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Original Article

Free convection of an electrically conducting fluid in a micro-channel with super-hydrophobic slip condition

Mani Ramanuja[1, 2], G. Gopi Krishna[2*], V. Naga Radhika[3], and K. Maruthi Prasad[4]

[1] *Department of Mathematics, GITAM Institute of Technology and Management, Bangalore, Karnatakta, 561203 India*

[2] *Department of Mathematics, Marri Laxman Reddy Institute of Technology and Management, Dundigal, Hyderabad, 500043 India*

[3] *Department of Mathematics, GITAM Institute of Technology and Management, Bangalore, Karnatakta, 561203 India*

[4] *Department of Mathematics, School of Science, GITAM Institute of Technology and Management, Hyderabad Campus, Hyderabad, Telangana, 502329 India*

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Abstract

The present investigation is about the incompressible natural convective fluid flow which is applied in the magnetic field. The vertical movement is applied over an isothermally heated open-ended vertical micro-channel. The super-hydrophobic slip, in addition to temperature jump situations under the effect of radiation and buoyancy forces, is applied on a vertical micro-channel. The model equations, momentum, and energy balanced are advanced and worked numerical by applying a Runge-Kutta fourth-order technique. Numerical results were obtained in both cases. The first case represents that the super-hydrophobic surface was heated, and the second case represents the no-slip surface which was unheated. It is established that the temperature profiles increase with the buoyancy parameter.

Keywords: MHD, Buoyancy parameter, super-hydrophobic slip, Radiation, heat transfer

1. Introduction

Fluid flow in the micro-channel is considered advantageous for many practical applications involving natural convection determined by internal heat generation. Its applications are material treating operations also manufacturing, high-power-density chips utilized inside computers, and alternative electronic and up-coming technologies. On the other hand, not many investigations have been performed on mixed and natural convection in vertical micro-tube or micro-channels.

*Corresponding author
Email address: drgopikrishnag@gmail.com

Its large applications in engineering efforts, the natural convection of a covered fluid has been given significant importance in recent times. Experimentally, a previous study on this issue isothermal parallel-plate channel was recorded by (Elenbaas, 1942), they analyzed the different thermal restriction conditions (Poots, 1961) incorporated flow qualities like net mass flow and wall Nusselt numbers. Fan, Hwang, Knieper and Hwang (1967) analyzed this system through identical heat flux lying on the wall and examined it with the help of a finite-difference method.

The current inspection intends to examine the influence of rarefaction lying on fluid-wall interaction in vertical parallel-plate micro-channels resting on steady and entirely developed natural convection. It should be noted that (Aung, 1972) examined that the designs of micro-heat exchangers and micro-pumps are significant. Soundalgekar, and Haldavenekar (1973) investigated different phases of the natural convection with the help of thermal conducting non-perfect plates. (Yang & Yu, 1974) examined the MHD convective micro-channel flow entrance hall problem numerically among pair parallel plates connected to an inclined axial gradient and temperature of pressure. Rudraiah, Venkatachalappa, and Malashetty (1982) studied the combined analytical and numerical influence of an enlarge in value of the temperature variation connecting the permeability parameter on the velocity of the plate mass flow and temperature.

The study of natural convection flow in micro-scale systems ought to be comprehensively researched by Gebhart, Jaluria, Mahajan, Sammakia and Yovanovich (1989). Gad-el-Hak (1999) examined that in a solid-fluid interface, traditional no-slip boundary surroundings must start to crack ahead of the linear stress-strain relationship results avoid. Harley, Huang, Bau, and Zemel, (1995), Jianqiang, Yu-chong, and Chih-Ming (1995) constituted that certain micro-flows having theoretical results will match the experimental evidence due to the slip-flow condition encouraged by the rarefaction influence and recently meant to be to a greater extent. Ezquerra Larrode, Housiadas, and Drossinos (2000), Yu, and Ameel (2001) examined temperature jump conditions with establishing the influence of interaction within fluid boundaries. Chen, and Weng (2006), Khadrawi, Othman, and Al-Nimr (2005) examined natural convection concerning heat influence over vertical micro-channels.

