22.1665	23.3539	19.4708	22.1517
1.3	Reduce Model	x_1, x_3, x_4	187.5984	180.4145	186.1372	183.8589	187.4841	188.7333	188.3828	178.7357	178.1715	180.0966
1.4	Reduce Model	x_1, x_2, x_4	4.0891	3.0739*	4.3532	4.2024	3.2500	3.7736*	3.4284	3.0051*	3.9698*	3.0625*
1.5	Reduce Model	x_1, x_2, x_3	3.6523*	3.5223	3.0520*	3.0239*	3.1006*	4.3708	3.1845*	4.6510	4.1804	6.6090
2.1	Reference Model		3.6523	3.0739	3.0520	3.0239	3.1006	3.7736	3.1845	3.0051	3.9698	3.0625
2.2	Reduce Model	x_2, x_4	-	28.4633	-	-	-	13.8603	-	21.4918	18.3272	20.1686
2.3	Reduce Model	x_1, x_4	-	178.7184	-	-	-	190.3235	-	179.5188	177.1285	178.3028
2.4	Reduce Model	x_2, x_3	23.3986	-	19.8004	27.4458	15.5144	-	20.3586	-	-	-
2.5	Reduce Model	x_1, x_3	186.3175	-	184.3828	182.0417	185.9572	-	186.5598	-	-	-
2.6	Reduce Model	x_1, x_2	2.7804*	1.7858*	2.3255*	2.2992*	1.2124*	2.9983*	1.5718*	2.6889*	3.2152*	4.6485
3.1	Reference Model		2.7804	1.7858	2.3255	2.2992	1.2124	2.9983	1.5718	2.6889	3.2152	
3.2	Reduce Model	x_2	22.4824	26.8948	18.8735	26.6091	13.6238	12.9335	18.8017	21.0888	17.6714	
3.3	Reduce Model	x_1	185.4321	178.1146	184.7840	182.4900	184.1618	189.5551	184.9615	179.2051	178.8081	

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	32	33	34	35	36	37	38	39	40
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9399	4.9492	5.0139	5.0256	4.7893	4.9541	5.1371	5.1382	4.9555	5.0407
1.2	Reduce Model	x_2, x_3, x_4	25.9832	23.2196	25.6024	19.6708	31.4177	21.1416	19.2221	21.1617	23.7421	22.0343
1.3	Reduce Model	x_1, x_3, x_4	178.3411	184.9974	186.9541	185.9285	177.8985	184.8442	187.9966	187.8295	180.7071	184.9817
1.4	Reduce Model	x_1, x_2, x_4	3.0348*	3.1268*	4.7573	4.2595	4.4452	6.3468	3.1115*	3.3313*	2.9916*	3.0653*
1.5	Reduce Model	x_1, x_2, x_3	3.7359	6.4856	3.0545*	3.3403*	3.5723*	3.0669*	3.1134	3.4240	4.9926	3.1796
2.1	Reference Model		3.0348	3.1268	3.0545	3.3403	3.5723	3.0669	3.1115	3.3313	2.9916	3.0653
2.2	Reduce Model	x_2, x_4	24.0907	21.3482	-	-	-	-	17.3282	19.3399	21.7726	20.1669
2.3	Reduce Model	x_1, x_4	176.7182	183.4029	-	-	-	-	186.0685	186.7769	181.6102	183.3489
2.4	Reduce Model	x_2, x_3	-	-	23.6707	18.1508	30.0622	19.4861	-	-	-	-
2.5	Reduce Model	x_1, x_3	-	-	184.9882	184.3713	176.7642	183.8840	-	-	-	-
2.6	Reduce Model	x_1, x_2	1.8942*	4.6329	2.7829*	2.5700*	3.4276*	4.4694	1.0837*	1.5504*	3.0433	1.2040*
3.1	Reference Model		1.8942		2.7829	2.5700	3.4276		1.0837	1.5504		1.2040
3.2	Reduce Model	x_2	22.7074		23.1286	17.4203	29.8133		15.2954	17.5380		18.5929
3.3	Reduce Model	x_1	175.5585		185.4195	184.0324	176.6705		184.2409	185.0849		182.3079

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	42	43	44	45	46	47	48	49	50
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.7487	5.1170	4.8845	4.9996	4.8959	4.9578	4.9480	5.1670	5.0237	4.9527
1.2	Reduce Model	x_2, x_3, x_4	31.0206	22.9982	24.1304	20.7285	28.4026	23.5213	23.7371	17.4152	21.0519	24.4700
1.3	Reduce Model	x_1, x_3, x_4	181.9648	181.2063	179.3638	186.4189	183.5109	173.4526	186.5340	184.7155	174.6055	182.0545
1.4	Reduce Model	x_1, x_2, x_4	3.0406*	3.0741*	6.0444	3.2020*	3.3298	3.3808*	3.3069*	4.3087	5.5247	3.7300
1.5	Reduce Model	x_1, x_2, x_3	3.3748	3.1200	4.7203*	5.1993	3.1763*	4.5028	4.7779	3.2914*	3.0717*	3.0001*
2.1	Reference Model		3.0406	3.0741	4.7203	3.2020	3.1763	3.3808	3.3069	3.2914	3.0717	3.0001
2.2	Reduce Model	x_2, x_4	29.8635	20.9528	-	18.8271	-	21.8709	22.0702	-	-	-
2.3	Reduce Model	x_1, x_4	180.2545	179.1880	-	185.1869	-	172.0687	184.8639	-	-	-
2.4	Reduce Model	x_2, x_3	-	-	23.8944	-	26.6073	-	-	15.6626	19.0661	22.5225
2.5	Reduce Model	x_1, x_3	-	-	179.1191	-	183.3331	-	-	183.1975	174.1782	180.1216
2.6	Reduce Model	x_1, x_2	1.6419*	1.0810*	5.5985	3.4777	1.6644*	2.8288*	3.0678*	2.3375*	3.5302	1.7660*
3.1	Reference Model		1.6419	1.0810			1.6644	2.8288	3.0678	2.3375		1.7660
3.2	Reduce Model	x_2	28.8924	19.1261			25.0071	21.8490	22.0680	14.8222		21.1047
3.3	Reduce Model	x_1	178.9346	178.7739			182.0214	171.8719	184.6639	183.5730		179.4645

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1241	4.8294	5.1044	5.0180	5.0044	5.0321	5.0324	5.0944	5.0808	5.0245
1.2	Reduce Model	x_2, x_3, x_4	18.7294	26.2475	19.3144	24.4851	24.7611	24.3839	22.1173	19.4193	19.7657	21.4832
1.3	Reduce Model	x_1, x_3, x_4	186.9654	177.2138	178.8354	184.2400	181.0293	180.8413	177.4244	182.1902	187.3955	181.2224
1.4	Reduce Model	x_1, x_2, x_4	5.1261	3.7282*	3.5354*	3.4258*	4.2242	3.3379	3.6949*	3.0787	3.0828*	3.3389
1.5	Reduce Model	x_1, x_2, x_3	3.1281*	6.2173	3.7940	3.9127	3.0272*	3.0260*	4.6137	3.0600*	5.6211	3.2212*
2.1	Reference Model		3.1281	3.7282	3.5354	3.4258	3.0272	3.0260	3.6949	3.0600	3.0828	3.2212
2.2	Reduce Model	x_2, x_4	-	25.1981	17.5451	22.6804	-	-	20.7081	-	17.8074	-
2.3	Reduce Model	x_1, x_4	-	176.1639	179.0484	183.3828	-	-	176.1087	-	185.4960	-
2.4	Reduce Model	x_2, x_3	16.8777	-	-	-	22.9021	22.4517	-	17.3850	-	20.3813
2.5	Reduce Model	x_1, x_3	184.9767	-	-	-	181.0979	179.3439	-	180.3637	-	179.8100
2.5	Reduce Model	x_1, x_2	3.2141	4.9978	2.1992*	2.3841*	2.2378*	1.3368*	3.2413*	1.0450*	3.6430	1.5154*
3.1	Reference Model				2.1992	2.3841	2.2378	1.3368	3.2413	1.0450		1.5154
3.2	Reduce Model	x_2			16.3069	21.8118	22.0470	20.7538	20.2936	15.3662		18.8388
3.3	Reduce Model	x_1			178.2361	182.6957	180.2957	177.7392	175.7977	178.3525		178.0964

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9055	4.9468	5.0259	4.9571	5.0185	5.0838	4.8547	4.8030	4.9719	5.1393
1.2	Reduce Model	x_2, x_3, x_4	25.9530	26.7767	23.1261	23.5655	20.4569	17.8889	29.3502	24.9306	23.3609	18.6755
1.3	Reduce Model	x_1, x_3, x_4	177.1072	180.9007	185.5764	183.0793	182.7432	181.4945	179.0220	183.1898	181.9408	186.7763
1.4	Reduce Model	x_1, x_2, x_4	3.0907*	3.1521*	3.0891*	3.2455*	3.4109*	4.4100	3.1564*	3.6036*	3.0025*	3.9844
1.5	Reduce Model	x_1, x_2, x_3	3.7822	3.5903	3.1816	3.7161	5.5715	3.5403*	5.4292	7.5519	3.3873	3.1749*
2.1	Reference Model		3.0907	3.1521	3.0891	3.2455	3.4109	3.5403	3.1564	3.6036	3.0025	3.1749
2.2	Reduce Model	x_2, x_4	24.1939	25.0911	21.2483	22.0968	18.7692	-	27.6431	23.6369	21.4498	-
2.3	Reduce Model	x_1, x_4	175.7739	180.0824	183.7463	181.4274	181.3247	-	178.8010	182.0951	183.5952	-
2.4	Reduce Model	x_2, x_3	-	-	-	-	-	16.9122	-	-	-	16.7237
2.5	Reduce Model	x_1, x_3	-	-	-	-	-	180.1486	-	-	-	184.8895
2.6	Reduce Model	x_1, x_2	2.0126*	1.8106*	1.2448*	2.0412*	3.8798	2.7298*	3.8179	6.4214	1.4494*	1.9783*
3.1	Reference Model		2.0126	1.8106	1.2448	2.0412		2.7298			1.4494	1.9783
3.2	Reduce Model	x_2	23.1422	23.6134	19.7970	22.0752		16.7074			19.7858	15.6206
3.3	Reduce Model	x_1	176.5841	179.9761	183.0313	180.3734		181.0167			182.9781	183.7027

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0467	4.9586	4.8975	5.1648	4.9104	5.0123	5.0503	5.0553	5.1141	4.9455
1.2	Reduce Model	x_2, x_3, x_4	21.7780	19.0742	31.0336	17.1606	30.3610	22.9027	24.0549	23.4202	20.4829	24.8565
1.3	Reduce Model	x_1, x_3, x_4	186.7229	180.0353	183.5344	186.8214	166.6695	180.3562	183.8661	181.2193	180.0021	180.4430
1.4	Reduce Model	x_1, x_2, x_4	4.2158	5.8814*	3.8910	3.2019*	3.1232	3.5747	3.3806	3.3293*	3.1419*	2.9897*
1.5	Reduce Model	x_1, x_2, x_3	3.1642*	6.2678	3.0718*	3.9433	3.0761*	3.5417*	3.1892*	4.1892	3.1443	6.8587
2.1	Reference Model		3.1642	5.8814	3.0718	3.2019	3.0761	3.5417	3.1892	3.3293	3.1419	2.9897
2.2	Reduce Model	x_2, x_4	-	20.4017	-	15.1908	-	-	-	21.6285	18.5096	22.9110
2.3	Reduce Model	x_1, x_4	-	182.6563	-	184.9503	-	-	-	179.4953	178.2546	181.9299
2.4	Reduce Model	x_2, x_3	19.9644	-	29.2980	-	28.5069	22.3666	22.2571	-	-	-
2.5	Reduce Model	x_1, x_3	184.8461	-	182.2185	-	165.5042	178.8823	183.7894	-	-	-
2.6	Reduce Model	x_1, x_2	2.2642*	7.2060	2.0661*	1.9571*	1.2701*	2.1987*	1.5406*	2.4326*	1.1867*	4.9195
3.1	Reference Model		2.2642		2.0661	1.9571	1.2701	2.1987	1.5406	2.4326	1.1867	
3.2	Reduce Model	x_2	18.9020		27.9836	13.9835	26.7611	20.8305	20.8523	20.7483	16.5279	
3.3	Reduce Model	x_1	185.7774		181.2223	183.6979	164.3214	179.8380	182.8382	178.7976	176.4163	

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.8866	4.9744	5.0774	4.7546	4.9700	5.1374	5.0972	5.0652	4.9066	5.0519
1.2	Reduce Model	x_2, x_3, x_4	27.4159	22.7757	21.2752	30.8997	25.0656	18.8389	19.3260	20.0466	23.3607	21.0254
1.3	Reduce Model	x_1, x_3, x_4	170.1222	179.1765	175.5516	182.6180	183.1468	171.7965	183.3136	181.8337	171.9264	180.2572
1.4	Reduce Model	x_1, x_2, x_4	2.9632*	3.2390*	3.2376	6.2266	3.1647*	3.4564*	3.6335	4.1370	3.4902*	3.3995
1.5	Reduce Model	x_1, x_2, x_3	6.3069	4.0013	3.1202*	2.9628*	3.1759	3.8647	3.3561*	3.6724*	4.1818	3.3461*
2.1	Reference Model		2.9632	3.2390	3.1202	2.9628	3.1647	3.4564	3.3561	3.6724	3.4902	3.3461
2.2	Reduce Model	x_2, x_4	25.4884	21.2414	-	-	23.3151	17.1563	-	-	22.5014	-
2.3	Reduce Model	x_1, x_4	169.6384	177.7390	-	-	181.8302	170.2060	-	-	172.1011	-
2.4	Reduce Model	x_2, x_3	-	-	19.3334	29.1870	-	-	17.8690	18.7194	-	19.3684
2.5	Reduce Model	x_1, x_3	-	-	173.7373	181.6172	-	-	181.6235	180.5372	-	178.5507
2.6	Reduce Model	x_1, x_2	4.3773	2.3328*	1.2797*	4.4257	1.3387*	2.1720*	1.8688*	2.7635*	2.6571*	1.6305*
3.1	Reference Model			2.3328	1.2797		1.3387	2.1720	1.8688	2.7635	2.6571	1.6305
3.2	Reduce Model	x_2		20.1383	17.7972		21.5206	15.8445	16.1668	17.6887	22.0117	17.4914
3.3	Reduce Model	x_1		176.8455	171.8808		180.0220	169.3368	180.3199	180.7836	173.6189	178.2996

Table D.3 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1713	5.0212	5.0706	4.9791	5.0316	5.1550	4.9650	4.9889	5.0249	4.7549
1.2	Reduce Model	x_2, x_3, x_4	17.6443	22.1371	18.8519	22.9978	24.5200	15.8533	26.8796	21.2606	23.1652	32.9388
1.3	Reduce Model	x_1, x_3, x_4	191.1209	184.6366	180.3015	175.9319	184.2413	182.5988	184.6616	179.5933	169.1504	180.2375
1.4	Reduce Model	x_1, x_2, x_4	3.2758	3.0288*	3.0433	3.3413*	3.4722*	3.1388*	3.8137*	3.5290	3.7581	3.0092*
1.5	Reduce Model	x_1, x_2, x_3	3.1076*	4.5423	3.0424*	3.7274	4.1094	3.9850	3.9420	3.2278*	3.2007*	3.2507
2.1	Reference Model		3.1076	3.0288	3.0424	3.3413	3.4722	3.1388	3.8137	3.2278	3.2007	3.0092
2.2	Reduce Model	x_2, x_4	-	20.1465	-	21.4027	22.9353	13.8443	25.6335	-	-	31.2519
2.3	Reduce Model	x_1, x_4	-	182.6539	-	174.8605	182.8093	181.2411	184.5684	-	-	180.4779
2.4	Reduce Model	x_2, x_3	15.6195	-	16.8654	-	-	-	-	19.4659	21.3484	-
2.5	Reduce Model	x_1, x_3	189.3641	-	178.3760	-	-	-	-	178.4863	167.9149	-
2.6	Reduce Model	x_1, x_2	1.2124*	2.5617*	1.0151*	2.2029*	2.6066*	1.9566*	2.6303*	1.7952*	1.9023*	1.5025*
3.1	Reference Model		1.2124	2.5617	1.0151	2.2029	2.6066	1.9566	2.6303	1.7952	1.9023	1.5025
3.2	Reduce Model	x_2	13.7197	19.5015	14.9872	20.7872	21.9576	12.8207	24.2388	17.9683	19.7756	29.5493
3.3	Reduce Model	x_1	187.4671	182.8200	176.8140	173.7443	182.0141	180.0551	183.4447	178.2255	166.7621	179.0971

Table D.4 The MC Values for Variable Selection When $\rho_{12} = 0.7$

Iteration	Model	Variables in the Model	Replicate									
			1	2	3	4	5	6	7	8	9	10
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.8762	5.0282	4.9044	5.0090	4.9707	4.7673	5.0730	4.7929	4.8646	4.7454
1.2	Reduce Model	x_2, x_3, x_4	24.5739	22.9067	27.7639	24.4483	24.1431	30.4612	21.1711	28.6134	27.1456	30.8748
1.3	Reduce Model	x_1, x_3, x_4	182.6643	188.4192	179.2733	177.6657	184.4411	183.6414	187.8509	182.2484	183.9848	174.5080
1.4	Reduce Model	x_1, x_2, x_4	3.3027*	4.4855	2.9463*	3.6351	3.2461	2.8693*	3.3453	3.4326*	4.0567*	2.9015
1.5	Reduce Model	x_1, x_2, x_3	4.8972	4.2995*	2.9608	3.2212*	3.1326*	4.7309	3.2520*	4.9660	5.4259	2.8588*
2.1	Reference Model		3.3027	4.2995	2.9463	3.2212	3.1326	2.8693	3.2520	3.4326	4.0567	2.8588
2.2	Reduce Model	x_2, x_4	23.4107	-	25.9081	-	-	28.6653	-	27.4779	26.0150	-
2.3	Reduce Model	x_1, x_4	181.0030	-	177.3562	-	-	181.8376	-	181.1855	184.7455	-
2.4	Reduce Model	x_2, x_3	-	22.0553	-	22.7333	22.2731	-	19.3630	-	-	29.0660
2.5	Reduce Model	x_1, x_3	-	188.2553	-	175.8192	182.9734	-	186.0352	-	-	172.7344
2.6	Reduce Model	x_1, x_2	3.3206	3.5444*	1.0019*	1.9297*	1.4489*	2.8418*	1.4686*	3.4448	4.3783	1.0149*
3.1	Reference Model			3.5444	1.0019	1.9297	1.4489	2.8418	1.4686			1.0149
3.2	Reduce Model	x_2		21.3646	24.0057	21.3801	20.5560	28.4666	17.5538			27.2218
3.3	Reduce Model	x_1		187.5029	176.2003	175.7833	182.0952	183.7430	184.3168			171.4435

