INVESTIGATION OF CREDIT RATING AGENCY PAYMENT SCHEME TO IMPROVE PROFITABILITY AND RATING QUALITY

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ABSTRACT

Title of Dissertation	Investigation of Credit Rating Agency Payment Scheme to Improve Profitability and Rating Quality
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This dissertation aims to examine the impact of incentive compensation mechanism on encouraging credit rating agencies (CRAs) to exert effort in the rating evaluation process and report ratings truthfully. Two analytical studies: one focusing on the CRA's effort and rating decisions, and the other focusing on the investor's information acquisition decision and the resulting CRA's behavior, are conducted.

The first research investigates how a performance-based mechanism can influence the credit rating agency's behavior of choosing her effort level when conducting a credit rating and announcing a truthful or biased rating based on the observed data. In this setting, the CRA can exert some effort to observe a signal and makes a decision to either truthfully report the rating or biasedly report (deflate or inflate) the rating in presence of the expected reputational loss realized when the rating is inaccurate. Two payment mechanisms: upfront fee and incentive pay are considered. Under the incentive pay scheme, the CRA charges a fixed fee to offset her information production cost and can claim the incentive fee only if the issued rating correctly predicts the outcome of the rated security. Our findings reveal that the incentive pay can encourage the CRA to exert a higher level of effort to produce a credible signal leading to rating accuracy improvement. Furthermore, the incentive pay scheme can more effectively incentivize the CRA to truthfully report the rating that consistently reflects the observed signal, compared to the upfront fee scheme.

The second research investigates the investor's decision on information acquisition for the rating and the CRA's decision on the truthfulness of the rating report. Two payment mechanisms: upfront fee and profit-sharing scheme, are considered. The study consists of three main parts: 1) investigation of dynamic interaction between the investor and the CRA, 2) comparison of optimal decisions of the investor and the CRA under the two payment schemes, and 3) decisions on information acquisition of the two types of investors: institutional and marginal investor. In our analysis, the CRA's behavior of rating policy adoption and the investor's behavior of requesting additional information accuracy level are characterized. The qualities of credit ratings resulting from the two payment schemes are compared to pinpoint which payment scheme is more effective in improving rating quality.

Ultimately, this dissertation provides key insights for policymakers as well as other agents involved in the credit rating process on how to design efficient payment schemes to improve the accuracy of ratings, and subsequently, the desirable outcome of uncertain financial instruments and projects.

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ABBREVIATIONS AND SYMBOLS

Abbreviations	**Equivalence**
CRA	Credit rating agency

SYMBOLS

T	The loan type
G	A good type of loans
B	A bad type of loans
R_G	Payoff of the good loan
R_B	Payoff of the bad loan
α	A fraction of good loans in the market
∅	a signal observed by the CRA
e	Effort level
$\Psi(e)$	A rating evaluation cost
θ	A cost parameter
σ_d	A rating deflation probability
σ_i	A rating inflation probability
$\tilde{r}$	An indicative rating
h	A high rating
l	A low rating
n	A project type
p_n	A success probability of a project type
$x_n(R)$	An expected payoff of each project type
α_g	A fraction of good project in the market
α_b	A fraction of bad project in the market
β	A retained share of an investor

$C(i)$	An information production cost
$\mu^{\emptyset}$	Rating strategy after observing a signal
$q(i)$	A signal accuracy of the CRA
I	An indicative function of misreporting probability

CHAPTER 1

INTRODUCTION

The outbreak of the global financial crisis in 2008 has brought about the intensive scrutiny of credit rating agencies' role in providing credible information on the riskiness of all kinds of debts' underlying assets. In particular, the "Big three" credit rating agencies (CRAs) - Fitch ratings, Moody's, and Standard and Poor's (S&P) - were accused of aggravating the financial crisis by providing overly optimistic evaluation of extremely risky assets, especially insolvent financial institution and mortgage-backed securities. In general, credit rating agencies are also perceived to be quite unregulated by various groups of people such as politicians, market participants, policymakers, and even rating agencies themselves. A major cause of inaccurate ratings issued by CRAs lies in the current business model of credit rating services where CRAs are paid for their rating task by the issuers of the rated financial products, also known as the "issuer-pay model." This payment model is most commonly adopted worldwide. Under the issuer-pay model, since CRAs are compensated by the issuers whose securities they are assessing, the CRAs tend to offer overly favorable ratings to attract fees from the issuers. If the rating agencies issue low ratings, the issuers can purchase ratings from other rating agencies who would offer better ratings. This behavior, known as "rating shopping," can eventually lead to low rating accuracy. To solve these problems embedded in the issuer-pay model, some policymakers propose a switch from the issuer-pay model to the investor-pay model. Under the investor-pay model, CRAs charge investors a subscription fee for access to their rating information. The investor-pay model, however, still cannot effectively improve the quality of credit ratings. Since under this payment model, CRAs receive a one-time fixed fee and are not responsible for any loss resulted from the failure of the products they have rated, the CRAs are not motivated to exert enough effort to acquire relevant information and provide accurate ratings. Accordingly, politicians and policymakers increasingly look for a more binding form of mechanisms that can better tie CRAs' performance with their compensation in order to improve the quality of credit ratings.

Aiming to address the problem of low credit rating accuracy, this dissertation is composed of two theoretical research studies regarding new compensation mechanisms that better align the incentives of the CRAs and the payer (e.g., issuers or investors) and discourage the CRAs' behavior of issuing biased ratings. The new compensation schemes are designed mainly based on the idea of tying the CRAs' performance with the payment that they will receive after the outcome of the rated securities have been realized. Different from existing literature, this dissertation takes CRA's conduct of issuing both inflated rating and deflated rating into consideration. Also, it examines the CRA's incentive towards giving out accurate ratings and that under the performance-based payment scheme. Ultimately, the investor's decision on requesting information and the CRA's decision on selecting rating policy are dynamically explored. This dissertation contributes to new academic knowledge of understanding impacts of performance based compensation scheme on the CRA's incentive towards improving rating quality and the investor's incentive towards acquiring information when soliciting ratings. For managerial implication, the findings can be used by policymakers to enforce performance based compensation policy and practitioners to make informed decisions under the proposed compensation scheme to prevent financial crises arising from inaccurate ratings.

The first research examines an incentive pay scheme as a mechanism to reduce the biased rating problem often observed with the traditional issuer-pay scheme. Under the incentive pay scheme, the CRA is compensated based on accuracy of the ratings she issues. We consider a setting where the CRA can potentially inflate or deflate ratings, as supposed to truthfully report ratings based on the best available information. We assume the CRA incurs the reputational loss and loss of the incentive pay whenever she issues inaccurate ratings. The focus of this paper is on the CRA's decision – how much effort to exert in the rating task and whether to report the rating truthfully or biasedly. To investigate the advantages of the incentive pay scheme over the conventional upfront fee scheme, we compare which compensation scheme better induces the CRA to truthfully report ratings and put more effort in the rating evaluation process. The results show that the incentive pay mechanism induces the CRA to put more effort in the rating evaluation process and issue more accurate rating, compared to the fixed upfront fee scheme.

The second research considers another performance-based mechanism in the context of investor-pay model. We scrutinize a profit-sharing scheme where the CRA gets a fixed fee to cover her rating production cost and later shares the profit or loss from the outcome of the rated securities once it has been realized. In this study, we examine the dynamic decision of both the investor (institutional or marginal investor) and the CRA. When purchasing a rating from the CRA, the investor can request any level of additional information accuracy to be obtained by the CRA. The CRA makes a rating decision whether to truthfully report the rating or biasedly report the rating (inflation or deflation). Our findings show that the institutional investor, restricted by the investment regulation allowing him to invest in only high-rated (investment grade) securities, would request additional information based on the size of profit-sharing proportion. More precisely, if a large amount of the financial outcome would be allocated to the CRA, then the investor would request a low level of additional information accuracy. If a medium amount would be distributed to the CRA, the investor would request a sufficiently high accuracy level. If a small amount would be shared with the CRA, the investor would request the maximum accuracy of the rating. For a marginal investor, who can invest in any securities, the decision to purchase a rating from the CRA depends on market condition. If the market is extremely favorable (high proportion of good securities), the investor would invest in securities without purchasing the rating. If the market is extremely unfavorable, the investor would neither invest in securities nor purchase the rating. In other cases, the investor would purchase the rating and his decision to request additional accuracy of the rating would depend on the size of profit-sharing proportion, similar to that situation for the institutional investor. Ultimately, it has been shown that the profit sharing scheme better induces the CRA to produce accurate ratings than the fixed upfront fee scheme does.

CHAPTER 2

PERFORMANCE-BASED MECHANISM WITH EFFORT AND REPUTATIONAL EFFECTS

2.1 Introduction

The role of the credit rating agencies (CRAs) has been widely questioned after a massive default of structured products causing the Hamburger crisis. Subsequently, CRAs' downgrade announcements of sovereign debts of nine Eurozone countries negatively affect the Euro currency and initiate the ongoing Eurozone crisis. Accordingly, CRAs have undergone under intensive scrutiny since credit ratings do not reflect credit risk embedded in financial assets, especially complex products. In principle, CRAs produce credit ratings to help investors price financial assets more accurately and reduce asymmetric information between issuers and investors. Credit ratings should be conceptually viewed as opinion but market participants (e.g., issuers, investors, creditors, and policy makers) have mainly relied on ratings to make informed decisions. Furthermore, ratings become a fixture of financial regulation such as banks' capital requirement (International Organization of Securities Commissions, 2008) allowing financial institutions to hold high-rated bonds to meet liquidity requirement. With investors' incremental demand for high-rated bonds to exploit regulatory benefit, issuers are motivated to shop for rating and CRAs distort ratings to satisfy issuer's objectives in exchange of fees. Besides, CRAs can indirectly force issuers, who refuse to solicit ratings, to purchase ratings by issuing unfavorable unsolicited ratings (Fulghieri, Strobl, & Xia, 2014). CRAs' incentive problems therefore deteriorate rating accuracy and usefulness.

The incentive problems arise mainly from conflict of interest and compensation scheme. Conflict of interest problem inherent in the issuer-pay business model inspires CRAs to inflate ratings to induce issuers to purchase ratings, and maintain long-term relationship with issuers. To alleviate the conflict of interest problem, switching to the investor-pay model is a proposed alternative but it is argued that CRAs may suffer from a free rider problem due to information leakage among investors and cannot earn sufficient income to survive in the rating business. Current payment schemes (e.g., the upfront fee, and rating contingent fee) can incentivize CRAs to distort ratings since CRAs are not financially affected from the outcome of securities they rates. Therefore, politicians and policymakers proposed that CRAs' compensation should be tied to their performance to ensure credible rating being produced.

Aiming to address the incentive problems and rating inaccuracy, an incentive pay scheme which ties the outcome of rated securities to CRAs' compensation is theoretically analyzed. This study examines the influence of the incentive pay and upfront fee scheme on the CRAs' incentive toward rating policy selection and rating accuracy improvement since the pros and cons of the incentive pay over upfront fee scheme have not been extensively studied. In our model, the CRA can choose a rating policy and the effort level to be put in the rating evaluation process. To determine a better payment scheme, the CRA's conduct on selecting rating policy and exerting effort to improve rating accuracy under the upfront fee scheme is compared to that under the incentive pay scheme. Our findings reveal that the incentive pay scheme provides better incentive for the CRA to adopt the full disclosure regime, hence resulting in more accurate ratings, than the upfront fee scheme does. Furthermore, the incentive pay scheme motivates the CRA to exert a higher effort level in the rating evaluation process than the upfront fee scheme does.

2.2 Literature Review

Existing researches have reported that both exogenous and endogenous factors have negative influence on CRAs' incentive to produce accurate ratings. Exogenous factors that worsen the incentive problems are rating shopping behavior of issuing firms (Skreta & Veldkamp, 2009), regulatory benefit of holding high rated securities

(Opp, Christian, Opp, Marcus, & Harris, 2013), the presence of naïve investors (Bolton, Freixas, & Shapiro, 2012), and increasing competition in the CRA industry (Camanho, Deb, & Liu, 2009). Endogenous factors that adversely affect rating quality of CRAs are the desire to develop long-term relationships with issuing firms (Mullard, 2012), inept rating models (Benmelech & Dlugosz, 2010), and analysts' biases (Ashcraft, Goldsmith-Pinkham, & Vickery, 2010).

A strand of theoretical researches builds on reputational concerns to discipline CRAs' behavior. Mathis, McAndrews, & Rochet. (2009) presenting a dynamic model of reputational concerns show that increasing business from structured products weaken CRAs' incentive to issue accurate ratings. Similarly, the reputational concerns cannot prevent CRAs from inflating ratings particularly when CRAs' main revenue derives from complex products (Wang, 2011). Increasing new entrants in the rating industry deteriorates rating quality when CRAs weigh expected fee income against reputational cost (Becker & Milbourn, 2011). With noisy private information, monopoly power motivates CRAs to deflate ratings to mislead market participants as if they have more precise private information, and increasing competition influences CRAs to inflate ratings to protect their market power (Mariano, 2012). Including marketing environment and reputational concerns into a dynamic rating model, rating quality negatively correlates with economic cycle (Bar-Isaac & Shapiro, 2013) and CRA's incentive toward issuing accurate ratings is distorted in boom periods, high fee revenue, and increasing numbers of naïve investors (Bolton, Freixas, & Shapiro, 2012). Built on a cheap-talk game, competition among CRAs can increase rating coarseness which reduces social welfare (Goel & Thakor, 2015). Extending from the cheap-talk model (Frenkel, 2015), CRAs intend to maintain double reputation, which is credibility reputation among investors and leniency reputation among issuers to maintain long-term business relationship, so that the CRAs tend to inflate ratings when few issuers exist in a market of structured products. Yao, Gu, & Chen (2017) show that CRAs, weighing current incomes obtained by issuing favorable ratings against reputation costs, tend to deflate ratings during recessions and inflate ratings during booms.

A line of theoretical studies considers alternative compensation schemes to improve rating accuracy. Bongaerts (2014) introduced the CRA co-investment system which requires CRAs to partially invest in securities they give out high rating to ensure rating accuracy. Practically, the system may deter the growth of debt markets because of CRAs' capital constraint. Kashyap and Kovrijnykh (2016) analyzes the influence of commission scheme on different business models (e.g., an investor-pay, issuer-pay, and social planner-pay model). They infer that under the commission scheme the investor-pay model can more effectively influence CRAs to improve rating quality than the issuer-pay model does. However, both studies do not consider CRAs' behavior of distorting ratings in their model setting. Proposed by Dodd-Frank Act 2010, the issuer risk retention (Ozerturk, 2015) requires issuers to retain a fraction of the securitized portfolios to discourage the issuers from pulling bad loans with good ones. However, this mechanism directly puts pressure on the issuers to ensure their loan creditworthiness but CRAs' incentive problems may not be exactly mitigated. Ozerturk (2014), taking CRAs' inflating rating behavior into consideration, investigates CRAs' incentive to improve rating quality under the upfront fee versus that under the rating contingent fee, where CRAs get paid only if they provide favorable ratings. The finding shows that the upfront fee induces CRAs to produce more or equal rating quality than the rating contingent fee does. In the present study, we examine a relevant problem to Ozerturk (2014), but consider a different scheme, incentive pay, where the payment to the CRA is tied to the accuracy of the ratings.

Key differences between this study and aforementioned literatures are twofold. Firstly, our model investigates the possibilities for the CRA to either inflate or deflate ratings, depending on the financial incentives, which allows the model to capture the reality to a better extent. Secondly, our work illustrates how the CRA's rating quality is affected by a new type of payment scheme, incentive pay, in comparison to the traditional scheme of upfront fee. This study provides yet another supporting piece of evidence that it might be beneficial to the community if the payments to the CRAs are allowed to be tied to the CRAs' performance.

2.3 Model

We consider a setting where an issuer (he) brings a risky loan portfolio to a credit rating agency (she) to be evaluated. Both the issuer and the CRA are rational and risk-neutral. The risk-free rate is zero. There are two types of loan portfolios: good (G) or bad (B). The loan type is given by $\mathrm{T} \in \{G, B\}$, and its return is given by R_T. A good portfolio generates a positive payoff, $R_G > 0$, and a bad one yields zero return, $R_B = 0$. It is public information that a fraction of good portfolios in the market is given by $\alpha \in [0, 1]$, and a fraction of bad portfolios is the marketing is given by $1 - \alpha$.

The CRA has rating evaluation technology that can help assess the portfolio quality by reporting a signal, $\emptyset \in \{g, b\}$, where g denotes a good quality and b signifies a bad quality of the portfolio. If the CRA does not exert effort in the rating evaluation, then the observed signal $\emptyset$ is accurate with probability of $\frac{1}{2}$. However, the CRA can increase signal accuracy by exerting effort, $e \in [0, \frac{1}{2}]$, in the rating evaluation process to improve signal accuracy. That is,

$$\Pr\{\emptyset = g|\mathrm{T} = G\} = \Pr\{\emptyset = b|\mathrm{T} = B\} = \frac{1}{2} + e \tag{1}$$

The CRA incurs the rating evaluation cost with effort level given by

$$\Psi(e) = \theta e^2 \quad \text{and} \quad \Psi(0) = 0 \tag{2}$$

where $\theta > 0$ is a cost parameter for hiring analysts and upgrading rating models to cope with increasing complexity of portfolios. After observing the signal $\emptyset$, the CRA announces the rating of the portfolio, $\tilde{r} \in \{h, l\}$, where h and l represent a high and low rating respectively. The CRA can make the reported rating consistent with the observed signal (the full disclosure regime) or contradicting with the actual signal (the rating inflation or rating deflation regime). The probability that the CRA announces a low rating while observing a good signal (deflation) is given by $\sigma_d \in [0, 1]$. That is,

$$\Pr\{\tilde{r} = l | \emptyset = g\} = 1 - \Pr\{\tilde{r} = h | \emptyset = g\} = \sigma_d \quad (3)$$

The probability that the CRA announces a high rating while observing a bad signal (inflation) is denoted by $\sigma_i \in [0, 1]$. That is,

$$\Pr\{\tilde{r} = h | \emptyset = b\} = 1 - \Pr\{\tilde{r} = l | \emptyset = b\} = \sigma_i \quad (4)$$

The CRA is informed to be compensated by the upfront or incentive pay schemes before the rating evaluation process begins. In addition to the payment scheme being used, the CRA's expected profit is also affected by the discounted reputational loss from the inaccuracy of its rating. This point will be further elaborated in Section 2.4. The sequence of a rating evaluation process is shown in Figure 2.1 as follows: 1) the CRA privately exerts the effort in observing a signal $\emptyset$ of the portfolio, 2) the CRA decides on a disclosure rule σ_d if observing $\emptyset = g$ (rating deflation probability), or σ_i if observing $\emptyset = b$ (rating inflation probability), and 3) the CRA announces a rating $\tilde{r}$

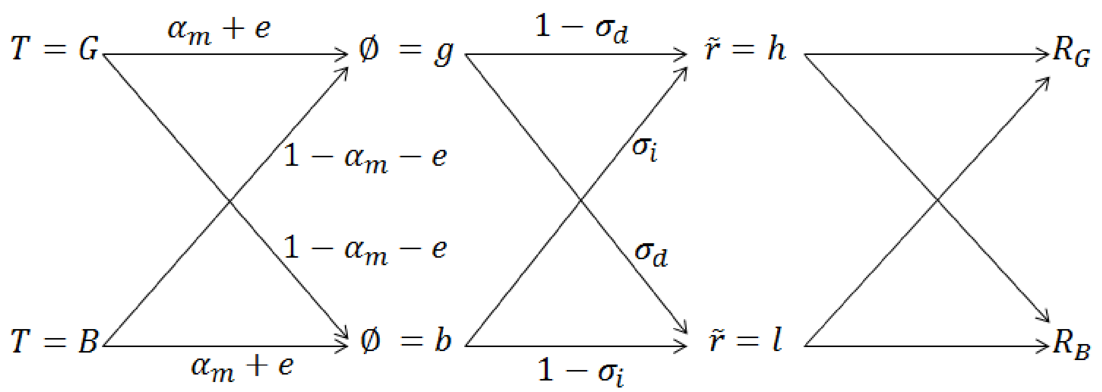

Figure 2.1 The Sequence of a Rating Valuation Process.

2.4 Credit Rating Agency Analysis

After the issuer brings the portfolio to the CRA for rating under a specific term of payment, the CRA decides on how much effort to exert in the information acquisition process, and then observes a signal of the portfolio. Suppose the CRA observes a good signal, $\emptyset = g$. Given the deflation probability σ_d, the CRA will announce a low rating (deflation) with probability σ_d, and will truthfully announce a high rating (full discloser) with probability $1 - \sigma_d$. The CRA exerting effort level e and observing $\emptyset = g$ will give an accurate rating with probability:

$$\mu^g(e, \sigma_d) = \frac{\alpha\left(\frac{1}{2}+e\right)+(\frac{1}{2}-e-\alpha)\sigma_d}{\frac{1}{2}-e+2\alpha e}, \tag{5}$$

and an inaccurate rating with probability $1 - \mu^g(e, \sigma_d) = \frac{\left(\frac{1}{2}-e\right)(1-\alpha)-(\frac{1}{2}-e-\alpha)\sigma_d}{\frac{1}{2}-e+2\alpha e}$, accordingly. Likewise, suppose the CRA observes a bad signal, $\emptyset = b$. With the inflation probability σ_i, the CRA will announce a high rating (inflation) with probability σ_i, and will truthfully announce a low rating (full disclosure) with probability $1 - \sigma_i$. The CRA exerting effort level e and observing a bad signal $\emptyset = b$ will announce an accurate rating for the portfolio with probability:

$$\mu^b(e, \sigma_i) = \frac{\left(\frac{1}{2}+e\right)(1-\alpha)-(\frac{1}{2}+e-\alpha)\sigma_i}{\frac{1}{2}+e-2\alpha e}, \tag{6}$$

and an inaccurate rating with probability $1 - \mu^b(e, \sigma_i) = \frac{\alpha\left(\frac{1}{2}-e\right)+\left(\frac{1}{2}+e-\alpha\right)\sigma_i}{\frac{1}{2}+e-2\alpha e}$, accordingly.

Prior to observing a signal for the type of the portfolio, the probability that the CRA will give an inaccurate rating is:

$$\varepsilon(e, \sigma_d, \sigma_i) = \left(\frac{1}{2} - e\right) - (\frac{1}{2} - e - \alpha)\sigma_d + (\frac{1}{2} + e - \alpha)\sigma_i; \tag{7}$$

while, the probability that the CRA will give an accurate rating, $\lambda(e, \sigma_d, \sigma_i)$, is:

$$1 - \varepsilon(e, \sigma_d, \sigma_i) = \left(\frac{1}{2} + e\right) + (\frac{1}{2} - e - \alpha)\sigma_d - (\frac{1}{2} + e - \alpha)\sigma_i \qquad (8)$$

Before the CRA announces the rating, she weighs discounted reputational loss from inaccurate ratings against the expected payment received from the issuer. The reputational loss is the discounted sum of future profits the CRA expects to lose whenever her assigned ratings do not reflect the actual performance of portfolios, and hence, causing diminished confidence of issuers and potential investors. The magnitude of this loss is denoted by $\tau > 0$, and is incurred when a high-rated portfolio defaults or a low-rated portfolio succeeds. The CRA's expected reputational loss from misreporting the rating, $\Gamma(e, \sigma_d, \sigma_i)$, is:

$$\tau\varepsilon(e, \sigma_d, \sigma_i) = \tau\left[\left(\frac{1}{2} - e\right) - (\frac{1}{2} - e - \alpha)\sigma_d + (\frac{1}{2} + e - \alpha)\sigma_i\right] \qquad (9)$$

2.4.1 The Upfront Fee Scheme

Under the upfront fee scheme, the CRA charges a one-time upfront fee, $f_u > 0$, no matter which rating is assigned to the portfolio. We will first investigate the CRA's rating behavior after observing a signal of the portfolio type. The CRA's expected payoff when exerting an effort e and observing a good signal, $\emptyset = g$, and adopting rating strategy σ_d is given by

$$\Pi_u^g(e, \sigma_d) = \max_{\sigma_d, e}[f_u - \Psi(e) - \tau(1 - \mu^g(e, \sigma_d))]. \qquad (10)$$

The CRA's expected payoff when exerting an effort e and observing a bad signal, $\emptyset = b$, and adopting rating strategy σ_i is given by

$$\Pi_u^b(e, \sigma_i) = \max_{\sigma_i, e}[f_u - \Psi(e) - \tau(1 - \mu^b(e, \sigma_i))]. \qquad (11)$$

The CRA's optimal rating strategy is characterized in Proposition 2.1 below.

Proposition 2.1 Under the upfront fee scheme, the CRA's optimal strategy on reporting ratings is characterized by a threshold on the exerted effort level $\bar{e}^{\emptyset}$ and the observed signal $\emptyset \in \{g, b\}$, where $\bar{e}^{g}(\alpha) = \frac{1}{2} - \alpha = -\bar{e}^{b}(\alpha)$, as follows:

1) If $e \leq \bar{e}^{g}$ and the CRA observes a good signal, it is optimal for the CRA to adopt the rating deflation strategy ($\sigma_d^* = 1$) and report a low rating. Otherwise, if $e > \bar{e}^{g}$, it is optimal for the CRA to adopt the full disclosure strategy ($\sigma_d^* = 0$) and report a high rating.

2) If $e \leq \bar{e}^{b}$ and the CRA observes a bad signal, it is optimal for the CRA to adopt the rating inflation strategy ($\sigma_i^* = 1$) and report a high rating. Otherwise, if $e > \bar{e}^{b}$, it is optimal for the CRA to adopt the full disclosure strategy ($\sigma_i^* = 0$) and report a low rating.

Proof. See Appendix A

If the CRA puts low effort level, $e \leq \bar{e}^{g}(\alpha)$, in the rating evaluation process and observes a good signal, then the observed signal has low accuracy, implying that the portfolio getting the good signal is actually likely to be of bad quality and will default later. Hence, she strategically deflates the rating ($\sigma_d^* = 1$). On the other hand, if she exerts sufficiently high effort level, $e > \bar{e}^{g}(\alpha)$, to produce a credible signal in the rating evaluation process, the observed good signal has high accuracy. Hence, it is best for the CRA to report a high rating truthfully ($\sigma_d^* = 0$) since it is highly possible that the portfolio receiving a good signal will succeed.

Similarly, if the CRA exerts low effort level, $e \leq \bar{e}^{b}(\alpha)$, in a rating production process and observes a bad signal, the observed signal is not very accurate. In this case, it is optimal for the CRA to strategically inflate the rating ($\sigma_i^* = 1$) since there is a decent chance that the portfolio getting the bad signal could turn out to be of good quality and will not default later. On the other hand, if she exerts sufficiently high effort level, $e > \bar{e}^{b}(\alpha)$, in the rating evaluation process, the observed bad signal is highly accurate. Hence, the CRA should report a low rating truthfully ($\sigma_i^* = 0$) since it is highly possible that the portfolio receiving a bad signal will fail.

When $\bar{e}^{g}(\alpha) < 0$ causing $\bar{e}^{b}(\alpha) \geq 0 > \bar{e}^{g}(\alpha)$, the CRA always reports a high rating when observing a good signal since any effort level exerted by the CRA is higher than the effort level threshold, $e \geq 0 > \bar{e}^{g}(\alpha)$. On the other hand, if the CRA observes a bad signal, she can either inflate a rating ($e \leq \bar{e}^b(\alpha)$) or report a rating truthfully ($e > \bar{e}^b(\alpha)$) since $\bar{e}^b(\alpha) \geq 0$. Similarly, if $\bar{e}^b(\alpha) < 0$ causing $\bar{e}^g(\alpha) \geq 0 > \bar{e}^b(\alpha)$, the CRA always report a low rating when observing a bad signal no matter how low the CRA's effort level being exerted in the rating assessment process is since $e \geq 0 > \bar{e}^b(\alpha)$. However, if the CRA observes a good signal, she may either deflate a rating ($e \leq \bar{e}^g(\alpha)$) or report a rating truthfully ($e > \bar{e}^g(\alpha)$) since $\bar{e}^g(\alpha) \geq 0$.

Next, we will analyze the CRA's behavior of reporting ratings when market conditions are given in Lemma 2.1.

Lemma 2.1

1) If the market condition is bad ($\alpha \leq \frac{1}{2}$), the CRA always reports the rating truthfully when observing a bad signal ($\sigma_i^* = 0$).

2) If the market condition is good ($\frac{1}{2} < \alpha$), the CRA always reports the rating truthfully when observing a good signal ($\sigma_d^* = 0$).

Proof. See Appendix A

In a bad market ($\alpha \leq \frac{1}{2}$ causing $\bar{e}^b(\alpha) < 0$), the chance that a portfolio is bad and will default is fundamentally high. Hence, as long as the CRA observes a bad signal, even with a low effort level exerted, she always reports a low rating to minimize the reputational loss. In a good market ($\frac{1}{2} < \alpha$ resulting in $\bar{e}^g(\alpha) < 0$), the default risk of a portfolio is inherently low. If the CRA observes a good signal, even with a low effort level exerted, then she should be even more certain that the portfolio has a high chance of success. Hence, she will always report a high rating.

So far, our results discuss the CRA's optimal rating behavior when the effort level is given. We will next analyze the CRA's decision on selecting the optimal effort level, which in turn determines the accuracy of the signal and the CRA's optimal rating policy after observing the signal. We will first analyze the CRA's decision in a bad market (Proposition 2.2), where her expected payoff under the upfront fee scheme is given by:

$$\Pi_u^{M_b}(e, \sigma_d, \sigma_i = 0) = \max_{\sigma_d, e}[f_u - \Psi(e) - \Gamma(e, \sigma_d, \sigma_i = 0)] \quad (12)$$

Proposition 2.2 In a bad market ($\alpha \leq \frac{1}{2}$), the CRA's behavior of issuing ratings is characterized by a reputational loss threshold $\bar{\tau}_u^b \coloneqq 2\theta(1 - 2\alpha)$ as follows:

1) If $\tau \geq \bar{\tau}_u^b$, it is optimal for the CRA to exert effort $e_u^* = \min(\frac{1}{2}, \frac{\tau}{2\theta})$ in the rating evaluation process and report ratings truthfully ($\sigma_d^* = 0$ and $\sigma_i^* = 0$).

2) If $\tau < \bar{\tau}_u^b$, it is optimal for the CRA to exert no effort ($e_u^* = 0$) in the rating evaluation process and always report a low rating ($\sigma_d^* = 1$ and $\sigma_i^* = 0$).

Proof. See Appendix A

When the reputational loss from inaccurate ratings is high ($\tau \geq \bar{\tau}_u^b$), the CRA has incentives to prevent the loss from issuing inaccurate rating. Thus, she exerts optimal effort level ($e_u^* = \min(\frac{1}{2}, \frac{\tau}{2\theta})$) to produce a credible signal and reports a rating in accordance with the observed signal ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) . On the other hand, when the reputational loss is modest ($\tau < \bar{\tau}_u^b$), the CRA has incentives to save on information production cost by exerting no effort in the evaluation process ($e_u^* = 0$), resulting in low accuracy of the observed signal. In this case, the CRA's rating decision is largely influenced by the market condition, rather than the observed low-accuracy signal. Whether the CRA actually observes a good or bad signal, she trusts the market condition more that the portfolio has a high default risk. Hence, the CRA always assigns a low rating (deflates rating, $\sigma_d^* = 1$, if observing a good signal, or reports truthfully, $\sigma_i^* = 0$, if observing a bad signal).

Next, we will characterize the CRA's decision on choosing the optimal effort level, and the optimal rating policy in a good market in Proposition 2.3. For this, we will analyze the CRA's expected payoff under the upfront fee scheme given by:

$$\Pi_u^{M_g}(e, \sigma_d = 0, \sigma_i) = \max_{\sigma_i, e}[f_u - \Psi(e) - \Gamma(e, \sigma_d = 0, \sigma_i)] \quad (13)$$

Proposition 2.3 In a good market ($\frac{1}{2} < \alpha$), the CRA's behavior of issuing ratings is characterized by a reputational loss threshold $\bar{\tau}_u^g \coloneqq 2\theta(2\alpha - 1)$ as follows:

1) If $\tau \geq \bar{\tau}_u^g$, it is optimal for the CRA to exert effort $e_u^* = \min(\frac{1}{2}, \frac{\tau}{2\theta})$ in the rating evaluation process and report ratings truthfully ($\sigma_i^* = 0$ and $\sigma_d^* = 0$).

2) If $\tau < \bar{\tau}_u^g$, it is optimal for the CRA to exert no effort ($e_u^* = 0$) in the rating evaluation process and always report a high rating ($\sigma_i^* = 1$ and $\sigma_d^* = 0$).

Proof. See Appendix A

When the reputational loss from inaccurate ratings is high ($\tau \geq \bar{\tau}_u^g$), the CRA has incentives to minimize the expected loss from misreporting by exerting optimal effort level $e_u^* = \min(\frac{1}{2}, \frac{\tau}{2\theta})$ to produce an accurate signal. Hence it is in the CRA's best interest to report a rating that truthfully reflects the observed signal. In contrast, when the reputational loss is modest ($\tau < \bar{\tau}_u^g$), the CRA tends to cut down information production cost by putting no effort in the evaluation process ($e_u^* = 0$), resulting in a low accuracy of the observed signal, since the expected loss is considerably low. Hence, the CRA's rating decision is highly influenced by the market condition instead of the observed-signal accuracy. No matter what the observed signal is, she believes that default risk is naturally low in a good market. Accordingly, she always inflates the rating when observing a bad signal ($\sigma_i^* = 1$) and truthfully reports the rating when observing a good signal ($\sigma_d^* = 0$).

2.4.2 The Incentive Pay Scheme

Under the incentive pay scheme, the CRA can claim an incentive fee, $f_i > 0$, only if the announced rating accurately predicts the credit risk of the assets. In other words, the CRA is compensated only if the high-rated portfolio succeeds or the low-rated portfolio defaults. To examine the CRA's incentive when observing $\emptyset = g$, we need to analyze the CRA's expected payoff when $\emptyset = g$ given by

$$\Pi_i^g(e, \sigma_d) = \max_{\sigma_d, e}[\mu^g(e, \sigma_d) f_i - \Psi(e) - \tau(1 - \mu^g(e, \sigma_d))] \tag{14}$$

Also, we will analyze the CRA's incentive when observing a bad signal. In this case, the CRA's expected payoff when $\emptyset = b$ given by

$$\Pi_i^b(e, \sigma_i) = \max_{\sigma_i, e}[\mu^b(e, \sigma_i) f_i - \Psi(e) - \tau(1 - \mu^b(e, \sigma_i))] \quad (15)$$

We will characterize the CRA's optimal rating regime when the effort level exerted by the CRA is given in Proposition 2.4 below.

Proposition 2.4 Under the incentive pay scheme, the CRA's optimal strategy on reporting ratings is characterized by a threshold on the exerted effort level $\bar{e}^{\emptyset}$ and the observed signal $\emptyset \in \{g, b\}$, where $\bar{e}^g(\alpha) = \frac{1}{2} - \alpha = -\bar{e}^b(\alpha)$, as follows:

1) If $e \leq \bar{e}^g$ and the CRA observes a good signal, it is optimal for the CRA to adopt the rating deflation strategy ($\sigma_d^* = 1$) and report a low rating. Otherwise, if $e > \bar{e}^g$, it is optimal for the CRA to adopt the full disclosure strategy ($\sigma_d^* = 0$) and report a high rating.

2) If $e \leq \bar{e}^b$ and the CRA observes a bad signal, it is optimal for the CRA to adopt the rating inflation strategy ($\sigma_i^* = 1$) and report a high rating. Otherwise, if $e > \bar{e}^b$, it is optimal for the CRA to adopt the full disclosure strategy ($\sigma_i^* = 0$) and report a low rating.

Proof. See Appendix A

If the CRA puts low effort level, $e \leq \bar{e}^g(\alpha)$, to observe a good signal in the rating evaluation process, the observed good signal is not very accurate. It is highly possible that the portfolio getting a good signal is likely of bad quality and will subsequently fail. Hence, the CRA strategically deflates the rating ($\sigma_d^* = 1$). On the other hand, if she exerts sufficiently high effort level, $e > \bar{e}^g(\alpha)$, to produce a credible signal when observing a good signal, the observed signal has high accuracy. Hence, the CRA reports a high rating truthfully ($\sigma_d^* = 0$).

If the CRA exerts low effort level, $e \leq \bar{e}^b(\alpha)$, in a rating production process when observing a bad signal. The observed bad signal is not very accurate and there is a decent chance that the portfolio getting a bad signal is of good quality and will not default later. Then, she strategically inflates the rating ($\sigma_i^* = 1$). On the other hand, if she puts sufficiently high effort level, $e > \bar{e}^b(\alpha)$, in the rating evaluation process to produce a credible signal when observing a bad signal. The CRA reports a low rating truthfully ($\sigma_i^* = 0$) since the observed signal is accurate.

If the CRA's effort level is higher than the effort threshold, $\bar{e}^g(\alpha) < 0$, when observing a good signal resulting in $\bar{e}^b(\alpha) \geq 0 > \bar{e}^g(\alpha)$, the CRA always reports a high rating. Accordingly, when observing a bad signal, she can either inflate the rating if $e \leq \bar{e}^b(\alpha)$ or report the rating truthfully if $e > \bar{e}^b(\alpha)$.

Similarly, if the CRA's effort level is higher than the effort level threshold , $\bar{e}^b(\alpha) < 0$,when observing a bad signal causing $\bar{e}^g(\alpha) \geq 0 > \bar{e}^b(\alpha)$, the CRA always reports a low rating in accordance with the observed bad signal. If she observes a good signal, she can deflate the rating if $e \leq \bar{e}^g(\alpha)$ or truthfully disclose the rating if $e > \bar{e}^g(\alpha)$.

We will next characterize the CRA's rating decision in different market conditions when the CRA is compensated by the incentive pay scheme in Lemma 2.2 below.

Lemma 2.2

1) If the market condition is bad ($\alpha \leq \frac{1}{2}$), the CRA always reports the rating truthfully when observing a bad signal ($\sigma_i^* = 0$).

2) If the market condition is good ($\frac{1}{2} < \alpha$), the CRA always reports the rating truthfully when observing a good signal ($\sigma_d^* = 0$).

Proof. See Appendix A

When the CRA observes a bad signal, $\emptyset = b$, in a bad market, she has no incentives to inflate a rating ($\sigma_i^* = 0$) no matter how low accuracy of the observed bad signal is. She is better off reporting a low rating because default risk of a portfolio having a bad signal and market risk in the bad market is fundamentally high. When the CRA observes a good signal, $\emptyset = g$, in a good market, she reports a high rating truthfully ($\sigma_d^* = 0$) at any effort level being exerted by the CRA. Since default risk of a portfolio and market risk in the good market is relatively low, it is in the CRA's best interest to report a good rating regardless of how low accuracy of the observed good signal is.