Naphon, and Khonseur (2009) analyzed the constant heat flux operating condition combined with pressure drop and heat transfer characteristics within a micro-channel heat sink. The numerical analysis examined the heat transfer characteristics for laminar flow inside the micro-channel heat sink. Xie, Liu, He, and Tao (2009) apply treating water as the running fluid to optimize the geometry for efficient heat dissipation. Khan (2013) examined the influence of the slip rate in the micro-channel. Upon being noticed that the no-slip condition of Navier-Stokes results equations failed to suitably model the momentum transferred as of fluid to channel surface. (Ahmad & Pop, 2014) investigated the impact of a heat transfer in a mixed convective boundary layer in a vertical channel implanted in a porous medium, (evaporate impact).

Andreozzi, and Manca (2015) inspected that the radiation consequence on natural convective flow in the direction of a vertical channel. They analyzed six different porous heat sink configurations and concluded with a sandwich configuration shows a better thermal efficiency. Khademi (2016) described the influence of thermal Radiation in a micro-channel of the tributary. The effect of thermal Radiation lying on the movement of nanofluid into a channel is examined by Das, Jana, and Makinde (2016), Lopez, Ibanez, Pantoja, Moreira, and Lastrer (2017) and discussed the impact of entropy generation and nanofluid MHD flow together with thermal Radiation with the help of permeable vertical micro-channel. In recent times, Howell, Siegel, and Menguc (2015), Tseng, Sikorski, Viskanta, and Chen (2011), Zhao, Tassou, and Lu (2008) established the Rosseland concept's experimental application. Using this design the macro-scale heat transfer where no-jump temperature and the no-slip velocity are compelling.

MHD flows are more useful in fluid dynamics research; this study of MHD is applicable in various magnetically controlled devices and sensor technologies, cooling of nuclear reactors, combustion, fire engineering, and so many others. It has been established that subjecting the conducting fluids to an externally applied magnetic field is efficient and serves as a control mechanism in most convective flow situations as used in many metal and semi-conductor industries. Jha, and Gwandu (2018) investigated the different magnetic parameters, and also the hydrodynamic slip lengths do not impact the fluid temperature, and the suction parameter increases the temperature. Sompong, and Suwannasri Pairin (2019) have numerically examined the heat transfer characteristics of axisymmetric flow past a rotating torus in a viscous incompressible fluid. Ramprasad, Subba Bhatta, and Mallikarjuna (2020) examined the viscous incompressible fluid flow through a divergent channel in a particulate suspension with magnetohydrodynamic (MHD) heat generation.

Ramanuja, Gopi Krishna, Kamala sree, and Naga Radhika (2020) examined free convection in MHD flow by a permeable medium in a vertical slit micro-channel through super-hydrophobic and the slip furthermore temperature bounce situations among a steady heat source. Jha, and Malgwi (2021) have presented that fully developed hydromagnetic natural convection flows in a vertical micro-channel which is in the existence of induced magnetic field and Hall current.

To validate the accuracy of our study, a comparison is made with the previous article by Jha, and Gwandu (2018); the possible effects of magnetism on the velocity, volume flow rate, and Nusselt number. Considering all the above, the current study aimed to synthesize a natural extension of our previous work to be quantitatively concerning various parameters like heat source/sink, magnetic parameter, buoyancy parameter, radiation parameter, and slip parameter on micro-channel.

An alternative isothermal surface heating is enforced towards a super-hydrophobic open-ended channel through viscosity dissipation and radiation on an electrically conducting MHD fluid flow. It is considered that the polarity pressure is not outstripping. This causes the fluid to raise the vertical micro-channel (influence of buoyancy) exclusive of overflowing within the micro-cavities at the super-hydrophobic wall. The study will have applications in the design and maintenance of both mini- and nano-science as well as in micro-devices and nano-technology.