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0806	4.9442	5.0906	4.8901	5.0087	4.9332	4.9542	4.8762	4.8739	4.9765
1.2	Reduce Model	x_2, x_3, x_4	22.3789	24.8015	18.8652	18.8653	21.0775	25.1400	22.4093	24.2805	27.5943	25.7539
1.3	Reduce Model	x_1, x_3, x_4	179.8123	184.5370	180.9428	186.0703	171.2940	180.4346	179.7770	169.9823	181.5706	177.1703
1.4	Reduce Model	x_1, x_2, x_4	3.0614*	2.9692*	4.2073	6.2476	3.2507*	3.2196*	6.4278	3.3713*	3.8889	3.1036*
1.5	Reduce Model	x_1, x_2, x_3	3.7211	4.3936	3.0778*	5.9350*	5.7920	4.3056	4.4292*	4.4593	3.1217*	3.3882
2.1	Reference Model		3.0614	2.9692	3.0778	5.9350	3.2507	3.2196	4.4292	3.3713	3.1217	3.1036
2.2	Reduce Model	x_2, x_4	20.3726	22.8287	-	-	19.5122	23.3674	-	22.8196	-	23.8463
2.3	Reduce Model	x_1, x_4	177.7841	183.9472	-	-	169.5529	178.7957	-	171.0134	-	176.8642
2.4	Reduce Model	x_2, x_3	-	-	16.8705	21.2165	-	-	21.4492	-	25.9992	-
2.5	Reduce Model	x_1, x_3	-	-	181.1883	188.2106	-	-	181.0190	-	181.5103	-
2.6	Reduce Model	x_1, x_2	1.7075*	2.4216*	2.1911*	7.5136	3.9722	2.6140*	5.5581	3.0195*	2.0695*	1.5017*
3.1	Reference Model		1.7075	2.4216	2.1911			2.6140		3.0195	2.0695	1.5017
3.2	Reduce Model	x_2	18.9938	21.9302	15.8809			22.4862		22.3402	25.2216	22.6817
3.3	Reduce Model	x_1	177.0342	183.5655	180.3104			179.7746		170.8056	181.5345	175.2443

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.7988	4.8851	4.8006	5.0659	4.9904	5.0116	4.9904	5.0470	4.9712	5.0566
1.2	Reduce Model	x_2, x_3, x_4	28.9259	28.0255	29.0667	17.9359	22.7003	24.1578	26.2521	22.2507	15.5084	20.9763
1.3	Reduce Model	x_1, x_3, x_4	178.4695	179.3913	176.4354	182.2041	173.4403	183.9510	184.8681	178.4880	181.9115	173.7287
1.4	Reduce Model	x_1, x_2, x_4	5.8163	5.0708	2.8816*	3.7956	3.3984	3.6434	4.2870	3.4103	5.4557*	4.1419
1.5	Reduce Model	x_1, x_2, x_3	3.2877*	2.9693*	4.1186	3.3400*	3.0146*	3.2087*	4.2584*	3.0833*	6.3679	3.2709*
2.1	Reference Model		3.2877	2.9693	2.8816	3.3400	3.0146	3.2087	4.2584	3.0833	5.4557	3.2709
2.2	Reduce Model	x_2, x_4	-	-	27.1887	-	-	-	-	-	16.2226	-
2.3	Reduce Model	x_1, x_4	-	-	174.8002	-	-	-	-	-	182.4438	-
2.4	Reduce Model	x_2, x_3	27.4437	26.1211	-	16.5350	20.7138	22.3829	25.3762	20.3908	-	19.0571
2.5	Reduce Model	x_1, x_3	177.3490	177.6481	-	180.4680	172.3831	182.4273	184.3396	180.0236	-	172.2877
2.6	Reduce Model	x_1, x_2	4.1919	3.1952	2.2011*	2.1429*	1.4272*	1.8418*	3.3245*	1.4349*	6.6056	2.4423*
3.1	Reference Model				2.2011	2.1429	1.4272	1.8418	3.3245	1.4349		2.4423
3.2	Reduce Model	x_2			26.6696	15.2077	19.7028	21.3738	24.4358	18.7623		18.1752
3.3	Reduce Model	x_1			174.2635	179.4937	171.9510	182.9487	183.8323	180.8086		171.4685

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0211	4.7712	4.7917	5.0430	5.0390	5.0933	5.0031	5.1218	4.9996	4.8819
1.2	Reduce Model	x_2, x_3, x_4	23.2046	34.5143	31.5039	20.8687	18.9322	18.5942	24.7373	20.0228	20.8115	24.0588
1.3	Reduce Model	x_1, x_3, x_4	187.1211	181.7982	179.8407	180.0961	187.4375	177.3463	180.9339	187.3071	183.0270	174.6696
1.4	Reduce Model	x_1, x_2, x_4	3.7345	2.8693*	4.6312*	3.0542*	4.5573	3.0881*	3.0179*	3.2729	3.4447*	3.4582*
1.5	Reduce Model	x_1, x_2, x_3	3.6586*	2.9079	4.9577	5.0731	3.4097*	4.1313	3.3160	3.1143*	3.9341	5.8227
2.1	Reference Model		3.6586	2.8693	4.6312	3.0542	3.4097	3.0881	3.0179	3.1143	3.4447	3.4582
2.2	Reduce Model	x_2, x_4	-	32.7637	31.1034	18.9683	-	16.5977	22.7884	-	19.4865	22.5489
2.3	Reduce Model	x_1, x_4	-	180.1644	181.4422	178.2674	-	175.4363	179.1281	-	181.8736	173.2220
2.4	Reduce Model	x_2, x_3	21.8782	-	-	-	17.3360	-	-	18.1270	-	-
2.5	Reduce Model	x_1, x_3	186.4564	-	-	-	187.4144	-	-	185.2995	-	-
2.6	Reduce Model	x_1, x_2	2.3216*	1.0075*	4.6634	3.0769	2.7807*	2.1241*	1.3220*	1.2502*	2.3220*	4.3164
3.1	Reference Model		2.3216	1.0075			2.7807	2.1241	1.3220	1.2502	2.3220	
3.2	Reduce Model	x_2	21.4320	30.8638			16.4738	15.5156	21.1546	16.2975	18.5756	
3.3	Reduce Model	x_1	186.0409	179.0068			186.7855	177.7064	179.1140	183.4493	180.7336	

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	42	43	44	45	46	47	48	49	50
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.8452	4.7855	5.0065	5.0228	4.9289	4.7348	5.0173	4.8156	5.0434	4.8438
1.2	Reduce Model	x_2, x_3, x_4	26.6101	32.1925	25.1209	20.2890	26.3330	32.3207	17.5164	27.9243	23.3888	30.7035
1.3	Reduce Model	x_1, x_3, x_4	184.3334	178.5347	184.8022	181.2498	187.6427	180.3803	189.8763	176.8188	186.1848	183.9168
1.4	Reduce Model	x_1, x_2, x_4	3.6065	3.2708*	3.0111*	3.6228*	3.9728	3.0364*	3.1672*	3.0419*	4.3091	4.2530
1.5	Reduce Model	x_1, x_2, x_3	3.5161*	3.3827	4.1394	3.6859	3.5856*	3.5022	6.1015	3.2616	3.0289*	2.9917*
2.1	Reference Model		3.5161	3.2708	3.0111	3.6228	3.5856	3.0364	3.1672	3.0419	3.0289	2.9917
2.2	Reduce Model	x_2, x_4	-	30.5003	23.2476	18.8447	-	30.5138	15.6972	26.2039	-	-
2.3	Reduce Model	x_1, x_4	-	177.0214	182.8786	180.4108	-	178.7022	188.2018	175.8727	-	-
2.4	Reduce Model	x_2, x_3	25.3604	-	-	-	25.1072	-	-	-	21.3739	29.5297
2.5	Reduce Model	x_1, x_3	183.7374	-	-	-	186.3340	-	-	-	184.4415	182.1234
2.6	Reduce Model	x_1, x_2	2.3143*	1.8499*	2.1453*	2.3765*	2.6380*	1.7834*	4.2391	1.4948*	2.2952*	2.3720*
3.1	Reference Model		2.3143	1.8499	2.1453	2.3765	2.6380	1.7834		1.4948	2.2952	2.3720
3.2	Reduce Model	x_2	24.3716	29.2681	22.2480	17.4021	24.1148	28.9628		25.1692	20.4155	28.7791
3.3	Reduce Model	x_1	183.3920	175.6034	183.2360	179.1743	185.3768	178.1293		174.9475	184.8420	182.6515

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0889	4.9357	5.0793	4.9760	4.9528	4.9810	4.8949	5.0218	5.0344	5.0367
1.2	Reduce Model	x_2, x_3, x_4	19.3563	26.2795	16.9058	18.3391	24.5836	24.2558	25.0473	20.7006	21.4684	20.9059
1.3	Reduce Model	x_1, x_3, x_4	187.4991	175.9473	188.7414	180.4145	184.3816	188.3402	180.5447	182.7034	178.7257	178.0140
1.4	Reduce Model	x_1, x_2, x_4	3.2447	3.1201	3.7772*	3.7808*	3.8694	3.3867	3.2516*	3.2962*	3.0953*	3.9710*
1.5	Reduce Model	x_1, x_2, x_3	3.0658*	3.0622*	4.2780	7.7830	3.0353*	3.1074*	5.6802	4.1772	3.1216	4.1746
2.1	Reference Model		3.0658	3.0622	3.7772	3.7808	3.0353	3.1074	3.2516	3.2962	3.0953	3.9710
2.2	Reduce Model	x_2, x_4	-	-	15.5260	17.0526	-	-	23.4869	19.1899	19.7587	19.7578
2.3	Reduce Model	x_1, x_4	-	-	190.3938	179.2264	-	-	180.1892	182.1005	177.7060	177.0106
2.4	Reduce Model	x_2, x_3	17.3210	24.3915	-	-	23.1252	22.4185	-	-	-	-
2.5	Reduce Model	x_1, x_3	186.0004	175.9774	-	-	182.7697	186.5159	-	-	-	-
2.6	Reduce Model	x_1, x_2	1.2212*	1.2518*	2.9470*	6.6974	1.9682*	1.5057*	3.9763	2.5954*	1.1934*	3.2494*
3.1	Reference Model		1.2212	1.2518	2.9470		1.9682	1.5057		2.5954	1.1934	3.2494
3.2	Reduce Model	x_2	15.4704	23.0296	14.5124		22.2362	20.8774		18.6320	17.9024	19.1525
3.3	Reduce Model	x_1	184.2579	174.5104	189.5746		184.2707	184.9293		181.4334	175.9181	178.7159

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9406	5.0496	5.0115	4.8991	4.9104	4.7843	4.9699	4.9891	4.8968	5.0012
1.2	Reduce Model	x_2, x_3, x_4	25.3053	23.4208	23.8456	27.4477	24.7197	31.4980	27.2953	21.0477	27.2769	23.5530
1.3	Reduce Model	x_1, x_3, x_4	186.4010	184.1607	180.0663	178.3284	185.0554	174.8057	187.0638	186.0364	179.9262	183.5342
1.4	Reduce Model	x_1, x_2, x_4	3.6093	3.1432*	3.0278*	3.0102*	3.0953*	4.4972	4.7946	4.1626	3.0465*	3.0391*
1.5	Reduce Model	x_1, x_2, x_3	2.9662*	3.4514	6.5759	3.7351	6.4419	3.9857*	3.0351*	3.2939*	3.5436	3.1741
2.1	Reference Model		2.9662	3.1432	3.0278	3.0102	3.0953	3.9857	3.0351	3.2939	3.0465	3.0391
2.2	Reduce Model	x_2, x_4	-	21.4800	21.8703	25.5722	22.8547	-	-	-	25.6383	21.7841
2.3	Reduce Model	x_1, x_4	-	182.4575	178.2969	176.7218	183.4712	-	-	-	178.2251	181.5531
2.4	Reduce Model	x_2, x_3	23.3734	-	-	-	-	30.6789	25.3814	19.5241	-	-
2.5	Reduce Model	x_1, x_3	184.4391	-	-	-	-	174.5792	185.1232	184.4654	-	-
2.6	Reduce Model	x_1, x_2	1.6349*	1.5524*	4.6161	1.9109*	4.5973	3.5952*	2.8429*	2.4635*	1.6883*	1.2061*
3.1	Reference Model		1.6349	1.5524		1.9109		3.5952	2.8429	2.4635	1.6883	1.2061
3.2	Reduce Model	x_2	22.3135	20.8325		24.2043		30.6563	24.8666	18.7350	24.2872	19.9123
3.3	Reduce Model	x_1	183.1447	180.8071		175.6047		174.7307	185.6199	184.0550	176.8185	182.1612

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1461	4.7366	4.9858	4.9033	5.0979	5.0984	4.9128	4.9964	4.6764	5.0364
1.2	Reduce Model	x_2, x_3, x_4	19.0696	33.1862	25.2420	23.2616	20.6231	22.7219	25.3018	23.8415	33.7070	24.2021
1.3	Reduce Model	x_1, x_3, x_4	185.7405	177.9645	184.6216	184.6380	188.1750	187.9593	180.7452	184.9069	181.8330	186.1677
1.4	Reduce Model	x_1, x_2, x_4	3.4534	4.4556	3.0270*	6.3044	3.0912*	3.3154*	2.9657*	3.0397*	3.0138*	3.1152*
1.5	Reduce Model	x_1, x_2, x_3	3.2507*	3.5849*	4.4542	3.0248*	3.0929	3.3925	4.9857	3.1226	3.3179	3.2785
2.1	Reference Model		3.2507	3.5849	3.0270	3.0248	3.0912	3.3154	2.9657	3.0397	3.0138	3.1152
2.2	Reduce Model	x_2, x_4	-	-	23.5351	-	18.7374	20.9133	23.3606	21.9794	32.6666	22.2787
2.3	Reduce Model	x_1, x_4	-	-	183.5139	-	186.2800	186.9379	181.6307	183.2788	180.1714	184.6821
2.4	Reduce Model	x_2, x_3	17.1538	31.8782	-	21.6283	-	-	-	-	-	-
2.5	Reduce Model	x_1, x_3	183.8524	176.8926	-	183.7008	-	-	-	-	-	-
2.6	Reduce Model	x_1, x_2	1.5234*	3.5123*	2.4795*	4.4350	1.0818*	1.5424*	3.0598	1.1667*	1.6296*	1.3289*
3.1	Reference Model		1.5234	3.5123	2.4795		1.0818	1.5424		1.1667	1.6296	1.3289
3.2	Reduce Model	x_2	15.4153	31.6937	23.1262		16.7248	19.1129		20.3952	31.7333	20.7002
3.3	Reduce Model	x_1	183.0063	176.8797	185.3623		184.4621	185.2526		182.2546	178.8635	184.2186

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.0677	5.0734	4.8352	4.9469	4.8385	4.9129	4.8969	4.8973	5.1035	5.1079
1.2	Reduce Model	x_2, x_3, x_4	24.9076	20.5417	26.2177	22.7066	30.8268	25.2824	27.9153	25.6182	19.3645	20.9270
1.3	Reduce Model	x_1, x_3, x_4	181.1705	186.4746	179.1043	186.4419	183.2878	173.2571	178.3331	186.5620	186.4610	183.9828
1.4	Reduce Model	x_1, x_2, x_4	3.0434*	3.5278*	5.8870	3.1253*	3.3269	3.3451*	2.9685*	3.3099*	3.0989*	3.6078
1.5	Reduce Model	x_1, x_2, x_3	3.0761	3.6710	4.7915*	5.1715	3.1444*	4.4293	6.9920	4.7466	3.5168	3.1403*
2.1	Reference Model		3.0434	3.5278	4.7915	3.1253	3.1444	3.3451	2.9685	3.3099	3.0989	3.1403
2.2	Reduce Model	x_2, x_4	22.8829	19.1421	-	20.7909	-	23.6366	26.1163	23.9975	17.6248	-
2.3	Reduce Model	x_1, x_4	179.1726	184.9068	-	185.1659	-	171.8792	176.4381	184.9491	185.0976	-
2.4	Reduce Model	x_2, x_3	-	-	26.0896	-	29.0512	-	-	-	-	19.1397
2.5	Reduce Model	x_1, x_3	-	-	178.9859	-	183.1333	-	-	-	-	182.9863
2.6	Reduce Model	x_1, x_2	1.0545*	2.1925*	5.5581	3.4176	1.6897*	2.7670*	5.0441	3.0862*	1.5022*	1.6346*
3.1	Reference Model		1.0545	2.1925			1.6897	2.7670		3.0862	1.5022	1.6346
3.2	Reduce Model	x_2	21.0565	17.9935			27.4819	23.6219		24.0306	15.9921	17.6300
3.3	Reduce Model	x_1	178.8053	185.1326			181.8730	171.6462		184.7646	184.9746	181.5679

Table D.4 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	5.1354	4.9651	4.9863	4.8977	5.0962	4.7814	4.6598	5.0686	4.7781	5.0368
1.2	Reduce Model	x_2, x_3, x_4	18.5575	25.2261	22.6459	26.6653	20.1091	28.2867	37.4700	20.8155	31.5556	23.0195
1.3	Reduce Model	x_1, x_3, x_4	184.7334	181.4490	174.4009	181.8677	186.9202	176.9554	176.1387	178.5746	180.2758	185.6241
1.4	Reduce Model	x_1, x_2, x_4	4.3234	3.2706	5.3139	3.7149	5.0996	3.7020*	2.8567	3.4368*	3.5918	3.8212
1.5	Reduce Model	x_1, x_2, x_3	3.2827*	3.1576*	3.0345*	2.9695*	3.0952*	6.0887	2.8050*	3.8347	2.9248*	3.0711*
2.1	Reference Model		3.2827	3.1576	3.0345	2.9695	3.0952	3.7020	2.8050	3.4368	2.9248	3.0711
2.2	Reduce Model	x_2, x_4	-	-	-	-	-	27.2631	-	18.9983	-	-
2.3	Reduce Model	x_1, x_4	-	-	-	-	-	175.9277	-	178.7866	-	-
2.4	Reduce Model	x_2, x_3	16.8617	23.4807	20.6808	24.7432	18.2522	-	35.6305	-	29.7262	21.0674
2.5	Reduce Model	x_1, x_3	183.2449	179.6413	174.0070	179.9587	184.9370	-	175.0003	-	178.4825	183.6187
2.6	Reduce Model	x_1, x_2	2.3709*	1.4543*	3.3304	1.7745*	3.1688	4.8926	0.9987*	2.1779*	1.7391*	1.8608*
3.1	Reference Model		2.3709	1.4543		1.7745			0.9987	2.1779	1.7391	1.8608
3.2	Reduce Model	x_2	16.1078	22.1489		23.3432			33.7742	17.7832	28.2157	20.7283
3.3	Reduce Model	x_1	183.6502	180.3722		179.3445			173.1623	178.0767	178.7318	182.3920