We have analyzed the CRA's optimal rating conduct when the effort level is given. In Proposition 2.5, we will characterize the CRA's optimal effort level and optimal rating regime after observing a signal in a bad market by analyzing her expected payoff given by

$$\Pi_i^{M_b}(e, \sigma_d, \sigma_i = 0) = \max_{\sigma_d, e}[\lambda(e, \sigma_d, \sigma_i = 0) f_i - \Psi(e) - \Gamma(e, \sigma_d, \sigma_i = 0)] \quad (16)$$

Proposition 2.5 In a bad market ($\alpha \leq \frac{1}{2}$), the CRA's behavior of issuing rating is characterized by a reputational threshold $\bar{\tau}_i^b \coloneqq 2\theta(1 - 2\alpha) - f_i$ as follows:

1) For $\tau \geq \bar{\tau}_i^b$, it is optimal for the CRA to exert effort $e_i^* = \min(\frac{1}{2}, \frac{(f_i + \tau)}{2\theta})$ in the rating evaluation process and report ratings truthfully ($\sigma_d^* = 0$ and $\sigma_i^* = 0$).

2) For $\tau < \bar{\tau}_i^b$, it is optimal for the CRA to exert no effort $e_i^* = 0$ in the rating evaluation process and always report low ratings ($\sigma_d^* = 1$ and $\sigma_i^* = 0$).

Proof. See Appendix A

In a bad market, when the reputational loss from inaccurate ratings are high ($\tau \geq \bar{\tau}_i^b$), the CRA exerts optimal effort level ($e_i^* = \min(\frac{1}{2}, \frac{(f_i + \tau)}{2\theta})$) to produce a credible signal for preventing the loss from misreporting. Accordingly, the CRA reports a rating that truthfully reflects the observed signal ($\sigma_d^* = 0$ and $\sigma_i^* = 0$). When the reputational loss is modest ($\tau < \bar{\tau}_i^b$), the CRA puts no effort level ($e_i^* = 0$) into producing a signal in order to save on information production cost. This results in low accuracy of the observed signal. In this case, the CRA's rating decision relies mainly on the market information where the default risk is naturally high. As a result, the CRA always reports a low rating, either when observing a good signal ($\sigma_d^* = 1$) or when observing a bad signal ($\sigma_i^* = 0$).

After characterizing the CRA's optimal effort level and rating strategy in the bad market, we will investigate the CRA's conduct of selecting optimal effort level and optimal rating policy in a good market in Proposition 2.6. For this, we will analyze the CRA's expected payoff under the incentive payment scheme given by

$$\Pi_i^{M_g}(e, \sigma_d = 0, \sigma_i) = \max_{\sigma_i, e}[\lambda(e, \sigma_d = 0, \sigma_i) f_i - \Psi(e) - \Gamma(e, \sigma_d = 0, \sigma_i)] \quad (17)$$

Proposition 2.6 In a good market ($\frac{1}{2} < \alpha$) the CRA's behavior of issuing ratings is characterized by a reputational cost threshold $\bar{\tau}_i^g \coloneqq 2\theta(2\alpha - 1) - f_i$ as follows:

1) For $\tau \geq \bar{\tau}_i^g$, it is optimal for the CRA to exert effort, $e_i^* = \min(\frac{1}{2}, \frac{(f_i+\tau)}{2\theta})$, in the rating evaluation process and report ratings truthfully ($\sigma_i^* = 0$ and $\sigma_d^* = 0$).

2) For $\tau < \bar{\tau}_i^g$, it is optimal for the CRA to exert no effort, $e_i^* = 0$, in the rating evaluation process and always report high ratings ($\sigma_i^* = 1$ and $\sigma_d^* = 0$).

Proof. See Appendix A

When the reputational loss from inaccurate ratings is high ($\tau \geq \bar{\tau}_i^g$), the CRA has incentives to put optimal effort level ($e_i^* = \min(\frac{1}{2}, \frac{(f_i+\tau)}{2\theta})$) in a rating evaluation process to produce a credible signal since misreporting incurs the expected high loss. Hence, the CRA reports a rating truthfully ($\sigma_i^* = 0$ and $\sigma_d^* = 0$) based on the observed signal. On the other hand, when the reputational loss is modest ($\tau < \bar{\tau}_i^g$), the CRA endogenously eliminates signal observation cost by exerting no effort ($e_i^* = 0$) to produce a signal. Due to low accuracy of the observed signal, the CRA's rating decision is based solely on the market risk which is relatively low in the good market. As a result, the CRA always reports a high rating, either when observing a bad signal ($\sigma_i^* = 1$) or when observing a good signal ($\sigma_d^* = 0$).

2.4.3 Comparisons of Induced Outcomes

It is earlier illustrated that the CRA's decision to truthfully report or strategically misreport ratings is characterized by the reputational loss threshold under both types of compensation schemes. It remains to determine which compensation scheme can better motivate the CRA to exert a higher effort level and implement the full disclosure regime, resulting in more accurate ratings. Proposition 2.7 discusses these results.

Proposition 2.7 In all market conditions,

1) The reputational loss threshold under the upfront fee scheme is higher than that under the incentive pay scheme ($\bar{\tau}_u^b > \bar{\tau}_i^b$ and $\bar{\tau}_u^g > \bar{\tau}_i^g$).

2) If the reputational loss threshold is sufficiently large, the CRA's optimal effort level under the incentive pay scheme is always higher than that under the upfront fee ($e_i^* > e_u^*$).

3) If the optimal effort level is exerted, the probability of accurate ratings being produced under the incentive pay scheme is higher than that under the upfront fee scheme ($\lambda(e_i^*, \sigma_d^* = 0, \sigma_i^* = 0) > \lambda(e_u^*, \sigma_d^* = 0, \sigma_i^* = 0)$).

Proof. See Appendix A

We have that the CRA's reputational loss threshold under the upfront fee is higher than that under the incentive pay scheme in both bad market ($\bar{\tau}_u^b > \bar{\tau}_i^b$) and good market ($\bar{\tau}_u^g > \bar{\tau}_i^g$). This implies that the incentive pay scheme better discourages the CRA from strategically misreport ratings than the upfront fee does regardless of market conditions. Also, the incentive pay scheme better induces the CRA to put higher effort level in the rating evaluation process ($e_i^* > e_u^*$) to produce more credible signals. As a result, the probability that the CRA's rating is accurate under the incentive pay scheme is higher than that under the upfront scheme ($\lambda(e_i^*, \sigma_d^* = 0, \sigma_i^* = 0) > \lambda(e_u^*, \sigma_d^* = 0, \sigma_i^* = 0)$). However, when the expected reputational loss is significantly low, the CRA exerts no effort in the rating evaluation process and strategically misreports the ratings because market risk is clearly identified (exceptionally low or high).

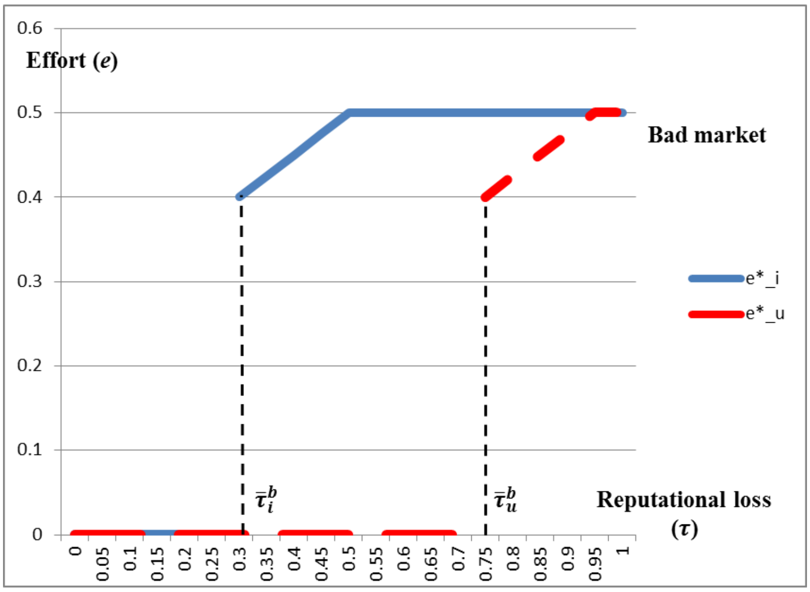

Figure 2.2 The CRA's Optimal Effort Level under the Upfront Scheme and the Incentive Pay Scheme with Respect to Reputational Cost (τ) when $\alpha = 0.3, \theta = 1, \sigma_d = \sigma_i = 0, \bar{\tau}_i^b = 0.3, \bar{\tau}_u^b = 0.8, f^u = f^i = 1$

Figure 2.2 illustrates that the CRA's decision to exert effort level in the rating evaluation process is influenced by the reputational loss in a bad market. When the reputational loss is sufficiently low, the CRA has incentive to maximize the expected profit by cutting down rating production cost. Hence, the CRA exerts minimum effort level, $e^* = 0$, in the rating evaluation process. When the reputational loss is sufficiently high, the CRA has incentive to minimize the loss from misreporting. The CRA compensated by the incentive pay scheme tends to put higher effort level in the rating evaluation process than the CRA compensated by the upfront fee scheme does. In other words, the CRA has incentive to avoid high reputational loss from misreporting by exerting optimal effort level to produce a credible signal and reporting a rating that truthfully reflects the observed rating. In addition, the incentive fee scheme more efficiently discourages the CRA from exerting no effort $e^* = 0$ in the rating evaluation process and giving out biased ratings since $\bar{\tau}_i^b < \bar{\tau}_u^b$. In other words, the CRA compensated by the incentive fee scheme incurs higher reputational loss than the CRA compensated by the upfront fee scheme does.

2.5 Conclusion

To address issues of inaccurate ratings announced by credit rating agencies, this study emphasizes mainly on which payment scheme between the upfront fee and incentive pay scheme can encourage the issuer-paid rating agency to truthfully report ratings and improve rating accuracy. In the model setting, the CRA picks the effort level exerted in the rating evaluation process and selects whether to report the ratings truthfully or strategically misreport the ratings. The CRA's behavior of issuing inflated ratings and deflated ratings are taken into account. Our findings reveal that the incentive pay scheme better incentivizes the CRA to exert higher effort level in the rating evaluation process and adopt the full disclosure regime than the upfront fee scheme does because the reputational loss threshold under the incentive pay scheme is lower than that under the upfront scheme. As a result, the incentive pay scheme better enhances the probability that the CRA issues accurate ratings than the upfront fee scheme.

A key finding of this study is that the proposed incentive pay scheme can effectively motivate the CRA to truthfully report ratings and exert higher effort level in the rating evaluation process to improve rating accuracy. However, the CRA under the incentive pay scheme may occasionally engage in strategic misreporting regime and puts no effort in the rating evaluation process when reputational loss is exceptionally low.

The insights from this study can guide regulators and policy developers in proposing the policy reform regarding compensation scheme to improve rating quality and potentially prevent financial crises caused by uninformative ratings. For the academia, this study illustrates the CRA's decision on selecting rating policy and exerting effort level in the rating evaluation process. Future studies may consider other payment schemes which can induce rating agencies to produce more accurate ratings.

CHAPTER 3

PROFIT SHARING MECHANISM WITH INVESTORS' DECISIONS ON INFORMATION LEVEL AND CONTRACT TERMS

3.1 Introduction

Securitized financial products, especially mortgage-backed security (MBS) and asset-backed security (ABS), have been blamed for partial initiation of the 2007-2009 financial crisis and massive downgrades of European sovereign debts ignited ongoing Eurozone crisis. The 2007-2009 financial crisis begins from banking sector as banks issue loans in a primary loan market. The loans are securitized and rated by credit rating agencies (CRAs) before selling these structured financial products to investors in the bond market. The CRAs are also inevitably blamed for the financial crisis because the announced ratings have significant impacts on issuing firm, potential investors, and economic welfare. The impact of CRAs has continuously captured both public and private attention due to onset of Eurozone financial crisis. The conclusion is drawn in the same direction that the ratings fail to foresee financing risk for subprime loan, and don't incorporate effect of moral hazard in organizations. Therefore, low quality of ratings has significantly contributed to the causation of the financial crises. The major concern about CRAs based on the issuer-pay model is conflict of interest centered mainly on three issues: 1) rating agencies' behavior of issuing biased ratings, 2) issuers' behavior of rating shopping, and 3) naïve investors (trusting investor clientele) who make investment decision based on credit ratings. However, the studies about CRAs' incentives towards issuing biased ratings have been widely investigated under the issuer-pay model but those have rarely been considered under the investor-pay business model. In this study, the CRAs' incentives towards producing accurate ratings under the investor-pay model are mainly explored on the basis of theoretical models.

Currently, there are two main business models that the CRAs have world-widely adopted namely an issuer-pay model and investor-pay model. The issuer-pay model, which collects fees from issuing firms, gains far more popularity among the CRAs and is also adopted by three major CRAs (Moody's, S&P, and Fitch Ratings). In contrast, a small CRA (Egan-Jones Ratings) has maintained the investor-pay model whose subscription fees derive from investors (White, 2013). However, the CRAs of these two models have accepted a similar payment scheme by charging upfront premium in exchange of credit ratings which can motivate them to assess credit ratings imprudently because their revenue is not tied with their performance. Accordingly, this study emphasizes mainly on which payment system (profit-sharing vs. pure upfront fee) can efficiently motivate the CRAs based on the investor-pay model to produce more accurate ratings and how an investor can strategically interact with the CRA to improve rating qualities.

This study aims to examine dynamic interaction between the investor and CRA under the investor-pay business model and analyze a more efficient payment scheme that provides incentive for the CRA to produce accurate ratings. To understand the dynamic interaction, expected profit of the investor is analyzed to identify the investor's incentive towards acquiring additional information accuracy level and expected profit of the CRA is investigated to illustrate the CRA's incentive towards adopting rating policy (a biased rating or full disclosure regime). To determine a more efficient payment scheme, optimal compensation of the CRA based on the profit-sharing scheme is compared with that of the CRA based on the upfront fee schedule.

3.2 Literature Review

3.2.1 Importance and Impacts of CRAs to Financial System

According to rising complexity of financial assets and markets, all market participants (investors, issuers, and government regulators) have turned to CRAs for opinions in investment decision, corporate financing and risk management (Baker and

Mansi, 2002). Credit ratings have conveyed credible information about quality of financial assets in the credit market and the ratings sometimes are blamed for increasing borrowing cost for firms that retrieve low ratings (Micu, Remolona, and Wooldridge, 2004). The role of CRAs is to reduce asymmetric information regarding creditworthiness of companies or countries between investors and issuers in the bond market. On the other side, CRAs perform as opinion makers to reduce asymmetric information between lenders and borrowers in the primary loan market when the quality of projects is ambiguous (Elkhoury, 2009). To reduce asymmetric information, CRAs evaluate the credit risk of both corporate and government entities based on publicly and non-publicly available information regarding the issuers, borrowers, industry, and macroeconomic environments which may be costly, time-consuming, and inaccessible for individuals to collect and analyze. The credit ratings convey opinion of the CRAs about ability of issuers or borrowers to meet their financial obligation on due date but the credit ratings are not recommendation to buy or sell securities or structured products (IOSCO, 2008). Nevertheless, the importance of credit ratings to market participants has been far more meaningful than an opinion from CRAs.

Credit ratings have impacts on purchasing decisions of potential buyers, on firm reputation, and on responsiveness of firms getting low-rated (Chatterji and Toffel, 2010). Credit ratings can economically enhance attractions in foreign investments and have strong impacts on dynamics of interest rates (Bozic and Magazzino, 2013). The ratings issued by CRAs play a key role in decisions of banks to lend money to firms and have negative relationship with borrowing rates of firms (Micu, Remolona, & Wooldridge, 2004). The ratings have also affected IPO pricing (An and Chan, 2008) in a way that IPOs issued by firms with credit ratings tend to be less subjected to underpricing than that issued by firms without credit ratings. Credit rating impact on strategic decision of capital structure of firms (Kisgen, 2006, 2009) is investigated to the extent that firms near an upgrade and downgrade ratings tend to opt for issuing equity instead of debt. The recent study on credit rating impacts on capital structure (Kemper & Rao, 2013) extended from Kisgen (2006, 2009) shows that credit ratings are one of the most critical factors that managers take into consideration when decide the target of capital structure. Kisgen and Strahan (2010) investigate the impact on yields of lower-rating bonds rated by other CRAs comparing with that of higher-rating bonds rated by Dominion Bond Rating Service (DBRS), the newly certified CRA for use in bond investment regulations by the U.S. Securities and Exchange Commission (SEC). The empirical results identify that bond-investment regulation has affected both yield of a firm and decision of some investors to hold bonds, and the rating-based regulation inevitably has an impact on the cost of capital of the firm. Influenced by such impacts of credit ratings, the credit ratings have not only had a significant impact on investment decision-making of market participants but also had a crucial influence on policymakers because credit ratings have been incorporated into international financial regulation for bank (Weber & Darbellay, 2008). Subsequently pension funds, insurance companies, and investment funds adopt the regulation (Partnoy, 2009). According to such effects, issuing firms have monitored the ratings they obtain and use them as one of criteria to judge their performance. Investors also regard publicly announced ratings as one of important factors to make investment decision in the bond market. Overall, the ratings are used by issuers, investors, borrowers, and government to make financial decision and have crucial effects on efficient market operation as well as market participants (Staikouras, 2012). Therefore, major concern about the credit rating is whether the CRAs provide accurate ratings.

3.2.2 Empirical Evidence of Rating Inflation

Many empirical researches are conducted to observe if credit ratings are accurate measure of default risk in different financial asset classes. Empirical evidence on rating bias caused by rating shopping and rating model error (Benmelech & Dlugosz, 2010) reveals that effect of rating shopping is unclear but rating models incorrectly identify default risk across financial assets, especially complex structured products. Rösch and Scheule (2011) conduct empirical research to evaluate historical performance of CRAs in providing accurate ratings for securitization before and during the financial crisis, and exhibits that the CRAs underestimate the systematic risk of underlying asset pools by providing overly high ratings. Empirical research investigates performance of CRAs in producing collateral debt obligation (CDO) ratings by analyzing 916 CDOs issued by one of the top three CRAs during 1997-

2007 (Griffin & Tang, 2011). The empirical results show that the top CRA allocates too much weight to subjective candidates such as manager experience or credit enhancement so that the credit rating process becomes more qualitative resulting in assigning overly high ratings. The rating model fails to evaluate risk characteristics of structured financial products while many investors take these inflated ratings as their benchmark in making investment decision (Brennan, Hein, & Poon, 2009; Mason & Rosner, 2007). Accordingly, the credit rating can be a double-edged sword to investment perception of financial market participants. The excessive impact of credit ratings drives investors, fund managers, and market participants to base their investment decisions solely on the credit rating by neglecting to conduct their own risk analysis and valuation of structured products (IOSCO, 2008). However, these empirical results suggest that the credit rating agencies produce inaccurate ratings because of their methodology. These do not focus on what incentives drive the CRAs to assign overly high ratings. Then, studies regarding influential factors that induce CRAs operated under the issuer-pay business model to issue biased ratings are increasingly examined.

3.2.3 Factors that Motivate CRAs to Issue Biased Ratings

Since almost all of CRAs operates under the issuer-pay business model, main interest is centered on influential factors that drive CRAs based on the issuer-pay model to produce biased ratings. Existing studies on the CRAs operated under the issuer-pay model reveal that both exogenous and endogenous factors can potentially motivate the CRAs to give out biased ratings.

The exogenous factors are such as issuing firms' behavior of rating shopping, regulatory reliance on credit ratings, and increasing competition in the rating industry. The issuing firms' behavior of rating shopping, derived from the issuers' ability to choose among CRAs, leads to overly optimistic ratings although all CRAs evaluate financial assets honestly (Skreta & Veldkamp, 2009). Study regarding effect of rating shopping on rating quality shows that CRAs assign overly high ratings to mortgage-backed security (MBS) tranches in order to attract investors, who can invest in the AAA tranches only (He, Qian, & Strahan, 2016). In particular, the rating shopping usually occurs when the investors do not update their belief on a possibility that the

CRAs may inflate ratings. The regulatory reliance on credit ratings, which provides regulatory benefits to investors who hold highly rated securities, may support rating inflation by motivating rational investors to demand high rated securities even if the high rating is informative (Ozerturk, 2014). Increasing competition in the CRA industry by allowing more new entrants into the industry can adversely affect qualities of ratings since rating shopping is facilitated (Becker & Milbourn, 2011) and the CRAs may offer overly high ratings to attract the issuing firms in exchange for fees (Sangiorgi, Sokobin, & Spatt, 2009). Study on impact of increasing competition on rating quality in the collateralized debt obligation (CDO) market reveals that CRAs tend to inflate ratings by making positive adjustment beyond their rating model especially when their competitors relax the assumptions of credit rating assessment (Griffin, Nickerson, & Tang, 2013).

The endogenous factors that induce the rating agencies to produce biased ratings consist of the CRAs' desire to develop long-term relationships with the issuing firms, inadequate rating model to assess qualities of underlying assets, and analysts' bias. According to desire to maintain long-term relationships with issuing firms, the CRAs tend to consistently offer overly high ratings to persuade issuing firms to repeat doing business with the rating agencies (Mullard, 2012). In addition, evidences reveal that the CRAs inflate ratings because they do not have adequate rating models to assess credit risk of underlying assets (Benmelech & Dlugosz, 2010), particularly complex assets such as mortgage-backed securities (MBS), asset-backed securities (ABS), collateralized debt obligation (CDO), and etc. To extract fees from issuing firms, the CRAs may intentionally make positive adjustment beyond their rating model by allocating too much weight on qualitative information resulting in inflated ratings (Ashcraft, Goldsmith-Pinkham, & Vickery, 2010; Griffin & Tang, 2012).

However, these studies do not offer explanation why CRAs occasionally inflate ratings to attract issuing firms to purchase ratings instead of consistently issuing inflated ratings. To decide when CRAs should inflate ratings, there should likely be some factors that CRAs may weigh against benefit of assigning inflated ratings. Therefore, incentives of CRAs towards issuing accurate ratings are developed on basis of conflict of interest to answer why CRAs have incentives to occasionally inflate ratings.

3.2.4 CRAs’ Incentives toward Issuing Accurate Ratings

The conflict of interest derived from CRA business models is one of main issues which potentially cause CRAs to inflate ratings. There are two models world-widely adopted by CRAs namely an issuer-pay model and investor-pay model (Caprio, 2012). The issuer-pay model, adopted by most of CRAs including the three major CRAs namely Moody’s, S&P, and Fitch ratings, is questioned about conflict of interest issue whether issuer fees subvert ethic of CRA in a process of rating analysis (Mullard, 2012). In contrast, the investor-pay model collects subscription fees from investors who need to know the real quality of firms or underlying financial assets so that the accurate rating is preferred. Accordingly, the issuer-pay model is blamed for altering CRA priorities from research-based ratings to generating higher profit and fees, resulting in rating inflation. Empirical research regarding conflict of interest stem from business model of CRAs in bond markets is conducted to investigate whether S&P gives out higher bond ratings after it switches from investor-pay to issuer-pay model in 1974 (Jiang, Harris Stanford, & Xie, 2012). The ratings under different business models, when S&P implements investor-pay model and issuer-pay model, are compared with Moody’s who has always adopted issuer-pay model. The evidence reveals that S&P under investor-pay model gives out lower ratings than Moody’s does while S&P under issuer-pay model assigns as high ratings as Moody’s does. The findings infer that issuer-pay model motivate CRAs to assign higher ratings as subsequently supported by Bonsall Iv (2014). Accordingly, theoretical studies about CRAs’ incentives towards issuing rating accuracy under the issuer-pay model are built on implicit factors namely issuers’ behavior of rating shopping, reputational cost, and impact of competition in the CRA industry.

Built on impact of issuer behavior of rating shopping on CRAs’ incentives, findings show that rating shopping influences CRAs to issue inflated ratings. A rational expectation model, in which issuers can purchase many ratings and disclose only the favorable rating to attract investment from investors, is developed to describe issuers’ behavior of rating shopping driven by rating processes which cause CRAs to inflate ratings especially when financial assets are complex (Skreta & Veldkamp, 2009). Additionally, rating shopping can be mitigated by investor-initiated rating system but information market may collapse because CRAs cannot survive in the market with such a small amount of subscription fees. Similarly, Sangiorgi and Spatt (2015) add that undisclosed ratings deteriorate information content of ratings leading to inefficient investment decision although investors take CRAs' behavior of biased rating into account. Building on a rational expectation model Opp, et al. (2013), their model characterizes a monopolistic CRA based on the issuer-pay model and issuing firms have more private information about their project types. The issuers' behavior of rating shopping incentivized by regulatory advantages of highly rated securities motivates CRAs to inflate ratings, especially when underlying assets of securities are sufficiently complex such as mortgage-backed securities (MBS), collateralized loan obligations (CDOs), and etc. Similar results by Ozerturk (2014) reveal that rating-contingent regulation adversely affects rating accuracy and complex securities are more likely exposed to inflated ratings. Their findings congruent with Cornaggia, Jess and Cornaggia, Kimberly J. (2013) describe that CRAs adjust their main model to provide positive ratings to issuing firms as discovered by Ashcraft, Goldsmith-Pinkham, Hull, and Vickery, (2011) and Griffin and Tang (2012).

Emphasizing on the impact of reputational concerns on CRAs' incentives towards issuing accurate ratings, results reveal that reputational concerns cannot keep rating quality in check. Mathis et al. (2009) and Wang (2011) show that CRAs have incentive to reduce rating accuracy when reputational concerns are lower than revenues mostly supplied by fees from structured products. In addition, CRAs tend to provide less accurate ratings in boom periods. Integrating naïve investors and reputational concerns in a model (Bar-Isaac & Shapiro, 2013; Bolton, Freixas, & Shapiro, 2012), inflated rating is pronounced in boom periods, high fee-income, and the presence of naïve investors, who are willing to pay premium for highly rated securities. CRAs inflate ratings to harvest from their reputation in favorable periods, and increase rating accuracy to build up their reputation in unfavorable periods. Mariano (2012) finds that reputational concerns are not sufficient to prevent the CRA from inflating ratings to secure its market power when private information is noisy and competition in CRA industries increases. With desire to reduce ex post risk of misevaluation that potentially damage reputation, the CRA has adverse incentive to issue either overly optimistic ratings to good-rated investment or overly pessimistic ratings to bad-rated investment so that ratings are less informative or coarse (Holden, Natvik, & Vigier, 2012). Building on cheap-talk concept (Goel & Thakor, 2015), the coarse rating reduces both social welfare and informativeness level, ultimately resulting in deteriorated level of rating accuracy. Extended from cheap-talk model (Frenkel, 2015), CRAs have strong incentive to maintain double reputation, which is credibility reputation among investors and leniency reputation among issuers, so that the CRAs tend to inflate ratings particularly when few issuers exist in a market of structured products.

Focused on impact of competition on CRA’s incentive, results show that increased competition deteriorates rating quality. Extended from competition effects of Mathis et al. (2009), CRAs weigh the trade-off between inflating ratings to boost current revenues and sustaining reputation to harvest from future profits, in lieu of rating shopping. In monopolistic market structure of rating industries, CRAs are prone to inflate rating when competition among rating agencies rises (Camanho, Deb, & Liu, 2009). Based on optimal contract between issuers and CRAs, ratings convey less credible information under increasing competition in the rating industry (Faure-Grimaud, Peyrache, & Quesada, 2009). Building on dynamic settings with existence of either naïve or sophisticated investors in an evolutionary-game-theory (Hirth, 2014), CRAs truthfully disclose information when enough sophisticated investors present and CRAs otherwise inflate rating when enough naïve investors appear. Building competition in conflict-of-interest context on a game theoretical framework, the CRA is adversely incentivized to inflate rating as degree of competition increases (Wang & Xu, 2015). In addition, all CRAs have incentive to truthfully report their ratings if the third party pays for CRAs. Accordingly, these studies infer that reputational cost, rating shopping behavior of issuing firms, and increasing competition in the CRA industry cannot assure rating accuracy. However, these aforementioned studies aim mainly to illustrate causation of CRAs’ incentives under the issuer-pay model and do not examine optimal contract to induce CRAs to produce accurate ratings. Different from this dissertation, optimal contract and performance-based payment for CRAs are developed to motivate CRAs to issue accurate ratings. The findings can be viewed as complementary solution of the previous findings. To address the problem of inflated ratings caused by conflict of interest, policymakers and scholars recommend reform policy for CRA business.

3.2.5 Policy Analysis about CRAs

According to the role of CRAs in contribution to the financial crisis, the CRAs become a main focus of the Dodd-Frank Act of 2010 which enforces transparency and accountability of the CRAs and financial institutions. There are some policy-oriented researches that recommend reconsidering business-model reform of CRAs (Medvedev & Fennell, 2011; White, 2010, 2013). An issuer-and-investor-pay compensation model is proposed with the intention to reduce conflict of interest from CRA business model in a long-term perspective and the U.S. Ratings Funds is set up as a government sponsor organization to pay fees for CRAs instead of issuers or investors (Kotecha, Ryan, Weinberger, & DiGiacomo, 2012; Kotecha, Weinberger, & Ryan, 2011). The CRAs have continuously received a substantial attention from both media and public due to their significant contribution to widespread of financial distress on Eurozone countries. The policymakers have attempted to enact regulation to solve the two critical issues which are corporate governance and conflict of interest in order to enforce CRAs to perform as reliable information intermediaries. The policymakers set the primary objective of the regulation to address the problem of conflict of interest derived from the business model of the credit rating agencies (Utzig, 2010). Creditworthiness information is tightly linked with financial stability and especially in the bond market which is dominated by institutional investors. The collapse of the bond market can potentially result in the financial crisis. New business models or payment system of the CRAs are thus welcome to sustain creditworthiness information.

3.2.6 Payment Schemes to CRAs and Impacts on Ratings

In search of the efficient business model (Medvedev & Fennell, 2011), the investor-pay model is the most likely practical model among four recommended business models (issuer-pay, investor-pay, government-pays, and platform model). However, it cannot completely eliminate rating shopping and align CRAs' incentives with investors' incentives (Nwogugu, 2013). Moreover, big-three CRAs in 1970s (Standard & Poor's (S&P), Moody's, and Fitch Group) transformed their business model from the investor-pay to issuer-pay model due to a free-ride problem amongst investors. The free-ride problem causes insufficient income for CRAs operated under the investor-pay model because information purchased by some investors is dispersed to other investors who do not subscribe to CRAs' service. However, insufficient revenue caused by the free-ride problem can be solved with the augmenting uses of ratings by institutions and efficient system of information diffusion (Richardson & White, 2009). Accordingly, the investor-pay model should be adpoted since it potentially aligns the incentives of CRAs with that of investors (Deb & Murphy, 2009). However, these studies focus solely on the business-model reform but do not consider the payment system reform to induce CRAs to improve rating accuracy.

The existing literature analyzing an adequate payment scheme for CRAs is quite rare. Mukhopadhyay (2003) studies incentive payments that induce CRAs to prevent a possible CRAs moral hazard problem, which may occur from unobservable rating evaluation standards. The findings reveal that CRA payments should be linked with expected return on a project and an incentive contract is tied with value of debt only. Poblete and Spulber (2012) examine optimal contract for CRAs and propose a critical ratio to determine what compensation for CRAs should be under different economic scenarios. The critical ratio is the hazard rate of shock multiplied by the marginal rate of technical substitution of the rating agent's effort for the shock. The finding suggests that bond is the optimal contract for CRAs when the critical ratio increases and a bonus with cap is the optimal contract for CRAs when the critical ratio decreases. Nevertheless, the compensation schemes proposed by Mukhopadhyay (2003) and Poblete and Spulber (2012) can unlikely be practicable to enforce because CRAs do not have efficient models to value risk embedded in complex structured products as empirically discovered by Brennan, et al. (2009), and Mason and Rosner (2007).

Bongaerts (2013) emphasizes particularly on derivation of optimal compensation from different fee-payer systems which consist of an investor-pay rating, an investor-produced rating, and CRA co-investment system. Investor-produced rating, initially implemented by the French credit insurer Coface, is that the rating producer is also the end user. In conclusion, the results suggest that each fee-payer system can improve social welfare in different ways but all these systems require support from strict regulatory. Nonetheless, the findings do not pinpoint which business model should be strictly enforced by policymakers. Ozerturk (2014) analyzes whether the upfront fee scheme or the rating-contingent fee scheme is more efficient payment system that can induce a CRA to improve quality of ratings. The finding reveals that both schemes motivate the CRA to provide similar quality of ratings if the CRA posts both fee and rating policy prior to soliciting the rating. On the other hand, if the CRAs posts only fee and sets rating policy after soliciting the rating, then the upfront fee scheme can improve rating quality better. Such CRA's behavior of implicitly adopting rating policy, however, may hardly be monitored or enforced in practice. Kashyap and Kovrijnykh (2016) analyze impact of both compensation scheme and business models of CRAs on accurate rating. They tie a commission scheme for CRAs with rating accuracy under different fee-payer systems when a social planner, an issuing firm, or investors order credit ratings. The finding shows that the investor-pay model is more efficient than the issuer-pay model in terms of rating accuracy level and social welfare.

Proposed by Dodd-Frank Act 2010, issuer risk retention ("skin-in-the-game") under the issuer-pay model requires issuer to retain a fraction of loan portfolios securitized to improve rating accuracy. The CRA's incentive to improve rating accuracy increases in a fraction of a loan portfolio retained by issuer and the issuer's skin-in-the-game should increase in complexity of securities' underlying assets (Ozerturk, 2015). However, the more a retained fraction of the loan portfolio, the less fund that the issuer gains to finance new investment opportunities and bares higher liquidity cost. These reasons may motivate the issuer to require only sufficiently good rating instead of highly good rating, and curb growth of structured product markets.

This dissertation, more relevant to the study of Kashyap and Kovrijnykh (2016), emphasizes on developing an optimal compensation contract to motivate CRAs based on the investor-pay model to exert efforts in producing more accurate ratings. This dissertation is different from previous studies in three important ways. First, if CRAs do not exert effort in a process of rating evaluation, previous studies assume probability that the CRA assigning high rating to a bad firm is equal to probability that the CRA issuing low rating to a good firm. Contrarily, this study assumes that the probability of the CRA giving high or low rating is conditional on historical default rates of the market. Accordingly, the investor's decision to acquire information from CRAs depends on past performance of each CRA and the CRA's different methodologies of information production can be taken into account. Second, CRAs are assumed to have limited liability in some studies such as Kashyap and Kovrijnykh (2016). In this study, CRAs do not have limited liability to reflect real practice since they pay penalty in case that they are sued. Third, previous studies (e.g., Bongaerts (2013), and Kashyap and Kovrijnykh (2016)) do not integrate CRAs' behavior of issuing biased ratings into the models while this dissertation does.

Overall existing literature mostly investigates the impacts and influence of bias rating on market participants and financial stability. In particular, several researches explore which factors cause bias ratings, and conflict of interest for CRAs when a traditional payment scheme (upfront fee) and where the issuer pays for the rating is used. There has been limited number of papers studying the effects of payment schemes on the quality of credit ratings. To the best of my knowledge, this dissertation differs from the existing researches in terms of payment scheme design. The payment scheme (performance-based payment) is modeled to reflect real practice as the CRA is compensated with one-time upfront and shared profit fee by using optimal contract concept. The upfront fee is collected after the CRA assigns ratings and the shared profit fee is realized after outcome of rated products reveals. In particular, this dissertation studies impact of the performance-based payment (profit-sharing scheme) on incentive of the CRA operated under the investor-pay model to produce accurate rating. In addition, the profit-sharing scheme is compared with pure upfront fee by using optimal contract concept to identify the most efficient payment system that incentivizes the CRA to produce accurate rating and generates higher expected profit for the investor.

3.3 Model

A two-period model of project investment decision is considered. In the market, there are three economic agents, namely firm, investor, and credit rating agency (CRA). These economic agents are rationale and the risk-free rate is zero. The investor (he) has sufficient fund to finance projects, and is categorized into the institutional investor and marginal investor. The institutional investor abides by investment regulation that allows him to make investment only in high-rated (investment-grade) projects. In this setup, a high rating assigned to projects means that it allows the issuing firm to raise funds from the institutional investor. It does not matter if the rating of the project is AAA, BBB, or C. In contrast, the marginal investor does not conform to such investment regulation, and can invest in any projects with or without acquiring rating information from the CRA. Once the investor solicits rating from the CRA and chooses additional information accuracy i, he always invests in high-rated projects only.

The CRA (she) has knowledge about information production technology, the demand of investors in the bond market, and monopolistic power, which can approximately reflect the oligopoly market structure of CRAs. The firm, owned by an entrepreneur who has no cash, is endowed with a risky project. The project, which may fail or succeed, requires an initial investment normalized to one and yields return at the end of the period. The net cash flow produced is $R > 1$ if the project succeeds. For simplicity, the net cash flow is zero if the project fails. The project types are classified into two types $n \in \{g, b\}$ where g and b are good and bad types of projects, respectively. Both types of projects are assumed to generate similar pattern of cash flows at the end of the period. A good project differs from a bad project by means of the probability of success denoted by p_n where $0 \leq p_b < p_g \leq 1$. The ex ante profit of each type of projects is given by:

$$x_n(R) = p_n R - 1 \tag{18}$$

We assume that only a good project can generate a positive expected payoff: $x_g(R) = p_g R - 1 > 0$; while a bad project yields a negative expected payoff: $x_b(R) = p_b R - 1 < 0$. Before assessing credit ratings, a fraction of good projects is given by α_g and a fraction of bad projects is given by $\alpha_b = (1 - \alpha_g)$; both are common knowledge in the market. The ex ante profit of the market, designated by $\alpha_g x_g(R) + \alpha_b x_b(R)$, implies the overall market condition. Favorable market condition is characterized by $\alpha_g x_g(R) + \alpha_b x_b(R) \geq 0$ while unfavorable market condition is signified by $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$.

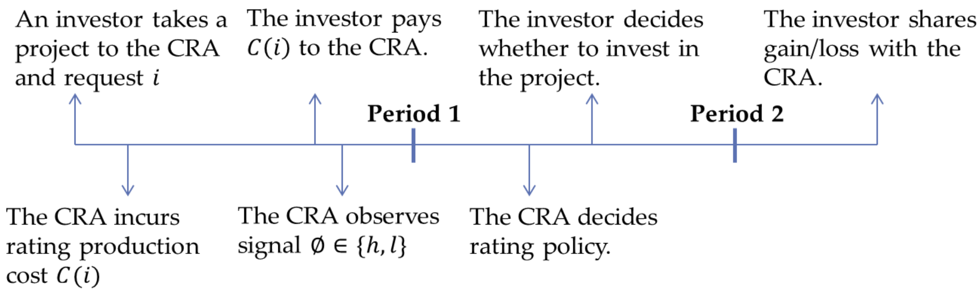

Figure 3.1 The Sequence of Events for Soliciting a Rating.