The solution of coupled linear ODE's equations is obtained numerically by applying the shooting technique coupled through the 4th order Runge-Kutta specified transformation technique. This work is concerned about consistent numerical results presented graphically and discussed quantitatively concerning various influence parameters like heat source parameter, magnetic parameter, buoyancy parameter, radiation parameter, and slip parameter.

2. Mathematical formation of the problem

Consider the study, with the assumption that the flow in an electrically conducting viscous is fully developed, incompressible, and laminar a distance h separating apart the two plates. A transverse consistent magnetic field B_0 strength is enforced across the exterior of the vertical micro-channel. The direction of flow that is parallel to the *x*-axis and ‘g’ is the induced gravity due to acceleration, while the y-axis which corresponds to the flow rate is normal to it (Figure 1). At the left wall T_1 temperatures ($y = L$) and hotter plate T_2 at the cooler ($y = 0$) right plate is wall provided. The thermo-physical properties of the fluid, excluding viscous in an incompressible fluid, are considered only in buoyancy terms and by adopting Boussinesq’s approximation.

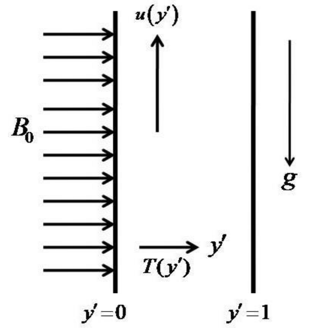

Figure 1. Configuration of the model

In view of the assumptions made above, the basic equations of motion and energy in the vertical micro-channel (Jha & Gwandu, 2018; Ramanuja, Gopi Krishna, Kamala Sree, & Naga Radhika, 2020; Rudraiah, Venkatachalappa, & Malashetty, 1982) are as follows.

Case I: The super-hydrophobic surface being heated

The Cartesian co-ordinate system is governed by the equations as below

$$v\frac{d^2u}{dy^{l^2}}+g\beta_0\left(T-T_0\right)-\frac{\sigma B_0^2\,u}{\rho}=0 \tag{1}$$

$$\frac{K_0}{\rho cp}\frac{d^2T}{dy^{l^2}}+\frac{\mu}{\rho cp}\left(\frac{du}{dy}\right)^2-\frac{1}{\rho cp}\frac{\partial q_r}{\partial y}+Q_0=0 \tag{2}$$

Their various boundary conditions are

$$\left.\begin{aligned} u\left(y^l\right)&=\lambda^l\frac{du}{dy^l}\\ T\left(y^l\right)&=T_h+\gamma^l\frac{dT}{dy^l}\end{aligned}\right\}\quad at\ \ y^l=0 \tag{3}$$

$$\left.\begin{aligned} u\left(y^l\right)&=0\\ T\left(y^l\right)&=T_0\end{aligned}\right\}\quad at\ \ y^l=L \tag{4}$$

By introducing, the Radiative heat flux below Roseland’s approximation (Brewster, 1992) has the subsequent appearance

$$q_r=-\left(\frac{4\sigma^*}{3k^*}\right)\frac{\partial T^4}{\partial y^l} \tag{5}$$

Further, we unspecified that the high-temperature difference through the stream is acceptable minute such as the purpose of the term expression T^4 might be used as a temperature-line function.

Where σ^* be the Stefan–Boltzmann stable also k^* be the indicate absorption coefficient. therefore getting higher T^4 within Taylor`s series concerning T_0 mention temperature T^4 term and ignore the higher power conditions, thus as following

$$T^4=4T_0^3-3T_0^4 \tag{6}$$

In an inspection of the equation (6), equation (5) takes the following form

$$q_r=-\left(\frac{16\sigma^*T_0^3}{3k^*}\right)\frac{\partial T}{\partial y^l} \tag{7}$$

Substituting equation (7) in equation (2) one can get

$$\frac{K_0}{\rho cp}\frac{d^2T}{dy^{l^2}}+\frac{\mu}{\rho cp}\left(\frac{du^l}{dy^l}\right)^2+\frac{1}{\rho C_p}\left(\frac{16\sigma^*T_0^3}{k^*}\right)\frac{d^2T}{dy^{l2}}+Q_0=0 \tag{8}$$