Table D.5 The MC Values for Variable Selection When $\rho_{12} = 0.8$

Iteration	**Model**	**Variables in the Model**	**Replicate**									
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.8276	5.0051	4.9985	4.8615	4.9705	4.9744	4.9265	4.7235	5.0464	5.0073
1.2	Reduce Model	x_2, x_3, x_4	26.5156	23.4547	23.7941	29.3683	25.6041	26.9297	25.6714	32.1390	21.8481	21.2016
1.3	Reduce Model	x_1, x_3, x_4	182.5536	169.1623	188.7992	179.2169	177.9314	182.7407	184.5629	183.6811	188.2351	186.8801
1.4	Reduce Model	x_1, x_2, x_4	3.1627*	3.0684*	4.4503	2.9193*	3.5608	3.3662*	3.2196	2.8442*	3.3354	4.7267*
1.5	Reduce Model	x_1, x_2, x_3	4.9037	3.8430	4.2637*	2.9290	3.2396*	3.4112	3.0825*	4.6675	3.2124*	6.0698
2.1	Reference Model		3.1627	3.0684	4.2637	2.9193	3.2396	3.3662	3.0825	2.8442	3.2124	4.7267
2.2	Reduce Model	x_2, x_4	25.2326	21.5114	-	27.5272	-	25.5882	-	30.3548	-	21.2308
2.3	Reduce Model	x_1, x_4	180.8199	167.2615	-	177.3142	-	181.3559	-	181.8928	-	187.9360
2.4	Reduce Model	x_2, x_3	-	-	22.9312	-	23.9115	-	23.8326	-	20.0266	-
2.5	Reduce Model	x_1, x_3	-	-	188.6294	-	176.1556	-	183.1035	-	186.4099	-
2.6	Reduce Model	x_1, x_2	3.2361	1.9156*	3.5059*	0.9863*	1.9211*	1.7828*	1.4134*	2.7992*	1.4482*	5.7041
3.1	Reference Model			1.9156	3.5059	0.9863	1.9211	1.7828	1.4134	2.7992	1.4482	
3.2	Reduce Model	x_2		20.2536	22.2408	25.6611	22.5311	24.3719	22.1283	30.1338	18.2337	
3.3	Reduce Model	x_1		166.5431	187.8743	176.1542	176.0794	180.3474	182.2411	183.7698	184.7140	

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.7637	5.0893	4.8292	4.7030	5.0465	4.9423	4.9892	4.9083	5.0024	5.0659
1.2	Reduce Model	x_2, x_3, x_4	29.5856	17.1294	28.3001	32.4233	23.3718	26.1836	24.3410	26.0198	26.4230	19.6064
1.3	Reduce Model	x_1, x_3, x_4	182.5401	185.4534	184.1884	174.4331	180.0380	183.4217	181.9786	184.6811	175.5309	181.1098
1.4	Reduce Model	x_1, x_2, x_4	3.3916*	3.9663	4.0728*	2.8868	3.0367*	4.3095	2.9991*	2.9477*	3.0276	4.1844
1.5	Reduce Model	x_1, x_2, x_3	4.8333	3.1854*	5.3892	2.8295*	3.7650	3.5571*	3.8774	4.4221	3.0265*	3.0591*
2.1	Reference Model		3.3916	3.1854	4.0728	2.8295	3.0367	3.5571	2.9991	2.9477	3.0265	3.0591
2.2	Reduce Model	x_2, x_4	28.4394	-	27.1928	-	21.3910	-	22.4772	24.0616	-	-
2.3	Reduce Model	x_1, x_4	181.4623	-	185.0266	-	178.0224	-	180.5734	184.1076	-	-
2.4	Reduce Model	x_2, x_3	-	15.2497	-	30.6226	-	24.9668	-	-	24.4499	17.6174
2.5	Reduce Model	x_1, x_3	-	186.1604	-	172.6690	-	184.7499	-	-	174.0483	181.3771
2.6	Reduce Model	x_1, x_2	3.3080*	2.0031*	4.3898	1.0131*	1.7666*	2.7415*	1.8832*	2.4648*	1.0514*	2.1746*
3.1	Reference Model		3.3080	2.0031		1.0131	1.7666	2.7415	1.8832	2.4648	1.0514	2.1746
3.2	Reduce Model	x_2	28.5302	14.0540		28.8018	20.0663	24.2903	21.6483	23.1901	22.6063	16.6211
3.3	Reduce Model	x_1	184.0628	184.9890		171.3909	177.3338	185.4162	179.7350	183.7856	173.1634	180.5013

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.8634	5.0479	4.9803	4.8947	4.9148	4.8401	4.8418	4.8850	4.8681	4.9401
1.2	Reduce Model	x_2, x_3, x_4	19.7377	24.3342	21.9237	26.5445	23.8954	25.7266	28.7388	29.0127	30.0364	26.8215
1.3	Reduce Model	x_1, x_3, x_4	186.3178	180.9710	171.3571	180.4606	179.8920	169.8235	181.6605	180.7901	172.6230	177.2876
1.4	Reduce Model	x_1, x_2, x_4	6.1596	3.1025*	3.2032*	3.2125*	6.3504	3.3256*	3.8101	2.9663*	2.9472*	3.0538*
1.5	Reduce Model	x_1, x_2, x_3	5.8219*	3.3405	5.8276	4.2563	4.3598*	4.3912	3.1150*	3.6374	3.1369	3.4135
2.1	Reference Model		5.8219	3.1025	3.2032	3.2125	4.3598	3.3256	3.1150	2.9663	2.9472	3.0538
2.2	Reduce Model	x_2, x_4	-	22.3809	20.3375	24.7926	-	24.2569	-	27.0828	28.1475	24.9044
2.3	Reduce Model	x_1, x_4	-	179.8813	169.6017	178.8506	-	170.8226	-	179.2937	171.0292	177.0075
2.4	Reduce Model	x_2, x_3	22.0532	-	-	-	22.8902	-	27.1831	-	-	-
2.5	Reduce Model	x_1, x_3	188.3906	-	-	-	181.0899	-	181.6129	-	-	-
2.6	Reduce Model	x_1, x_2	7.3332	1.4010*	3.9922	2.5965*	5.4588	2.9390*	2.0154*	1.7039*	1.2114*	1.5144*
3.1	Reference Model			1.4010		2.5965		2.9390	2.0154	1.7039	1.2114	1.5144
3.2	Reduce Model	x_2		20.6071		23.8905		23.7388	26.3575	25.5689	26.5673	23.8677
3.3	Reduce Model	x_1		178.2263		179.8279		170.5850	181.6120	178.0512	170.2104	175.4515

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.7555	4.8535	4.8987	4.7630	5.0364	4.9652	4.9712	4.9373	4.9601	5.0192
1.2	Reduce Model	x_2, x_3, x_4	30.4475	28.9102	23.9805	30.1350	18.7873	23.3559	25.3567	21.4445	27.2408	22.9790
1.3	Reduce Model	x_1, x_3, x_4	178.4893	179.6092	179.9077	176.5786	182.3246	173.6141	184.2780	182.8484	185.1520	178.8477
1.4	Reduce Model	x_1, x_2, x_4	5.8339	5.1068	3.7555	2.8593*	3.7760	3.4102	3.6429	4.8469	4.2737	3.4509
1.5	Reduce Model	x_1, x_2, x_3	3.2467*	2.9448*	3.0727*	4.1930	3.3539*	2.9923*	3.2013*	3.5684*	4.1568*	3.0758*
2.1	Reference Model		3.2467	2.9448	3.0727	2.8593	3.3539	2.9923	3.2013	3.5684	4.1568	3.0758
2.2	Reduce Model	x_2, x_4	-	-	-	28.2704	-	-	-	-	-	-
2.3	Reduce Model	x_1, x_4	-	-	-	174.9561	-	-	-	-	-	-
2.4	Reduce Model	x_2, x_3	28.9681	27.0232	22.1516	-	17.4811	21.3745	23.6194	19.9319	26.2954	21.1601
2.5	Reduce Model	x_1, x_3	177.3667	177.8699	178.5615	-	180.6331	172.5432	182.7800	182.0773	184.5677	180.3699
2.6	Reduce Model	x_1, x_2	4.2132	3.2367	1.9859*	2.2912*	2.1707*	1.4410*	1.8743*	3.6398	3.2469*	1.4940*
3.1	Reference Model				1.9859	2.2912	2.1707	1.4410	1.8743		3.2469	1.4940
3.2	Reduce Model	x_2			20.9884	27.8802	16.1558	20.4424	22.6832		25.3786	19.5721
3.3	Reduce Model	x_1			177.6534	174.5256	179.6717	172.1320	183.3247		184.0796	181.1590

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9500	5.0279	4.9973	4.8435	4.7249	4.7525	4.8205	4.9835	5.0166	5.0114
1.2	Reduce Model	x_2, x_3, x_4	16.5808	21.8394	23.9367	25.7636	36.1666	32.8478	31.4006	21.0119	21.8262	20.1064
1.3	Reduce Model	x_1, x_3, x_4	181.7522	173.8932	187.4421	175.3445	181.8494	180.1516	179.9710	187.5440	180.1252	187.5516
1.4	Reduce Model	x_1, x_2, x_4	5.3580*	4.0430	3.6666	3.4470*	2.8419*	4.6114*	3.0567	3.0144*	3.0429*	4.4553
1.5	Reduce Model	x_1, x_2, x_3	6.0777	3.2076*	3.6310*	5.8300	2.8655	4.7840	2.9350*	4.2594	5.0048	3.3138*
2.1	Reference Model		5.3580	3.2076	3.6310	3.4470	2.8419	4.6114	2.9350	3.0144	3.0429	3.3138
2.2	Reduce Model	x_2, x_4	17.2452	-	-	24.4993	34.4441	32.4571	-	19.1401	19.9309	-
2.3	Reduce Model	x_1, x_4	182.2145	-	-	174.2823	180.2337	181.7663	-	185.5997	178.3266	-
2.4	Reduce Model	x_2, x_3	-	19.9076	22.6075	-	-	-	29.5252	-	-	18.4404
2.5	Reduce Model	x_1, x_3	-	172.4443	186.7726	-	-	-	179.3532	-	-	187.4135
2.6	Reduce Model	x_1, x_2	6.2532	2.2973*	2.2523*	4.1572	0.9833*	4.5135*	1.1563*	2.3094*	3.0128*	2.6316*
3.1	Reference Model			2.2973	2.2523		0.9833	4.5135	1.1563	2.3094	3.0128	2.6316
3.2	Reduce Model	x_2		18.9444	22.1232		32.5599	32.8287	27.6454	18.3942	20.3886	17.5136
3.3	Reduce Model	x_1		171.5501	186.2986		179.1167	181.7882	177.5701	185.1441	181.0273	186.7313

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.7836	5.0710	4.9752	5.0964	4.9636	4.8604	4.7947	4.7533	5.0183	4.9216
1.2	Reduce Model	x_2, x_3, x_4	31.9808	19.3430	25.4927	20.7693	21.7549	24.7920	28.2294	33.2667	22.0611	22.8791
1.3	Reduce Model	x_1, x_3, x_4	177.1316	177.5198	181.2192	187.5865	183.1652	174.8332	184.5536	178.5803	180.8841	174.4758
1.4	Reduce Model	x_1, x_2, x_4	2.9659*	3.0637*	2.9961*	3.2418	3.4942*	3.4857*	3.5892	3.2255*	3.0825*	2.9616*
1.5	Reduce Model	x_1, x_2, x_3	4.3094	4.0304	3.2906	3.1003*	3.9634	5.6321	3.5230*	3.3874	3.5193	4.8375
2.1	Reference Model		2.9659	3.0637	2.9961	3.1003	3.4942	3.4857	3.5230	3.2255	3.0825	2.9616
2.2	Reduce Model	x_2, x_4	30.2187	17.3434	23.5656	-	20.5711	23.3142	-	31.5891	20.1644	21.2457
2.3	Reduce Model	x_1, x_4	175.6893	175.6027	179.4141	-	182.1358	173.4368	-	177.0546	179.3059	172.9290
2.4	Reduce Model	x_2, x_3	-	-	-	18.8854	-	-	27.0457	-	-	-
2.5	Reduce Model	x_1, x_3	-	-	-	185.5904	-	-	184.0003	-	-	-
2.6	Reduce Model	x_1, x_2	2.4813*	2.0217*	1.3054*	1.2307*	2.4302*	4.1733	2.3559*	1.8415*	1.5841*	2.8778*
3.1	Reference Model		2.4813	2.0217	1.3054	1.2307	2.4302		2.3559	1.8415	1.5841	2.8778
3.2	Reduce Model	x_2	30.1124	16.1832	21.9385	17.0507	19.7415		26.1059	30.4068	18.6084	22.0013
3.3	Reduce Model	x_1	175.2247	177.7522	179.4103	183.7338	181.0563		183.6827	175.6772	178.6911	172.8585

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9782	4.7630	4.9930	4.8997	4.6956	4.9911	4.7723	5.0142	4.8034	5.0483
1.2	Reduce Model	x_2, x_3, x_4	25.7832	26.6437	21.4002	27.4065	33.5899	18.3608	29.5459	24.3614	31.9378	20.7235
1.3	Reduce Model	x_1, x_3, x_4	185.2355	181.1255	181.2305	187.8611	180.5320	189.9924	176.6973	186.4191	184.1696	187.6541
1.4	Reduce Model	x_1, x_2, x_4	2.9945*	4.5323	3.5928*	3.8639	3.0086*	3.1620*	3.0207*	4.2689	4.3212	3.2466
1.5	Reduce Model	x_1, x_2, x_3	4.1908	3.4839*	3.6593	3.5258*	3.4931	6.1048	3.2257	3.0119*	2.9686*	3.0372*
2.1	Reference Model		2.9945	3.4839	3.5928	3.5258	3.0086	3.1620	3.0207	3.0119	2.9686	3.0372
2.2	Reduce Model	x_2, x_4	23.9402	-	19.9534	-	31.7954	16.5528	27.8516	-	-	-
2.3	Reduce Model	x_1, x_4	183.3297	-	180.3871	-	178.8656	188.3350	175.7757	-	-	-
2.4	Reduce Model	x_2, x_3	-	25.8293	-	26.1500	-	-	-	22.3588	30.8133	18.7072
2.5	Reduce Model	x_1, x_3	-	179.8533	-	186.5196	-	-	-	184.6836	182.3945	186.1843
2.6	Reduce Model	x_1, x_2	2.2054*	3.1364*	2.3482*	2.4979*	1.7856*	4.2630	1.4810*	2.2655*	2.4565*	1.2370*
3.1	Reference Model		2.2054	3.1364	2.3482	2.4979	1.7856		1.4810	2.2655	2.4565	1.2370
3.2	Reduce Model	x_2	22.9722	26.0734	18.5009	25.0782	30.2597		26.8451	21.3764	30.1540	16.8977
3.3	Reduce Model	x_1	183.7330	180.8524	179.1516	185.4859	178.3172		174.8619	185.0767	183.0014	184.4966

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.8921	5.0451	4.9493	4.9147	5.1084	4.9479	4.9405	4.8915	4.8608	4.9849
1.2	Reduce Model	x_2, x_3, x_4	27.8811	18.2711	19.2722	25.7733	19.2298	23.5802	25.7736	24.2870	26.4135	21.7952
1.3	Reduce Model	x_1, x_3, x_4	175.8758	188.8554	180.4867	184.5697	188.5829	174.6932	188.4510	180.8541	180.5320	182.7874
1.4	Reduce Model	x_1, x_2, x_4	3.0945	3.7833*	3.7638*	3.8975	3.0705*	5.1400	3.3508	3.5135	3.1437*	3.3237*
1.5	Reduce Model	x_1, x_2, x_3	3.0461*	4.1833	7.6885	2.9979*	3.7949	3.2184*	3.0435*	3.2274*	5.5203	4.2253
2.1	Reference Model		3.0461	3.7833	3.7638	2.9979	3.0705	3.2184	3.0435	3.2274	3.1437	3.3237
2.2	Reduce Model	x_2, x_4	-	16.8960	17.9826	-	17.2501	-	-	-	24.7709	20.4143
2.3	Reduce Model	x_1, x_4	-	190.5730	179.3082	-	187.2829	-	-	-	180.1911	182.2137
2.4	Reduce Model	x_2, x_3	26.0166	-	-	24.3102	-	21.9915	23.9083	22.6740	-	-
2.5	Reduce Model	x_1, x_3	175.9180	-	-	182.9721	-	173.5881	186.6297	179.1801	-	-
2.6	Reduce Model	x_1, x_2	1.2540*	2.8928*	6.6116	1.9958*	1.7593*	3.4883	1.4484*	1.8095*	3.7545	2.7260*
3.1	Reference Model		1.2540	2.8928		1.9958	1.7593		1.4484	1.8095		2.7260
3.2	Reduce Model	x_2	24.7036	15.8054		23.5047	16.2190		22.3767	21.7423		19.9842
3.3	Reduce Model	x_1	174.4693	189.6977		184.5841	186.2532		185.0501	177.7786		181.6477