In period 0, the firm endowed with a risky project seeks fund from the investor. In period 1, the investor asks the CRA to verify the quality of the project and specifies additional information accuracy i to make an investment decision. The CRA evaluates the project quality gaining signal $\emptyset \in \{l, h\}$ and charges the investor information production cost $C(i)$. In period 2, the CRA reports an indicative rating $\tilde{r}$ and the investor decides to finance a high-rated project only. If the high-rated project succeeds, the investor realizes retained share β and the CRA gets profit sharing, $1 - \beta \geq 0$, at the end of period. If the project fails, the investor claims all earnings while both the firm and CRA gains zero profit sharing at the end of period. Since the firm and CRA proportionately share profit and loss in all economic circumstances, we hereafter model the firm and CRA as one agent referred to as the "CRA" whose objective is to maximize the ex ante expected payoff from the project (see Fig 1).

The sequence of a rating evaluation process consists of 5 steps. First, the investor determines additional level of information accuracy i and solicits a rating. Second, the CRA observes a private signal $\emptyset$ and incurs information production cost $C(i)$ increasing in i that directly passes on to the investor. Third, the CRA endogenously forms rating-reported strategy $\mu^{\emptyset}$ after observing signal $\emptyset$. Her strategy can be either the full disclosure F or biased rating regime B, or $\mu^{\emptyset} \in \{F, B\}$. The full disclosure regime is the probability that the CRA reports an indicative rating $\tilde{r}$ consistent with a signal observed from a project. Contrarily, the biased rating strategy represents the probability that the CRA assigns an indicative rating $\tilde{r}$ contradicted to a signal observed from a project. Since the signal and rating-reported regime are privately known, the CRA may have incentive to misreport the rating in exchange of higher benefits or to induce the investor to purchase rating (e.g., rating shopping). Hence, let $I\left(\mu^{\emptyset}(i)\right)$ be an indicator function denoting the probability that the CRA misreports where $I\left(\mu^{\emptyset}(i)\right) = \begin{cases} 0 \; if \, \mu^{\emptyset}(i) = F \\ 1 \; if \; \mu^{\emptyset}(i) = B \end{cases}$. Fourth, the CRA informs an indicative rating $\tilde{r} \in \{h, l\}$ where h and l are high and low rating respectively. Fifth, the investor chooses to invest only in h-rated securities (see Fig. 3.2).

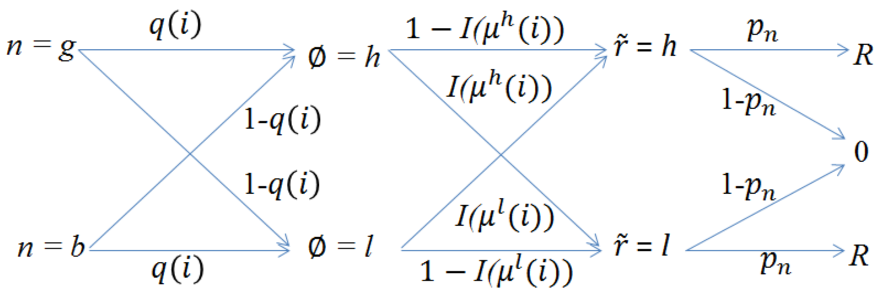

Figure 3.2 The Rating Process Conditional on Each Type of Projects.

In practice, the CRA acquires both qualitative and quantitative data to evaluate success probability of each project. Accordingly, there exists cost of information production in a rating process $C(i)$ which is strictly increasing convex in the level of information production i. The cost function of information production is:

$$C(i) = ai^2 \qquad \text{and} \qquad a > 0 \tag{19}$$

where a represents the CRA's cost for conducting due diligence and improving analytical models to match the complexity of each asset class being rated. The quality of rating evaluation depends on the level of information production, $i \geq 0$, and $\lim_{i \to 1-\alpha_m} C(i) = +\infty$ which is a sufficient condition to illustrate that the project quality can never be clearly verified. The probability that the CRA evaluates the project type correctly increases as the level of additional information accuracy is increasingly acquired.

$$\Pr\{\emptyset = h | n = g\} = \Pr\{\emptyset = l | n = b\} = q(i) = \alpha_m + i \tag{20}$$

where $\alpha_m = \max\{\alpha_g, \alpha_b\}$ and $0 \leq i \leq 1 - \alpha_m$ implies historical proportion of good and bad projects in the market. If the additional level of information production i is zero, the signal is thus uninformative. The signal accuracy of the CRA is given by $q(i) = \alpha_m + i$.

3.4 Analysis

In this study, the profit-sharing payment scheme is analyzed to investigate whether it can align incentive of the investor with that of the CRA operated under the investor-pay business model, and prevent the CRA from misreporting the rating. From the model settings, necessary probabilities used in this analysis are derived as follows.

The probability of the project to succeed before rating evaluation is:

$$\pi_{sb} = \alpha_g p_g + \alpha_b p_b$$

The probability of the project to get signal of *h* after assessing rating is:

$$\pi_{ah}(i) = \alpha_g q(i) + \alpha_b (1 - q(i))$$

The probability of the project to get signal of *l* after assessing rating is:

$$\pi_{al}(i) = 1 - \pi_{ah}(i) = \alpha_g (1 - q(i)) + \alpha_b q(i)$$

The probability of the project to get an indicative rating of h after assessing rating is:

$$\pi_h(i) = \alpha_g [q(i)(1 - I\left(\mu^h(i)\right)) + (1 - q(i)) I\left(\mu^l(i)\right)] + \alpha_b [q(i) I\left(\mu^l(i)\right) + \left(1 - q(i)\right)(1 - I\left(\mu^h(i)\right))]$$

The probability of the project to get an indicative rating of l after assessing rating is:

$$\pi_l(i) = \alpha_g\left[q(i)I\left(\mu^h(i)\right) + (1-q(i))\left(1 - I\left(\mu^l(i)\right)\right)\right] + \alpha_b[q(i)(1 - I\left(\mu^l(i)\right)) + (1-q(i))I\left(\mu^h(i)\right)]$$

The success probability given the high-rating project after rating evaluation is:

$$\pi_{ahs}(i) = \frac{\alpha_g p_g\left[q(i)(1-I\left(\mu^h(i)\right)) + (1-q(i))I\left(\mu^l(i)\right)\right] + \alpha_b p_b[q(i)I\left(\mu^l(i)\right) + (1-q(i))(1-I\left(\mu^h(i)\right))]}{\pi_h(i)}$$

The default probability given the high-rating project after rating evaluation is:

$$\pi_{ahd}(i) = \frac{\alpha_g(1-p_g)\left[q(i)\left(1 - I\left(\mu^h(i)\right)\right) + (1-q(i))I\left(\mu^l(i)\right)\right]}{\pi_h(i)} + \frac{\alpha_b(1-p_b)[q(i)I\left(\mu^l(i)\right) + (1-q(i))(1 - I\left(\mu^h(i)\right))]}{\pi_h(i)}$$

The success probability given the low-rating project after rating evaluation is:

$$\pi_{als}(i) = \frac{\alpha_g p_g\left[q(i)I\left(\mu^h(i)\right) + (1-q(i))(1-I\left(\mu^l(i)\right))\right] + \alpha_b p_b[q(i)(1-I\left(\mu^l(i)\right)) + (1-q(i))I\left(\mu^h(i)\right)]}{\pi_l(i)}$$

The default probability given the low-rating project after rating evaluation is:

$$\pi_{ald}(i) = \frac{\alpha_g(1-p_g)\left[q(i)I\left(\mu^h(i)\right) + (1-q(i))(1-I\left(\mu^l(i)\right))\right] + \alpha_b(1-p_b)[q(i)(1-I\left(\mu^l(i)\right)) + (1-q(i))I\left(\mu^h(i)\right)]}{\pi_l(i)}$$

3.4.1 Credit Rating Agency Problem

Taking the investor's level of additional information accuracy i as given, the CRA can maximize her profit by selecting rating-reported strategy $I\left(\mu^{\emptyset}(i)\right)$, either full disclosure or biased rating regime. Under the full disclosure regime, the CRA truthfully reports an indicative rating in accordance with a signal being observed. If the CRA adopts biased rating regime, she reports an indicative rating contradicted to a signal being observed. Biased rating regime represents both rating deflation and rating deflation regime. The rating deflation is that the CRA observes a signal of h but reports an indicative rating of l. The rating inflation occurs when the CRA observes a signal of l but reports an indicative rating of h. The CRA can collect the upfront fee in period 1 and profit sharing in period 2 from the investor. Once the investor solicits rating from the CRA, he finances the project only if it gets a high-rating (investment-grade). Therefore, the ex-ante expected profit maximization that determines the CRA's behavior is:

$$\Pi_{CRA} = max_{I\left(\mu^{h}(i)\right),I\left(\mu^{l}(i)\right)}[\pi_{ah}(i)\left[1 - I\left(\mu^{h}(i)\right)\right]x_g(R) + \pi_{al}(i)I\left(\mu^{l}(i)\right)x_b(R)][1 - \beta] \quad (21)$$

To clearly understand the CRA's behavior, it is necessary to study the optimal choice of rating-reported regimes which potentially affects the CRA's profit maximization.

Proposition 3.1 Given the requested additional information accuracy level $i \in [0,1 - \alpha_m]$, the CRA's optimal response when observing a signal $\emptyset \in \{l, h\}$ is characterized by a threshold $\bar{\iota}^{\emptyset}$, such that

1) If $i < \bar{\iota}^{\emptyset}$, it is optimal to misreport the project type.
2) If $i \geq \bar{\iota}^{\emptyset}$, it is optimal to truthfully report the project type.

Furthermore, when the market is unfavorable (negative ex ante expected profit of the market), $\bar{\iota}^{l} < \min\{0, \bar{\iota}^{h}\}$ equivalent to $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$. When the market is favorable (nonnegative ex ante expected profit of the market), $\bar{\iota}^l \geq \min\{0, \bar{\iota}^h\}$ equivalent to $\alpha_g x_g(R) + \alpha_b x_b(R) \geq 0$.

Proof. See Appendix B

From proposition 3.1, it is revealed that when the CRA observes a type of a project, she makes a decision to choose optimal rating policy between a full disclosure and biased rating regime based on a threshold of an information accuracy level $\bar{\iota}^\emptyset$. The CRA observes a high signal of a project, $\emptyset = h$, and reports a high rating (the full disclosure regime) only if the additional information accuracy level i requested by the investor is sufficiently high, $i \geq \bar{\iota}^h$. Otherwise, the CRA misreports the project type (the rating deflation regime), $i < \bar{\iota}^h$. In this case, she realizes that the observed high signal may not be correct or the high-signaled project can be a bad type with decent probability (coarse rating analysis). Thus, it is optimal for her to implement deflated rating regime because she runs high risk of realizing negative profit sharing from announcing a high rating on a bad project that later defaults. Similarly, the CRA observes a low signal of the project, $\emptyset = l$, and reports a low rating only if $i \geq \bar{\iota}^l$. Otherwise, the CRA misreports the project type (the rating inflation regime), $i < \bar{\iota}^l$. In this case, she realizes that the observed low signal may likely be incorrect or a good type of project is assigned a low rating. The CRA may adopt rating inflation regime since it is highly possible that the observed low-type project later succeeds and she gains positive profit sharing from the project outcome.

To understand the CRA's behavior of adopting rating policy in different market conditions, market conditions are classified as follows. Bad market designated by $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$ incorporates both extremely unfavorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, and favorable market, $-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$. Additionally, good market designated by $\alpha_g x_g(R) + \alpha_b x_b(R) \geq 0$ includes both favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, and extremely favorable market, $\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] \leq \alpha_g x_g(R) + \alpha_b x_b(R)$.

In bad market, $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$, it follows that $\bar{\iota}^l < min\{0, \bar{\iota}^h\}$. The CRA always truthfully reports the type of the project if she observes the low type of the project, $\emptyset = l$, since the threshold of additional information accuracy level $\bar{\imath}^l$ is less than zero. However, the CRA may misreport the type of the project when she observes the high type of the project, $\emptyset = h$. In other words, the CRA may adopt rating deflation regime since she is exposed to a high probability of realizing negative profit sharing from the project outcome due to conducting coarse rating analysis, $i < \bar{\imath}^h$, in bad market condition.

In good market, $\alpha_g x_g(R) + \alpha_b x_b(R) \geq 0$, it follows that $\bar{\imath}^l \geq min\{0, \bar{\imath}^h\}$. The CRA always reports the h-type of the project if she observes h-type of the project $\emptyset = h$. However, the CRA may misreport the type of the project if she observes l-type project, $\emptyset = l$. In particular, the CRA may adopt rating inflation regime since the observed low signal may be misevaluated, $i < \bar{\imath}^l$, or a good project type receiving low rating is likely to succeed in good market condition so that the CRA expects a high probability of gaining positive profit sharing from the project outcome. Note that since $\bar{\imath}^l > 0$ and $\bar{\imath}^h > 0$ cannot occur at the same time, it is impossible for the CRA to simultaneously adopt biased rating for both high and low signal.

3.4.2 Investor Problem

Conditional on the CRA strategies, the investor can acquire additional information accuracy level i to maximize his profit. There are five possible alternatives for the investor to make an investment decision: 1) acquire rating and invest in the project given high-rated, 2) acquire rating and invest in the project given low-rated, 3) don't acquire rating and invest in the project, 4) do not acquire rating and don't invest in the project, and 5) acquire rating but don't invest. The exante expected payoff of the investor in each alternative is shown as follows:

1) acquire rating and invest in the project given high-rated:

$$\Pi_{INV}^{\tilde{r}=h}(i) = \left[\pi_{ah}(i)[1 - I\left(\mu^h(i)\right)]x_n(R) + \pi_{al}(i)I\left(\mu^l(i)\right)x_n(R)\right]\beta - C(i)$$

2) acquire rating and invest in the project given low-rated:

$$\Pi_{INV}^{\tilde{r}=l}(i) = \Big[\pi_{ah}(i)I\Big(\mu^h(i)\Big)x_n(R) + \pi_{al}(i)[1 - I\Big(\mu^l(i)\Big)]x_n(R)\Big]\beta - C(i) < 0$$

3) do not acquire rating and invest in the project: $\Pi_{INV}^{sb} = \pi_{sb}R - 1$

4) do not acquire rating and don't invest in the project: $\Pi_{INV}^{0} = 0$

5) acquire rating but don't invest, the expected payoff is $0 - C(i)$.

The second alternative is irrational because the investor relies on rating when quality of the project is unjustified. Accordingly, the rational investor shouldn't fund the low-quality project confirmed by the credit rating agency. The fifth alternative clearly yields the negative expected profit deriving from cost of acquiring rating that is not used intentionally.

Since the investor in this study is categorized into the institutional investor and marginal investor because of different investment regulations each type of investors complies with. We first examine the institutional investor's decision to make investment, and second analyze the marginal investor's decision to make investment in the project.

3.4.2.1 Institutional Investor Problem

First, the institutional investor' investment behavior is studied. The institutional investor must acquire rating information from the CRA to make investment decision due to compliance with investment regulation. Since he always seeks investing opportunities regardless of market conditions, only the first alternative is considered to maximize his expected profit. The institutional investor's incentive to acquire additional information accuracy can be explained by the expected profit maximization. Accordingly, the ex ante expected profit of the institutional investor is given by

$$\Pi_{INV}^{\tilde{r}=h}(i) = \max_i \Big[\Big[\pi_{ah}(i)\Big[1 - I\Big(\mu^h(i)\Big)\Big]x_n(R) + \pi_{al}(i)I\Big(\mu^l(i)\Big)x_n(R)\Big]\beta - C(i)\Big] \tag{22}$$

Proposition 3.2 The optimal additional information accuracy i^* is characterized by market conditions and three thresholds of investor's retained share $\underline{\beta}$, $\bar{\beta}$, and $\bar{\bar{\beta}}$ as follows.

1) When the CRA can switch rating policy between biased rating and full disclosure regime ($\bar{\iota}^h \geq 0 > \bar{\iota}^l$, or $\bar{\iota}^l \geq 0 > \bar{\iota}^h$) in extremely unfavorable, $\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, or extremely favorable market $\alpha_g x_g(R) + \alpha_b x_b(R) \geq \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$,

(1) if $\beta \leq \bar{\bar{\beta}}$, it is optimal to not request any additional accuracy ($i^* = 0$).

(2) if $\beta > \bar{\bar{\beta}}$, it is optimal to request the maximum accuracy ($i^* = 1 - \alpha_m$).

2) When the CRA can switch rating policy between biased rating and full disclosure regime ($\bar{\iota}^h \geq 0 > \bar{\iota}^l$ or $\bar{\iota}^l \geq 0 > \bar{\iota}^h$) in unfavorable, $-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$, and favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m[\alpha_g x_g(R) + \alpha_b x_b(R)]$, or the CRA always adopts full disclosure regime ($0 > \bar{\iota}^h > \bar{\iota}^l$ or $0 > \bar{\iota}^l > \bar{\iota}^h$),

(1) if $\beta \leq \underline{\beta}$, it is optimal to not request any additional accuracy ($i^* = 0$).

(2) if $\underline{\beta} < \beta < \bar{\beta}$, it is optimal to request $i^* = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$.

(3) if $\beta \geq \bar{\beta}$, it is optimal to request the maximum accuracy ($i^* = 1 - \alpha_m$).

Proof. See Appendix B

It is straightforward in Proposition 3.2 that the investor considers either acquiring no additional accuracy level, $i^* = 0$, or requesting maximum additional accuracy level, $i^* = 1 - \alpha_m$, based on a portion of retained share β he gets to keep if the CRA can mix rating policy in extremely unfavorable or extremely favorable market. If the investor keeps a small portion of retained share, $\beta \leq \bar{\bar{\beta}}$, he is better off not to acquire any additional accuracy level $i^* = 0$. In this case, the investor shifts a large portion of risk to the CRA so that the CRA is directly affected by gain or loss of the risky project she rates. Thus, the CRA tends to assign rating in consistent with a signal she observes to hedge herself against severe loss from the outcome of the project. If the investor decides to keep a large portion of retained share, $\beta > \bar{\bar{\beta}}$, he needs to acquire maximum additional accuracy level, $i^* = 1 - \alpha_m$, to minimize default risk of the project and pays high cost of information production to incentivize the CRA in order to produce high accurate rating.

In the case that the CRA always adopts full disclosure regime, or the CRA can mix rating policy in unfavorable or favorable market, the institutional investor's incentive to acquire the optimal level of additional information accuracy i^* is determined by the investor's retained share β. If the investor keeps a small portion of retained share, $\beta \leq \underline{\beta}$, or a large portion of profit sharing $\beta - 1$ is given to the CRA, there is no need for the investor to request any additional information accuracy level, $i^* = 0$. In this case, the investor needs not to acquire any accuracy level $i^* = 0$ because the CRA is responsible for a significant amount of gain/loss derived from the project outcome. In other words, a large portion of default risk is shifted to the CRA so that she is directly induced to issue accurate rating to capture shared profit and avoid loss from the project outcome. If a middle portion of profit is shared with the CRA, $\underline{\beta} < \beta < \overline{\beta}$, the investor acquires sufficient information accuracy level, $i^* = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$. In this case, the investor needs to monitor the CRA more closely since she may occasionally inflate rating according to baring less amount of risk, especially in a good market due to less default risk shifted to the CRA than that retained by the investor. If a small portion of profit is allocated to the CRA, $\beta \geq \overline{\beta}$, the investor needs to request maximum information accuracy. In this case, it is straightforward that the investor must exert high effort to monitor the CRA since her performance may be insignificantly affected by the project outcome according to a little amount of profit that she shared.

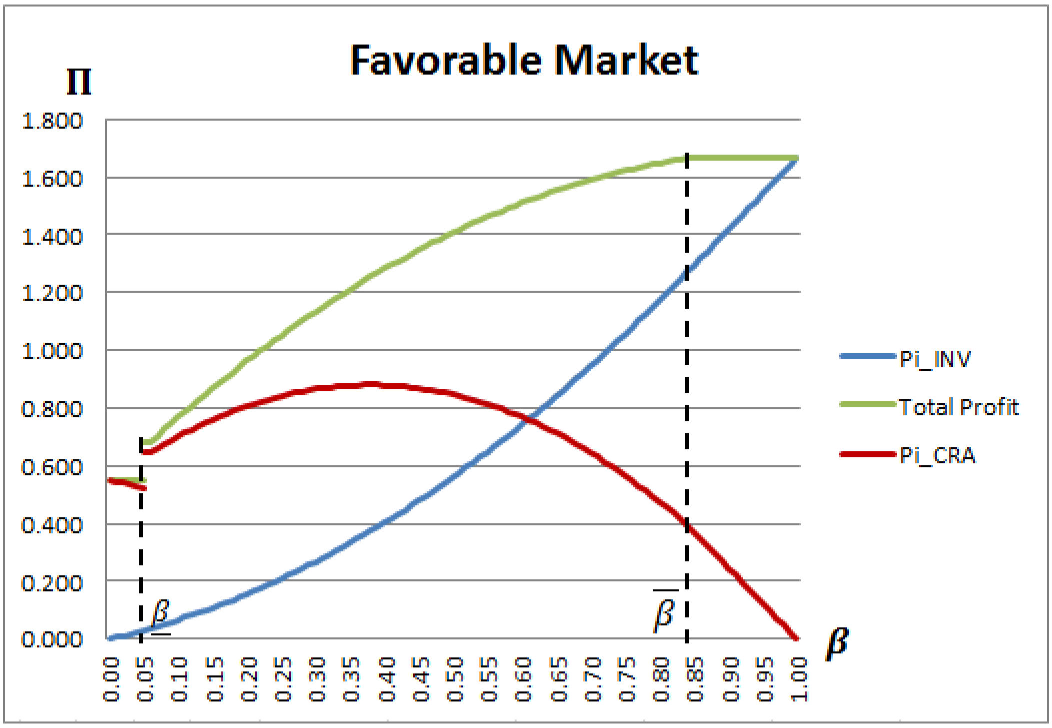

Figure 3.3 The Institutional Investor's and CRA's Expected Profit with Respect to Profit-Sharing Parameter β when $\alpha_g = 0.55, \alpha_b = 0.45, x_g = 4.5, x_b = -4, a = 4.$

Figure 3.3 exhibits the influence of shares of the project return on the investor's and CRA's expected profit. In this case, the market is favorable, $0 \leq [\alpha_g x_g(R) + \alpha_b x_b(R)] < \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$. For $\beta \leq \underline{\beta}$, the investor does not request additional information accuracy, $i^* = 0$. This provides an opportunity for the CRA to maximize her expected profit by saving on rating production cost and inflate a rating. Although the rating has low accuracy, the CRA believes that default risk of the project is inherently low in the favorable market condition. Note that the CRA's expected profit from the project is decreasing in β as she receives a smaller share of the profit. For $\underline{\beta} < \beta < \overline{\beta}$, the investor has incentive to request $i^* = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$ that is monotonically increasing in β. The issued rating becomes increasingly more accurate leading to increase in the expected net profit that increases with β. Although the CRA gains less shared profit of the project as β increases, the CRA's expected profit also increases because the total expected profit of the project increases with β. For $\beta \geq \overline{\beta}$, the investor tends to request the maximum accuracy, $i^* = 1 - \alpha_m$. The CRA's expected profit declines because the investor takes a larger share of the profit as β increases.

Moreover, notice that a minimum threshold of the investor's retained share $\underline{\beta} = \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ in unfavorable market, $\bar{\iota}^l < min\{0, \bar{\iota}^h\}$, is different from the minimum threshold of the investor's retained share $\underline{\beta} = \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ in favorable market, $\bar{\iota}^h < min\{0, \bar{\iota}^l\}$. Given no additional information accuracy requested by the institutional investor, $i^* = 0$, the retained share of the investor β in unfavorable market must be greater than that in favorable market since $\underline{\beta}$ in unfavorable market, $\bar{\iota}^l < min\{0, \bar{\iota}^h\}$, is greater than $\underline{\beta}$ in favorable market, $\bar{\iota}^h < min\{0, \bar{\iota}^l\}$. This implies that if the investor requests no additional accuracy level, $i^* = 0$, he can give less amount of profit sharing to the CRA in unfavorable market than that in favorable market to induce the CRA to issue an accurate rating. Since the CRA knows that there is a higher default probability in unfavorable market and the portion of profit sharing she contains is exposed to higher chance of loss than gain from the project outcome, thus only little amount of profit sharing can keep her working effectively. Therefore, the institutional investor can maintain higher retained share of profits β in unfavorable market than that in favorable market to induce high quality of rating from the CRA if he does not request positive additional information accuracy level, $i^* = 0$.

3.4.2.2 Marginal Investor Problem

Second, we consider the marginal investor's investment behavior. Different from the institutional investor, the marginal investor can invest in the project with or without acquiring information from the CRA. Thus, the marginal investor would form investment strategy that generates profit based on three rational decisions: (i) acquire rating and invest in the project if it gets high rating, $\Pi_{INV}^{\tilde{r}=h}$, (ii) don't acquire rating and invest in the project, Π_{INV}^{sb}, (iii) don't acquire rating and don't invest in the project Π_{INV}^{0}. To examine the marginal investor's incentive to acquire additional information accuracy level, the expected profits of the marginal investor are given by:

$$\Pi_{INV} = max_i[\underbrace{\left[\pi_{ah}(i)\left[1 - I\left(\mu^h(i)\right)\right]x_n(R) + \pi_{al}(i)I\left(\mu^l(i)\right)x_n(R)\right]\beta - C(i)}_{\Pi_{INV}^{\tilde{r}=h}(i)},$$

$$\underbrace{\pi_{sb}R - 1}_{\Pi_{INV}^{sb}}, \underbrace{0}_{\Pi_{INV}^{0}}\ ] \qquad (23)$$

Proposition 3.3 For a marginal investor who can decide whether to get ratings from the CRA,

1) If the cost of information acquisition is sufficiently low, it is optimal to get ratings from the CRA by requesting the information accuracy level as characterized in Proposition 3.2

2) If the cost of information acquisition is too high,

(1) It is optimal to not invest when the market is extremely unfavorable and unfavorable (negative ex ante expected profit).

(2) It is optimal to invest without getting ratings from the CRA when the market is extremely favorable and favorable (nonnegative ex ante expected profit).

Proof. See Appendix B

From proposition 3.3, the marginal investor's incentive toward purchasing rating information to form investment strategy is revealed. In bad market, it is optimal for the investor to consider only two investment strategies between purchasing rating information from the CRA and neither purchasing rating information nor investing in the project. If the information cost a is sufficiently high, the investor is better off not investing in the project and not soliciting rating from the CRA. Otherwise, he decides to solicit rating to make decision on investment strategy since gain from the project outcome cannot offset high information cost for producing accurate rating. In extremely unfavorable market, the investor tends to keep a large portion of retained share, $\beta > \bar{\bar{\beta}}$, and requests maximum accuracy level, $i^* = 1 - \alpha_m$ if the information cost a is sufficiently low. In this case, he demands high accurate rating to make investment decision since risk is exceptionally high in the market. Otherwise, he would rather not solicit rating and not invest in the market. In unfavorable market, if the information cost a is sufficiently low, the investor solicits rating from the CRA and requests sufficiently high information accuracy level, $i^* \in \{i_0, 1-\alpha_m\}$. In the case that the investor decides to keep a medium portion of retained share, $\underline{\beta} < \beta < \overline{\beta}$, he acquires additional accuracy, $i^* = i_0$, only if the information cost a is lower than the threshold of information cost $\bar{a}_1 = \frac{-[\alpha_g x_g(R) - \alpha_b x_b(R)]^2 \beta}{4[[\alpha_g x_g(R) - \alpha_b x_b(R)]\alpha_m + \alpha_b x_b(R)]}$. In this case, the CRA and investor bare equally likely similar level of default risk so that the CRA is induced to produce accurate rating to avoid loss from the project outcome. In case that the investor decides to keep a large portion of retain share, $\beta \geq \overline{\beta}$, he is likely to acquire the maximum information accuracy level, $i^* = 1 - \alpha_m$, only if the information cost a is lower than the threshold of information cost $\bar{a}_2 = \frac{\alpha_g x_g(R)\beta}{(1-\alpha_m)^2}$. In this case, the investor bares most of default risk so that he needs to incentivize the CRA by paying highest information cost in exchange of accurate rating. It is straightforward that the information cost a, which can vary by financial asset classes (the more complex products, the higher information cost becomes), is the key factor for the investor to decide whether to solicit rating from the CRA. In the case that the investor solicits rating, he demands sufficiently high accuracy level and relies on rating information from the CRA to make investment decision. He requires in-depth analysis from the CRA since default risk is exceptionally high in extremely unfavorable and unfavorable market. Without rating information, it is in his best interest to avoid potential loss by not investing in the project due to high default risk in bad market.

In good market, it is optimal for the investor to consider only two investment strategies between purchasing rating information from the CRA and investing in the project without purchasing rating information from the CRA. If the information cost a is sufficiently high, the investor is better off investing in the project without soliciting rating from the CRA since information cost of producing accurate rating may exceed loss from the project outcome. Otherwise, the investor decides to solicit rating to make decision on investment strategy. In extremely favorable market, the investor tends to keep a large portion of retained share, $\beta > \bar{\bar{\beta}}$, and requests maximum accuracy level, $i^* = 1 - \alpha_m$, if the information cost is sufficiently low, $a < \bar{a}_2 := \frac{\alpha_g x_g(R)[\beta-1]-\alpha_b x_b(R)}{(1-\alpha_m)^2}$. In this case, he requires high accurate rating to make investment decision and to hedge a significant portion of retained share (risk) on hands since it is highly possible for the CRA to inflate rating. In contrast, the investor is better off making investment without purchasing rating information if the CRA charges too high information cost, $a \geq \bar{a}_2 := \frac{\alpha_g x_g(R)[\beta-1]-\alpha_b x_b(R)}{(1-\alpha_m)^2}$ since the information cost is more expensive than loss from the project which is exceptionally low in extremely favorable market. In favorable market, the investor solicits rating from the CRA and requests sufficiently high information accuracy level, $i^* \in \{i_0, 1-\alpha_m\}$, if information cost a is sufficiently low. In the case that the investor decides to keep a medium portion of retained share, $\underline{\beta} < \beta < \overline{\beta}$, he acquires additional accuracy $i^* = i_0$ only if the information cost a is lower than the threshold of information cost $\bar{a}_1 = \frac{\left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]^2 \beta^2}{4\left(\alpha_g x_g(R)+\alpha_b x_b(R)[1-\beta]-\alpha_m\left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]\beta\right)}$. In this case, the investor shares equally likely similar level of risk with the CRA to discourage the CRA from inflating rating. If he decides to keep a large portion of retain share, $\beta \geq \overline{\beta}$, he is likely to acquire the maximum information accuracy level $i^* = 1-\alpha_m$ only if the information cost is lower than the threshold of information cost, $a < \bar{a}_2 = \frac{\alpha_g x_g(R)[\beta-1]-\alpha_b x_b(R)}{(1-\alpha_m)^2}$. The investor, in this case, ensures accurate rating by paying high information cost to discourage the CRA from inflating rating and keeps a significant portion of retain share since low default risk in favorable market. It is straightforward that the information cost a is the vital factor for the marginal investor to decide whether to solicit rating from the CRA. In case of too high information cost, it implies that the project quality is highly complex. Thus, the investor does not solicit rating since rating accuracy is unlikely assured and expected loss from the project is low. He is better off making investment without soliciting rating from the CRA. In case of low information cost, the investor solicits rating. He knows that it is highly possible for the CRA to inflate rating so that he tends to request sufficiently high additional accuracy to ensure rating quality. Then, he can make investment decision based on solicited rating in either extremely favorable or favorable market.

From Proposition 3.1 to 3.3, we already learned about how the CRA decides to implement rating policy (the full disclosure or biased rating), and how the (institutional and marginal) investor makes a decision to request additional information accuracy i in different market conditions. Additionally, the dynamic interaction between the (institutional and marginal) investor and the CRA is analyzed when the profit-sharing payment scheme is exerted.

3.4.2.3 Comparisons of Induced Outcomes

We now consider both advantages and disadvantages of the profit-sharing payment scheme over the pure upfront fee. In particular, we investigate solely a case when the investor purchases rating information from the CRA in order to make investment decision. Based on results derived from Proposition 2, the ex ante expected profit of the investor under the profit-sharing scheme depends on four market conditions and the CRA's behavior of rating policy implementation.

Firstly, if the CRA always adopts full disclosure regime, $0 > \bar{\iota}^h > \bar{\iota}^l$, or $0 > \bar{\iota}^l > \bar{\iota}^h$, the ex-ante expected profit of the investor similar across all market conditions is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m \beta + \alpha_b x_b(R)\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^*=i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^*=1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

Secondly, if the CRA can mix rating policy, $\bar{\iota}^h \geq 0 > \bar{\iota}^l$, or $\bar{\iota}^l \geq 0 > \bar{\iota}^h$, the ex ante expected profit of the investor depends on four market conditions as follows.

In extremely unfavorable market, the ex-ante expected profit of the investor is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0) = 0 & if\ \beta \leq \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \\ \Pi_{INV}^*(i^*=1-\alpha_m) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2 & if\ if\ \beta > \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \end{cases}$$

In unfavorable market, the ex-ante expected profit of the investor is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(0) = 0 & if\ \beta \le \underline{\beta} \\ \Pi_{INV}^*(i_0) = \Big[\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big][i_0 + \alpha_m] + \alpha_b x_b(R)\Big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \ge \bar{\beta} \end{cases}$$

Where $\underline{\beta} = \frac{2a\bar{\imath}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ and $\bar{\beta} = \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

In favorable market, the ex-ante expected profit of the investor is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \big[\alpha_g x_g(R) + \alpha_b x_b(R)\big]\beta & if\ \beta \le \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \Big[\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big][i_0 + \alpha_m] + \alpha_b x_b(R)\Big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \ge \bar{\beta} \end{cases}$$

Where $\underline{\beta} = \frac{2a\bar{\imath}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ and $\bar{\beta} = \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

In extremely favorable market, the ex-ante expected profit of the investor is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \big[\alpha_g x_g(R) + \alpha_b x_b(R)\big]\beta & if\ \beta \le \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \\ \Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta > \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \end{cases}.$$

Under the pure upfront fee, the ex-anted expected profit of the investor, obtained by substituting $\beta = 1$ into $\Pi_{INV}^{\tilde{r}=h}(i)$ equation or equation (9), is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i, \beta = 1) = \max_{i} \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R) \right] (i + \alpha_m) + \alpha_b x_b(R) - C(i) \right]$$

To examine which payment scheme (the profit-sharing vs. pure upfront fee) is more efficient in terms of keeping the CRA to produce accurate rating and generating more profit to the investor, the ex-ante expected profits of the investor under two payment schemes when the investor acquires optimal additional accuracy levels i^* are compared. Under the pure upfront fee scheme, the CRA is indifferent between implementing the full disclosure regime and biased rating regime since her expected profit becomes zero. Firstly, the CRA is assumed to always implements full disclosure regime.

Proposition 3.4 If the CRA always announces truthful ratings, then the pure upfront fee scheme is always more profitable to the investor than the profit-sharing scheme.

Proof. See Appendix B

Under the pure upfront fee, the investor has incentive to acquire an optimal level of additional information accuracy $i_f^* \coloneqq \frac{\alpha_g x_g(R) - \alpha_b x_b(R)}{2a} \geq 0$ to maximize his expected profit and the expected profit of the investor $\Pi_{INV}^{\tilde{r}=h}(i, \beta = 1)$ is concave in additional information accuracy level i. As revealed in Proposition 2, it is optimal for the investor to acquire additional information accuracy level $i^* \in \{0, i_0, 1 - \alpha_m\}$ under the profit-sharing payment scheme regardless of market conditions.

From Proposition 3.4, the expected profits under two payment systems of the investor, when acquiring the optimal level of additional information accuracy, are compared to determine an efficient payment scheme that can efficiently assure rating accuracy and generate more profit to the investor. The expected profit of the investor under the pure upfront fee is always higher than that of the investor under the profit-sharing scheme as long as the CRA always sticks with the full disclosure regime regardless of market conditions. This result is quite intuitive because the profit-sharing payment scheme is designed in such a way that the expected profit of the investor is partially shared with the CRA to prevent her from issuing biased rating or to incentivize her to produce accurate rating under each market conditions. If the CRA always truthfully reports rating information, the partial profit shared with the CRA is in turn disadvantages as it unnecessarily lowers the expected profit of the investor. In other words, there is no need to give extra benefits to motivate the CRA to truthfully report ratings since she always does. Therefore, the upfront fee is an efficient payment scheme since it generates the higher expected profit for the investor than the profit-sharing scheme does in this case.

Secondly, we analyze which payment scheme is more efficient when the CRA is assumed to implement biased rating (the rating inflation regime, or rating deflation regime) because the CRA's expected profit is zero under the pure upfront fee scheme.

Proposition 3.5 If the CRA implements biased ratings,

1) In extremely unfavorable and unfavorable market, the profit-sharing scheme is more profitable than the pure upfront fee scheme.

2) In favorable market, when $\beta \leq \underline{\beta}$ (the investor requests $i^* = 0$), the pure upfront fee is more profitable to the investor than the profit-sharing scheme. When $\beta > \underline{\beta}$ (the investor requests $i^* \in \{i_0, 1 - \alpha_m\}$), the information cost a determines which payment scheme produces higher expected profit.

(1) If the cost of information production is sufficiently low, the profit-sharing scheme is more profitable to the investor than the pure upfront fee scheme.

(2) If the cost of information production is too high, the pure upfront fee is more profitable to the investor than the profit-sharing scheme.

(3) In extremely favorable market, when $\beta \leq \bar{\bar{\beta}}$ (the investor requests $i^* = 0$), the pure upfront fee is more profitable to the investor than the profit-sharing scheme. When $\beta > \bar{\bar{\beta}}$ (the investor requests $i^* = 1 - \alpha_m$), the profit-sharing scheme is more profitable to the investor than the pure upfront fee if the information cost a is sufficiently low. Otherwise, the pure upfront fee is more profitable to the investor.