Using the below dimensionless similarity variables

$$\begin{aligned}&\left(Y,\gamma,\lambda\right)=\ (y',\gamma',\lambda')/L,\quad \theta=(\mathrm{T}-\mathrm{T}_0)/(T_h-T_0), M^2=\sigma B_0^2L^2/pv,\\ &U=uv\left[\mathrm{g}\,\beta_0L^2(\mathrm{T}_h-T_0)\right]^{-1},\ \beta=\frac{Q_0h^2}{K_0(T_h-T_0)}, N=\ \frac{\rho g^2\beta_0{}^2h^4(T_1-T_0)}{K_0v}\end{aligned} \tag{9}$$

The leading equations (1) & (8) along with given boundary conditions (3) and (4) becomes

$$\frac{d^2U}{dy^2}+\theta-M^2U=0 \tag{10}$$

$$\left(1+\frac{3}{4}R\right)\frac{d^2\theta}{dy^2}+N\left(\frac{dU}{dY}\right)^2+\beta=0 \tag{11}$$

When $R = 0$, $N = 0$ and $\beta = 0$ the equation (11) coincide with Jha and Gwandu, 2018

$$\left.\begin{aligned}\theta(y)&=0\\ U(Y)&=0\end{aligned}\right\}\ at\ \ Y=1 \tag{12}$$

$$\left.\begin{aligned}\theta\left(Y\right)&=1+\gamma\frac{d\theta}{dy}\\ U\left(Y\right)&=\lambda\frac{du}{dy}\end{aligned}\right\}\ at\ \ Y=0 \tag{13}$$

Case II: The no-slip surface being heated

Leading equations are stated in situation (1) & (8). Equation (9) was appropriate to transform them keen on Equations (10) & (11). We now, worked out Equations (10) and (11), using the concomitant limit conditions

$$\left.\begin{aligned} T\left(y^{l}\right) &= T_0 + \gamma^{l}\frac{dT}{dy^{l}} \\ u\left(y^{l}\right) &= \lambda^{l}\frac{du}{dy^{l}} \end{aligned}\right\} \quad at \quad y^{l} = 0 \tag{14}$$

$$\left.\begin{aligned} T\left(y^{l}\right) &= T_h \\ u\left(y^{l}\right) &= 0 \end{aligned}\right\} \quad at \ y^{l} = L \tag{15}$$

$$\left.\begin{aligned} T\left(Y\right) &= \gamma\frac{d\theta}{dY} \\ u\left(Y\right) &= \lambda\frac{dU}{dY} \end{aligned}\right\} \quad at \quad Y = 0 \tag{16}$$

$$\left.\begin{aligned} T\left(Y\right) &= 1 \\ u\left(Y\right) &= 0 \end{aligned}\right\} \quad at \ Y = 1 \tag{17}$$

3. Solution of the Problem

Nonlinear Eqs coupled (10) and (11) create a boundary value problem (BVP) together with boundary conditions (12)-(13) and (16)-(17). It is not possible to solve these equations exactly by BVP. Therefore, we use the initial value problem (IVP) in applying the Runge-Kutta fourth-order (MATLAB solver, bvp4c package software) to solve these equations. Equations (10)-(11) have been translated by substitution into the following system of ODE's of the first order.

$$\left.\begin{aligned} &\left(f,\ f^{1},\ \theta,\ \theta^{1}\right) = \left(X_1,\ X_2,\ X_3, X_4\right) \\ &\frac{dX_1}{dY} = X_2, \\ &\frac{dX_2}{dY} = \left[M^2 X_1 - X_3\right] \\ &\frac{dX_3}{dY} = X_4 \\ &\frac{dX_4}{dY} = \frac{1}{\left[1+\frac{3}{4}R\right]}\left[-N\left(X_2\right)^2 - \beta\right] \end{aligned}\right\} \tag{18}$$

The initial conditions in terms of X_1, X_2, X_3, X_4 are

$$\left.\begin{aligned} X_1(0) &= \lambda X_2 \\ X_1(1) &= 0 \\ X_3(0) &= 1+\gamma X_4 \\ X_3(1) &= 0 \end{aligned}\right\} \tag{19}$$

The above equations (18) are solved applying the fourth-order Runge-Kutta technique through shooting method area under discussion matter to the boundary conditions (19).