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	82	83	84	85	86	87	88	89	90
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9108	4.9953	5.0078	5.0400	4.9070	4.8356	4.7764	5.0143	4.9759	5.0272
1.2	Reduce Model	x_2, x_3, x_4	25.5142	22.8081	21.8598	22.7404	26.2016	26.7603	29.2779	24.6400	25.0205	22.9383
1.3	Reduce Model	x_1, x_3, x_4	177.7907	178.7452	178.0832	178.8700	186.6849	175.1280	175.1373	184.4302	180.2505	174.2997
1.4	Reduce Model	x_1, x_2, x_4	3.3661	3.0637*	3.9769*	3.3205	3.6465	2.9659	3.4822*	3.1140*	3.0004*	3.5714
1.5	Reduce Model	x_1, x_2, x_3	2.9921*	3.1015	4.1729	3.1717*	2.9456*	2.9226*	3.9335	3.3352	6.5427	3.0422*
2.1	Reference Model		2.9921	3.0637	3.9769	3.1717	2.9456	2.9226	3.4822	3.1140	3.0004	3.0422
2.2	Reduce Model	x_2, x_4	-	21.1123	20.7166	-	-	-	28.0103	22.7060	23.0520	-
2.3	Reduce Model	x_1, x_4	-	177.7440	177.1184	-	-	-	174.8207	182.7375	178.5061	-
2.4	Reduce Model	x_2, x_3	23.6959	-	-	20.9859	24.2906	25.3723	-	-	-	21.1250
2.5	Reduce Model	x_1, x_3	175.8402	-	-	179.1148	184.7374	173.2445	-	-	-	173.7370
2.6	Reduce Model	x_1, x_2	1.4450*	1.1804*	3.2853	1.4672*	1.6853	1.0514*	2.5910*	1.4412*	4.5826	1.6233*
3.1	Reference Model		1.4450	1.1804	3.2853	1.4672	1.6853	1.0514	2.5910	1.4412		1.6233
3.2	Reduce Model	x_2	22.0002	19.2735	20.1529	19.3725	23.3334	23.6324	27.2851	21.9482		19.7515
3.3	Reduce Model	x_1	174.7954	175.9761	178.8436	177.4919	183.5278	171.3651	175.5201	181.0172		172.5655

Table D.5 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	4.9044	4.8438	4.8716	4.8839	4.7409	4.9372	4.9631	4.9420	4.8659	4.9639
1.2	Reduce Model	x_2, x_3, x_4	30.0206	23.5065	28.2138	25.5338	32.9549	28.3069	21.8733	25.4280	28.1407	24.9799
1.3	Reduce Model	x_1, x_3, x_4	172.4333	179.6259	178.5350	185.2949	174.8486	187.3642	186.3091	178.2250	180.1276	183.5619
1.4	Reduce Model	x_1, x_2, x_4	3.1881	2.9122*	2.9937*	3.0711*	4.4870	4.8434	4.0707	4.5129	3.0139*	3.0029*
1.5	Reduce Model	x_1, x_2, x_3	2.9509*	7.1302	3.7442	6.4043	3.9910*	3.0240*	3.2536*	2.9784*	3.5772	3.1522
2.1	Reference Model		2.9509	2.9122	2.9937	3.0711	3.9910	3.0240	3.2536	2.9784	3.0139	3.0029
2.2	Reduce Model	x_2, x_4	-	21.8875	26.3497	23.6723	-	-	-	-	26.4952	23.1985
2.3	Reduce Model	x_1, x_4	-	177.7184	176.9392	183.7158	-	-	-	-	178.4319	181.5848
2.4	Reduce Model	x_2, x_3	28.1464	-	-	-	32.1831	26.4070	20.3354	23.4795	-	-
2.5	Reduce Model	x_1, x_3	170.7752	-	-	-	174.6632	185.4459	184.7203	176.2718	-	-
2.6	Reduce Model	x_1, x_2	1.2345*	5.2142	1.9322*	4.5630	3.6317*	2.9111*	2.3576*	2.5566*	1.7201*	1.1878*
3.1	Reference Model		1.2345		1.9322		3.6317	2.9111	2.3576	2.5566	1.7201	1.1878
3.2	Reduce Model	x_2	26.3541		24.9995		32.2098	25.9317	19.4806	22.7726	25.2095	21.3405
3.3	Reduce Model	x_1	171.5126		175.8621		174.8429	186.0053	184.2340	176.1056	177.0965	182.2068

Table D.6 The T_D Values for Variable Selection When $\rho_{12} = 0.3$

Iteration	Model	Variables in the Model	Replicate									
			1	2	3	4	5	6	7	8	9	10
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.4767	8.4396	8.6989	8.8256	8.6651	8.5886	8.6388	8.9892	8.9914	8.7580
1.3	Reduce Model	x_1, x_3, x_4	9.8328	9.8335	9.8389	9.8396	9.8319	9.8300	9.8365	9.8394	9.8389	9.8409
1.4	Reduce Model	x_1, x_2, x_4	-9.3882*	-4.9726	-7.5371*	-10.0209*	-1.5526	-8.2218*	3.6941	-4.1432	-5.6452	0.0407
1.5	Reduce Model	x_1, x_2, x_3	-7.6313	-9.6935*	2.3986	-2.6076	-7.0694*	2.0315	-1.9425*	-8.9159*	-9.4082*	-6.0709*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	6.9026	6.9119	7.3468	7.6291	-	7.1838	-	-	-	-
2.3	Reduce Model	x_1, x_4	9.7157	9.7173	9.7298	9.7297	-	9.7090	-	-	-	-
2.4	Reduce Model	x_2, x_3	-	6.8243	-	-	7.4204	-	7.2150	7.9410	7.9799	7.4660
2.5	Reduce Model	x_1, x_3	-	9.7158	-	-	9.7141	-	9.7271	9.7301	9.7267	9.7319
2.6	Reduce Model	x_1, x_2	-8.5046*	-7.4386*	-1.5663*	-6.2247*	-4.0222*	-2.1065*	1.0697*	-6.7755*	-7.6238*	-2.8463*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	5.5564	5.4023	6.3898	6.4016	6.2339	6.2250	6.4223	6.9353	6.9004	6.4248
3.3	Reduce Model	x_1	9.6007	9.5991	9.6276	9.6220	9.5991	9.5953	9.6238	9.6231	9.6174	9.6258

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0219	9.0663	9.1063	9.0275	8.3145	8.5908	8.8807	9.1815	8.7353	9.1088
1.3	Reduce Model	x_1, x_3, x_4	9.8435	9.8376	9.8381	9.8359	9.8397	9.8356	9.8370	9.8415	9.8386	9.8411
1.4	Reduce Model	x_1, x_2, x_4	-8.2656*	-9.7892*	-0.9483*	-9.6131*	-1.4903	-7.9535*	-9.4640*	-8.7202*	-2.9691	-3.1092
1.5	Reduce Model	x_1, x_2, x_3	5.5947	-3.5678	3.1271	-8.4505	-7.1558*	-2.8649	-6.7713	-6.1738	-9.8258*	-9.6135*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.9898	8.1322	7.9003	8.0566	-	7.1002	7.7358	8.2964	-	-
2.3	Reduce Model	x_1, x_4	9.7368	9.7242	9.7307	9.7209	-	9.7219	9.7264	9.7323	-	-
2.4	Reduce Model	x_2, x_3	-	-	-	-	6.6529	-	-	-	7.4385	8.2461
2.5	Reduce Model	x_1, x_3	-	-	-	-	9.7309	-	-	-	9.7264	9.7314
2.6	Reduce Model	x_1, x_2	2.0784	-6.6748*	1.8789	-9.0265*	-4.7327	-5.5416	-8.1222	-7.5071	-6.3770	-6.3738
3.1	Reference Model	x_1, x_2, x_3, x_4		-		-	-	-	-	-	-	-
3.2	Reduce Model	x_2		6.9847		7.0321	5.3281	5.9852	6.5672	7.2990	6.2570	7.2262
3.3	Reduce Model	x_1		9.6135		9.6080	9.6225	9.6097	9.6160	9.6249	9.6179	9.6261

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.3273	8.5655	8.6645	8.8577	8.1046	8.5583	8.7186	9.0014	8.8568	9.2145
1.3	Reduce Model	x_1, x_3, x_4	9.8383	9.8384	9.8399	9.8362	9.8336	9.8339	9.8352	9.8369	9.8408	9.8399
1.4	Reduce Model	x_1, x_2, x_4	-9.6477	-9.5350*	-1.4844	-10.1496*	-8.5419	-4.9435*	-6.0379	-9.4794*	-9.0967*	-1.4342
1.5	Reduce Model	x_1, x_2, x_3	-10.4100*	-6.7474	-9.2832*	-1.3696	-9.3840*	-3.8375	-9.2825*	-5.6311	3.7817	-5.9487*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	7.1212	-	7.7087	-	7.1540	-	7.9805	7.6769	-
2.3	Reduce Model	x_1, x_4	-	9.7276	-	9.7248	-	9.7183	-	9.7230	9.7311	-
2.4	Reduce Model	x_2, x_3	6.6057	-	7.2607	-	6.1686	-	7.4172	-	-	8.2288
2.5	Reduce Model	x_1, x_3	9.7261	-	9.7295	-	9.7167	-	9.7195	-	-	9.7306
2.6	Reduce Model	x_1, x_2	-10.0383*	-8.0812*	-5.2464*	-5.4179*	-8.9882*	-4.0236*	-7.8233*	-7.4124*	-0.5887*	-3.0041*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	4.9429	5.8244	6.0317	6.5534	4.3638	6.0289	6.1095	6.8575	6.6853	7.3311
3.3	Reduce Model	x_1	9.6132	9.6162	9.6224	9.6157	9.6009	9.6087	9.6043	9.6103	9.6303	9.6247

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	32	33	34	35	36	37	38	39	40
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.7064	8.9946	8.5533	8.8816	8.9155	8.8418	8.4763	8.9836	8.7398	8.3465
1.3	Reduce Model	x_1, x_3, x_4	9.8406	9.8377	9.8355	9.8380	9.8388	9.8354	9.8407	9.8343	9.8406	9.8361
1.4	Reduce Model	x_1, x_2, x_4	3.5031	-8.3416*	-6.5650*	-9.6929*	3.1699	-9.3871*	-4.1056	-1.4294	-2.4098	-3.7331
1.5	Reduce Model	x_1, x_2, x_3	-9.1778*	-1.6422	5.2802	0.0987	-0.7190*	-2.7291	-9.8280*	-8.1963*	-10.2472*	-9.4054*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	7.9663	7.0837	7.7417	-	7.6580	-	-	-	-
2.3	Reduce Model	x_1, x_4	-	9.7246	9.7207	9.7284	-	9.7215	-	-	-	-
2.4	Reduce Model	x_2, x_3	7.4088	-	-	-	7.6370	-	6.9513	7.9475	7.5178	6.7110
2.5	Reduce Model	x_1, x_3	9.7313	-	-	-	9.7306	-	9.7304	9.7180	9.7305	9.7219
2.6	Reduce Model	x_1, x_2	-0.8946*	-4.8717*	1.5680*	-4.1688*	1.1526*	-6.0925*	-7.0783*	-4.6976*	-6.2387*	-6.7809*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.4459	6.8191	6.5473	6.6281	6.8678	6.4618	5.6019	6.9339	6.3108	5.4300
3.3	Reduce Model	x_1	9.6300	9.6193	9.6200	9.6214	9.6275	9.6112	9.6251	9.6057	9.6235	9.6115

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.6175	8.4864	9.1890	8.7235	9.0402	8.9096	8.8974	8.5070	8.7236	8.9484
1.3	Reduce Model	x_1, x_3, x_4	9.8295	9.8384	9.8407	9.8403	9.8384	9.8388	9.8364	9.8393	9.8383	9.8363
1.4	Reduce Model	x_1, x_2, x_4	-9.4913*	0.0037	-8.7639*	-9.4918*	-9.8082*	-7.5419*	-8.4984	-7.8141	-9.2518*	-7.8365
1.5	Reduce Model	x_1, x_2, x_3	-7.6558	-9.7044*	-2.8878	-9.0124	-9.5229	-4.9367	-10.1301*	-9.5274*	-6.4948	-8.8625*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.1939	-	8.2939	7.4214	8.0777	7.7841	-	-	7.4316	-
2.3	Reduce Model	x_1, x_4	9.7076	-	9.7319	9.7296	9.7258	9.7282	-	-	9.7256	-
2.4	Reduce Model	x_2, x_3	-	6.9647	-	-	-	-	7.7881	6.9753	-	7.8488
2.5	Reduce Model	x_1, x_3	-	9.7258	-	-	-	-	9.7224	9.7276	-	9.7234
2.6	Reduce Model	x_1, x_2	-8.6018*	-4.1640*	-5.7283*	-9.2545*	-9.6662*	-6.1957*	-9.3223*	-8.7973*	-7.8456*	-8.3552*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	5.8344	5.9586	7.2733	6.1978	7.1002	6.7280	6.6505	5.5321	6.2173	6.8083
3.3	Reduce Model	x_1	9.5853	9.6215	9.6253	9.6195	9.6133	9.6187	9.6079	9.6189	9.6173	9.6128

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	52	53	54	55	56	57	58	59	60
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.6515	8.7199	8.5469	8.7744	8.8551	9.1846	8.2775	9.3450	8.8545	8.3460
1.3	Reduce Model	x_1, x_3, x_4	9.8380	9.8396	9.8210	9.8359	9.8364	9.8410	9.8373	9.8433	9.8326	9.8400
1.4	Reduce Model	x_1, x_2, x_4	-8.2346*	-3.4942	-7.3438*	-9.8649*	-9.4499*	-4.6181	-9.5604*	-9.6517*	-9.1081*	-3.6116
1.5	Reduce Model	x_1, x_2, x_3	1.7404	-8.8906*	-6.2742	-6.0750	-5.2127	-9.2544*	-5.5472	-9.5779	-9.0503	-5.7230*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.2603	-	7.0967	7.5219	7.6934	-	6.5140	8.7107	7.6955	-
2.3	Reduce Model	x_1, x_4	9.7262	-	9.6953	9.7254	9.7254	-	9.7237	9.7356	9.7183	-
2.4	Reduce Model	x_2, x_3	-	7.4045	-	-	-	8.3331	-	-	-	6.8200
2.5	Reduce Model	x_1, x_3	-	9.7280	-	-	-	9.7317	-	-	-	9.7330
2.6	Reduce Model	x_1, x_2	-2.3631*	-6.3381*	-7.1363*	-8.0606*	-7.3284*	-6.8985*	-7.6435*	-9.6091*	-9.0529*	-5.0326*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.2293	6.1522	5.7706	6.3173	6.5157	7.3865	5.0417	8.0402	6.5189	5.4596
3.3	Reduce Model	x_1	9.6207	9.6209	9.5741	9.6139	9.6169	9.6239	9.6107	9.6273	9.6036	9.6267

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	62	63	64	65	66	67	68	69	70
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.8459	8.6310	8.8762	8.6934	8.4767	8.3332	8.1472	9.0263	8.6980	8.1426
1.3	Reduce Model	x_1, x_3, x_4	9.8362	9.8341	9.8407	9.8319	9.8280	9.8376	9.8374	9.8402	9.8365	9.8295
1.4	Reduce Model	x_1, x_2, x_4	-8.4369*	-9.8555*	-7.6596*	-8.7749	-8.1946*	-9.5060*	-5.9303*	-5.5745*	-6.9300	-8.3136*
1.5	Reduce Model	x_1, x_2, x_3	-3.6987	-9.8235	-6.0945	-9.3411*	-5.9311	-7.2014	-5.1571	-4.7557	-8.4938*	-0.4394
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.6497	7.2244	7.6929	-	6.9445	6.6536	6.3839	7.9591	-	6.3484
2.3	Reduce Model	x_1, x_4	9.7218	9.7173	9.7320	-	9.7057	9.7241	9.7252	9.7324	-	9.7106
2.4	Reduce Model	x_2, x_3	-	-	-	7.3430	-	-	-	-	7.3205	-
2.5	Reduce Model	x_1, x_3	-	-	-	9.7128	-	-	-	-	9.7230	-
2.6	Reduce Model	x_1, x_2	-6.1078*	-9.8033*	-7.1436*	-9.0617*	-7.0958*	-8.3246*	-5.7067*	-5.5417*	-7.6243*	-4.3378*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.5552	5.8216	6.5715	6.0654	5.5942	5.2858	4.9707	7.0186	5.9739	5.1968
3.3	Reduce Model	x_1	9.6095	9.5997	9.6226	9.5925	9.5847	9.6107	9.6136	9.6241	9.6113	9.5939

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	72	73	74	75	76	77	78	79	80
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.8368	8.6731	8.9282	7.9270	8.5125	8.6211	8.5776	8.4304	8.9756	8.4075
1.3	Reduce Model	x_1, x_3, x_4	9.8287	9.8352	9.8297	9.8356	9.8361	9.8423	9.8302	9.8378	9.8412	9.8411
1.4	Reduce Model	x_1, x_2, x_4	-5.9438	-0.8620	-7.6153	-8.3347	-8.4750*	-9.9692*	-9.8943*	-2.4243	-10.3451*	-7.6239*
1.5	Reduce Model	x_1, x_2, x_3	-9.2163*	-9.1491*	-8.2178*	-8.7213*	0.6467	5.0395	-7.9673	-9.9654*	-6.9723	1.6545
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	-	-	7.0007	7.2048	7.1552	-	7.9733	6.7883
2.3	Reduce Model	x_1, x_4	-	-	-	-	9.7255	9.7339	9.7104	-	9.7319	9.7319
2.4	Reduce Model	x_2, x_3	7.6286	7.3034	7.8025	5.8977	-	-	-	6.8555	-	-
2.5	Reduce Model	x_1, x_3	9.7073	9.7226	9.7088	9.7201	-	-	-	9.7253	-	-
2.6	Reduce Model	x_1, x_2	-7.7165*	-4.3219*	-7.7849*	-8.5786*	-3.1646*	0.7999*	-8.9819*	-6.0806*	-8.6560*	-2.1068*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.4446	6.1651	6.6741	4.0457	5.8854	6.4879	5.8239	5.6785	6.9645	5.7506
3.3	Reduce Model	x_1	9.5868	9.6136	9.5874	9.6035	9.6175	9.6354	9.5897	9.6150	9.6282	9.6290