Proof. See Appendix B

From Proposition 3.5, we aim to determine an efficient payment scheme for the investor when the CRA is assumed to give out biased ratings regardless of market conditions. To do so, we compare the expected profit of the investor under the profit-sharing payment scheme with that under the pure upfront fee. In extremely unfavorable market, the investor under profit-sharing scheme chooses between acquiring $i^* = 0$ and $i^* = 1 - \alpha_m$ for the purpose of preventing high default risk caused by inaccurate rating. If the investor acquires $i^* = 0$ and allocates a large portion of sharing profit to the CRA, a large portion of default risk is shifted to the CRA. The CRA is incentivized to produce accurate rating since she knows that default risk is exceptionally high. If the investor requests $i^* = 1 - \alpha_m$ paying sufficiently high information cost, the CRA gets a small portion of sharing profit. To maximize her profit, the CRA needs to minimize potential loss from the project outcome arrived from a bad project getting h-rating.

In unfavorable market, the investor under profit-sharing scheme can acquire $i^* = 0$, $i^* = i_0$, or $i^* = 1 - \alpha_m$ to discourage the CRA from issuing inaccurate rating. If the investor request $i^* = 0$ or $i^* = i_0$, the CRA bares a large portion of sharing profit or a medium portion of sharing profit respectively. According to risk shifted to the CRA, she is discouraged from issuing biased rating to minimize potential loss since she is responsible for the loss/gain from the project outcome while risk is sufficiently high. If the investor requests $i^* = 1 - \alpha_m$, he pays high information cost and shares a little portion of profit with the CRA to discourage the CRAs' behavior of issuing biased rating. In this case, the investor demands high accurate rating from the CRA since he retains a large portion of retained profit (high risk) while the CRA tends to minimize loss.

In favorable market, the profit sharing scheme generates higher profit to the investor only if information cost is sufficiently low. If the investor requests $i^* = 0$ and allots a large portion of profit-sharing to the CRA, the CRA is highly responsible for gain/loss of the project outcome. The large amount of shared profit likely generates profit to the CRA and in turn reduces profit of the investor since default risk is sufficiently low. Thus, the pure upfront fee is more profitable to the investor in this case. If the information cost is sufficiently low, the investor acquires $i^* = i_0$ or $i^* = 1 - \alpha_m$ to discourage the CRA from inflating rating. If the investor requests $i^* = i_0$ and allocates a medium portion of shared profit to the CRA, the CRA tends to produce more accurate rating. Since the CRA holds a smaller portion of share profit, the only way for her to maintain her expected profit is to reduce potential loss incurred in the portion of shared profit. However, the profit-sharing scheme yields less profit than the pure upfront fee only if information cost is sufficiently high, $a > \hat{a}_1 = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2 \beta^2}{4(\alpha_g x_g(R) + \alpha_b x_b(R)[1-\beta] - \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta)}$, since information cost the CRA charges exceeds expected loss caused by the CRA's behavior of inflating rating. If the investor acquires $i^* = 1 - \alpha_m$ paying high information cost and shares a little portion of profit with the CRA, the CRA is increasingly induced to issue high accurate rating. To maximize her expected profit, the CRA needs to minimize potential loss in the portion of shared profit. However, the profit-sharing scheme generates less profit than the pure upfront fee only if information cost is sufficiently high, $a > \hat{a}_2 = \frac{\alpha_g x_g(R)[\beta - 1] - \alpha_b x_b(R)}{(1-\alpha_m)^2}$.

For instance, figure 3.4 illustrates that the investor offering the profit-sharing scheme to the CRA requests $i^* = i_0$ while the investor offering the pure upfront scheme to the CRA requests $i^* = i_i = 0$ in the favorable market, and the information cost charged by the CRA is sufficiently low, $a < \hat{a}_1$. When the market condition is favorable and the investor requests $i^* = i_i = 0$, the CRA compensated by the upfront fee scheme has incentive to save rating production cost and inflate the rating. Thus, the investor's expected profit under the upfront fee scheme is independent of information cost a. Contrarily, the investor's expected profit under the profit-sharing contract is convexly decreasing in the information cost because the investor has incentive to request $i^* = i_0$. If the information cost charged by the CRA is sufficiently low, $a < \hat{a}_1$, the profit-sharing scheme generates higher expected profit to the investor since the high rating accuracy provides better informative guideline for him in making investment decision resulting in higher expected profit for the investor. However, the profit sharing scheme generates lower expected profit for the investor than the upfront fee scheme when the CRA charges sufficiently high information cost. This is because the investor's expected profit declines with a.

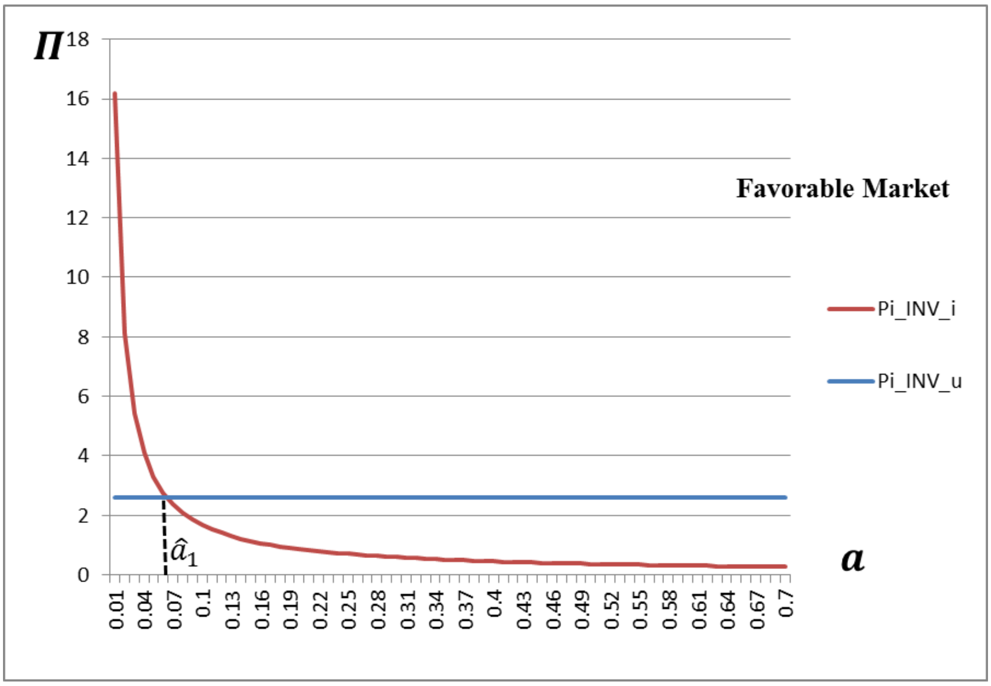

Figure 3.4 The (Institutional or Marginal) Investor's Expected Profit under the Profit-Sharing Scheme vs. that under the Upfront Fee Scheme where $\alpha_g = 0.55$, $\alpha_b = 0.45$, $x_g = 8$, $x_b = -4$, $\beta = 0.2$

In extremely favorable market, if the investor acquires $i^* = 0$ paying no information cost, a large portion of gain/loss from the project outcome is allotted to the CRA. Although the CRA inflates rating, the expected profit of the portion of shared profit is far beyond expected loss since default risk is exceptionally low. Thus, the profit-sharing scheme is less profitable to the investor in this case. If the investor requests $i^* = 1 - \alpha_m$ paying high information cost and sharing a small portion of profit with the CRA, the CRA's behavior of assigning inflated rating is discouraged. Accordingly, the only way for the CRA to maximize expected profit of the profit-sharing portion is to minimize potential loss arisen from a bad project receiving *h*-rating. However, the profit-sharing scheme generates less profit than the pure upfront fee only if information cost is sufficiently high, $a > \hat{a}_2 = \frac{\alpha_g x_g(R)[\beta-1]-\alpha_b x_b(R)}{(1-\alpha_m)^2}$.

This is quite intuitive since the profit-sharing scheme is designed to provide extra benefit to incentivize the CRA to provide accurate rating which implements the investor's investment decision making especially in bad market.

However, the investor needs to pay only sufficiently low information analysis cost which in turn leaving sufficiently high incentive for the CRA to effectively minimize potential loss from a bad project obtaining *h*-rating in a portion of shared profit in good market. Otherwise, the investor under the profit-sharing scheme is worse off if he pays for information analysis cost too high.

3.4.2.4 Optimal Retained Share

Since the advantages and disadvantages of both the profit-sharing scheme and pure upfront fee are already illustrated in the previous section, we in this section analyze a portion of the retained share the investor tends to keep when employing the profit-sharing payment scheme. We already learned from Proposition 2 that the investor tends to strategically request optimal additional accuracy level i^* depending on a portion of the retained share β in different market conditions, resulting in different expected for him as follows:

In extremely unfavorable and extremely favorable market,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) & if\ \beta \leq \bar{\bar{\beta}} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) & if\ \beta > \bar{\bar{\beta}} \end{cases}$$

Where $\bar{\bar{\beta}} \coloneqq \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$ in extremely unfavorable market, and $\bar{\bar{\beta}} \coloneqq \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$ in extremely favorable market

In unfavorable and favorable market,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

Where $\underline{\beta} = \frac{4a\bar{\imath}^h}{\alpha_g x_g(R) - \alpha_b x_b(R)}$ in unfavorable market, $\underline{\beta} = \frac{4a\bar{\imath}^l}{\alpha_g x_g(R) - \alpha_b x_b(R)}$ in favorable market, and $\bar{\beta} = \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$ in both market conditions

To derive the optimal retained share β^*, we need to investigate the investor's incentive toward acquiring additional information accuracy level i in each interval of the retained shares β. Thus, the expected profit of the investor in each interval of the retained share β is compared.

Proposition 3.6 Under profit-sharing scheme, the investor's expected profit monotonically increases in the retained share, β.

Proof. See Appendix B

From Proposition 3.6, we begin to observe the investor's incentive toward maintaining the optimal retained shares β^* in each interval of the retained share β while ensuring accurate ratings assigned by the CRA. Thus, we have that the investor acquires optimal additional accuracy level i^* and keeps the optimal retained shares β^* in different market conditions as follows:

In extremely unfavorable market, the investor requests $i^*, \beta^* = \begin{cases} i^* = 0, \beta^* = 0 & if\ \beta \leq \bar{\bar{\beta}} \\ i^* = 1 - \alpha_m, \beta^* \rightarrow 1 & if\ \beta > \bar{\bar{\beta}} \end{cases}$. If he acquires $i^* = 0$, his expected profit is zero and the indicative rating is uninformative. He is better off retaining as low risk as possible, $\beta^* = 0$, since default risk is exceptionally high in the market. In contrast, accurate rating (informative rating) is produced and he may gain positive expected profit if he request $i^* = 1 - \alpha_m$ paying sufficiently high information cost and giving very slim shared profit to the CRA. Thus, the investor is incentivized to keep high retained share, $\beta^* = \beta \rightarrow 1$. The investor tends to acquire $i^*, \beta^* = \begin{cases} i^* = 0, \beta^* = 0 & if\ \beta \leq \underline{\beta} \\ i^* = i_0, \beta^* = \bar{\beta} & if\ \underline{\beta} < \beta < \bar{\beta} \\ i^* = 1 - \alpha_m, \beta^* \rightarrow 1 & if\ \beta \geq \bar{\beta} \end{cases}$ in unfavorable market. He tends to contain a sufficiently small portion of the retained share $\beta^* = 0$ if he does not request any additional information accuracy level $i^* = 0$ having zero expected profit. In this case, the investor knows that rating quality is low and the CRA needs to be motivated by a large portion of the shared profit, $1 - \underline{\beta}$, hedging a significant portion of risk for him. When the investor acquires a sufficiently high level of additional information accuracy $i^* = i_0$, he tends to keep the a slightly larger portion of retained share $\beta^* = \bar{\beta}$. In this case, the investor pays higher cost of information production for the

CRA including a larger portion of shared profit $1-\bar{\beta}$ to motivate her to prudently evaluate ratings. Since the project quality may be ambiguous, the investor requires more careful rating production from the CRA to make investment decision. In addition, the investor can also have another protection from loss by allotting the portion of shared profit to the CRA, $1-\bar{\beta}$. If the investor requests the maximum level of additional information accuracy $i^* = 1-\alpha_m$, he tends to maintain almost all profit the project generates, $\beta^* = \beta \rightarrow 1$. With sufficiently high information cost and a very slim portion of shared profit given to the CRA, the rating accuracy is ensured. Thus, he is confident to keep a sufficiently large portion of retained share.

The investor has incentive to acquire $i^*, \beta^* = \begin{cases} i^* = 0, \beta^* = \underline{\beta} & if\ \beta \leq \underline{\beta} \\ i^* = i_0, \beta^* = \bar{\beta} & if\ \underline{\beta} < \beta < \bar{\beta} \\ i^* = 1-\alpha_m, \beta^* \rightarrow 1 & if\ \beta \geq \bar{\beta} \end{cases}$ in favorable market. He contains a sufficiently small portion of the retained share $\beta^* = 0$ if he does not request any additional information accuracy level $i^* = 0$. Since the rating is uninformative, he hedges against risk by retaining very small portion of retained profit $\beta^* = \underline{\beta}$ and giving away a large portion of the profit-sharing scheme to the CRA, $1-\underline{\beta}$. When the investor acquires a sufficiently high level of additional information accuracy $i^* = i_0$, he tends to keep a slightly larger portion of retained share $\beta^* = \bar{\beta}$. Since rating is more informative and default risk is sufficiently low, his expected profit can be maximized by allocating a slightly lower portion of shared profit to the CRA $1-\bar{\beta}$. If the investor requests the maximum level of additional information accuracy $i^* = 1-\alpha_m$, he tends to maintain almost all profit the project generates, $\beta^* = \beta \rightarrow 1$. Since the investor is certain about rating accuracy, he tends to maximize his expected profit by keeping a significantly high portion of retained share. In addition, the investor acquires $i^*, \beta^* = \begin{cases} i^* = 0, \beta^* = \bar{\bar{\beta}} & if\ \beta \leq \bar{\bar{\beta}} \\ i^* = 1-\alpha_m, \beta^* \rightarrow 1 & if\ \beta > \bar{\bar{\beta}} \end{cases}$ in extremely favorable market. If he acquires $i^* = 0$, he tends to keep a large portion of shared profit $\beta^* = \bar{\bar{\beta}}$. Although the indicative rating is uninformative, default risk is extremely low in the market. Thus, he is better off retaining a large portion of shared profit $\beta^* = \bar{\bar{\beta}}$ to maximize his profit. If he request $i^* = 1-\alpha_m$ paying sufficiently high information cost to ensure rating accuracy, the investor tends to keep high retained share, $\beta^* = \beta \rightarrow 1$. In this case, the investor may have optimistic view of the project or market conditions so that he is better off paying high cost of the rating production for the CRA, and retaining almost all of the project profit due to exceptionally low risk of losses.

Ultimately, we analyze one best optimal retained share β^* from the investor's three alternatives of additional accuracy acquisitions i^* that gives out different optimal retained shares β^*. To do so, we compare the investor's expected profits of each alternative. The results reveal that it is in the best interest of the investor to acquire the maximum level of additional information accuracy $i^* = 1 - \alpha_m$ and retain a significantly large portion of the shared profit $\beta^* \rightarrow 1$. This implies that the profit-sharing payment scheme can potentially urge the investor to request the maximum accuracy level from the CRA, while the CRA is induced to effectively evaluate credit ratings. In other words, the profit-sharing scheme can align incentive of issuing firms with that of investors. The result is intuitive for the investor to acquire high accurate rating in order to minimize risk, and invest in good projects to maximize his expected profit in every market condition.

3.5 Conclusion

This study analyzes how profit sharing scheme under the investor-pay model influences the CRA's incentive to produce accurate ratings and the (institutional and marginal) investor's incentive to acquire additional information accuracy level. In model setting, the CRA has control over the information disclosure level that allows her to form rating-reported strategy (e.g., the full disclosure, and biased rating regime). Based on the expected profit maximization, she sets the information accuracy threshold $\bar{\iota}^{\emptyset}$ as criteria to make a strategic decision on rating reported policy. If the issuer acquires the additional information accuracy i less than the additional accuracy threshold $\bar{\iota}^{\emptyset}$, the CRA tends to adopt the biased rating regime (e.g., inflated-rating or deflated-rating). Contrarily, the CRA tends to implement the full disclosure regime when the issuer acquires the additional information accuracy i greater than or equal the additional accuracy threshold $\bar{\iota}^{\emptyset}$.

Taken CRA's behavior of issuing biased rating into account, the (institutional or marginal) investor makes decision to choose level of additional information accuracy i that benefits him most. The institutional investor complies with investment regulation while the marginal investor does not. The institutional investor considers a portion of the retained shares β as indicators to make a decision to request additional information accuracy level which induces the CRA to truthfully report rating regardless of market conditions. The marginal investor considers cost of information production a as a factor to make investment decisions. If the cost of information is sufficiently low, he tends to acquire optimal additional accuracy levels $i^* \in \{0, i_0, 1 - \alpha_m\}$ regardless of market conditions. Otherwise, he makes no investment in unfavorable market, and makes investment without purchasing rating from the CRA in favorable market. Taken both the CRA's behavior of choosing the optimal rating-reported policy and the investor's incentive toward requesting optimal additional accuracy levels, dynamic interaction between the investor and CRA reveals that the investor tends to request optimal additional accuracy levels $i^* \in \{0, i_0, 1 - \alpha_m\}$ regardless of market conditions, and the optimal rating-reported regime for the CRA is either the full disclosure or inflated rating regime. Thus, the profit- sharing scheme under the investor-pay model can effectively align the (institution and marginal) investors' incentive with the (issuing firm) CRA's incentive resulting in improved rating accuracy.

In addition, the efficient payment scheme based on the investor's expected profit maximization is analyzed by comparing the profit-sharing scheme with the pure upfront fee. Conditional on the full disclosure regime always implemented by the CRA, the pure upfront fee is more efficient since the shared profit is not needed to motivate the CRA to truthfully issue rating information. In contrast, if the CRA always adopt the rating inflation regime, the profit-sharing scheme is more efficient in unfavorable market. In favorable market, if the investor decides to not acquire any additional accuracy level $i^* = 0$, the pure upfront fee is more efficient. If the investor decides to acquire sufficiently high additional accuracy level $i^* \in \{i_0, 1 - \alpha_m\}$, the cost of information a determines an efficient payment scheme. The pure upfront fee is more efficient only if the cost of information is too high. Otherwise, the profit-sharing is more efficient. Furthermore, the investor's incentive toward maintaining an optimal retained profit is analyzed. The findings show that the investor has incentive to retain the maximum of the retained profit $\beta^* \to 1$ regardless of market conditions. This implies that the investor tends to acquire the maximum level of additional information accuracy $i^* = 1 - \alpha_m$ under the profit-sharing scheme in both unfavorable and favorable market. Therefore, the profit-sharing scheme can effectively align incentive of the investor with that of the (issuing firm) CRA resulting in rating accuracy improvement regardless of market conditions.

Key findings of this study are that the profit-sharing scheme can effectively induce the CRA to improve rating accuracy regardless of market conditions. It can also prevent the institutional investor from facilitating the CRA to give out inflated rating (the inflated ratings provide investment alternatives and diversification opportunities for the institutional investor). In addition, it allows some flexibility for the investor to shift risk to the CRA when the rating accuracy is reasonably doubted. However, the information cost a is needed to maintain at a sufficiently low level to incentivize the marginal investor to purchase rating information from the CRA. For managerial implications, the profit-sharing scheme can be beneficial to regulators as well as financial market participants in making investment decision and preventing unforeseen financial crises causes by inaccurate credit ratings. For the academia, the understanding of factors that influence both the CRA's and the investor's behavior are illustrated, and dynamic interaction between the investor and CRA is examined. Future studies may look into another payment scheme (e.g., risk retention) to understand its rating accuracy improvement and its influence on behavior of the investor as well as that of the CRA.

CHAPTER 4

CONCLUSION

From chapter 2 and 3, we examine effect of incentive compensation mechanisms, the incentive pay scheme and the profit sharing scheme, on the credit rating agency's decision toward issuing the rating that truthfully reflects outcome of rated securities. The rating agency's conduct of strategically issuing the inflated rating and the deflated rating is also taken into the setting of consideration. To pinpoint superiority of these mechanisms, we compare the proposed compensation mechanism with the traditional compensation mechanism, the fixed upfront fee scheme. Findings reveal that these two compensation mechanisms better encourage the rating agency to produce high rating accuracy than the fixed upfront fee scheme does.

In chapter 2, we emphasize that the incentive pay scheme and reputational loss play an important role in the credit rating agency's decision on exerting sufficiently high effort in the rating assessment process leading to rating accuracy improvement. The rating agency in this study is operating under the issuer-pay model that potentially deteriorates rating accuracy due to the conflict of interest problem. The rating agency can decide on effort level, exerted in the rating assessment process, and rating-reported strategy, either the full disclosure regime or the biased rating regime. Findings reveal that, regardless of the incentive pay scheme or the upfront fee scheme, the rating agency can strategically misreport the rating if she exerts very low effort in the rating evaluation process (the exerted effort is lower than the effort threshold). On the contrary, the rating agency always report the rating truthfully if she exerts sufficiently high effort level in the rating assessment process.

In the favorable market, if the rating agency exerts very low effort level to produce the rating, she strategically inflates the rating since she relies on information of the market condition. Otherwise, she reports the rating truthfully in the case that sufficiently high effort level is exerted. In the unfavorable market, the rating agency strategically deflates the rating because her rating decision mainly relies on information of the market condition if she puts very low effort in the rating assessment process. Contrarily, the rating agency truthfully reports the rating if she exerts sufficiently high effort level to produce the rating.

To point out which compensation scheme is better, we examine the rating agency's decision on effort exertion level and rating regime selection. The incentive pay scheme influences the rating agency to exert higher effort level to produce a credible signal and report the rating in accordance with the observed signal. The incentive pay scheme can also better induce the rating agency to adopt the full disclosure regime than the upfront fee scheme does because the rating agency incurs higher loss from the incentive pay and the reputational loss if the issued rating is inaccurate. It is clear that the incentive pay scheme is better than the upfront fee scheme. Although the rating agency has less incentive to strategically misreport the rating, there is a slim chance that the rating agency can biasedly report the rating.

In chapter 3, we investigate the profit sharing scheme impact on the rating agency's decision on rating policy selection and rating accuracy improvement. In this study, the rating agency is operating under the investor-pay model that allows the investor, particularly the institutional investor, to shop for ratings leading to biased rating issuance. The rating agency can strategically choose rating policy while the institutional or marginal investor decides on the additional information accuracy level. Additionally, the rating agency's behavior of adopting rating inflation regime and rating deflation regime is taken into consideration. Findings show that if the institutional or marginal investor purchases a rating and requests very low additional accuracy level, the rating agency tends to strategically misreport the rating in order to save on rating production cost. On the contrary, if the investor orders a rating and acquires sufficiently high additional accuracy, the rating agency tends to truthfully report the rating to avoid expected loss from misreporting. Furthermore, the investor's decision to request additional accuracy level depends on a proportion of profit shared with the rating agency. If a profit shared proportion is small, the investor has incentive to request maximum accuracy level because he bares high default risk. If a medium proportion of profit is shared with the rating agency, the investor requests sufficiently high additional accuracy level since default risk is equally shared between both parties. In the case that a large proportion of profit is allocated to the rating agency, the investor tends to acquire very low additional accuracy level because the rating agency bares high risk of default.

Different from the institutional investor, the marginal investor has two more options when making investment decision because he can decide to invest or not to invest in securities without purchasing a rating from the rating agency. Hence, the marginal investor’s decision to purchase a rating based on information cost charged by the rating agency. If the information cost charged by the rating agency is high, the investor chooses either not to invest or invest in securities without purchasing the rating. Without rating procurement, the investor’s investment decision is based on the market condition. If the market is exceptionally favorable, the investor has incentive to invest in securities. Otherwise, the investor does not invest in securities if the market is exceptionally unfavorable.

Next, we examine pros and cons of the profit sharing scheme by comparing it with the upfront fee scheme. If the rating agency always reports the rating truthfully, the upfront fee scheme generates higher expected payoff for the investor than the profit sharing scheme does. However, this case unlikely holds in practice because the evidence show that the rating agency occasionally reports biased ratings. When the rating agency implements biased ratings, the profit sharing scheme is more profitable than the upfront fee scheme is. Lastly, the investor has incentive to request maximum additional accuracy level when the profit sharing scheme is contracted.

From findings of both researches, these imply that the performance-related compensation mechanism can better align incentives of credit rating agencies with the both types of investors and better reduce biased rating issuance than the upfront fee scheme does. Future studies may investigate a compensation scheme that complies with the risk retention regulation proposed by some policymakers to align incentives of credit rating agencies with that of the investors, ensuring high rating accuracy.

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APPENDICES

APPENDIX A

PERFORMANCE-BASED MECHANISM WITH EFFORT AND REPUTATIONAL EFFECTS

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PERFORMANCE-BASED MECHANISM WITH EFFORT AND REPUTATIONAL EFFECTS

Proof of Proposition 2.1: We will show that there exists an effort threshold $\bar{e}^g$ where $\bar{e}^g(\alpha) = \frac{1}{2} - \alpha = -\bar{e}^b(\alpha)$ that characterizes the CRA's decision to report ratings. Suppose that the CRA observes $\emptyset = g$, substituting (2) and (5) into (10), the CRA's expected payoff under the upfront fee scheme is:

$$\Pi_u^g(e, \sigma_d) = f_u - \theta e^2 - \tau\left[(1-\alpha)(\frac{1}{2} - e) + (\alpha + e - \frac{1}{2})\sigma_d\right]$$

where $$\frac{\partial \Pi_u^g(e,\sigma_d)}{\partial \sigma_d} = -\tau\left[\alpha - (\frac{1}{2} - e)\right]$$

There exists an effort threshold, $\bar{e}^g(\alpha) = \frac{1}{2} - \alpha$. When $e \leq \bar{e}^g(\alpha)$, the CRA reports a low rating when observing a good signal, $\sigma_d^* = 1$, since $\frac{\partial \Pi_u^g(e,\sigma_d)}{\partial \sigma_d} \geq 0$. When $e > \bar{e}^g(\alpha)$, she reports a high rating when observing a good signal, $\sigma_d^* = 0$, since $\frac{\partial \Pi_u^g(e,\sigma_d)}{\partial \sigma_d} < 0$.

Suppose that the CRA observes, $\emptyset = b$, substituting (2) and (6) into (111), the CRA's expected payoff under the upfront fee scheme is:

$$\Pi_u^b(e, \sigma_i) = f_u - \theta e^2 - \tau\left[\alpha(\frac{1}{2} - e) + (e - \alpha + \frac{1}{2})\sigma_i\right]$$

Where $$\frac{\partial \Pi_u^b(e,\sigma_i)}{\partial \sigma_i} = -\tau\left[\frac{1}{2} + e - \alpha\right]$$

There exists an effort threshold, $\bar{e}^b(\alpha) = \alpha - \frac{1}{2}$. When $e \le \bar{e}^b(\alpha)$, the CRA reports a high rating when observing a bad signal, $\sigma_i^* = 1$, since $\frac{\partial \Pi_u^b(e,\sigma_i)}{\partial \sigma_i} \ge 0$. When $e > \bar{e}^b(\alpha)$, the CRA reports a low rating when observing a bad

signal, $\sigma_i^* = 0$, since $\frac{\partial \Pi_u^b(e,\sigma_i)}{\partial \sigma_i} < 0$.

Proof of Lemma 2.1: We will show that the CRA's optimal rating policy is either the full disclosure ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating deflation regime ($\sigma_d^* = 1$ and $\sigma_i^* = 0$) in a bad market. In contrast, the CRA's optimal rating policy is either the full disclosure ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating inflation regime ($\sigma_d^* = 0$ and $\sigma_i^* = 1$) in a good market.

We learned from Proposition 1 that $\bar{e}^g(\alpha) = \frac{1}{2} - \alpha = -\bar{e}^b(\alpha)$. If the market is bad ($\alpha \le \frac{1}{2}$), then $\bar{e}^g(\alpha) \ge 0 > \bar{e}^b(\alpha)$. When the CRA observes a good signal, she can report either a high rating ($\sigma_d^* = 0$) or a low rating ($\sigma_d^* = 1$). When the CRA observes a bad signal, she always reports a low rating ($\sigma_i^* = 0$). This implies that the optimal rating policy is either the full disclosure regime ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating deflation regime ($\sigma_d^* = 1$ and $\sigma_i^* = 0$).

If the market is good ($\frac{1}{2} < \alpha$), then $\bar{e}^b(\alpha) > 0 > \bar{e}^g(\alpha)$. When the CRA observes a bad signal, she can report either a high rating ($\sigma_i^* = 1$) or a low rating ($\sigma_i^* = 0$). When the CRA observes a good signal, she always reports rating truthfully ($\sigma_d^* = 0$). This implies that the optimal rating policy is either the full disclosure regime ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating inflation regime ($\sigma_d^* = 0$ and $\sigma_i^* = 1$).

Proof of proposition 2.2: In a bad market ($\alpha \le \frac{1}{2}$), there exists a reputational loss threshold $\bar{\tau}_u^b$ for the CRA to choose between the rating deflation regime and the full disclosure regime.

Substituting equation (2) and (9) into equation (12), the CRA's expected payoff under the upfront fee scheme is:

$$\Pi_u^{M_b}(e, \sigma_d, \sigma_i = 0) = \max_{\sigma_d, e}[f_u - \Psi(e) - \Gamma(e, \sigma_d, \sigma_i = 0)]$$

$$\Pi_u^{M_b}(e,\sigma_d,\sigma_i=0)=f_u-\theta e^2-\ \tau\ \left[\left(\frac{1}{2}-e\right)+(\alpha+e-\frac{1}{2})\sigma_d\right]$$

where $$\frac{\partial\Pi_u^{M_b}(e,\sigma_d,\sigma_i=0)}{\partial\sigma_d}=-\tau\left[\alpha-(\tfrac{1}{2}-e)\right]$$

Since the CRA's expected payoff function is linear in σ_d, the optimal rating regime for the CRA is either the full disclosure ($\sigma_d^*=0$), or the rating deflation

($\sigma_d^*=1$).

If $\sigma_d^*=0$, we have that $\Pi_u^{M_b}(e,\sigma_d^*=0,\sigma_i=0)$ is concave in e with a unique solution to the first-order condition given by $e_u^*=\frac{\tau}{2\theta}$. Accordingly, the CRA's expected payoff under the upfront fee scheme is given by

$$\Pi_u^{M_b}\left(e_u^*=\frac{\tau}{2\theta},\sigma_d^*=0,\sigma_i=0\right)=f_u+\frac{\tau^2}{4\theta}-\frac{\tau}{2}$$

If $\sigma_d^*=1$, we have that $\Pi_u^{M_b}(e,\sigma_d^*=1,\sigma_i=0)$ is independent of the CRA's effort level in information production process, $e_u^*=0$. Accordingly, the CRA's expected payoff is given by

$$\Pi_u^{M_b}(e_u^*=0,\sigma_d^*=1,\sigma_i=0)=f_u-\ \tau\ \alpha$$

Comparing $\Pi_u^{M_b}\left(e_u^*=\frac{\tau}{2\theta},\sigma_d^*=0,\sigma_i=0\right)$ with $\Pi_u^{M_b}(e_u^*=0,\sigma_d^*=1,\sigma_i=0)$,

there exists the threshold of reputational cost $\bar{\tau}_u^b\coloneqq 2\theta(1-2\alpha)$ such that it is optimal for the CRA to adopt the full disclosure regime ($\sigma_d^*=0$ and $\sigma_i^*=0$) if $\tau\geq\bar{\tau}_u^b$. It is optimal for the CRA to implement the rating inflation regime ($\sigma_d^*=1$ and $\sigma_i^*=0$) if $\tau<\bar{\tau}_u^b$.

Proof of proposition 2.3: In a good market, $\frac{1}{2}\leq\alpha$, there exists a reputational loss threshold $\bar{\tau}_u^g$ for the CRA to choose between the rating inflation regime and the full disclosure regime.

Substituting equation (2) and (9) into equation (13), the CRA's expected payoff under the upfront fee scheme is:

$$\Pi_u^{M_g}(e, \sigma_i, \sigma_d = 0) = f_u - \theta e^2 - \tau \left[\left(\frac{1}{2} - e\right) + (\frac{1}{2} + e - \alpha)\sigma_i\right]$$

where $$\frac{\partial \Pi_u^{M_g}(e, \sigma_i, \sigma_d = 0)}{\partial \sigma_i} = -\tau\left[(\frac{1}{2} + e) - \alpha\right]$$

Since the CRA's expected payoff function is linear in σ_i, the optimal rating regime for the CRA is either the full disclosure ($\sigma_i^* = 0$), or the rating inflation ($\sigma_i^* = 1$).

If $\sigma_i^* = 0$, we have that $\Pi_u^{M_g}(e, \sigma_i^* = 0, \sigma_d = 0)$ is concave in e with a unique solution to the first-order condition given by $e_u^* = \frac{\tau}{2\theta}$. Accordingly, the CRA's expected payoff under the upfront fee scheme is given by

$$\Pi_u^{M_g}\left(e_u^* = \frac{\tau}{2\theta}, \sigma_i^* = 0, \sigma_d = 0\right) = f_u + \frac{\tau^2}{4\theta} - \frac{\tau}{2}$$

If $\sigma_i^* = 1$, we have that $\Pi_u^{M_g}(e, \sigma_i^* = 1, \sigma_d = 0)$ is independent of the CRA's effort level in the rating evaluation process, $e_u^* = 0$. Accordingly, the CRA's expected payoff is given by

$$\Pi_u^{M_g}(e_u^* = 0, \sigma_i^* = 1, \sigma_d = 0) = f_u - \tau(1 - \alpha)$$

Comparing $\Pi_u^{M_g}\left(e_u^* = \frac{\tau}{2\theta}, \sigma_i^* = 0, \sigma_d = 0\right)$ with $\Pi_u^{M_g}(e_u^* = 0, \sigma_i^* = 1, \sigma_d = 0)$,

there exists the threshold of reputational cost $\bar{\tau}_u^g \coloneqq 2\theta(2\alpha - 1)$ such that it is optimal for the CRA to adopt the full disclosure regime ($\sigma_i^* = 0$ and $\sigma_d^* = 0$) if $\tau \geq \bar{\tau}_u^g$. It is optimal for the CRA to implement the rating inflation regime ($\sigma_i^* = 1$ and $\sigma_d^* = 0$) if $\tau < \bar{\tau}_u^g$.

Proof of proposition 2.4: We will show that there exists an effort threshold $\bar{e}^g$ where $\bar{e}^g(\alpha) = \frac{1}{2} - \alpha = -\bar{e}^b(\alpha)$ that characterizes the CRA's decision to report ratings.

Suppose that the CRA observes $\emptyset = g$, substituting (2), and (5) into (14), the CRA's expected payoff under the incentive pay scheme is:

$$\Pi_i^g(e,\sigma_d) = \left[\alpha\left(\frac{1}{2}+e\right) + (\frac{1}{2} - e - \alpha)\sigma_d\right] f_i - \theta e^2 - \tau\left[\left(\frac{1}{2} - e\right)(1-\alpha) - \left(\frac{1}{2} - e - \alpha\right)\sigma_d\right]$$

where $$\frac{\partial \Pi_i^g(e,\sigma_d)}{\partial \sigma_d} = (f_i + \tau)(\frac{1}{2} - e - \alpha)$$

There exists an effort threshold, $\bar{e}^g(\alpha) = \frac{1}{2} - \alpha$. When $e \leq \bar{e}^g(\alpha)$, the CRA reports a low rating when observing a good signal, $\sigma_d^* = 1$, since $\frac{\partial \Pi_i^g(e,\sigma_d)}{\partial \sigma_d} \geq 0$. When $e > \bar{e}^g(\alpha)$, she reports a high rating when observing a good signal, $\sigma_d^* = 0$, since $\frac{\partial \Pi_i^g(e,\sigma_d)}{\partial \sigma_d} < 0$.

Suppose that the CRA observes, $\emptyset = b$, substituting (2) and (6) into (15), the CRA's expected payoff under the incentive pay scheme is:

$$\Pi_i^b(e,\sigma_i) = \left[\left(\frac{1}{2}+e\right)(1-\alpha) - (\frac{1}{2} + e - \alpha)\sigma_i\right] f_i - \theta e^2$$
$$- \tau\left[\alpha\left(\frac{1}{2} - e\right) + (\frac{1}{2} + e - \alpha)\sigma_i\right]$$

Where $$\frac{\partial \Pi_i^b(e,\sigma_i)}{\partial \sigma_i} = (f_i + \tau)(\alpha - e - \frac{1}{2})$$

There exists an effort threshold, $\bar{e}^b(\alpha) = \alpha - \frac{1}{2}$. When $e \leq \bar{e}^b(\alpha)$, the CRA reports a high rating when observing a bad signal, $\sigma_i^* = 1$, since $\frac{\partial \Pi_i^b(e,\sigma_i)}{\partial \sigma_i} \geq 0$. When $e > \bar{e}^b(\alpha)$, the CRA reports a low rating when observing a bad signal, $\sigma_i^* = 0$, since $\frac{\partial \Pi_i^b(e,\sigma_i)}{\partial \sigma_i} < 0$.

Proof of Lemma 2.2: We will show that the CRA's optimal rating policy is either the full disclosure ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating deflation regime ($\sigma_d^* = 1$ and $\sigma_i^* = 0$) in a bad market. In contrast, the CRA's optimal rating policy is either the full disclosure ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating inflation regime ($\sigma_d^* = 0$ and $\sigma_i^* = 1$) in a good market.

We learned from Proposition 2.4 that $\bar{e}^g(\alpha) = \frac{1}{2} - \alpha = -\bar{e}^b(\alpha)$. If the market is bad, $\alpha \leq \frac{1}{2}$, then $\bar{e}^g(\alpha) \geq 0 > \bar{e}^b(\alpha)$. When the CRA observes a good signal, she can report either a high rating ($\sigma_d^* = 0$) or a low rating ($\sigma_d^* = 1$). When the CRA observes a bad signal, she always reports a low rating ($\sigma_i^* = 0$). This implies that the optimal rating policy is either the full disclosure regime ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating deflation regime ($\sigma_d^* = 1$ and $\sigma_i^* = 0$).

If the market is good, $\frac{1}{2} < \alpha \leq 1$, then $\bar{e}^b(\alpha) > 0 > \bar{e}^g(\alpha)$. When the CRA observes a bad signal, she can report either a high rating ($\sigma_i^* = 1$) or a low rating ($\sigma_i^* = 0$). When the CRA observes a good signal, she always reports rating truthfully ($\sigma_d^* = 0$). This implies that the optimal rating policy is either the full disclosure regime ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) or the rating inflation regime ($\sigma_d^* = 0$ and $\sigma_i^* = 1$).