4. Skin Friction

The Skin Friction at two vertical wall $Y = 0$ and $Y = 1$ given by

$$\tau_{0,1} = \left(\frac{dU}{dY}\right)_{Y=0,1} \tag{20}$$

5. Validation

Table 1 (Case-I & Case-II) illustrates a comparison of the current results with the already published work of Jha, and Gwandu (2018) for the several values of *M*. It is noticed that there is a good correlation between both the results up to the desired accuracy, and that validates the convergence criteria of the present methodology adopted.

Table 1. Comparison of skin friction in the limiting case when $R = 0$, $N = 0$ and $\beta = 0$

M	Jha & Gwandu (2018) $R = 0$, $N = 0$ and $\beta = 0$	Present work $R = 1$, $N = 1$ and $\beta = 1$
Case I: The super-hydrophobic surface being heated		
1	0.067668	0.096672
1.5	0.054960	0.078143
2	0.043688	0.061788
2.5	0.034714	0.048840
Case II: The no-slip surface being heated		
1	0.132121	0.161733
1.5	0.104390	0.127875
2	0.080159	0.098398
2.5	0.061290	0.075470

6. Results and Discussion

This article represents that free convection of an electrically conducting fluid in a micro-channel with super-hydrophobic slip condition. It aims to inspect the natural convection through a heat flow within a micro-channel. The leading equations (10) & (11) with both cases boundary conditions (12)-(13) and (16) - (17) were solved by the Runge-Kutta (R.K) scheme by using the shooting method. The computations were carried out by taking, $1 \le M \le 2.5$, $1 \le R \le 4$, $1 \le \beta \le 2.5$, $0.5 \le \lambda \le 2$, $1 \le N \le 15$. A solution was obtained in both cases. In the first case, the super-hydrophobic surface was heated, and in the second case, the no-slip surface was heated.

In order to achieve the decision, numerical estimation is accepted, which is exposed by making an allowance used for diverse values of non-dimensional governing parameters. We probed into the influences of governing parameters like Heat source parameter β, Radiation parameter *R*, Magnetic parameter *M*, Buoyancy parameter *N*, Slip parameter λ.

Case I: The super-hydrophobic surface being heated

We consider in case-I the influence of the different set of boundary conditions. These are depicted in Figure 2 and Figure 3 temperature and velocity profiles for discrete values of buoyancy parameter. In Figure 4 and Figure 5 velocity and temperature profiles for discrete values of the Radiation parameter are depicted. Figure 6 indicate velocity profiles on magnetic parameter. Figure 7 illustrates the temperature profiles on the heat source parameter. And Figure 8 illustrates rate profiles on slip parameters.

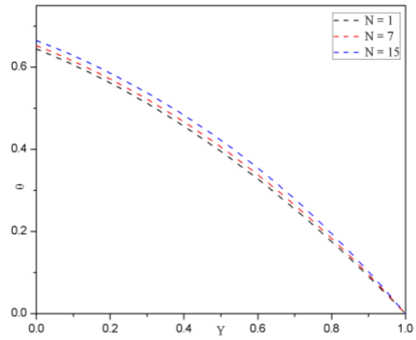

Figure 2. Temperature profiles corresponding to various values of buoyancy parameter *N*