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	82	83	84	85	86	87	88	89	90
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0663	8.7913	8.7977	8.9181	8.9213	8.3980	8.9694	8.7643	8.7313	8.5829
1.3	Reduce Model	x_1, x_3, x_4	9.8373	9.8393	9.8417	9.8377	9.8387	9.8382	9.8401	9.8324	9.8343	9.8401
1.4	Reduce Model	x_1, x_2, x_4	-2.5620*	-7.4107*	-0.3109*	-9.6847*	-9.6390	-8.1656	-9.7574*	-9.1133*	-9.5101*	-8.2314
1.5	Reduce Model	x_1, x_2, x_3	-1.2702	-5.6151	0.1851	-8.9710	-9.8680*	-9.8912*	-9.0357	-6.1696	4.7468	-9.5758*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.9690	7.5586	7.5788	7.8149	-	-	7.9237	7.5336	7.4358	-
2.3	Reduce Model	x_1, x_4	9.7272	9.7293	9.7366	9.7248	-	-	9.7294	9.7162	9.7175	-
2.4	Reduce Model	x_2, x_3	-	-	-	-	7.8223	6.7499	-	-	-	7.1224
2.5	Reduce Model	x_1, x_3	-	-	-	-	9.7286	9.7254	-	-	-	9.7290
2.6	Reduce Model	x_1, x_2	-1.9319*	-6.6876*	-0.2054*	-9.2902*	-9.7453*	-9.0546*	-9.4076*	-7.6383*	0.4884*	-8.9346*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.0753	6.4182	6.6873	6.6539	6.7072	5.1879	6.8695	6.3439	6.8512	5.7645
3.3	Reduce Model	x_1	9.6190	9.6212	9.6331	9.6111	9.6214	9.6127	9.6183	9.5996	9.6120	9.6178

Table D.6 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	92	93	94	95	96	97	98	99	100
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.5899	8.2344	8.7149	8.7189	8.9642	8.5539	8.6634	8.8051	8.2298	8.8112
1.3	Reduce Model	x_1, x_3, x_4	9.8374	9.8403	9.8432	9.8338	9.8348	9.8395	9.8389	9.8366	9.8390	9.8388
1.4	Reduce Model	x_1, x_2, x_4	-9.8676*	5.6720	-6.0132*	-9.9096*	-9.6827*	-9.5713*	-7.8287*	-10.0626*	-10.0117*	3.6516
1.5	Reduce Model	x_1, x_2, x_3	-4.0926	5.0525	3.2251	-7.6413	-8.0579	-9.5226	-5.2299	-4.0462	-7.0231	5.9915
2.1	Reference Model	x_1, x_2, x_3, x_4	-		-	-	-	-	-	-	-	
2.2	Reduce Model	x_2, x_4	7.1380		7.3549	7.4074	7.9363	7.0992	7.3031	7.5909	6.4123	
2.3	Reduce Model	x_1, x_4	9.7238		9.7367	9.7168	9.7196	9.7298	9.7276	9.7223	9.7269	
2.4	Reduce Model	x_2, x_3	-		-	-	-	-	-	-	-	
2.5	Reduce Model	x_1, x_3	-		-	-	-	-	-	-	-	
2.6	Reduce Model	x_1, x_2	-7.0639*		-0.0213*	-8.7861*	-8.8673*	-9.5597*	-6.5278*	-7.0694*	-8.5648*	
3.1	Reference Model	x_1, x_2, x_3, x_4	-		-	-	-	-	-	-	-	
3.2	Reduce Model	x_2	5.8739		6.3832	6.0300	6.8500	5.6286	6.1230	6.5244	4.8359	
3.3	Reduce Model	x_1	9.6120		9.6364	9.6007	9.6078	9.6181	9.6169	9.6105	9.6153	

Table D.7 The T_D Values for Variable Selection When $\rho_{12} = 0.4$

Iteration	Model	Variables in the Model	Replicate									
			1	2	3	4	5	6	7	8	9	10
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.7590	8.8273	8.6873	8.8630	8.4616	8.6287	8.6841	9.0196	9.0935	8.2959
1.3	Reduce Model	x_1, x_3, x_4	9.8403	9.8337	9.8337	9.8397	9.8421	9.8299	9.8365	9.8395	9.8377	9.8359
1.4	Reduce Model	x_1, x_2, x_4	-2.0928	-5.9680	-9.2476*	-9.6948*	3.5474	-8.0068*	3.6658	-4.0529	-9.4991*	-5.7371
.5	Reduce Model	x_1, x_2, x_3	-2.9241*	-8.6512*	-6.3595	-2.5371	3.0220	2.0187	-1.9112*	-8.6390*	-3.4643	-8.3550*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-		-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	7.3328	7.7028		7.2567	-	-	8.1860	-
2.3	Reduce Model	x_1, x_4	-	-	9.7159	9.7299		9.7087	-	-	9.7245	-
2.4	Reduce Model	x_2, x_3	7.4864	7.6112	-	-		-	7.2753	7.9976	-	6.6091
2.5	Reduce Model	x_1, x_3	9.7318	9.7161	-	-		-	9.7267	9.7303	-	9.7209
2.6	Reduce Model	x_1, x_2	-2.9916*	-7.1388*	-7.8301*	-6.0357*		-2.0580*	1.0410*	-6.5762*	-6.4790*	-6.8297*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-		-	-	-	-	-
3.2	Reduce Model	x_2	6.4673	6.4405	6.0339	6.4831		6.2844	6.4693	7.0060	7.0445	5.1215
3.3	Reduce Model	x_1	9.6235	9.6017	9.5993	9.6218		9.5943	9.6224	9.6230	9.6135	9.6067

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.6120	9.1668	8.3703	8.5339	8.9600	9.2038	8.7728	9.1348	8.3841	8.8027
1.3	Reduce Model	x_1, x_3, x_4	9.8304	9.8400	9.8397	9.8363	9.8420	9.8416	9.8387	9.8413	9.8383	9.8411
1.4	Reduce Model	x_1, x_2, x_4	-3.4716	-0.7087*	-1.4754	-6.4685*	-3.7586	-8.4664*	-2.9067	-3.0142	-9.3214	-7.5862
1.5	Reduce Model	x_1, x_2, x_3	-8.2830*	1.0537	-6.9801*	-6.1447	-6.1820*	-6.0038	-9.5312*	-9.2979*	-10.0628*	-8.8437*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	8.0534	-	7.0216	-	8.3384	-	-	-	-
2.3	Reduce Model	x_1, x_4	-	9.7351	-	9.7233	-	9.7325	-	-	-	-
2.4	Reduce Model	x_2, x_3	7.1754	-	6.7501	-	7.8686	-	7.5124	8.2953	6.7165	7.5997
2.5	Reduce Model	x_1, x_3	9.7105	-	9.7307	-	9.7343	-	9.7264	9.7316	9.7262	9.7313
2.6	Reduce Model	x_1, x_2	-5.6465*	-0.0161*	-4.6277*	-6.0587*	-4.9254*	-7.2943*	-6.1861*	-6.1729*	-9.7009*	-8.2819*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	5.9355	7.3381	5.4305	5.6777	6.8763	7.3539	6.3391	7.2812	5.0960	6.3786
3.3	Reduce Model	x_1	9.5925	9.6314	9.6219	9.6100	9.6275	9.6251	9.6175	9.6261	9.6131	9.6211

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.8918	8.5993	8.8078	9.0289	8.8922	8.9959	8.6314	9.2359	8.7507	8.7184
1.3	Reduce Model	x_1, x_3, x_4	9.8363	9.8339	9.8356	9.8370	9.8409	9.8407	9.8398	9.8401	9.8407	9.8385
1.4	Reduce Model	x_1, x_2, x_4	-9.8055*	-4.8041*	-9.9852*	-9.1864*	-8.8133*	-1.0070	-3.1040	-1.3961	3.4782	-8.7797*
1.5	Reduce Model	x_1, x_2, x_3	-1.3270	-3.7372	3.7724	-5.4709	3.7525	-9.5408*	-7.9974*	-5.7778*	-8.9261*	-8.2622
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.7748	7.2127	7.5989	8.0338	7.7445	-	-	-	-	7.4140
2.3	Reduce Model	x_1, x_4	9.7249	9.7179	9.7202	9.7231	9.7312	-	-	-	-	9.7272
2.4	Reduce Model	x_2, x_3	-	-	-	-	-	7.9772	7.2430	8.2621	7.4942	-
2.5	Reduce Model	x_1, x_3	-	-	-	-	-	9.7302	9.7291	9.7307	9.7313	-
2.6	Reduce Model	x_1, x_2	-5.2558*	-3.9172*	-0.7642*	-7.1902*	-0.5705*	-4.9859*	-5.4915*	-2.9279*	-0.8746*	-8.7000*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.6207	6.0978	6.7558	6.9233	6.7346	6.9390	6.0229	7.3670	6.5071	6.1417
3.3	Reduce Model	x_1	9.6154	9.6078	9.6142	9.6101	9.6296	9.6240	9.6206	9.6245	9.6292	9.6149

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	32	33	34	35	36	37	38	39	40
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.1080	8.9166	8.7944	8.9497	8.9047	8.3798	8.7335	8.5216	9.0567	8.9409
1.3	Reduce Model	x_1, x_3, x_4	9.8417	9.8381	9.8389	9.8389	9.8333	9.8361	9.8326	9.8384	9.8388	9.8389
1.4	Reduce Model	x_1, x_2, x_4	-9.2907*	-9.4063*	-9.4120*	3.1333	-7.5281*	-3.6290	1.5948	0.0042	-4.9655*	-7.3399*
1.5	Reduce Model	x_1, x_2, x_3	-1.9546	0.1029	-8.8174	-0.6859*	-1.7496	-9.0818*	-9.0570*	-9.3902*	3.5605	-4.8125
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.1970	7.8109	7.5719	-	7.7392	-	-	-	7.9514	7.8425
2.3	Reduce Model	x_1, x_4	9.7323	9.7285	9.7269	-	9.7165	-	-	-	9.7282	9.7282
2.4	Reduce Model	x_2, x_3	-	-	-	7.6759	-	6.7710	7.4268	7.0305	-	-
2.5	Reduce Model	x_1, x_3	-	-	-	9.7303	-	9.7216	9.7158	9.7256	-	-
2.6	Reduce Model	x_1, x_2	-5.5330*	-4.0578*	-9.1200*	1.1204*	-4.7156*	-6.5653*	-2.9910*	-4.0280*	0.3098*	-6.0257*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.0035	6.6965	6.3939	6.9054	6.7369	5.4995	6.3469	6.0137	7.0555	6.7984
3.3	Reduce Model	x_1	9.6252	9.6212	9.6155	9.6264	9.6028	9.6107	9.6032	9.6205	9.6245	9.6183

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	42	43	44	45	46	47	48	49	50
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.9580	8.9310	8.6193	9.0332	9.0567	8.8615	8.9077	8.6946	8.4410	9.1633
1.3	Reduce Model	x_1, x_3, x_4	9.8372	9.8365	9.8395	9.8370	9.8384	9.8395	9.8396	9.8381	9.8358	9.8393
1.4	Reduce Model	x_1, x_2, x_4	-3.1430	-8.2070	-9.4083*	-9.1338*	-8.8676*	-9.6215*	-8.2140*	-7.9608*	-2.5070	-8.2238*
1.5	Reduce Model	x_1, x_2, x_3	-9.6551*	-9.7868*	1.6014	-5.2328	-6.4563	-9.1889	-5.5499	1.7153	-6.9537*	1.8307
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	7.2061	8.0551	8.0798	7.7194	7.7559	7.3365	-	8.2997
2.3	Reduce Model	x_1, x_4	-	-	9.7279	9.7237	9.7273	9.7281	9.7284	9.7261	-	9.7297
2.4	Reduce Model	x_2, x_3	7.9110	7.8537	-	-	-	-	-	-	6.9928	-
2.5	Reduce Model	x_1, x_3	9.7259	9.7224	-	-	-	-	-	-	9.7212	-
2.6	Reduce Model	x_1, x_2	-6.4276*	-9.0036*	-2.8397*	-7.1127*	-7.6364*	-9.4070*	-6.8505*	-2.3128*	-4.9905*	-2.0251*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.8772	6.7398	6.2027	6.9959	7.0649	6.5977	6.8465	6.2963	5.8887	7.3643
3.3	Reduce Model	x_1	9.6153	9.6078	9.6204	9.6152	9.6182	9.6179	9.6181	9.6200	9.6107	9.6261

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	52	53	54	55	56	57	58	59	60
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.8846	8.5116	8.7658	8.6017	9.2065	8.3273	9.1871	8.8781	8.6711	8.9767
1.3	Reduce Model	x_1, x_3, x_4	9.8384	9.8379	9.8397	9.8377	9.8411	9.8373	9.8295	9.8362	9.8341	9.8382
1.4	Reduce Model	x_1, x_2, x_4	-9.3414*	-5.0320	-3.4122	2.5465	-4.4942	-9.2568*	-8.6708	-8.1704*	-9.5276*	-9.5187*
1.5	Reduce Model	x_1, x_2, x_3	-7.3714	-9.1841*	-8.6630*	3.4078	-8.9605*	-5.3871	-8.9267*	-3.6104	-9.5003	4.3580
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-		-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.7447	-	-		-	6.6074	-	7.7077	7.3008	7.9380
2.3	Reduce Model	x_1, x_4	9.7301	-	-		-	9.7236	-	9.7217	9.7173	9.7295
2.4	Reduce Model	x_2, x_3	-	6.9829	7.4932		8.3732	-	8.3664	-	-	-
2.5	Reduce Model	x_1, x_3	-	9.7246	9.7282		9.7319	-	9.7090	-	-	-
2.6	Reduce Model	x_1, x_2	-8.3051*	-7.2433*	-6.1821*		-6.6899*	-7.4087*	-8.8507*	-5.9354*	-9.4786*	-0.0069*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-		-	-	-	-	-	-
3.2	Reduce Model	x_2	6.5291	5.6647	6.2569		7.4360	5.1541	7.5403	6.6230	5.9312	6.9663
3.3	Reduce Model	x_1	9.6210	9.6118	9.6208		9.6237	9.6102	9.5882	9.6089	9.5995	9.6278

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	62	63	64	65	66	67	68	69	70
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.7326	9.2231	8.5196	8.5703	8.6453	8.9065	8.3838	8.8037	9.0565	8.7450
1.3	Reduce Model	x_1, x_3, x_4	9.8319	9.8437	9.8280	9.8366	9.8364	9.8335	9.8397	9.8391	9.8404	9.8366
1.4	Reduce Model	x_1, x_2, x_4	-8.5240	3.8177	-7.9231*	-6.4373	-3.7591	-6.6545*	-8.6536	-9.5740*	-5.3892*	-6.7321
1.5	Reduce Model	x_1, x_2, x_3	-9.0776*	-9.1991*	-5.7434	-7.9378*	-6.2513*	-3.0083	-9.4437*	-1.9109	-4.6184	-8.2572*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	7.0184	-	-	7.7565	-	7.5765	8.0055	-
2.3	Reduce Model	x_1, x_4	-	-	9.7053	-	-	9.7190	-	9.7269	9.7324	-
2.4	Reduce Model	x_2, x_3	7.4184	8.4197	-	7.1236	7.2677	-	6.7137	-	-	7.4093
2.5	Reduce Model	x_1, x_3	9.7127	9.7375	-	9.7222	9.7226	-	9.7286	-	-	9.7231
2.6	Reduce Model	x_1, x_2	-8.8044*	-0.6045*	-6.8652*	-7.2771*	-4.9961*	-5.0572*	-9.0526*	-5.5689*	-5.3713*	-7.4113*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.1676	7.4342	5.6858	5.7379	6.0745	6.7245	5.0973	6.3663	7.0819	6.0892
3.3	Reduce Model	x_1	9.5922	9.6405	9.5839	9.6078	9.6118	9.6097	9.6162	9.6185	9.6239	9.6112

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.8712	9.2774	8.7175	8.9580	7.9658	8.7342	9.0221	8.5750	8.7234	8.5598
1.3	Reduce Model	x_1, x_3, x_4	9.8287	9.8423	9.8353	9.8297	9.8355	9.8315	9.8418	9.8350	9.8302	9.8361
1.4	Reduce Model	x_1, x_2, x_4	-5.7803	-8.8991*	-0.8326	-7.4152	-8.1216	-2.4603	0.1780	-5.7135	-8.5527	-8.2118*
1.5	Reduce Model	x_1, x_2, x_3	-8.9071*	-7.5422	-8.8630*	-8.0090*	-8.4920*	-9.2170*	4.0425	-9.1116*	-9.2415*	0.6439
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-		-	-	-
2.2	Reduce Model	x_2, x_4	-	8.5357	-	-	-	-		-	-	7.0869
2.3	Reduce Model	x_1, x_4	-	9.7363	-	-	-	-		-	-	9.7254
2.4	Reduce Model	x_2, x_3	7.6915	-	7.3885	7.8585	5.9633	7.4373		7.1052	7.4231	-
2.5	Reduce Model	x_1, x_3	9.7072	-	9.7226	9.7087	9.7196	9.7154		9.7211	9.7090	-
2.6	Reduce Model	x_1, x_2	-7.4637*	-8.2458*	-4.2053*	-7.5797*	-8.3533*	-5.8970*		-7.5381*	-8.9179*	-3.0630*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-		-	-	-
3.2	Reduce Model	x_2	6.5280	7.7194	6.2491	6.7496	4.1300	6.2636		5.8574	6.1272	5.9625
3.3	Reduce Model	x_1	9.5864	9.6292	9.6132	9.5871	9.6027	9.6037		9.6063	9.5876	9.6169

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	82	83	84	85	86	87	88	89	90
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.6246	9.0946	8.9556	9.0034	8.7637	8.6338	8.2783	8.7457	8.6046	8.8480
1.3	Reduce Model	x_1, x_3, x_4	9.8302	9.8374	9.8389	9.8403	9.8343	9.8401	9.8403	9.8376	9.8396	9.8389
1.4	Reduce Model	x_1, x_2, x_4	-9.5775*	-2.5160*	-9.3009	-9.4720*	-9.2502*	-7.9784	5.6416	-8.4440*	-9.2788*	3.6214
1.5	Reduce Model	x_1, x_2, x_3	-7.7338	-1.2419	-9.5206*	-8.7814	4.7193	-9.2773*	5.0247	-2.6222	-9.2371	5.9650
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-		-	-	
2.2	Reduce Model	x_2, x_4	7.2463	8.0119	-	7.9911	7.4984	-		7.4735	7.1966	
2.3	Reduce Model	x_1, x_4	9.7103	9.7271	-	9.7297	9.7174	-		9.7248	9.7299	
2.4	Reduce Model	x_2, x_3	-	-	7.8875	-	-	7.2222		-	-	
2.5	Reduce Model	x_1, x_3	-	-	9.7288	-	-	9.7291		-	-	
2.6	Reduce Model	x_1, x_2	-8.7028*	-1.8834*	-9.4029*	-9.1371*	0.4705*	-8.6549*		-5.3599*	-9.2701*	
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-		-	-	
3.2	Reduce Model	x_2	5.9460	7.1237	6.8009	6.9679	6.8855	5.8988		6.3731	5.7701	
3.3	Reduce Model	x_1	9.5895	9.6187	9.6216	9.6186	9.6106	9.6177		9.6140	9.6182	