Proof of proposition 2.5: In a bad market ($\alpha \leq \frac{1}{2}$), there exists a reputational loss threshold $\bar{\tau}_i^b$ for the CRA to choose between the rating deflation regime and the full disclosure regime.

Substituting equation (2), (7), and (9) into equation (16), the CRA's expected payoff under the incentive pay scheme is:

$$\Pi_i^{M_b}(e, \sigma_d, \sigma_i = 0) = \left[\left(\frac{1}{2} + e\right) - (\alpha + e - \frac{1}{2})\sigma_d\right] f_i - \theta e^2 - \tau\left[\left(\frac{1}{2} - e\right) + (\alpha + e - \frac{1}{2})\sigma_d\right]$$

where $$\frac{\partial \Pi_i^{M_b}(e, \sigma_d, \sigma_i = 0)}{\partial \sigma_d} = (f_i + \tau)(\frac{1}{2} - \alpha - e)$$

Since the CRA's expected payoff function is linear in σ_d, the optimal rating regime for the CRA is either the full disclosure ($\sigma_d^* = 0$), or the rating deflation ($\sigma_d^* = 1$).

If $\sigma_d^* = 0$, we have that $\Pi_i^{M_b}(e, \sigma_d^* = 0, \sigma_i = 0)$ is concave in e with a unique solution to the first-order condition given by $e_i^* = \frac{(f_i+\tau)}{2\theta}$. Accordingly, the CRA's expected payoff under the incentive pay scheme is given by

$$\Pi_i^{M_b}\left(e_i^* = \frac{(f_i + \tau)}{2\theta}, \sigma_d^* = 0, \sigma_i = 0\right) = \frac{(f_i + \tau)^2}{4\theta} + \frac{(f_i - \tau)}{2}$$

If $\sigma_d^* = 1$, we have that $\Pi_i^{M_b}(e, \sigma_d^* = 1, \sigma_i = 0)$ is independent of the CRA's effort level in information production process, $e_i^* = 0$. Accordingly, the CRA's expected payoff is given by

$$\Pi_i^{M_b}(e_i^* = 0, \sigma_d^* = 1, \sigma_i = 0) = (1 - \alpha)f_i - \tau\,\alpha$$

Comparing $\Pi_i^{M_b}\left(e_i^* = \frac{(f_i+\tau)}{2\theta}, \sigma_d^* = 0, \sigma_i = 0\right)$ with $\Pi_i^{M_b}(e_i^* = 0, \sigma_d^* = 1, \sigma_i = 0)$, there exists the threshold of reputational cost $\bar{\tau}_i^b \coloneqq 2\theta(1 - 2\alpha) - f_i$ such that it is optimal for the CRA to adopt the full disclosure regime ($\sigma_d^* = 0$ and $\sigma_i^* = 0$) if $\tau \geq \bar{\tau}_i^b$. It is optimal for the CRA to implement the rating inflation regime ($\sigma_d^* = 1$ and $\sigma_i^* = 0$) if $\tau < \bar{\tau}_i^b$.

Proof of proposition 2.6: In a good market ($\frac{1}{2} < \alpha$), there exists a reputational loss threshold $\bar{\tau}_i^g$ for the CRA to choose between the rating inflation regime and the full disclosure regime.

Substituting equation (2), (7), and (9) into equation (17), the CRA's expected payoff under the incentive pay scheme is:

$$\Pi_i^{M_g}(e,\sigma_i,\sigma_d=0)$$
$$=\left[\left(\frac{1}{2}+e\right)-(\frac{1}{2}+e-\alpha)\sigma_i\right]f_i-\theta e^2-\tau\left[\left(\frac{1}{2}-e\right)+(\frac{1}{2}+e-\alpha)\sigma_i\right]$$

where $$\frac{\partial\Pi_i^{M_g}(e,\sigma_i,\sigma_d=0)}{\partial\sigma_i}=-(f_i+\tau)(\frac{1}{2}+e-\alpha)$$

Since the CRA's expected payoff function is linear in σ_i, the optimal rating regime for the CRA is either the full disclosure ($\sigma_i^*=0$), or the rating inflation ($\sigma_i^*=1$).

If $\sigma_i^*=0$, we have that $\Pi_i^{M_g}(e,\sigma_i^*=0,\sigma_d=0)$ is concave in e with a unique solution to the first-order condition given by $e_i^*=\frac{(f_i+\tau)}{2\theta}$. Accordingly, the CRA's expected payoff is:

$$\Pi_i^{M_g}\left(e_i^*=\frac{(f_i+\tau)}{2\theta},\sigma_i^*=0,\sigma_d=0\right)=\frac{(f_i+\tau)^2}{4\theta}+\frac{(f_i-\tau)}{2}$$

If $\sigma_i^*=1$, we have that $\Pi_i^{M_g}(e,\sigma_i^*=1,\sigma_d=0)$ is independent of the CRA's effort level in the rating evaluation process, $e_i^*=0$. Accordingly, the CRA's expected payoff is given by

$$\Pi_i^{M_g}(e_i^*=0,\sigma_i^*=1,\sigma_d=0)=\alpha f_i-\tau(1-\alpha)$$

Comparing $\Pi_i^{M_g}\left(e_i^*=\frac{(f_i+\tau)}{2\theta},\sigma_i^*=0,\sigma_d=0\right)$ with $\Pi_i^{M_g}(e_i^*=0,\sigma_i^*=1,\sigma_d=0)$, there exists the threshold of reputational cost $\bar{\tau}_i^g\coloneqq 2\theta(2\alpha-1)-f_i$ such that it is optimal for the CRA to adopt the full disclosure regime ($\sigma_i^*=0$ and $\sigma_d^*=0$) if $\tau\geq\bar{\tau}_i^g$. It is optimal for the CRA to implement the rating inflation regime ($\sigma_i^*=1$ and $\sigma_d^*=0$) if $\tau<\bar{\tau}_i^g$.

Proof of proposition 2.7: In all market conditions, we will show that the incentive pay scheme is more efficient than the upfront fee in terms of encouraging the CRA to report ratings truthfully and putting higher effort level in the rating evaluation process, resulting in higher probability that the CRA produces accurate ratings.

1) We earlier had that $\bar{\tau}_u^b = \bar{\tau}_u^g = 2\theta(1-2\alpha)$ and $\bar{\tau}_i^b = \bar{\tau}_i^g = 2\theta(1-2\alpha) - f_i$. It is simple to see that $\bar{\tau}_u^b > \bar{\tau}_i^b$ and $\bar{\tau}_u^g > \bar{\tau}_i^g$. Accordingly, the reputational loss threshold under the upfront fee scheme is higher than that under the incentive pay scheme.

2) When $\tau > \bar{\tau}_u^\phi$ and $\tau > \bar{\tau}_i^\phi$, we had that $e_i^* = \frac{(f_i+\tau)}{2\theta} > e_u^* = \frac{\tau}{2\theta}$. Then, the CRA's optimal effort level under the incentive pay scheme is always higher than that under the upfront fee.

3) Knowing that when the CRA exerts an optimal effort level under the upfront scheme (e_u^*) and under the incentive pay scheme (e_i^*), she will report ratings truthfully ($\sigma_d^* = 0$ and $\sigma_i^* = 0$). Accordingly, the probability that the CRA produces accurate ratings under the incentive pay scheme from equation (7) is $\lambda(e_i^*, \sigma_d^* = 0, \sigma_i^* = 0) = \frac{f_i+\tau+\theta}{2\theta}$, and that under the upfront scheme from equation (10) is $\lambda(e_u^*, \sigma_d^* = 0, \sigma_i^* = 0) = \frac{\tau+\theta}{2\theta}$. It is simple to see that $\lambda(e_i^*, \sigma_d^* = 0, \sigma_i^* = 0) > \lambda(e_u^*, \sigma_d^* = 0, \sigma_i^* = 0)$.

APPENDIX B

PROFIT-SHARING MECHANISM WITH INVESTOR'S DECISIONS ON INFORMATION LEVEL AND CONTRACT TERMS

APPENDIX B

PROFIT-SHARING MECHANISM WITH INVESTOR'S DECISIONSON INFORMATION LEVEL AND CONTRACT TERMS

Proof of proposition 3.1: Knowing that an investor invests in the project only if the project gets high rating, the CRA can maximize her profit by implementing the full disclosure or bias rating regime after observing the type of each project.

Case 1: when the CRA observes $\emptyset = l$, the CRA decides whether to report high rating (bias rating) or low rating (full disclosure). The CRA's expected profit when $\emptyset = l$ is given by

$$\Pi_{CRA}^{\emptyset=l}(\mu^l(i)) = \left[\alpha_g\big(1-q(i)\big)I(\mu^l(i))x_g(R) + \alpha_b q(i)I(\mu^l(i))x_b(R)\right](1-\beta)$$

If the CRA reports low rating (Full disclosure $I(\mu^l(i)) = 0$), then her expected profit is given by

$$\Pi_{CRA}^{\emptyset=l}(F) \; = C(i) - C(i) = 0$$

Because all information production cost is covered by the investor but he will not invest in the low-rated project. If the CRA reports the high rating (Biased rating $I(\mu^l(i)) = 1$), her expected profit is given by

$$\Pi_{CRA}^{\emptyset=l}(B) = \left[\alpha_g\big(1-q(i)\big)x_g(R) + \alpha_b q(i)x_b(R)\right](1-\beta)$$

The CRA adopts the full disclosure regime if $\Pi_{CRA}^{\emptyset=l}(F) \geq \Pi_{CRA}^{\emptyset=l}(B)$:

$$0 \geq \left[\alpha_g\big(1-q(i)\big)x_g(R) + \alpha_b q(i)x_b(R)\right](1-\beta)$$

Thus, $$i \geq \bar{\iota}^l = \frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} - \alpha_m$$

The CRA adopts the bias rating regime if $\Pi_{CRA}^{\emptyset=l}(F) < \Pi_{CRA}^{\emptyset=l}(B)$:

Thus, $$i < \bar{\iota}^l = \overbrace{\frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}}^{\leq 1} - \alpha_m \text{ while } \bar{\iota}^l \leq 1 - \alpha_m$$

Hence, when the CRA observes $\emptyset = l$, he reports low rating (Full disclosure) if and only if $i \geq \bar{\iota}^{l} = \frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} - \alpha_m$. Otherwise, the CRA adopts the bias rating regime.

Case 2: when the CRA observes $\emptyset = h$, the CRA decides whether to report low rating (bias rating) or high rating (full disclosure). The CRA's expected profit when $\emptyset = h$ is given by:

$$\Pi_{CRA}^{\emptyset=h}(\mu^h(i)) = \big[\alpha_g q(i)(1 - I(\mu^h(i)))x_g(R) + \alpha_b(1 - q(i))(1 - I(\mu^h(i)))x_b(R)\big](1 - \beta)$$

If the CRA reports high rating (Full disclosure $I(\mu^h(i)) = 0$), her expected profit is given by

$$\Pi_{CRA}^{\emptyset=h}(F) = \big[\alpha_g q(i)x_g(R) + \alpha_b\big(1 - q(i)\big)x_b(R)\big](1 - \beta)$$

If the CRA reports low rating (Bias rating $I(\mu^h(i)) = 1$), her expected profit is given by

$$\Pi_{CRA}^{\emptyset=h}(B) \ = C(i) - C(i) = 0$$

Because all information production cost is covered by the investor but he will not invest in the low-rated project. The CRA adopts the full disclosure regime only if $\Pi_{CRA}^{\emptyset=h}(F) \geq \Pi_{CRA}^{\emptyset=h}(B)$ so that:

$$\big[\alpha_g q(i)x_g(R) + \alpha_b(1 - q(i))x_b(R)\big](1 - \beta) \geq 0$$

Thus, $$i \geq \bar{\iota}^{h} = -\underbrace{\frac{\alpha_b x_b(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}}_{\leq 1} - \alpha_m \text{ while } \bar{\iota}^{h} \leq 1 - \alpha_m$$

The CRA adopts the bias rating regime only if $\Pi_{CRA}^{\emptyset=h}(F) < \Pi_{CRA}^{\emptyset=h}(B)$:

Thus, $$i < \bar{\iota}^{h} = \overbrace{\frac{-\alpha_b x_b(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}}^{\leq 1} - \alpha_m \text{ while } \bar{\iota}^{h} \leq 1 - \alpha_m$$

Hence, when the CRA observes $\emptyset = h$, she reports high rating (Full disclosure) if and only if $i \geq \bar{\imath}^h = -\left[\frac{\alpha_b x_b(R)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} + \alpha_m\right]$. Otherwise, the CRA implements the bias rating regime.

From $\alpha_m = \max\{\alpha_g, \alpha_b\}$ knowing that $\alpha_m \geq \frac{1}{2}$, we lastly consider two market conditions: (1) bad market, and (2) good market.

1) Bad market denoted by $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$ includes both extremely unfavorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, and favorable market, $-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$.

Find position of $\bar{\imath}^l$:

$$\bar{\imath}^l = \frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} - \alpha_m \lesseqgtr 0$$

$$\alpha_g x_g(R) - \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] \lesseqgtr 0 \qquad \text{----(A1)}$$

When substitute $\alpha_m = \frac{1}{2}$ into (A1), we get $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$ meaning that $\bar{\imath}^l < 0$.

When substitute $\alpha_m = 1$ into (A1), we get $\alpha_b x_b(R) < 0$ implying that $\bar{\imath}^l < 0$.

Find position of $\bar{\imath}^h$:

$$\bar{\imath}^h - \bar{\imath}^l = \frac{-\alpha_b x_b(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} - \alpha_m - \frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} + \alpha_m$$

$$\bar{\imath}^h - \bar{\imath}^l \coloneqq -\frac{[\alpha_g x_g(R)+\alpha_b x_b(R)]}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} > 0 \text{ so that } \bar{\imath}^h > \bar{\imath}^l$$

From $\bar{\imath}^l < 0$ and $\bar{\imath}^h > \bar{\imath}^l$, we have that $\bar{\imath}^l < \min\{0, \bar{\imath}^h\}$ in bad market.

2) Favorable market denoted by $\alpha_g x_g(R) + \alpha_b x_b(R) \geq 0$ includes both favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, and extremely favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \geq \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$.

Find position of $\bar{\imath}^h$:

$$\bar{\iota}^h = \frac{-\alpha_b x_b(R)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} - \alpha_m \lesseqgtr 0$$

$-\alpha_b x_b(R) - \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right] \lesseqgtr 0$ ----(A2)

When substitute $\alpha_m = \frac{1}{2}$ into (A2), thus $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R)$ meaning that $\bar{\iota}^h \leq 0$.

When substitute $\alpha_m = 1$ into (A2), thus $-\alpha_g x_g(R) \leq 0$ implying that $\bar{\iota}^h \leq 0$.

Find position of $\bar{\iota}^l$:

$$\bar{\iota}^l - \bar{\iota}^h = \frac{\alpha_g x_g(R)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} - \alpha_m + \frac{\alpha_b x_b(R)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} + \alpha_m$$

$$\bar{\iota}^l - \bar{\iota}^h \coloneqq \frac{\alpha_g x_g(R) + \alpha_b x_b(R)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} \geq 0 \text{ so that } \bar{\iota}^l \geq \bar{\iota}^h$$

From $\bar{\iota}^h \leq 0$ and $\bar{\iota}^l \geq \bar{\iota}^h$, we have that $\bar{\iota}^h \leq \min\{0, \bar{\iota}^l\}$ in favorable market.

Accordingly, if market is bad, $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$, then $\bar{\iota}^l < 0$ meaning that the CRA always assigns l-rating when she observes l-signals (Full disclosure). On the other hands, the CRA chooses between the full disclosure and biased rating depending on additional information accuracy level requested by the investor when she observes h-signals since $\bar{\iota}^h \geq 0$.

If market is good, $\alpha_g x_g(R) + \alpha_b x_b(R) \geq 0$, then $\bar{\iota}^h \leq 0$ causing the CRA to always report h-rating (Full disclosure) when she observes h-signals. On the other hands, the CRA adopts either the full disclosure or biased rating regime conditional on additional information accuracy level requested by the investor when she observes l-signals because $\bar{\iota}^h \geq 0$.

Since both $\bar{\iota}^l$ and $\bar{\iota}^h$ cannot be greater than zero at the same time so that it is impossible for the CRA to simultaneously adopt bias rating regime for both high and low signals.

Proof of proposition 3.2: Knowing that the investor invests only in h-rated project, we consider 4 market conditions: (i) extremely unfavorable market, (ii) unfavorable market, (iii) favorable market, and (iv) extremely favorable market, respectively. The expected profit of the investor is given by

$$\Pi_{INV}^{\tilde{r}=h}(i) = \max_i \Big[[\pi_{ah}(i)[1 - I(\mu^h(i))]x_n(R) + \pi_{ah}(i)I(\mu^h(i))x_n(R)]\beta - C(i)\Big]$$

$$\Pi_{INV}^{\tilde{r}=h}(i) = \max_i \Bigg[\Big[\alpha_g x_g(R)\Big[q(i)\Big(1 - I(\mu^h(i))\Big) + \big(1 - q(i)\big)I(\mu^l(i))\Big] +$$

$$\alpha_b x_b(R)\Big[\big(1 - q(i)\big)\Big(1 - I(\mu^h(i))\Big) + q(i)I(\mu^l(i))\Big]\Big]\beta - C(i)\Bigg] \qquad \text{----(A3)}$$

Where $q(i) = i + \alpha_m$

1) Extremely unfavorable market,

$\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]$.

We now assume that $\bar{\imath}^h \geq 0 > \bar{\imath}^l$ and have from proposition 1 that $\bar{\imath}^l < min\{0, \bar{\imath}^h\}$. To determine the optimal level of additional information accuracy i^*, we consider two intervals: 1.1) $0 \leq i < \bar{\imath}^h$ and 1.2) $\bar{\imath}^h \leq i \leq 1 - \alpha_m$.

1.1) For $0 \leq i < \bar{\imath}^h$. The CRA observes l-signal, $\emptyset = l$, and reports low rating since $\bar{\imath}^l < 0$ resulting in $I(\mu^l(i)) = 0 = I(F)$. In contrast, the CRA observes h-signal, $\emptyset = h$, but reports low rating since $i < \bar{\imath}^h$ resulting in $I(\mu^h(i)) = 1 = I(B)$. Hence, the investor's expected profit from (A3) when the CRA adopts rating deflation regime is given by

$$\Pi_{INV}^{\tilde{r}=h}(i) = -C(i) \qquad \text{----(A4)}$$

$$\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = -C'(i) < 0$$

Therefore, the investor's expected profit is monotonically decreasing in i so that the optimal additional information accuracy level i^* is zero, which generates $\Pi_{INV}^*(i^* = 0) = -C(0) = -a(0)^2 = 0$.

1.2) For $\bar{\imath}^h \leq i \leq 1 - \alpha_m$. The CRA observes l-signal, $\emptyset = l$, and reports a low rating since $i > 0 > \bar{\imath}^l$ resulting in $I(\mu^l(i)) = 0 = I(F)$. The CRA observes h-signal, $\emptyset = h$, and reports high rating since $i > \bar{\imath}^h$ resulting in $I(\mu^h(i)) = 0 = I(F)$. Hence, the CRA implements full disclosure and the investor's expected profit based on investing in high-rated projects only from (A3) is given by

$$\Pi_{INV}^{\tilde{r}=h}(i) = \beta\pi_{ah}(i)x_n(R) - C(i)$$

$$\Pi_{INV}^{\tilde{r}=h} = \max_i \Big[\big[[\alpha_g x_g(R) - \alpha_b x_b(R)][i + \alpha_m] + \alpha_b x_b(R)\big]\beta - C(i)\Big] \text{----(A5)}$$

where $C(i) = ai^2$

We can derive the first and second derivatives of $\Pi_{INV}^{\tilde{r}=h}$ with respect to the decision variable, i, as follows.

First differentiation: $\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = \underbrace{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]\beta}_{>0} - \underbrace{C'(i)}_{>0}$

Second differentiation: $\frac{d^2\Pi_{INV}^{\tilde{r}=h}}{di^2} = -C''(i) < 0$

Then, the investor's profit $\Pi_{INV}^{\tilde{r}=h}$ is concave in additional information accuracy level i.

First order condition: $\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = \big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]\beta - 2ai = 0$

Then, the optimal additional information accuracy is $i^* = i_0 := \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$.

In extremely unfavorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]$, we perform continuity test to check if the investor's expected profit function is continuous at the point of $\bar{\iota}^h$, a conjunction point, by

$$\Pi_{INV}^{\tilde{r}=h}(i = \bar{\iota}^h) = \Pi_{INV}^{\tilde{r}=h}(i = \bar{\iota}^h) = -a\bar{\iota}^{h^2}$$

Therefore, the investor's expected profit function is continuous at the point of $\bar{\iota}^h$.

To check if the optimal additional information accuracy level i_0 is a boundary or interior solution, we consider three cases: (i) $i_0 \leq \bar{\iota}^h$, (ii) $\bar{\iota}^h < i_0 < 1 - \alpha_m$, and (iii) $i_0 \geq 1 - \alpha_m$.

1.2.1) If $i_0 \leq \bar{\iota}^h$, we can further derive that $\beta \leq \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

The investor has incentive to request $i^* = \bar{\iota}^h$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = \bar{\iota}^h$ into equation (A5),

$$\Pi_{INV}^{*}(i^{*}=\bar{\imath}^{h})=\left[[\alpha_g x_g(R)-\alpha_b x_b(R)][\bar{\imath}^{h}+\alpha_m]+\alpha_b x_b(R)\right]\beta-a(\bar{\imath}^{h})^2$$

$$\Pi_{INV}^{*}(i^{*}=\bar{\imath}^{h})=-a\bar{\imath}^{h^2}=-a\left[\frac{\alpha_b x_b(R)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}+\alpha_m\right]^2$$

Accordingly, if $i_0 \le \bar{\imath}^{h}$, the investor's expected profit is $\Pi_{INV}^{*}(i^{*}=\bar{\imath}^{h})=-a\bar{\imath}^{h^2}=-a\left[\frac{\alpha_b x_b(R)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}+\alpha_m\right]^2$, which is not optimal in equilibrium.

To examine the investor's incentive of requesting the optimal additional accuracy level i^{*} in extremely unfavorable market if $i_0 \le \bar{\imath}^{h}$ equivalent to $\beta \le \frac{2a\bar{\imath}^{h}}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$, the investor's expected profits under the full disclosure and biased rating regime are compared:

$$\Pi_{INV}^{*}(i^{*}=0)-\Pi_{INV}^{*}(i^{*}=\bar{\imath}^{h})=0-[-a\bar{\imath}^{h^2}]$$

Knowing that $a>0$ and $\bar{\imath}^{h}\ge 0$, $a\bar{\imath}^{h^2}>0$

Hence, the investor always requests $i^{*}=0$ yielding $\Pi_{INV}^{*}(i^{*}=0)=0$ in an interval of $i_0 \le \bar{\imath}^{h}$ equivalent to $\beta \le \frac{2a\bar{\imath}^{h}}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$, since $\Pi_{INV}^{*}(i^{*}=0) > \Pi_{INV}^{*}(i^{*}=\bar{\imath}^{h})$.

1.2.2) If $\bar{\imath}^{h} < i_0 < 1-\alpha_m$, we can further derive that $\frac{2a\bar{\imath}^{h}}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} < \beta < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

The investor has incentive to request $i^{*}=i_0$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset=\{l,h\}$ she observes:

Substitute $i^{*}=i_0$ into equation (A5),

$$\Pi_{INV}^{*}(i^{*}=i_0)=\left[\alpha_g x_g(R)[i_0+\alpha_m]+\alpha_b x_b(R)\left[\left(1-(i_0+\alpha_m)\right)\right]\right]\beta-C(i_0)$$

Accordingly, if $\bar{\imath}^{h} < i_0 < 1-\alpha_m$, the investor's expected profit is $\Pi_{INV}^{*}(i^{*}=i_0)=\left[[\alpha_g x_g(R)-\alpha_b x_b(R)][i_0+\alpha_m]+\alpha_b x_b(R)\right]\beta-C(i_0)$.

To examine the investor's incentive of requesting the optimal additional accuracy level i^* in extremely unfavorable market if $\frac{2a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} < \beta < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$, the investor's expected profits under the full disclosure and biased rating regime are compared:

$$\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0)$$
$$= \Big[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\Big]\beta - C(i_0)$$

Substitute $i_0 = \frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]\beta}{2a}$ in $\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0)$, we have that

$$\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0) = \begin{cases} > 0 \; if \; \beta > \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} \\ \leq 0 \; if \; \beta \leq \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} \end{cases}.$$

Let $\underline{\beta} = \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$ and $\bar{\beta} = \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$. To ensure that $\underline{\beta}$ is in an interval of $\frac{2a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} < \beta < \bar{\beta}$, $\underline{\beta}$ must satisfy: (i) $\underline{\beta} > \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$ and (ii) $\underline{\beta} < \bar{\beta}$.

(i) To check if $\underline{\beta} > \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$:

$$\frac{4a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} > \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$$

Then, we have that $\underline{\beta} > \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

(ii) To check if $\underline{\beta} < \bar{\beta}$:

$$\frac{4a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} \gtreqless \frac{2a(1 - \alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$$

Substitute $\bar{\iota}^h = \frac{-\alpha_b x_b(R)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} - \alpha_m$ in,

$$-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] \geq [\alpha_g x_g(R) + \alpha_b x_b(R)]$$

Then, we have that $\underline{\beta} \coloneqq \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} \geq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

Accordingly, $\underline{\beta}$ is not in an interval of $\bar{\iota}^h < i_0 < 1 - \alpha_m$. Thus, the investor always requests $i^* = 0$ if $\frac{2a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} < \beta < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$ since $\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0) < 0$.

1.2.3) If $i_0 \geq 1-\alpha_m$, we can further derive that $\beta \geq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

The investor has incentive to request $i^* = 1-\alpha_m$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = 1-\alpha_m$ into equation (A5),

$$\Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2$$

Accordingly, if $i_0 \geq 1-\alpha_m$, the investor's expected profit is $\Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m)$.

To examine the investor's incentive of requesting the optimal additional accuracy level i^* in extremely unfavorable market if $\beta \geq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$, the investor's expected profits under the full disclosure and biased rating regime are compared:

$$\Pi_{INV}^*(i^* = 1-\alpha_m) - \Pi_{INV}^*(i^* = 0) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2$$

Thus, we have that $\Pi_{INV}^*(1-\alpha_m) - \Pi_{INV}^*(i^* = 0) = \begin{cases} > 0 \; if \; \beta > \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \\ \leq 0 \; if \; \beta \leq \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \end{cases}$.

Let $\bar{\bar{\beta}} = \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$. We need to check if $\bar{\bar{\beta}}$ is in an interval of $\beta \geq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$ by

$$-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] \geq [\alpha_g x_g(R) + \alpha_b x_b(R)]$$

Then, we have that $\beta \geq \bar{\bar{\beta}}$. Accordingly, the investor acquires $i^* = \begin{cases} > 0 \; if \; \beta > \bar{\bar{\beta}} \\ \leq 0 \; if \; \beta \leq \bar{\bar{\beta}} \end{cases}$ since $\Pi_{INV}^*(i^* = i_0) - \Pi_{INV}^*(i^* = 0) = \begin{cases} > 0 \; if \; \beta > \bar{\bar{\beta}} \\ \leq 0 \; if \; \beta \leq \bar{\bar{\beta}} \end{cases}$.

Lastly in extremely unfavorable market,

$\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, we now assume $\bar{\iota}^h < 0$, and learned from Proposition 1 that $\bar{\iota}^l < min\{0, \bar{\iota}^h\}$. When $\bar{\iota}^h < \bar{\iota}^l < 0$, any level of additional information accuracy i the investor acquires can induce the CRA to adopt the full disclosure regime. To determine the optimal level of additional information accuracy i^*, we only consider one interval of $0 \leq i \leq 1-\alpha_m$.

Hence, the CRA implements full disclosure and the investor's expected profit based on investing in high-rated projects only from (A5) is given by

$$\Pi_{INV}^{\tilde{r}=h} = \max_{i}\left[\left[[\alpha_g x_g(R) - \alpha_b x_b(R)][i + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i)\right]$$

where $C(i) = ai^2$

We can derive the first and second derivatives of $\Pi_{INV}^{\tilde{r}=h}$ with respect to the decision variable, i, as follows.

First differentiation: $$\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = \underbrace{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\beta}_{>0} - \underbrace{C'(i)}_{>0}$$

Second differentiation: $$\frac{d^2\Pi_{INV}^{\tilde{r}=h}}{di^2} = -C''(i) < 0$$

Then, the investor's profit $\Pi_{INV}^{\tilde{r}=h}$ is concave in additional information accuracy level i.

First order condition: $$\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\beta - 2ai = 0$$

Then, the optimal additional information accuracy is $i^* = i_0 := \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$.

To check if the optimal additional information accuracy level i_0 is a boundary or interior solution, we consider three cases: (i) $i_0 \leq 0$, (ii) $0 < i_0 < 1 - \alpha_m$, and (iii) $i_0 \geq 1 - \alpha_m$.

1) If $i_0 \leq 0$, we can further derive that $\beta \leq 0$.

Knowing that by definition $0 \leq \beta \leq 1$ and $i_0 \geq 0$, it is possible only if $i_0 = 0$ and $\beta = 0$. In this case, the issuer has incentive to choose $i^* = i_0 = 0$ to maximize his expected profit while the CRA adopts full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = 0$ into equation (A5),

$$\Pi_{INV}^{*}(i^* = 0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m\beta + \alpha_b x_b(R)\beta$$

Accordingly, the investor's expected profit is $\Pi_{INV}^{*}(i^* = 0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m\beta + \alpha_b x_b(R)\beta$ if $\beta \leq 0$.

2) If $0 < i_0 < 1 - \alpha_m$, we can further derive that $0 < \beta < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

The investor has incentive to request $i^* = i_0$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = i_0$ into equation (A5),

$$\Pi_{INV}^{*}(i^* = i_0) = \left[\alpha_g x_g(R)[i_0 + \alpha_m] + \alpha_b x_b(R)\left[\left(1 - (i_0 + \alpha_m)\right)\right]\right]\beta - C(i_0)$$

Accordingly, the investor's expected profit is $\Pi_{INV}^{*}(i^{*}=i_{0})=\big[[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)][i_{0}+\alpha_{m}]+\alpha_{b}x_{b}(R)\big]\beta-C(i_{0})$ if $0<\beta<\frac{2a(1-\alpha_{m})}{[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)]}$.

3) If $i_{0}\geq 1-\alpha_{m}$, we can further derive that $\beta\geq\frac{2a(1-\alpha_{m})}{[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)]}$.

The investor has incentive to request $i^{*}=1-\alpha_{m}$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset=\{l,h\}$ she observes:

Substitute $i^{*}=1-\alpha_{m}$ into equation (A5),

$$\Pi_{INV}^{*}(i^{*}=1-\alpha_{m})=\alpha_{g}x_{g}(R)\beta-a(1-\alpha_{m})^{2}$$

Accordingly, the investor's expected profit is $\Pi_{INV}^{*}(i^{*}=1-\alpha_{m})=\alpha_{g}x_{g}(R)\beta-C(1-\alpha_{m})$ if $\beta\geq\frac{2a(1-\alpha_{m})}{[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)]}$.

If $\bar{\iota}^{h}\geq 0>\bar{\iota}^{l}$, the CRA can either deflate rating or truthfully report rating.

$$\Pi_{INV}^{\tilde{r}=h}(i^{*})=\begin{cases}\Pi_{INV}^{*}(i^{*}=0)=0 & if\ \beta\leq\bar{\bar{\beta}}:=\dfrac{a(1-\alpha_{m})^{2}}{\alpha_{g}x_{g}(R)}\\ \Pi_{INV}^{*}(i^{*}=1-\alpha_{m})=\alpha_{g}x_{g}(R)\beta-a(1-\alpha_{m})^{2} & if\ if\ \beta\leq\bar{\bar{\beta}}:=\dfrac{a(1-\alpha_{m})^{2}}{\alpha_{g}x_{g}(R)}\end{cases}$$

If $0>\bar{\iota}^{h}>\bar{\iota}^{l}$, the CRA always adopts full disclosure regime.

$$\Pi_{INV}^{\tilde{r}=h}(i^{*})=\begin{cases}\Pi_{INV}^{*}(i^{*}=0)=\big[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\big]\alpha_{m}\beta+\alpha_{b}x_{b}(R)\beta & if\ \beta\leq\underline{\beta}\\ \Pi_{INV}^{*}(i^{*}=i_{0})=\Big[\big[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\big][i_{0}+\alpha_{m}]+\alpha_{b}x_{b}(R)\Big]\beta-C(i_{0}) & if\ \underline{\beta}<\beta<\bar{\beta}\\ \Pi_{INV}^{*}(i^{*}=1-\alpha_{m})=\alpha_{g}x_{g}(R)\beta-C(1-\alpha_{m}) & if\ \beta\geq\bar{\beta}\end{cases}$$

1) Unfavorable market, $-\alpha_{m}\big[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\big]<\alpha_{g}x_{g}(R)+\alpha_{b}x_{b}(R)<0$. We now assume that $\bar{\iota}^{h}\geq 0>\bar{\iota}^{l}$ and have from proposition 1 that $\bar{\iota}^{l}<min\{0,\bar{\iota}^{h}\}$. To determine the optimal level of additional information accuracy i^{*}, we consider two intervals: 1.1) $0\leq i<\bar{\iota}^{h}$ and 1.2) $\bar{\iota}^{h}\leq i\leq 1-\ \alpha_{m}$.

2.1) <u>For $0\leq i<\bar{\iota}^{h}$</u>. The CRA observes l-signal, $\emptyset=l$, and reports low rating since $\bar{\iota}^{l}<0$ resulting in $I(\mu^{l}(i))=0=I(F)$. In contrast, the CRA observes h-signal, $\emptyset=h$, but reports low rating since $i<\bar{\iota}^{h}$ resulting in $I(\mu^{h}(i))=$

$1 = I(B)$. Hence, the investor's expected profit from (A3) when the CRA adopts rating deflation regime is given by

$$\Pi_{INV}^{\tilde{r}=h}(i) = -C(i) \qquad \text{----(A4)}$$

$$\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = -C'(i) < 0$$

Therefore, the investor's expected profit is monotonically decreasing in i so that the optimal additional information accuracy level i^* is zero, which generates $\Pi_{INV}^*(i^* = 0) = -C(0) = -a(0)^2 = 0$.

2.2) For $\bar{\iota}^h \leq i \leq 1 - \alpha_m$. The CRA observes l-signal, $\emptyset = l$, and reports a low rating since $i > 0 > \bar{\iota}^l$ resulting in $I(\mu^l(i)) = 0 = I(F)$. The CRA observes h-signal, $\emptyset = h$, and reports high rating since $i \geq \bar{\iota}^h$ resulting in $I(\mu^h(i)) = 0 = I(F)$. Hence, the CRA implements full disclosure and the investor's expected profit based on investing in high-rated projects only from (A3) is given by

$$\Pi_{INV}^{\tilde{r}=h}(i) = \beta\pi_{ah}(i)x_n(R) - C(i)$$

$$\Pi_{INV}^{\tilde{r}=h} = \max_i \Big[\big[[\alpha_g x_g(R) - \alpha_b x_b(R)][i + \alpha_m] + \alpha_b x_b(R)\big]\beta - C(i)\Big] \qquad \text{(A5)}$$

where $C(i) = ai^2$, we can derive the first and second derivatives of $\Pi_{INV}^{\tilde{r}=h}$ with respect to the decision variable, i, as follows.

First differentiation: $\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = \underbrace{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]\beta}_{>0} - \underbrace{C'(i)}_{>0}$

Second differentiation: $\frac{d^2\Pi_{INV}^{\tilde{r}=h}}{di^2} = -C''(i) < 0$

Then, the investor's profit $\Pi_{INV}^{\tilde{r}=h}$ is concave in additional information accuracy level i.

First order condition: $\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = \big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]\beta - 2ai = 0$

Then, the optimal additional information accuracy is $i^* = i_0 := \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$.

In unfavorable market, $-\alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$, we perform continuity test to check if the investor's expected profit function is continuous at the point of $\bar{\iota}^h$, a conjunction point, by

$$\Pi_{INV}^*(i^* = 0)\big|_{i^* = \bar{\iota}^h} = \Pi_{INV}^*(i^* = \bar{\iota}^h) = -a{\bar{\iota}^h}^2$$

Therefore, the investor's expected profit function is continuous at the point of $\bar{\iota}^h$.

To check if the optimal additional information accuracy level i_0 is a boundary or interior solution, we consider three cases: (i) $i_0 \le \bar{\iota}^h$, (ii) $\bar{\iota}^h < i_0 < 1 - \alpha_m$, and (iii) $i_0 \ge 1 - \alpha_m$.

2.2.1) If $i_0 \le \bar{\iota}^h$, we can further derive that $\beta \le \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

The investor has incentive to request $i^* = \bar{\iota}^h$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = \bar{\iota}^h$ into equation (A5),

$$\Pi_{INV}^*(i^* = \bar{\iota}^h) = -a\bar{\iota}^{h^2} = -a\left[\frac{\alpha_b x_b(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} + \alpha_m\right]^2$$

Accordingly, if $i_0 \le \bar{\iota}^h$, the investor's expected profit is $\Pi_{INV}^*(i^* = \bar{\iota}^h) = -a\bar{\iota}^{h^2} = -a\left[\frac{\alpha_b x_b(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} + \alpha_m\right]^2$, which is not optimal in equilibrium.

To examine the investor's incentive of requesting the optimal additional accuracy level i^* in unfavorable market if $\beta \le \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$, the investor's expected profits under the full disclosure and biased rating regime are compared:

$$\Pi_{INV}^*(i^* = 0) - \Pi_{INV}^*(i^* = \bar{\iota}^h) = 0 - [-a\bar{\iota}^{h^2}]$$

Knowing that $a > 0$ and $\bar{\iota}^h \ge 0$, $\quad a\bar{\iota}^{h^2} > 0$

Hence, the investor always requests $i^* = 0$ yielding $\Pi_{INV}^*(i^* = 0) = 0$ in an interval of $i_0 \le \bar{\iota}^h$ equivalent to $\beta \le \frac{2a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$, since $\Pi_{INV}^*(i^* = 0) > \Pi_{INV}^*(i^* = \bar{\iota}^h)$.