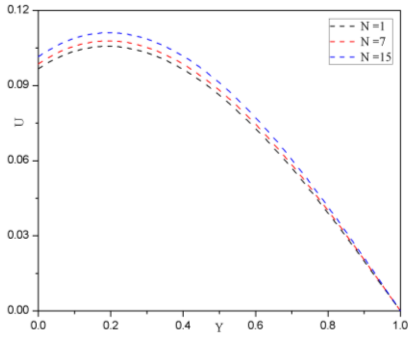

Figure 3. Velocity profiles corresponding to various values of buoyancy parameter *N*

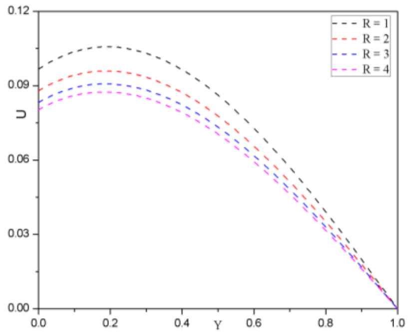

Figure 4. Velocity profiles corresponding to various values of radiation parameter *R*

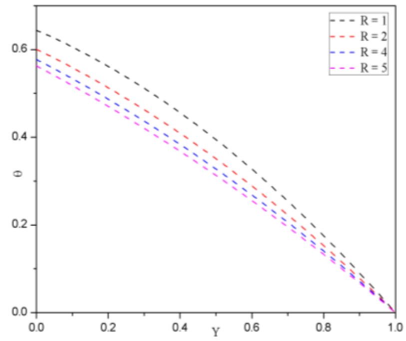

Figure 5. Temperature profiles corresponding to various values of radiation parameter *R*

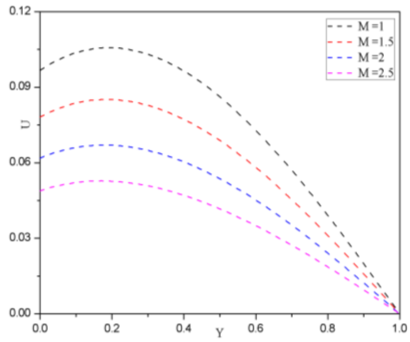

Figure 6. Velocity profiles corresponding to various values of heat magnetic parameter *M*

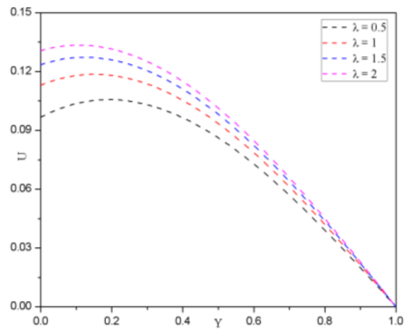

Figure 7. Temperature profiles corresponding to various values of heat source parameter *β*

Figure 2 displays the temperature for dissimilar values of buoyancy parameter *N*. As well as temperature jump is observed in the buoyancy parameter. It is clear through this figure that fluid temperature can be enhanced by putting through the micro-channel walls to symmetric wall heat. By reason of the consequences of the buoyancy parameter, the temperature is enhanced by the addition of significant heat due to viscous dissipations. Figure 3 exhibits the significance of velocity profiles of dissimilar values of buoyancy parameter *N*. This is because the enhancement in velocity slip preceded the independence of fluid molecules, thereby resulting in an increase of fluid velocity. This influence of buoyancy is to increases the velocity and by greater heat addition as a result of viscous dissipations. Figure 4 depicts the effect of velocity profiles for discrete values of Radiation parameter *R*. It is obvious that the velocity enhances the Radiation effect on the micro-channel in the case of the super-hydrophobic surface which is heated. This occurs because the temperature helps in increasing the kinetic energy and accordingly the velocity enhances. Figure 5 depicts the profiles of temperature for discrete values of the Radiation parameter *R*. We perceived that the Radiation parameter raises the fluid temperature in a vertical micro-channel in the case of the super-hydrophobic surface when heated. Figure 6 indicates velocity profiles for dissimilar values of the magnetic parameter *M*. The velocity of heat transfer is enlarge with a raise in the magnetic parameter. It is clear that heat flow in the direction of the Magnetic parameter obstructs the flow velocity.