Table D.7 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	92	93	94	95	96	97	98	99	100
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.6732	8.9107	8.8294	8.7393	9.1713	8.8409	8.6790	8.6126	8.9302	8.3979
1.3	Reduce Model	x_1, x_3, x_4	9.8332	9.8402	9.8361	9.8394	9.8437	9.8345	9.8366	9.8368	9.8330	9.8291
1.4	Reduce Model	x_1, x_2, x_4	-8.5158*	-4.2316	-8.9351*	-6.1823	-3.1690	-8.1785	-7.1113	1.7322	-9.2853*	-9.2359
1.5	Reduce Model	x_1, x_2, x_3	-5.9981	-4.8623*	-3.3435	-6.8481*	-4.1885*	-8.9282*	-9.6681*	-0.9441*	-8.6120	-9.3547*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.3518	-	7.6485	-	-	-	-	-	7.8859	-
2.3	Reduce Model	x_1, x_4	9.7156	-	9.7210	-	-	-	-	-	9.7170	-
2.4	Reduce Model	x_2, x_3	-	7.6994	-	7.3203	8.1631	7.6441	7.3189	7.3257	-	6.7355
2.5	Reduce Model	x_1, x_3	-	9.7312	-	9.7290	9.7384	9.7219	9.7228	9.7262	-	9.7066
2.6	Reduce Model	x_1, x_2	-7.3033*	-4.8418*	-6.0521*	-6.2732*	-2.8489*	-8.5889*	-8.4352*	0.3099*	-8.9938*	-9.2950*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.1162	6.7304	6.5963	6.0790	7.2797	6.5435	5.9662	6.4917	6.8181	5.1789
3.3	Reduce Model	x_1	9.5997	9.6225	9.6077	9.6197	9.6356	9.6093	9.6126	9.6179	9.5990	9.5889

Table D.8 The T_D Values for Variable Selection When $\rho_{12} = 0.5$

Iteration	Model	Variables in the Model	Replicate									
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.7970	9.0354	8.8697	9.1118	8.7328	8.9044	8.4670	8.5155	8.6733	8.7382
1.3	Reduce Model	x_1, x_3, x_4	9.8404	9.8375	9.8339	9.8419	9.8338	9.8399	9.8347	9.8422	9.8300	9.8367
1.4	Reduce Model	x_1, x_2, x_4	-2.0535	-9.3290*	-5.8564	-6.1670*	-9.0319*	-9.3822*	-3.6217	3.5095	-7.8356*	3.6357
1.5	Reduce Model	x_1, x_2, x_3	-2.8656*	-9.2330	-8.3801*	0.8858	-6.1596	-2.4294	-9.3493*	2.9731	2.0325	-1.8971*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-		-	-
2.2	Reduce Model	x_2, x_4	-	8.0692	-	8.1128	7.4242	7.7853	-		7.3387	-
2.3	Reduce Model	x_1, x_4	-	9.7237	-	9.7339	9.7162	9.7302	-		9.7087	-
2.4	Reduce Model	x_2, x_3	7.5374	-	7.6880	-	-	-	6.8824		-	7.3509
2.5	Reduce Model	x_1, x_3	9.7316	-	9.7165	-	-	-	9.7210		-	9.7266
2.6	Reduce Model	x_1, x_2	-2.9318*	-9.2838*	-6.9388*	-2.5157*	-7.6198*	-5.8301*	-6.4872*		-2.0032*	1.0094*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-		-	-
3.2	Reduce Model	x_2	6.5294	7.0717	6.5422	7.1478	6.1506	6.5761	5.5366		6.3546	6.5314
3.3	Reduce Model	x_1	9.6228	9.6102	9.6019	9.6332	9.5994	9.6219	9.6069		9.5936	9.6215

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0525	9.1229	8.3542	8.0350	8.6554	9.2666	9.1939	8.4357	8.9119	8.5867
1.3	Reduce Model	x_1, x_3, x_4	9.8398	9.8380	9.8360	9.8384	9.8305	9.8419	9.8403	9.8398	9.8364	9.8364
1.4	Reduce Model	x_1, x_2, x_4	-4.0035	-9.2325*	-5.5698	1.6670	-3.4474	-9.2164*	-0.6789*	-1.5018	-9.1491*	-6.2754*
1.5	Reduce Model	x_1, x_2, x_3	-8.3755*	-3.2995	-8.0664*	3.9130	-8.0612*	-8.9349	0.9690	-6.8837*	-7.6864	-5.9608
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-		-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	8.2446	-		-	8.5480	8.0866	-	7.8029	7.1094
2.3	Reduce Model	x_1, x_4	-	9.7250	-		-	9.7328	9.7351	-	9.7217	9.7232
2.4	Reduce Model	x_2, x_3	8.0596	-	6.7142		7.2564	-	-	6.8654	-	-
2.5	Reduce Model	x_1, x_3	9.7306	-	9.7209		9.7105	-	-	9.7307	-	-
2.6	Reduce Model	x_1, x_2	-6.4022*	-6.2604*	-6.6088*		-5.5448*	-9.0710*	-0.0491*	-4.5755*	-8.4650*	-5.8731*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-		-	-	-	-	-	-
3.2	Reduce Model	x_2	7.0846	7.1102	5.2505		6.0235	7.8032	7.3755	5.5526	6.6657	5.7896
3.3	Reduce Model	x_1	9.6232	9.6140	9.6065		9.5919	9.6237	9.6309	9.6216	9.6094	9.6096

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.9939	9.2285	8.8138	9.1632	8.4515	8.1512	8.8495	8.9299	8.6458	8.8465
1.3	Reduce Model	x_1, x_3, x_4	9.8422	9.8420	9.8388	9.8416	9.8385	9.8397	9.8413	9.8365	9.8340	9.8357
1.4	Reduce Model	x_1, x_2, x_4	-3.7324	-8.2370*	-2.8527	-2.8573	-8.9592	-5.9425*	-7.3526	-9.4624*	-4.6517*	-9.6231*
1.5	Reduce Model	x_1, x_2, x_3	-6.0220*	-5.8320	-9.2560*	-8.9945*	-9.6996*	-2.9684	-8.6002*	-1.2379	-3.6252	3.7468
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	8.3859	-	-	-	6.3585	-	7.8489	7.2813	7.6741
2.3	Reduce Model	x_1, x_4	-	9.7331	-	-	-	9.7323	-	9.7252	9.7177	9.7204
2.4	Reduce Model	x_2, x_3	7.9263	-	7.5939	8.3493	6.8488	-	7.6905	-	-	-
2.5	Reduce Model	x_1, x_3	9.7345	-	9.7266	9.7321	9.7264	-	9.7316	-	-	-
2.6	Reduce Model	x_1, x_2	-4.8383*	-7.0933*	-6.0077*	-5.9423*	-9.3372*	-4.5133*	-8.0362*	-5.0609*	-3.7962*	-0.7311*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.9474	7.4163	6.4315	7.3420	5.2798	4.9731	6.5012	6.6982	6.1804	6.8058
3.3	Reduce Model	x_1	9.6273	9.6256	9.6175	9.6265	9.6134	9.6240	9.6214	9.6154	9.6073	9.6133

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	32	33	34	35	36	37	38	39	40
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0574	8.9287	9.0273	8.6782	9.2582	8.8032	8.6021	8.7592	8.9531	8.8381
1.3	Reduce Model	x_1, x_3, x_4	9.8372	9.8411	9.8409	9.8400	9.8404	9.8409	9.8387	9.8387	9.8383	9.8391
1.4	Reduce Model	x_1, x_2, x_4	-8.9210*	-8.5503*	-0.9304	-3.0866	-1.3235	3.4617	-9.0732*	-8.4508*	-9.1429*	-9.1279*
1.5	Reduce Model	x_1, x_2, x_3	-5.3018	3.7259	-9.2176*	-7.7704*	-5.5777*	-8.7117*	-9.0645	-7.9697	0.1255	-8.5792
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.0894	7.8150	-	-	-	-	7.1760	7.4880	7.8834	7.6563
2.3	Reduce Model	x_1, x_4	9.7234	9.7315	-	-	-	-	9.7262	9.7274	9.7289	9.7272
2.4	Reduce Model	x_2, x_3	-	-	8.0380	7.3277	8.2963	7.5968	-	-	-	-
2.5	Reduce Model	x_1, x_3	-	-	9.7305	9.7293	9.7312	9.7317	-	-	-	-
2.6	Reduce Model	x_1, x_2	-6.9761*	-0.5584*	-4.8096*	-5.3802*	-2.8136*	-0.8562*	-9.0779*	-8.3780*	-3.9431*	-8.8577*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.9923	6.7876	7.0016	6.1191	7.4051	6.5852	5.7280	6.2446	6.7698	6.5129
3.3	Reduce Model	x_1	9.6103	9.6292	9.6239	9.6205	9.6247	9.6287	9.6127	9.6149	9.6212	9.6158

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	42	43	44	45	46	47	48	49	50
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.2558	8.8743	8.9889	8.7625	9.1535	8.9427	8.9582	8.4210	8.7770	8.9897
1.3	Reduce Model	x_1, x_3, x_4	9.8439	9.8349	9.8391	9.8408	9.8414	9.8335	9.8421	9.8361	9.8327	9.8392
1.4	Reduce Model	x_1, x_2, x_4	-8.4489*	-9.2520*	3.0720	-8.2240*	-7.2707	-7.3046*	-7.8575*	-3.4920	1.4960	-5.4126
1.5	Reduce Model	x_1, x_2, x_3	-6.6033	-9.0417	-0.5919*	0.6439	-8.0686*	-1.7380	-0.6190	-8.7420*	-8.8624*	-9.1814*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.4520	7.7208	-	7.4785	-	7.8083	7.8236	-	-	-
2.3	Reduce Model	x_1, x_4	9.7369	9.7184	-	9.7312	-	9.7167	9.7336	-	-	-
2.4	Reduce Model	x_2, x_3	-	-	7.7227	-	8.2407	-	-	6.8466	7.5143	7.9545
2.5	Reduce Model	x_1, x_3	-	-	9.7302	-	9.7338	-	-	9.7215	9.7160	9.7272
2.6	Reduce Model	x_1, x_2	-7.5861*	-9.1386*	1.0964*	-3.0103*	-7.5230*	-4.6100*	-4.0016*	-6.3241*	-3.0161*	-7.3672*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.6576	6.5739	6.9530	6.5063	7.2918	6.8144	6.7993	5.5893	6.4237	6.7980
3.3	Reduce Model	x_1	9.6302	9.6037	9.6257	9.6256	9.6252	9.6027	9.6279	9.6101	9.6024	9.6168

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	52	53	54	55	56	57	58	59	60
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.5646	9.0901	8.6231	8.9748	8.9925	8.9689	8.8467	8.6321	8.6673	8.8092
1.3	Reduce Model	x_1, x_3, x_4	9.8384	9.8390	9.8337	9.8391	9.8374	9.8367	9.8344	9.8361	9.8396	9.8371
1.4	Reduce Model	x_1, x_2, x_4	0.0066	-4.8485*	-7.0144*	-7.2109*	-3.1863	-7.8927	-5.9701*	-9.0548	-9.1386*	-7.8121
1.5	Reduce Model	x_1, x_2, x_3	-9.1075*	3.5148	-5.6822	-4.7034	-9.3326*	-9.4513*	-1.5687	-9.1395*	1.5958	-8.4145*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	8.0064	7.1975	7.9072	-	-	7.6357	-	7.2999	-
2.3	Reduce Model	x_1, x_4	-	9.7284	9.7190	9.7284	-	-	9.7187	-	9.7281	-
2.4	Reduce Model	x_2, x_3	7.1119	-	-	-	7.9788	7.9284	-	7.2117	-	7.6723
2.5	Reduce Model	x_1, x_3	9.7255	-	-	-	9.7262	9.7227	-	9.7209	-	9.7237
2.6	Reduce Model	x_1, x_2	-3.9045*	0.2846*	-6.4129*	-5.8983*	-6.2984*	-8.6775*	-3.8768*	-9.0956*	-2.7553*	-8.1677*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.0850	7.1031	5.9278	6.8766	6.9593	6.8421	6.6358	5.7751	6.2813	6.5113
3.3	Reduce Model	x_1	9.6197	9.6241	9.6038	9.6182	9.6153	9.6080	9.6052	9.6043	9.6196	9.6092

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	62	63	64	65	66	67	68	69	70
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0660	9.0885	8.9083	8.9444	8.7452	8.4939	9.1904	9.0419	8.9292	8.5539
1.3	Reduce Model	x_1, x_3, x_4	9.8372	9.8387	9.8398	9.8398	9.8382	9.8359	9.8396	9.8433	9.8387	9.8379
1.4	Reduce Model	x_1, x_2, x_4	-8.8372*	-8.5600*	-9.3075*	-7.9496*	-7.6760*	-2.5958	-8.0274*	-5.3636*	-9.0979*	-4.8447
1.5	Reduce Model	x_1, x_2, x_3	-5.0716	-6.2224	-8.8404	-5.4858	1.6790	-6.8255*	1.7472	5.3395	-7.1953	-8.9009*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.1189	8.1393	7.8113	7.8217	7.4274	-	8.3527	7.8610	7.8341	-
2.3	Reduce Model	x_1, x_4	9.7241	9.7277	9.7286	9.7287	9.7263	-	9.7301	9.7365	9.7306	-
2.4	Reduce Model	x_2, x_3	-	-	-	-	-	7.0812	-	-	-	7.0627
2.5	Reduce Model	x_1, x_3	-	-	-	-	-	9.7212	-	-	-	9.7247
2.6	Reduce Model	x_1, x_2	-6.8730*	-7.3656*	-9.0757*	-6.6722*	-2.2660*	-4.9696*	-2.0557*	2.4625	-8.0921*	-7.0035*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-		-	-
3.2	Reduce Model	x_2	7.0758	7.1434	6.7211	6.9288	6.3784	5.9834	7.4150		6.6524	5.7640
3.3	Reduce Model	x_1	9.6155	9.6185	9.6185	9.6182	9.6195	9.6103	9.6260		9.6215	9.6116

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	72	73	74	75	76	77	78	79	80
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.8177	8.6486	9.2307	8.3882	9.2102	8.8939	8.9481	8.9137	8.7161	9.0090
1.3	Reduce Model	x_1, x_3, x_4	9.8399	9.8378	9.8414	9.8374	9.8298	9.8368	9.8382	9.8364	9.8342	9.8384
1.4	Reduce Model	x_1, x_2, x_4	-3.2940	2.5471	-4.3617	-8.9574*	-8.4453	-6.3400	-7.6896	-7.9190*	-9.1952*	-9.2248*
1.5	Reduce Model	x_1, x_2, x_3	-8.4557*	3.3204	-8.6778*	-5.2026	-8.6819*	-6.5089*	-8.6619*	-3.5321	-9.1837	4.3006
2.1	Reference Model	x_1, x_2, x_3, x_4	-		-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-		-	6.7231	-	-	-	7.7730	7.3873	8.0014
2.3	Reduce Model	x_1, x_4	-		-	9.7237	-	-	-	9.7218	9.7175	9.7298
2.4	Reduce Model	x_2, x_3	7.5946		8.4174	-	8.4121	7.8321	7.8516	-	-	-
2.5	Reduce Model	x_1, x_3	9.7286		9.7324	-	9.7094	9.7229	9.7275	-	-	-
2.6	Reduce Model	x_1, x_2	-6.0166*		-6.4823*	-7.1636*	-8.6123*	-6.1886*	-8.1262*	-5.7596*	-9.1548*	-0.0468*
3.1	Reference Model	x_1, x_2, x_3, x_4	-		-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.3791		7.4921	5.2958	7.6066	6.7641	6.7860	6.7003	6.0561	7.0088
3.3	Reduce Model	x_1	9.6212		9.6241	9.6101	9.5887	9.6129	9.6163	9.6087	9.5997	9.6269

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	82	83	84	85	86	87	88	89	90
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.1226	8.9017	8.7776	9.2517	9.0129	8.5723	8.6228	8.6939	8.9460	8.7726
1.3	Reduce Model	x_1, x_3, x_4	9.8329	9.8370	9.8321	9.8441	9.8395	9.8280	9.8367	9.8365	9.8337	9.8359
1.4	Reduce Model	x_1, x_2, x_4	-9.1710*	-8.3341*	-8.2840	3.7328	-8.4453*	-7.6395*	-6.3437	-3.7912	-6.4751*	-7.6806
1.5	Reduce Model	x_1, x_2, x_3	3.5618	-4.2208	-8.8410*	-8.8859*	-8.3891	-5.5190	-7.7259*	-6.1181*	-2.9513	-7.9605*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.2385	7.7704	-	-	7.9540	7.1114	-	-	7.8242	-
2.3	Reduce Model	x_1, x_4	9.7164	9.7236	-	-	9.7286	9.7052	-	-	9.7192	-
2.4	Reduce Model	x_2, x_3	-	-	7.5063	8.4747	-	-	7.2192	7.3499	-	7.4731
2.5	Reduce Model	x_1, x_3	-	-	9.7129	9.7382	-	-	9.7223	9.7225	-	9.7209
2.6	Reduce Model	x_1, x_2	-0.9356*	-6.0962*	-8.5661*	-0.6510*	-8.4964*	-6.6101*	-7.1119*	-4.9518*	-4.9329*	-7.9974*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.2318	6.6139	6.2869	7.4870	6.9144	5.8042	5.8685	6.1728	6.8060	6.2372
3.3	Reduce Model	x_1	9.6094	9.6115	9.5924	9.6406	9.6164	9.5834	9.6077	9.6114	9.6097	9.6080