2.2.2) If $\bar{\iota}^h < i_0 < 1 - \alpha_m$, we can further derive that $\frac{2a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} < \beta < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

The investor has incentive to request $i^* = i_0$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = i_0$ into equation (A5),

$$\Pi_{INV}^*(i^* = i_0) = \left[\alpha_g x_g(R)[i_0 + \alpha_m] + \alpha_b x_b(R)\left[\left(1 - (i_0 + \alpha_m)\right)\right]\right]\beta - C(i_0)$$

Accordingly, if $\bar{\imath}^h < i_0 < 1 - \alpha_m$, the investor's expected profit is $\Pi^*_{INV}(i^* = i_0) = \big[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\big]\beta - C(i_0)$.

To examine the investor's incentive of requesting the optimal additional accuracy level i^* in unfavorable market if $\frac{2a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} < \beta < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$, the investor's expected profits under the full disclosure and biased rating regime are compared. It follows that

$$\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0) = \begin{cases} > 0 \ if \ \beta > \frac{4a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} \\ \leq 0 \ if \ \beta \leq \frac{4a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} \end{cases}.$$

Let $\underline{\beta} = \frac{4a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$ and $\bar{\beta} = \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$. To ensure that $\underline{\beta}$ is in an interval of $\frac{2a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} < \beta < \bar{\beta}$, $\underline{\beta}$ must satisfy: (i) $\underline{\beta} > \frac{2a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$ and (ii) $\underline{\beta} < \bar{\beta}$.

(i) To check if $\underline{\beta} > \frac{2a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$:

$$\frac{4a\bar{\imath}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} > \frac{2a\bar{\imath}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$$

Then, we have that $\underline{\beta} > \frac{2a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

(iii) To check if $\underline{\beta} < \bar{\beta}$ where $\bar{\imath}^h = \frac{-\alpha_b x_b(R)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} - \alpha_m$:

$$\frac{4a\bar{\imath}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} \gtrless \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$$

Then, we have that $\underline{\beta} := \frac{4a\bar{\imath}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

Accordingly, $\underline{\beta}$ is in an interval of $\bar{\imath}^h < i_0 < 1 - \alpha_m$ .Thus, the investor has incentive to acquire $i^* = \begin{cases} 0 \quad if \ \beta \leq \underline{\beta} \\ i_0 \ if \ \underline{\beta} < \beta < \bar{\beta} \end{cases}$ since $\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0) = \begin{cases} \leq 0 \quad if \ \beta \leq \underline{\beta} \\ > 0 \ if \ \underline{\beta} < \beta < \bar{\beta} \end{cases}$.

2.2.3) If $i_0 \geq 1 - \alpha_m$, we can further derive that $\beta \geq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

The investor has incentive to request $i^* = 1 - \alpha_m$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = 1 - \alpha_m$ into equation (A5),

$$\Pi^*_{INV}(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - a(1 - \alpha_m)^2$$

Accordingly, if $i_0 \geq 1 - \alpha_m$, the investor's expected profit is $\Pi^*_{INV}(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m)$.

To examine the investor's incentive of requesting the optimal additional accuracy level i^* in unfavorable market if $\beta \geq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$, the investor's expected profits under the full disclosure and biased rating regime are compared:

$$\Pi^*_{INV}(i^* = 1 - \alpha_m) - \Pi^*_{INV}(i^* = 0) = \alpha_g x_g(R)\beta - a(1 - \alpha_m)^2$$

Thus, we have that $\Pi^*_{INV}(1 - \alpha_m) - \Pi^*_{INV}(i^* = 0) = \begin{cases} > 0 \; if \; \beta > \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \\ \leq 0 \; if \; \beta \leq \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \end{cases}$.

Let $\bar{\bar{\beta}} = \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$. We need to check if $\bar{\bar{\beta}}$ is in an interval of $\beta \geq \bar{\beta} := \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ by $\frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \lesseqgtr \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

Then, we have that $\bar{\bar{\beta}} \leq \bar{\beta}$ and the investor has incentive to request $i^* = 1 - \alpha_m$ if $\beta \geq \overline{\beta}$ since $\Pi^*_{INV}(i^* = 1 - \alpha_m) - \Pi^*_{INV}(i^* = 0) > 0$.

Lastly in unfavorable market, $-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$, we now assume $\bar{\iota}^h < 0$, and learned from Proposition 1 that $\bar{\iota}^l < min\{0, \bar{\iota}^h\}$. When $\bar{\iota}^h < \bar{\iota}^l < 0$, any level of additional information accuracy i the investor acquires can induce the CRA to adopt the full disclosure regime. To characterize the optimal level of additional information accuracy i^*, we only consider one interval of $0 \leq i \leq 1 - \alpha_m$.

Hence, the same result as previously proven in extremely unfavorable market holds.

If $\bar{\iota}^h \geq 0 > \bar{\iota}^l$, the CRA can either deflate rating or truthfully report rating.

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0) = 0 & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^*=i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

If $0 > \bar{\imath}^h > \bar{\imath}^l$, the CRA always adopts full disclosure regime.

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m\beta + \alpha_b x_b(R)\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^*=i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

2) Favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]$.

We now assume that $\bar{\imath}^l \geq 0$ and have from proposition 1 that $\bar{\imath}^h \leq min\{0, \bar{\imath}^l\}$. To determine the optimal level of additional information accuracy i^*, we consider two intervals: 1) $0 \leq i < \bar{\imath}^l$ and 2) $\bar{\imath}^l \leq i \leq 1 - \alpha_m$.

3.1) For $0 \leq i < \bar{\imath}^l$. The CRA observes h-signal, $\emptyset = h$, and reports high rating since $\bar{\imath}^h < 0$ resulting in $I(\mu^h(i)) = 0 = I(F)$. In contrast, the CRA observes l-signal, $\emptyset = l$, but reports high rating since $i < \bar{\imath}^l$ resulting in $I(\mu^l(i)) = 1 = I(B)$. Hence, the CRA adopts rating inflation regime and the investor's expected profit from (A3) is given by

$$\Pi_{INV}^{\tilde{r}=h} = \max_i \left[\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta - C(i)\right]$$

$$\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = -C'(i) < 0$$

Thus, the investor's profit is monotonically decreasing in i for $0 \leq i \leq \bar{\imath}^l$. The optimal additional information accuracy level i^* requested by the investor is the minimum level ($i^* = 0$) which generates $\Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta$.

3.2) For $\bar{\imath}^l \leq i \leq 1 - \alpha_m$. The CRA observes h-signal, $\emptyset = h$, and reports high rating since $i > 0 > \bar{\imath}^h$ resulting in $I(\mu^h(i)) = 0 = I(F)$. The CRA observes l-signal, $\emptyset = l$, and reports low rating since $i \geq \bar{\imath}^l$ resulting in $I(\mu^l(i)) = 0 = I(F)$. Hence, the CRA implements full disclosure and the investor's expected profit is given by

$$\Pi_{INV}^{\tilde{r}=h} = \max_i \left[\left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i)\right] \quad \text{----(A5)}$$

where $C(i) = ai^2$

We can derive the first and second derivatives of $\Pi_{INV}^{\tilde{r}=h}$ with respect to the decision variable, i, as follows.

First differentiation: $\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = \underbrace{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}_{>0} - \underbrace{C'(i)}_{>0}$

Second differentiation: $\frac{d^2\Pi_{INV}^{\tilde{r}=h}}{di^2} = -C''(i) < 0$

Then, the investor's profit $\Pi_{INV}^{\tilde{r}=h}$ is concave in additional information accuracy level i.

First order condition: $\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = [\alpha_g x_g(R) - \alpha_b x_b(R)]\beta - 2ai = 0$

Then, the optimal additional information accuracy is $i^* = i_0 := \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$.

In favorable market, $0 \le \alpha_g x_g(R) + \alpha_b x_b(R) \le \alpha_m[\alpha_g x_g(R) + \alpha_b x_b(R)]$, we perform continuity test to check if the investor's expected profit function is continuous at the point of $\bar{\iota}^l$, a conjunction point, by

$$\Pi_{INV}^*(i^* = 0)|_{i^*=\bar{\iota}^l} = \Pi_{INV}^*(i^* = \bar{\iota}^l) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta - C(\bar{\iota}^l)$$

Therefore, the investor's expected profit function is continuous at the point of $\bar{\iota}^l$.

To check if the optimal additional information accuracy level i_0 is a boundary or interior solution, we consider three cases: (i) $i_0 \le \bar{\iota}^l$, (ii) $\bar{\iota}^l < i_0 < 1 - \alpha_m$, and (iii) $i_0 \ge 1 - \alpha_m$.

3.2.1) If $i_0 \le \bar{\iota}^l$, we can further derive that $\beta \le \frac{2a\bar{\iota}^l}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

The investor has incentive to request $i^* = \bar{\iota}^l$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = i_0$ into equation (A5),

$$\Pi_{INV}^*(i^* = \bar{\iota}^l) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta - C(\bar{\iota}^l)$$

Accordingly, we obtain that the investor's expected profit is $\Pi_{INV}^*(i^* = \bar{\iota}^l) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta - C(\bar{\iota}^l)$ if $i_0 \le \bar{\iota}^l$.

To examine the investor's incentive of requesting additional information accuracy level i if $i_0 \le \bar{\iota}^l$ in favorable market, $0 \le \alpha_g x_g(R) + \alpha_b x_b(R) \le \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, the investor's expected profits under the full disclosure and rating inflation regime are compared. It follows that $\Pi_{INV}^*(i^* = 0) - \Pi_{INV}^*(i^* = \bar{\iota}^l) >$

0. Hence, the investor always requests $i^* = 0$ yielding $\Pi^*_{INV}(i^* = 0) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta$ in an interval of $i_0 \leq \bar{\iota}^l$ equivalent to $\beta \leq \frac{2a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$, since $\Pi^*_{INV}(i^* = 0) > \Pi^*_{INV}(i^* = \bar{\iota}^l)$.

3.2.2) If $\bar{\iota}^l < i_0 < 1 - \alpha_m$, we can further derive that $\frac{2a\bar{\iota}^l}{\alpha_g x_g(R) - \alpha_b x_b(R)} < \beta < \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

The investor has incentive to request $i^* = i_0$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = i_0$ into equation (A5),

$$\Pi^*_{INV}(i^* = i_0) = \left[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0)$$

Accordingly, we obtain that the investor's expected profit is $\Pi^*_{INV}(i^* = i_0) = \left[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0)$ if $\bar{\iota}^l < i_0 < 1 - \alpha_m$.

To examine the investor's incentive of requesting additional information accuracy level i if $\bar{\iota}^l < i_0 < 1 - \alpha_m$ in favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) \leq \alpha_m\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]$, the investor's expected profits under the full disclosure and rating inflation regime are compared. It follows that $\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0) = \begin{cases} > 0 \;\; if\; \beta > \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} \\ \leq 0 \; if\; \beta \leq \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} \end{cases}$.

Let $\underline{\beta} = \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ and $\bar{\beta} = \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$. To ensure that $\underline{\beta}$ is in an interval of $\frac{2a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} < \beta < \bar{\beta}$, $\underline{\beta}$ must satisfy: (i) $\underline{\beta} > \frac{2a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ and (ii) $\underline{\beta} < \bar{\beta}$.

(ii) To check if $\underline{\beta} > \frac{2a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$:

$$\frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} > \frac{2a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$$

Then, we have that $\underline{\beta} > \frac{2a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

(iv) To check if $\underline{\beta} < \bar{\beta}$:

$$\frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} \gtrless \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$$

Substitute $\bar{\iota}^l = \frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} - \alpha_m$ in,

$$\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right] < \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]$$

Then, we have that $\underline{\beta} \coloneqq \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

Accordingly, $\underline{\beta}$ is in an interval of $\bar{\iota}^l < i_0 < 1 - \alpha_m$. Thus, the investor has incentive to acquire $i^* = \begin{cases} 0 & if\ \beta \leq \underline{\beta} \\ i_0 & if\ \underline{\beta} < \beta < \bar{\beta} \end{cases}$ since $\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0) = \begin{cases} \leq 0 & if\ \beta \leq \underline{\beta} \\ > 0 & if\ \underline{\beta} < \beta < \bar{\beta} \end{cases}$.

3.2.3) If $i_0 \geq 1 - \alpha_m$, we can further derive that $\beta \geq \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

The investor has incentive to request $i^* = 1 + \alpha_m$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = 1 + \alpha_m$ into equation (A5),

$$\Pi^*_{INV}(i^* = 1 - \alpha_m) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i^* + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i^*)$$

Accordingly, the investor's expected profit is $\Pi^*_{INV}(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m)$ if $i_0 \geq 1 - \alpha_m$.

To examine the investor's incentive of requesting additional information accuracy level i if $i_0 \geq 1 - \alpha_m$ in favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) \leq -2\alpha_b x_b(R)$, the investor's expected profits under the full disclosure and rating inflation regime are compared:

$$\Pi^*_{INV}(i^* = 1 - \alpha_m) - \Pi^*_{INV}(i^* = 0) = -\alpha_b x_b(R)\beta - a(1 - \alpha_m)^2$$

Thus, we have that $\Pi^*_{INV}(i^* = 1 - \alpha_m) - \Pi^*_{INV}(i^* = 0) = \begin{cases} > 0\ if\ \beta > \frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \\ \leq 0\ if\ \beta \leq \frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \end{cases}$.

To ensure that $\Pi^*_{INV}(i^* = 1 - \alpha_m) - \Pi^*_{INV}(i^* = 0) > 0$ in an interval of $\beta \geq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$, we need to check if $\frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \lesseqgtr \bar{\beta} \coloneqq \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$. It follows that $\frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} < \frac{2a(1-\alpha_m)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

Accordingly, we have that the investor requests $i^* = 1 - \alpha_m$ if $\beta \geq \bar{\beta}$ since $\Pi^*_{INV}(i^* = 1 - \alpha_m) - \Pi^*_{INV}(i^* = 0) > 0$.

Lastly in favorable market, $0 \leq [\alpha_g x_g(R) + \alpha_b x_b(R)] < \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$, we now assume that $\bar{\iota}^l < 0$ and have from proposition 1 that $\bar{\iota}^h < min\{0, \bar{\iota}^l\}$. When $\bar{\iota}^h < \bar{\iota}^l < 0$, any level of additional information accuracy i the investor acquires can induce the CRA to implement the full disclosure regime. To determine the optimal level of additional information accuracy i^*, we only consider one interval of $0 \leq i \leq 1 - \alpha_m$.

Hence, the same result as previously derived in extremely bad market holds.

If $\bar{\iota}^l \geq 0 > \bar{\iota}^h$, the CRA can either deflate rating or truthfully report rating.

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \left[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

If $0 > \bar{\iota}^l > \bar{\iota}^h$, the CRA always adopts full disclosure regime.

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = [\alpha_g x_g(R) - \alpha_b x_b(R)]\alpha_m\beta + \alpha_b x_b(R)\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \left[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

3) Extremely favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \geq \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$.

We now assume that $\bar{\iota}^l \geq 0$ and have from proposition 1 that $\bar{\iota}^h \leq min\{0, \bar{\iota}^l\}$. To determine the optimal level of additional information accuracy i^*, we consider two intervals: 4.1) $0 \leq i < \bar{\iota}^l$ and 4.2) $\bar{\iota}^l \leq i \leq 1 - \alpha_m$.

4.1) <u>For $0 \leq i < \bar{\iota}^l$</u>. The CRA observes h-signal, $\emptyset = h$, and reports high rating since $\bar{\iota}^h < 0$ resulting in $I(\mu^h(i)) = 0 = I(F)$. In contrast, the CRA observes l-signal, $\emptyset = l$, but reports high rating since $i < \bar{\iota}^l$ resulting in $I(\mu^l(i)) = 1 = I(B)$. Hence, the CRA adopts rating inflation regime and the investor's expected profit from (A3) is given by

$$\Pi_{INV}^{\tilde{r}=h} = \max_i \left[[\alpha_g x_g(R) + \alpha_b x_b(R)]\beta - C(i)\right]$$

$$\frac{d\Pi_{INV}^{\tilde{r}=h}}{di} = -C'(i) < 0$$

Thus, the investor's profit is monotonically decreasing in i for $0 \leq i \leq \bar{\iota}^l$. The optimal additional information accuracy level i^* requested by the investor is the minimum level ($i^* = 0$) which generates $\Pi^*_{INV}(i^* = 0) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta$.

4.2) For $\bar{\iota}^l \leq i \leq 1 - \alpha_m$. The CRA observes h-signal, $\emptyset = h$, and reports high rating since $i > 0 > \bar{\iota}^h$ resulting in $I(\mu^h(i)) = 0 = I(F)$. The CRA observes l-signal, $\emptyset = l$, and reports low rating since $i \geq \bar{\iota}^l$ resulting in $I(\mu^l(i)) = 0 = I(F)$. Hence, the CRA implements full disclosure and the investor's expected profit is given by

$$\Pi^{\tilde{r}=h}_{INV} = \max_i \left[[[\alpha_g x_g(R) - \alpha_b x_b(R)][i + \alpha_m] + \alpha_b x_b(R)]\beta - C(i)\right] \qquad \text{----(A5)}$$

where $C(i) = ai^2$

We can derive the first and second derivatives of $\Pi^{\tilde{r}=h}_{INV}$ with respect to the decision variable, i, as follows.

First differentiation: $\frac{d\Pi^{\tilde{r}=h}_{INV}}{di} = \underbrace{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}_{>0} - \underbrace{C'(i)}_{>0}$

Second differentiation: $\frac{d^2\Pi^{\tilde{r}=h}_{INV}}{di^2} = -C''(i) < 0$

Then, the investor's profit $\Pi^{\tilde{r}=h}_{INV}$ is concave in additional information accuracy level i.

First order condition: $\frac{d\Pi^{\tilde{r}=h}_{INV}}{di} = [\alpha_g x_g(R) - \alpha_b x_b(R)]\beta - 2ai = 0$

Then, the optimal additional information accuracy is $i^* = i_0 := \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$.

In extremely favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) > -2\alpha_b x_b(R)$, we perform continuity test to check if the investor's expected profit function is continuous at the point of $\bar{\iota}^l$, a conjunction point, by

$$\Pi^*_{INV}(i^* = 0)|_{i^*=\bar{\iota}^l} = \Pi^*_{INV}(i^* = \bar{\iota}^l) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta - C(\bar{\iota}^l)$$

Therefore, the investor's expected profit function is continuous at the point of $\bar{\iota}^l$.

To check if the optimal additional information accuracy level i_0 is a boundary or interior solution, we consider three cases: (i) $i_0 \leq \bar{\iota}^l$, (ii) $\bar{\iota}^l < i_0 < 1 - \alpha_m$, and (iii) $i_0 \geq 1 - \alpha_m$.

4.2.1) If $i_0 \leq \bar{\iota}^l$, we can further derive that $\beta \leq \frac{2a\bar{\iota}^l}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

The investor has incentive to request $i^* = \bar{\iota}^l$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = i_0$ into equation (A5),

$$\Pi_{INV}^*(i^* = \bar{\iota}^l) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta - C(\bar{\iota}^l)$$

Accordingly, we obtain that the investor's expected profit is $\Pi_{INV}^*(i^* = \bar{\iota}^l) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta - C(\bar{\iota}^l)$ if $i_0 \leq \bar{\iota}^l$.

To examine the investor's incentive of requesting additional information accuracy level i if $i_0 \leq \bar{\iota}^l$ in favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) > -2\alpha_b x_b(R)$, the investor's expected profits under the full disclosure and rating inflation regime are compared:

$$\Pi_{INV}^*(i^* = 0) - \Pi_{INV}^*(i^* = \bar{\iota}^l) \coloneqq a\bar{\iota}^{l^2} > 0$$

Hence, the investor always requests $i^* = 0$ yielding $\Pi_{INV}^*(i^* = 0) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta$ in an interval of $i_0 \leq \bar{\iota}^l$ equivalent to $\beta \leq \frac{2a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$, since $\Pi_{INV}^*(i^* = 0) > \Pi_{INV}^*(i^* = \bar{\iota}^l)$.

4.2.2) If $\bar{\iota}^l < i_0 < 1 - \alpha_m$, we can further derive that $\frac{2a\bar{\iota}^l}{\alpha_g x_g(R) - \alpha_b x_b(R)} < \beta < \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

The investor has incentive to request $i^* = i_0$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = i_0$ into equation (A5),

$$\Pi_{INV}^*(i^* = i_0) = [[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)]\beta - C(i_0)$$

Accordingly, we obtain that the investor's expected profit is $\Pi_{INV}^*(i^* = i_0) = [[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)]\beta - C(i_0)$ if $\bar{\iota}^l < i_0 < 1 - \alpha_m$.

To examine the investor's incentive of requesting additional information accuracy level i if $\bar{\iota}^l < i_0 < 1 - \alpha_m$ in favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) > -2\alpha_b x_b(R)$, the investor's expected profits under the full disclosure and rating inflation regime are compared. It follows that $\Pi_{INV}^*(i^* = i_0) - \Pi_{INV}^*(i^* = 0) = \begin{cases} > 0 \ \ if\ \beta > \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} \\ \leq 0 \ if\ \beta \leq \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} \end{cases}$.

Let $\underline{\beta} = \frac{4a\bar{\imath}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ and $\bar{\beta} = \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$. To ensure that $\underline{\beta}$ is in an interval of $\frac{2a\bar{\imath}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} < \beta < \bar{\beta}$, $\underline{\beta}$ must satisfy: (i) $\underline{\beta} > \frac{2a\bar{\imath}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ and (ii) $\underline{\beta} < \bar{\beta}$.

(i) To check if $\underline{\beta} > \frac{2a\bar{\imath}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$:

$$\frac{4a\bar{\imath}^l}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} > \frac{2a\bar{\imath}^l}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}$$

Then, we have that $\underline{\beta} > \frac{2a\bar{\imath}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

(ii) To check if $\underline{\beta} < \bar{\beta}$:

$$\frac{4a\bar{\imath}^l}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} \gtrless \frac{2a(1-\alpha_m)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}$$

Substitute $\bar{\imath}^l = \frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} - \alpha_m$ in,

$$\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right] \geq \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]$$

Then, we have that $\underline{\beta} \geq \bar{\beta}$.

Accordingly, $\underline{\beta}$ is in NOT an interval of $\bar{\imath}^l < i_0 < 1 - \alpha_m$. Thus, the investor acquires $i^* = 0$ if $\underline{\beta} < \beta < \bar{\beta}$ since $\Pi^*_{INV}(i^* = i_0) - \Pi^*_{INV}(i^* = 0) < 0$.

4.2.3) If $i_0 \geq 1 - \alpha_m$, we can further derive that $\beta \geq \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

The investor has incentive to request $i^* = 1 + \alpha_m$ to motivate the CRA to adopt full disclosure regime regardless of signal $\emptyset = \{l, h\}$ she observes:

Substitute $i^* = 1 + \alpha_m$ into equation (A5),

$$\Pi^*_{INV}(i^* = 1 - \alpha_m) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i^* + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i^*)$$

Accordingly, the investor's expected profit is $\Pi^*_{INV}(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m)$ if $i_0 \geq 1 - \alpha_m$.

To examine the investor's incentive of requesting additional information accuracy level i if $i_0 \geq 1 - \alpha_m$ in extremely favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) > -2\alpha_b x_b(R)$, the investor's expected profits under the full disclosure and rating inflation regime are compared. It follows that $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)-\Pi_{INV}^{*}(i^{*}=0)=\begin{cases} >0\ if\ \beta>\frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \\ \leq 0\ if\ \beta\leq\frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \end{cases}$.

Let $\bar{\bar{\beta}}=\frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)}$. We need to check if $\bar{\bar{\beta}}$ is in an interval of $\beta\geq\frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$ by $\frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)}\lesseqgtr\frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$. Thus, we obtain that $\bar{\bar{\beta}}\geq\overline{\beta}:=\frac{2a(1-\alpha_m)}{[\alpha_g x_g(R)-\alpha_b x_b(R)]}$.

Accordingly, we have that the investor requests $i^{*}=\begin{cases} 0 & if\ \beta<\bar{\bar{\beta}} \\ 1-\alpha_m & if\ \beta\geq\bar{\bar{\beta}} \end{cases}$ since $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)-\Pi_{INV}^{*}(i^{*}=0)=\begin{cases} <0\ if\ \beta<\bar{\bar{\beta}} \\ \geq 0\ if\ \beta\geq\bar{\bar{\beta}} \end{cases}$.

Lastly, in extremely favorable market, $[\alpha_g x_g(R)+\alpha_b x_b(R)]\geq\alpha_m[\alpha_g x_g(R)-\alpha_b x_b(R)]$, we now assume that $\bar{\imath}^{l}<0$ and have from proposition 1 that $\bar{\imath}^{h}<min\{0,\bar{\imath}^{l}\}$. When $\bar{\imath}^{h}<\bar{\imath}^{l}<0$, any level of additional information accuracy i the investor acquires can induce the CRA to adopt the full disclosure regime. To determine the optimal level of additional information accuracy i^{*}, we only consider one interval of $0\leq i\leq 1-\ \alpha_m$.

Hence, the same result as previously derived in extremely bad market holds.

Accordingly, we can summarize, in extremely favorable market, the optimal accuracy level i^{*} the investor acquires $i^{*}=\begin{cases} 0 & if\ \beta\leq\bar{\bar{\beta}} \\ 1-\alpha_m & if\ \beta\geq\bar{\bar{\beta}} \end{cases}$ if $\bar{\imath}^{l}\geq 0>\bar{\imath}^{h}$, and $i^{*}=\begin{cases} 0 & if\ if\ \beta\leq\underline{\beta} \\ i_0 & if\ \underline{\beta}<\beta<\bar{\beta} \\ 1-\alpha_m & if\ \beta\geq\bar{\beta} \end{cases}$ if $0>\bar{\imath}^{l}>\bar{\imath}^{h}$. The investor gains the expected profit as follows:

If $\bar{\imath}^{l}\geq 0>\bar{\imath}^{h}$, the CRA can either inflate rating or truthfully report rating.

$$\Pi_{INV}^{\tilde{r}=h}(i^{*})=$$

$$\begin{cases} \Pi_{INV}^{*}(i^{*}=0)=[\alpha_g x_g(R)+\alpha_b x_b(R)]\beta & if\ \beta\leq\bar{\bar{\beta}}:=\frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \\ \Pi_{INV}^{*}(i^{*}=1-\alpha_m)=\alpha_g x_g(R)\beta-C(1-\alpha_m) & if\ \beta>\bar{\bar{\beta}}:=\frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \end{cases}.$$

If $0>\bar{\imath}^{l}>\bar{\imath}^{h}$, the CRA always adopts full disclosure regime.

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0) = [\alpha_g x_g(R) - \alpha_b x_b(R)]\alpha_m\beta + \alpha_b x_b(R)\beta & if\ \beta \le \underline{\beta} \\ \Pi_{INV}^*(i^*=i_0) = \big[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0+\alpha_m] + \alpha_b x_b(R)\big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^*=1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \ge \bar{\beta} \end{cases}$$

Hence, we can summarize results of Proposition 2 as follows:

- If the CRA always implements full disclosure regime, $0 > \bar{\iota}^h > \bar{\iota}^l$ or $0 > \bar{\iota}^l > \bar{\iota}^h$, the investor requests $i^* = \begin{cases} 0 & if\ if\ \beta \le \underline{\beta} \\ i_0 & if\ \underline{\beta} < \beta < \bar{\beta} \\ 1-\alpha_m & if\ \beta \ge \bar{\beta} \end{cases}$.
- If $\bar{\iota}^h \ge 0 > \bar{\iota}^l$or $\bar{\iota}^l \ge 0 > \bar{\iota}^l$,

1. Extremely unfavorable market $-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] \ge \alpha_g x_g(R) + \alpha_b x_b(R)$:

If the CRA can switch rating policy from rating deflation to full disclosure regime, $\bar{\iota}^h \ge 0 > \bar{\iota}^l$, the investor has incentive to acquire $i^* = \begin{cases} 0 & if\ \beta \le \bar{\bar{\beta}} \\ 1-\alpha_m & if\ \beta \ge \bar{\bar{\beta}} \end{cases}$.

2. Unfavorable market $-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$:

If the CRA can switch rating policy from rating deflation to full disclosure regime, $\bar{\iota}^h \ge 0 > \bar{\iota}^l$, the investor always requests $i^* = \begin{cases} 0 & if\ if\ \beta \le \underline{\beta} \\ i_0 & if\ \underline{\beta} < \beta < \bar{\beta} \\ 1-\alpha_m & if\ \beta \ge \bar{\beta} \end{cases}$ where $\underline{\beta} = \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

3. Favorable market $0 \le \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$:

If the CRA can switch rating policy from rating inflation to full disclosure regime, $\bar{\iota}^h \ge 0 > \bar{\iota}^l$, the investor always requests $i^* = \begin{cases} 0 & if\ if\ \beta \le \underline{\beta} \\ i_0 & if\ \underline{\beta} < \beta < \bar{\beta} \\ 1-\alpha_m & if\ \beta \ge \bar{\beta} \end{cases}$ where $\underline{\beta} = \frac{4a\bar{\iota}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$.

4. Extremely favorable market $0 \le \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$:

If the CRA can switch rating policy from rating inflation to full disclosure regime, $\bar{\imath}^l \geq 0 > \bar{\imath}^h$, the investor has incentive to acquire $i^* = \begin{cases} 0 & if\ \beta \leq \bar{\bar{\beta}} \\ 1-\alpha_m & if\ \beta \geq \bar{\bar{\beta}} \end{cases}$.

Proof of proposition 3.3: We investigate the marginal investor's incentive toward additional information accuracy acquisition under three investment strategies. The marginal investor's expected profits under each investment strategy are given by

$$\Pi_{INV} = max\left[\underbrace{[\pi_{ah}(i)[1-I_h(i)]x_g(R)+\pi_{al}(i)I_l(i)x_b(R)]\beta - C(i)}_{\Pi_{INV}^{\tilde{r}=h}(i)}, \underbrace{\pi_{sb}R-1}_{\Pi_{INV}^{sb}}, \underbrace{0}_{\Pi_{INV}^{0}}\right]$$

To characterize the optimal level of additional information accuracy the investor tends to acquire, we compare the investor's expected profits of each investment strategy under extremely unfavorable, unfavorable, favorable, and extremely favorable market respectively.

1. Extremely unfavorable market, $-\alpha_m[\alpha_g x_g(R)+\alpha_b x_b(R)] \geq \alpha_g x_g(R) + \alpha_b x_b(R)$.

From Proposition 2, the investor's expected profit in extremely unfavorable market is:

If $\bar{\imath}^h \geq 0 > \bar{\imath}^l$,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0)=0 & if\ \beta \leq \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \\ \Pi_{INV}^*(i^*=1-\alpha_m) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2 & if\ if\ \beta > \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \end{cases}$$

If $0 > \bar{\imath}^h > \bar{\imath}^l$,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0) = [\alpha_g x_g(R)-\alpha_b x_b(R)]\alpha_m\beta + \alpha_b x_b(R)\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^*=i_0) = \big[[\alpha_g x_g(R)-\alpha_b x_b(R)][i_0+\alpha_m]+\alpha_b x_b(R)\big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^*=1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

1.1) Comparing Π_{INV}^{sb} with Π_{INV}^{0}:

$$\Pi_{INV}^{sb} - \Pi_{INV}^{0} = [\alpha_g p_g + \alpha_b p_b]R - 1$$

Thus, $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$ since $-\alpha_m[\alpha_g x_g(R)+\alpha_b x_b(R)] < 0$

Accordingly, the investor neither purchases rating information from the CRA nor invests in the project in extremely unfavorable market, $-\alpha_m\left[\alpha_g x_g(R)+\alpha_b x_b(R)\right] \geq \alpha_g x_g(R)+\alpha_b x_b(R)$. Notice that the investor makes investment decision by comparing between $\Pi_{INV}^{\tilde{r}=h}(i^*)$ and Π_{INV}^{0} in extremely unfavorable market.

1.2) We compare $\Pi_{INV}^{\tilde{r}=h}(i)$ with Π_{INV}^{0} while $\Pi_{INV}^{\tilde{r}=h}(i)$ depends on level of $i=i^* \in \{0, i_0, 1-\alpha_m\}$ in extremely unfavorable market.

1.2.1) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^*=0)$ with Π_{INV}^{0}:

Since the investor gains different expected profits $\Pi_{INV}^{\tilde{r}=h}(i^*=0)$ depending on threshold of additional accuracy level, it can be separated into two cases: (i) $\bar{\iota}^h \geq 0 > \bar{\iota}^l$ and (ii) $0 > \bar{\iota}^h > \bar{\iota}^l$.

(i) If $\bar{\iota}^h \geq 0 > \bar{\iota}^l$, $\quad \Pi_{INV}^{\tilde{r}=h}(i^*=0) = \Pi_{INV}^{0} = 0$

Then, the investor is indifferent between investing with no additional information accuracy and not investing because both strategies do not generate profits for the investor.

(ii) If $0 > \bar{\iota}^h > \bar{\iota}^l$, $\quad \Pi_{INV}^{\tilde{r}=h}(i^*=0) - \Pi_{INV}^{0} = \left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]\alpha_m\beta + \alpha_b x_b(R)\beta$

Then, $\quad \Pi_{INV}^{\tilde{r}=h}(i^*=0) - \Pi_{INV}^{0} = \begin{cases} > 0 \; if \; 0 > \bar{\iota}^h \\ \leq 0 \; if \; 0 \leq \bar{\iota}^h \end{cases}$

Accordingly, we obtain that $\Pi_{INV}^{\tilde{r}=h}(i^*=0) > \Pi_{INV}^{0}$ since $0 > \bar{\iota}^h > \bar{\iota}^l$.

Then, the investor always solicits rating information from the CRA with no additional information accuracy level $i^*=0$. In other words, the investor purchases rating information $i^*=0$ to make investment decision if the CRA always truthfully reports rating information.

1.2.2) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^*=i_0)$ with Π_{INV}^{0} when $0 > \bar{\iota}^h > \bar{\iota}^l$:

$\Pi_{INV}^{\tilde{r}=h}(i^*=i_0) - \Pi_{INV}^{0} = \frac{\left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]^2\beta^2}{4a} + \left[\left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]\alpha_m + \alpha_b x_b(R)\right]\beta$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i^*=i_0) - \Pi_{INV}^{0}$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i^*=i_0) - \Pi_{INV}^{0}\right]}{\partial a} = -\frac{\left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]^2\beta^2}{4a^2} \leq 0$$

Hence, $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^0$ is decreasing in information production cost a. This implies that there exists a threshold of information production cost $\bar{a}_1$which makes $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) = \Pi_{INV}^0$.

Then, $$\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^0 = 0$$

$$a = \bar{a}_1 := \frac{-\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta}{4\left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m + \alpha_b x_b(R)\right]}$$

Accordingly, $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^0 = \begin{cases} > 0 \; if \; a < \bar{a}_1 \\ \leq 0 \; if a \geq \bar{a}_1 \end{cases}$. The investor acquires $i^* = i_0$ only if the cost of information acquisition is less than $\bar{a}_1 = \frac{-\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta}{4\left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m + \alpha_b x_b(R)\right]}$. Otherwise, the investor neither acquires rating information nor invests in the unfavorable market.

1.2.3) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m)$ with Π_{INV}^0:

$$\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^0 = \alpha_g x_g(R)\beta - a(1 - \alpha_m)^2$$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^0$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^0\right]}{\partial a} = -(1 - \alpha_m)^2 \leq 0$$

Therefore, $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^0$ is decreasing in information production cost a. This implies that there exists a threshold of information production cost $\bar{a}_1$which makes $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) = \Pi_{INV}^0$.

Then, $$\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^0 = 0$$

$$a = \bar{a}_2 = \frac{\alpha_g x_g(R)\beta}{(1 - \alpha_m)^2} \geq 0$$

Accordingly, $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^0 = \begin{cases} > 0 \; if \; a < \bar{a}_2 \\ \leq 0 \; if \; a \geq \bar{a}_2 \end{cases}$. The investor acquires $i^* = 1 - \alpha_m$ only if the cost of information production is less than $\bar{a}_2 = \frac{\alpha_g x_g(R)\beta}{(1-\alpha_m)^2}$. Otherwise, the investor neither purchases rating information nor invests in the unfavorable market.

2. Unfavorable market, $-\alpha_m\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$.

From Proposition 2, the investor's expected profit in unfavorable market is:

If $\bar{\iota}^h \geq 0 > \bar{\iota}^l$:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0)=0 & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^*=i_0)=\Big[\big[\alpha_g x_g(R)-\alpha_b x_b(R)\big][i_0+\alpha_m]+\alpha_b x_b(R)\Big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^*=1-\alpha_m)=\alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

If $0 > \bar{\iota}^h > \bar{\iota}^l$,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^*=0)=\big[\alpha_g x_g(R)-\alpha_b x_b(R)\big]\alpha_m\beta+\alpha_b x_b(R)\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^*=i_0)=\Big[\big[\alpha_g x_g(R)-\alpha_b x_b(R)\big][i_0+\alpha_m]+\alpha_b x_b(R)\Big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^*=1-\alpha_m)=\alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

2.1) Comparing Π_{INV}^{sb} with Π_{INV}^0:

The same result as previously proven in 1.1) of extremely unfavorable market case holds. The investor neither solicits rating information from the CRA nor invests in financial asset in favorable market, $-\alpha_m\big[\alpha_g x_g(R)+\alpha_b x_b(R)\big] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$ since $\Pi_{INV}^{sb} < \Pi_{INV}^0$. Thus, he makes investment decision by comparing between $\Pi_{INV}^{\tilde{r}=h}(i)$ and Π_{INV}^0 in unfavorable market.

2.2) We compare $\Pi_{INV}^{\tilde{r}=h}(i)$ with Π_{INV}^0 while $\Pi_{INV}^{\tilde{r}=h}(i)$ depends on level of $i = i^* \in \{0, i_0, 1-\alpha_m\}$ in unfavorable market.

2.2.1) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^*=0)$ with Π_{INV}^0:

The same result as previously proven in 1.2.1) of extremely unfavorable market case holds. Therefore, the investor always solicits rating information from the CRA with no additional information accuracy level $i^* = 0$. In other words, the investor purchases rating information if the information cost is sufficiently low and the CRA always truthfully reports rating information.

2.2.2) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^*=i_0)$ with Π_{INV}^0:

The same result as previously proven in 1.2.2) of extremely unfavorable market case holds. Since $\Pi_{INV}^{\tilde{r}=h}(i^*=i_0) - \Pi_{INV}^0 = \begin{cases} > 0\ if\ a < \bar{a}_1 \\ \leq 0\ if a \geq \bar{a}_1 \end{cases}$, the investor acquires $i^* = i_0$ only if the cost of information acquisition is less than

$\bar{a}_1 = \frac{-\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta}{4\left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m + \alpha_b x_b(R)\right]}$. Otherwise, the investor neither acquires rating information nor invests in the unfavorable market.