Figure 7 depicts the for temperature profiles discrete values of heat source parameter *β*. It has been noticed, when heated, the temperature in the channel walls rises with the heat source parameter for the super-hydrophobic surface. A super-hydrophobic surface gives a grater velocity of fluid flow concluded the micro-channel.

Figure 8 reveals that velocity profiles for dissimilar values of slip parameter λ. while the velocity slip parameter raises, the velocity profile also increases. Enhancement in the slip length λ can enhance the flow velocity.

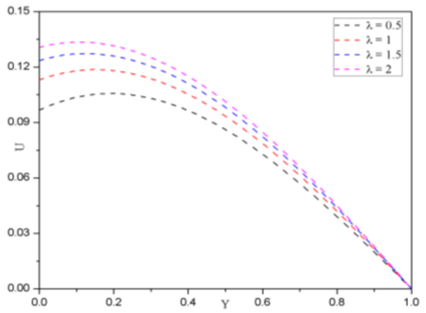

Figure 8. Velocity profiles corresponding to various values of slip parameter λ

Case II: The no-slip surface being heated

We deal with Case-II to investigate the impact of the different sets of boundary conditions lying on heat transfer velocity in more detail. The no-slip surface is heated from Figure 9 to Figure 14. From Figure 9 it is perceived that as the heat source parameter β enhances, velocity enhances. The no-slip boundary situation or no-velocity-off set boundary condition hypothesized that the rate of the fluid layer is in through influence with the boundary and is indistinguishable from the rate of this boundary. There is no related faction among the boundary and the fluid layer, and hence present is no slip.

Figure 10 indicates the influence of velocity profiles for dissimilar values of magnetic parameter M. The Lorentz strength is formed. This strength tends to slow down the flow and as an affect the rate profile diminishes. Figure 11 we examine that enhance in the Radiation parameter R consequences in increasing temperature inside the boundary. The heat transfer coefficient reduces on the cold wall and enhances on the heated wall after raising the radiation parameter. Figure 12 reveals the velocity profiles for dissimilar values of Radiation parameter R. Velocity decreases when the radiation parameter is enhanced. As radiation parameter R indicates the relative contribution of conduction heat transfer to radiation heat transfer, it is observed from Figure 13 that when energy emitted out (Radiation parameter R) increases, the velocity of flow decreases. Figure 13 illustrates the velocity profiles for discrete values N. We examined that when the buoyancy parameter is enhanced velocity raise. This is due to the high rate and exterior of the super-hydrophobic surface's roughness (high friction) and high rate with slight or no roughness. Figure 14 reveals the temperature profiles for discrete readings of buoyancy parameter N. Hence we conclude the buoyancy parameter depends upon the density of the fluid. The temperature jump is to encumber heat loss during the super-hydrophobic surface and therefore to enhance the temperature increase or buoyancy in the fluid. This due to N enhances the temperature for different also increases resulting in large values of N natural convection which transfer more heat.

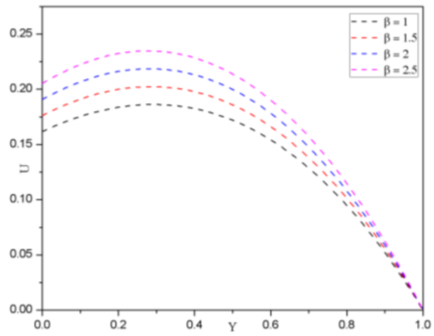

Figure 9. Velocity profiles corresponding to various values of heat source parameter β

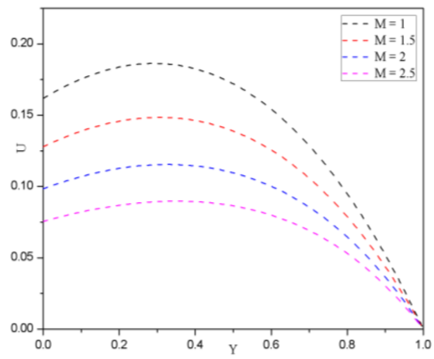

Figure 10. Velocity profiles corresponding to various values of magnetic parameter M