Table D.8 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	92	93	94	95	96	97	98	99	100
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.4356	8.8500	8.5941	8.9105	8.9755	8.2191	9.0893	8.8014	8.9090	9.3003
1.3	Reduce Model	x_1, x_3, x_4	9.8398	9.8393	9.8354	9.8347	9.8388	9.8350	9.8406	9.8368	9.8288	9.8427
1.4	Reduce Model	x_1, x_2, x_4	-8.4162	-9.2778*	-9.3640	-7.6356*	-7.1858*	-9.0487*	-5.1386*	-6.5122	-5.6420	-8.6206*
1.5	Reduce Model	x_1, x_2, x_3	-9.1905*	-1.8136	-9.3682*	-5.5824	-3.8614	-4.8725	-4.4635	-8.0454*	-8.5821*	-7.3085
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	7.6685	-	7.7726	7.9073	6.3760	8.0555	-	-	8.5807
2.3	Reduce Model	x_1, x_4	-	9.7273	-	9.7195	9.7273	9.7194	9.7327	-	-	9.7369
2.4	Reduce Model	x_2, x_3	6.8150	-	7.1445	-	-	-	-	7.5181	7.7605	-
2.5	Reduce Model	x_1, x_3	9.7287	-	9.7196	-	-	-	-	9.7235	9.7074	-
2.6	Reduce Model	x_1, x_2	-8.8069*	-5.3805*	-9.3661*	-6.3175*	-5.3448*	-7.0783*	-5.1643*	-7.1988*	-7.2169*	-7.9844*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	5.2368	6.4663	5.7176	6.7461	6.8967	4.9213	7.1519	6.2320	6.6212	7.7828
3.3	Reduce Model	x_1	9.6163	9.6185	9.6032	9.6045	9.6168	9.6034	9.6241	9.6116	9.5864	9.6299

Table D.9 The T_D Values for Variable Selection When $\rho_{12} = 0.7$

Iteration	Model	Variables in the Model	Replicate									
			1	2	3	4	5	6	7	8	9	10
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0077	8.8873	9.1283	8.9708	8.9663	9.2354	8.7673	9.1765	9.1146	9.2499
1.3	Reduce Model	x_1, x_3, x_4	9.8413	9.8413	9.8385	9.8350	9.8402	9.8444	9.8400	9.8429	9.8424	9.8392
1.4	Reduce Model	x_1, x_2, x_4	-7.0592*	-2.0120	-8.9225*	-5.8373	-7.7054	-8.9108*	-7.4861	-6.0650*	-3.2914*	-8.6766
1.5	Reduce Model	x_1, x_2, x_3	0.0709	-2.8031*	-8.8561	-7.8509*	-8.2516*	-0.1282	-7.9266*	0.5730	1.6871	-8.8801*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	7.6763	-	7.8454	7.8524	-	7.4600	-	-	8.4543
2.3	Reduce Model	x_1, x_4	-	9.7317	-	9.7208	9.7302	-	9.7287	-	-	9.7278
2.4	Reduce Model	x_2, x_3	7.6902	-	8.2541	-	-	8.0692	-	8.0145	7.8668	-
2.5	Reduce Model	x_1, x_3	9.7338	-	9.7267	-	-	9.7424	-	9.7402	9.7372	-
2.6	Reduce Model	x_1, x_2	-3.0998*	-2.8607*	-8.8912*	-6.6272*	-7.8893*	-4.0864*	-7.8357*	-2.6996*	-0.8689*	-8.7788*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.8989	6.6911	7.3577	6.7934	6.7817	7.3869	6.2072	7.2778	7.0781	7.6962
3.3	Reduce Model	x_1	9.6243	9.6230	9.6129	9.6044	9.6195	9.6366	9.6162	9.6344	9.6327	9.6149

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.8419	9.0028	8.5809	8.6463	8.7788	9.0210	8.8774	8.9947	9.1281	9.0368
1.3	Reduce Model	x_1, x_3, x_4	9.8348	9.8409	9.8353	9.8430	9.8307	9.8386	9.8377	9.8330	9.8407	9.8355
1.4	Reduce Model	x_1, x_2, x_4	-8.7567*	-8.8922*	-3.4571	3.3963	-7.7056*	-7.7376*	3.5663	-6.7458*	-4.1083	-8.3450*
1.5	Reduce Model	x_1, x_2, x_3	-5.6578	-2.0826	-8.8379*	2.7943	2.1736	-2.4307	-1.9621*	-1.5590	-7.9580*	-6.9744
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-		-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.6487	7.9852	-		7.5381	7.9550	-	7.8921	-	8.0518
2.3	Reduce Model	x_1, x_4	9.7179	9.7321	-		9.7098	9.7262	-	9.7188	-	9.7216
2.4	Reduce Model	x_2, x_3	-	-	7.1064		-	-	7.5618	-	8.2040	-
2.5	Reduce Model	x_1, x_3	-	-	9.7220		-	-	9.7275	-	9.7322	-
2.6	Reduce Model	x_1, x_2	-7.2222*	-5.4021*	-6.1444*		-1.8701*	-4.9039*	0.9373*	-3.8321*	-6.2056*	-7.6913*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-		-	-	-	-	-	-
3.2	Reduce Model	x_2	6.4429	6.8079	5.7907		6.5363	6.8650	6.7164	6.7980	7.2743	7.0481
3.3	Reduce Model	x_1	9.6016	9.6238	9.6075		9.5938	9.6181	9.6212	9.6053	9.6250	9.6059

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.1852	9.1412	9.1892	8.4989	8.8850	8.9580	9.0548	8.8441	8.2429	8.7597
1.3	Reduce Model	x_1, x_3, x_4	9.8405	9.8391	9.8392	9.8367	9.8326	9.8390	9.8401	9.8347	9.8387	9.8312
1.4	Reduce Model	x_1, x_2, x_4	2.7772	0.6069	-8.8458*	-5.2960	-6.9686	-5.9078	-2.7246	-7.1055	1.4530*	-3.6279
1.5	Reduce Model	x_1, x_2, x_3	-6.8407*	-8.8013*	-2.7270	-7.5199*	-8.7882*	-8.0663*	-2.8512*	-8.6436*	3.4366	-7.8507*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	8.3794	-	-	-	-	-	6.8448	-
2.3	Reduce Model	x_1, x_4	-	-	9.7272	-	-	-	-	-	9.7275	-
2.4	Reduce Model	x_2, x_3	8.2751	8.2748	-	6.9827	7.7381	7.8581	7.9887	7.6534	-	7.4595
2.5	Reduce Model	x_1, x_3	9.7307	9.7270	-	9.7221	9.7149	9.7272	9.7297	9.7224	-	9.7116
2.6	Reduce Model	x_1, x_2	-1.1118*	-3.4365*	-5.7564*	-6.2042*	-7.8796*	-6.9917*	-3.2827*	-7.9169*	2.3883	-5.5843*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-		-
3.2	Reduce Model	x_2	7.2170	7.2016	7.2618	5.5910	6.6264	6.8075	7.0797	6.4356		6.2471
3.3	Reduce Model	x_1	9.6263	9.6182	9.6168	9.6077	9.5982	9.6183	9.6203	9.6116		9.5923

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.9051	9.3289	9.2580	8.7558	8.6021	8.5554	8.9861	8.6694	8.7652	8.9827
1.3	Reduce Model	x_1, x_3, x_4	9.8407	9.8432	9.8415	9.8359	9.8405	9.8329	9.8373	9.8386	9.8387	9.8361
1.4	Reduce Model	x_1, x_2, x_4	-5.4129	-8.8669*	-0.5764*	-8.7993*	-1.7491	-8.8287*	-8.8012*	-8.0468	-6.8400*	-6.3106*
1.5	Reduce Model	x_1, x_2, x_3	-5.7863*	-8.6850	0.5532	0.0985	-7.0369*	-3.8249	-7.4076	-8.7693*	-4.4054	2.5831
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	8.6750	8.1709	-	7.1426	-	-	7.2961	-	-
2.3	Reduce Model	x_1, x_4	-	9.7353	9.7362	-	9.7305	-	-	9.7258	-	-
2.4	Reduce Model	x_2, x_3	7.7440	-	-	7.5338	-	7.1078	7.9072	-	7.4267	7.7627
2.5	Reduce Model	x_1, x_3	9.7312	-	-	9.7246	-	9.7195	9.7253	-	9.7272	9.7250
2.6	Reduce Model	x_1, x_2	-5.7344*	-8.7726*	-0.2229*	-3.9191*	-4.7068*	-6.4147*	-8.1379*	-8.4497*	-5.7269*	-1.0229*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.8025	7.9944	7.4743	6.4543	5.8824	5.7151	6.8772	5.9503	6.2941	6.9333
3.3	Reduce Model	x_1	9.6222	9.6274	9.6313	9.6111	9.6222	9.6025	9.6115	9.6117	9.6151	9.6111

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	42	43	44	45	46	47	48	49	50
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0977	9.2760	9.0030	8.7201	9.0712	9.2867	8.4725	9.1506	8.9092	9.2286
1.3	Reduce Model	x_1, x_3, x_4	9.8430	9.8409	9.8396	9.8371	9.8431	9.8432	9.8424	9.8391	9.8397	9.8428
1.4	Reduce Model	x_1, x_2, x_4	-5.3635	-6.9505*	-8.8596*	-6.0113*	-3.9467	-7.9236*	-8.1781*	-8.1645*	-2.8048	-2.3077
1.5	Reduce Model	x_1, x_2, x_3	-5.8207*	-6.3680	-3.4345	-5.6975	-5.8759*	-5.5135	2.8236	-7.0663	-8.8404*	-8.5151*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	8.4684	7.9958	7.3417	-	8.5010	6.8911	8.2490	-	-
2.3	Reduce Model	x_1, x_4	-	9.7310	9.7279	9.7240	-	9.7352	9.7337	9.7280	-	-
2.4	Reduce Model	x_2, x_3	7.9475	-	-	-	8.0637	-	-	-	7.7875	8.4760
2.5	Reduce Model	x_1, x_3	9.7360	-	-	-	9.7356	-	-	-	9.7282	9.7344
2.6	Reduce Model	x_1, x_2	-5.4912*	-6.6797*	-6.1458*	-5.6282*	-4.8871*	-6.7781*	-1.3921*	-7.5984*	-5.7777*	-5.4061*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.9888	7.6503	6.8638	6.0922	7.1191	7.5696	6.0799	7.3676	6.6586	7.4858
3.3	Reduce Model	x_1	9.6297	9.6205	9.6180	9.6102	9.6282	9.6285	9.6271	9.6169	9.6189	9.6291

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	52	53	54	55	56	57	58	59	60
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.6269	9.0671	8.3804	8.5705	8.9909	8.9690	9.0256	8.7501	8.7986	8.7595
1.3	Reduce Model	x_1, x_3, x_4	9.8395	9.8356	9.8404	9.8376	9.8406	9.8423	9.8396	9.8380	9.8352	9.8346
1.4	Reduce Model	x_1, x_2, x_4	-8.1748	-8.1741	-5.4523*	-5.0305*	-4.5163	-7.0231	-7.4660*	-7.6070*	-8.5699*	-4.3294*
1.5	Reduce Model	x_1, x_2, x_3	-9.0003*	-8.4475*	-3.0970	5.2810	-8.6225*	-8.3731*	2.2350	-3.3299	-8.4498	-3.3839
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	6.7325	7.1071	-	-	8.0518	7.4538	7.5804	7.4581
2.3	Reduce Model	x_1, x_4	-	-	9.7333	9.7244	-	-	9.7297	9.7262	9.7200	9.7184
2.4	Reduce Model	x_2, x_3	7.1993	8.0898	-	-	8.0030	7.9279	-	-	-	-
2.5	Reduce Model	x_1, x_3	9.7282	9.7224	-	-	9.7302	9.7335	-	-	-	-
2.6	Reduce Model	x_1, x_2	-8.5924*	-8.2971*	-4.2801*	2.3610	-6.5242*	-7.7422*	-1.8320*	-5.0514*	-8.4844*	-3.5260*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-		-	-	-	-	-	-
3.2	Reduce Model	x_2	5.7721	7.1535	5.3968		6.8908	6.8280	6.8911	6.3039	6.3368	6.4015
3.3	Reduce Model	x_1	9.6157	9.6074	9.6248		9.6244	9.6238	9.6247	9.6145	9.6032	9.6077

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	62	63	64	65	66	67	68	69	70
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0252	8.9097	8.9431	9.1179	9.0063	9.2590	9.0988	8.7832	9.1128	8.9282
1.3	Reduce Model	x_1, x_3, x_4	9.8421	9.8383	9.8366	9.8381	9.8420	9.8385	9.8418	9.8408	9.8391	9.8390
1.4	Reduce Model	x_1, x_2, x_4	-5.7165	-8.3743*	-8.9697*	-8.5570*	-8.1697*	-1.0627	-0.6095	-3.2617	-8.4330*	-8.6783*
1.5	Reduce Model	x_1, x_2, x_3	-8.8268*	-6.9104	3.7125	-4.9602	3.6867	-3.2923*	-8.6919*	-7.4837*	-5.9497	-8.0587
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	7.7754	7.8653	8.2098	7.9682	-	-	-	8.2174	7.8527
2.3	Reduce Model	x_1, x_4	-	9.7254	9.7220	9.7251	9.7330	-	-	-	9.7271	9.7265
2.4	Reduce Model	x_2, x_3	8.0356	-	-	-	-	8.2291	8.1778	7.5230	-	-
2.5	Reduce Model	x_1, x_3	9.7328	-	-	-	-	9.7280	9.7323	9.7306	-	-
2.6	Reduce Model	x_1, x_2	-7.3099*	-7.6516*	-0.6710*	-6.6239*	-0.5437*	-2.3495*	-4.3921*	-5.3497*	-7.2082*	-8.3861*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.1	Reduce Model	x_2	6.9526	6.8106	6.9453	7.1431	6.9087	7.4351	7.1514	6.3427	7.1630	6.7132
2.2	Reduce Model	x_1	9.6231	9.6114	9.6130	9.6122	9.6296	9.6195	9.6254	9.6215	9.6147	9.6174

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.5795	9.3059	9.0131	8.9371	8.7206	8.8602	9.0312	8.9447	9.3265	8.9541
1.3	Reduce Model	x_1, x_3, x_4	9.8371	9.8416	9.8400	9.8419	9.8396	9.8395	9.8393	9.8399	9.8454	9.8398
1.4	Reduce Model	x_1, x_2, x_4	-7.2089	-1.0474	-8.7473*	3.4669	-8.6897*	-7.8326*	-8.7553*	-8.7117*	-7.8923*	-8.5659*
1.5	Reduce Model	x_1, x_2, x_3	-8.1526*	-5.0865*	-1.9849	-8.4699*	-8.6820	-7.4609	0.2411	-8.3312	-6.2940	-7.7931
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	8.0291	-	7.4078	7.6756	8.0408	7.8640	8.5827	7.8644
2.3	Reduce Model	x_1, x_4	-	-	9.7296	-	9.7280	9.7288	9.7306	9.7289	9.7398	9.7289
2.4	Reduce Model	x_2, x_3	7.1069	8.3658	-	7.8650	-	-	-	-	-	-
2.5	Reduce Model	x_1, x_3	9.7227	9.7332	-	9.7335	-	-	-	-	-	-
2.6	Reduce Model	x_1, x_2	-7.7685*	-2.4454*	-5.3321*	-0.8220*	-8.6955*	-7.8015*	-3.7125*	-8.5219*	-7.1492*	-8.2465*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	5.6913	7.4880	7.0198	6.8059	6.0678	6.5101	6.9351	6.8102	7.8449	6.7883
3.3	Reduce Model	x_1	9.6088	9.6270	9.6237	9.6295	9.6152	9.6167	9.6228	9.6180	9.6340	9.6190

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	82	83	84	85	86	87	88	89	90
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.9793	8.7234	9.0855	8.8956	9.2327	9.0309	9.1348	9.0493	8.6199	8.7407
1.3	Reduce Model	x_1, x_3, x_4	9.8360	9.8392	9.8400	9.8420	9.8425	9.8344	9.8381	9.8432	9.8385	9.8368
1.4	Reduce Model	x_1, x_2, x_4	-8.8705*	-6.6485*	2.8367	-8.0848*	-6.8221	-6.9806*	-8.7831*	-7.1812*	-8.7346*	-6.3548
1.5	Reduce Model	x_1, x_2, x_3	-8.7261	-5.9446	-0.1512*	0.7120	-7.7432*	-1.8146	4.5269	-0.5312	-6.7813	-8.5625*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	7.9361	7.4350	-	7.7487	-	7.9741	8.2722	7.9852	7.2285	-
2.3	Reduce Model	x_1, x_4	9.7204	9.7273	-	9.7333	-	9.7181	9.7248	9.7354	9.7263	-
2.4	Reduce Model	x_2, x_3	-	-	7.8438	-	8.3957	-	-	-	-	7.4607
2.5	Reduce Model	x_1, x_3	-	-	9.7312	-	9.7358	-	-	-	-	9.7234
2.6	Reduce Model	x_1, x_2	-8.7926*	-6.1473*	1.0845*	-2.9265*	-7.1360*	-4.4946*	0.2413*	-3.6829*	-7.7955*	-7.5136*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.8873	6.2612	7.0884	6.7664	7.5051	7.0052	7.3018	6.9846	5.8592	6.2257
3.3	Reduce Model	x_1	9.6066	9.6177	9.6257	9.6275	9.6280	9.6038	9.6190	9.6296	9.6155	9.6085

Table D.9 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.5383	9.0181	8.8829	9.0897	8.6833	9.1686	9.3986	8.7442	9.2614	8.8919
1.3	Reduce Model	x_1, x_3, x_4	9.8367	9.8385	9.8334	9.8403	9.8389	9.8399	9.8423	9.8343	9.8421	9.8395
1.4	Reduce Model	x_1, x_2, x_4	-3.1267	-7.6288	1.0106	-5.0612	0.0109	-4.6607*	-8.7045	-7.0540*	-5.2265	-5.0849
1.5	Reduce Model	x_1, x_2, x_3	-8.1039*	-8.1820*	-8.6759*	-8.7519*	-8.7398*	3.3604	-8.9580*	-5.1110	-8.6186*	-8.7277*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	-	-	-	8.1409	-	7.4263	-	-
2.3	Reduce Model	x_1, x_4	-	-	-	-	-	9.7298	-	9.7197	-	-
2.4	Reduce Model	x_2, x_3	7.0690	7.9746	7.7337	8.1576	7.3431	-	8.8137	-	8.5319	7.7539
2.5	Reduce Model	x_1, x_3	9.7225	9.7259	9.7173	9.7293	9.7264	-	9.7342	-	9.7330	9.7275
2.6	Reduce Model	x_1, x_2	-5.8163*	-7.9315*	-3.3409*	-6.9730*	-3.7577*	0.1873*	-8.8399*	-6.1135*	-7.0011*	-6.9776*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	5.8620	7.0173	6.6251	7.0673	6.3018	7.2278	8.2105	6.2090	7.6591	6.7549
3.3	Reduce Model	x_1	9.6106	9.6169	9.6023	9.6194	9.6194	9.6244	9.6244	9.6047	9.6267	9.6152