2.2.3) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m)$ with Π_{INV}^0:

The same result as previously proven in 1.2.3) of extremely unfavorable market case holds. Since $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^0 = \begin{cases} > 0 \; if \; a < \bar{a}_2 \\ \leq 0 \; if \; a \geq \bar{a}_2 \end{cases}$, the investor acquires $i^* = 1 - \alpha_m$ only if the cost of information acquisition is less than $\bar{a}_2 = \frac{\alpha_g x_g(R)\beta}{(1-\alpha_m)^2}$. Otherwise, the investor neither purchases rating information nor invests in the unfavorable market.

3. Favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]$.

From Proposition 2, the investor's expected profit in favorable market is:

If $\bar{\imath}^l \geq 0 > \bar{\imath}^h$,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta & if \; \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if \; \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if \; \beta \geq \bar{\beta} \end{cases}$$

If $0 > \bar{\imath}^l > \bar{\imath}^h$,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m\beta + \alpha_b x_b(R)\beta & if \; \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if \; \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if \; \beta \geq \bar{\beta} \end{cases}$$

3.1) Comparing Π_{INV}^{sb} with Π_{INV}^0:

$$\Pi_{INV}^{sb} - \Pi_{INV}^0 \coloneqq \alpha_g x_g(R) + \alpha_b x_b(R) \geq 0$$

Hence, the investor always invests in financial assets without acquiring rating information from the CRA if the market condition is favorable. Notice that the investor makes investment decision by comparing between $\Pi_{INV}^{\tilde{r}=h}(i)$ and Π_{INV}^{sb} in favorable market.

3.2) We compare $\Pi_{INV}^{\tilde{r}=h}(i)$ with Π_{INV}^{sb} while $\Pi_{INV}^{\tilde{r}=h}(i)$ depends on level of $i = i^* \in \{0, i_0, 1 - \alpha_m\}$ in favorable market.

3.2.1) Comparing Π_{INV}^{sb} with $\Pi_{INV}^{\tilde{r}=h}(i^* = 0)$:

Since the investor gains different expected profits $\Pi_{INV}^{\tilde{r}=h}(i^* = 0)$ depending on threshold of additional accuracy level, it can be separated into two cases: (i) $\bar{\imath}^l \geq 0 > \bar{\imath}^h$ and (ii) $0 > \bar{\imath}^l > \bar{\imath}^h$.

(i) If $\bar{\imath}^l \geq 0 > \bar{\imath}^h$,

$$\Pi_{INV}^{sb} - \Pi_{INV}^{\tilde{r}=h}(i^* = 0) = \alpha_g x_g(R) + \alpha_b x_b(R) - [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta$$

$$1 \geq \beta$$

Then, the investor would rather invest in the risky project without purchasing rating information from the CRA than invest in the risky project with no additional accuracy acquisition, $i^* = 0$, in the favorable market since $\Pi_{INV}^{sb} \geq \Pi_{INV}^{\tilde{r}=h}(i^* = 0)$.

(ii) If $0 > \bar{\imath}^l > \bar{\imath}^h$,

$$\Pi_{INV}^{sb} - \Pi_{INV}^{\tilde{r}=h}(i^* = 0)$$

$$= [\alpha_g x_g(R) + \alpha_b x_b(R)] - [\alpha_g x_g(R) - \alpha_b x_b(R)]\alpha_m\beta - \alpha_b x_b(R)\beta$$

Then, $$\Pi_{INV}^{sb} - \Pi_{INV}^{\tilde{r}=h}(i^* = 0) = \begin{cases} > 0 \; if \; \beta < 1 - \frac{\bar{\imath}^l}{\bar{\imath}^h} \\ \leq 0 \; if \; \beta \geq 1 - \frac{\bar{\imath}^l}{\bar{\imath}^h} \end{cases}$$

Accordingly, it is possible only in the case that $\Pi_{INV}^{sb} \leq \Pi_{INV}^{\tilde{r}=h}(i^* = 0)$ since $\frac{\bar{\imath}^l}{\bar{\imath}^h} > 1$.

Then, the investor chooses to solicit rating information from the CRA with no additional information accuracy level $i^* = 0$. In other words, the investor solicits rating information if the information cost is sufficiently low and the CRA always adopts full disclosure regime.

3.2.2) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0)$ with Π_{INV}^{sb}:

$\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^{sb} = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2 \beta^2}{4a} + [\alpha_g x_g(R) - \alpha_b x_b(R)]\alpha_m\beta - \alpha_g x_g(R) + \alpha_b x_b(R)(\beta - 1)$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^{sb}$ with respect to a as follow.

$$\frac{\partial[\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^{sb}]}{\partial a} = -\frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2 \beta^2}{4a^2} \leq 0$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^{sb}$ is decreasing in cost of information production a. This implies that there exists a threshold of information production cost $\bar{a}_1$which makes $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) = \Pi_{INV}^{sb}$.

Then, $$\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^{sb} = 0$$

$$a = \bar{a}_1 = \frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta^2}{4\left(\alpha_g x_g(R) + \alpha_b x_b(R)[1-\beta] - \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\beta\right)}$$

Therefore, $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^{sb} = \begin{cases} > 0 \; if \; a < \bar{a}_1 \\ \leq 0 \; if a \geq \bar{a}_1 \end{cases}$. The investor acquires $i^* = i_0$ only if the cost of information acquisition is less than $\bar{a}_1 = \frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta^2}{4\left(\alpha_g x_g(R) + \alpha_b x_b(R)[1-\beta] - \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\beta\right)}$. Otherwise, the investor invests in the risky project without purchasing rating information from the CRA in the favorable market.

3.2.3) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m)$ with Π_{INV}^{sb}:

$$\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^{sb} = \alpha_g x_g(R)[\beta - 1] - a(1 - \alpha_m)^2 - \alpha_b x_b(R)$$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^{sb}$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^{sb}\right]}{\partial a} = -(1 - \alpha_m)^2 \leq 0$$

Accordingly, $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^{sb}$ is decreasing in information production cost a. This implies that there exists the threshold of information production cost a which makes $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) = \Pi_{INV}^{sb}$.

$$\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^{sb} = 0$$

$$a = \bar{a}_2 = \frac{\alpha_g x_g(R)[\beta - 1] - \alpha_b x_b(R)}{(1 - \alpha_m)^2}$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^{sb} = \begin{cases} > 0 \; if \; a < \bar{a}_2 \\ \leq 0 \; if a \geq \bar{a}_2 \end{cases}$. The investor acquires $i^* = 1 - \alpha_m$ only if the cost of information production is less than $\bar{a}_2 = \frac{\alpha_g x_g(R)[\beta-1] - \alpha_b x_b(R)}{(1-\alpha_m)^2}$. Otherwise, the investor invests in the project without purchasing information from the CRA in the favorable market.

4. Extremely favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \geq \alpha_m\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]$.

From Proposition 2, the investor's expected profit in extremely favorable market is:

If $\bar{\iota}^l \geq 0 > \bar{\iota}^h$,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta & if\ \beta \leq \bar{\bar{\beta}} \coloneqq \dfrac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \\ \Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta > \bar{\bar{\beta}} \coloneqq \dfrac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \end{cases}.$$

If $0 > \bar{\iota}^l > \bar{\iota}^h$,

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m\beta + \alpha_b x_b(R)\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

4.1) Comparing Π_{INV}^{sb} with Π_{INV}^{0}:

The same result as earlier proven in 3.1) of favorable market case holds. Hence, the investor always invests in financial assets without acquiring rating information from the CRA if the market condition is favorable. Notice that the investor makes investment decision by comparing between $\Pi_{INV}^{\tilde{r}=h}(i)$ and Π_{INV}^{sb} in extremely favorable market.

4.2) We compare $\Pi_{INV}^{\tilde{r}=h}(i)$ with Π_{INV}^{sb} while $\Pi_{INV}^{\tilde{r}=h}(i)$ depends on level of $i = i^* \in \{0, i_0, 1-\alpha_m\}$ in extremely favorable market.

4.2.1) Comparing Π_{INV}^{sb} with $\Pi_{INV}^{\tilde{r}=h}(i^* = 0)$:

The same result as earlier proven in 3.2.1) of favorable market case holds. Therefore, the investor chooses to solicit rating information from the CRA with no additional information accuracy level $i^* = 0$. In other words, he solicits rating information if the information cost is sufficiently low and the CRA always adopts full disclosure regime.

4.2.2) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0)$ with Π_{INV}^{sb} while $\bar{\iota}^l > 0 > \bar{\iota}^h$:

The same result as earlier proven in 3.2.2) of favorable market case holds. Since $\Pi_{INV}^{\tilde{r}=h}(i^* = i_0) - \Pi_{INV}^{sb} = \begin{cases} > 0\ if\ a < \bar{a}_1 \\ \leq 0\ if a \geq \bar{a}_1 \end{cases}$, the investor acquires $i^* = i_0$ only if the cost of information acquisition is less than $\bar{a}_1 = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2 \beta^2}{4(\alpha_g x_g(R) + \alpha_b x_b(R)[1-\beta] - \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta)}$. Otherwise, he invests in the risky project without purchasing rating information from the CRA in the favorable market.

4.2.3) Comparing $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m)$ with Π_{INV}^{sb}:

The same result as earlier proven in 3.2.3) of favorable market case holds. Since $\Pi_{INV}^{\tilde{r}=h}(i^* = 1 - \alpha_m) - \Pi_{INV}^{sb} = \begin{cases} > 0\ if\ a < \bar{a}_2 \\ \leq 0\ if a \geq \bar{a}_2 \end{cases}$, the investor acquires $i^* = 1 - \alpha_m$ only if the cost of information acquisition is less than $\bar{a}_2 = \frac{\alpha_g x_g(R)[\beta - 1] - \alpha_b x_b(R)}{(1-\alpha_m)^2}$. Otherwise, he invests in the project without purchasing information from the CRA in the favorable market.

Hence in Proposition 3, we can summarize the optimal level of additional information accuracy i^* the investor tends to request in proposition 3 as follow.

1) Extremely unfavorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$.

If the CRA can switch rating policy between rating deflation and full disclosure regime, $\bar{\iota}^h \geq 0 > \bar{\iota}^l$, the investor requests $i^* = 1 - \alpha_m$ if $a < \bar{a}_2 := \frac{\alpha_g x_g(R)\beta}{(1-\alpha_m)^2}$. Otherwise he neither purchases rating information nor invests in the financial asset.

If the CRA always adopts full disclosure regime, $0 > \bar{\iota}^h > \bar{\iota}^l$, the investors always purchases rating information from the CRA to make investment decision as follows.

$$i^* = \begin{cases} i_0 \quad if\ a < \bar{a}_1 := \dfrac{-[\alpha_g x_g(R) - \alpha_b x_b(R)]^2 \beta}{4\left[[\alpha_g x_g(R) - \alpha_b x_b(R)]\alpha_m + \alpha_b x_b(R)\right]} \\ 1 - \alpha_m \qquad\qquad if\ a < \bar{a}_2 := \dfrac{\alpha_g x_g(R)\beta}{(1-\alpha_m)^2} \end{cases}$$

Otherwise, the investor neither purchases information from the CRA nor invests in the project.

2) Unfavorable market, $-\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$.

The investor may invest in the risky assets and acquires additional information accuracy i, depending on the cost of information production as follow.

$$i^* = \begin{cases} i_0 \quad if\ a < \bar{a}_1 := \dfrac{-\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta}{4\left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m + \alpha_b x_b(R)\right]} \\ 1-\alpha_m \quad if\ a < \bar{a}_2 := \dfrac{\alpha_g x_g(R)\beta}{(1-\alpha_m)^2} \end{cases}$$

Otherwise, the investor neither purchases information from the CRA nor invests in the project.

3) Favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) < \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]$.

The investor tends to invest in the risky assets and acquires additional information accuracy i, from the CRA, depending on the cost of information production as follow.

$$i^* = \begin{cases} i_0 \ \ if\ a < \bar{a}_1 := \dfrac{-\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta^2}{4\left(\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m \beta - \alpha_g x_g(R) + \alpha_b x_b(R)(\beta - 1)\right)} \\ 1-\alpha_m \quad if\ a < \bar{a}_2 := \dfrac{\alpha_g x_g(R)[\beta - 1] + \alpha_b x_b(R)}{(1-\alpha_m)^2} \end{cases}$$

Otherwise, the investor invests in the risky project without acquiring information from the CRA.

4) Extremely favorable market $\alpha_g x_g(R) + \alpha_b x_b(R) \geq \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]$.

If the CRA can switch rating policy between rating inflation and full disclosure regime, $\bar{\iota}^l \geq 0 > \bar{\iota}^h$, the investor requests $i^* = 1 - \alpha_m$ if $a < \bar{a}_2 := \frac{\alpha_g x_g(R)\beta}{(1-\alpha_m)^2}$. Otherwise, it is in the best interest of the investor to invest in the financial asset without purchasing information from the CRA.

If the CRA always adopts full disclosure regime, $0 > \bar{\iota}^l > \bar{\iota}^h$, the investors always solicits rating information from the CRA to make investment decision as follow.

$$i^* = \begin{cases} i_0 \ \ if\ a < \bar{a}_1 := \dfrac{-\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2 \beta^2}{4\left(\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m \beta - \alpha_g x_g(R) + \alpha_b x_b(R)(\beta - 1)\right)} \\ 1-\alpha_m \quad if\ a < \bar{a}_2 := \dfrac{\alpha_g x_g(R)[\beta - 1] + \alpha_b x_b(R)}{(1-\alpha_m)^2} \end{cases}$$

Otherwise, the investor invests in the risky project without acquiring information from the CRA.

Proof of proposition 3.4:

From (A3), the investor's expected profit under pure upfront fee scheme, $\beta = 1$, is given by

$$\Pi_{INV}^{\tilde{r}=h}(i,\beta=1) = \max_i \Big[\alpha_g x_g(R)\Big[q(i)\Big(1 - I(\mu^h(i))\Big) + \big(1-q(i)\big)I(\mu^l(i))\Big] + \alpha_b x_b(R)\Big[\big(1-q(i)\big)\Big(1 - I(\mu^h(i))\Big) + q(i)I(\mu^l(i))\Big] - C(i)\Big] \quad \text{----}$$

(B1)

The CRA's expected profit under pure upfront fee, $\beta = 1$, is given by

$$\Pi_{CRA}(\beta = 1) = 0$$

Under the pure upfront fee scheme, the CRA is indifferent between implementing the full disclosure and biased rating regime since she gets only fixed cost of information production. In this proposition, we shall now assume that the CRA under pure upfront fee always implements full disclosure regime, $I(\mu^l(i)) = I(\mu^h(i)) = 0$ under different market conditions.

To illustrate the investor's incentive under different payment schemes, we compare the investor's expected profit under profit-sharing scheme with that under pure upfront fee.

From (B1), the Investor's expected profit under pure upfront fee scheme is given by

$$\Pi_{INV}^{\tilde{r}=h}(i,\beta=1) = \max_i \Big[\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big](i + \alpha_m) + \alpha_b x_b(R) - C(i)\Big] \quad \text{(B2)}$$

We can derive the first and second differentiation of $\Pi_{INV}^{\tilde{r}=h}(i,\beta=1)$ with respect to i as follow.

First differentiation: $\frac{\partial \Pi_{INV}^{\tilde{r}=h}(i,\beta=1)}{\partial i} = \alpha_g x_g(R) - \alpha_b x_b(R) - 2ai$

Second differentiation: $\frac{\partial^2 \Pi_{INV}^{\tilde{r}=h}(i,\beta=1)}{\partial i^2} = -2a \leq 0$

From the first and second differentiation, the investor's expected profit under pure upfront fee scheme $\Pi_{INV}^{\tilde{r}=h}(i,\beta=1)$ is concave with respect to additional information accuracy level i. Thus, we can obtain a unique solution of i by using the first-order condition.

The first-order condition: $\frac{\partial \Pi_{INV}^{\tilde{r}=h}(i,\beta=1)}{\partial i} = \alpha_g x_g(R) - \alpha_b x_b(R) - 2ai = 0$

$$i = i_f^* = \frac{\alpha_g x_g(R) - \alpha_b x_b(R)}{2a} \geq 0$$

Hence, the investor has incentive to acquire the optimal level of additional information accuracy i_f^* to maximize his expected profit under pure upfront fee scheme while the CRA implements the full disclosure regime. From (B2), the investor's expected profit is given by

$$\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right](i_f^* + \alpha_m) + \alpha_b x_b(R) - C(i_f^*)$$

$$\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2}{4a} + \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right] + \alpha_b x_b(R) \quad \text{----(B3)}$$

From proposition 2, if the CRA always truthfully reports rating ($0 > \bar{\iota}^h > \bar{\iota}^l$, or $0 > \bar{\iota}^l > \bar{\iota}^h$), the investor has incentive to request the optimal level of additional accuracy, $i^* \in \{0, i_0, 1 - \alpha_m\}$, regardless of market conditions. Accordingly, we compare the investor's expected profit under pure upfront fee scheme with that under profit sharing scheme under two cases: 4.1) $0 > \bar{\iota}^h > \bar{\iota}^l$, and 4.2) $0 > \bar{\iota}^l > \bar{\iota}^h$.

4.1) For $0 > \bar{\iota}^h > \bar{\iota}^l$.

The investor's expected profit under profit-sharing scheme, when $0 > \bar{\iota}^h > \bar{\iota}^l$, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m \beta + \alpha_b x_b(R)\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

Where $\underline{\beta} = \frac{4a\bar{\iota}^h}{\alpha_g x_g(R) - \alpha_b x_b(R)}$, and $\bar{\beta} = \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

4.1.1) $\Pi_{INV}^{\tilde{r}=h}\left(i = i_f^*, \beta = 1\right) - \Pi_{INV}^*(i^* = 0)$ if $\beta \leq \frac{4a\bar{\iota}^h}{\alpha_g x_g(R) - \alpha_b x_b(R)}$

$$= \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2}{4a} + \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right](1 - \beta) + \alpha_b x_b(R)[1 - \beta]$$

Then, $\Pi_{INV}^{\tilde{r}=h}\left(i = i_f^*, \beta = 1\right) - \Pi_{INV}^*(i^* = 0) = \begin{cases} > 0\ if\ \beta < 1 - \frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}{4a\bar{\iota}^h} \\ \leq 0\ if\ \beta \geq 1 - \frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}{4a\bar{\iota}^h} \end{cases}$

We have that $1-\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4a\bar{\iota}^h} \geq 1$ since $0 > \bar{\iota}^h$. Thus, the investor's expected profit under pure upfront fee is always greater than that under profit-sharing scheme, $\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) > \Pi_{INV}^*(i^* = 0)$, since $\beta < 1 - \frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4a\bar{\iota}^h}$ always.

4.1.2) $\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) - \Pi_{INV}^*(i^* = i_0)$ if $\frac{4a\bar{\iota}^h}{\alpha_g x_g(R)-\alpha_b x_b(R)} < \beta < \frac{2a(1-\alpha_m)}{\alpha_g x_g(R)-\alpha_b x_b(R)}$ where $i_0 = \frac{\alpha_g x_g(R)-\alpha_b x_b(R)}{2a}\beta$ and $C(i_0) = a{i_0}^2$:

$$= \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2}{4a}(1 + \beta) + \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)] + \alpha_b x_b(R)$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) - \Pi_{INV}^*(i^* = i_0) = \begin{cases} > 0 \; if \; \beta > \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} - 1 \\ \leq 0 \; if \; \beta \leq \frac{4a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} - 1 \end{cases}$

From $0 > \bar{\iota}^h > \bar{\iota}^l$, we obtain that $\frac{4a\bar{\iota}^h}{[\alpha_g x_g(R)-\alpha_b x_b(R)]} - 1 < 0$ always. Hence, the investor's expected profit under pure upfront fee is always greater than that under profit-sharing scheme, $\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) > \Pi^*_{INV}(i^* = i_0)$, since by definition $0 \leq \beta \leq 1$.

4.1.3) $\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)$ if $\beta \geq \frac{2a(1-\alpha_m)}{\alpha_g x_g(R)-\alpha_b x_b(R)}$:

$$= \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2}{4a} + \alpha_m \alpha_g x_g(R) + [1 - \alpha_m][\alpha_b x_b(R) + a(1 - \alpha_m)] - \alpha_g x_g(R)\beta$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i = i_f^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m) =$

$$\begin{cases} > 0 \; if \; \beta < \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2}{4a\alpha_g x_g(R)} + \alpha_m + \frac{[\alpha_b x_b(R) + a(1 - \alpha_m)]}{\alpha_g x_g(R)}[1 - \alpha_m] \\ \leq 0 \; if \; \beta \geq \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2}{4a\alpha_g x_g(R)} + \alpha_m + \frac{[\alpha_b x_b(R) + a(1 - \alpha_m)]}{\alpha_g x_g(R)}[1 - \alpha_m] \end{cases}$$

We need to check if $\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]^2}{4a\alpha_g x_g(R)} + \alpha_m + \frac{[\alpha_b x_b(R)+a(1-\alpha_m)]}{\alpha_g x_g(R)}[1 - \alpha_m] \leq 1$,

Knowing that $\alpha_m \coloneqq \max\{\alpha_g, \alpha_b\} \geq \frac{1}{2}$;

If $\alpha_m = \frac{1}{2}$, $\quad \frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]^2}{4a\alpha_g x_g(R)} + \frac{1}{2} + \frac{[\alpha_b x_b(R)+a(1-\alpha_m)]}{\alpha_g x_g(R)}\left[1 - \frac{1}{2}\right] \lesseqgtr 1$

Then, $$\left(\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right] - a\right)^2 > 0 \qquad \text{----(B4)}$$

If $\alpha_m = 1$, $$\frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2}{4a\alpha_g x_g(R)} + 1 + \frac{\left[\alpha_b x_b(R) + a(1-\alpha_m)\right]}{\alpha_g x_g(R)}[1-1] \lesseqgtr 1$$

Then, $$\frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2}{4a\alpha_g x_g(R)} > 0 \qquad \text{----(B5)}$$

From (B4) and (B5), we have that $$\frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2}{4a\alpha_g x_g(R)} + \alpha_m + \frac{\left[\alpha_b x_b(R) + a(1-\alpha_m)\right]}{\alpha_g x_g(R)} [1-\alpha_m] > 1$$

Hence, the investor's expected profit under pure upfront fee is always greater than that under profit-sharing scheme, $\Pi_{INV}^{\tilde{r}=h}\left(i = i_f^*, \beta = 1\right) > \Pi^*_{INV}(i^* = 1-\alpha_m)$, since by definition $0 \le \beta \le 1$ and $1 < \frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]^2}{4a\alpha_g x_g(R)} + \alpha_m + \frac{\left[\alpha_b x_b(R) + a(1-\alpha_m)\right]}{\alpha_g x_g(R)}$ $[1-\alpha_m]$ always.

4.2) For $0 > \bar{\imath}^l > \bar{\imath}^h$.

The investor's expected profit under profit-sharing scheme, when $0 > \bar{\imath}^h > \bar{\imath}^l$, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi^*_{INV}(i^*=0) = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\alpha_m\beta + \alpha_b x_b(R)\beta & if\ \beta \le \underline{\beta} \\ \Pi^*_{INV}(i^*=i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0+\alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi^*_{INV}(i^*=1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \ge \bar{\beta} \end{cases}$$

Where $\underline{\beta} = \frac{4a\bar{\imath}^l}{\alpha_g x_g(R) - \alpha_b x_b(R)}$, and $\bar{\beta} = \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

Since $0 > \bar{\imath}^h$, the same result as 4.1) in Proposition 4 holds when comparing the investor's expected profit $\Pi_{INV}^{\tilde{r}=h}(i^*)$ if $0 > \bar{\imath}^l > \bar{\imath}^h$ with that under upfront fee scheme. Hence the investor's expected profit under pure upfront fee is always greater than that under profit-sharing scheme.

Hence in Proposition 4, we can summarize that the investor gains higher expected profit under pure upfront fee scheme than that under profit-sharing payment scheme if the CRA operated under pure upfront fee consistently adopts the full disclosure regime.

Proof of proposition 3.5: we will compare the fixed upfront fee scheme ($\beta = 1$) with profit-sharing scheme. Assumption: the CRA always adopts biased rating regime.

From (A3), the investor's expected profit under pure upfront fee scheme, $\beta = \mathbf{1}$, is given by

$$\Pi_{INV}^{\tilde{r}=h}(i, \beta = 1) = \max_i \left[\alpha_g x_g(R)\left[q(i)\left(1 - I(\mu^h(i))\right) + \left(1 - q(i)\right)I(\mu^l(i))\right] + \alpha_b x_b(R)\left[\left(1 - q(i)\right)\left(1 - I(\mu^h(i))\right) + q(i)I(\mu^l(i))\right] - C(i)\right] \quad \text{----(B1)}$$

The CRA's expected profit under pure upfront fee, $\beta = \mathbf{1}$, is given by

$$\Pi_{CRA}(\beta = 1) = 0$$

The CRA is indifferent between implementing the full disclosure and biased rating regime since she receives only fixed cost of information production. We learned from interaction between the investor and CRA in proposition 1 and 2 that if the CRA decides to adopt the biased rating regime to maximize her expected profit, it may be optimal for her to adopt rating inflation but never be optimal for her to do rating deflation regime.

To illustrate the investor's incentive under different payment schemes, we compare the investor's expected profit under profit-sharing scheme with that under pure upfront fee. In this proposition, we assume that the CRA under pure upfront fee always implements rating inflation regime, $I\left(\mu^l(i)\right) = 1$ and $I(\mu^h(i)) = 0$.

From (B3), the Investor's expected profit under pure upfront fee is given by

$$\Pi_{INV}^{\tilde{r}=h}(i, \beta = 1) = \max_i\left[\alpha_g x_g(R) + \alpha_b x_b(R) - ai^2\right] \quad \text{----(B6)}$$

We can derive the first and second differentiation of $\Pi_{INV}^{\tilde{r}=h}(i, \beta = 1)$ with respect to i as follow.

First differentiation: $\frac{\partial \Pi_{INV}^{\tilde{r}=h}(i,\beta=1)}{\partial i} = -2ai \leq 0$

Second differentiation: $\frac{\partial^2 \Pi_{INV}^{\tilde{r}=h}(i,\beta=1)}{\partial i^2} = -2a \leq 0$

From the first and second differentiation, the investor's expected profit under pure upfront fee scheme $\Pi_{INV}^{\tilde{r}=h}(i, \beta = 1)$ is concave with respect to additional information accuracy level i. Thus, we can obtain a unique solution of i by using the first-order condition.

First-order condition: $\frac{\partial \Pi_{INV}^{\tilde{r}=h}(i,\beta=1)}{\partial i} = -2ai = 0$

$$i = i_i^* = 0$$

Hence, it is optimal for the investor to not acquire additional information accuracy, $i_i^* = 0$, to maximize his expected profit under pure upfront fee while the CRA implements the rating inflation regime. From (B6), the investor's expected profit is given by

$$\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right] - C(i_i^*)$$

$$\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) = \alpha_g x_g(R) + \alpha_b x_b(R) \qquad \text{----(B7)}$$

From proposition 2, if the CRA always adopts biased ratings ($\bar{\imath}^h \geq 0 > \bar{\imath}^l$, or $\bar{\imath}^l \geq 0 > \bar{\imath}^h$), the investor tends to request the optimal level of additional accuracy, $i^* \in \{0, i_0, 1-\alpha_m\}$, based on market conditions. Accordingly, we compare the investor's expected profit under pure upfront fee scheme with that under profit sharing scheme under four market conditions as follows.

1) Extremely unfavorable market, $-\alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right] \geq \alpha_g x_g(R) + \alpha_b x_b(R)$.

1.1 For $\bar{\imath}^h \geq 0 > \bar{\imath}^l$.

The investor's expected profit under profit-sharing scheme if the CRA can mix rating deflation with full disclosure regime, $\bar{\imath}^h \geq 0 > \bar{\imath}^l$, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(0) = 0 & if\ \beta \leq \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \\ \Pi_{INV}^*(1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta > \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \end{cases}$$

1.1.1) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 0)$ if $\beta \leq \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$:

$$= \alpha_g x_g(R) + \alpha_b x_b(R) - 0$$

In extremely unfavorable market, $-\alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right] \geq \alpha_g x_g(R) + \alpha_b x_b(R)$, we have that $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$.

Hence, the investor's expected profit under profit-sharing scheme when $i^* = 0$ is always greater than that under pure upfront fee since $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) < \Pi_{INV}^*(i^* = 0)$.

1.1.2) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)$ if $\beta > \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$:

$$= \alpha_g x_g(R) + \alpha_b x_b(R) - \alpha_g x_g(R)\beta + a(1 - \alpha_m)^2$$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)\right]}{\partial a} = (1 - \alpha_m)^2 \geq 0$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)$ is increasing in cost of information production a. This implies that there exists a threshold of information production cost $\hat{a}_2$which makes $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) = \Pi_{INV}^*(i^* = 1 - \alpha_m)$.

$$\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m) = 0$$

$$a = \hat{a}_2 = \frac{\alpha_g x_g(R)[\beta - 1] - \alpha_b x_b(R)}{(1 - \alpha_m)^2}$$

Then, $$\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m) = \begin{cases} < 0 \; if \; a < \hat{a}_2 \\ \geq 0 \; if a \geq \hat{a}_2 \end{cases}$$

We need to check if $\hat{a}_2$ is in an interval of $a < \frac{\alpha_g x_g(R)}{(1-\alpha_m)^2}\beta$ equivalent to $\beta > \bar{\bar{\beta}} \coloneqq \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$.

$$\hat{a}_2 \coloneqq \frac{\alpha_g x_g(R)[\beta - 1] - \alpha_b x_b(R)}{(1 - \alpha_m)^2} \lesseqgtr \frac{\alpha_g x_g(R)}{(1 - \alpha_m)^2}\beta$$

$$0 > \alpha_g x_g(R) + \alpha_b x_b(R)$$

It follows that $\hat{a}_2$ is not in an interval of $a < \frac{\alpha_g x_g(R)}{(1-\alpha_m)^2}\beta$ since $\hat{a}_2 > \frac{\alpha_g x_g(R)}{(1-\alpha_m)^2}\beta$. Thus, condition under unfavorable market that makes $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) \geq \Pi_{INV}^*(i^* = 1 - \alpha_m)$ should be impossible as long as $\beta > \bar{\bar{\beta}}$ equivalent to $a < \frac{\alpha_g x_g(R)}{(1-\alpha_m)^2}\beta$.

Accordingly, the investor's expected profit under profit-sharing scheme when the investor acquires $i^* = 1 - \alpha_m$ is always greater than that under pure upfront fee when the investor acquires $i_i^* = 0$ since the cost of information production a is less than $\hat{a}_2$.

2) Unfavorable market, $-\alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$.

2.1) For $\bar{\iota}^h \geq 0 > \bar{\iota}^l$.

The investor's expected profit under profit-sharing scheme if the CRA can mix rating deflation with full disclosure regime, $\bar{\imath}^h \geq 0 > \bar{\imath}^l$, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = 0 & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \Big[\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big][i_0 + \alpha_m] + \alpha_b x_b(R)\Big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

Where $\underline{\beta} = \frac{4a\bar{\imath}^h}{\alpha_g x_g(R) - \alpha_b x_b(R)}$, and $\bar{\beta} = \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$.

2.1.1) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 0)$ if $\beta \leq \frac{4a\bar{\imath}^h}{\alpha_g x_g(R) - \alpha_b x_b(R)}$:

$$= \alpha_g x_g(R) + \alpha_b x_b(R) - 0$$

From unfavorable market, $-\alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$, we have that $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$.

Hence, the investor's expected profit under profit-sharing payment scheme is always greater than that under pure upfront fee since $\alpha_g x_g(R) + \alpha_b x_b(R) < 0$ in unfavorable market.

2.1.2) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = i_0)$ if $\frac{4a\bar{\imath}^h}{\alpha_g x_g(R) - \alpha_b x_b(R)} < \beta < \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$

where $i_0 = \frac{\alpha_g x_g(R) - \alpha_b x_b(R)}{2a}\beta$ and $C(i_0) = a{i_0}^2$:

$$= \alpha_g x_g(R) + \alpha_b x_b(R) - \frac{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]^2\beta^2}{4a} - \alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]\beta - \alpha_b x_b(R)\beta$$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = i_0)$ with respect to a as follow.

$$\frac{\partial\big[\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = i_0)\big]}{\partial a} = \frac{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]^2\beta^2}{4a^2} \geq 0$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = i_0)$ is increasing in cost of information production a. This implies that there exists a threshold of information production cost $\hat{a}_1$which makes $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) = \Pi_{INV}^*(i^* = i_0)$.

$$\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = i_0) = 0$$

$$a = \hat{a}_1 = \frac{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]^2\beta^2}{4\big(\alpha_g x_g(R) + \alpha_b x_b(R)[1 - \beta] - \alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]\beta\big)}$$

Thus, $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)=\begin{cases}<0\ if\ a<\hat{a}_1\\ \geq 0\ if a\geq \hat{a}_1\end{cases}$

We need to check if $\hat{a}_1$ is in an interval of $\underline{\beta}<\beta<\bar{\beta}$ equivalent to $\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{2(1-\alpha_m)}\beta<a<\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4\bar{\iota}^h}\beta$.

$$\hat{a}_1 \lesseqgtr \frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4\bar{\iota}^h}\beta:$$

$$0 \lesseqgtr \frac{\bar{\iota}^l}{\bar{\iota}^h}-1$$

Then, we obtain that $\hat{a}_1>\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4\bar{\iota}^h}\beta$.

It follows that $\hat{a}_1$is not in the interval of $\underline{\beta}<\beta<\bar{\beta}$ since $\hat{a}_1>\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4\bar{\iota}^h}\beta$. Thus, condition under unfavorable market that makes $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)\geq\Pi_{INV}^*(i^*=i_0)$ should be impossible as long as $\underline{\beta}<\beta<\bar{\beta}$, equivalent to $\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{2(1-\alpha_m)}\beta<a<\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4\bar{\iota}^h}\beta$.

Accordingly, the investor's expected profit under profit-sharing scheme when the investor acquires $i^*=i_0$ is always greater than that under pure upfront fee since the cost of information production a is always less than $\hat{a}_1$.

2.1.3) $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=1-\alpha_m)$ if $\beta\geq\frac{2a(1-\alpha_m)}{\alpha_g x_g(R)-\alpha_b x_b(R)}$:

$$=\alpha_g x_g(R)+\alpha_b x_b(R)-\alpha_g x_g(R)\beta+a(1-\alpha_m)^2$$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=1-\alpha_m)$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=1-\alpha_m)\right]}{\partial a}=(1-\alpha_m)^2\geq 0$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=1-\alpha_m)$ is increasing in cost of information production a. This implies that there exists a threshold of information production cost $\hat{a}_2$which makes $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)=\Pi_{INV}^*(i^*=1-\alpha_m)$.

$$\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=1-\alpha_m)=0$$

$$a=\hat{a}_2=\frac{\alpha_g x_g(R)[\beta-1]-\alpha_b x_b(R)}{(1-\alpha_m)^2}$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=1-\alpha_m)=\begin{cases}<0\ if\ a<\hat{a}_2\\ \geq 0\ if a\geq \hat{a}_2\end{cases}$

We need to check if $\hat{a}_2$ is in an interval of $\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{2(1-\alpha_m)}\beta \leq a$ equivalent to $\beta \geq \bar{\beta}$. It follows that $\hat{a}_2$ is not in an interval of $\beta \geq \bar{\beta}$ since $\hat{a}_2 > \frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{2(1-\alpha_m)}\beta$. Thus, condition under unfavorable market that makes $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) \geq \Pi_{INV}^*(i^* = 1 - \alpha_m)$ should be impossible as long as $\beta \geq \bar{\beta}$ equivalent to $a \leq \frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{2(1-\alpha_m)}\beta$.

Accordingly, the investor's expected profit under profit-sharing scheme when the investor acquires $i^* = 1 - \alpha_m$ is always greater than that under pure upfront fee since the cost of information production a is less than $\hat{a}_2$.

In extremely unfavorable market, we can summarize that the profit-sharing scheme always yields higher benefit to the investor than the pure upfront fee. In unfavorable market, the profit-sharing scheme generates higher benefit to the investor only if the information cost is sufficiently low. Otherwise, the pure upfront fee generates higher benefit to the investor.

3) Favorable market, $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) \leq \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$.

3.1) For $\bar{\iota}^l \geq 0 > \bar{\iota}^h$.

The investor's expected profit under profit-sharing scheme if the CRA can mix rating inflation with full disclosure regime, $\bar{\iota}^l \geq 0 > \bar{\iota}^h$, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \big[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

Where $\underline{\beta} = \frac{4a\bar{\iota}^l}{\alpha_g x_g(R)-\alpha_b x_b(R)}$, and $\bar{\beta} = \frac{2a(1-\alpha_m)}{\alpha_g x_g(R)-\alpha_b x_b(R)}$.

3.1.1) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 0)$ if $\beta \leq \frac{4a\bar{\iota}^l}{\alpha_g x_g(R)-\alpha_b x_b(R)}$:

$$\alpha_g x_g(R) + \alpha_b x_b(R) - [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta \geq 0$$
$$1 \geq \beta$$

The investor's expected profit under pure upfront fee is always greater than that under profit-sharing payment scheme as long as the investor's retained share β is less than 1.

3.1.2) $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)$ if $\frac{4a\bar{\iota}^l}{\alpha_g x_g(R)-\alpha_b x_b(R)}\beta<\frac{2a(1-\alpha_m)}{\alpha_g x_g(R)-\alpha_b x_b(R)}$:

$=\alpha_g x_g(R)+\alpha_b x_b(R)-\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]^2\beta^2}{4a}-\alpha_m[\alpha_g x_g(R)-\alpha_b x_b(R)]\beta-\alpha_b x_b(R)\beta$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)\right]}{\partial a}=\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]^2\beta^2}{4a^2}\geq 0$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)$ is increasing in cost of information production a. This implies that there exists a threshold of information production cost $\hat{a}_1$which makes $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)=\Pi_{INV}^*(i^*=i_0)$.

$$\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)=0$$

$$a=\hat{a}_1=\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]^2\beta^2}{4\left(\alpha_g x_g(R)+\alpha_b x_b(R)[1-\beta]-\alpha_m[\alpha_g x_g(R)-\alpha_b x_b(R)]\beta\right)}$$

Thus, $$\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)=\begin{cases}>0\ if\ a>\hat{a}_1\\ \leq 0\ if a\leq\hat{a}_1\end{cases}$$

We need to check if $\hat{a}_1$ is in an interval of $\underline{\beta}<\beta<\bar{\beta}$ equivalent to $\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{2(1-\alpha_m)}\beta<a<\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4\bar{\iota}^l}\beta$.