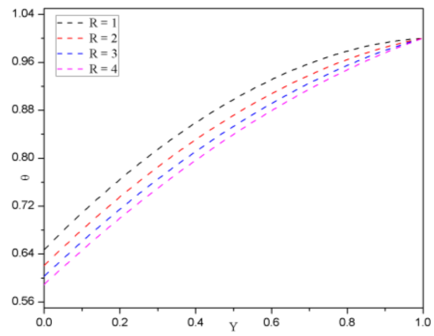

Figure 11. Temperature profiles for various values of radiation parameter R

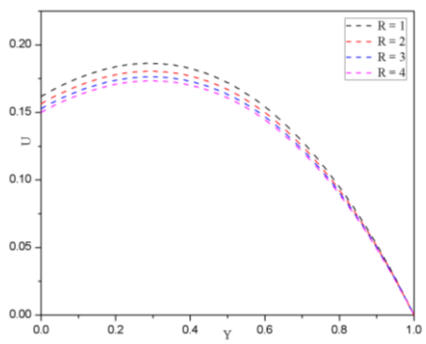

Figure 12. Velocity profiles corresponding to various values of radiation parameter R

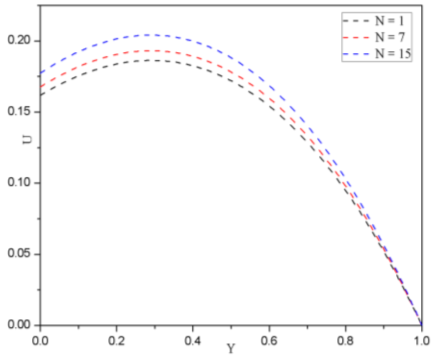

Figure 13. Velocity profiles corresponding to various values of buoyancy parameter

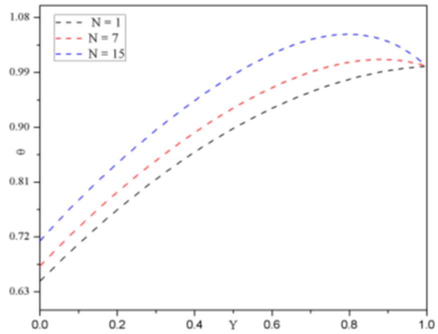

Figure 14. Temperature profiles corresponding to various values of buoyancy parameter *N*

7. Conclusions

This paper described the buoyancy effects free convective of an electrically conducting fluid in a micro-channel with super-hydrophobic slip condition. In the first case, the super-hydrophobic surfaces were heated, and in the second case, the no-slip surface was heated with the help of vertical and open-ended micro-channel. The results of model width on channel height ratio on the potent slip length are interpreted as analyzed for distinct shear-free fractions. The significant results are outlined as follows:

- In super-hydrophobic slip condition and no-slip, the velocity drops through raising values of slip parameter γ.
- At constant slip conditions $\lambda = 1$ and $\gamma = 1$ velocity decrease was observed with the increase of Hartman Number *M*.
- Fluid temperature decreases with decrease in thermal radiation.
- It is found that the temperature profiles increase with the buoyancy parameter.

Nomenclature

B_0	Magnetic field intensity
cp	Specific heat at constant pressure
γ^l	Dimensional temperature jump conditions
λ^l	Dimensional slip length
g	Gravitational acceleration
k	Thermal conductivity
k^*	Mean absorption coefficient
γ	temperature jump parameter
λ	slip length parameter
M	Magnetic field parameter
q_r	Radiative heat flux
R	Radiation parameter
T_0	Reference temperature
u	Dimensional Velocity of the flow
ρ	Density
σ^*	Stefan–Boltzmann constant
V	Kinematic viscosity
N	buoyancy parameter
U	Velocity of the flow
y^l	Dimensional horizontal position of the plates

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