Table D.10 The T_D Values for Variable Selection When $\rho_{12} = 0.8$

Iteration	Model	Variables in the Model	Replicate									
			1	2	3	4	5	6	7	8	9	10
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0980	8.9224	8.9414	9.1875	9.0308	9.0839	9.0435	9.2842	8.8188	8.7874
1.3	Reduce Model	x_1, x_3, x_4	9.8423	9.8292	9.8422	9.8395	9.8361	9.8391	9.8413	9.8454	9.8408	9.8409
1.4	Reduce Model	x_1, x_2, x_4	-7.5585*	-8.5809*	-2.0406	-8.8898*	-6.0264	-7.0777*	-7.5742	-8.8239*	-7.4281	-0.9828*
1.5	Reduce Model	x_1, x_2, x_3	0.2556	-4.8243	-2.8375*	-8.8453	-7.5899*	-6.8552	-8.2267*	-0.1989	-8.0083*	2.7767
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.2228	7.8107	-	8.3810	-	8.1309	-	8.5748	-	7.6277
2.3	Reduce Model	x_1, x_4	9.7334	9.7068	-	9.7276	-	9.7273	-	9.7395	-	9.7315
2.4	Reduce Model	x_2, x_3	-	-	7.7504	-	7.9740	-	8.0567	-	7.5885	-
2.5	Reduce Model	x_1, x_3	-	-	9.7332	-	9.7208	-	9.7317	-	9.7302	-
2.6	Reduce Model	x_1, x_2	-3.1673*	-6.6909*	-2.8883*	-8.8688*	-6.5577*	-6.9977*	-7.8141*	-4.1015*	-7.8388*	1.0315*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.0880	6.6482	6.7974	7.5474	6.9482	7.2157	7.0154	7.5060	6.3534	6.8462
3.3	Reduce Model	x_1	9.6266	9.5853	9.6244	9.6158	9.6073	9.6156	9.6224	9.6388	9.6185	9.6246

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			11	12	13	14	15	16	17	18	19	20
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.2108	8.4016	9.1598	9.2941	8.9069	9.0618	8.9704	9.0617	9.0584	8.6512
1.3	Reduce Model	x_1, x_3, x_4	9.8438	9.8382	9.8433	9.8402	9.8357	9.8403	9.8383	9.8418	9.8338	9.8360
1.4	Reduce Model	x_1, x_2, x_4	-6.1588*	-4.5507	-3.0432*	-8.5718	-8.7629*	-2.4319	-8.8250*	-8.7995*	-8.7522	-3.4329
1.5	Reduce Model	x_1, x_2, x_3	0.2371	-8.2974*	1.6846	-8.8463*	-5.3171	-6.0176*	-4.5872	-1.8158	-8.7571*	-8.7421*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.2985	-	8.0409	-	7.7861	-	7.9278	8.1080	-	-
2.3	Reduce Model	x_1, x_4	9.7371	-	9.7381	-	9.7198	-	9.7258	9.7338	-	-
2.4	Reduce Model	x_2, x_3	-	6.7359	-	8.6023	-	7.9999	-	-	8.0965	7.2495
2.5	Reduce Model	x_1, x_3	-	9.7281	-	9.7291	-	9.7326	-	-	9.7166	9.7233
2.6	Reduce Model	x_1, x_2	-2.9449*	-6.5728*	-0.7829*	-8.7100*	-7.0444*	-4.5676*	-6.7486*	-5.2003*	-8.7555*	-6.0905*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.3530	5.2801	7.1600	7.8283	6.6242	7.0505	6.8478	6.9539	7.1216	5.9589
3.3	Reduce Model	x_1	9.6359	9.6153	9.6342	9.6177	9.6043	9.6257	9.6130	9.6260	9.5999	9.6090

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			21	22	23	24	25	26	27	28	29	30
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.7256	8.9569	8.8411	9.0854	8.9657	9.0648	9.1716	9.1721	9.2056	9.0862
1.3	Reduce Model	x_1, x_3, x_4	9.8438	9.8363	9.8314	9.8395	9.8387	9.8338	9.8415	9.8399	9.8350	9.8364
1.4	Reduce Model	x_1, x_2, x_4	3.2971	-8.5699*	-7.8317*	-7.5443*	3.5146	-6.7993*	-4.3538	-8.6960*	-8.7476*	-8.4002*
1.5	Reduce Model	x_1, x_2, x_3	2.6223	-7.4547	2.3282	-2.4752	-2.0969*	-1.6950	-7.8556*	-5.3833	-7.8461	-6.6703
2.1	Reference Model	x_1, x_2, x_3, x_4		-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4		7.8825	7.6605	8.0786	-	8.0310	-	8.3376	8.4017	8.1552
2.3	Reduce Model	x_1, x_4		9.7222	9.7112	9.7279	-	9.7201	-	9.7290	9.7189	9.7234
2.4	Reduce Model	x_2, x_3		-	-	-	7.7073	-	8.2884	-	-	-
2.5	Reduce Model	x_1, x_3		-	-	-	9.7289	-	9.7337	-	-	-
2.6	Reduce Model	x_1, x_2		-8.0050*	-1.7906*	-4.8449*	0.8553*	-3.9478*	-6.2603*	-7.0985*	-8.3140*	-7.5680*
3.1	Reference Model	x_1, x_2, x_3, x_4		-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2		6.7870	6.6532	7.0233	6.8506	6.9630	7.3908	7.3477	7.5339	7.1897
3.3	Reduce Model	x_1		9.6066	9.5952	9.6201	9.6224	9.6068	9.6269	9.6175	9.6034	9.6086

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.2361	9.1747	8.9748	9.2263	8.5899	8.9270	9.0206	8.8233	9.0982	8.8935
1.3	Reduce Model	x_1, x_3, x_4	9.8415	9.8399	9.8391	9.8401	9.8374	9.8334	9.8401	9.8400	9.8409	9.8356
1.4	Reduce Model	x_1, x_2, x_4	2.9202	0.8143	-4.8612	-8.7991*	-5.2397	-6.8164	-5.6579	-0.3088	-2.6515	-6.8175
1.5	Reduce Model	x_1, x_2, x_3	-6.8721*	-8.7012*	-8.2703*	-2.2392	-7.2923*	-8.8087*	-7.8222*	-5.9145*	-3.1734*	-8.6048*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	-	-	8.4565	-	-	-	-	-	-
2.3	Reduce Model	x_1, x_4	-	-	-	9.7291	-	-	-	-	-	-
2.4	Reduce Model	x_2, x_3	8.3774	8.3446	7.8582	-	7.1560	7.8259	7.9789	7.5072	8.0620	7.7548
2.5	Reduce Model	x_1, x_3	9.7326	9.7286	9.7275	-	9.7234	9.7163	9.7292	9.7298	9.7308	9.7241
2.6	Reduce Model	x_1, x_2	-0.9953*	-3.2582*	-6.4050*	-5.4536*	-6.0567*	-7.8198*	-6.7438*	-2.4964*	-3.3682*	-7.7614*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.3195	7.2698	6.6934	7.3472	5.8192	6.7471	6.9729	6.4446	7.1707	6.5739
3.3	Reduce Model	x_1	9.6286	9.6200	9.6162	9.6196	9.6095	9.6002	9.6211	9.6224	9.6216	9.6139

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			41	42	43	44	45	46	47	48	49	50
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	8.3904	8.8232	8.9489	9.0655	9.3669	9.2957	9.2505	8.7820	8.8264	8.7076
1.3	Reduce Model	x_1, x_3, x_4	9.8390	9.8320	9.8414	9.8374	9.8443	9.8426	9.8409	9.8418	9.8365	9.8412
1.4	Reduce Model	x_1, x_2, x_4	1.2416*	-3.9404	-5.6549	-6.1878*	-8.8428*	-0.5065*	-8.0949	-8.7562*	-8.7544*	-2.0592
1.5	Reduce Model	x_1, x_2, x_3	2.9322	-7.9672*	-5.8308*	2.6916	-8.7310	0.1084	-8.6767*	-2.8007	-0.0389	-7.3959*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	6.9963	-	-	7.9805	8.7540	8.2238	-	7.5289	7.6218	-
2.3	Reduce Model	x_1, x_4	9.7274	-	-	9.7245	9.7375	9.7376	-	9.7322	9.7217	-
2.4	Reduce Model	x_2, x_3	-	7.5905	7.8229	-	-	-	8.4932	-	-	7.3743
2.5	Reduce Model	x_1, x_3	-	9.7130	9.7323	-	-	-	9.7320	-	-	9.7329
2.6	Reduce Model	x_1, x_2	1.9432	-5.8316*	-5.8434*	-1.2298*	-8.7851*	-0.4152*	-8.4281*	-5.6886*	-4.0020*	-4.9748*
3.1	Reference Model	x_1, x_2, x_3, x_4		-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2		6.3962	6.9026	7.0757	8.1145	7.5400	7.7026	6.2712	6.5964	6.1111
3.3	Reduce Model	x_1		9.5938	9.6238	9.6180	9.6307	9.6327	9.6213	9.6232	9.6122	9.6235

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			51	52	53	54	55	56	57	58	59	60
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.2704	8.6276	9.0257	8.7320	8.8351	9.0214	9.1637	9.3053	8.8404	8.9131
1.3	Reduce Model	x_1, x_3, x_4	9.8400	9.8335	9.8381	9.8393	9.8396	9.8367	9.8443	9.8416	9.8369	9.8350
1.4	Reduce Model	x_1, x_2, x_4	-8.3484*	-8.7492*	-8.7940*	-8.0284	-6.3521*	-6.0964*	-5.2470	-7.0387*	-8.5774*	-8.8906*
1.5	Reduce Model	x_1, x_2, x_3	-1.8153	-4.1626	-7.4164	-8.6677*	-4.0623	2.2087	-5.5837*	-6.1962	-6.4814	-0.2891
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.5088	7.2014	8.0384	-	7.5704	7.8733	-	8.5301	7.6327	7.8335
2.3	Reduce Model	x_1, x_4	9.7291	9.7155	9.7250	-	9.7288	9.7224	-	9.7324	9.7229	9.7190
2.4	Reduce Model	x_2, x_3	-	-	-	7.4315	-	-	8.0635	-	-	-
2.5	Reduce Model	x_1, x_3	-	-	-	9.7272	-	-	9.7383	-	-	-
2.6	Reduce Model	x_1, x_2	-4.9363*	-6.5374*	-8.1303*	-8.3865*	-5.3269*	-1.2397*	-5.3120*	-6.6388*	-7.5347*	-4.2945*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.5886	5.8789	6.9972	6.1293	6.4691	6.9864	7.1545	7.7315	6.4075	6.7727
3.3	Reduce Model	x_1	9.6195	9.6036	9.6138	9.6137	9.6175	9.6115	9.6331	9.6226	9.6095	9.6040

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			61	62	63	64	65	66	67	68	69	70
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0379	9.1161	8.8053	9.1160	9.3207	8.5666	9.2083	8.9648	9.2663	8.7418
1.3	Reduce Model	x_1, x_3, x_4	9.8406	9.8430	9.8377	9.8438	9.8441	9.8431	9.8400	9.8404	9.8438	9.8404
1.4	Reduce Model	x_1, x_2, x_4	-8.8745*	-0.8442	-6.0101*	-4.3096	-7.9079*	-8.0965*	-8.0836*	-2.8557	-1.8445	-7.8034
1.5	Reduce Model	x_1, x_2, x_3	-3.0979	-5.7026*	-5.6809	-5.9901*	-5.3911	2.8906	-7.0571	-8.7749*	-8.4183*	-8.7497*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	8.0697	-	7.4976	-	8.5704	7.0759	8.3657	-	-	-
2.3	Reduce Model	x_1, x_4	9.7298	-	9.7250	-	9.7372	9.7350	9.7298	-	-	-
2.4	Reduce Model	x_2, x_3	-	8.0665	-	8.1485	-	-	-	7.9036	8.5509	7.4348
2.5	Reduce Model	x_1, x_3	-	9.7350	-	9.7368	-	-	-	9.7297	9.7364	9.7300
2.6	Reduce Model	x_1, x_2	-5.9777*	-3.3505*	-5.6242*	-5.1348*	-6.7110*	-1.3020*	-7.5540*	-5.7849*	-5.0915*	-8.2784*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.9484	7.1192	6.3007	7.2284	7.6630	6.2391	7.5352	6.8005	7.5701	6.1073
3.3	Reduce Model	x_1	9.6202	9.6307	9.6116	9.6296	9.6312	9.6280	9.6196	9.6207	9.6318	9.6184

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			71	72	73	74	75	76	77	78	79	80
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.1339	8.5372	8.6607	9.0504	8.6054	8.9421	9.0450	8.9908	9.0875	8.8322
1.3	Reduce Model	x_1, x_3, x_4	9.8366	9.8412	9.8383	9.8416	9.8396	9.8345	9.8433	9.8398	9.8403	9.8389
1.4	Reduce Model	x_1, x_2, x_4	-8.0996	-5.2111*	-4.9722*	-4.1989	-8.8054*	0.6130	-7.0045	-6.0372	-7.7873*	-7.2813*
1.5	Reduce Model	x_1, x_2, x_3	-8.3286*	-3.3455	5.2197	-8.6042*	-5.4103	-7.6459*	-8.4786*	-7.4730*	1.9313	-2.9497
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	6.9999	7.2604	-	7.1613	-	-	-	8.1765	7.6102
2.3	Reduce Model	x_1, x_4	-	9.7347	9.7254	-	9.7287	-	-	-	9.7311	9.7279
2.4	Reduce Model	x_2, x_3	8.2243	-	-	8.1218	-	7.8214	8.0829	7.8724	-	-
2.5	Reduce Model	x_1, x_3	9.7243	-	-	9.7321	-	9.7185	9.7353	9.7285	-	-
2.6	Reduce Model	x_1, x_2	-8.1997*	-4.2757*	2.2992	-6.3614*	-7.1334*	-2.7722*	-7.7759*	-6.9060*	-2.1844*	-4.6473*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-		-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.3525	5.7166		7.0398	5.8939	6.7358	7.0474	6.9271	7.0113	6.5085
3.3	Reduce Model	x_1	9.6102	9.6265		9.6270	9.6169	9.6050	9.6264	9.6165	9.6256	9.6167

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.0408	8.8893	8.8298	8.8742	9.0690	9.1060	9.2000	8.9785	9.0048	8.8888
1.3	Reduce Model	x_1, x_3, x_4	9.8374	9.8361	9.8353	9.8351	9.8430	9.8375	9.8389	9.8392	9.8375	9.8323
1.4	Reduce Model	x_1, x_2, x_4	-6.8391	-8.5351*	-4.1713*	-7.4832	-5.3812	-8.5941	-5.7740*	-8.3541*	-8.7719*	-6.2312
1.5	Reduce Model	x_1, x_2, x_3	-8.6457*	-8.3631	-3.2669	-8.1828*	-8.7822*	-8.7963*	-3.5029	-7.3138	3.7081	-8.7485*
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	7.7644	7.5728	-	-	-	8.1992	7.9175	7.9905	-
2.3	Reduce Model	x_1, x_4	-	9.7218	9.7195	-	-	-	9.7285	9.7272	9.7238	-
2.4	Reduce Model	x_2, x_3	8.0761	-	-	7.7091	8.1275	8.2420	-	-	-	7.7610
2.5	Reduce Model	x_1, x_3	9.7233	-	-	9.7215	9.7346	9.7235	-	-	-	9.7150
2.6	Reduce Model	x_1, x_2	-7.7680*	-8.4241*	-3.3852*	-7.7923*	-7.1217*	-8.6989*	-4.7305*	-7.8384*	-0.6628*	-7.4179*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	6.9963	6.6055	6.5501	6.5627	7.0730	7.3450	7.2589	6.9912	7.0459	6.6107
3.3	Reduce Model	x_1	9.6095	9.6059	9.6091	9.6056	9.6258	9.6082	9.6207	9.6140	9.6141	9.5965

Table D.10 (Continued)

Iteration	Model	Variables in the Model	Replicate									
			91	92	93	94	95	96	97	98	99	100
1.1	Reference Model: Full Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
1.2	Reduce Model	x_2, x_3, x_4	9.1987	8.9644	9.1491	9.0468	9.3002	9.1396	8.8424	9.0304	9.1482	9.0058
1.3	Reduce Model	x_1, x_3, x_4	9.8340	9.8402	9.8388	9.8427	9.8395	9.8427	9.8415	9.8370	9.8400	9.8398
1.4	Reduce Model	x_1, x_2, x_4	-7.6937	-8.8444*	-8.5286*	-8.1348*	-0.9398	-0.3207	-3.5545	-1.5772	-8.4608*	-8.7382*
1.5	Reduce Model	x_1, x_2, x_3	-8.8109*	4.7894	-4.7968	3.6728	-3.0873*	-8.5619*	-7.5163*	-8.7990*	-5.6410	-8.0489
2.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
2.2	Reduce Model	x_2, x_4	-	7.9360	8.2740	8.0511	-	-	-	-	8.2900	-
2.3	Reduce Model	x_1, x_4	-	9.7289	9.7266	9.7344	-	-	-	-	9.7288	-
2.4	Reduce Model	x_2, x_3	8.3989	-	-	-	8.2927	8.2590	7.6378	8.0456	-	7.9527
2.5	Reduce Model	x_1, x_3	9.7168	-	-	-	9.7297	9.7341	9.7319	9.7225	-	9.7313
2.6	Reduce Model	x_1, x_2	-8.2664*	0.6238*	-6.5235*	-0.5590*	-2.1915*	-4.1490*	-5.5289*	-5.0118*	-7.0723*	-8.4051*
3.1	Reference Model	x_1, x_2, x_3, x_4	-	-	-	-	-	-	-	-	-	-
3.2	Reduce Model	x_2	7.5374	6.8975	7.2236	6.9777	7.5187	7.2418	6.4771	6.8927	7.2515	6.9462
3.3	Reduce Model	x_1	9.6035	9.6250	9.6142	9.6306	9.6218	9.6273	9.6230	9.6094	9.6170	9.6199

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