(i) When $\hat{a}_1\lesseqgtr\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{4\bar{\iota}^l}\beta$, we have that $\hat{a}_1\leq\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]\beta}{4\bar{\iota}^l}$ since $\bar{\iota}^h\leq\bar{\iota}^l$.

(ii) When $\hat{a}_1\lesseqgtr\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]}{2(1-\alpha_m)}\beta$, we then obtain that $\bar{a}_1$ can be more or less than $\frac{[\alpha_g x_g(R)-\alpha_b x_b(R)]\beta}{2(1-\alpha_m)}$.

Thus, condition of $\Pi_{INV}^{\tilde{r}=h}(i=i_i^*,\beta=1)-\Pi_{INV}^*(i^*=i_0)=\begin{cases}<0\ if\ a<\hat{a}_1\\ \geq 0\ if a\geq\hat{a}_1\end{cases}$ is satisfied. The investor's expected profit under profit-sharing scheme is greater than that under pure upfront fee if and only if cost of information production is less than the threshold of information production cost, $a<\hat{a}_1$. In other words, the sharing-profit scheme provides higher profit only if the information production cost is sufficiently low.

3.1.3) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)$ if $\beta \geq \frac{2a(1-\alpha_m)}{\alpha_g x_g(R) - \alpha_b x_b(R)}$:

$$= \alpha_g x_g(R) + \alpha_b x_b(R) - \alpha_g x_g(R)\beta + a(1 - \alpha_m)^2$$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)\right]}{\partial a} = (1 - \alpha_m)^2 \geq 0$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m)$ is increasing in cost of information production a. This implies that there exists a threshold of information production cost $\hat{a}_2$which makes $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) = \Pi_{INV}^*(i^* = 1 - \alpha_m)$.

$$\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m) = 0$$

$$a = \hat{a}_2 = \frac{\alpha_g x_g(R)[\beta - 1] - \alpha_b x_b(R)}{(1 - \alpha_m)^2}$$

Then, we have that $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m) = \begin{cases} < 0 \; if \; a < \hat{a}_2 \\ \geq 0 \; if a \geq \hat{a}_2 \end{cases}$.

We need to check if $\hat{a}_2$ is in an interval of $\beta \geq \bar{\beta}$ equivalent to $\frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}{2(1-\alpha_m)}\beta \leq a$.

$$\hat{a}_2 \lesseqgtr \frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}{2(1 - \alpha_m)}\beta$$

$$\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta + \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]\beta \lesseqgtr 2\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]$$

Knowing that $\alpha_m = \max\{\alpha_g, \alpha_b\} \geq \frac{1}{2}$ and $0 \leq \alpha_g x_g(R) + \alpha_b x_b(R) \leq -2\alpha_b x_b(R)$;

If $\alpha_m = \frac{1}{2}$, then $\quad \left[3\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta \lesseqgtr 4\left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]$

If $\alpha_m = 1$, then $\quad \beta \lesseqgtr \frac{\alpha_g x_g(R) + \alpha_b x_b(R)}{\alpha_g x_g(R)}$

Accordingly, $\hat{a}_2$ can be more or less than $\frac{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}{2(1-\alpha_m)}\beta$ or there is a possibility that $\hat{a}_2$ is in an interval of $\beta \geq \bar{\beta}$. Thus, condition of $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m) = \begin{cases} < 0 \; if \; a < \hat{a}_2 \\ \geq 0 \; if a \geq \hat{a}_2 \end{cases}$ is satisfied. The investor's expected profit under profit-sharing scheme is greater than that under pure upfront fee if and only if cost of information production is less than the threshold of information production cost, $a < \hat{a}_2$. In other words, the sharing-profit scheme provides higher profit only if the information production cost is sufficiently low.

4) Favorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \geq \alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]$.

4.1) For $\bar{\iota}^l \geq 0 > \bar{\iota}^h$.

The investor's expected profit under profit-sharing scheme if the CRA can mix rating inflation with full disclosure regime, $\bar{\iota}^l \geq 0 > \bar{\iota}^h$, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta & \text{if } \beta \leq \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \\ \Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2 & \text{if } \beta > \bar{\bar{\beta}} := \dfrac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \end{cases}.$$

4.1.1) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 0)$ if $\beta \leq \bar{\bar{\beta}} := \frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)}$:

$$\alpha_g x_g(R) + \alpha_b x_b(R) - \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta \geq 0$$

$$1 \geq \beta$$

The investor's expected profit under pure upfront fee is always greater than that under profit-sharing payment scheme as long as the investor's retained share β is less than 1.

4.1.2) $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1-\alpha_m)$ if $\beta > \bar{\bar{\beta}} := \frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)}$:

$$= \alpha_g x_g(R) + \alpha_b x_b(R) - \alpha_g x_g(R)\beta + a(1-\alpha_m)^2$$

We can derive the first differentiation of $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1-\alpha_m)$ with respect to a as follow.

$$\frac{\partial\left[\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1-\alpha_m)\right]}{\partial a} = (1-\alpha_m)^2 \geq 0$$

Then, $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1-\alpha_m)$ is increasing in cost of information production a. This implies that there exists a threshold of information production cost $\hat{a}_2$which makes $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) = \Pi_{INV}^*(i^* = 1-\alpha_m)$.

$$\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1-\alpha_m) = 0$$

$$a = \hat{a}_2 = \frac{\alpha_g x_g(R)[\beta - 1] - \alpha_b x_b(R)}{(1-\alpha_m)^2}$$

Then, we have that $\Pi_{INV}^{\tilde{r}=h}(i = i_i^*, \beta = 1) - \Pi_{INV}^*(i^* = 1 - \alpha_m) = \begin{cases} \leq 0 \; if \; a \leq \hat{a}_2 \\ > 0 \; if a > \hat{a}_2 \end{cases}$.

We need to check if $\hat{a}_2$ is in an interval of $\frac{-\alpha_b x_b(R)}{(1-\alpha_m)^2}\beta > a$ equivalent to $\beta > \bar{\bar{\beta}}$. It follows that $\hat{a}_2 \leq \frac{-\alpha_b x_b(R)}{(1-\alpha_m)^2}\beta$ or $\hat{a}_2$ is in an interval of $\beta > \bar{\bar{\beta}}$.

Accordingly, the investor's expected profit under profit-sharing scheme is greater than that under pure upfront fee if and only if cost of information production is less than the threshold of information production cost, $a \leq \hat{a}_2$. In other words, the sharing-profit scheme provides higher profit only if the information production cost is sufficiently low.

Hence in extremely favorable and favorable market, we can conclude that if the investor decides not to acquire any additional information accuracy $i^* = 0$, he obtains higher expected profit under the pure upfront fee than that under the profit-sharing scheme. In addition, If the investor decides to request sufficiently high additional accuracy level $i^* \in \{i_0, 1 - \alpha_m\}$, he gains higher expected profit under profit-sharing payment scheme only if cost of information production a is sufficiently low.

Proof of Proposition 3.6: optimal retained profit (β^*) for profit-sharing scheme

We already learned from proposition 2 that the investor, when purchases rating information from the CRA, decides to strategically acquire the optimal level of additional information accuracy, $i^* \in \{0, i_0, 1 - \alpha_m\}$, and makes investment decision based on four market conditions. Then, we will show results of optimal retained share β^* allocated to the CRA in each market condition.

1) Extremely unfavorable market, $\alpha_g x_g(R) + \alpha_b x_b(R) \leq -\alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]$.

   Since the investor's expected profit depends on level of additional information accuracy thresholds, we thus need to consider two cases of the accuracy thresholds: 1.1) $\bar{\iota}^h \geq 0 > \bar{\iota}^l$, and 1.2) $0 > \bar{\iota}^h > \bar{\iota}^l$.

1.1) For $\bar{\iota}^h \geq 0 > \bar{\iota}^l$.

The investor's expected profit $\Pi_{INV}^{*}(i^{*})$, when the CRA mixes rating deflation with full disclosure regime, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^{*}) = \begin{cases} \Pi_{INV}^{*}(0) = 0 & if\ \beta \leq \bar{\bar{\beta}} \coloneqq \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \\ \Pi_{INV}^{*}(1-\alpha_m) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2 & if\ \beta > \bar{\bar{\beta}} \coloneqq \dfrac{a(1-\alpha_m)^2}{\alpha_g x_g(R)} \end{cases}$$

1.1.1) For $\beta \leq \bar{\bar{\beta}}$.

The optimal expected profit of the investor is given by $\Pi_{INV}^{*}(i^{*}=0)=0$. When we derive the first and second differentiation of $\Pi_{INV}^{*}(i^{*}=0)$ with respect to β, the optimal retained share β^{*} is obviously zero. Furthermore, the investor's expected profit, when he requests, $i^{*}=0$, and keeps, $\beta^{*}=0$, is given by

$$\Pi_{INV}^{*}(i^{*}=0, \beta^{*}=0)=0$$

1.1.2) For $\beta < \bar{\bar{\beta}}$.

The optimal expected profit of the investor is given by

$$\Pi_{INV}^{*}(i^{*}=1-\alpha_m) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2$$

We can derive the first differentiation of $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ with respect to β as follow.

$$\frac{\partial \Pi_{INV}^{*}(i^{*}=1-\alpha_m)}{\partial \beta} = \alpha_g x_g(R) \geq 0$$

Accordingly, the investor's expected profit when requesting $i^{*}=1-\alpha_m$ is monotonically increasing in retained share β. Thus, the optimal retained share is close to one, $\beta^{*} \to 1$.

We need to perform continuity test to check if the investor's expected profit between $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ are continuous at the point of $\beta = \bar{\bar{\beta}} \coloneqq \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$.

The investor's expected profit when requesting $i^{*}=0$ at the point of $\underline{\beta}$ is given by

$$\Pi_{INV}^{*}(i^{*}=0, \beta=\bar{\bar{\beta}}) = -C(i) = 0$$

The investor's expected profit when acquiring $i^{*}=1-\alpha_m$ at the point of $\bar{\bar{\beta}}$ is

$$\Pi_{INV}^{*}(i^{*}=1-\alpha_m, \beta=\bar{\bar{\beta}}) = \alpha_g x_g(R)\bar{\bar{\beta}} - a(1-\alpha_m)^2$$

Substitute $\bar{\bar{\beta}} = \frac{a(1-\alpha_m)^2}{\alpha_g x_g(R)}$ in $\Pi_{INV}^{*}\left(i^{*} = 1-\alpha_m, \beta = \bar{\bar{\beta}}\right)$,

$$\Pi_{INV}^{*}\left(i^{*} = 1-\alpha_m, \beta = \bar{\bar{\beta}}\right) = 0$$

Accordingly, the investor's expected profit between $\Pi_{INV}^{*}(i^{*} = 0)$ and $\Pi_{INV}^{*}(i^{*} = 1 - \alpha_m)$ are continuous at the point of $\beta = \bar{\bar{\beta}}$ since $\Pi_{INV}^{*}\left(i^{*} = 0, \beta = \bar{\bar{\beta}}\right) = \Pi_{INV}^{*}\left(i^{*} = 1 - \alpha_m, \beta = \bar{\bar{\beta}}\right)$.

Hence, we can conclude that it is optimal for the investor to choose retained share β close to one ($\beta^{*} \to 1$) in extremely unfavorable market since $\Pi_{INV}^{*}(i^{*} = 0)$ and $\Pi_{INV}^{*}(i^{*} = 1 - \alpha_m)$ are continuous at the point of $\beta = \bar{\bar{\beta}}$ and $\Pi_{INV}^{*}(i^{*} = 1 - \alpha_m)$ is increasing in retained share β.

1.2) For $0 > \bar{\iota}^{h} > \bar{\iota}^{l}$.

We learned from Proposition 2 that the CRA always implements full disclosure regime. Thus, the investor determines optimal retained share, β^{*}, based on his expected profit given by:

$$\Pi_{INV}^{\tilde{r}=h} = \max_{\beta}\left[\left[[\alpha_g x_g(R) - \alpha_b x_b(R)][i + \alpha_m] + \alpha_b x_b(R)\right]\beta - ai^2\right]$$

We can derive the first and second derivatives of $\Pi_{INV}^{\tilde{r}=h}$ with respect to the decision variable, β, as follows.

First differentiation: $\frac{\partial \Pi_{INV}^{\tilde{r}=h}}{\partial \beta} = \left[\alpha_g x_g(R) - \alpha_b x_b(R)\right](i + \alpha_m) + \alpha_b x_b(R)$

Thus, the investor's expected function is a linear function of retained share β so that the optimal solution must be boundary. In other words, the optimal retained share that the investor chooses to maximize his expected profit should be either nearly one ($\beta \to 1$) or nearly zero ($\beta \to 0$).

To check if retained share close to zero ($\beta \to 0$) is optimal for the investor, we first consider the case that the investor's expected profit is decreasing in retained share, $\frac{\partial \Pi_{INV}^{\tilde{r}=h}}{\partial \beta} < 0$.

$$\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right](i + \alpha_m) + \alpha_b x_b(R) < 0$$

$$i < \bar{\iota}^{h} := \frac{-\alpha_b x_b(R)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} - \alpha_m$$

From $0 > \bar{\iota}^h > \bar{\iota}^l$, the retained share close to zero ($\beta \to 0$) cannot be optimal for the investor because he can never acquire additional accuracy level, $i < \bar{\iota}^h$, since $\bar{\iota}^h < 0$.

To check if retained share close to one ($\beta \to 1$) is optimal for the investor, we subsequently consider the case that the investor's expected profit is increasing in retained share, $\frac{\partial \Pi_{INV}^{\tilde{r}=h}}{\partial \beta} \geq 0$.

$$\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right](i + \alpha_m) + \alpha_b x_b(R) \geq 0$$

$$i \geq \bar{\iota}^h := \frac{-\alpha_b x_b(R)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} - \alpha_m$$

Thus, the retained share close to one is optimal for the investor, $\beta^* \to 1$, as long as he purchases rating information from the CRA, $i \geq 0$.

2) Unfavorable market, $-\alpha_m\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right] < \alpha_g x_g(R) + \alpha_b x_b(R) < 0$. Since the investor's expected profit depends on level of additional information accuracy thresholds, we thus need to consider two cases of the accuracy thresholds: 2.1) $\bar{\iota}^h \geq 0 > \bar{\iota}^l$, and 2.2) $0 > \bar{\iota}^h > \bar{\iota}^l$.

2.1) For $\bar{\iota}^h \geq 0 > \bar{\iota}^l$.

The investor's expected profit $\Pi_{INV}^*(i^*)$, when the CRA mixes rating deflation with full disclosure regime, is given by:

$$\Pi_{INV}^*(i^*) = \begin{cases} \Pi_{INV}^*(i^* = 0) = -C(0) = 0 & if\ \beta \leq \underline{\beta} \\ \Pi_{INV}^*(i^* = i_0) = \left[\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right][i_0 + \alpha_m] + \alpha_b x_b(R)\right]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m) & if\ \beta \geq \bar{\beta} \end{cases}$$

Where $\underline{\beta} := \frac{4a\bar{\iota}^h}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]}$ and $\bar{\iota}^h = \frac{\alpha_g x_g(R)}{\left[\alpha_g x_g(R) - \alpha_b x_b(R)\right]} - \alpha_m$.

Accordingly, we consider three cases of $\Pi_{INV}^*(i^*)$ to characterize the optimal retained profit β^*.

2.1.1) For $\beta \leq \underline{\beta}$.

The optimal expected profit of the investor is given by $\Pi_{INV}^*(i^* = 0) = 0$. When we derive the first and second differentiation of $\Pi_{INV}^*(i^* = 0)$ with respect to β, the optimal retained share β^* is obviously zero. Furthermore, the investor's expected profit, when he requests, $i^* = 0$, and keeps, $\beta^* = 0$, is given by

$$\Pi_{INV}^*(i^* = 0, \beta^* = 0) = 0$$

2.1.2) For $\beta \geq \bar{\beta}$.

The optimal expected profit of the investor is given by

$$\Pi_{INV}^*(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - a(1 - \alpha_m)^2$$

We can derive the first differentiation of $\Pi_{INV}^*(i^* = 1 - \alpha_m)$ with respect to β as follow.

$$\frac{\partial \Pi_{INV}^*(i^* = 1 - \alpha_m)}{\partial \beta} = \alpha_g x_g(R) \geq 0$$

Accordingly, the investor's expected profit when the investor requests $i^* = 1 - \alpha_m$ is monotonically increasing in retained share β. Thus, the optimal retained share is one, $\beta^* = 1$.

2.1.3) For $\underline{\beta} < \beta < \bar{\beta}$.

The optimal expected profit of the investor is given by

$$\Pi_{INV}^*(i^* = i_0) = \Big[\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big][i_0 + \alpha_m] + \alpha_b x_b(R)\Big]\beta - C(i_0)$$

Substitute $i_0 = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta}{2a}$ and $C(i) = ai^2$ into $\Pi_{INV}^*(i^* = i_0)$.

$$\Pi_{INV}^*(i^* = i_0) = \frac{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]^2 \beta^2}{4a} + \alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]\beta + \alpha_b x_b(R)\beta$$

We can derive the first and second differentiation of $\Pi_{INV}^*(i^* = i_0)$ with respect to β as follow.

$$\frac{\partial \Pi_{INV}^*(i^* = i_0)}{\partial \beta} = \beta\frac{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]^2}{2a} + \alpha_m\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big] + \alpha_b x_b(R)$$

$$\frac{\partial^2 \Pi_{INV}^*(i^* = i_0)}{\partial \beta^2} = \frac{\big[\alpha_g x_g(R) - \alpha_b x_b(R)\big]^2}{2a} \geq 0$$

From the first and second differentiation, the optimal expected profit when the investor requests $i^* = i_0$ is convex in retained share β. Thus, we can obtain a minimal retained share β_0 from the first-order condition by

$\frac{\partial \Pi_{INV}^{*}(i^{*}=i_{0})}{\partial \beta}=\beta \frac{\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]^{2}}{2a}+\alpha_{m}\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]+\alpha_{b} x_{b}(R)=$ 0

From $\bar{\imath}^{h}=-\left(\frac{\alpha_{b} x_{b}(R)}{\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]}+\alpha_{m}\right)$;

$$\beta=\frac{2a}{\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]}\left(\frac{-\alpha_{b} x_{b}(R)}{\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]}-\alpha_{m}\right)$$

Then, the minimal retained profit is $\beta=\beta_{0}:=\frac{2a\bar{\imath}^{h}}{\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]}$ or $\beta_{0}=\frac{1}{2}\underline{\beta}$.

From $\bar{\imath}^{h} \geq 0>\bar{\imath}^{l}$ in unfavorable market, it is straightforward to see that the minimal retained share, $\beta_{0}=\frac{1}{2}\underline{\beta}$, is not in an interval of $\beta_{0}<\underline{\beta}<\bar{\beta}$ since $\bar{\imath}^{h} \geq 0$.

Accordingly, we have that the retained share is maximized at the point of $\bar{\beta}$, $\beta^{*}=\bar{\beta}$, since $\Pi_{INV}^{*}(i^{*}=i_{0})$ is increasingly convex in retained share β in an interval of $\underline{\beta}<\beta<\bar{\beta}$.

From $\beta_{0}:=\frac{2a\bar{\imath}^{h}}{\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)} \geq 0$ since $\bar{\imath}^{h} \geq 0$, we have that β_{0} and $\underline{\beta}$ are greater than zero. Accordingly, we need to perform continuity test to examine if the investor's expected profit between $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=i_{0})$ are continuous at the point of $\beta=\underline{\beta}$, and if the investor's expected profit between $\Pi_{INV}^{*}(i^{*}=i_{0})$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_{m})$ are continuous at the point of $\beta=\bar{\beta}$.

Firstly, we check whether or not $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=i_{0})$ are continuous at the point of $\beta=\underline{\beta}$ as follow.

The investor's expected profit when requesting $i^{*}=0$ at the point of $\underline{\beta}$ is given by

$$\Pi_{INV}^{*}\left(i^{*}=0, \beta=\underline{\beta}\right)=-C(i)=0 \qquad \text{----(B8)}$$

The investor's expected profit when acquiring $i^{*}=i_{0}$ at the point of $\underline{\beta}$ is given by

$$\Pi_{INV}^{*}\left(i^{*}=i_{0}, \beta=\underline{\beta}\right)=4a\bar{\imath}^{h}\left(\frac{-\alpha_{b} x_{b}(R)}{\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]}-\alpha_{m}+\alpha_{m}+\frac{\alpha_{b} x_{b}(R)}{\left[\alpha_{g} x_{g}(R)-\alpha_{b} x_{b}(R)\right]}\right)$$

$$\Pi_{INV}^{*}\left(i^{*}=i_{0}, \beta=\underline{\beta}\right)=0 \qquad \text{----(B9)}$$

From (B8) and (B9), we have that the investor's expected profits are continuous at the point of $\beta=\underline{\beta}$ since $\Pi_{INV}^{*}\left(i^{*}=0, \beta=\underline{\beta}\right)=\Pi_{INV}^{*}\left(i^{*}=i_{0}, \beta=\underline{\beta}\right)$.

Next, we examine whether or not $\Pi_{INV}^{*}(i^{*}=i_0)$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ are continuous at the point of $\beta=\bar{\beta}:=\frac{2a(1-\alpha_m)}{\alpha_g x_g(R)-\alpha_b x_b(R)}$ as follow.

The investor's expected profit when requesting $i^{*}=i_0$ at the point of $\bar{\beta}$ is given by

$$\Pi_{INV}^{*}\left(i^{*}=i_0,\beta=\underline{\beta}\right)=a(1-\alpha_m)\left[\frac{\alpha_g x_g(R)-\alpha_b x_b(R)}{\alpha_g x_g(R)-\alpha_b x_b(R)}+\alpha_m+\frac{2\alpha_b x_b(R)}{\alpha_g x_g(R)-\alpha_b x_b(R)}\right]$$

$$\Pi_{INV}^{*}\left(i^{*}=i_0,\beta=\underline{\beta}\right)=a(1-\alpha_m)\left[\frac{\alpha_g x_g(R)+\alpha_b x_b(R)}{\alpha_g x_g(R)-\alpha_b x_b(R)}+\alpha_m\right] \qquad \text{----(B10)}$$

The investor's expected profit when acquiring $i^{*}=1-\alpha_m$ at the point of $\bar{\beta}$ is given by

$$\Pi_{INV}^{*}(i^{*}=1-\alpha_m)|_{\beta=\bar{\beta}}=a(1-\alpha_m)\left[\frac{\alpha_g x_g(R)+\alpha_b x_b(R)}{\alpha_g x_g(R)-\alpha_b x_b(R)}+\alpha_m\right] \qquad \text{----(B11)}$$

From (B10) and (B11), we have that the investor's expected profits are continuous at $\beta=\bar{\beta}$ since$\Pi_{INV}^{*}\left(i^{*}=i_0,\beta=\underline{\beta}\right)=\Pi_{INV}^{*}(i^{*}=1-\alpha_m)|_{\beta=\bar{\beta}}$.

Accordingly, we have that $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=i_0)$ are continuous at the point of $\underline{\beta}$ while $\Pi_{INV}^{*}(i^{*}=i_0)$ is increasingly convex in the retained share β. In addition, $\Pi_{INV}^{*}(i^{*}=i_0)$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ are continuous at the point of $\bar{\beta}$ while $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ is monotonically increasing in the retained share β. Hence, we can conclude that the optimal retained share $\beta^{*}$the investor keeps is close to 1.

2.2) For $0>\bar{\iota}^{h}>\bar{\iota}^{l}$.

The same result as 1.2) in Proposition 6 holds since the investor's expected profit $\Pi_{INV}^{\tilde{r}=h}$ when the CRA always implements full disclosure regime is the same.

Hence, we can conclude that it is optimal for the investor to keep retained share β close to one, $\beta^{*}\rightarrow 1$, in extremely unfavorable and unfavorable market.

3) Favorable market, $0\leq\alpha_g x_g(R)+\alpha_b x_b(R)<\alpha_m\left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]$.

Since the investor's expected profit depends on level of additional information accuracy thresholds, we thus need to consider two cases of the accuracy thresholds: 3.1) $\bar{\iota}^{l}\geq 0>\bar{\iota}^{h}$, and 3.2) $0>\bar{\iota}^{l}>\bar{\iota}^{h}$.

3.1) For $\bar{\iota}^{l}\geq 0>\bar{\iota}^{h}$.

The investor's expected profit $\Pi_{INV}^{*}(i^{*})$, when the CRA mixes rating inflation with full disclosure regime, is given by:

$$\Pi^*_{INV}(i^*) = \begin{cases} \Pi^*_{INV}(i^* = 0) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta & if\ \beta \le \underline{\beta} \\ \Pi^*_{INV}(i^* = i_0) = \big[[\alpha_g x_g(R) - \alpha_b x_b(R)][i_0 + \alpha_m] + \alpha_b x_b(R)\big]\beta - C(i_0) & if\ \underline{\beta} < \beta < \bar{\beta} \\ \Pi^*_{INV}(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m) & if\ \beta \ge \bar{\beta} \end{cases}$$

Where $\underline{\beta} := \frac{4a\bar{\imath}^l}{[\alpha_g x_g(R) - \alpha_b x_b(R)]}$ and $\bar{\imath}^l = \frac{\alpha_g x_g(R)}{[\alpha_g x_g(R) - \alpha_b x_b(R)]} - \alpha_m$.

3.1.1) For $\beta \le \underline{\beta}$.

The optimal expected profit of the investor is given by:

$$\Pi^*_{INV}(i^* = 0) = [\alpha_g x_g(R) + \alpha_b x_b(R)]\beta$$

We can derive the first differentiation of $\Pi^*_{INV}(i^* = 0)$ with respect to β as follow.

$$\frac{\partial \Pi^*_{INV}(i^* = 0)}{\partial \beta} = \alpha_g x_g(R) + \alpha_b x_b(R) \ge 0$$

Accordingly, the investor's expected profit $\Pi^*_{INV}(i^* = 0)$ is monotonically increasing in the retained share β so that the optimal retained share β^* is $\underline{\beta}$. The investor's expected profit when he acquires $i^* = 0$ and keeps $\beta^* = \underline{\beta}$ is given by:

$$\Pi^*_{INV}\left(i^* = 0, \beta^* = \underline{\beta}\right) = 4a\bar{\imath}^l \left[\frac{\alpha_g x_g(R) + \alpha_b x_b(R)}{\alpha_g x_g(R) - \alpha_b x_b(R)}\right]$$

3.1.2) For $\beta \ge \bar{\beta}$.

The optimal expected profit of the investor is given by

$$\Pi^*_{INV}(i^* = 1 - \alpha_m) = \alpha_g x_g(R)\beta - C(1 - \alpha_m)$$

We can derive the first differentiation of $\Pi^*_{INV}(i^* = 1 - \alpha_m)$ with respect to β. It follows that $\frac{\partial \Pi^*_{INV}(i^* = 1 - \alpha_m)}{\partial \beta} = \alpha_g x_g(R) \ge 0$.

Thus, the optimal expected profit when the investor requests $i^* = 1 - \alpha_m$ is monotonically increasing in retained share β. The optimal retained share is close to one, $\beta^* \to 1$.

3.1.3) For $\underline{\beta} < \beta < \bar{\beta}$.

The optimal expected profit of the investor is given by

$$\Pi^*_{INV}(i^* = i_0) = \frac{[\alpha_g x_g(R) - \alpha_b x_b(R)]^2 \beta^2}{4a} + \alpha_m[\alpha_g x_g(R) - \alpha_b x_b(R)]\beta + \alpha_b x_b(R)\beta$$

We can derive the first and second differentiation of $\Pi^*_{INV}(i^* = i_0)$ with respect to β as follow.

$$\frac{\partial \Pi_{INV}^{*}(i^{*}=i_{0})}{\partial \beta}=\beta\frac{\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]^{2}}{2a}+\alpha_{m}\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]+\alpha_{b}x_{b}(R)$$

$$\frac{\partial^{2} \Pi_{INV}^{*}(i^{*}=i_{0})}{\partial \beta^{2}}=\frac{\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]^{2}}{2a}\geq 0$$

Accordingly, the optimal expected profit when the investor requests $i^{*}=i_{0}$ is convex in retained share β. Thus, we can obtain a minimal retained share β_{0} from the first-order condition by

$$\frac{\partial \Pi_{INV}^{*}(i^{*}=i_{0})}{\partial \beta}=\beta\frac{\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]^{2}}{2a}+\alpha_{m}\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]+\alpha_{b}x_{b}(R)=0$$

$$\beta=\frac{2a}{\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)}\left[\frac{-\alpha_{b}x_{b}(R)}{\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)}-\alpha_{m}\right]$$

Then, the minimal retained share is $\beta=\beta_{0}:=\frac{2a\bar{\iota}^{h}}{\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]}$.

In favorable market, $0\leq\alpha_{g}x_{g}(R)+\alpha_{b}x_{b}(R)\leq\alpha_{m}\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]$, it is simple to see that the minimal retained share, $\beta_{0}<0$, is not in an interval of $\beta_{0}<\underline{\beta}<\bar{\beta}$ since $\bar{\iota}^{h}<0$.

Accordingly, we have that the retained share is maximized at the point of $\bar{\beta}$, $\beta^{*}=\bar{\beta}$, since $\Pi_{INV}^{*}(i^{*}=i_{0})$ is increasingly convex in retained share β in an interval of $\underline{\beta}<\beta<\bar{\beta}$.

From $\underline{\beta}:=\frac{4a\bar{\iota}^{l}}{\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]}\geq 0$ where $\bar{\iota}^{l}\geq 0$, we need to perform continuity test to examine if the investor's expected profit between $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=i_{0})$ are continuous at the point of $\beta=\underline{\beta}$, and if the investor's expected profit between $\Pi_{INV}^{*}(i^{*}=i_{0})$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_{m})$ are continuous at the point of $\beta=\bar{\beta}$.

Firstly, we check whether or not $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=i_{0})$ are continuous at the point of $\beta=\underline{\beta}$ where $\underline{\beta}=\frac{4a\bar{\iota}^{l}}{\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]}$ and $\bar{\iota}^{l}=\frac{\alpha_{g}x_{g}(R)}{\left[\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)\right]}-\alpha_{m}$ as follow.

The investor's expected profit when requesting $i^{*}=0$ at the point of $\underline{\beta}$ is given by

$$\Pi_{INV}^{*}\left(i^{*}=0,\beta=\underline{\beta}\right)=4a\bar{\iota}^{l}\left(\frac{\alpha_{g}x_{g}(R)+\alpha_{b}x_{b}(R)}{\alpha_{g}x_{g}(R)-\alpha_{b}x_{b}(R)}\right) \quad \text{----(B12)}$$

The investor's expected profit when requesting $i^{*}=i_{0}$ at the point of $\underline{\beta}$ is given by

$$\Pi_{INV}^{*}\left(i^{*}=i_0,\beta=\underline{\beta}\right)=4a\bar{\iota}^{l}\left(\frac{\alpha_g x_g(R)+\alpha_b x_b(R)}{\alpha_g x_g(R)-\alpha_b x_b(R)}\right) \qquad \text{----(B13)}$$

From (B12) and (B13), we have that the investor's expected profits are continuous at the point of $\beta=\underline{\beta}$ since $\Pi_{INV}^{*}\left(i^{*}=0,\beta=\underline{\beta}\right)=\Pi_{INV}^{*}\left(i^{*}=i_0,\beta=\underline{\beta}\right)$.

Next, we examine whether or not $\Pi_{INV}^{*}(i^{*}=i_0)$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ are continuous at the point of $\beta=\bar{\beta}:=\frac{2a(1-\alpha_m)}{\alpha_g x_g(R)-\alpha_b x_b(R)}$ as follow.

The investor's expected profit when requesting $i^{*}=i_0$ at the point of $\bar{\beta}$ is given by

$$\Pi_{INV}^{*}\left(i^{*}=i_0,\beta=\bar{\beta}\right)=a(1-\alpha_m)\left[\frac{\alpha_g x_g(R)+\alpha_b x_b(R)}{\alpha_g x_g(R)-\alpha_b x_b(R)}+\alpha_m\right] \qquad \text{----(B14)}$$

The investor's expected profit when acquiring $i^{*}=1-\alpha_m$ at the point of $\bar{\beta}$ is

$$\Pi_{INV}^{*}(i^{*}=1-\alpha_m)|_{\beta=\bar{\beta}}=a(1-\alpha_m)\left[\frac{\alpha_g x_g(R)+\alpha_b x_b(R)}{\alpha_g x_g(R)-\alpha_b x_b(R)}+\alpha_m\right] \qquad \text{----(B15)}$$

From (B14) and (B15), we have that the investor's expected profits are continuous at $\beta=\bar{\beta}$ since $\Pi_{INV}^{*}\left(i^{*}=i_0,\beta=\bar{\beta}\right)=\Pi_{INV}^{*}\left(i^{*}=1-\alpha_m,\beta=\bar{\beta}\right)$.

Accordingly, we have that $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=i_0)$ are continuous at the point of $\underline{\beta}$ while $\Pi_{INV}^{*}(i^{*}=i_0)$ is increasingly convex in the retained share β. In addition, $\Pi_{INV}^{*}(i^{*}=i_0)$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ are continuous at the point of $\bar{\beta}$ while $\Pi_{INV}^{*}(i^{*}=1-\alpha_m)$ is monotonically increasing in the retained share β. Hence, we can conclude that the optimal retained share β^{*} the investor keeps is close to 1.

3.2) For $0>\bar{\iota}^{l}>\bar{\iota}^{h}$.

The same result as 1.2) in Proposition 6 holds since the investor's expected profits $\Pi_{INV}^{*}(i^{*})$ when the CRA always adopts full disclosure regime are similar for all market conditions.

4) Extremely favorable market, $\alpha_g x_g(R)+\alpha_b x_b(R)\geq\alpha_m\left[\alpha_g x_g(R)-\alpha_b x_b(R)\right]$.

   Since the investor's expected profit depends on level of additional information accuracy thresholds, we thus need to consider two cases of the accuracy thresholds: 4.1) $\bar{\iota}^{l}\geq 0>\bar{\iota}^{h}$, and 4.2) $0>\bar{\iota}^{l}>\bar{\iota}^{h}$.

4.1) For $\bar{\iota}^{l}\geq 0>\bar{\iota}^{h}$.

The investor's expected profit $\Pi_{INV}^{*}(i^{*})$, when the CRA mixes rating inflation with full disclosure regime, is given by:

$$\Pi_{INV}^{\tilde{r}=h}(i^*) =$$

$$\begin{cases} \Pi_{INV}^*(0) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta & if\ \beta \le \bar{\bar{\beta}} := \frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \\ \Pi_{INV}^*(1-\alpha_m) = \alpha_g x_g(R)\beta - a(1-\alpha_m)^2 & if\ \beta > \bar{\bar{\beta}} := \frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)} \end{cases}$$

4.1.1) For $\beta \le \bar{\bar{\beta}}$.

The optimal expected profit of the investor is given by:

$$\Pi_{INV}^*(i^* = 0) = \left[\alpha_g x_g(R) + \alpha_b x_b(R)\right]\beta$$

We can derive the first differentiation of $\Pi_{INV}^*(i^* = 0)$ with respect to β as follow.

$$\frac{\partial \Pi_{INV}^*(i^* = 0)}{\partial \beta} = \alpha_g x_g(R) + \alpha_b x_b(R) \ge 0$$

Accordingly, the investor's expected profit $\Pi_{INV}^*(i^* = 0)$ is monotonically increasing in the retained share β so that the optimal retained share β^* is $\bar{\bar{\beta}}$.

4.1.2) For $\beta \ge \bar{\beta}$.

The optimal expected profit of the investor is given by

$$\Pi_{INV}^*(i^* = 1-\alpha_m) = \alpha_g x_g(R)\beta - C(1-\alpha_m)$$

We can derive the first differentiation of $\Pi_{INV}^*(i^* = 1-\alpha_m)$ with respect to β. It follows that $\frac{\partial \Pi_{INV}^*(i^*=1-\alpha_m)}{\partial \beta} = \alpha_g x_g(R) \ge 0$.

Thus, the optimal expected profit when the investor requests $i^* = 1-\alpha_m$ is monotonically increasing in retained share β. The optimal retained share is close to one, $\beta^* \to 1$.

We need to perform continuity test to check if the investor's expected profit between $\Pi_{INV}^*(i^* = 0)$ and $\Pi_{INV}^*(i^* = 1-\alpha_m)$ are continuous at the point of $\beta = \bar{\bar{\beta}} := \frac{a(1-\alpha_m)^2}{-\alpha_b x_b(R)}$.

The investor's expected profit when requesting $i^* = 0$ at the point of $\bar{\bar{\beta}}$ is given by

$$\Pi_{INV}^*\left(i^* = 0, \beta = \bar{\bar{\beta}}\right) = \left[\frac{\alpha_g x_g(R)}{-\alpha_b x_b(R)} - 1\right] a(1-\alpha_m)^2$$

The investor's expected profit when acquiring $i^* = 1-\alpha_m$ at the point of $\bar{\bar{\beta}}$ is given by

$$\Pi_{INV}^*\left(i^* = 1-\alpha_m, \beta = \bar{\bar{\beta}}\right) = \left[\frac{\alpha_g x_g(R)}{-\alpha_b x_b(R)} - 1\right] a(1-\alpha_m)^2$$

Accordingly, the investor's expected profit between $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_{m})$ are continuous at the point of $\beta=\bar{\bar{\beta}}$ since $\Pi_{INV}^{*}\left(i^{*}=0,\beta=\bar{\bar{\beta}}\right)=\Pi_{INV}^{*}\left(i^{*}=1-\alpha_{m},\beta=\bar{\bar{\beta}}\right)$.

Hence, we can conclude that it is optimal for the investor to choose retained share β close to one, $\beta^{*}\rightarrow 1$, in extremely favorable market since $\Pi_{INV}^{*}(i^{*}=0)$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_{m})$ are continuous at point of $\bar{\bar{\beta}}$ and $\Pi_{INV}^{*}(i^{*}=1-\alpha_{m})$ is increasing in β.

4.2) For $0>\bar{\iota}^{l}>\bar{\iota}^{h}$.

The same result as 1.2) in Proposition 3.6 holds since the investor's expected profits $\Pi_{INV}^{*}(i^{*})$ when the CRA always adopts full disclosure regime are similar for all market conditions. Therefore, it is in the best interest of the investor to keep retained share β close to one, $\beta^{*}\rightarrow 1$, regardless of market conditions.

BIOGRAPHY

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